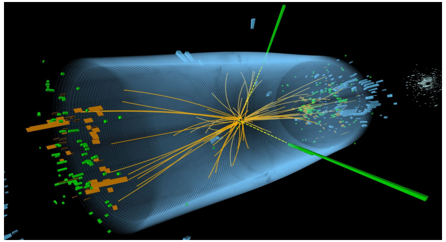
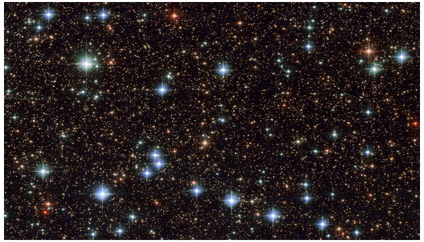
Novel results in perturbation theory for LSS

 $\sim$ 

Henrique Rubira

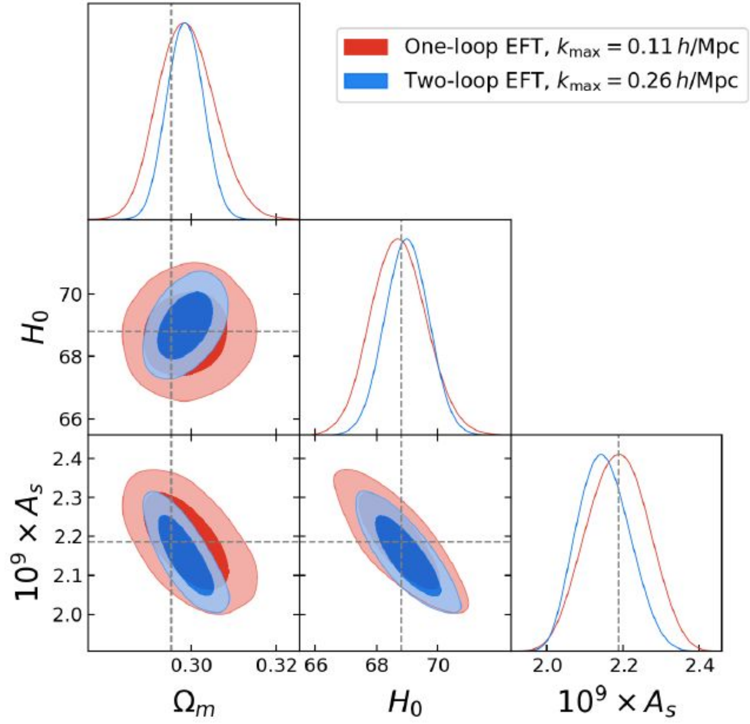
(Cambridge/LMU fellow)

Berkeley, Feb 2026

henrique.rubira@lmu.de

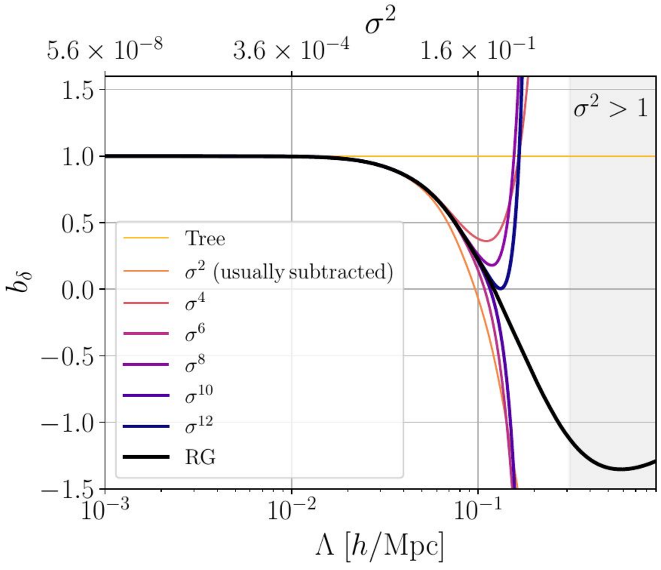
Message to take home

First two-loop MCMC!



Callan-Symanzik equation:

$$\left.\frac{db_a}{d\Lambda}\right|_{1L} = -b_b s_{ba} \frac{d\sigma^2_\Lambda}{d\Lambda} \,,$$



Part I: Introduction

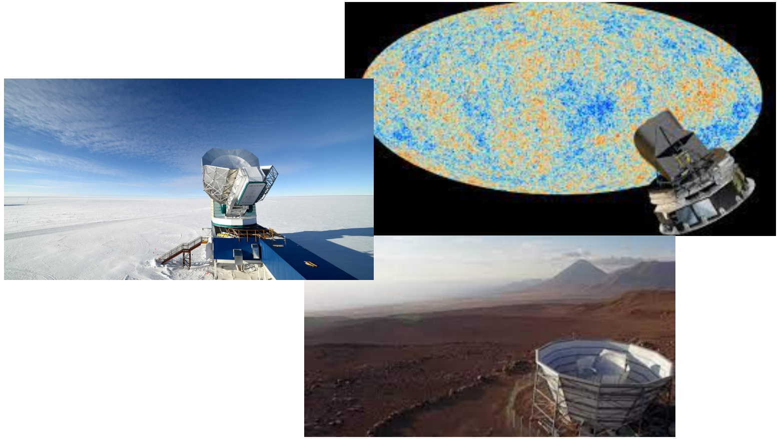
Part II: Towards two-loop EFT

Part III: The renormalization group for LSS

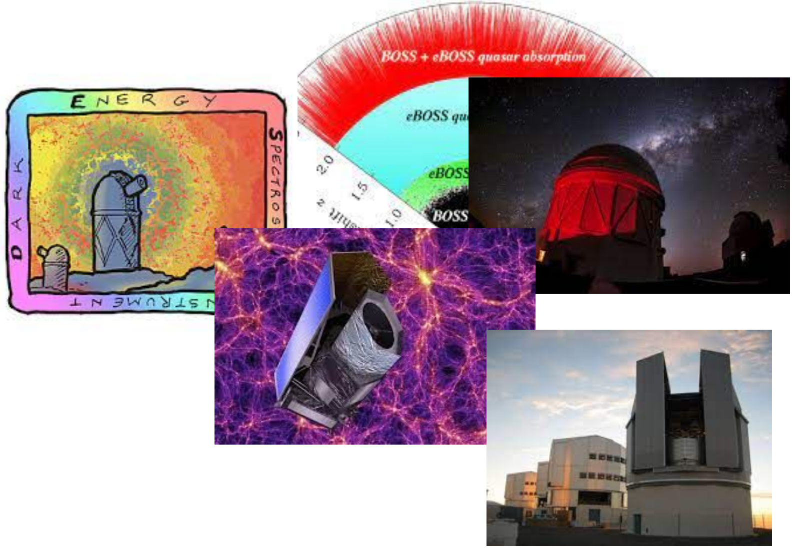
Part IV: Multi-tracer

Cosmology in the era of (subpercent) precise data

CMB
(Planck, ACT, SPT, SO, CMB-S4, Litebird)



LSS
[(e)BOSS, DESI, Euclid, DES, Kids, LSST, SPHEREx]



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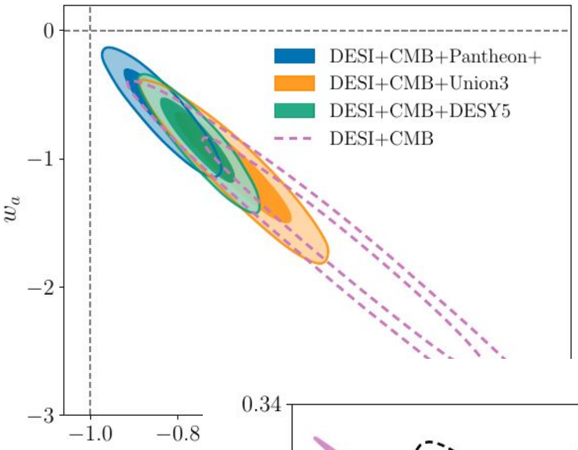
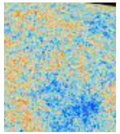
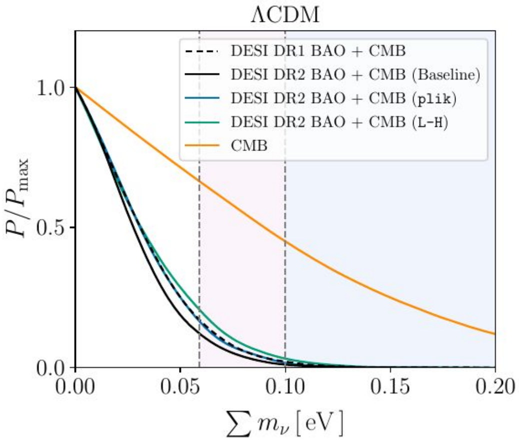
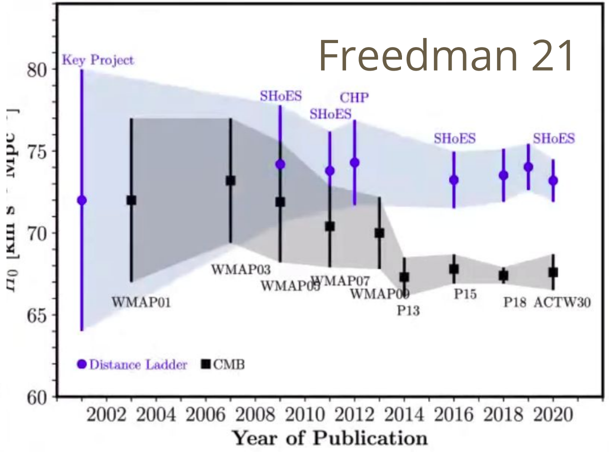
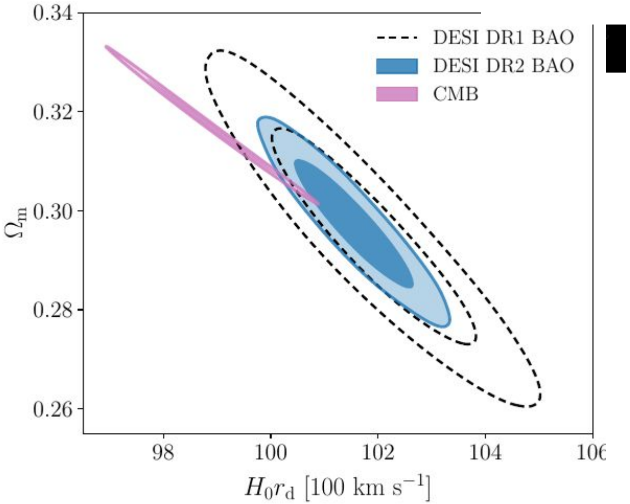


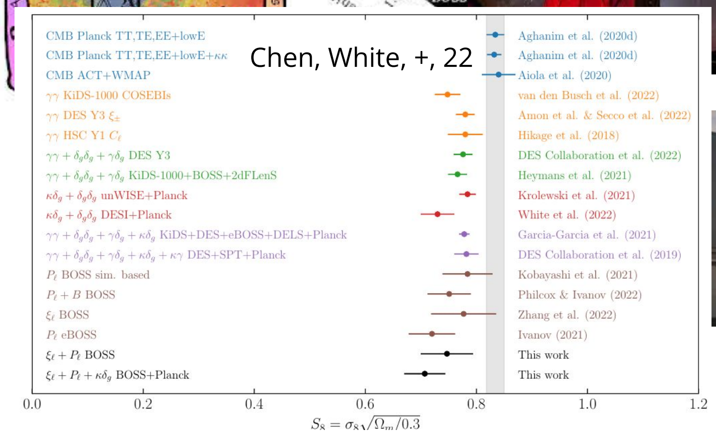
, CMB-S

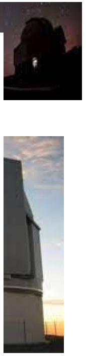












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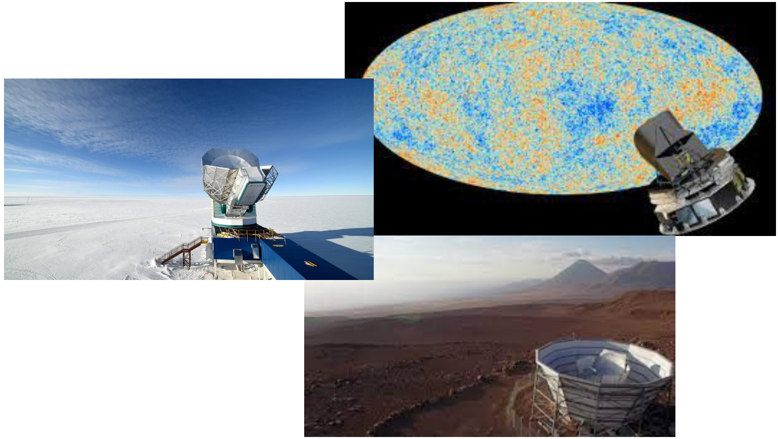

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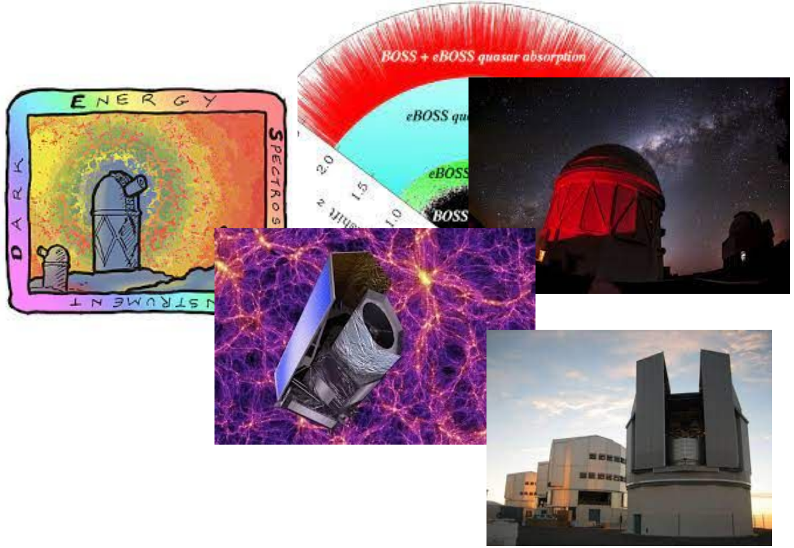


After precise data, how to provide precise theory?

