Note Title 6/7/2013

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Spherical Cows in the Sky with The Fab 4

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based off [arXiv:1310.5058] w/ N. Kaloper

Outline: Feb 4

↳ What is it?

↳ Spherical Solutions & Bounds

↳ Additional Considerations

Gravity

$$S = \frac{M_P^2}{2}\int d^4x \sqrt{g}\, R + \mathcal{L}_m + \Lambda$$

"Everybody agrees curvature is minimized."

diffeomorphism invariant

2 derivative, constructed from metric

HKT Theorem, Weinberg Theorem, ...

GR is the <u>unique</u> diffeomorphism-invariant theory of a spin-2 field

<corollary>

Modifying gravity means

breaking diffeomorphism invariance

OR (and)

introducing new degrees of freedom.

Proof

"~~Everybody~~ agrees curvature is minimized."

Some people

Diffs broken

\+ other stuff

"Everybody agrees curvature is minimized."

New DOFs

Horndeski

↳ Most general ALLOWABLE scalar – tensor theory

$$\mathcal{L} = K(\phi, X)$$

$$+ G_3(\phi, X)\Box\phi$$

$$+ G_4(\phi, X) R + G_{4,X}\left(\Box\phi^2 - \phi_{;\mu\nu}^2\right)$$

$$+ G_5(\phi, X) G_{\mu\nu}\phi^{\mu}\phi^{\nu} - \frac{G_{5,X}}{6}\left(\Box\phi^3 - 3\Box\phi\,\phi_{;\mu\nu}^2 + 2\phi_{;\mu\nu}^3\right)$$

↳ 4 functions of ϕ & X

$X = \frac{1}{2}\partial\phi^2$

Fab 4

$\boxed{\begin{array}{l} \hookrightarrow \text{Admits Minkowski solutions } \forall \Lambda \\ \hookrightarrow \text{Even through phase transitions} \end{array}}$

$$S = \int d^4x \sqrt{g} \; V_1(\phi) R + V_{KB}(\phi) G_{\mu\nu} \partial^\mu \phi \partial^\nu \phi + V_{GB}(\phi) \widetilde{GB}$$

$$+ V_H(\phi) \epsilon^2 \left(R_{\alpha\beta\gamma\delta}, \partial_\alpha \partial_\beta \phi, \partial_\alpha \phi \partial_\beta \phi \right) + \Lambda$$

Why?

↳ Each term posesses

LINEAR galilean sym.

$$\phi \to \phi + b_\mu x^\mu + c$$

↳ Evades Weinberg's NO GO theorem

↳ Integrate out ϕ for nonlocal theory

Why Not?

↳ Irrel. ops. $\propto \frac{1}{H_0}$

↳ Other Horndeskis $\propto \frac{1}{\Lambda}$?

NOT an EFT

Solar system

↳ 5th forces

- Equivalence Principle Violations $\qquad r_V = (r_S L^2)^{1/3} > 100\,AU$
- Anomalous Precession
- etc.

↳ Strong Coupling $\qquad r_Q = (\ell_p L^2)^{1/3} < mm$

$$\boxed{\mathcal{L} = -\partial\phi^2 - L^3\partial\phi^2\Box\phi - L^7\partial\phi^2\Box\phi + \ldots}$$

Solar system

$$V_{\text{Brans-Dicke}}(\phi) \longrightarrow e^{\phi} + \text{conformal xfm } \bar{g}_{\mu\nu} \rightarrow e^{\phi} g_{\mu\nu}$$

$$S = M_P^2 \int d^4x \sqrt{g}\, R - \partial\phi^2 + \phi T_\mu{}^\mu$$

$$+ \tilde{V}_{KB}(\phi) \left(G_{\mu\nu} \phi^\mu \phi^\nu + \epsilon^2 (\partial^2\phi, \partial\phi^2) + \partial\phi^4 \right)$$

$$+ \tilde{V}_{GB}(\phi) \left(GB + G_{\mu\nu} (\phi^\mu \phi^\nu + \phi^{\mu\nu}) + \epsilon^2 (\partial^2\phi^2) \right)$$

$$+ \tilde{V}_H(\phi) \left(\epsilon^2 (R, \partial^2\phi, \partial\phi^2) + \epsilon^2 (\partial\phi^2, \partial^2\phi^2) - \partial\phi^2 \epsilon^2 (\partial\phi^2, \partial^2\phi) \right)$$

3 Simplifications

↳ Schwarzschild Metric, radial solutions

↳ $V_{KB}(\phi) \to L_{KB}^2$ $\quad V_{GB}(\phi) \to L_{GB}^3\phi$ $\quad V_H(\phi) \to L_H^4$

