Local gravity and the Cosmos

Using local tests of modified gravity to probe cosmological physics Tristan L. Smith Caltech



Reasons we may want to modify Einstein's general relativity:

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Reasons we may want to modify Einstein's general relativity:





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- Reasons we may want to modify Einstein's general relativity:
 - * Current epoch of accelerated expansion





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- Reasons we may want to modify Einstein's general relativity:
 - * Current epoch of accelerated expansion
 - * We know that Einstein's GR isn't the full story because it cannot be quantized





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- * Generic result of modifying gravity:
 - Create a new degree of freedom
 - * May connect local dynamics to cosmology!



* Start with the standard Einstein-Hilbert action

$$S = \frac{1}{2\kappa} \int \mathrm{d}^4 x \sqrt{-g} R + S_m$$

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* Start with the standard Einstein-Hilbert action

$$S = \frac{1}{2\kappa} \int \mathrm{d}^4 x \sqrt{-g} R + S_m$$

* Modify by adding an extra term



$$S = \frac{1}{2\kappa} \int \mathrm{d}^4 x \sqrt{-g} [\mathbf{R} + \mathbf{f}(\mathbf{R})] + S_m$$

* Vary the action to obtain the field equation

$[1 + f'(R)]G_{\mu\nu} = \kappa T_{\mu\nu}$ $-\frac{1}{2}g_{\mu\nu}[Rf'(R) - f + \Box f'(R)] + \nabla_{\mu}\nabla_{\nu}f'(R)$

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 $G_{\mu\nu} = \kappa T_{\mu\nu}$

$$S = \frac{1}{2\kappa} \int \mathrm{d}^4 x \sqrt{-g} [\mathbf{R} + \mathbf{f}(\mathbf{R})] + S_m$$

* Vary the action to obtain the field equation

 $-\frac{1}{2}g_{\mu\nu}[Rf'(R)] =$

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 $\underbrace{+\nabla_{\mu}\nabla_{\nu}f'(R)}$

$$S = \frac{1}{2\kappa} \int \mathrm{d}^4 x \sqrt{-g} [\mathbf{R} + \mathbf{f}(\mathbf{R})] + S_m$$

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$$S = \frac{1}{2\kappa} \int \mathrm{d}^4 x \sqrt{-g} [\mathbf{R} + \mathbf{f}(\mathbf{R})] + S_m$$

* Look at the trace of the full field equation:

$$\Box f'(R) = \frac{1}{3} \Big[\kappa T + R[1 - f'(R)] + 2f(R) \Big]$$

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GR terms imply algebraic relation between T and R

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* Look at the trace of the full field equation:

$$\Box f'(R) = \frac{1}{3} \Big[\kappa T + R[1 - f'(R)] + 2f(R) \Big]$$

f(R) implies a <u>non-linear differential</u> relation between R and T

New scalar propagating degree of freedom!

A preview: weak-field f(R)

- * A lot of the literature on weak-field f(R) is confusing
- * Reinterpret weak-field f(R) as taking a pressureless source (w=0) and giving it $w_{\rm eff} \neq 0$
- * The exact value of w_{eff} depends on the solution to the trace of the field equation
- Present the condition that allows us to determine the solution to the trace equation
- * As discussed in Hu and Sawicki, viable cosmology and Solar System tests restricts us to f(R) theories where

$$|f(R_{\rm cos})| \lesssim \kappa \rho_{\rm crit} \ll \kappa \rho$$

 $f'(R_{\rm cos}) \ll 1$

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* Consider small deviations away from a flat metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$|h_{\mu\nu}| \ll 1$$



Linearize the field equations sourced by a perfect fluid at rest

$$T_{\mu\nu} = (\rho + P)\delta_{\alpha 0}\delta_{\beta 0} + P\eta_{\mu\nu}$$

Linearized field equations are given by:

 $\begin{array}{c} \textbf{General} \\ \textbf{relativity} \\ P \neq 0 \end{array}$

f(R) gravity

P = 0



Linearized field equations are given by:





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Linearized field equations are given by:





* Interpreted in terms of GR, f(R) gravity produces an effective pressure: $\tilde{P} = \frac{1}{2}\Gamma$ $\tilde{\rho} \equiv \rho - \frac{1}{2}\Gamma$