Can theory catch up?

Model-independent **tools** to analyze data.

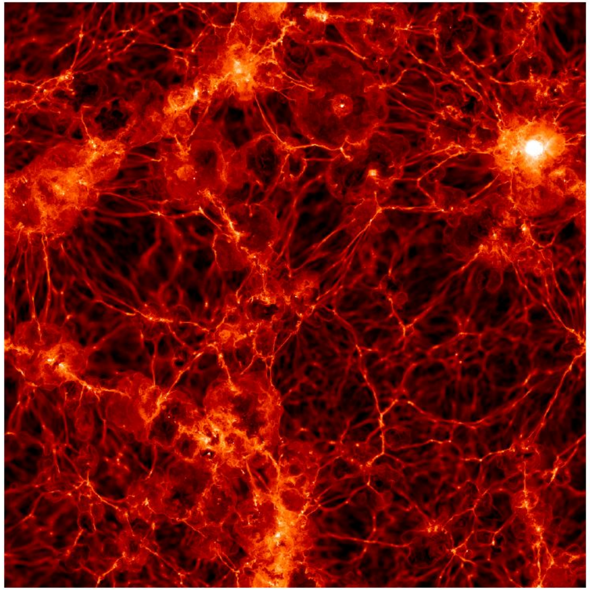
We learnt a lot from HEP analysis in the last 50yrs. Now that cosmology is becoming a precise science, can we import HEP those methods?

QFT to describe n-pt functions

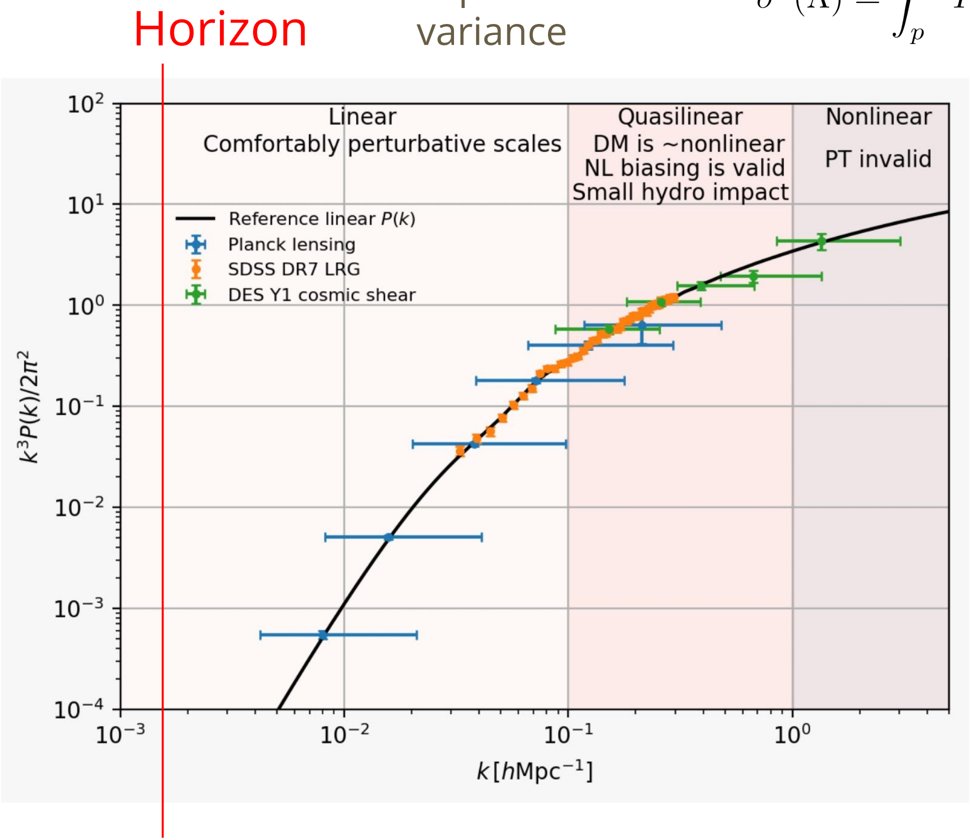
The system and its scales

Perturbative expansion in the variance

$$\sigma^2(\Lambda) = \int_p^{\Lambda} P_{\delta^{(1)}}(p)$$



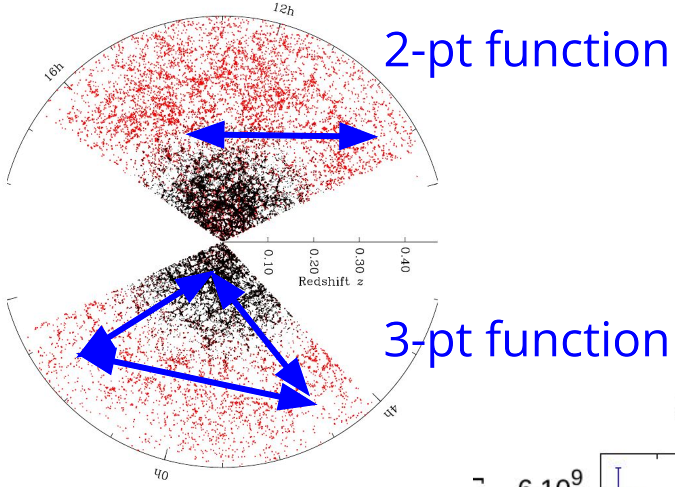
From Illustris simulation, Haiden++15



Credit: Nick Kokron

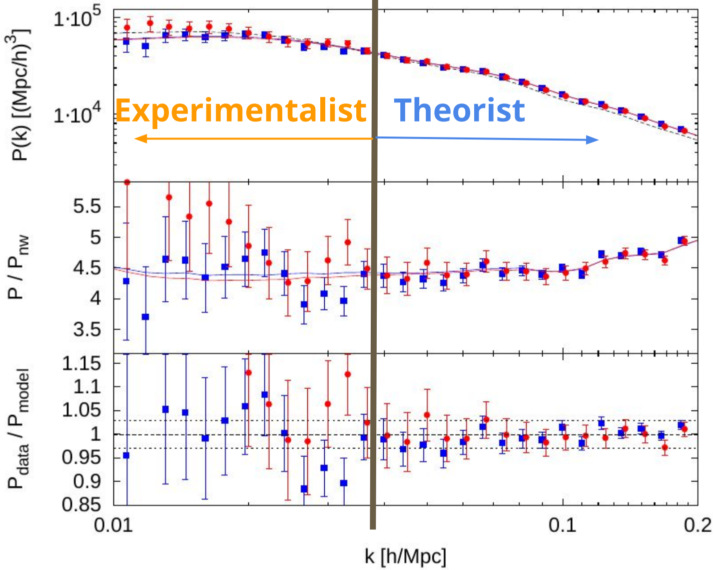
N-pt functions

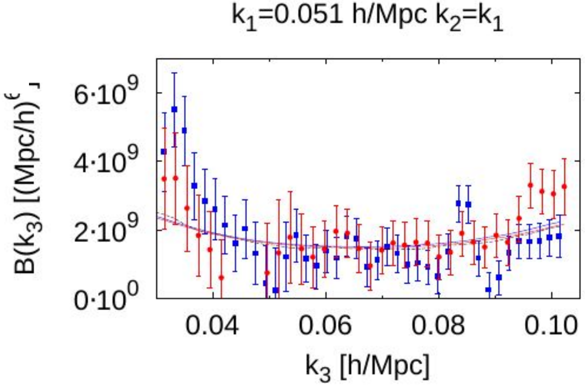
Gil-Marin+, 2014, SDSS BOSS data



Theory (yes)
Data (no)

Theory (no)
Data (yes)





Is there a way to push towards non-linear scales from 'first principles'?

The (smoothed) EoM

Overview on perturbation theory for LSS

$$\partial_\tau \delta + \theta = -\int_{\mathbf{p}_1}\int_{\mathbf{p}_2} \delta_D\left(\mathbf{p_2} - (\mathbf{k} - \mathbf{p}_1)\right)\theta_{\mathbf{p_1}}\delta_{\mathbf{p_2}}\alpha(\mathbf{p_1}, \mathbf{p_2})$$

$$\partial_\tau \theta + \mathcal{H}\theta + \frac{3}{2}\Omega_m \mathcal{H}^2\delta = -\int_{\mathbf{p}_1}\int_{\mathbf{p}_2} \delta_D\left(\mathbf{p_2} - (\mathbf{k} - \mathbf{p}_1)\right)\theta_{\mathbf{p_1}}\theta_{\mathbf{p_2}}\beta(\mathbf{p_1}, \mathbf{p_2})$$

Perturbative solution

$$\boxed{\delta(\mathbf{x},\tau) = \sum_n a^n(\tau)\delta^{(n)}(\mathbf{x})}$$

Time and space factorize when

$$\Omega_m = 1$$

$$\delta^{(n)}(\mathbf{k}) = \int_{\mathbf{q}_{1\ldots n}} \delta_{\rm D}(\mathbf{q}_{1\ldots n} - \mathbf{k})\, F^{(n)}(\mathbf{q}_1, \ldots, \mathbf{q}_n)\, \delta^{(1)}(\mathbf{q}_1)\ldots\delta^{(1)}(\mathbf{q}_n)$$

Baumann, Nicolis, Senatore, Zaldarriaga, Simonovic, Ivanov, Pajer, Baldauf, Philcox, Garny, Vlah, Schmidt, D'Amico, Zhang, Kokron, Wadekar, Chen, Scoccimarro, Lewandowski, White, **HR**, (many others)

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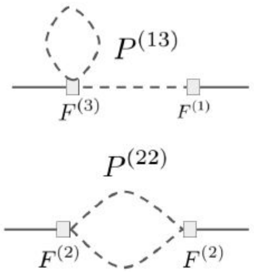
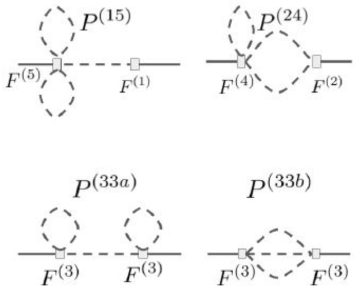
$$\delta^{(n)}(\mathbf{k}) = \int_{\mathbf{q}_{1\ldots n}} \delta_{\mathrm{D}}(\mathbf{q}_{1\ldots n} - \mathbf{k})\, F^{(n)}(\mathbf{q}_1, \ldots, \mathbf{q}_n)\, \delta^{(1)}(\mathbf{q}_1) \ldots \delta^{(1)}(\mathbf{q}_n)$$

1-loop



2-loop



How these loops look like:

$$P^{(13)}_{ab} = 3 \int_p K^{(1)}_a(k) K^{(3)}_b(\mathbf{k}, \mathbf{p}, -\mathbf{p}) P^{\mathrm{lin}}(k) P^{\mathrm{lin}}(p)\,,$$

$$P^{(22)}_{ab} = 2 \int_p K^{(2)}_a(\mathbf{k}-\mathbf{p}, \mathbf{p}) K^{(2)}_b(\mathbf{k}-\mathbf{p}, \mathbf{p}) P^{\mathrm{lin}}(|\mathbf{k}-\mathbf{p}|) P^{\mathrm{lin}}(p)\,,$$

$$P^{(15)}_{ab} = 15 \int_{pq} K^{(1)}_a(k) K^{(5)}_b(\mathbf{k}, \mathbf{p}, -\mathbf{p}, \mathbf{q}, -\mathbf{q}) P^{\mathrm{lin}}(k) P^{\mathrm{lin}}(p) P^{\mathrm{lin}}(q)\,,$$

$$P^{(24)}_{ab} = 12 \int_{pq} K^{(2)}_a(\mathbf{k}-\mathbf{p}, \mathbf{p}) K^{(4)}_b(\mathbf{k}-\mathbf{p}, \mathbf{p}, \mathbf{q}, -\mathbf{q}) P^{\mathrm{lin}}(|\mathbf{k}-\mathbf{p}|) P^{\mathrm{lin}}(p) P^{\mathrm{lin}}(q)\,,$$