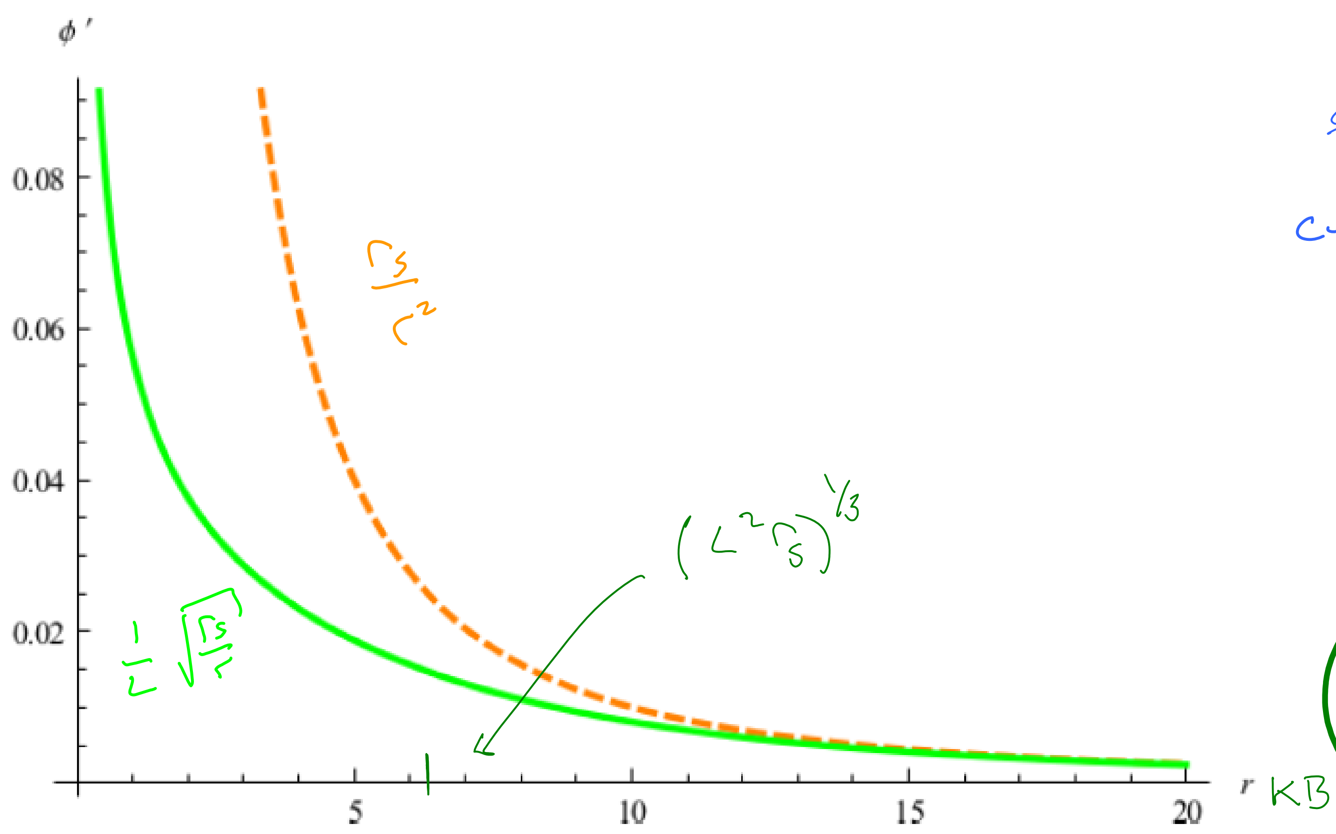
↳ discard $\mathcal{O}\left(\frac{r_s}{r}\right)$ (recovers Galilean sym ✓)

$$\phi' + L_{KB}^2\left(\frac{13}{r}\phi'^2 + 12\phi'^3\right) + L_{GB}^2\left(\frac{12}{r}\phi'^2 + 8\phi'^3 - \frac{r_s^2}{r^5}\right)$$

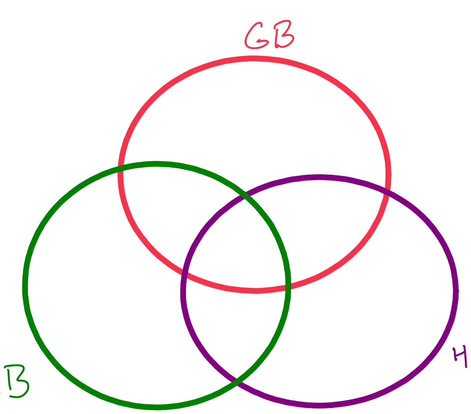
$$+ L_H^4\left(\frac{3r_s}{r^4}\phi'^2 + \frac{8}{r^2}\phi'^3 - \frac{10}{r}\phi'^4\right) = \frac{r_s}{r^2}$$

Kinetic Braiding

$$\boxed{\phi' - L_{KB}^2\left(\frac{12}{r}\phi'^2 + 12\phi'^3\right) = \frac{r_S}{r^2}}$$

