Analytic covariance of the redshift-space galaxy power spectrum using PT

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with Roman Scoccimarro

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UC Berkeley, Oct 2019

Motivation: Power Spectrum Covariance

- We estimate cosmological parameters (θ) from galaxy surveys using the power spectrum

$$p(P_d|\theta) = \frac{1}{(2\pi)^{n/2}\sqrt{\det C}} \exp\left[-\frac{1}{2}(P_d - P(\theta))^T C^{-1} (P_d - P(\theta))\right]$$

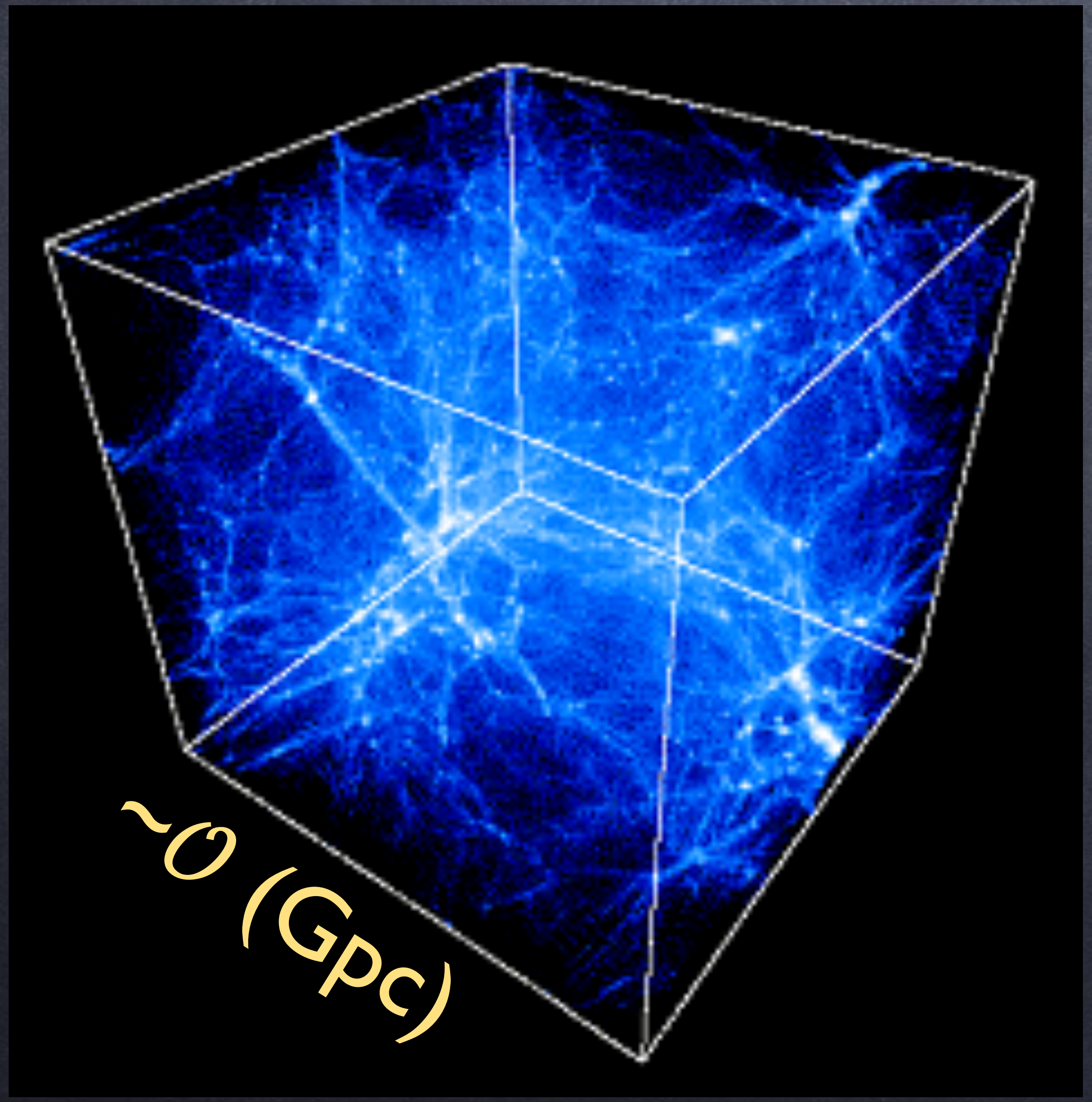
$$C(k_1, k_2) = \langle \hat{P}(k_1)\hat{P}(k_2)\rangle - \langle \hat{P}(k_1)\rangle\langle \hat{P}(k_2)\rangle$$

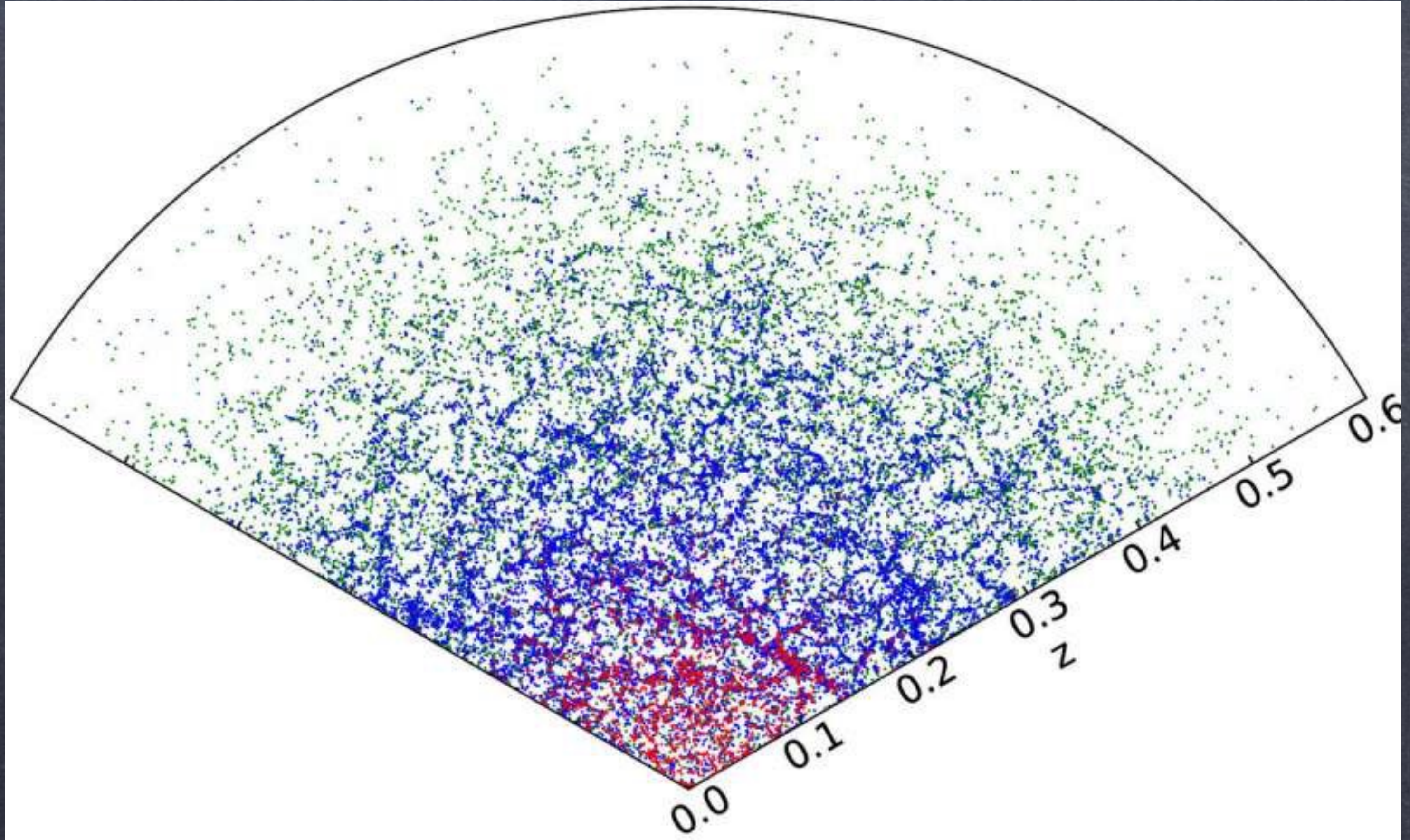
- Robust knowledge of the covariance matrix is crucial for parameter estimation

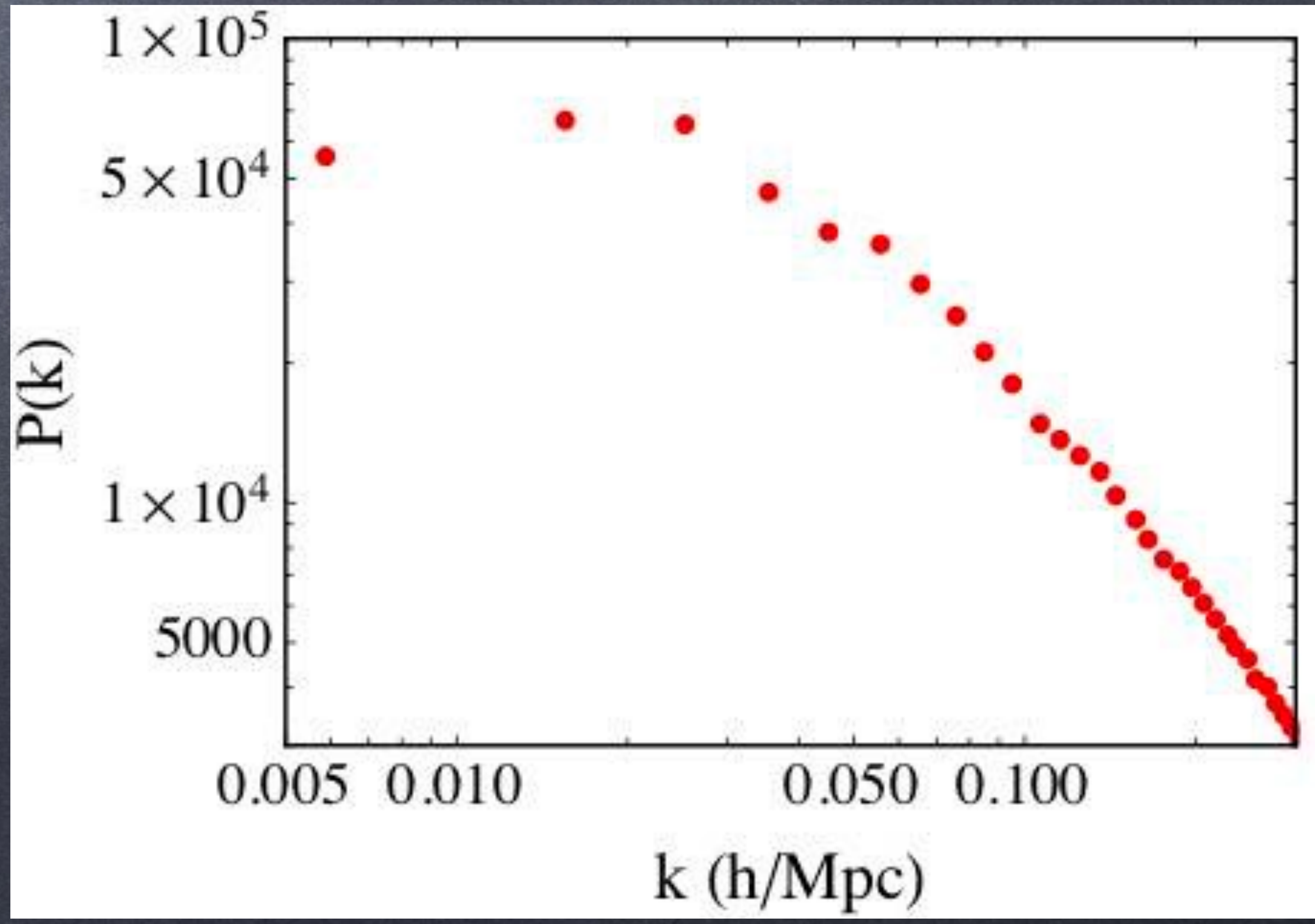
Covariance from mock catalogs

- Need to simulate mock surveys (~ thousands) to compute the covariance matrix

$$C_{1,2} = \frac{1}{N_{\text{mock}} - 1} \sum_{i=1}^{N_{\text{mock}}} (\hat{P}_i(k_1) - \bar{P}(k_1))(\hat{P}_i(k_2) - \bar{P}(k_2))$$







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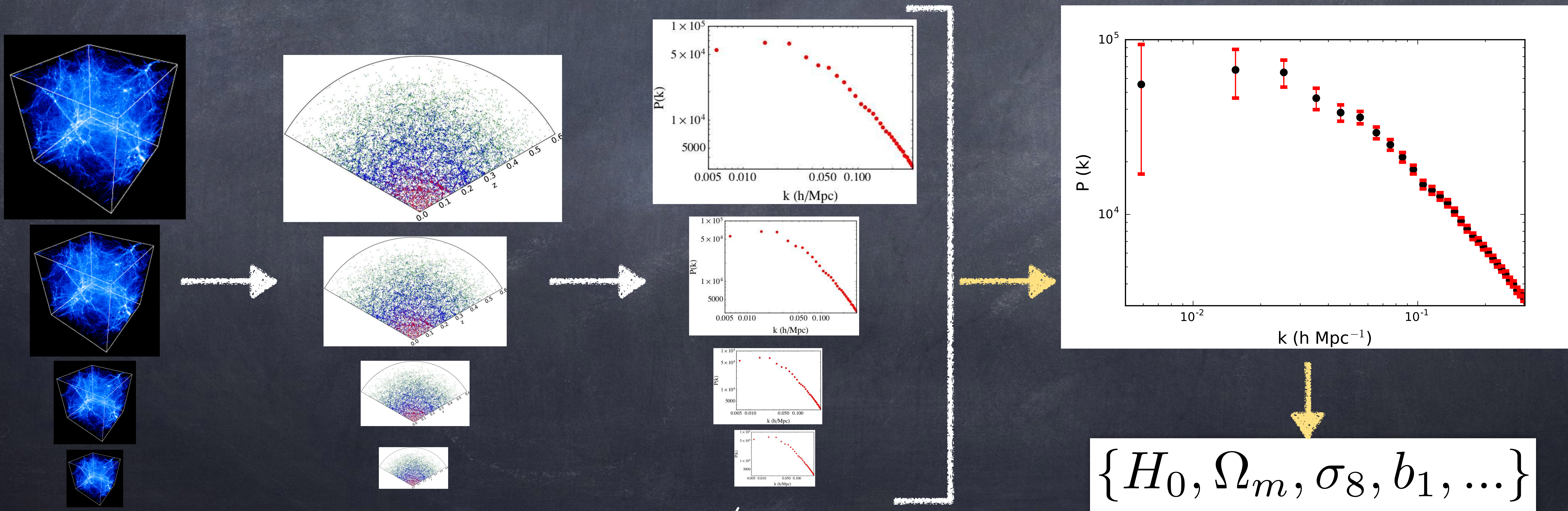


Covariance from mock catalogs

- As survey volume increases, mock catalogs become tougher to simulate (LSST, DESI, Euclid and others)

$$C_{1,2} = \frac{1}{N_{\rm mock} - 1} \sum_{i=1}^{N_{\rm mock}} (\hat{P}_i(k_1) - \bar{P}(k_1))(\hat{P}_i(k_2) - \bar{P}(k_2))$$

- Mocks suffer from sampling noise (error $\propto$ $1/\sqrt{N_{\rm mocks}}$)

  (noise can make matrix harder to invert)