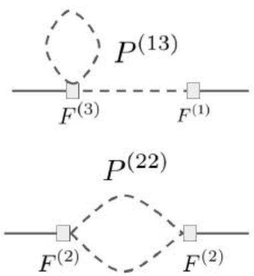
$$P^{(33),I}_{ab} = 9 \int_{pq} K^{(3)}_a(\mathbf{k}, \mathbf{p}, -\mathbf{p}) K^{(3)}_b(\mathbf{k}, \mathbf{q}, -\mathbf{q}) P^{\mathrm{lin}}(k) P^{\mathrm{lin}}(p) P^{\mathrm{lin}}(q)\,,$$

$$P^{(33),II}_{ab} = 6 \int_{pq} K^{(3)}_a(\mathbf{k}-\mathbf{p}-\mathbf{q}, \mathbf{p}, \mathbf{q}) K^{(3)}_b(\mathbf{k}-\mathbf{p}-\mathbf{q}, \mathbf{p}, \mathbf{q}) P^{\mathrm{lin}}(|\mathbf{k}-\mathbf{p}-\mathbf{q}|) P^{\mathrm{lin}}(p) P^{\mathrm{lin}}(q).$$

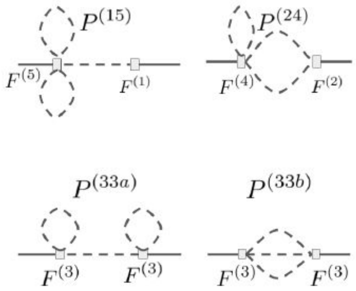
Overview on perturbation theory for LSS

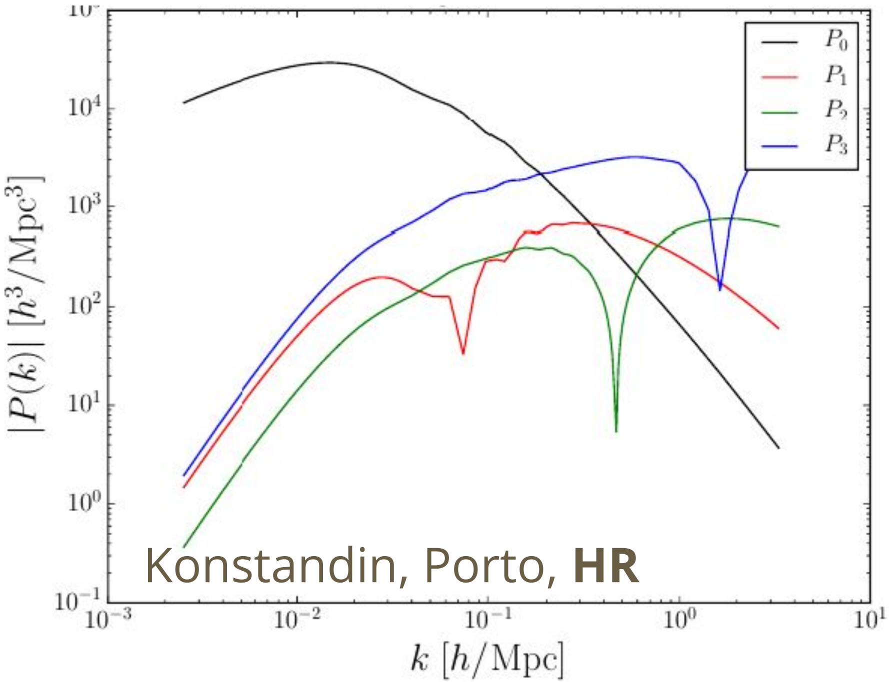
$$\delta^{(n)}(\mathbf{k}) = \int_{\mathbf{q}_{1\ldots n}} \delta_{\mathrm{D}}(\mathbf{q}_{1\ldots n} - \mathbf{k})\, F^{(n)}(\mathbf{q}_1, \ldots, \mathbf{q}_n)\, \delta^{(1)}(\mathbf{q}_1) \ldots \delta^{(1)}(\mathbf{q}_n)$$

1-loop

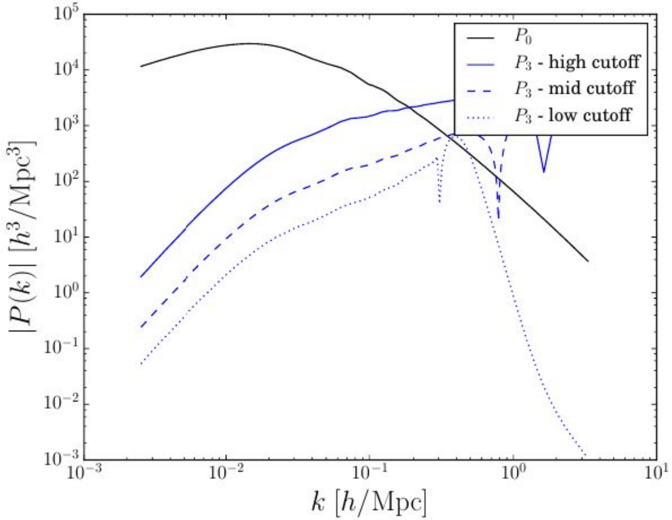


2-loop



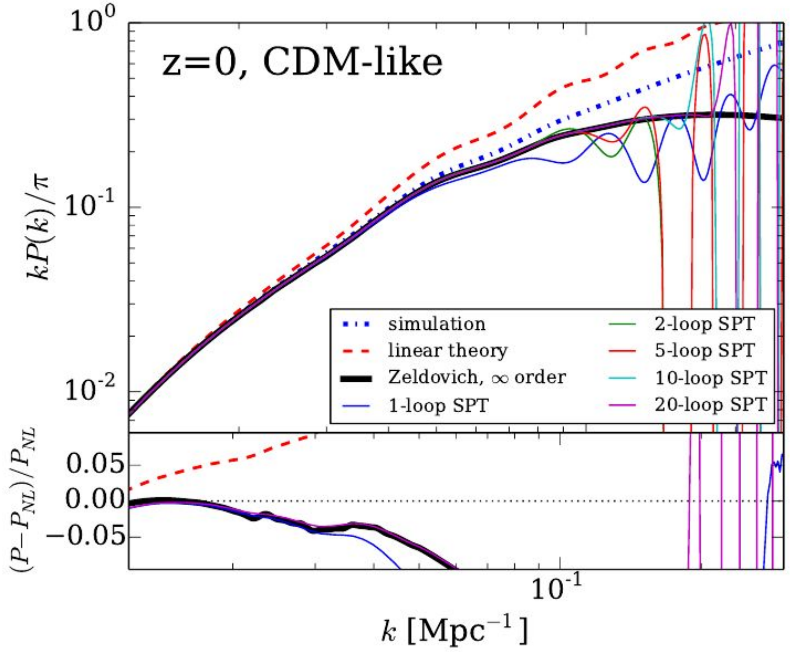


Konstandin, Porto, **HR**



Overview on perturbation theory for LSS

Even if we go to higher-loop orders and it converges, it does not converge to what you want...



McQuinn and White, 15

Overview on perturbation theory for LSS

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$$\partial_\tau \delta + \theta = - \int_{\mathbf{p}_1}^{\Lambda} \int_{\mathbf{p}_2} \delta_D \left(\mathbf{p_2} - (\mathbf{k} - \mathbf{p}_1)\right) \theta_{\mathbf{p_1}} \delta_{\mathbf{p_2}} \alpha(\mathbf{p_1}, \mathbf{p_2})$$

$$\partial_\tau \theta + \mathcal{H}\theta + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta = - \int_{\mathbf{p}_1}^{\Lambda} \int_{\mathbf{p}_2} \delta_D \left(\mathbf{p_2} - (\mathbf{k} - \mathbf{p}_1)\right) \theta_{\mathbf{p_1}} \theta_{\mathbf{p_2}} \beta(\mathbf{p_1}, \mathbf{p_2})$$

\+ small-scale contributions

Perturbative solution

$$\delta(\mathbf{x}, \tau) = \sum_n a^n(\tau) \delta^{(n)}(\mathbf{x})$$

\+ counter-terms (Λ)

$$\delta^{(n)}(\mathbf{k}) = \int_{\mathbf{q}_{1\ldots n}}^{\Lambda} \delta_{\mathrm{D}}(\mathbf{q}_{1\ldots n} - \mathbf{k}) \, F^{(n)}(\mathbf{q}_1, \ldots, \mathbf{q}_n) \, \delta^{(1)}(\mathbf{q}_1) \ldots \delta^{(1)}(\mathbf{q}_n)$$

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Baumann, Nicolis, Senatore, Zaldarriaga, Simonovic, Ivanov, Pajer, Baldauf, Philcox, Garny, Vlah, Schmidt, D'Amico, Zhang, Kokron, Wadekar, Chen, Scoccimarro, Lewandowski, White, **HR**, (many others)

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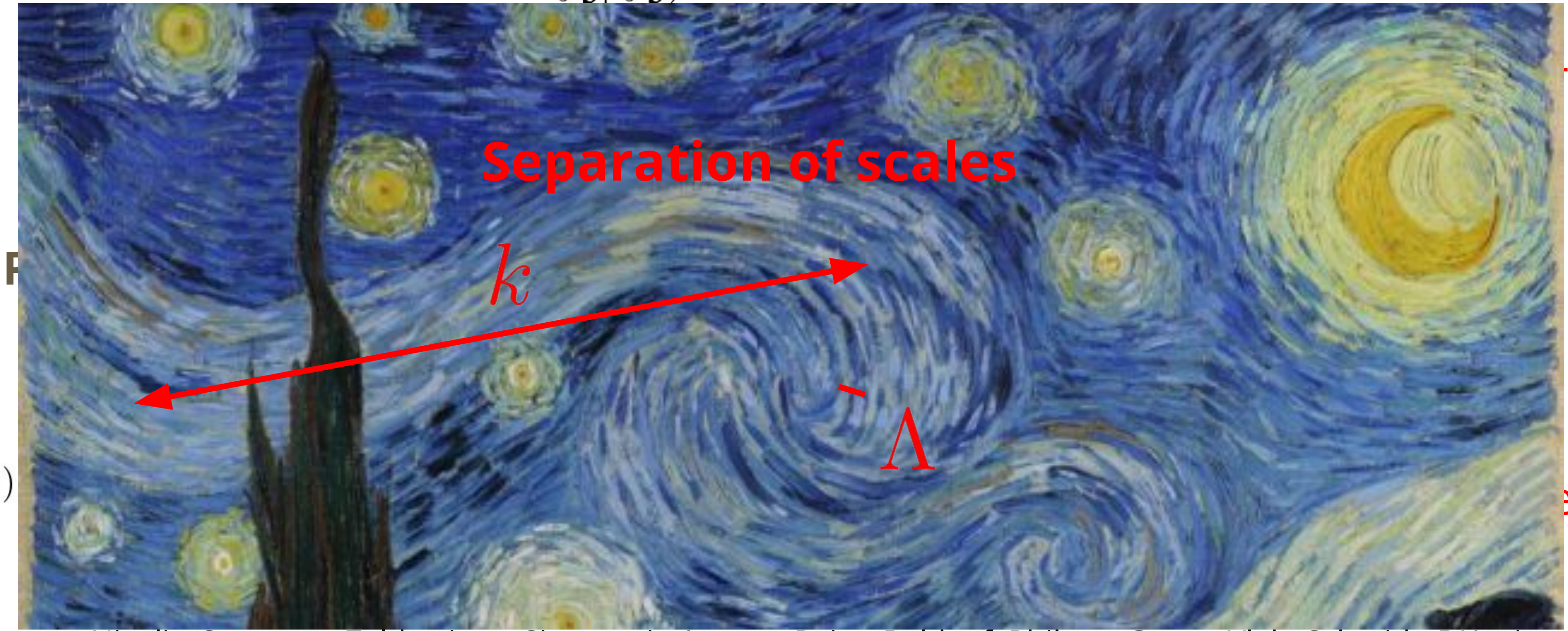


-scale
butions

F

(Λ)

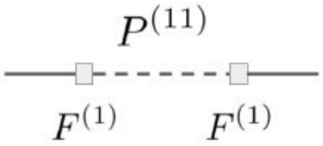
$\delta^{(n)}$

er-terms
(Λ)

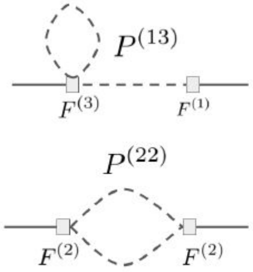
Baumann, Nicolis, Senatore, Zaldarriaga, Simonovic, Ivanov, Pajer, Baldauf, Philcox, Garny, Vlah, Schmidt, D'Amico, Zhang, Kokron, Wadekar, Chen, Scoccimarro, Lewandowski, White, **HR**, (many others)