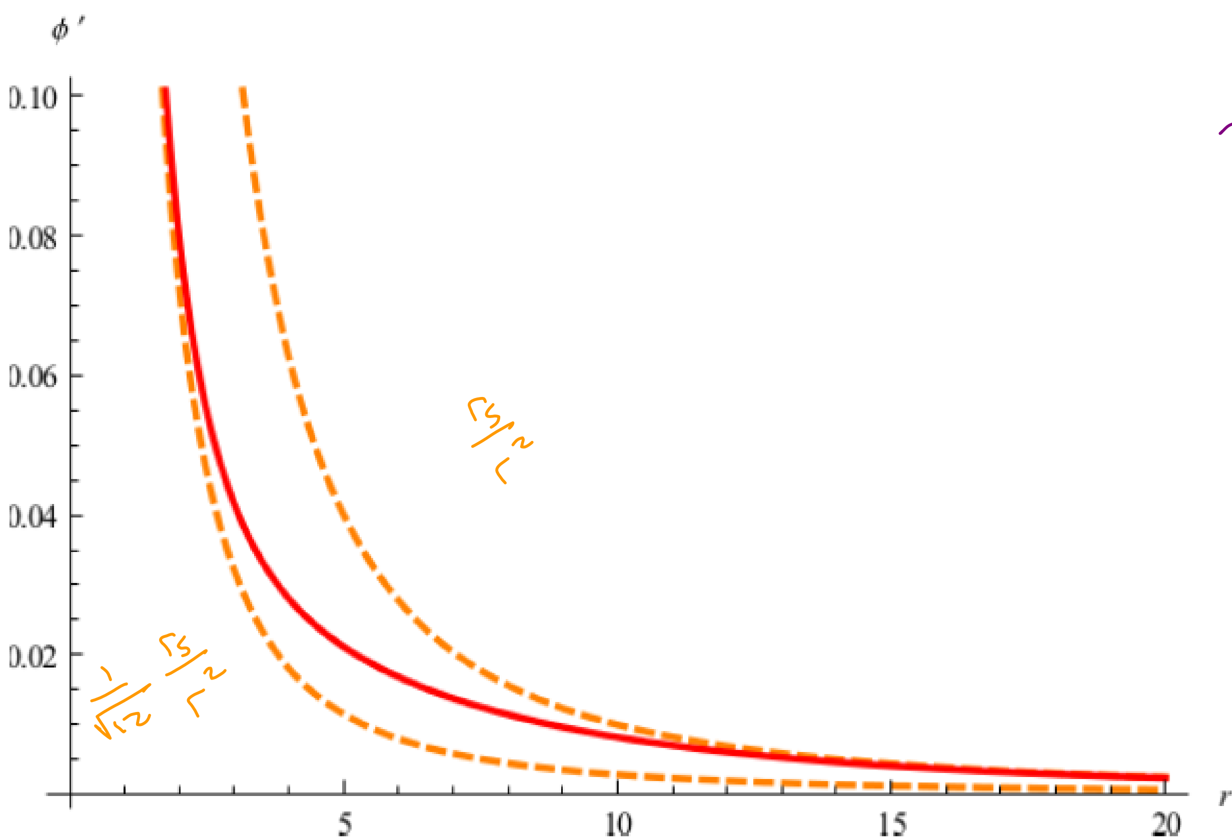


similar to cubic galileon



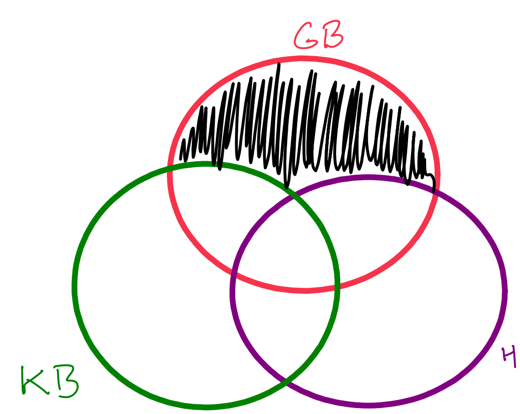
Gauss-Bonnet

$$\boxed{\phi' + L_{GB}^2\left(\frac{12}{r}\phi'^2 + 8\phi'^3 - \frac{r_s^2}{r^5}\right) = \frac{r_s}{r^2}}$$



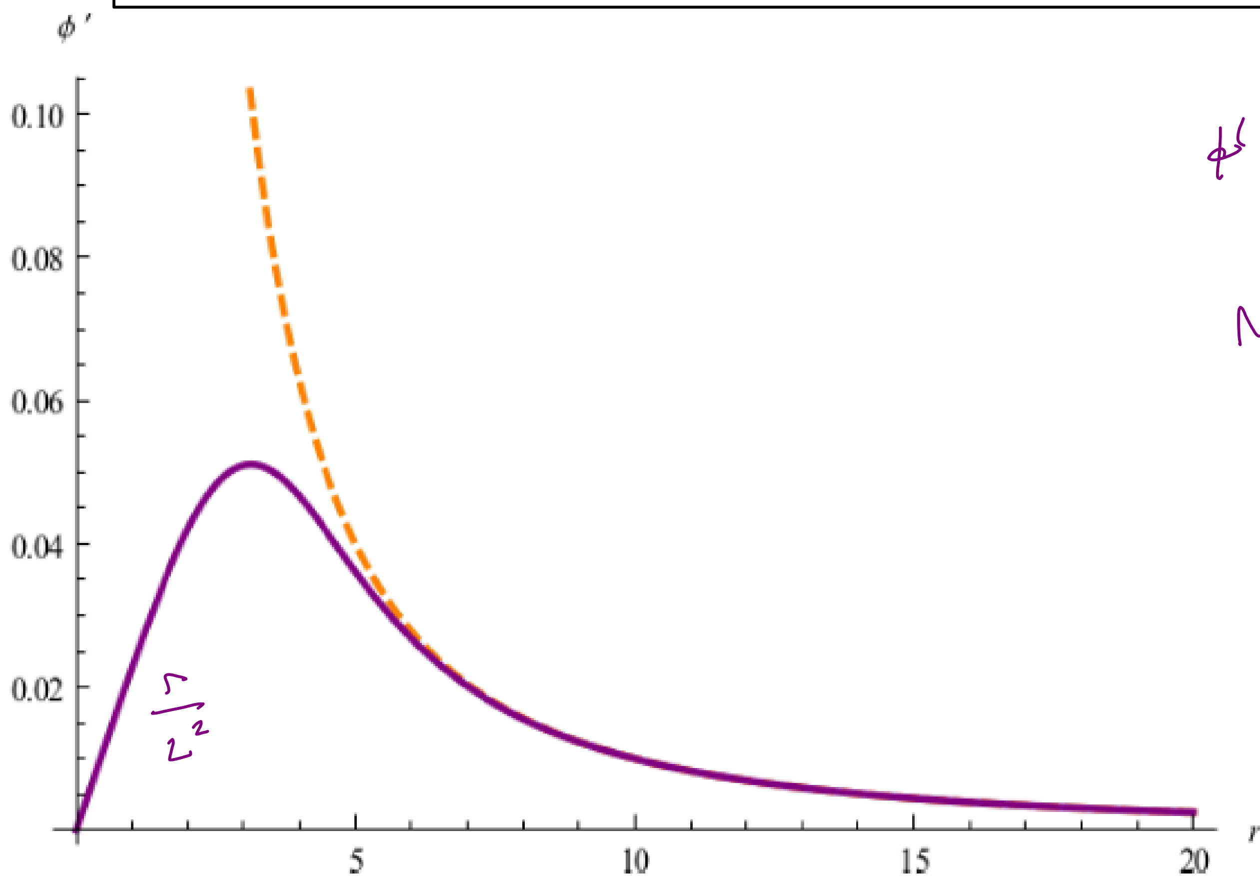
$\sim \frac{r_s}{r^2} \quad \forall r$!