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In f(R) gravity we started with a pressureless source



- * f(R) gravity produces an effective pressure $\tilde{P} = \frac{1}{2}\Gamma$
- * Note: \tilde{P} <u>does not</u> affect hydrostatic equilibrium $\longrightarrow \tilde{P}$ <u>only</u> seen through motion of test bodies Vilja
- * We can re-express this as an effective equation of state $w_{\text{eff}} = \frac{\tilde{P}}{\tilde{\rho}} = \frac{1}{2\rho/\Gamma - 1}$ $\Gamma = -\frac{1}{3}\left[T + \frac{R}{\kappa}\right]$

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* w_{eff} is determined by the trace of the field equation:

$$\nabla^2 f'(R) = \frac{1}{3} [\kappa T + R] = \frac{\partial V_{\text{eff}}}{\partial f'(R)}$$

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$$R = -\kappa T$$

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* GR extremum is stable if curvature is positive

$$\frac{\partial^2 V_{\text{eff}}}{\partial [f'(R)]^2} = \frac{1}{3} f''(R)^{-1} > 0$$

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* <u>If</u> the system reaches the stable GR minimum:

 $R = -\kappa T$

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* <u>If</u> the system reaches the stable GR minimum:

$$R = -\kappa T$$

$$\longrightarrow \Gamma = -\frac{1}{3} \left[T + \frac{R}{\kappa} \right] = 0$$

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* <u>If</u> the system reaches the stable GR minimum:



* ... and we regain GR

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- * We have established a stable GR minimum, but the theory <u>does not necessarily</u> attain this limit
- * Two different behaviors of the trace of the field equation: $\nabla^2 f' = \frac{1}{3}[\kappa T + R]$ <u>Linear</u> $R = R_0 + \delta R$ <u>Non-linear</u>

- * We have established a stable GR minimum, but the theory <u>does not necessarily</u> attain this limit
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- * We have established a stable GR minimum, but the theory <u>does not necessarily</u> attain this limit
- * Two different behaviors of the trace of the field equation: $\nabla^2 f' = \frac{1}{3}[\kappa T + R]$ <u>Linear</u> $R = R_0 + \delta R$ <u>Non-linear</u> $|R| \ll |\kappa T|$ $R = -\kappa T$ $w_{\text{eff}} = \frac{1}{2\rho/\Gamma - 1} = \frac{1}{5}$ $w_{\text{eff}} = \frac{1}{2\rho/\Gamma - 1} = 0$

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* f(R) theories take a source with zero pressure and generate an effective equation of state

$$w_{
m eff} = 1/5$$

or
 $w_{
m eff} = 0$

- * The ability to linearize the trace equation determines w_{eff}
- * The ability to linearize the trace equation depends on the size of

$$\left|\frac{\delta R}{R_0}\right|$$

* One can show that the linearization of a localized source is determined by the ratio

$$\left|\frac{\delta R}{R_0}\right| = \frac{2}{3} \left|\Phi_N \frac{1}{R_0 f''(R_0)}\right|$$

Chiba, Smith, and Erickcek (2007)

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* One can show that the linearization of a localized source is determined by the ratio

$$\left|\frac{\delta R}{R_0}\right| = \frac{2}{3} \left| \Phi_N \frac{1}{R_0 f''(R_0)} \right| \qquad \begin{array}{c} \mbox{Chiba, Smith, and} \\ \mbox{Erickcek (2007)} \end{array} \right|$$

* To connect to measurements, we can write $w_{
m eff}$ in terms of the PPN parameter γ

$$\frac{\delta R}{R_0} \bigg| = \frac{2}{3} \bigg| \Phi_N \frac{1}{R_0 f''(R_0)} \bigg| \begin{cases} \ll 1 \Rightarrow & \gamma = 1/2 & \text{Chiba, Smith, and} \\ \gtrsim 1 \Rightarrow & \gamma = 1 & \text{Hu and Sawicki (2007)} \end{cases}$$

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γ and the environemnt

* For a given localized source (star/galaxy) the value of γ depends on two quantites:

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* The local Newtonian potential

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- * The local Newtonian potential
- * Background curvature R_0 and functional form f(R)
- * The background curvature, in turn, may depend on redshift

* For a given localized source (star/galaxy) the value of γ depends on two quantites:

$$\left| \frac{\delta R}{R_0} \right| = \frac{2}{3} \left| \Phi_N \frac{1}{R_0 f''(R_0)} \right|$$

* This theory implies that it may be interesting to measure γ in various environments and at various redshifts...