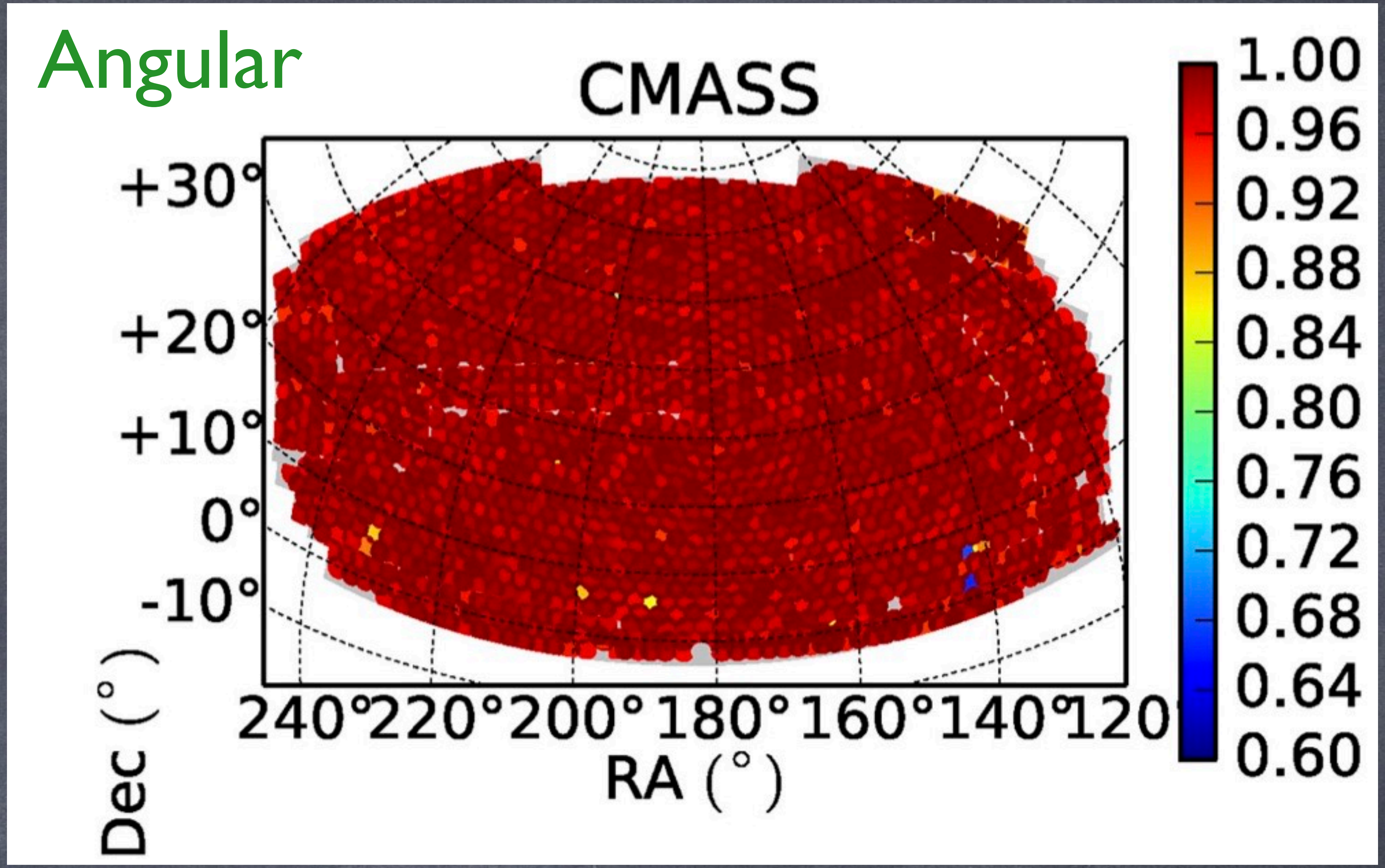
$$p(P_d|\theta) = \frac{1}{(2\pi)^{n/2}\sqrt{\det C(\theta)}} \exp\left[-\frac{1}{2}(P_d - P(\theta))^T C(\theta)^{-1}(P_d - P(\theta))\right]$$

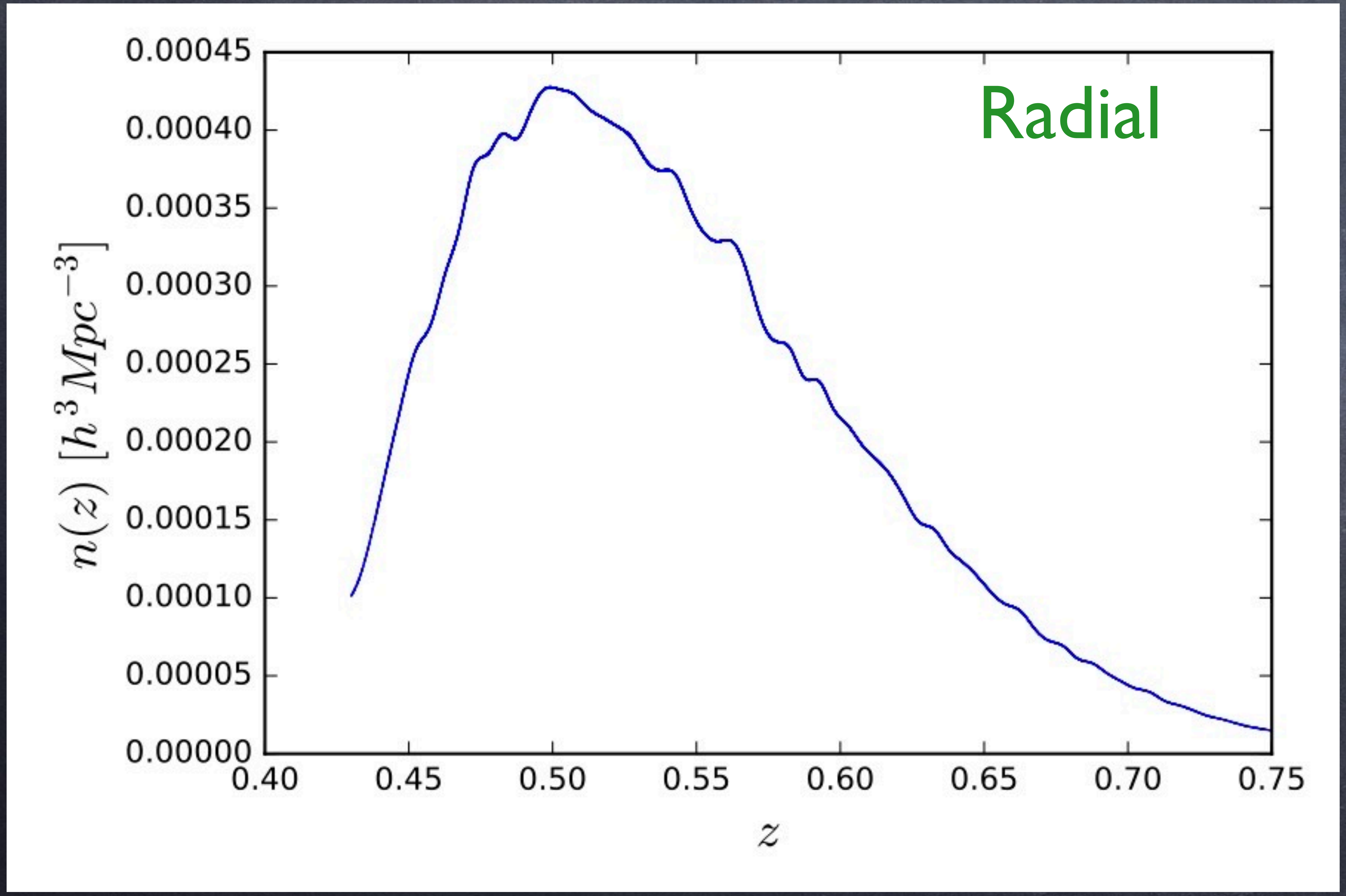
- Dependence of covariance on cosmology and bias parameters is computationally prohibitive

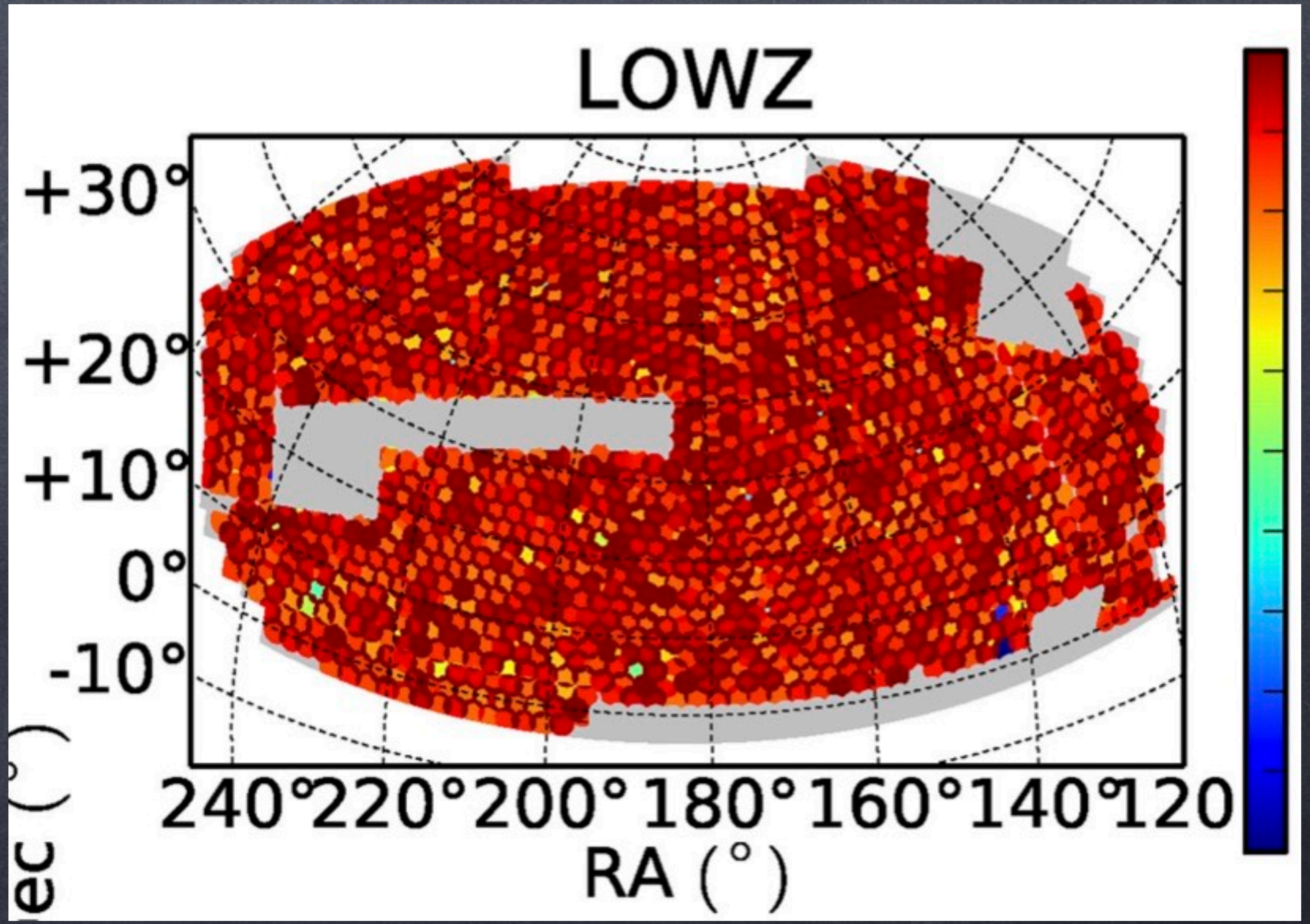
What are the challenges for an analytic method?

Challenge I:
Highly non-trivial survey window

$$\delta_W(\mathbf{x}) \equiv W(\mathbf{x})\delta(\mathbf{x})$$







SDSS DR12
Reid et al. 2015

Survey window enters Covariance

$$C(k_1, k_2) = \langle \hat{P}(k_1)\hat{P}(k_2)\rangle - \langle \hat{P}(k_1)\rangle\langle \hat{P}(k_2)\rangle$$

$$\begin{aligned}\langle \hat{P}(k_1)\hat{P}(k_2)\rangle &= \frac{1}{V_2^2}\int_{\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2} \langle \delta_W(\mathbf{k}_1)\delta_W(-\mathbf{k}_1)\delta_W(\mathbf{k}_2)\delta_W(-\mathbf{k}_2)\rangle \\ &= \frac{1}{V^2{}_2}\int_{\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2,\mathbf{p}_1,\mathbf{p}_1',\mathbf{p}_2,\mathbf{p}_2'} W(\mathbf{k}_1-\mathbf{p}_1)W(-\mathbf{k}_1-\mathbf{p}_1')W(\mathbf{k}_2-\mathbf{p}_2)W(-\mathbf{k}_2-\mathbf{p}_2') \\ &\quad \times \langle \delta(\mathbf{p}_1)\delta(\mathbf{p}_1')\delta(\mathbf{p}_2)\delta(\mathbf{p}_2')\rangle\end{aligned}$$

18 dimensional integral

Challenge II: 4-point function (Trispectrum)

- Need to calculate non-linear structure formation in PT

$$\mathbf{C}(k_1, k_2) = \langle\delta(k_1)\delta(-k_1)\delta(k_2)\delta(-k_2)\rangle - \langle\delta(k_1)\delta(-k_1)\rangle\langle\delta(k_2)\delta(-k_2)\rangle$$

$$\mathbf{C}^{\mathrm{G}}(k_1, k_2) \simeq 2\,\langle\delta(k_1)\delta(-k_2)\rangle\,\langle\delta(k_2)\delta(-k_1)\rangle$$

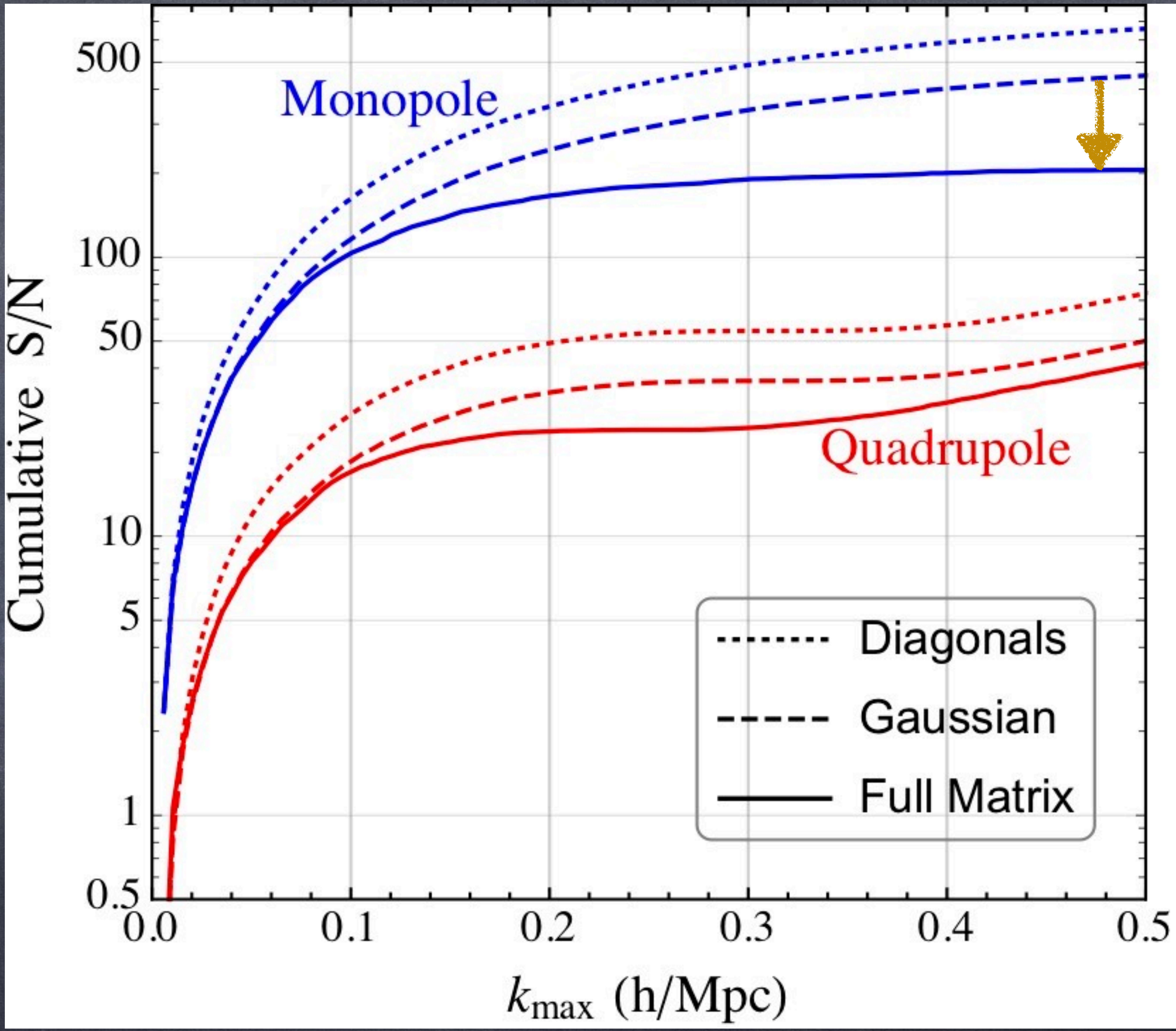
$$\mathbf{C}^{\mathrm{NG}}(k_1, k_2) = \langle\delta(k_1)\delta(-k_2)\delta(k_2)\delta(-k_1)\rangle_c$$