No Vainshteining!



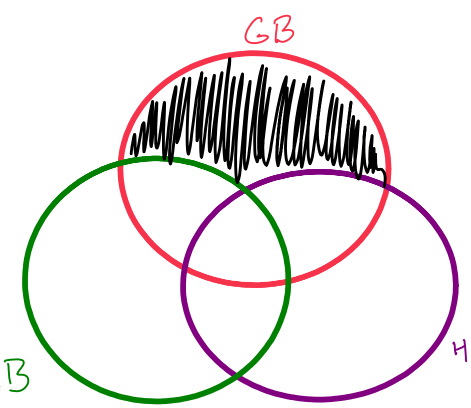
Horndeski

$$\boxed{\phi' + L_H^4\left(\frac{3r_s}{r^4}\phi'^2 + \frac{8}{r^2}\phi'^3 - \frac{10}{r}\phi'^4\right) = \frac{r_s}{r^2}}$$



$\phi' \propto \frac{r}{L^2}$

No precession!

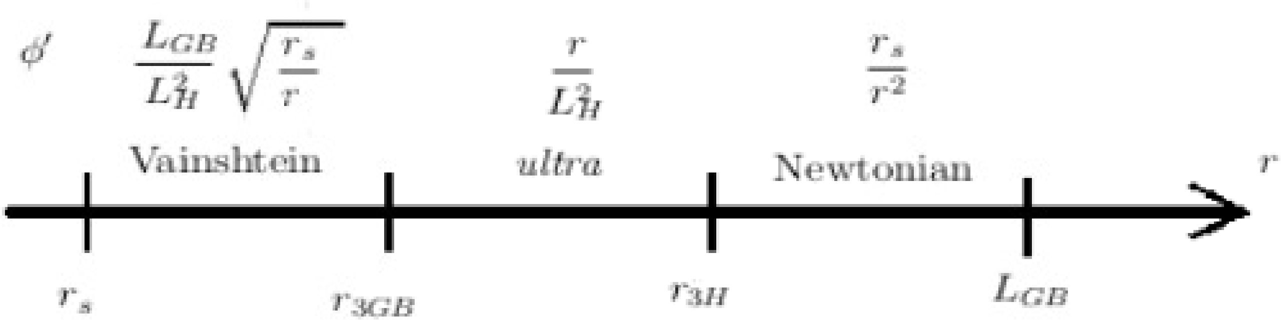


Combined

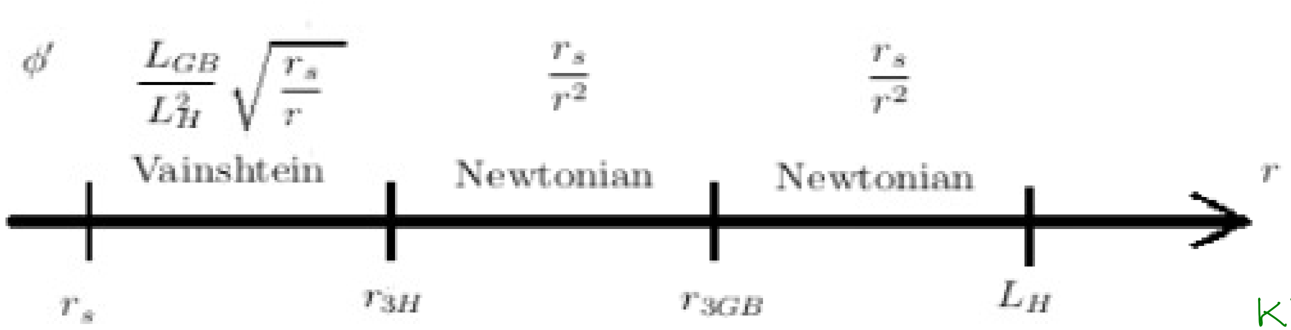
KB subdominant

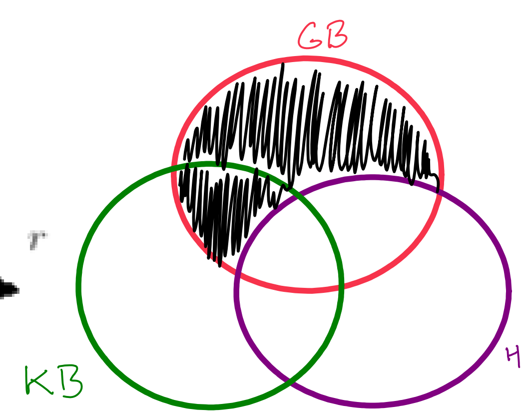
GB+H $\longrightarrow$ business as usual

$L_H > L_{GB}$



$L_{GB} > L_H$





Validity

$$\mathcal{L} = \sum_{nk} L_{nk}^{2k} \phi^n \partial\phi^{2k} \approx \phi^{\text{smallest}} \partial\phi^{\text{smallest}}$$

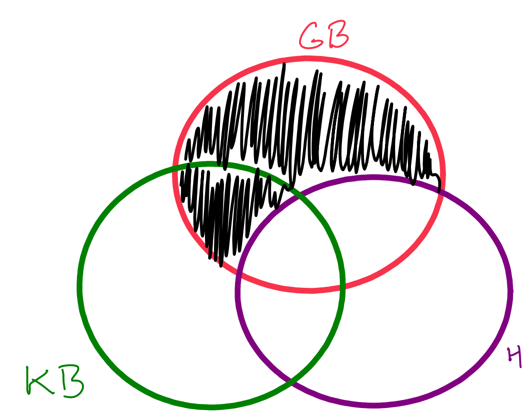
$$\phi < \frac{r_s}{r} < 1$$

want: $L_{n\,k+1}^{2k+2} \partial\phi^{2k+2} << L_{nk}^{2k} \partial\phi^{2k}$

$\frac{r_s}{r^2}$: invalid!