Measurement of γ

★ We can measure from observations of galaxies

* Directly measure the velocity dispersion (spectra)

 $\sigma_{\rm obs}$

* Measure the redshift and Einstein radius -

 $\theta_E = (1+\gamma)2\pi\sigma_{\rm obs}^2 [D_{\rm LS}(z)/D_{\rm S}(z)]$



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Measurement of γ

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Measurement of γ

* f(R) gravity would predict:

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* Two populations of \,\gamma\,
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some systems will have $~~\gamma=1/2$

others will have $~~\gamma=1$

correlated with local environment and possibly redshift...

Measurement of γ : preliminary analysis

* Data from SDSS and HST for 15 elliptical lensing galaxies (SLACS survey, Bolton et al.)



Measurement of γ : preliminary analysis

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Measurement of γ : preliminary analysis

- Data from SDSS and HST for 15 elliptical lensing galaxies (SLACS survey, Bolton et al.)
- 1σ <u>rejection</u> of the hypothesis that all points come from single distribution
- By the end of the year 70 more galaxies



- If bimodality persists with 70, statistically significant



f(R) conclusions

- * f(R) gravity is a theory constructed in order to explain the current epoch of accelerated expansion
- * Discussed the weak-field limit of f(R) gravity:
 - * Showed that a pressureless (w=0) source, f(R) gravity can generate a $w_{\rm eff} \neq 0$
 - * Leads to an environmentally dependent γ (= 1/2 or 1)
 - * May probe this using current and future galaxy lens surveys

Chern-Simons gravity

The first observational constraint to



Chern



Simons

Gravity

arXiv:0708.0001 TLS, A. Erickcek, R. R. Caldwell, M. Kamionkowski

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* Chern-Simons gravity is defined by the action

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[-\frac{1}{2\kappa} R + \frac{\ell}{12} \theta \mathbf{R} \tilde{\mathbf{R}} - \frac{1}{2} (\partial \theta)^2 - V(\theta) + \mathcal{L}_{\mathrm{mat}} \right]$$

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usual GR term

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$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2\kappa} R + \frac{\ell}{12} \theta R \tilde{R} - \frac{1}{2} (\partial \theta)^2 - V(\theta) + \mathcal{L}_{mat} \right]$$

usual GR term
Chern-Simons term
$$R \tilde{R} \equiv \frac{1}{2} \epsilon_{\sigma\tau\gamma\delta} R^{\beta}_{\ \alpha}{}^{\gamma\delta} R^{\alpha}_{\ \beta}{}^{\sigma\tau}$$

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* Chern-Simons gravity is defined by the action

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Chern-Simons (CS) gravity: motavations

- * <u>Higher energy (curvature)</u> correction to EH Lagrangian
- * May produce interesting effects (parity violation) in the CMB/ gravitational-wave detection
- Produces lepton number current anomaly which may lead to matter-antimatter asymmetry
- Lue, Wang, and Kamionkowski (1999) Jackiw and Pi (2003) Seto (2006)

Alexander, Peskin, and Sheikh-Jabbari (2006)

- It is a 'natural' consequence of the effective 4D string action Green and Schwartz (1985) Campbell et al. (1991)
- * As we did before, we ask whether local tests of gravity can detect effects of CS gravity

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Linearization of CS gravity

* Linearizing the equations about a flat metric:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

 $|h_{\mu\nu}| \ll 1$

* Define the trace-reversed metric perturbation

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$$

* Specialize to a gauge where

$$\partial^{\mu}\bar{h}_{\mu\nu}=0$$

* Consider the case where $\theta(t)$ and we neglect $\ddot{\theta}(t)$

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Gravito-magnetism

* We can write the field equations in analogy to electromagnetism:

Vector potential: $A_{\mu} \equiv -\frac{1}{4}\bar{h}_{\mu 0}$ Four-current: $J_{\mu} \equiv -T_{\mu 0}$ Gravito-electric field: $E^i \equiv \partial_i A_0 - \partial_0 A_i$ Gravito-magnetic field: $B^i \equiv \epsilon^{0ijk} \partial_j A_k$