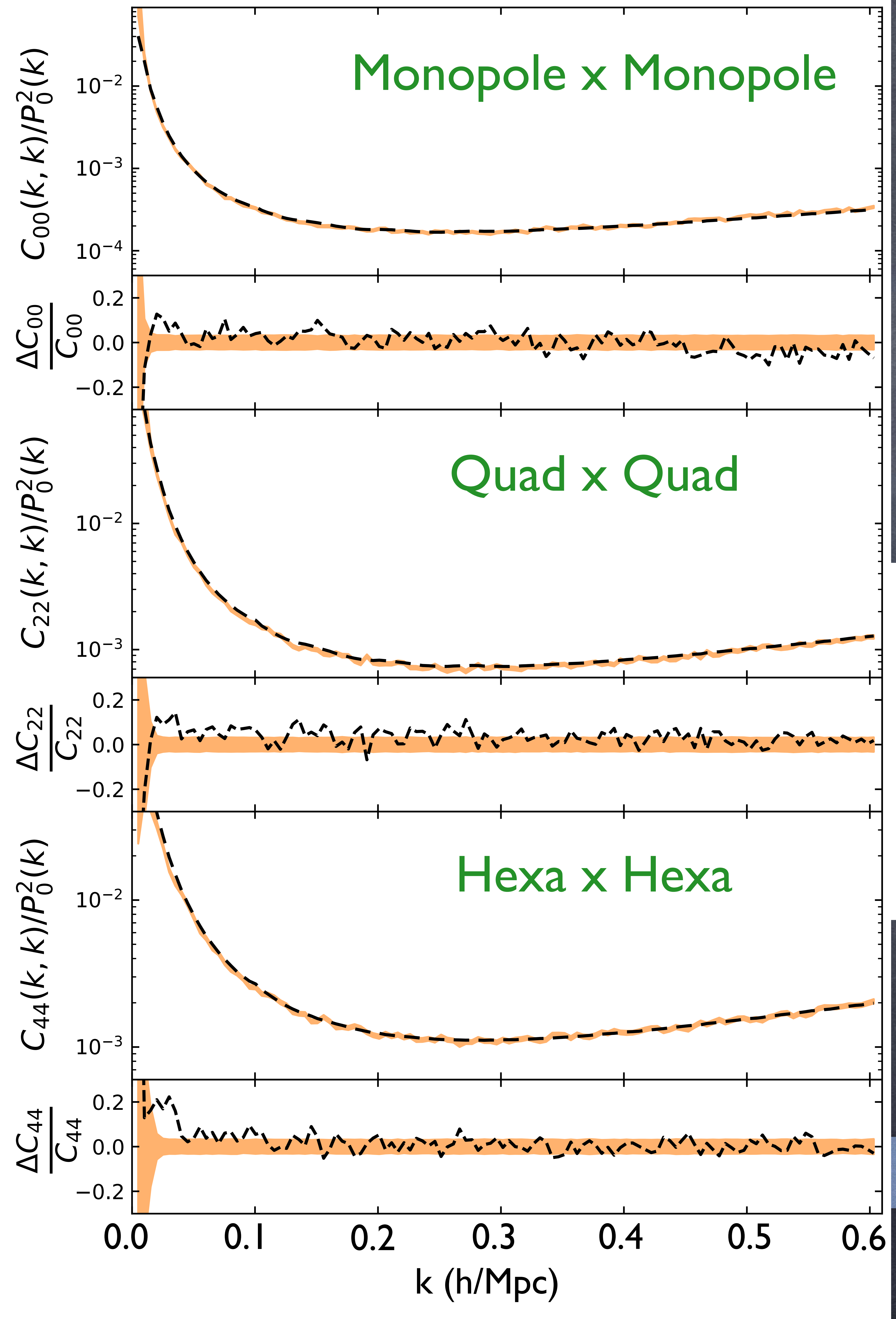
- Need to model:

1. RSD
2. Shot Noise
3. Bias (Linear, non-linear and non-local)
4. Effect of Super-Survey modes

Is it necessary to model the NG part?

NG part causes
40% leakage of info.
at k=0.3 h/Mpc





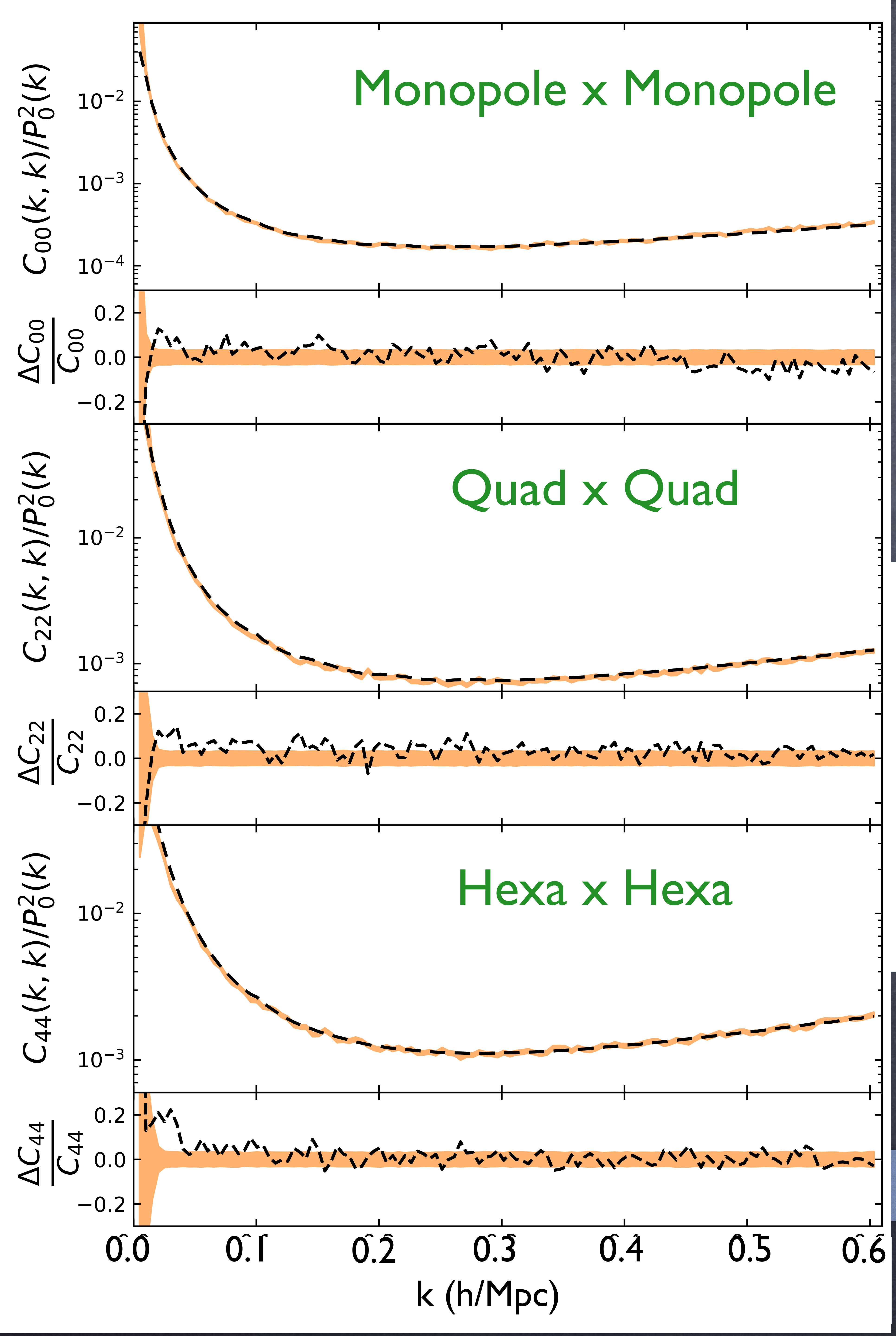
Results

Diagonal auto-covariance

- - - - ***Our analytic method***

Patchy Mocks (state-of-the-art mocks used for SDSS BOSS parameter estimation)

DW & Scoccimarro 19



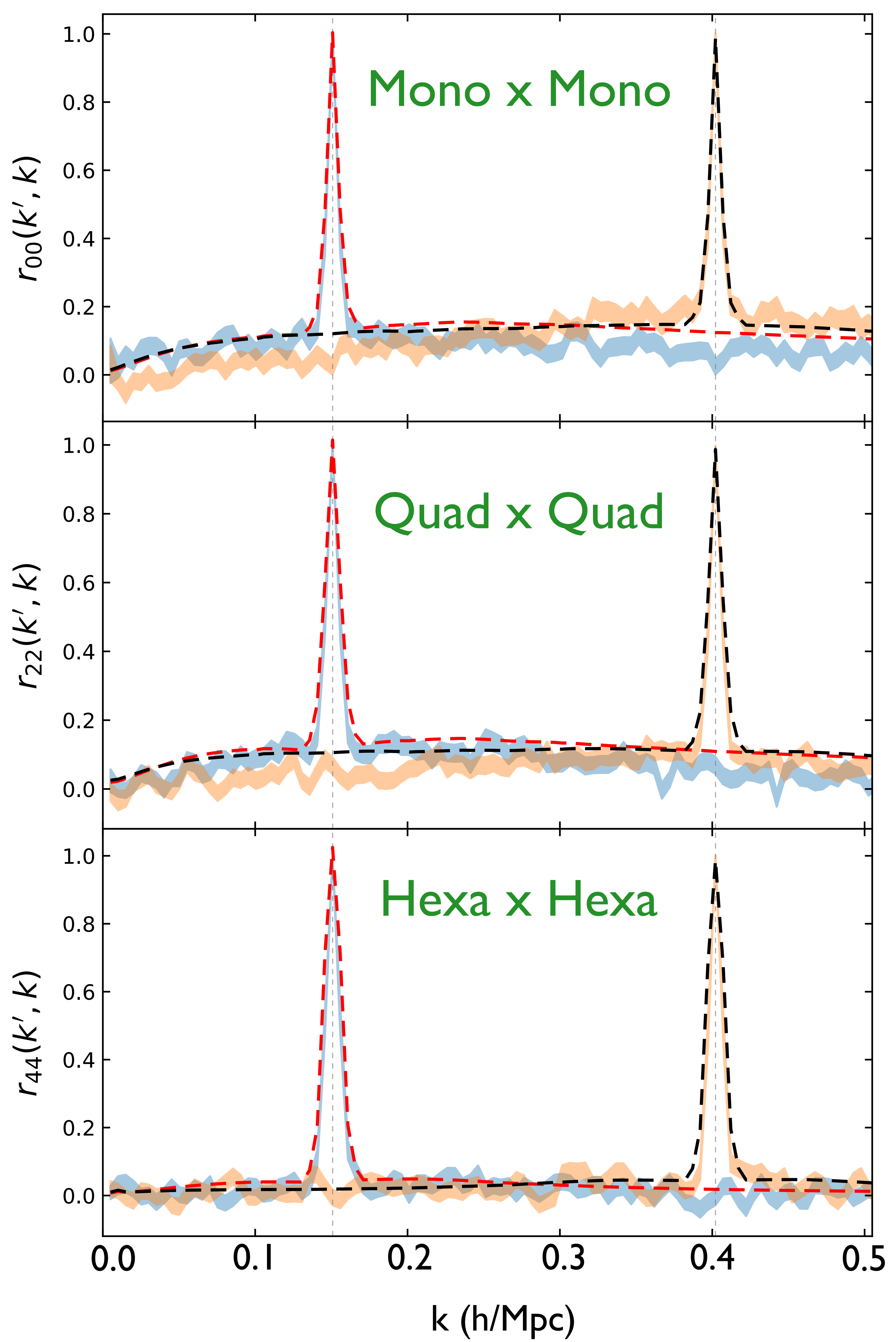
Results

Diagonal auto-covariance

- - - - - ***Our analytic method (≤ MINUTE)***

Patchy Mocks (~ ***MONTHS***) (state-of-the-art mocks used for SDSS BOSS parameter estimation)

DW & Scoccimarro 19

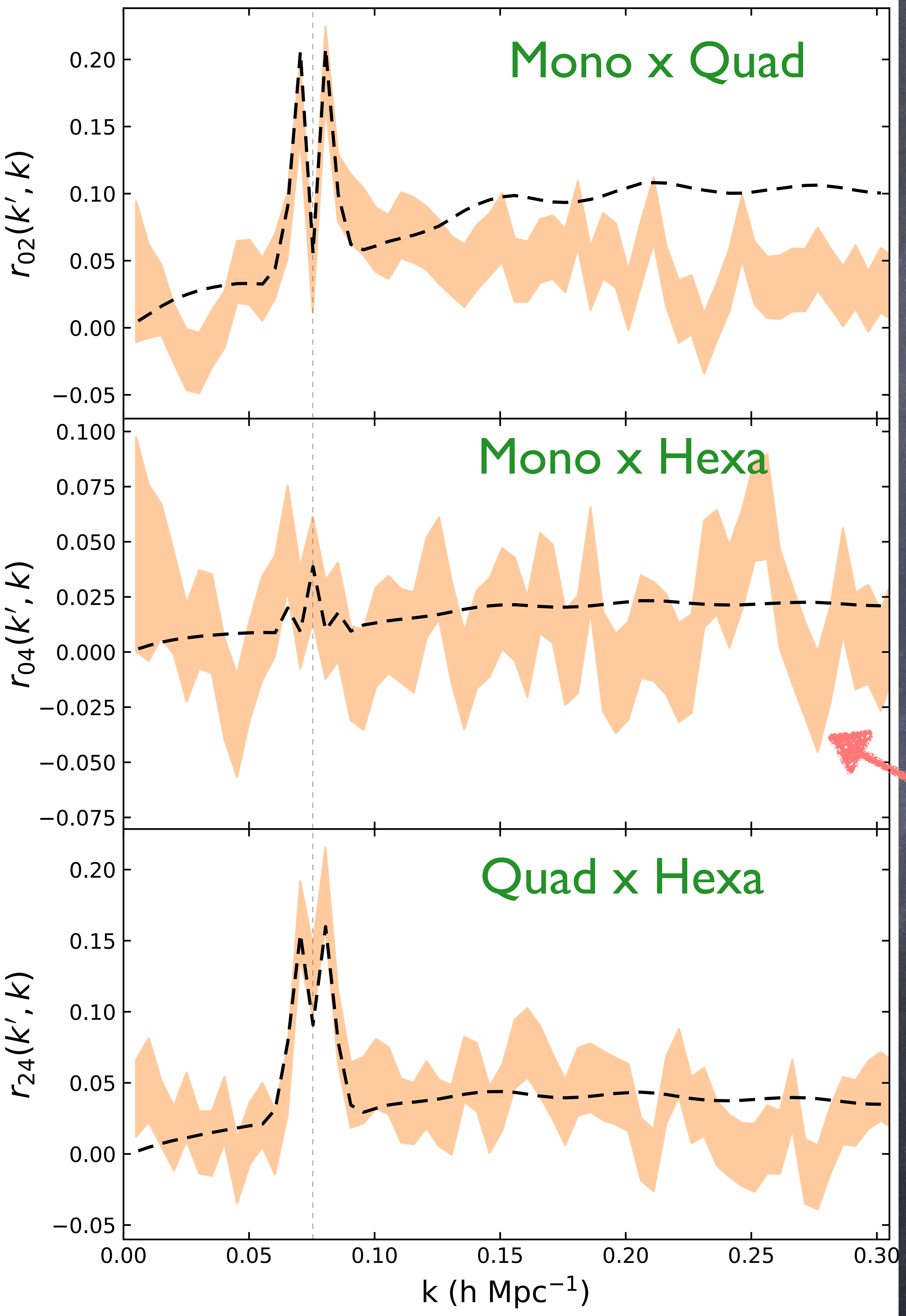


Results

Off-diagonal auto-covariance

(2 rows compared in fig)

DW & Scoccimarro 19



Results

Off-diagonal cross-covariance

High sampling noise in mocks

1. Gaussian Covariance

$$C(k_1, k_2) = \langle \hat{P}(k_1)\hat{P}(k_2)\rangle - \langle \hat{P}(k_1)\rangle\langle \hat{P}(k_2)\rangle$$

$$\mathbf{C}^{\mathrm{G}}(k_1, k_2) \simeq 2\,\langle \delta_W(k_1)\delta_W(-k_2)\rangle\,\langle \delta_W(k_2)\delta_W(-k_1)\rangle$$

1. Gaussian Covariance

Contains all dependence on cosmology and bias parameters

$$\mathbf{C}^{\mathrm{G}}_{\ell_1\ell_2}(k_1,k_2) \simeq \sum_{\ell_1',\ell_2'} P_{\ell_1'}(k_2)P_{\ell_2'}(k_1)\left\{\frac{(2\ell_1+1)(2\ell_2+1)}{\mathrm{I}_{22}^2}\int_{\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2,\mathbf{x}_1,\mathbf{x}_2} W_{22}(\mathbf{x}_1)W_{22}(\mathbf{x}_2)\, e^{-i(\mathbf{x}_1-\mathbf{x}_2)\cdot(\mathbf{k}_1-\mathbf{k}_2)} \right.$$

$$\left. \times\mathcal{L}_{\ell_1}(\hat{\mathbf{x}}_1\cdot\hat{\mathbf{k}}_1)\mathcal{L}_{\ell_1'}(\hat{\mathbf{x}}_1\cdot\hat{\mathbf{k}}_1)\mathcal{L}_{\ell_2'}(\hat{\mathbf{x}}_2\cdot\hat{\mathbf{k}}_2)\left[\mathcal{L}_{\ell_2}(\hat{\mathbf{x}}_2\cdot\hat{\mathbf{k}}_2)+\mathcal{L}_{\ell_2}(\hat{\mathbf{x}}_1\cdot\hat{\mathbf{k}}_1)\right]\right\}$$