$\frac{1}{L}\sqrt{\frac{r_s}{r}}$: valid!

$\frac{r}{L^2}$: valid if $\frac{L_{k+1}^{k+1}}{L_k^k} << r_s$



Schwarzschild

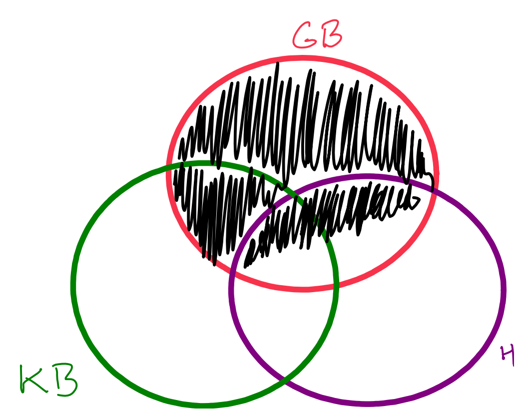
$$M_p^2 \left(1 + L_H^4 \phi'^2 \phi''\right) \Delta \Psi = \phi'^2 + L_H^4 \phi'^2 \phi'' + \rho_{matter}$$

want to be small

Only an issue if L_{GB} & $L_H \neq 0$

↳ Neutron stars

$L_{GB}^3 \lesssim L_H^2$ km



Bounds

↳ Lunar Laser Ranging $\delta\theta < 2\cdot 10^{-11} / \text{orbit}$

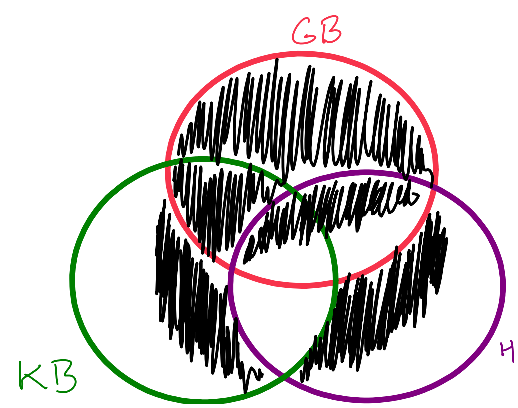
$L_{GB}^3 \lesssim L_H^2 \times .1\,\text{km}$

↳ Uranus's orbit $\left| \frac{M_{\odot\uparrow}}{M_\oplus} - 1 \right| < 10^{-6}$

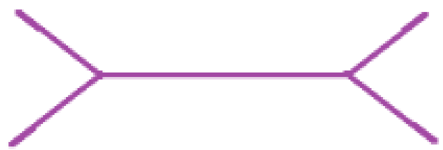
$L_H \gtrsim 10^7\,\text{AU}$

↳ $r_V > 100\,\text{AU}$

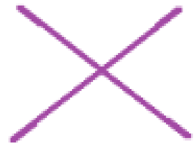
$L_{KB} \gtrsim 10^7\,\text{AU}$



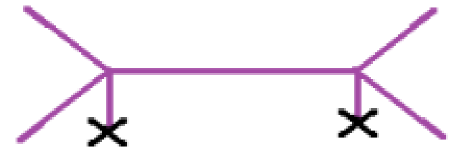
Strong Coupling



$\sim r_{Q_1}^6\, p^6$



$\sim r_{Q_2}^4\, p^4$



$\sim r_{Q_3}^4\, p^4$

$$\boxed{2 \to 2 \text{ amplitude} > 1 \implies \text{strong coupling}}$$

$$r_Q = \max\left\{ \frac{L_{KB}^{4/3}\, l_P^{1/3}}{\sqrt{z}},\ \frac{L_H^{4/3}\, l_P^{1/3}}{z^{1/3}},\ \frac{L_4^{4/3}\, r_s^{1/3}\, l_P^{1/3}}{r\, z^{1/2}} \right\}$$

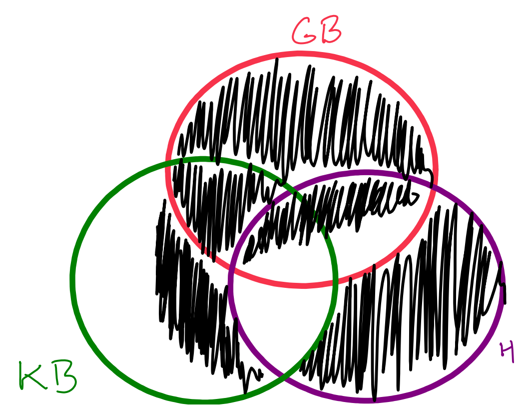
Strong Coupling

$$\mathcal{L}^{(2)} = -\frac{1}{2}\left(1 + Z_{KB} + Z_{GB} + Z_H\right)\partial\phi^2$$

$$\boxed{\begin{aligned} Z_{KB} &\sim L_{KB}^2 \frac{\phi'}{r} \\ Z_{GB} &\sim L_{GB}^2 \frac{\phi'}{r} \\ Z_H &\sim \left(L_H^2 \frac{\phi'}{r}\right)^2 \end{aligned}}$$

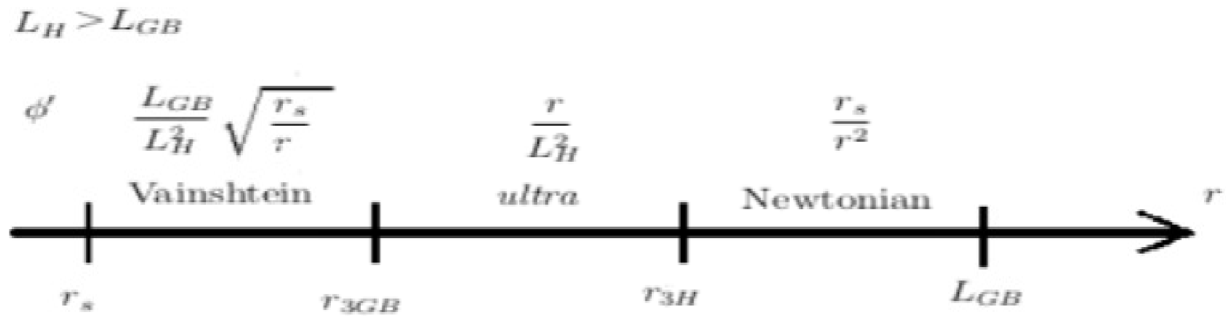
$\rightarrow$ $\boxed{\text{KB only: } Z \sim \left(\frac{r_V}{r}\right)^{3/2}}$