Overview on perturbation theory for LSS

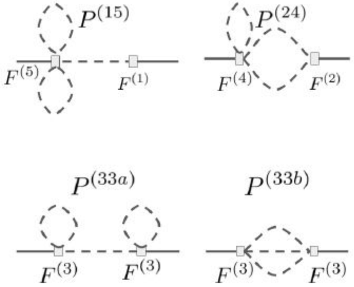
Linear:



1-loop



2-loop



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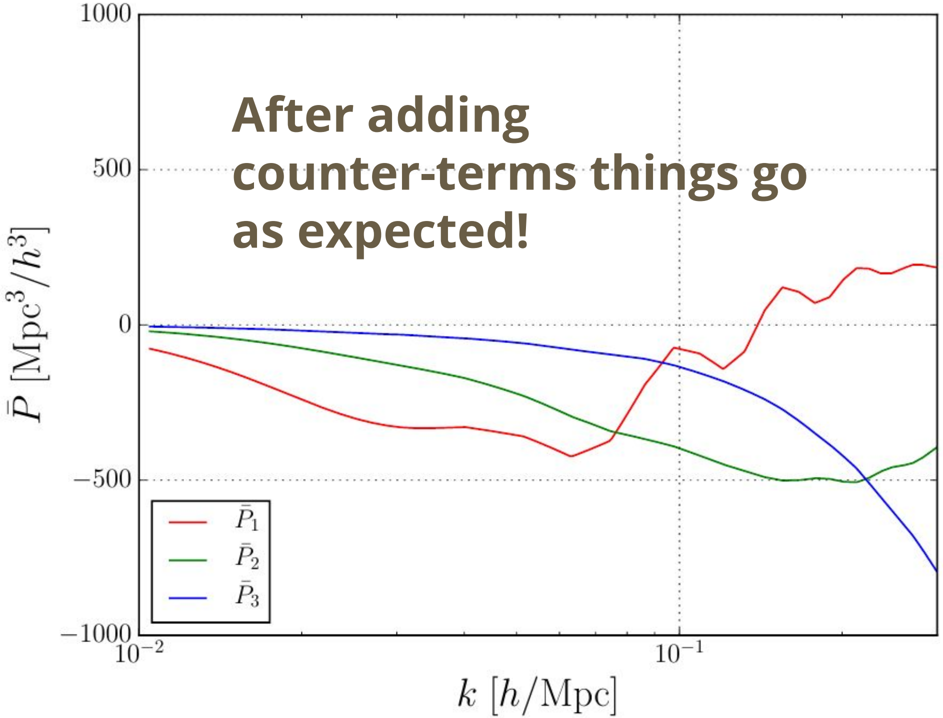
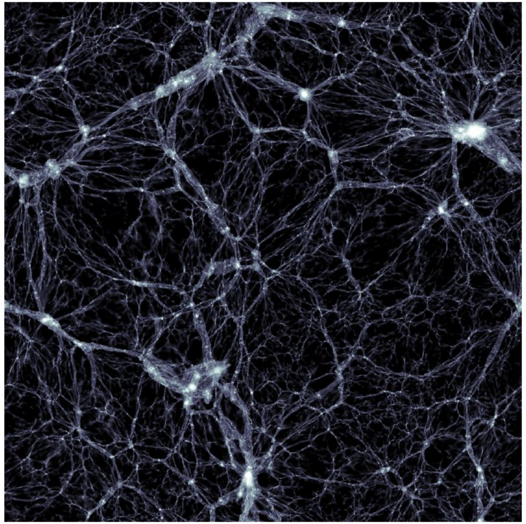


After adding counter-terms things go as expected!

Another important ingredient: bias expansion

The galaxy bias expansion

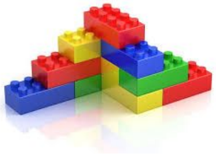
From Illustris simulation,
Haiden, Steinhauser, Vogelsberger,
Genel, Springel, Torrey, Hernquist, 15

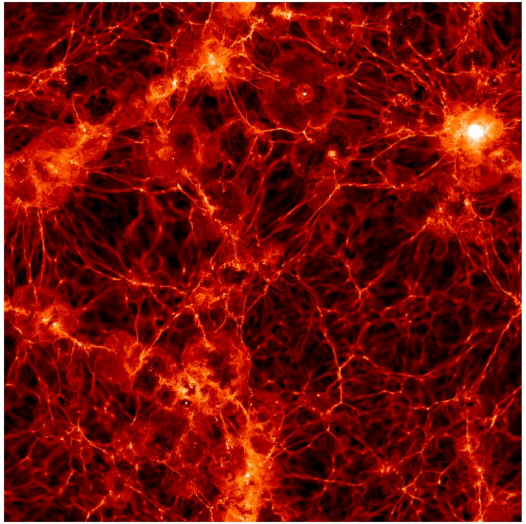


(a) dark matter

Symmetry: Gravity

$$\partial_i \partial_j \Phi_g$$





(b) baryons

Stochastic field

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\,\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

Bias

Bias review: Desjacques, Jeong, Schmidt

The galaxy bias expansion

Important: those are the same parameters for all n-pt functions

In a nutshell, it is an **Operator Product Expansion (OPE)**

$$\delta_{g,\mathrm{det}}(\boldsymbol{x},\tau) = \sum_{O} b_O(\tau) O(\boldsymbol{x},\tau)$$

$$\delta_K^{ij}(\partial_i\partial_j\Phi_g) \propto \delta$$

$$\left[\delta_K^{ij}(\partial_i\partial_j\Phi_g)\right]^2 \propto \delta^2$$

$$(\partial_i\partial_j\Phi_g)(\partial^i\partial^j\Phi_g) \propto \mathcal{G}_2$$

First order:	$\delta\,;$
Second order:	$\delta^2,\ \mathcal{G}_2\,;$
Third order:	$\delta^3,\ \delta\,\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3\,;$

One slide about stochasticity

Rubira and Schmidt 2025

non-minimal

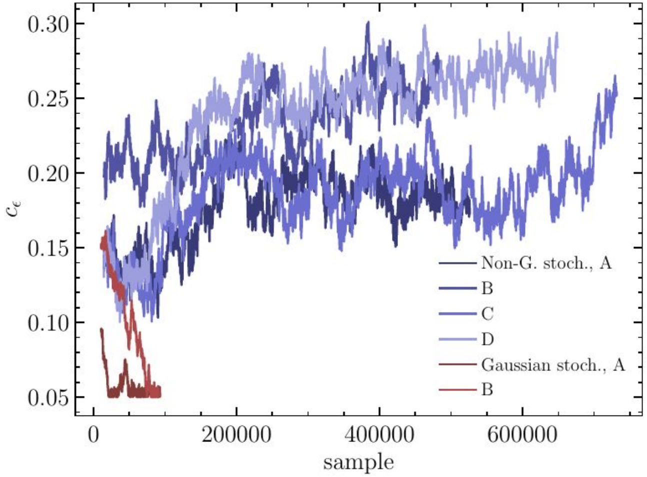
$$\delta_g(\boldsymbol{x},\tau) = \delta_{g,\mathrm{det}} + \epsilon^{\mathrm{n-min}}(\boldsymbol{x},\tau) + \sum_O \epsilon_O^{\mathrm{n-min}}(\boldsymbol{x},\tau) O(\boldsymbol{x},\tau)$$

$$\{\epsilon^{\mathrm{n-min}}, \epsilon_O^{\mathrm{n-min}}\}$$

Gaussian

$$\delta_g(\boldsymbol{x},\tau) = \sum_{m=0}^{\infty} \sum_{\mathbb{1},O} b_O^{\{m\}}(\tau) \left[\epsilon_{\mathrm{G}}(\boldsymbol{x})\right]^m O(\boldsymbol{x},\tau)$$

$$\epsilon_{\mathrm{G}}(\boldsymbol{x}) \sim \mathcal{N}(0,1)$$



One Gaussian field is enough!

Where we stand now

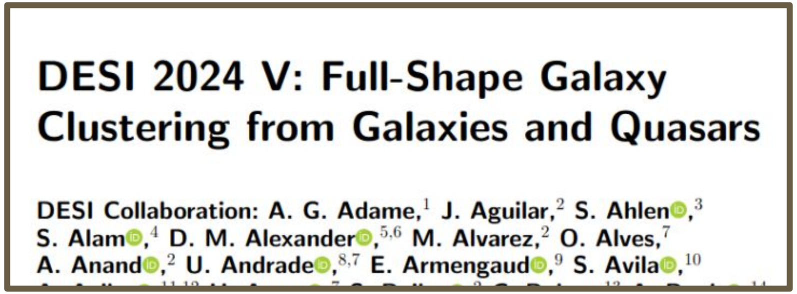
$$\{\text{tree}, 1L, \sout{2L, \ldots}\} \times$$

$$\{\text{real}, \text{redshift}\} \times$$

$$\{\text{Field}, 2\text{pt}, 3\text{pt}, \ldots\} \times$$

$$\{\text{BOSS}, \text{DESI}, \text{DES}, \text{cross}, \ldots\} \times$$

$$\{\text{LCDM}, \ldots\}$$


DESI 2024 V: Full-Shape Galaxy Clustering from Galaxies and Quasars

DESI Collaboration: A. G. Adame,[1] J. Aguilar,[2] S. Ahlen,[3] S. Alam,[4] D. M. Alexander,[5,6] M. Alvarez,[2] O. Alves,[7] A. Anand,[2] U. Andrade,[8,7] E. Armengaud,[9] S. Avila,[10]

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Part III: The renormalization group for LSS

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Fast two-loop evaluation

1-Loop: 2dim integral (~seconds)
2-Loop: 5dim integral (~minutes)
3-Loop: 8dim integral (~week)

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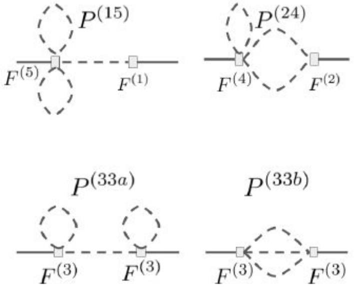


Idea: PCA expand the linear spec

$$P_L^{\Theta}(k) = \sum_{i=1}^{N_b} w_i(\Theta) v_i(k)$$

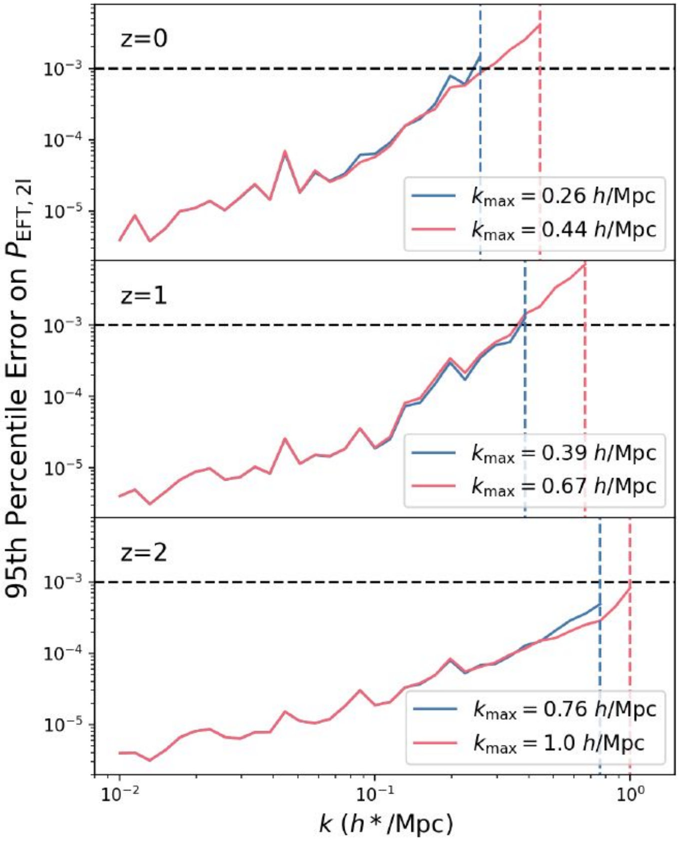
↓

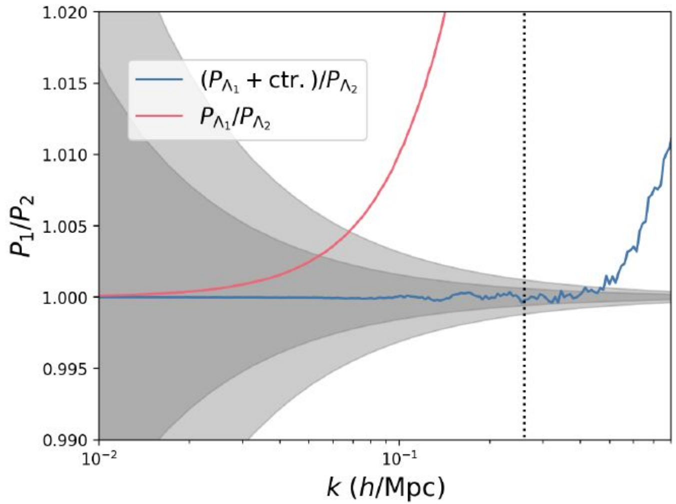
$$P^{\Theta}_{\text{1-loop}}(k,\mu) = \text{const.}(k,\mu) + \mathcal{S}^l_i(k,\mu) w_i(\Theta) + \mathcal{S}^q_{ij}(k,\mu) w_i(\Theta) w_j(\Theta)$$