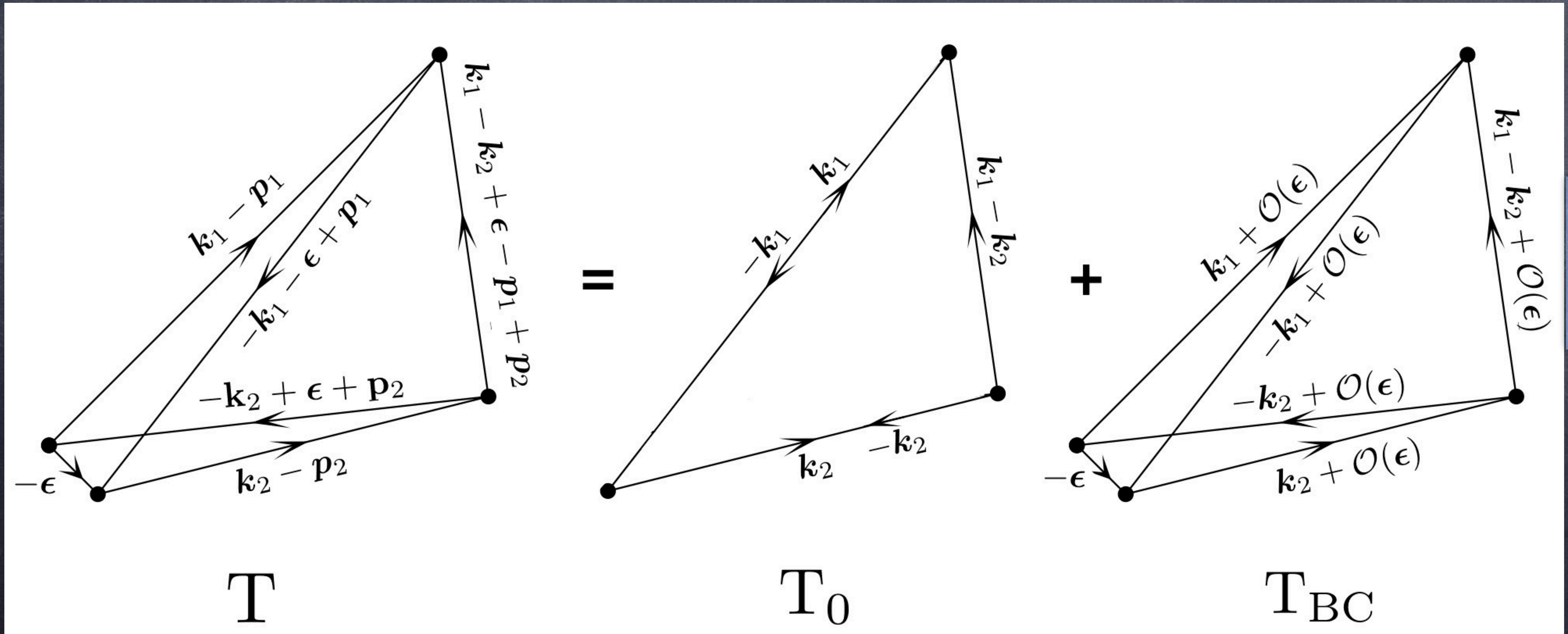
{Computed from survey random catalog}

See also:
Li et al. 2019

2. Non-Gaussian covariance

$$\mathbf{C}^{\mathrm{T}}(k_1,k_2) = \frac{1}{\mathrm{I}_{22}^2}\int_{\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2,\boldsymbol{\epsilon},\mathbf{p}_1,\mathbf{p}_2} W_{11}(\mathbf{p}_1)W_{11}(\mathbf{p}_2)W_{11}(\boldsymbol{\epsilon}-\mathbf{p}_1)W_{11}(-\boldsymbol{\epsilon}-\mathbf{p}_2) \times T(\mathbf{k}_1-\mathbf{p}_1, -\mathbf{k}_1-\boldsymbol{\epsilon}+\mathbf{p}_1, \mathbf{k}_2-\mathbf{p}_2, -\mathbf{k}_2+\boldsymbol{\epsilon}+\mathbf{p}_2)\ .$$

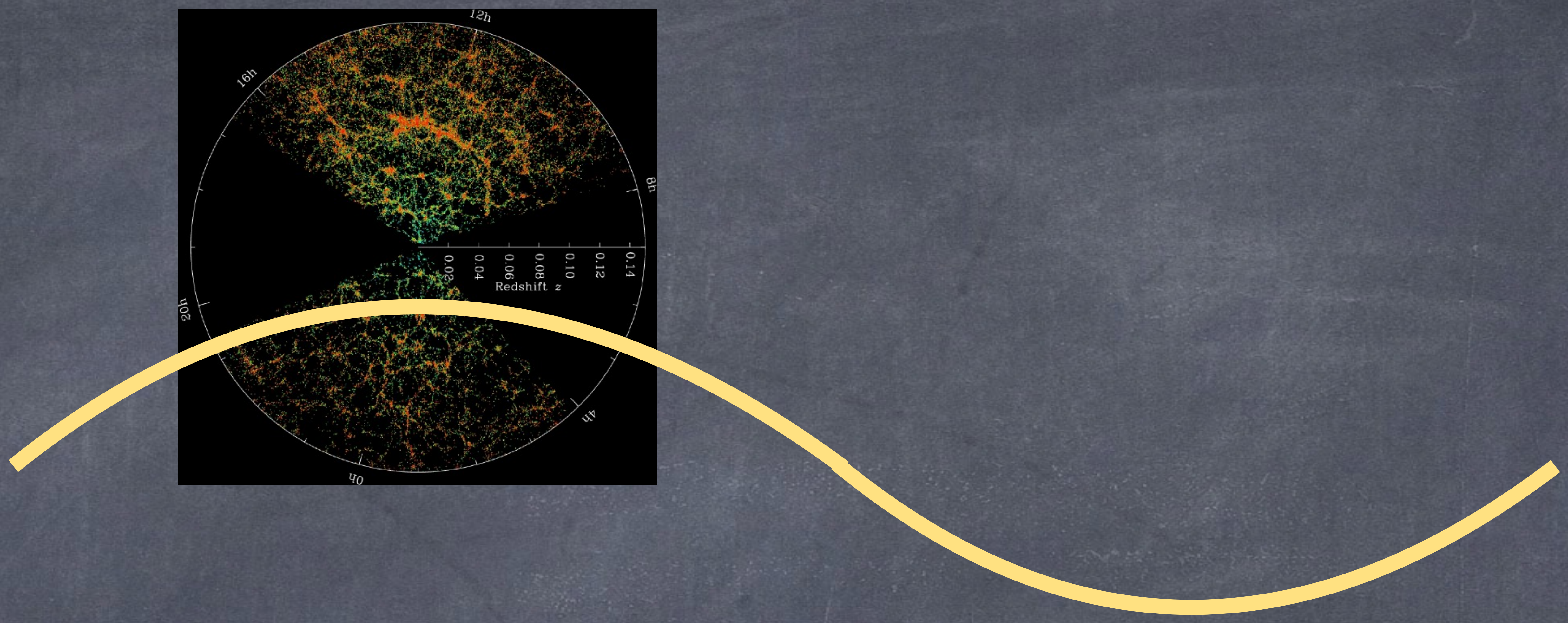
can be split into an ordinary (T_0) and a long-mode contribution (T_{BC})



Beat Coupling effect: Hamilton, Rimes & Scoccimarro (2006)

Beat Coupling (BC)/ Super-sample (SSC)

- Large-scale (super-survey) modes couple to small-scale fluctuations



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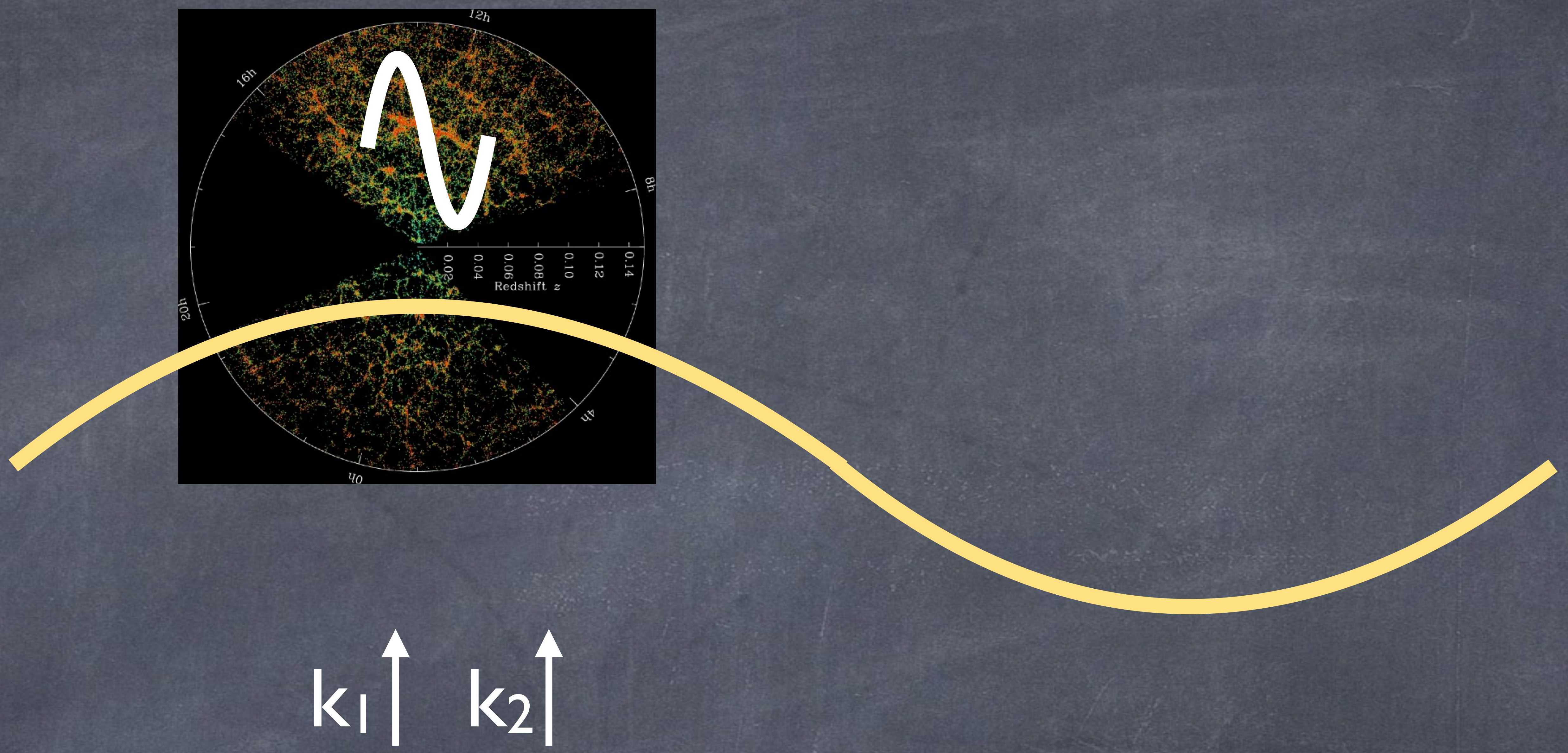
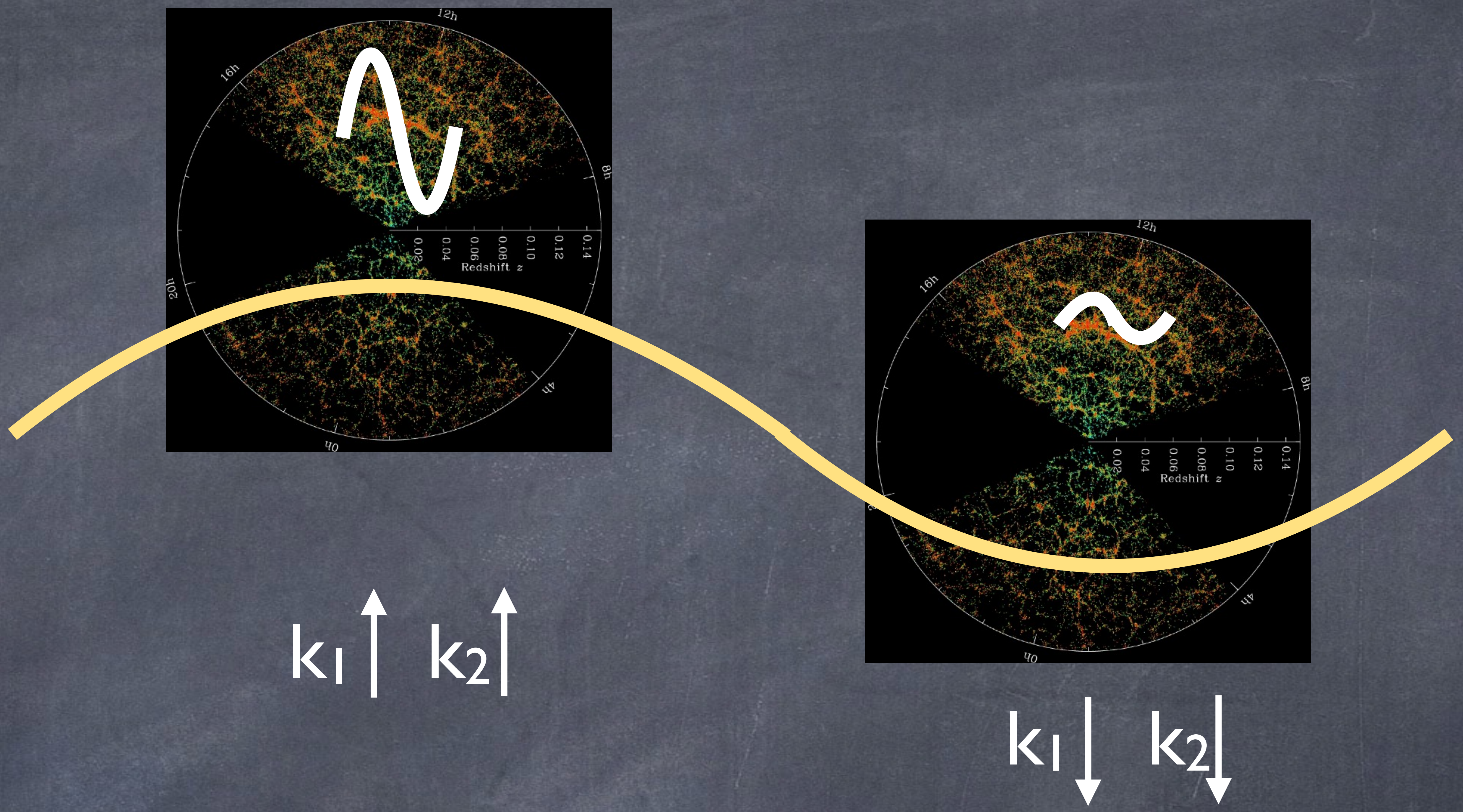


Beat Coupling (BC)/ Super-sample (SSC)

- Large-scale (super-survey) modes couple to small-scale fluctuations
- Causes substantial contribution to non-diagonal covariance $C(k_1,k_2)$

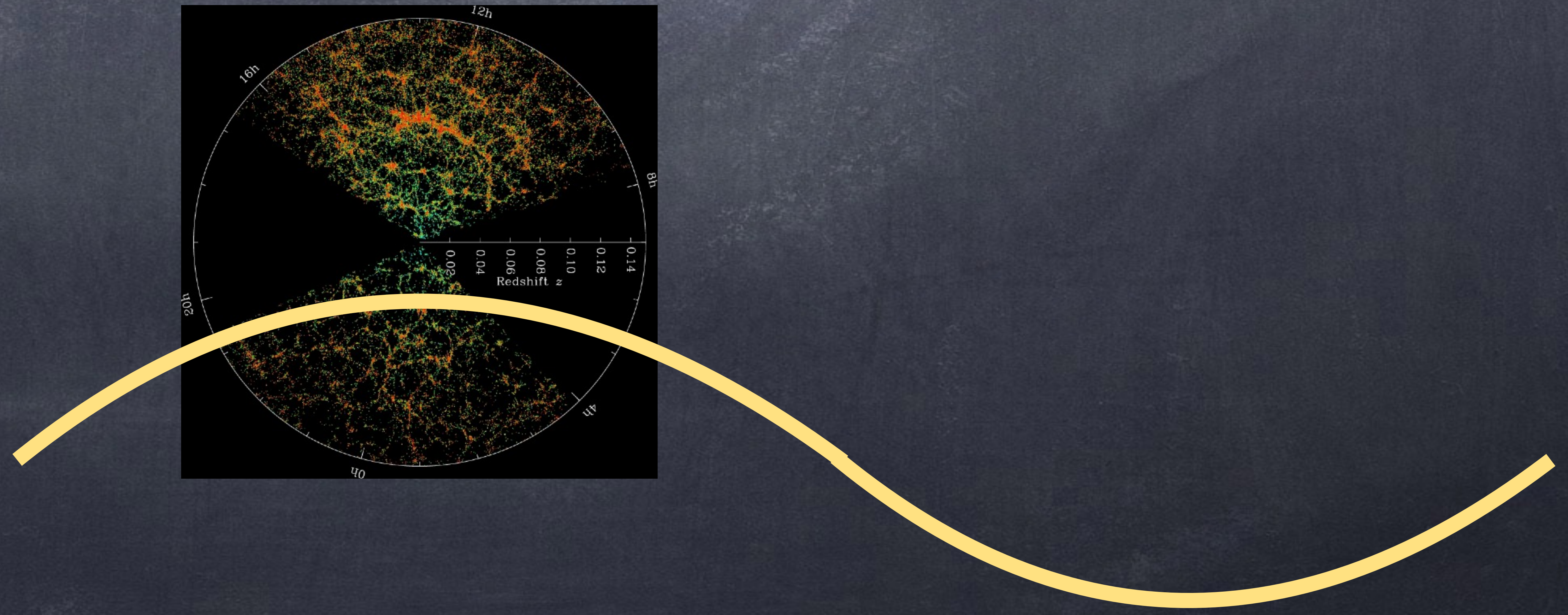


Beat Coupling (BC)