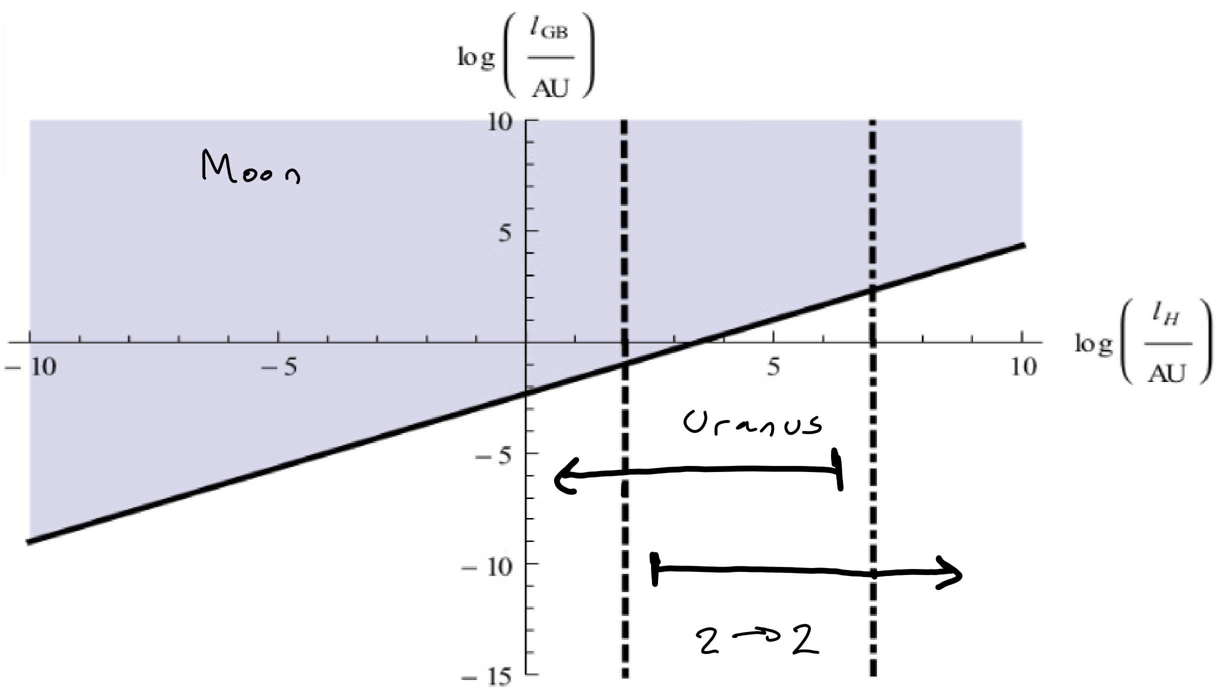
$\rightarrow$ $\boxed{\text{H only: } Z \sim 1}$

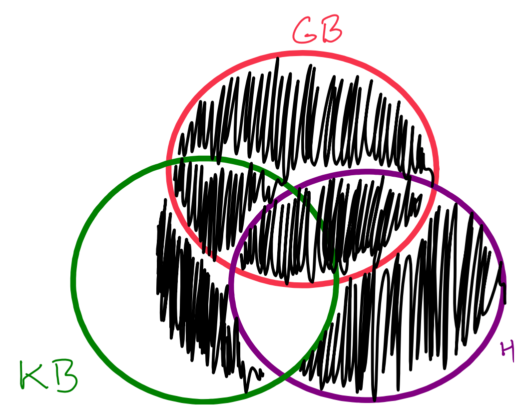


Strong Coupling

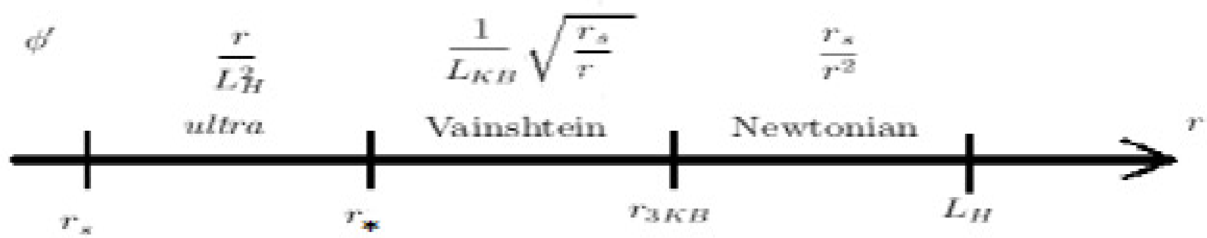
Can completely rule out GB!