Gravito-magnetism

* We can write the field equations in analogy to electromagnetism:

 $\begin{array}{lll} \mbox{Gauss' Law:} & \vec{\nabla} \cdot \vec{E} = 4\pi G\rho \\ \mbox{Faraday's Law:} & \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \mbox{No gravito-magnetic} & \vec{\nabla} \cdot \vec{B} = 0 \\ \mbox{monopoles:} & \vec{\nabla} \cdot \vec{B} = 0 \\ \mbox{Ampere's Law:} & \vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} - \frac{1}{m_{\rm cs}} \vec{B} = 4\pi G \vec{J} \end{array}$

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Lorentz force law

* Besides have analogous field equations

 $\begin{array}{lll} \mbox{Gauss' Law:} & \vec{\nabla}\cdot\vec{E}=4\pi G\rho \\ \mbox{Faraday's Law:} & \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \\ \mbox{No gravito-magnetic} & \vec{\nabla}\cdot\vec{B}=0 \\ \mbox{monopoles:} & \vec{\nabla}\times\vec{B}-\frac{\partial\vec{E}}{\partial t}-\frac{1}{m_{\rm cs}}\Box\vec{B}=4\pi G\vec{J} \end{array}$

* The geodesic equation can be written as a Lorentz force law:

$$\vec{a} = -\vec{E} - 4\vec{v} \times \vec{B}$$

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Only Ampere's law is changed



* <u>Only</u> Ampere's law is altered

- * To look for an effect of CS gravity we need to produce a gravito-magnetic field
- * Where are we going to find a mass current to generate a gravito-magnetic field?

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The rotating earth

Credit: NASA, JPL, Doug Ellison

The rotating earth and CS gravity

* The rotation of the earth generates the mass current

$$\vec{J} = \rho \vec{\omega} \times \vec{r} \Theta (R_{\oplus} - r)$$
$$= \rho r \omega \Theta (R_{\oplus} - r) \hat{\phi}$$



$$\vec{\nabla} \times \vec{B} - \frac{1}{m_{\rm cs}} \nabla^2 \vec{B} = 4\pi G \vec{J}$$

Tristan Smith, Caltech

 $\vec{()}$

The rotating earth and CS gravity



The rotating earth and CS gravity



Seeing the gravito-magentic field

In gravito-magnetism motion of a satellite is dominated by the usual Newtonian force:



small perturbing force

LAGEOS

* Perturb about a Keplerian orbit

 Look at perturbed motion that builds up in time (secular motion)

LAGEOS I & II

LAGEOS I & II are two satellites with several retroreflectors orbiting the earth

LAGEOS II



Launched 1974 and 1992

* Laser ranging measures their orbits very accurately www.signale.de/lageos

LAGEOS I

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LAGEOS I & II



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Gravity Probe-B

* Gravity Probe-B (GPB) measures precession of gyroscopes due to gravito-magnetic field



Gravity Probe-B

* If GPB is able to confirm the GR result to 10%



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Final limits to m_{cs}

* Current observations place the limit

$$m_{cs} \ge 0.001 \text{ km}^{-1} = 10^{-22} \text{ GeV}$$

* Future observations may improve this (GPB) by an order of magnitude

$$m_{cs} \ge 0.01 \text{ km}^{-1} = 10^{-21} \text{ GeV}$$

Double Pulsars!

* Another place to look for gravito-magnetic effects is in double pulsars



Double Pulsars!

- * Another place to look for gravito-magnetic effects is in double pulsars
- More complicated; two sources of mass current: rotation and orbital motion
- Gravito-magnetic field larger by an order of magnitude
- Orbital motion may improve constraints considerably (causes oscillation in semi-major axis)



Duncan Lorimer and Maura McLaughlin

Conclusions

- * First ever observable constraints on the theory
- * Violation of parity may be observable in the CMB/ direct detection of gravitational-waves
- * May participate in the matter anti-matter asymmetry
- * CS gravity is a higher energy modification of GR
- Future work to improve constraints through observations of double pulsars

Dark energy and modifications of gravity