Hamilton, Rimes & Scoccimarro (2006)

Contribution of "beat-modes" (ϵ) to the covariance:

$$T(\mathbf{k}_1-\mathbf{p}_1,-\mathbf{k}_1-\boldsymbol{\epsilon}+\mathbf{p}_1,\mathbf{k}_2-\mathbf{p}_2',-\mathbf{k}_2-\boldsymbol{\epsilon}+\mathbf{p}_2)=4P(\epsilon)[P_L(\mathbf{k}_1-\mathbf{p}_1)F_2(\mathbf{k}_1-\mathbf{p}_1,\boldsymbol{\epsilon})$$
$$+P_L(-\mathbf{k}_1-\mathbf{p}_1')F_2(-\mathbf{k}_1-\mathbf{p}_1',\boldsymbol{\epsilon})]\times[P_L(\mathbf{k}_2-\mathbf{p}_2)F_2(\mathbf{k}_2-\mathbf{p}_2,-\boldsymbol{\epsilon})+P_L(-\mathbf{k}_2-\mathbf{p}_2')F_2(-\mathbf{k}_2-\mathbf{p}_2',-\boldsymbol{\epsilon})]$$



Super-sample covariance approach (SSC)

Takada & Hu (2013)

$$T(\mathbf{k}_1-\mathbf{p}_1,-\mathbf{k}_1-\boldsymbol{\epsilon}+\mathbf{p}_1,\mathbf{k}_2-\mathbf{p}_2',-\mathbf{k}_2-\boldsymbol{\epsilon}+\mathbf{p}_2)=4P(\epsilon)[P_L(\mathbf{k}_1-\mathbf{p}_1)F_2(\mathbf{k}_1-\mathbf{p}_1,\boldsymbol{\epsilon}) \\ +P_L(-\mathbf{k}_1-\mathbf{p}_1')F_2(-\mathbf{k}_1-\mathbf{p}_1',\boldsymbol{\epsilon})]\times[P_L(\mathbf{k}_2-\mathbf{p}_2)F_2(\mathbf{k}_2-\mathbf{p}_2,-\boldsymbol{\epsilon})+P_L(-\mathbf{k}_2-\mathbf{p}_2')F_2(-\mathbf{k}_2-\mathbf{p}_2',-\boldsymbol{\epsilon})]$$

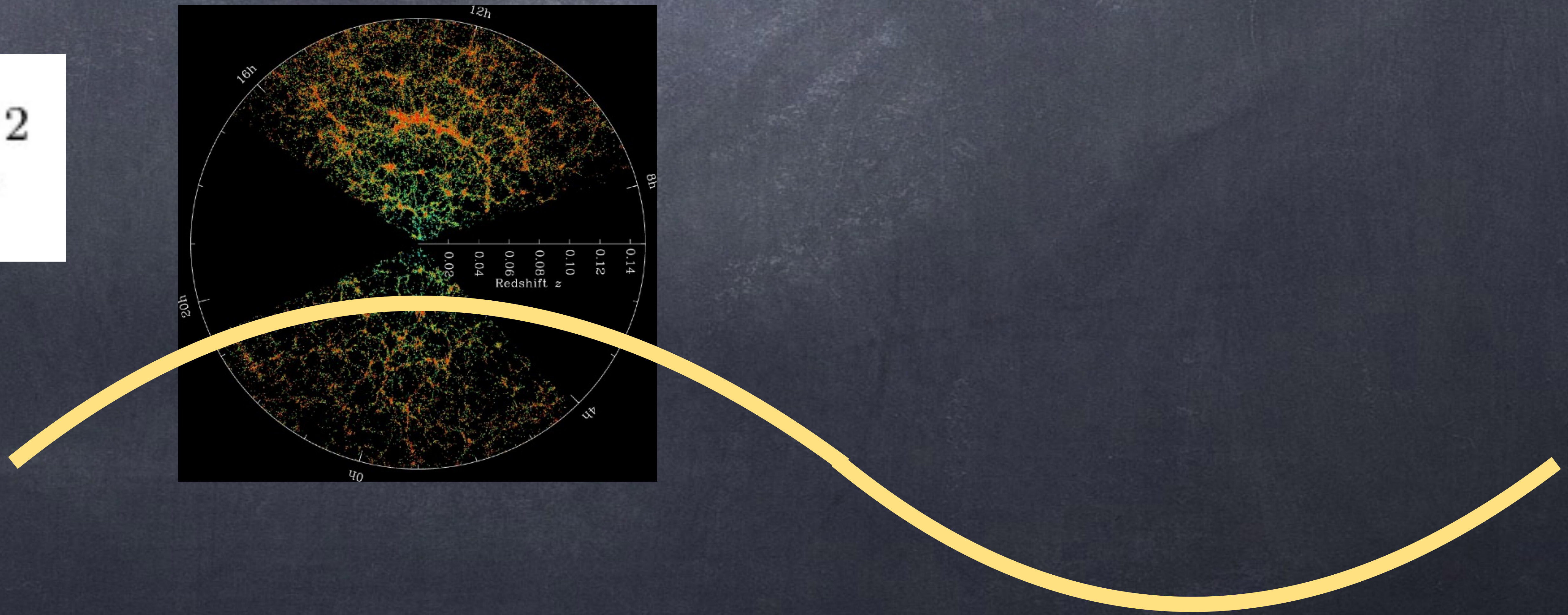
- Contribution of beat-mode covariance can be written as product of power spectrum responses

$$C^{\mathrm{SSC}}(k_1,k_2)=\sigma_W^2\left[\frac{\partial P(k_1)}{\partial\delta_b}\right]\left[\frac{\partial P(k_2)}{\partial\delta_b}\right]$$

$$\sigma_W^2=\frac{1}{V_W^2}\int\frac{d^3\boldsymbol{\epsilon}}{(2\pi)^3}P(\epsilon)|W(\boldsymbol{\epsilon})|^2$$

RMS fluctuations over window volume

($\sigma_W^2 \downarrow$ as $V_W \uparrow$)



Local Average Effect

de Putter et al. (2012)

- Fluctuations in galaxy surveys are normalized by the survey mean density (NOT true mean)

  $\Rightarrow$ Beat-mode effect on covariance damped by ~90%

$$\hat{\delta} \equiv \frac{\Delta\rho}{(\bar{\rho})_{\rm survey}} \equiv \hat{\delta} \equiv \frac{\delta(\mathbf{k})}{1+\delta_b}$$

$$\frac{\partial}{\partial \delta_b} P(k) \to \frac{\partial}{\partial \delta_b} \frac{P(k)}{(1+\delta_b)^2} = \frac{\partial P(k)}{\partial \delta_b} - 2P(k)$$

$$C^{\rm SSC}(k_1, k_2) = \sigma_W^2 \frac{\partial P(k_1)}{\partial \delta_b} \frac{\partial P(k_2)}{\partial \delta_b}$$

Final ‘simplified’ NG covariance

$$
\begin{aligned}
\mathbf{C}^{\mathrm{T}}_{\ell_1\ell_2}(k_1,k_2) =& \frac{1}{\mathrm{I}_{22}^2}\int_{\boldsymbol{\epsilon}} P_{\mathrm{L}}(\epsilon)\left\{4\int_{\hat{\mathbf{k}}_{\ell_1},\mathbf{p}_1} W_{11}(\mathbf{p}_1)W_{11}(\boldsymbol{\epsilon}-\mathbf{p}_1)P_{\mathrm{L}}(|\mathbf{k}_1-\mathbf{p}_1|)Z_1(\mathbf{k}_1-\mathbf{p}_1)Z_2(\mathbf{k}_1-\mathbf{p}_1,\boldsymbol{\epsilon})\right\}\\
&\times\left\{4\int_{\hat{\mathbf{k}}_{\ell_2},\mathbf{p}_2} W_{11}(\mathbf{p}_2)W_{11}(-\boldsymbol{\epsilon}-\mathbf{p}_2)P_{\mathrm{L}}(|\mathbf{k}_2-\mathbf{p}_2|)Z_1(\mathbf{k}_2-\mathbf{p}_2)Z_2(\mathbf{k}_2-\mathbf{p}_2,-\boldsymbol{\epsilon})\right\}\\
&+\frac{1}{\mathrm{I}_{22}^2}\int_{\hat{\mathbf{k}}_{\ell_1},\hat{\mathbf{k}}_{\ell_2},\boldsymbol{\epsilon}} |W_{22}(\boldsymbol{\epsilon})|^2\Big\{\Big[8P_{\mathrm{L}}^2(\mathbf{k}_1)Z_1^2(\mathbf{k}_1)P_{\mathrm{L}}(\mathbf{k}_1+\mathbf{k}_2)Z_2^2(-\mathbf{k}_1,\mathbf{k}_1+\mathbf{k}_2)+(\mathbf{k}_1\leftrightarrow\mathbf{k}_2)\Big]\\
&+16P_{\mathrm{L}}(\mathbf{k}_1)Z_1(\mathbf{k}_1)P_{\mathrm{L}}(\mathbf{k}_2)Z_1(\mathbf{k}_2)P_{\mathrm{L}}(\mathbf{k}_1+\mathbf{k}_2)Z_2(-\mathbf{k}_1,\mathbf{k}_1+\mathbf{k}_2)Z_2(-\mathbf{k}_2,\mathbf{k}_1+\mathbf{k}_2)\\
&+\Big[12\,Z_1^2(\mathbf{k}_1)P_{\mathrm{L}}^2(\mathbf{k}_1)Z_1(\mathbf{k}_2)P_{\mathrm{L}}(\mathbf{k}_2)Z_3(\mathbf{k}_1,-\mathbf{k}_1,\mathbf{k}_2)+(\mathbf{k}_1\leftrightarrow\mathbf{k}_2)\Big]\Big\}\\
&+P_{\ell_1}(k_1)P_{\ell_2}(k_2)\sigma_{10}^2\\
&-\frac{P(k_1)P_{\ell_2}(k_2)}{\mathrm{I}_{10}\mathrm{I}_{22}}\int_{\boldsymbol{\epsilon}} W_{10}(-\boldsymbol{\epsilon})W_{22}(\boldsymbol{\epsilon})[P(\epsilon)Z_1(\boldsymbol{\epsilon})\mathcal{Z}_{21}(k_1,\ell_1,\hat{\boldsymbol{\epsilon}}\cdot\hat{\mathbf{n}})+P_{\ell_1}(k_1)b_2]\\
&-\frac{P(k_2)P_{\ell_1}(k_1)}{\mathrm{I}_{10}\mathrm{I}_{22}}\int_{\boldsymbol{\epsilon}} W_{10}(-\boldsymbol{\epsilon})W_{22}(\boldsymbol{\epsilon})[P(\epsilon)Z_1(\boldsymbol{\epsilon})\mathcal{Z}_{21}(k_2,\ell_2,\hat{\boldsymbol{\epsilon}}\cdot\hat{\mathbf{n}})+P_{\ell_2}(k_2)b_2]\\
&+\frac{1}{\mathrm{I}_{10}^2}\left\langle\left(\sum_j^{N_g}+\bar{\alpha}^2\sum_j^{N_r}\right)\right\rangle+\frac{2}{\mathrm{I}_{10}}\left\langle\left(\sum_i^{N_g}+\bar{\alpha}^2\sum_i^{N_r}\right)\left(\sum_{i'(\neq i)}^{N_g}-\bar{\alpha}\sum_{i'(\neq i)}^{N_r}\right)w_iw_{i'}e^{-i\mathbf{k}_1\cdot(\mathbf{x}_i-\mathbf{x}_{i'})}\right\rangle\\
&+\frac{1}{\mathrm{I}_{22}^2}\int_{\hat{\mathbf{k}}_{\ell_1},\hat{\mathbf{k}}_{\ell_2}}\left\langle\left(\sum_i^{N_g}-\alpha\sum_i^{N_r}\right)\left(\sum_{j(\neq i)}^{N_g}+\alpha^2\sum_{j(\neq i)}^{N_r}\right)\left(\sum_{j'(\neq j)(\neq i)}^{N_g}-\alpha\sum_{j'(\neq j)(\neq i)}^{N_r}\right)w_iw_j^2w_{j'}e^{-i\mathbf{k}_1\cdot\mathbf{x}_i}e^{i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{x}_j}\right\rangle_c\\
&+\frac{2}{\mathrm{I}_{22}^2}\int_{\hat{\mathbf{k}}_{\ell_1},\hat{\mathbf{k}}_{\ell_2}}\left\langle\left(\sum_i^{N_g}+\alpha^2\sum_i^{N_r}\right)\left(\sum_{j(\neq i)}^{N_r}+\alpha^2\sum_{j(\neq i)}^{N_r}\right)w_i^2w_j^2e^{-i(\mathbf{k}_1-\mathbf{k}_2)\cdot(\mathbf{x}_i-\mathbf{x}_j)}\right\rangle_c
\end{aligned}
$$

Beat-Coupling

Ordinary Trispectrum (T_0)

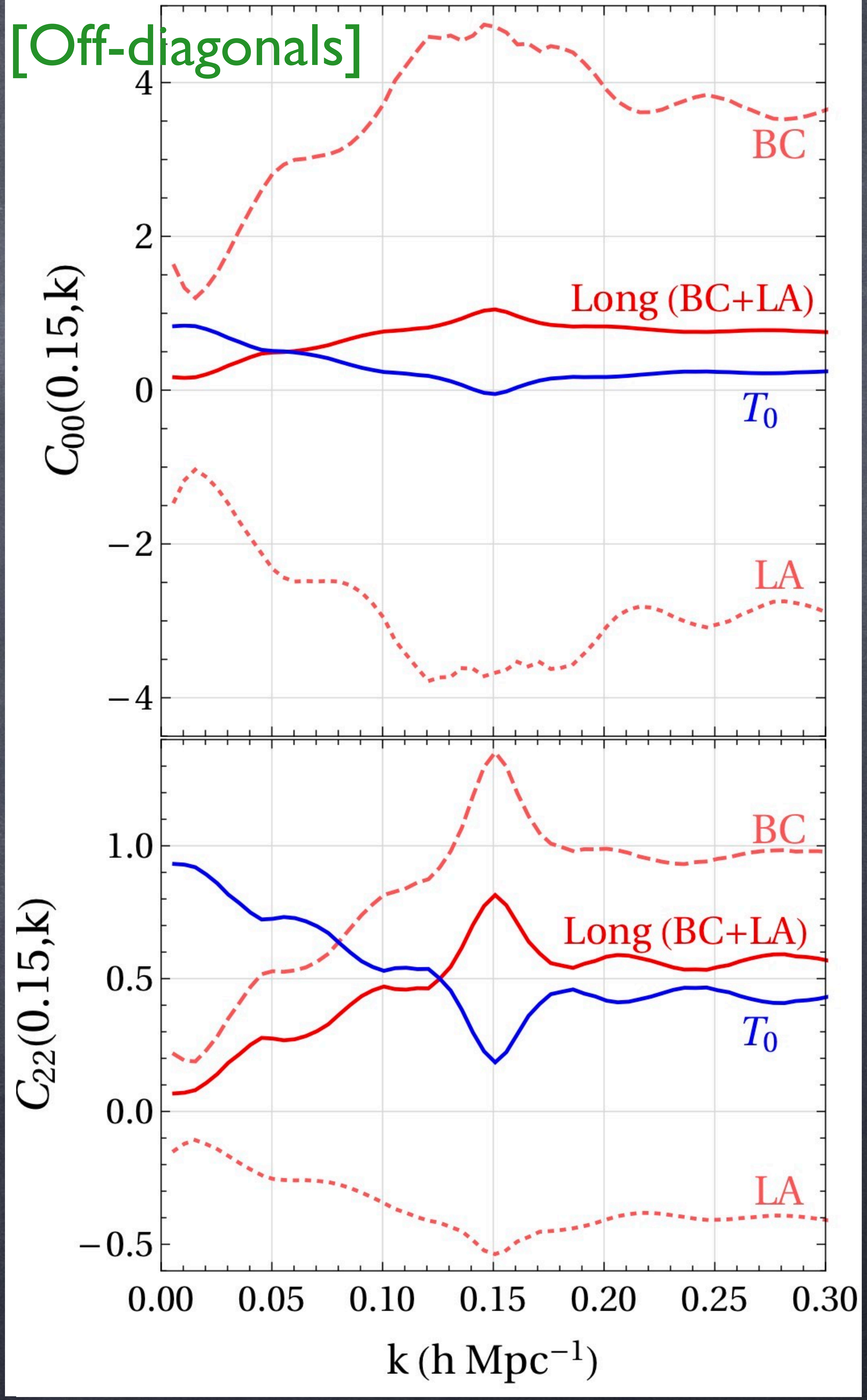
Local Average effect

Shot noise contribution

DW & Scoccimarro 19

Comparison of Non-Gaussian effects

Ordinary trispectrum (T_0) in redshift-space is relatively more important at low-k