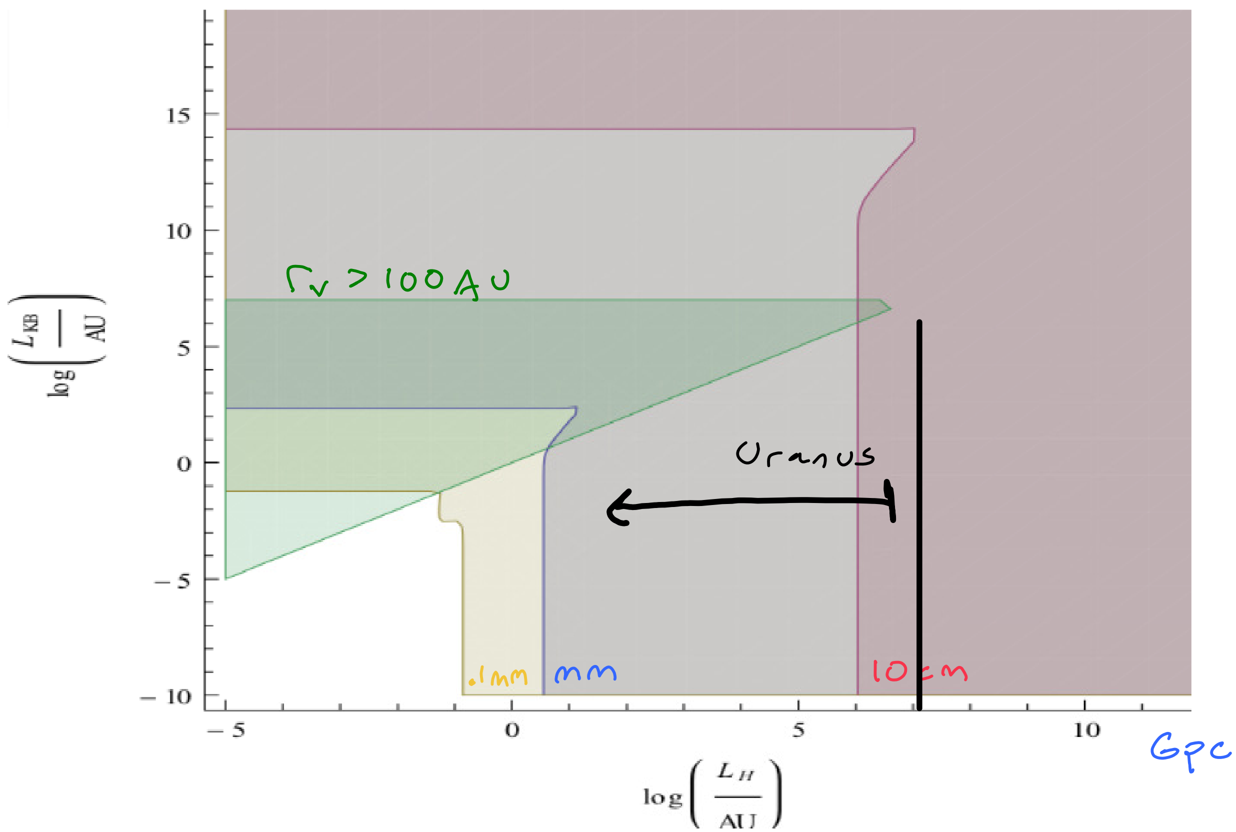




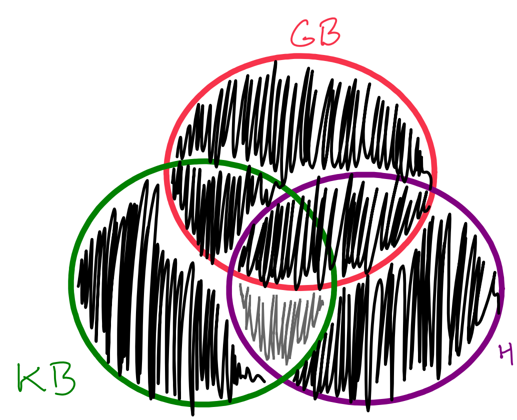


Strong Coupling





$$r_\star = \left(\frac{L_{KI}}{L_{KB}}\right)^{4/3} r_{3KB}$$



5th Fab?

$$\Delta \mathcal{L} = F(G^{\mu\nu} \phi_\mu \phi_\nu)$$

↳ Also 2nd order on cosmological backgrounds

↳ Ghosts elsewhere?

Take $F(X) = \frac{1}{M^{10}} X^2 + \dots \longrightarrow \frac{(\partial\phi^2 \Box \phi)^2}{M^{10}}$

↳ small fluctuations $\phi = \Phi(r) + \delta\phi$

5th Fab?