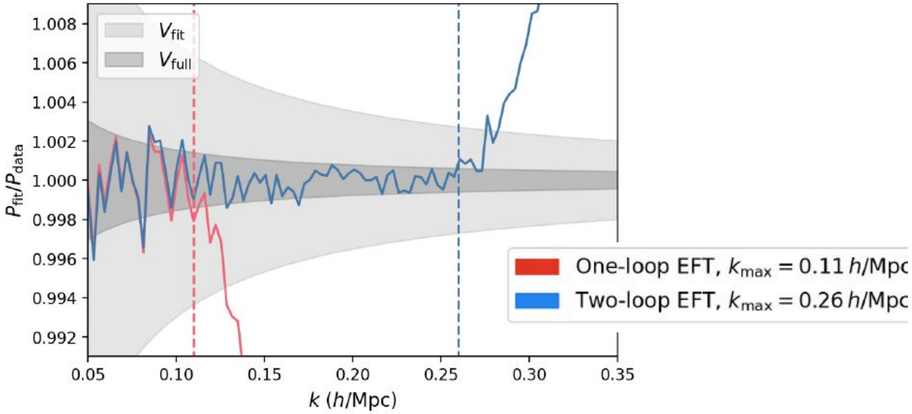
	Default	
Θ	Emulation range for $w_i(\Theta)$	Grid size for SVD
ω_c	[0.095,0.145]	30
ω_b	[0.0202,0.0238]	15
n_s	[0.91,1.01]	15
$10^9 A_s$	-	$10^9 A_s^* = 2$
h	[0.55,0.8]	$h^* = 0.7$
z	-	$z^* = 0$

Bakx, **HR**, Chisari, Vlah 2025;

Fast two-loop evaluation



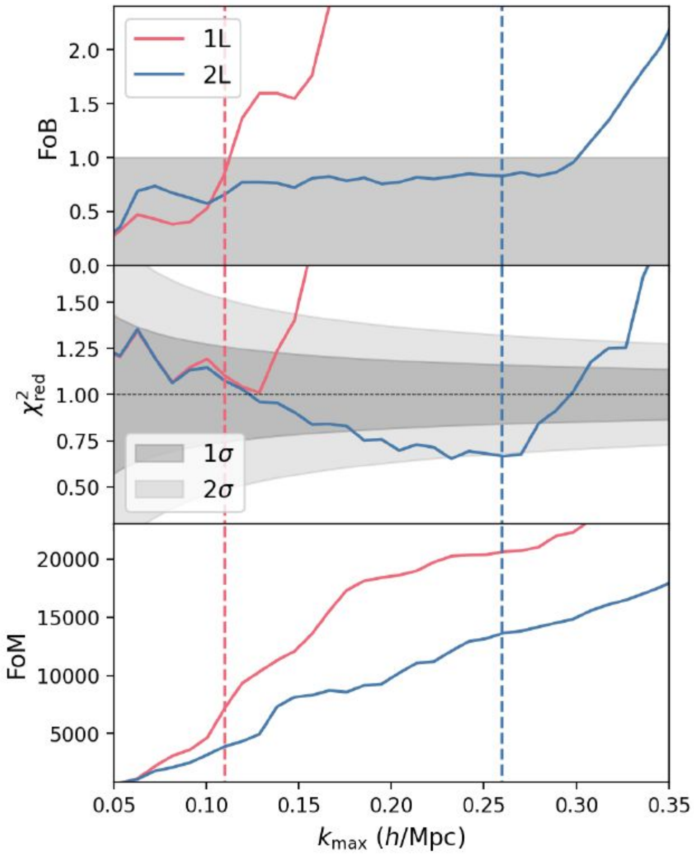


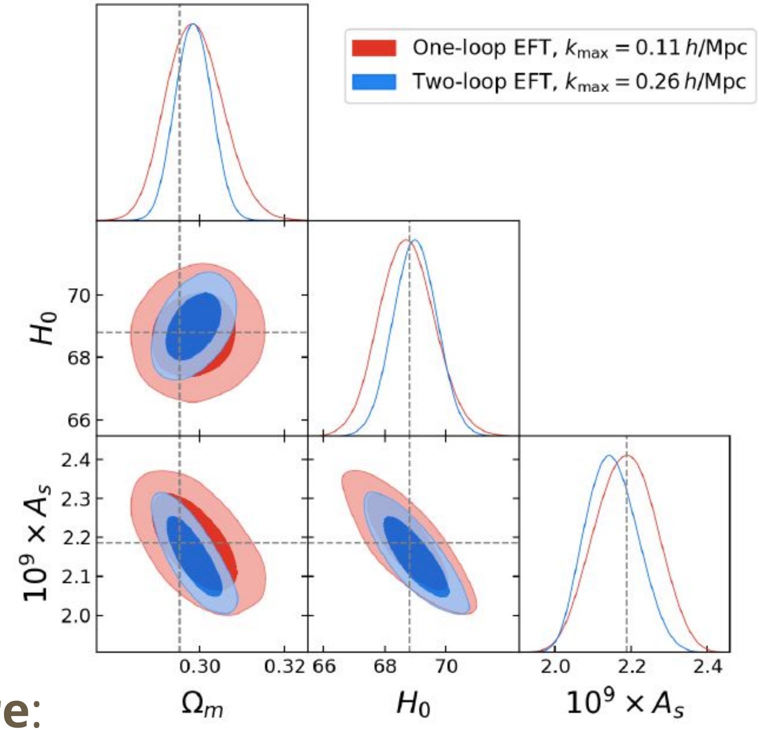


Bakx, **HR**, Chisari, Vlah 2025;

Fast two-loop evaluation

Bakx, **HR**, Chisari, Vlah 2025;





The future:

- bias expansion (we only did matter)
- Redshift space distortions
- Applying to data

Pause to drink some water

A lot of work to go to higher loops!

Can we make our life simpler?
(by complicating it first)

... Or on how to use a **one-loop** (renormalization group) to get **information about higher-loop** terms 'for free'

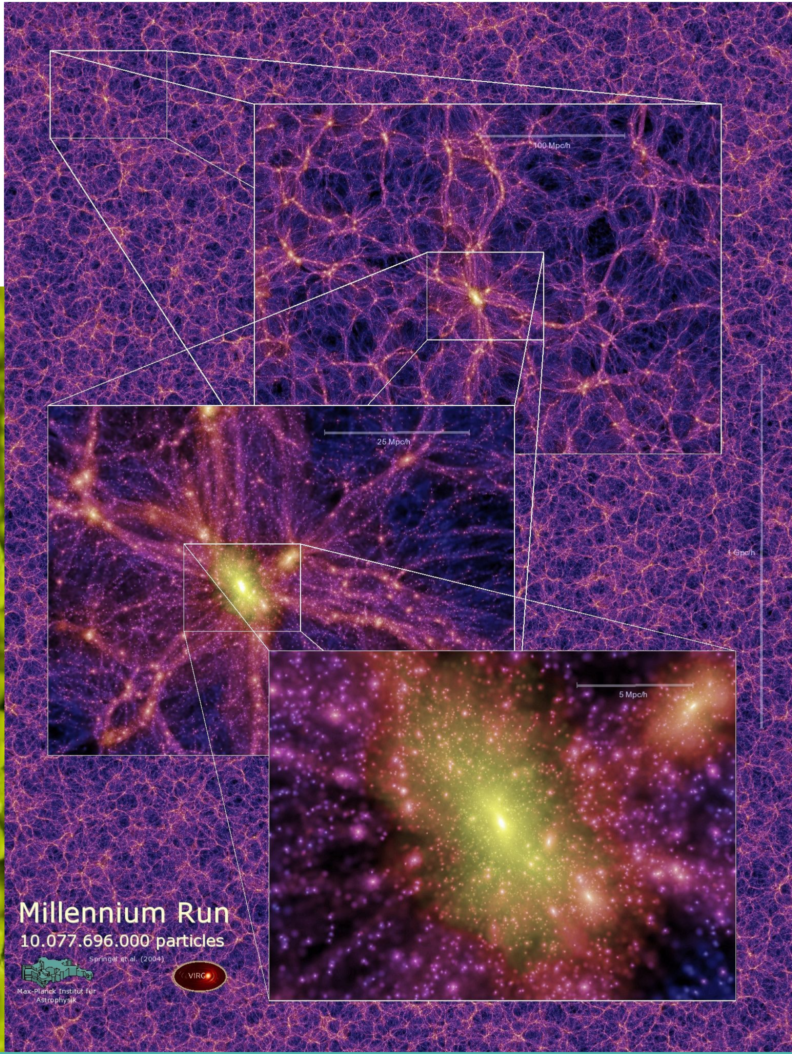
Part I: Introduction

Part II: Towards two-loop EFT

Part III: The renormalization group for LSS

Part IV: Multi-tracer

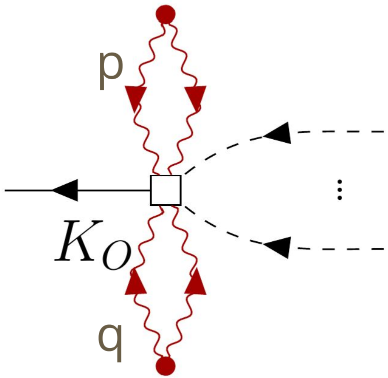
How things change with scale? (from food to galaxies)



How things change with scale?

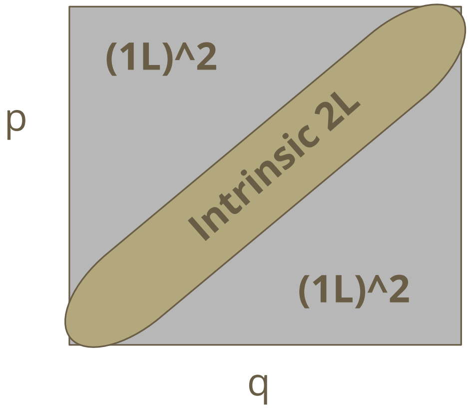
One-loop (RG) to get **information about higher-loop** terms 'for free'

Intuition: **(1loop)^n ~ n-loop**
(for some part of the integrals domain)



Method of regions:
(Beneke and Smirnov)

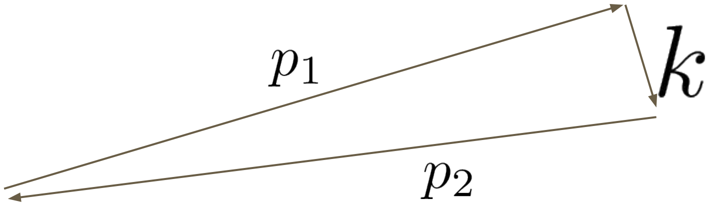
- p>>q (or q<<p): Absorbed by (1loop)^2
- p~q: Intrinsic 2-loop



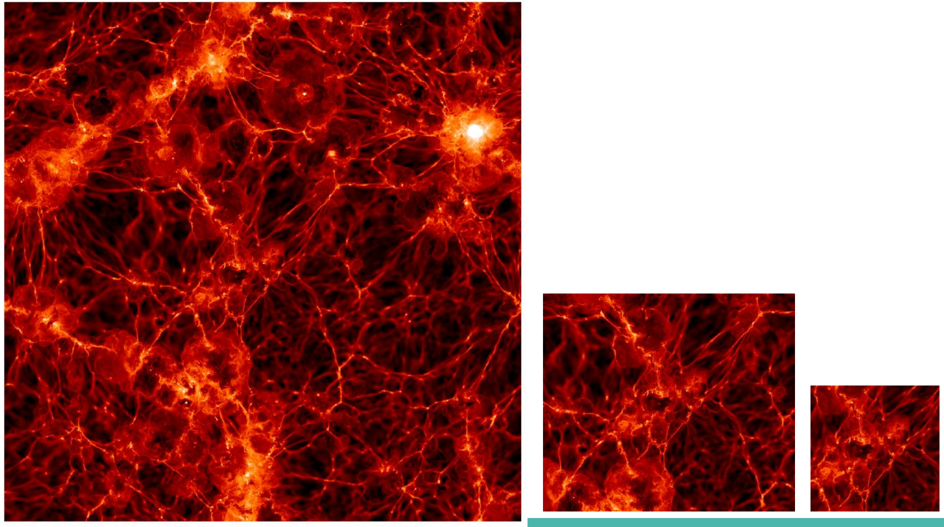
Renormalizing the bias parameters

$$\delta_{g,\mathrm{det}}(\boldsymbol{x},\tau) = \sum_O b_O^{\Lambda}(\tau)\, O_{\Lambda}(\boldsymbol{x},\tau)$$

$$\delta^2_{\Lambda}(k) = \int^{\Lambda} (2\pi)^3 \delta_{\mathrm{D}}(k - p_1 - p_2)\delta(p_1)\delta(p_2)$$



First order:	δ ;
Second order:	$\delta^2,\ \mathcal{G}_2$;
Third order:	$\delta^3,\ \delta\,\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3$;



Contribution from arbitrarily small scales!