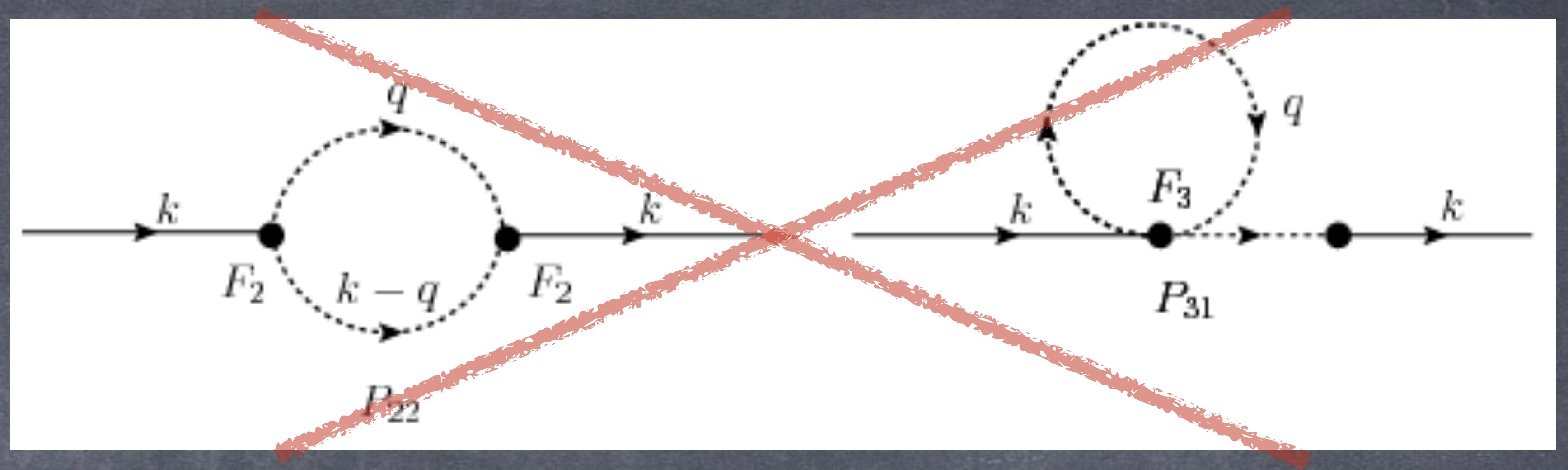


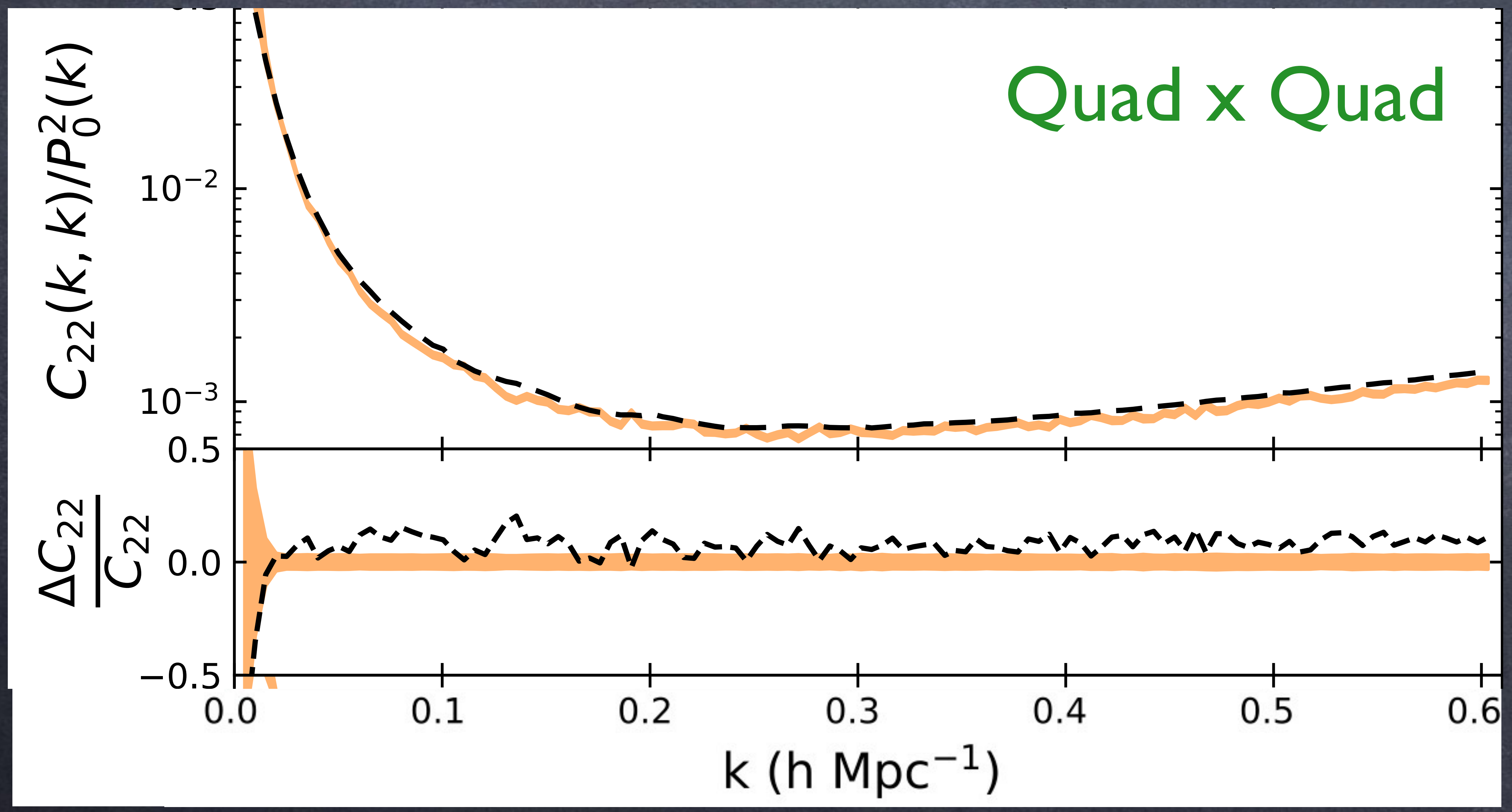
DW & Scoccimarro 19

See also: Mohammed et al. 2016

Even if we just use linear and tree-level PT (no Loops or FOG),

our analytic approach works very well at high-k, why!?





Shot Noise

- We observe particles (galaxies) instead of fields
- Affects results at high-k:

$$P(k) \; < \frac{1}{\bar{n}}$$

Why does analytic work so well?

Shot noise (dashed) dominates at high-k

$$\mathbf{C}(k_1,k_2) = \langle \delta(k_1)\delta(-k_1)\delta(k_2)\delta(-k_2)\rangle - \langle \delta(k_1)\delta(-k_1)\rangle\langle \delta(k_2)\delta(-k_2)\rangle$$

$$\mathbf{C}^{\mathrm{G}}(k_1,k_2) \simeq 2\,\langle \delta(k_1)\delta(-k_2)\rangle\,\langle \delta(k_2)\delta(-k_1)\rangle$$

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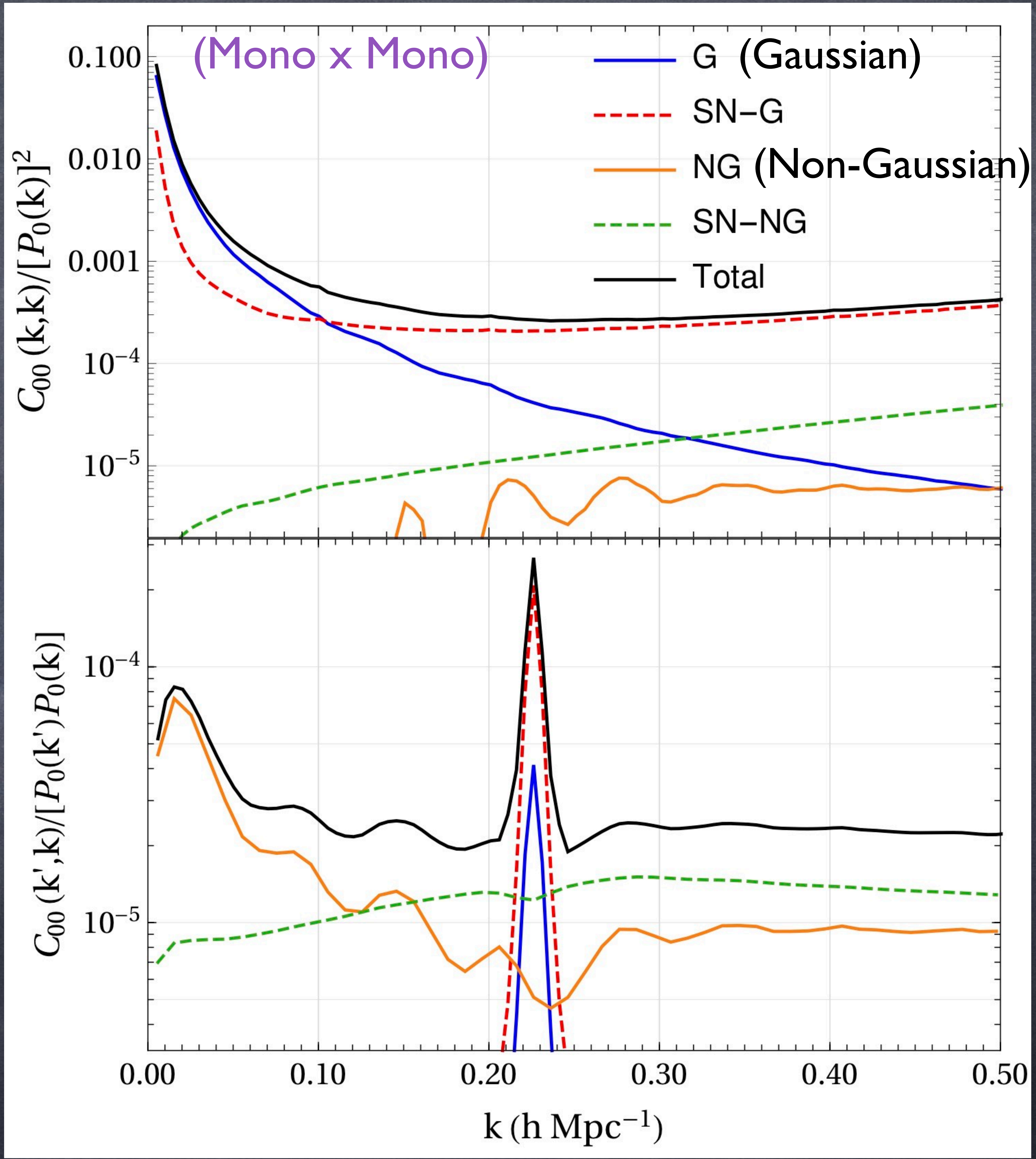

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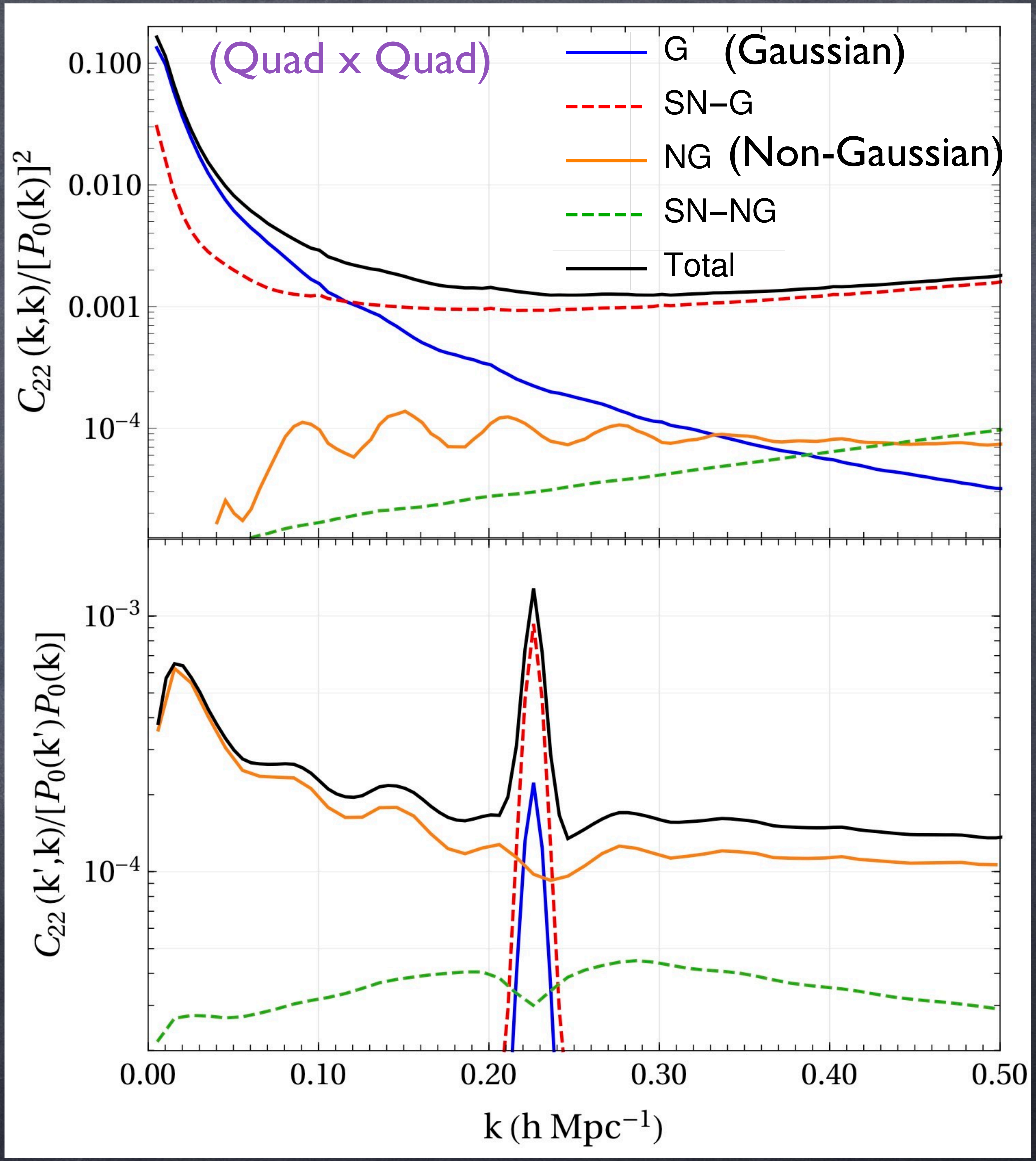


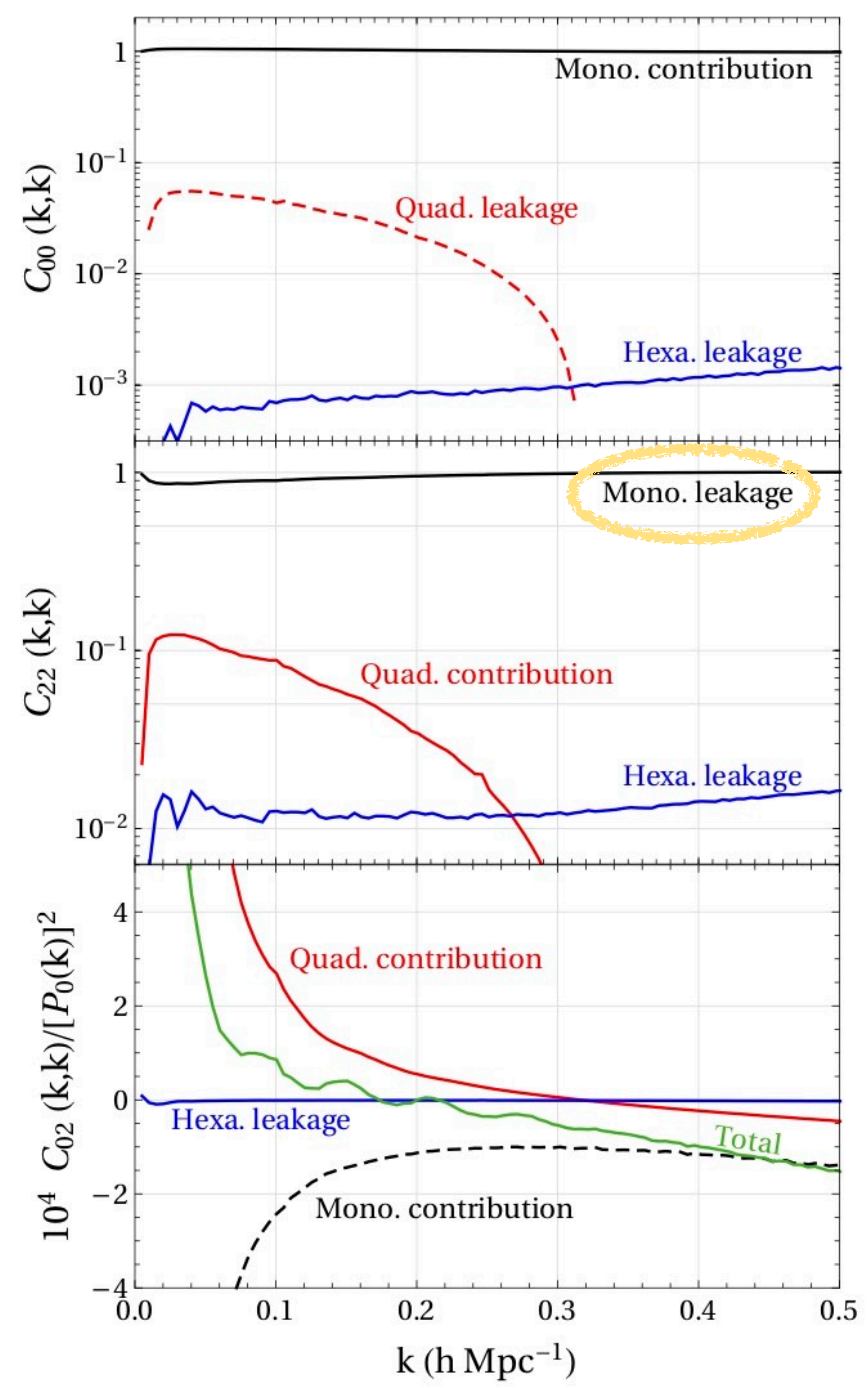
Why does analytic work so well?