↳ ghosts develop @ $M_g = \dfrac{M^5}{\max \bar{\phi}'^2} > \Lambda_Q$

↳ $\Delta\mathcal{L}$ does not affect background profile

below crossover scale $\phi' \sim (r_s M_p)^{1/5} M^2$

$M > 10^{16}\Lambda$ to avoid ghosts $\Rightarrow r_{crossover} < r_s$

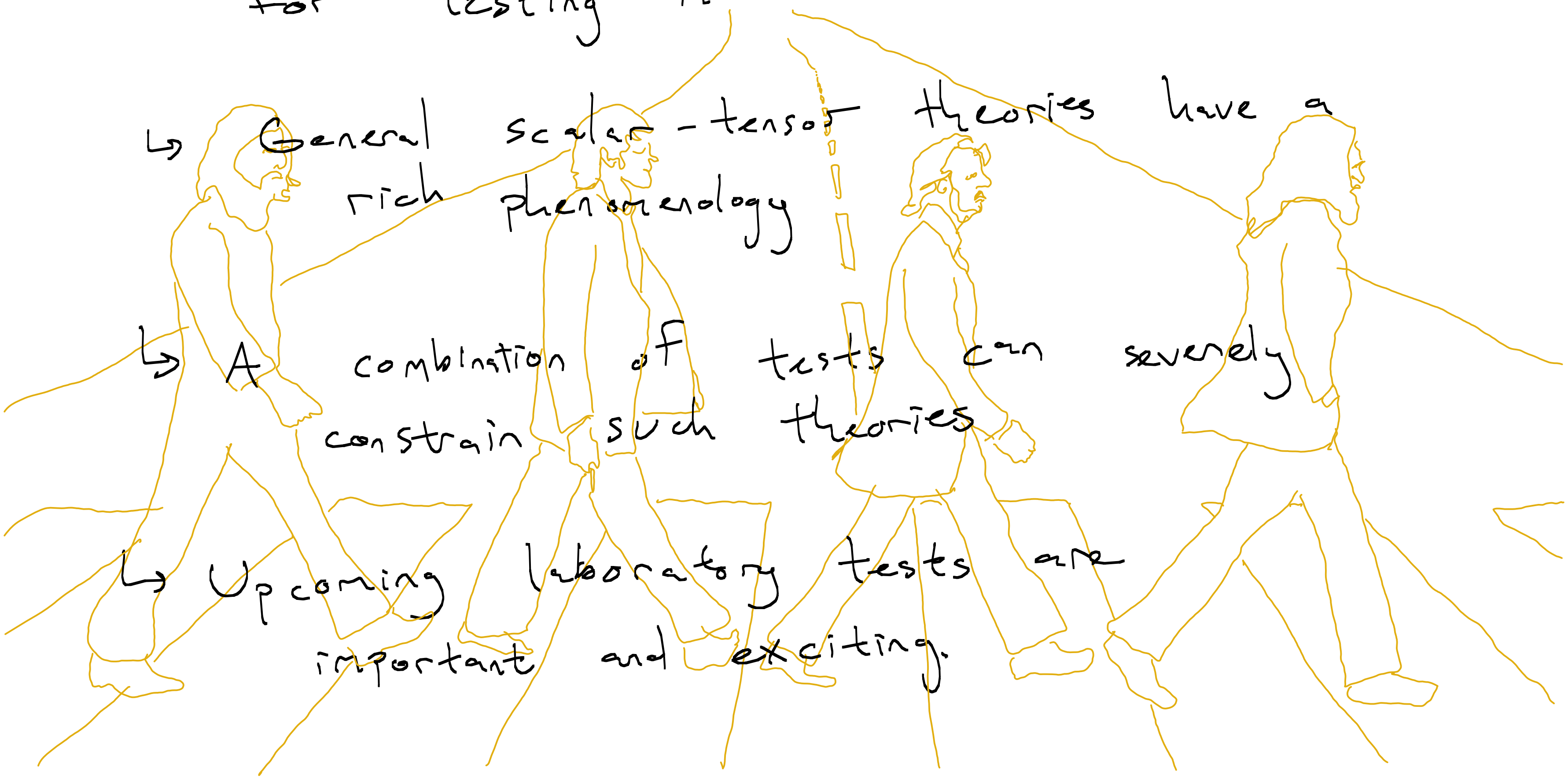
Just 5th Fab: $M > 10^3\,eV$

Vainshtein profile: $M > 10^{-5}\,eV$

ultraVainshtein profile: $M > 10^{-8}\,eV$

Conclusions

- The Solar system remains a good arena for testing modifications to GR
- General scalar-tensor theories have a rich phenomenology
- A combination of tests can severely constrain such theories
- Upcoming laboratory tests are important and exciting.



Where Does It come from?

Integrating out a heavy field?

$$\mathcal{L} = X\left(1 + \frac{\sigma}{\mu} + q\frac{\sigma^2}{\mu^2}\right) - \partial\sigma^2 - M^2\sigma^2 \qquad X = \partial\phi^2$$

$$\mathcal{L}_{eff} \longrightarrow X + \frac{1}{\mu^2 M^2}X^2 + \frac{q}{(\mu^2 M^2)^2}X^3 + \frac{1}{\mu^2 M^4}X\Box X + \cdots$$

Where Does It come from?

Integrating out a heavy field?

$$\mathcal{L} = X\left(1 + \left[\frac{\sigma}{\mu} + q\frac{\sigma^2}{\mu^2}\right) - 2\sigma^2 - M^2\sigma^2\right] \times N \qquad X = 2\phi^2$$

$$\mathcal{L}_{eff} \longrightarrow X + \underbrace{\frac{N}{\mu^2 M^2}}_{\frac{1}{H_0^4}} X^2 + \underbrace{\frac{N q}{(\mu^2 M^2)^2}}_{(mm)^8} X^3 + \underbrace{\frac{N}{\mu^2 M^4}}_{(mm)^6} X \Box X + \ldots$$

$\Rightarrow \quad N \sim 10^{232} \qquad \mu \sim mm^{-1} \qquad M \sim 10^{20} M_p$

+ indices...

Where Does It come from?

KK Lovelock?

$$\int d^7x \sqrt{g_7}\, M_7 \,\mathrm{Lovelock}_3 \rightarrow M_7^{?} \int d^4x\, L_{KT}^4 \left(\mathcal{L}_{KT} + \partial\phi^2 G_{\mu\nu}\phi^\mu\phi^\nu + \dots \right)$$

$$L_{KT}^4 = \frac{\sqrt{3}\, M_7}{M_p^2}$$

← super Hubble

— OR —

← Out of control

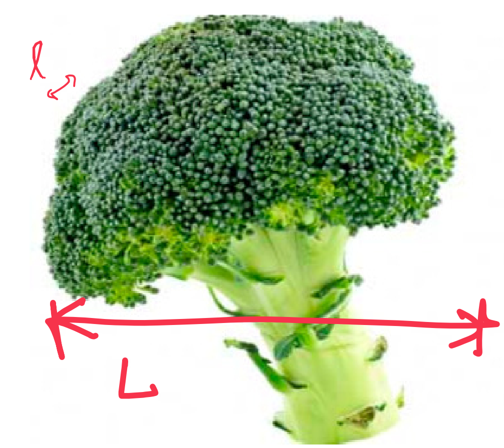
$M_7 \sim 10^{-120}$ GeV!

$8 \rightarrow 4$?

$$M_p^2 L_{KT}^4 = \Omega = \frac{N}{\ell^2} = \left(\frac{L}{\ell}\right)^8 \frac{1}{\ell^2}$$

$\ell = M_{GUT}^{-1}$

$L = $ mm



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