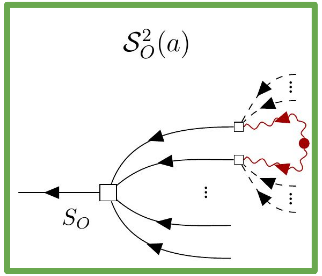
The equations

one-loop:

$$\left.\frac{db_a}{d\Lambda}\right|_{1\mathrm{L}} = -b_b s^{1\mathrm{L}}_{ba}\frac{d\sigma^2_\Lambda}{d\Lambda}$$

HR, Schmidt, 23

$s^O_{O'}$	δ	δ^2	$\mathcal{G}_2$	δ^3	$\mathcal{G}_3$	Γ_3	$\delta\mathcal{G}_2$
$\mathbb{1}$	-	-	-	-	-	-	-
δ	-	$68/21$	-	3	-	-	$-4/3$
δ^2	-	$8126/2205$	-	$68/7$	-	-	$-376/105$
$\mathcal{G}_2$	-	$254/2205$	-	-	-	-	$116/105$



a	$c^{(3)}_{a\delta}$	$d^{(5)}_a$	$\tilde{d}^{(5)}_a$
$\mathrm{tr}[\Pi^{[1]}]$	0	0	0
$\mathrm{tr}[(\Pi^{[1]})^2]$	$\frac{68}{63}$	$\frac{862}{1575}$	$\frac{376}{6615}$
$(\mathrm{tr}[\Pi^{[1]}])^2$	$\frac{68}{63}$	$\frac{862}{1575}$	$\frac{376}{6615}$
$(\mathrm{tr}[\Pi^{[1]}])^3$	1	$\frac{70739}{33075}$	$\frac{4}{105}$
$\mathrm{tr}[(\Pi^{[1]})^2]\mathrm{tr}[\Pi^{[1]}]$	$\frac{5}{9}$	$\frac{2917}{2205}$	$\frac{716}{1323}$
$\mathrm{tr}[(\Pi^{[1]})^3]$	$\frac{1}{3}$	$\frac{30263}{33075}$	$\frac{1748}{2205}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	$\frac{41}{63}$	$\frac{134957}{99225}$	$\frac{148}{441}$
$(\mathrm{tr}[\Pi^{[1]}])^4$	0	$\frac{272}{105}$	0
$\mathrm{tr}[(\Pi^{[1]})^3]\mathrm{tr}[\Pi^{[1]}]$	0	$\frac{82}{105}$	$\frac{5}{21}$
$\mathrm{tr}[(\Pi^{[1]})^2](\mathrm{tr}[\Pi^{[1]}])^2$	0	$\frac{6352}{4725}$	$\frac{4}{21}$
$(\mathrm{tr}[(\Pi^{[1]})^2])^2$	0	$\frac{592}{675}$	$\frac{8}{63}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{16112}{19845}$	$\frac{3706}{6615}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{18166}{14175}$	$\frac{373}{735}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{12814}{11025}$	$\frac{401}{2205}$
$\mathrm{tr}[\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{27784}{19845}$	$\frac{232}{315}$
$(\mathrm{tr}[\Pi^{[1]}])^5$	0	1	0
$\mathrm{tr}[(\Pi^{[1]})^3](\mathrm{tr}[\Pi^{[1]}])^2$	0	$\frac{11}{45}$	0
$\mathrm{tr}[(\Pi^{[1]})^2](\mathrm{tr}[\Pi^{[1]}])^3$	0	$\frac{7}{15}$	0
$\mathrm{tr}[(\Pi^{[1]})^3]\mathrm{tr}[(\Pi^{[1]})^2]$	0	$\frac{31}{225}$	0
$\mathrm{tr}[\Pi^{[1]}](\mathrm{tr}[(\Pi^{[1]})^2])^2$	0	$\frac{163}{675}$	0
$(\mathrm{tr}[\Pi^{[1]}])^2\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{47}{105}$	$\frac{2}{21}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{173}{675}$	$\frac{2}{63}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{391}{1575}$	$\frac{5}{63}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{197}{735}$	$\frac{82}{441}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{5057}{11025}$	$\frac{110}{441}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{643}{2835}$	$\frac{104}{945}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{5477}{14175}$	$\frac{4}{35}$
$\mathrm{tr}[\Pi^{[2]}\Pi^{[3]}]$	0	$\frac{8629}{19845}$	$\frac{491}{2205}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[4]}]$	0	$\frac{33403}{198450}$	$\frac{74}{6615}$

two-loop:

$$\left.\frac{db_\delta}{d\Lambda}\right|_{2\mathrm{L}} = -30 b_b \tilde{d}^{(5)}_b \frac{d\sigma^2_\Lambda}{d\Lambda}\int_0^\Lambda dq\, \frac{q^2 P^{\mathrm{lin}}(q)}{2\pi^2} g(q/\Lambda)\,,$$

Bakx, Garny, **HR**, Vlah

The (one-loop) solutions

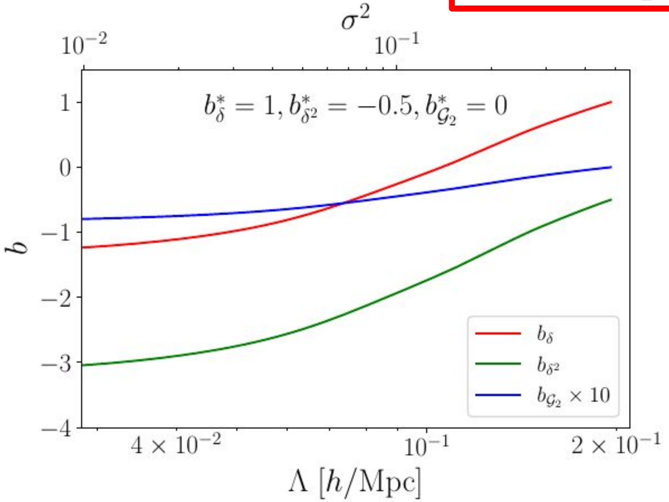
Solutions (one-loop)

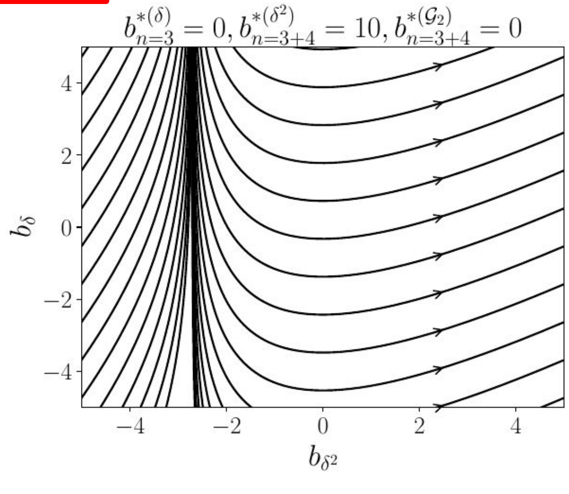
$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3}b^*_{\mathcal{G}_2\delta}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205}b_{\delta^2} + \frac{17}{7}b^*_{\delta^3} - \frac{376}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205}b_{\delta^2} + \frac{116}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$

HR, Schmidt 23





Why should you care?

What do the solutions of the RG tell us?

Bakx, Garny, **HR**, Vlah

We can always diagonalize the bias basis

$$\frac{db_i^{\text{diag}}}{d\sigma^2} = \lambda_i b_i^{\text{diag}}$$

$$b_a(\sigma^2) = p_{ai} e^{\lambda_i(\sigma^2 - \sigma_*^2)} c_i$$

If we stop at second-order, we find:

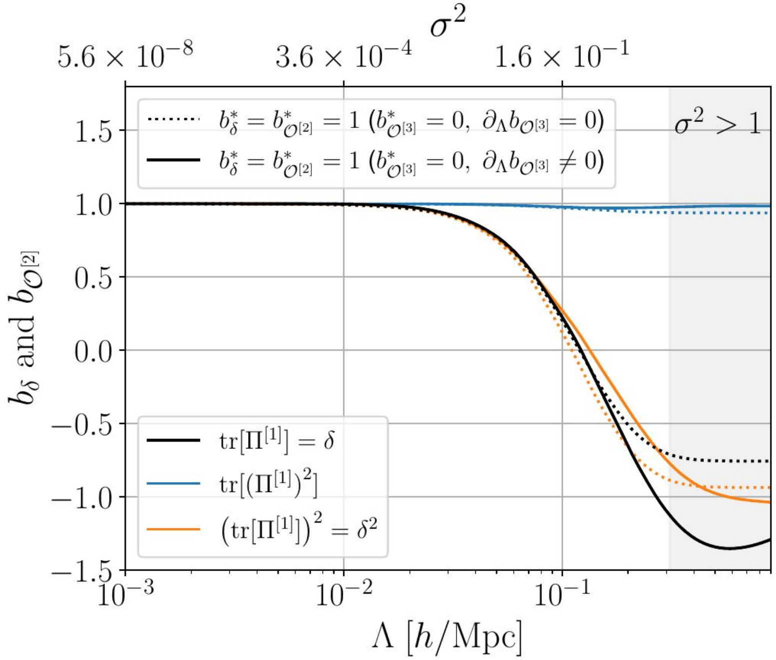
$$\{\lambda_1, \lambda_2, \lambda_3\} \simeq \{0, 0, -3.69\}$$

Marginal *Relevant*

Extending to third-order:

Irrelevant

$$\{0, 0, 0, -12.6, -3.44, -2.01, 0.220\}$$



Some parameters are supersensitive to the UV

Why should you care II?

Resumming terms with the RG equations

Bakx, Garny, **HR**, Vlah

1Loop RG eq.

$$\frac{db_a}{d\sigma^2} = -\bar{s}^{1\mathrm{L}}_{ac} b_c$$

Solution

$$b_a(\sigma^2) =$$

$$= b_a^* - (\sigma^2 - \sigma_*^2)\bar{s}^{1\mathrm{L}}_{ac} b_c^* + \frac{1}{2}(\sigma^2 - \sigma_*^2)^2 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bc} b_c^* - \frac{1}{6}(\sigma^2 - \sigma_*^2)^3 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bd}\bar{s}^{1\mathrm{L}}_{dc} b_c^* + \ldots$$

1-loop (1-loop)^2 (1-loop)^3

Resumming terms with the RG equations

Bakx, Garny, **HR**, Vlah

1Loop RG eq.

$$\frac{db_a}{d\sigma^2} = -\bar{s}^{1\mathrm{L}}_{ac} b_c$$

Solution

$$b_a(\sigma^2) = \left[e^{-\bar{s}^{1\mathrm{L}}\times(\sigma^2-\sigma_*^2)}\right]_{ac} b_c^*$$

RG resums the series!

$$= b_a^* - (\sigma^2 - \sigma_*^2)\bar{s}^{1\mathrm{L}}_{ac} b_c^* + \frac{1}{2}(\sigma^2 - \sigma_*^2)^2 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bc} b_c^* - \frac{1}{6}(\sigma^2 - \sigma_*^2)^3 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bd}\bar{s}^{1\mathrm{L}}_{dc} b_c^* + \ldots$$

1-loop

(1-loop)^2

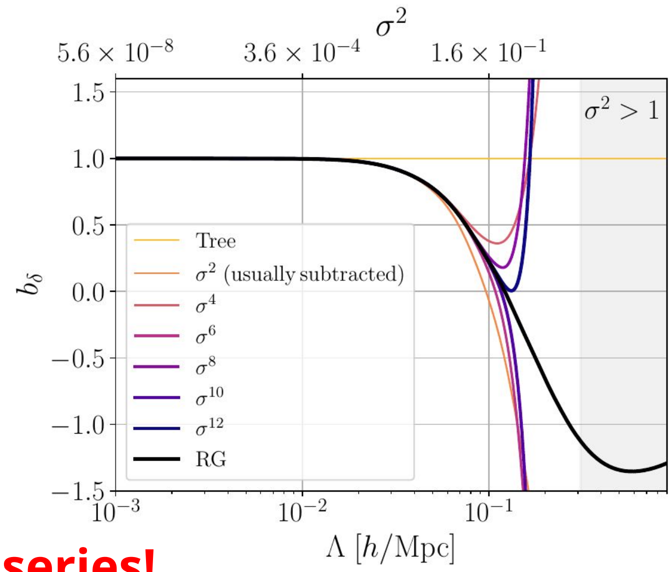
(1-loop)^3

Resumming terms with the RG equations

Bakx, Garny, **HR**, Vlah

1Loop RG eq.

$$\frac{db_a}{d\sigma^2} = -\bar{s}^{1\mathrm{L}}_{ac} b_c$$



Solution

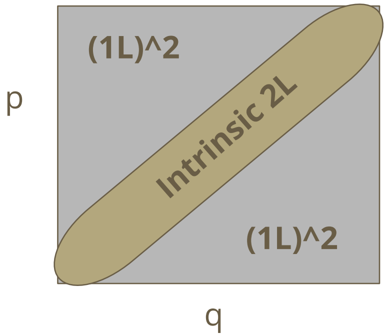
$$b_a(\sigma^2) = \left[e^{-\bar{s}^{1\mathrm{L}} \times (\sigma^2 - \sigma^2_*)}\right]_{ac} b^*_c$$

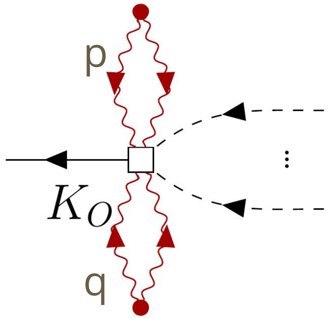
RG resums the series!

$$= b^*_a - (\sigma^2 - \sigma^2_*)\bar{s}^{1\mathrm{L}}_{ac} b^*_c + \frac{1}{2}(\sigma^2 - \sigma^2_*)^2 \bar{s}^{1\mathrm{L}}_{ab} \bar{s}^{1\mathrm{L}}_{bc} b^*_c - \frac{1}{6}(\sigma^2 - \sigma^2_*)^3 \bar{s}^{1\mathrm{L}}_{ab} \bar{s}^{1\mathrm{L}}_{bd} \bar{s}^{1\mathrm{L}}_{dc} b^*_c + \ldots$$

1-loop | (1-loop)^2 | (1-loop)^3

Partial conclusion





1-loop RG resums part of higher-loop contributions (p>>q or q<<p regions)

...But is the other part ('intrinsic 2-loop', p~q region) small?