<u>Leakage</u> due to
anisotropic survey window
& radial LOS

↓

Monopole (⇒ shot noise) leaks

into multipole covariance
at high-k

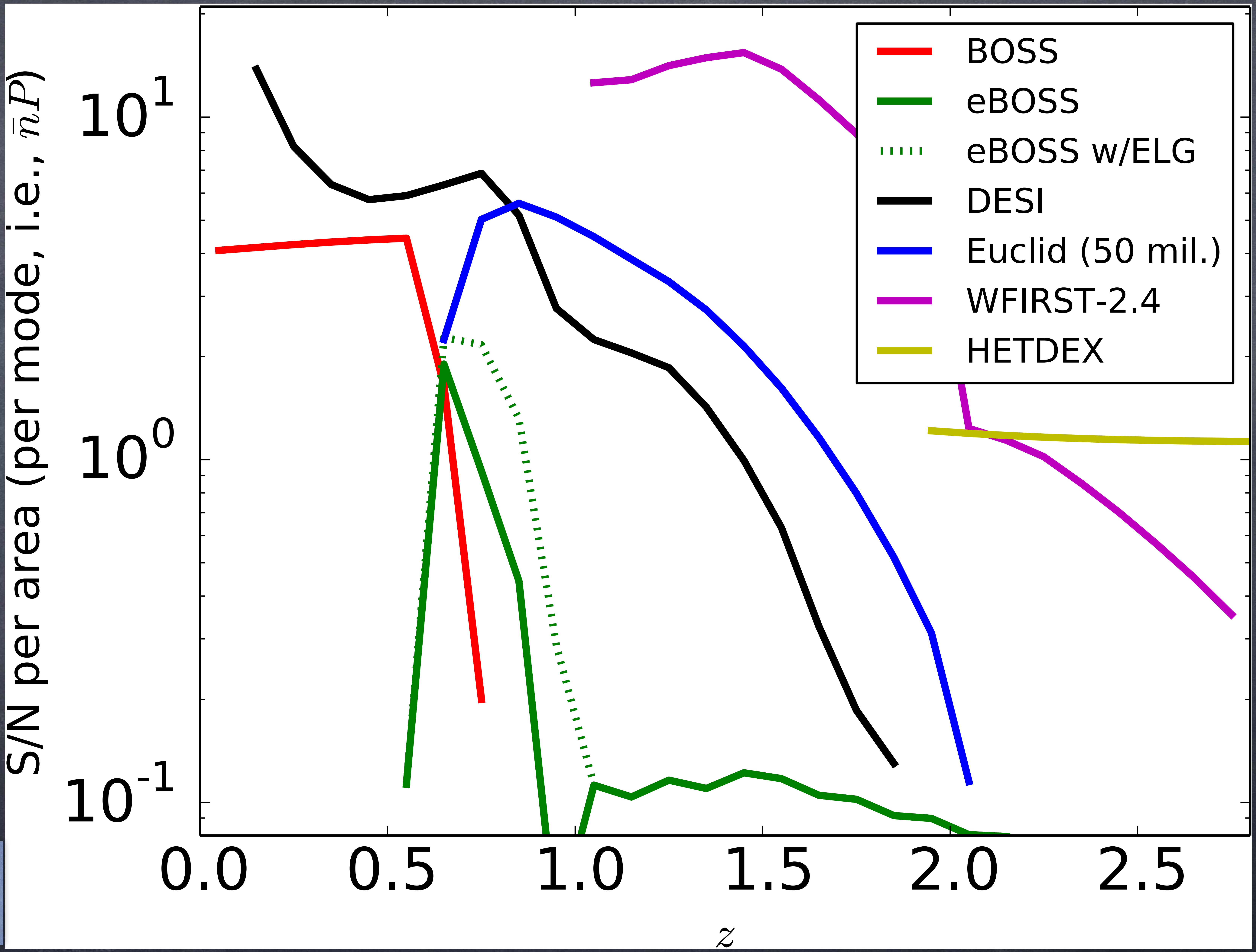


Shot noise level for future surveys

Redshift surveys typically designed with

$$\bar{n}P(k_{\mathrm{BAO}}) \simeq \text{few}$$

Font-Ribera et al. 2014



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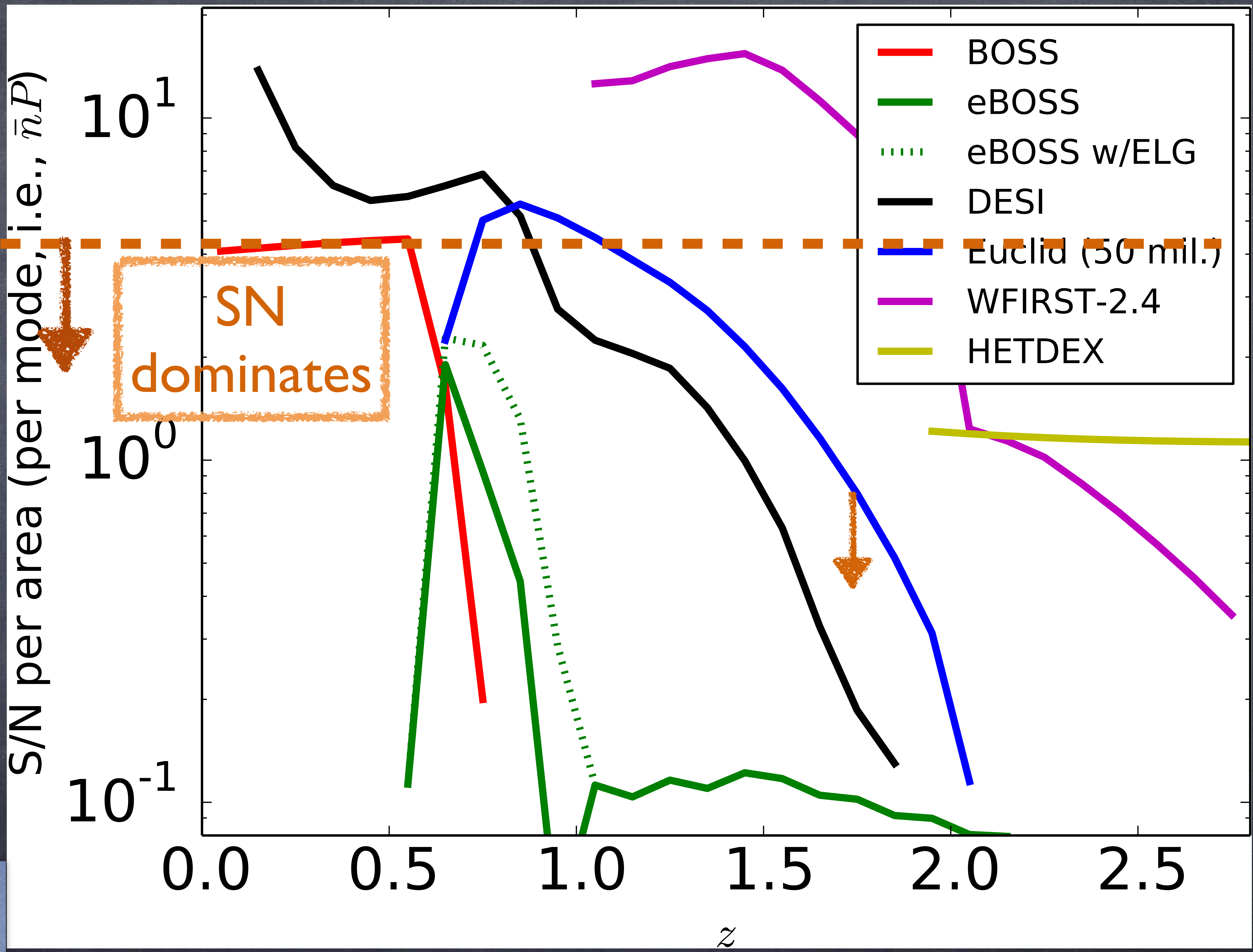

Font-Ribera et al. 2014

Important differences between our approach and rest of literature

Literature

1. Estimator motivated by simulations

$$\hat{\delta} \equiv \frac{\delta(\mathbf{k})}{1+\delta_b}$$

Our

1. FKP estimator (motivated from surveys)

$$\hat{\delta}^{\rm FKP}(\mathbf{x}) \equiv \frac{w(\mathbf{x})[n_g(\mathbf{x}) - \alpha n_s(\mathbf{x})]}{[\alpha \int d^3x \, w^2(\mathbf{x})\bar{n}(\mathbf{x}) n_s(\mathbf{x})]^{1/2}}$$

where $\alpha = N_g / N_s$ and simplifies to

$$\hat{\delta}^{\rm FKP} \simeq \frac{1}{\sqrt{\rm I_{22}}} \frac{W(\mathbf{x})\delta(\mathbf{x})}{(1+\delta_{\rm N_g})^{1/2}}$$

Literature

1. Estimator motivated by simulations

$$\hat{\delta} \equiv \frac{\delta(\mathbf{k})}{1+\delta_b}$$

2. Normalized by a matter mode

[responses for tides, bias, shot noise need to be added separately]

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[includes bias, shot noise and non-linearities like tides]

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3. Stronger normalization of long mode effects

$$\frac{\partial}{\partial\delta_b}P(k) \rightarrow \frac{\partial}{\partial\delta_b}\frac{P(k)}{(1+\delta_b)^2} = \frac{\partial P(k)}{\partial\delta_b} - 2P(k)$$

~0.7 P(k)
at k=0.1 h/Mpc

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2. Normalized by a *galaxy mode*

[includes bias, shot noise and non-linearities like tides]

3. Weaker normalization of long mode effects

$$\frac{\partial\hat{P}^{\rm FKP}}{\partial\delta_{\rm N_g}} \rightarrow \frac{\partial}{\partial\delta_{\rm N_g}}\frac{P}{(1+\delta_{\rm N_g})} = \frac{\partial P}{\partial\delta_{\rm N_g}} - P(k)$$

~1.7 P(k)
at k=0.1 h/Mpc

Literature

1. Estimator motivated by simulations

$$\hat{\delta} \equiv \frac{\delta(\mathbf{k})}{1+\delta_b}$$

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[responses for tides, bias, shot noise need to be added separately]

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Our Approach

1. FKP estimator (motivated from surveys)

$$\hat{\delta}^{\mathrm{FKP}} \simeq \frac{1}{\sqrt{\mathrm{I}_{22}}} \frac{W(\mathbf{x})\delta(\mathbf{x})}{(1+\delta_{\mathrm{N_g}})^{1/2}}$$

2. Normalized by a *galaxy mode*

[includes bias, shot noise and non-linearities like tides]

3. Weaker normalization of long mode effects

Effect of long mode on covariance is
5 x stronger
than literature

SSC approach assumes a top-hat window

$$C^{\mathrm{SSC}}(k_1, k_2) = \sigma_W^2 \left[\frac{\partial P(k_1)}{\partial \delta_b}\right] \left[\frac{\partial P(k_2)}{\partial \delta_b}\right]$$

$$\sigma_W^2 = \frac{1}{V_W^2} \int \frac{d^3\boldsymbol{\epsilon}}{(2\pi)^3} P(\epsilon) |W(\boldsymbol{\epsilon})|^2$$

RMS fluctuations
over window volume

- In the presence of a local-average effect, we get three different types of σ_W^2

$$\frac{1}{\mathrm{I}_{22}^2} \int_{\boldsymbol{\epsilon}} P(\epsilon) |W_{22}(\boldsymbol{\epsilon})|^2 \qquad \frac{1}{\mathrm{N_g I_{22}}} \int_{\boldsymbol{\epsilon}} P(\epsilon) W_{22}(\boldsymbol{\epsilon}) \bar{n}(-\boldsymbol{\epsilon}) \qquad \langle \delta_{\mathrm{N_g}}^2 \rangle \equiv \frac{1}{\mathrm{I}_{10}^2} \int_{\boldsymbol{\epsilon}} P(\epsilon) |\bar{n}(\boldsymbol{\epsilon})|^2$$