If YES: amaaaazing, 1-loop RG is doing something

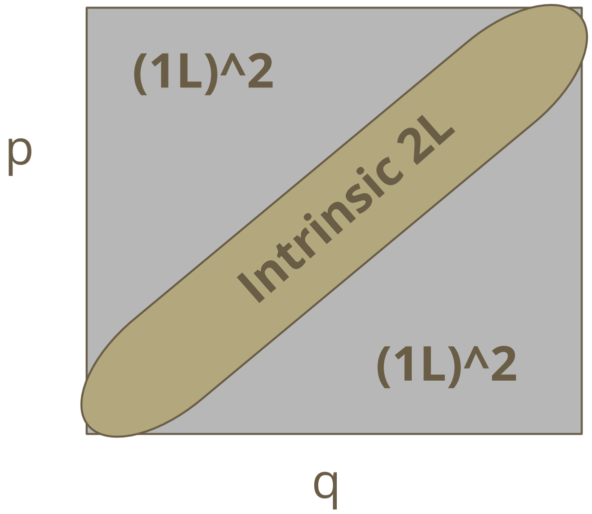
If NO: out, I have to calculate the loops anyway to get most of info

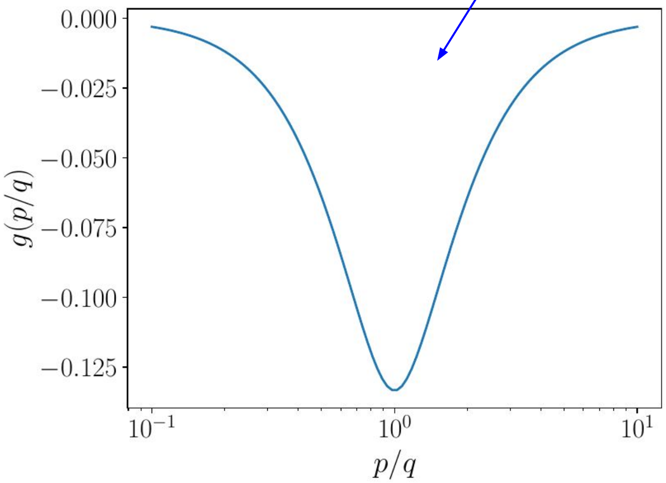
The (two-loop) solutions

The equations

two-loop:

$$\left.\frac{db_\delta}{d\Lambda}\right|_{2\mathrm{L}} = -30 b_b \tilde{d}_b^{(5)} \frac{d\sigma_\Lambda^2}{d\Lambda} \int_0^\Lambda dq \frac{q^2 P^{\mathrm{lin}}(q)}{2\pi^2} g(q/\Lambda)\,,$$





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$\mathrm{tr}[(\Pi^{[1]})^3]\mathrm{tr}[(\Pi^{[1]})^2]$	0	$\frac{31}{225}$	0
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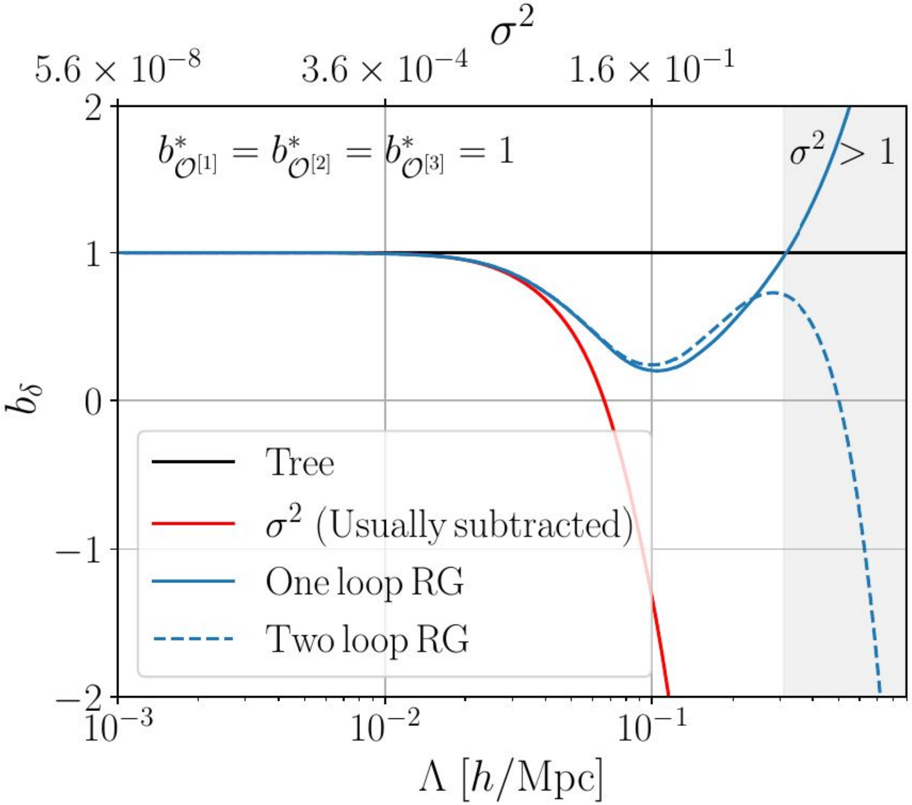
Bakx, Garny, **HR**, Vlah

Solutions (two-loop)

Bakx, Garny, **HR**, Vlah

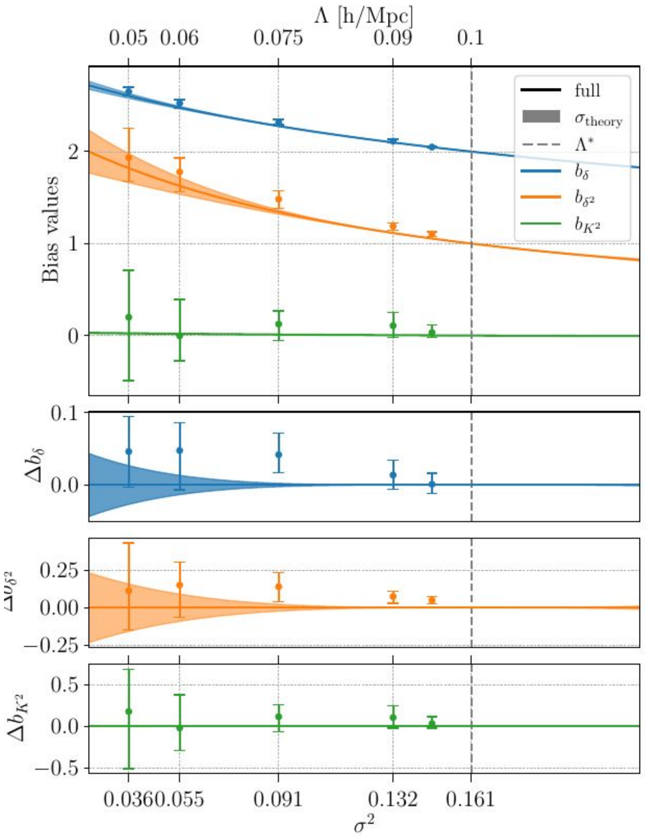


'Intrinsic' two-loop part is small!

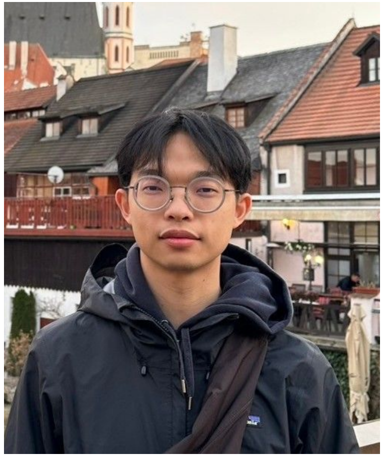
Why should you care III?

Measuring the bias running

Hsiang-Ming (Harry) Huang,
HR,
Fabian Schmidt,
Preliminary



Connecting the
field-level and
n-point functions!

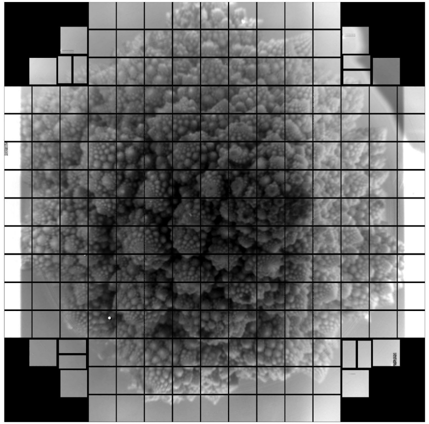


Conclusions

- Cross-check for EFT inference;
- Systematic renormalization (+ stochastic +PNG);
- More information from resummation? TBD!
- Still to be understood:

    1) RG stability when going to higher-order
    2) scales in between which RG can operate