$$W_{22}(\mathbf{x}) = \bar{n}^2(\mathbf{x}) w_{\mathrm{FKP}}^2(\mathbf{x})$$

These differ by up to 35% for the SDSS window

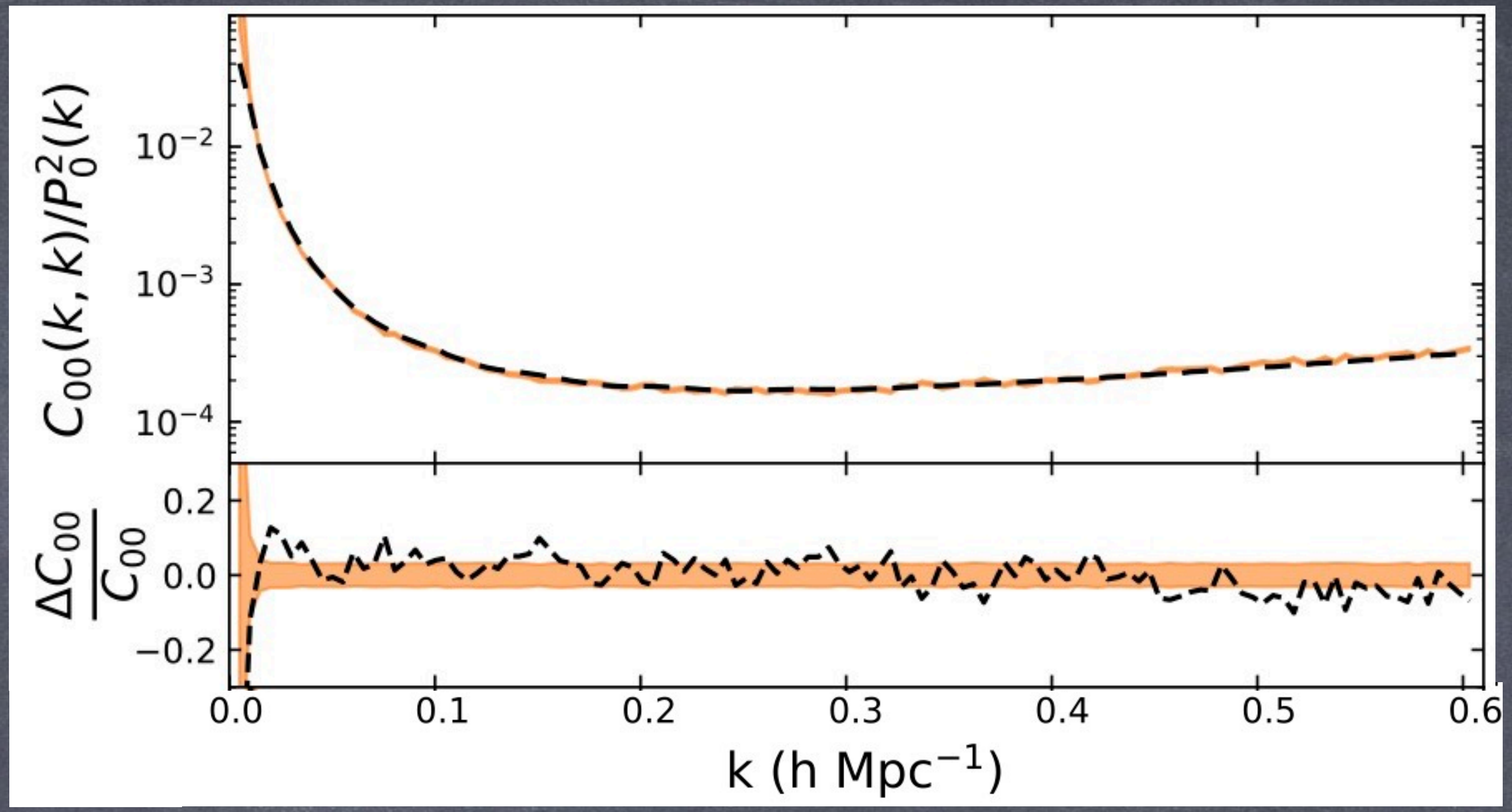
Summary

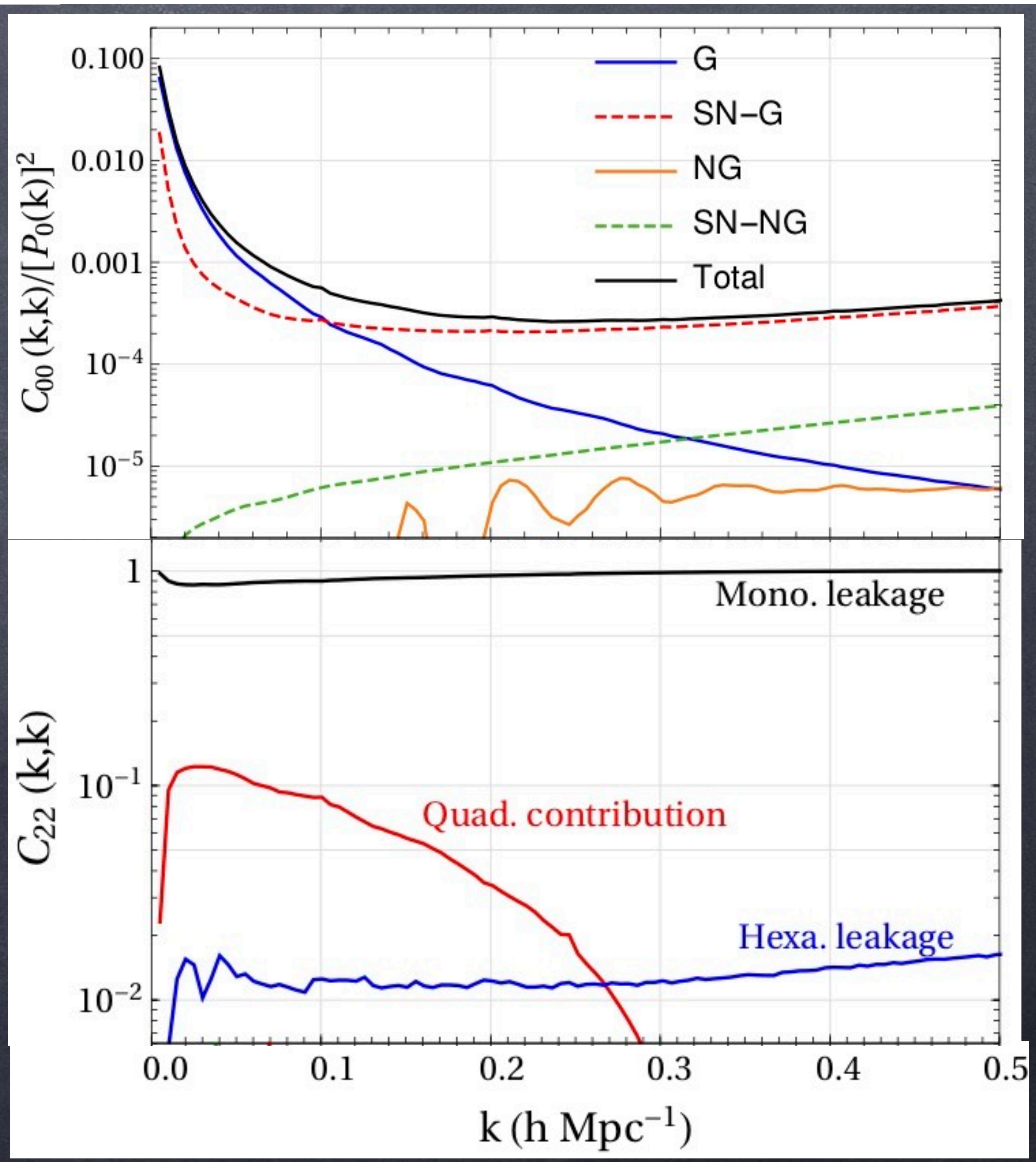
- Analytic covariance is an excellent alternative to mocks

1. Monopole Leakage simplifies RSD modeling

2. Shot noise dominates at high-k
   (Dominates over loops and FOG)

- *Mocks spend computer time calculating a slightly noisy covariance plus shot noise*





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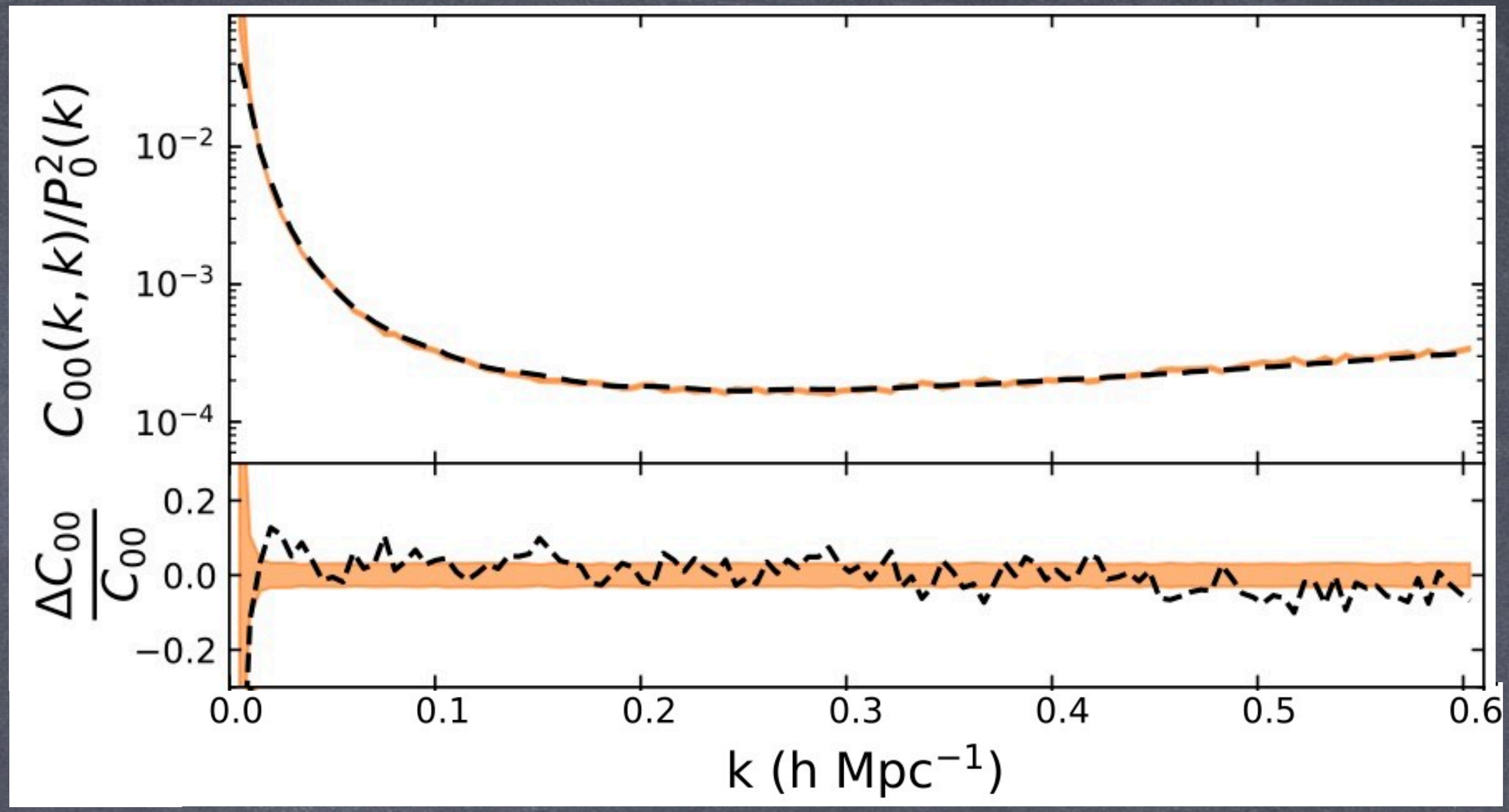
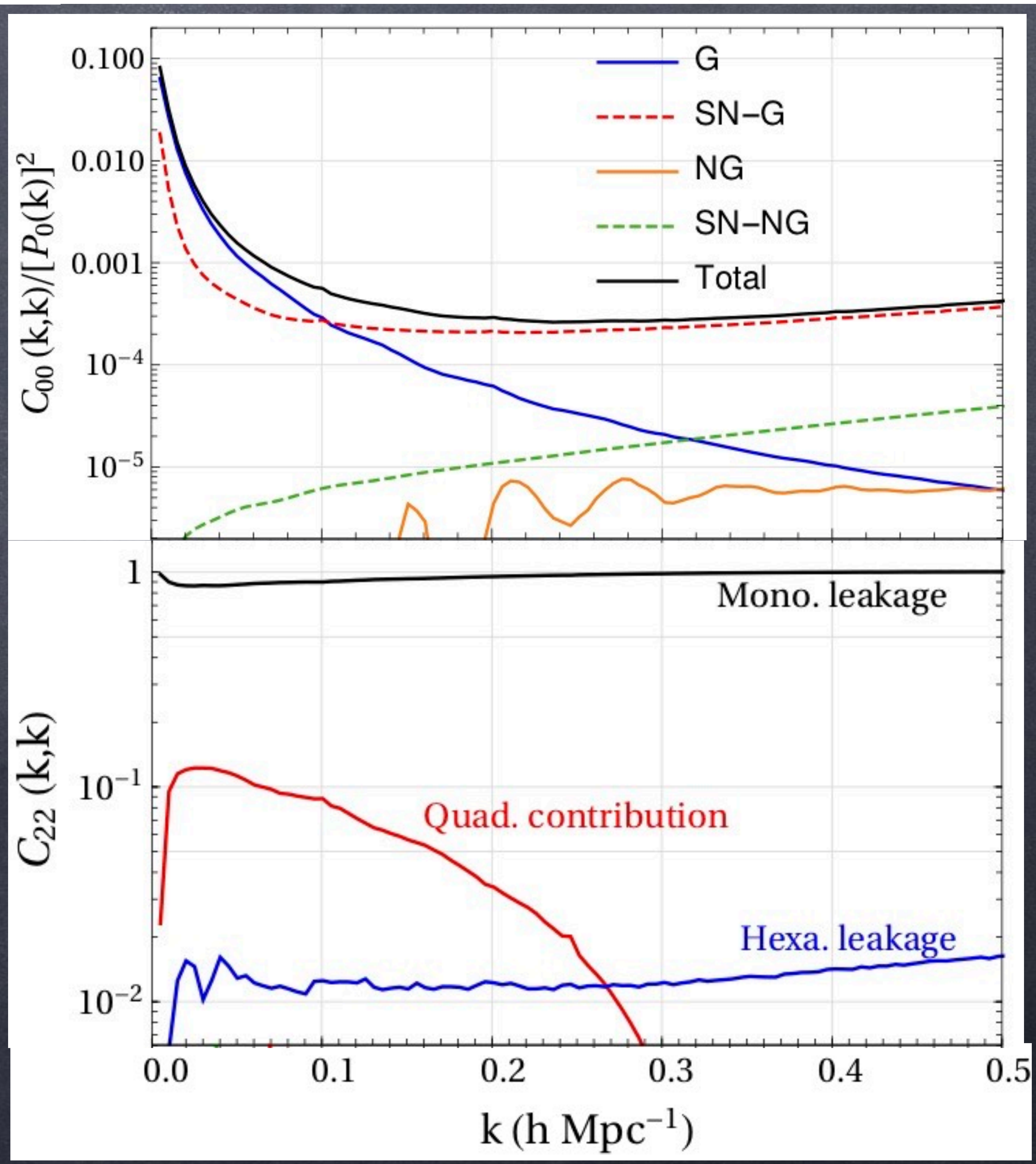
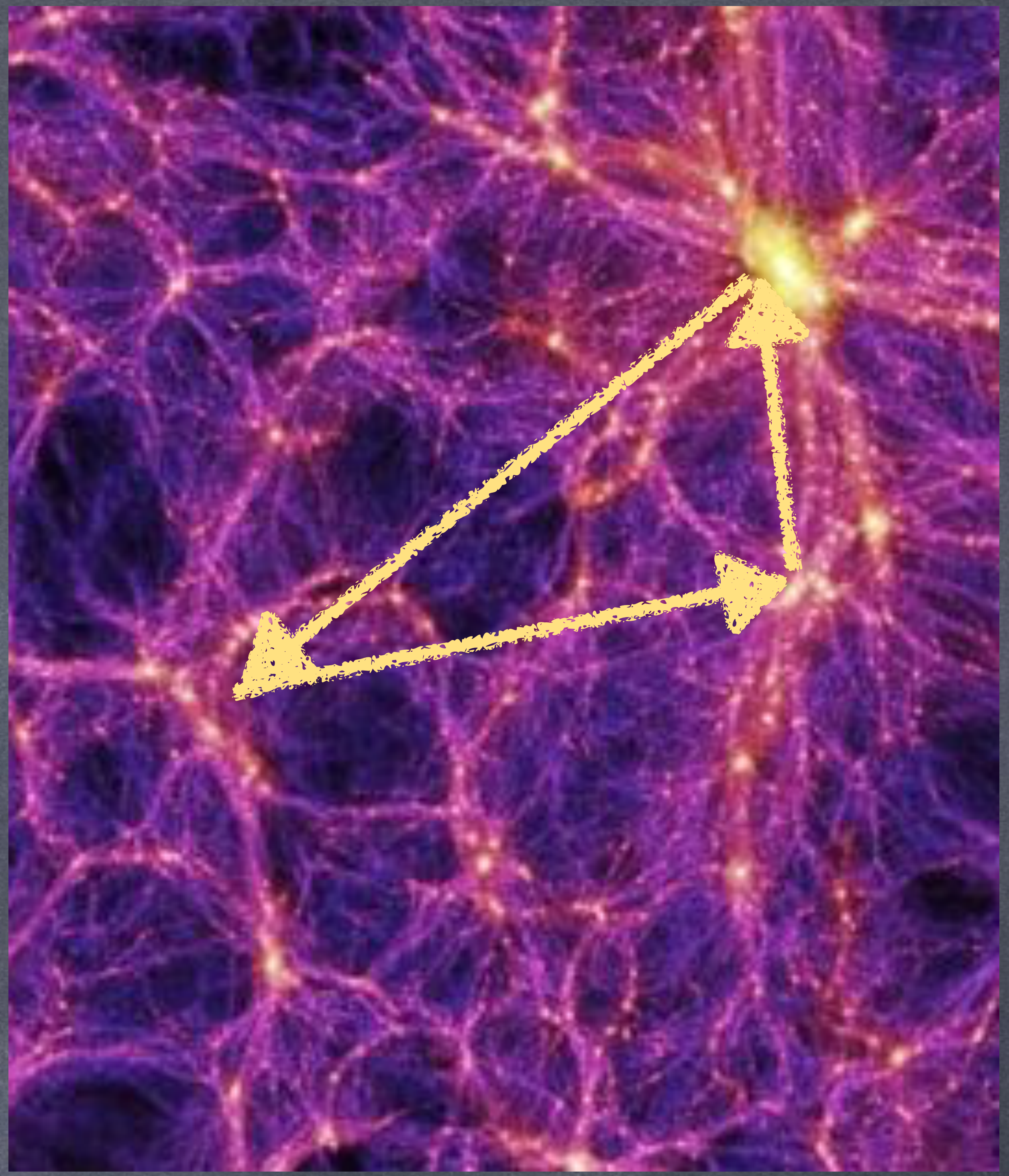
Next

- Bispectrum covariance
  (Simulations are computationally prohibitive)
- Redo BOSS analysis with parameter-dependent analytic covariance





Analytic covariance is crucial for BISPECTRUM



- Number of triangles to estimate ~ 6000 !!
- Number of mock simulations: $\mathcal{O}(1000)$
- Bottleneck for BOSS
  - Gil-Marin et al 17 could only use ~800 triangles

True vs FKP shot noise

$$\hat{P}_0(k) = \frac{1}{\mathrm{I}_{22}}\left[\int \frac{d\Omega_k}{4\pi}|F_0(\mathbf{k})|^2 - \mathrm{N}_0\right]$$

$$F_0(\mathbf{k}) = \left(\sum_{j=1}^{\mathrm{N_g}} - \alpha \sum_{j=1}^{\mathrm{N}_r}\right) w_j e^{i\mathbf{k}\cdot\mathbf{x}_j} \qquad \alpha = \frac{\mathrm{N_g}}{\mathrm{N}_r}$$

$$\mathrm{N}_0^{\mathrm{true}} \equiv \left(\sum_{j=1}^{\mathrm{N_g}} + \alpha^2 \sum_{j=1}^{\mathrm{N}_r}\right) w_j^2$$

$$\mathrm{N}_0^{\mathrm{FKP}} \equiv \left(\alpha \sum_{j=1}^{\mathrm{N}_r} + \alpha^2 \sum_{j=1}^{\mathrm{N}_r}\right) w_j^2$$

100% change in covariance at k~0.4 !

DW & Scoccimarro 19