First images of Rubin



A short rest for your eyes

Part I: Introduction

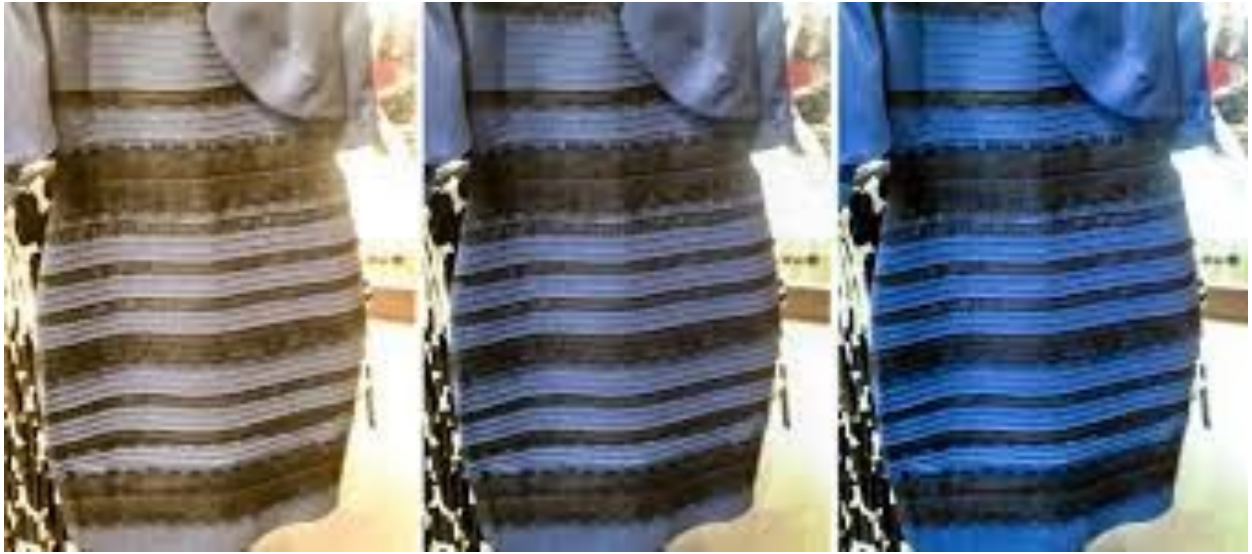
Part II: Towards two-loop EFT

Part III: The renormalization group for LSS

Part IV: Multi-tracer

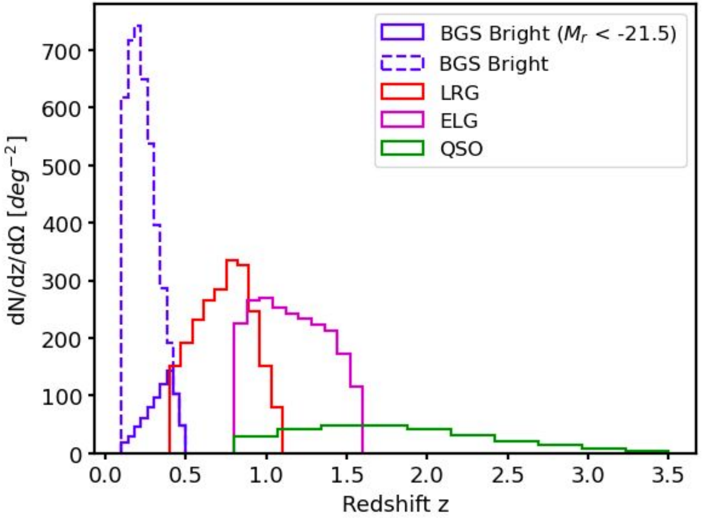
Motivation I

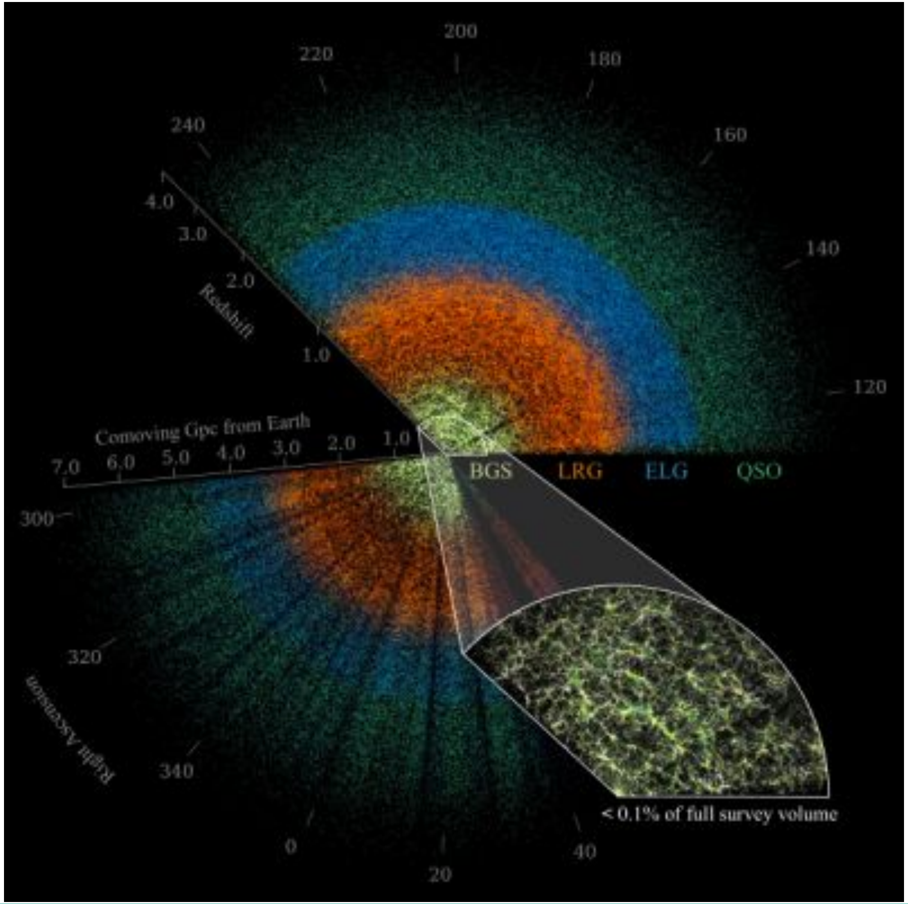
Motivation I

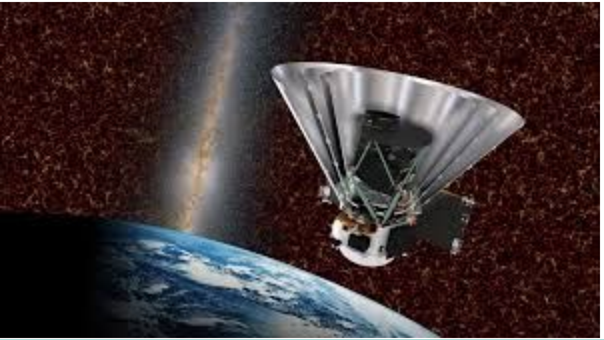


Motivation I

DESI collaboration







SPHEREx

Linear multi-tracer

Seljak 08, McDonald and Seljak 09

$$\rho = \rho^A + \rho^B \Longrightarrow \delta = \frac{\overline{n}^A}{\overline{n}}\delta^A + \frac{\overline{n}^B}{\overline{n}}\delta^B$$

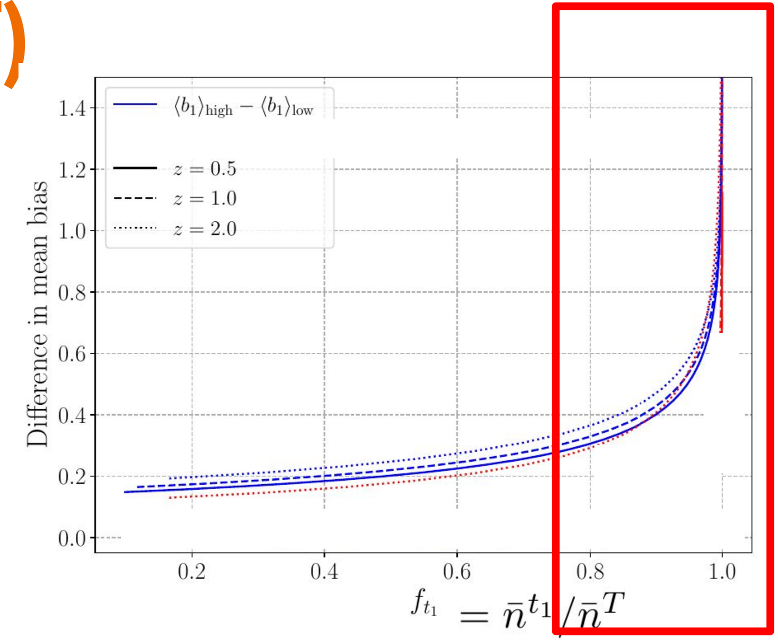
cross-stoch

$$\begin{cases} P^{AA} = \langle \delta^A \delta^A \rangle' = (b_1^A + f\mu^2)^2 P_L + \frac{1}{\overline{n}^A}(1 + c_0^{AA}) \\ P^{AB} = \langle \delta^A \delta^B \rangle' = (b_1^A + f\mu^2)(b_1^B + f\mu^2) P_L + \frac{1}{\sqrt{\overline{n}^A \overline{n}^B}} c_0^{AB} \\ P^{BB} = \langle \delta^B \delta^B \rangle' = (b_1^B + f\mu^2)^2 P_L + \frac{1}{\overline{n}^B}(1 + c_0^{BB}) \end{cases}$$

Three problems (with linear MT)

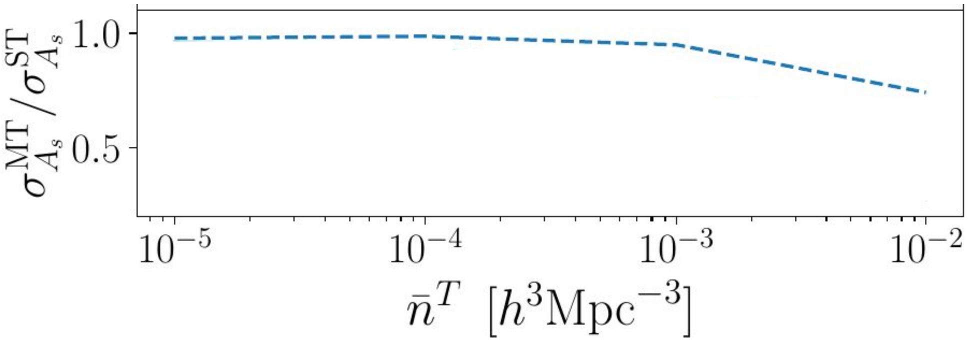
1) Finding tracers with different linear bias;

$$|b_1^A - b_1^B|$$



Three problems (with linear MT)

2) Very high number density (low shot noise)



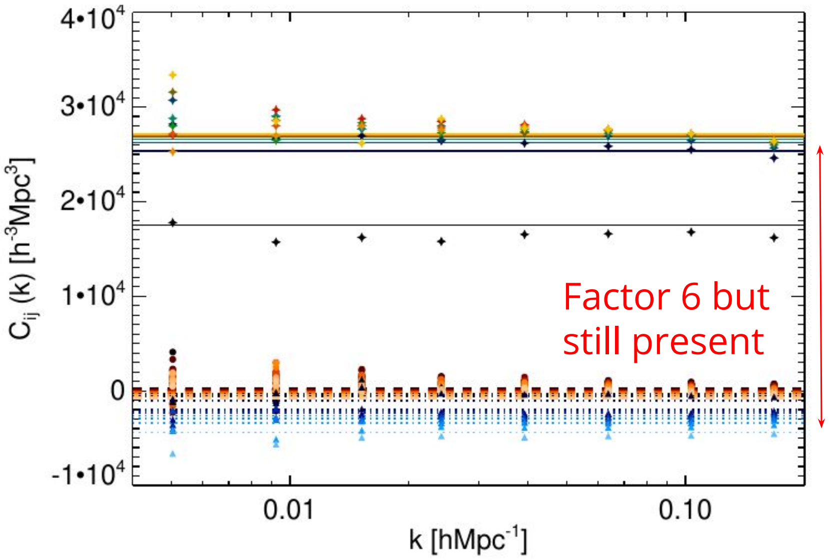
Three problems (with linear MT)

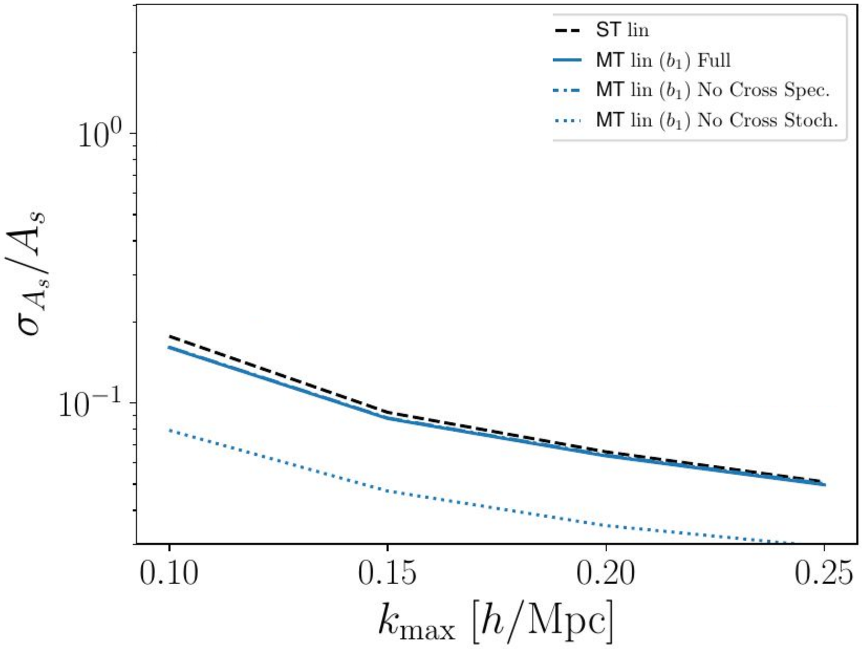
3) Cross-stochasticity

$$
\begin{cases}
P^{AA} = \langle \delta^A \delta^A \rangle' = (b_1^A + f\mu^2)^2 P_L + \frac{1}{\bar{n}^A}(1 + c_0^{AA}) \\
P^{AB} = \langle \delta^A \delta^B \rangle' = (b_1^A + f\mu^2)(b_1^B + f\mu^2) P_L + \frac{1}{\sqrt{\bar{n}^A \bar{n}^B}} c_0^{AB} \\
P^{BB} = \langle \delta^B \delta^B \rangle' = (b_1^B + f\mu^2)^2 P_L + \frac{1}{\bar{n}^B}(1 + c_0^{BB})
\end{cases}
$$

Rubira and Conteddu 25

Hamaus, Seljak+ 2010





Solving all those problems: Beyond linear theory

Bias expansion

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

Find samples with different non-linear bias!

$$b_{\mathcal{O}}^{t_1} = b_{\mathcal{O}}^{\mathrm{ST}} \mp \frac{\Delta b}{2} \quad \text{and} \quad b_{\mathcal{O}}^{t_2} = b_{\mathcal{O}}^{\mathrm{ST}} \pm \frac{\Delta b}{2}$$

With Thiago Mergulhão+

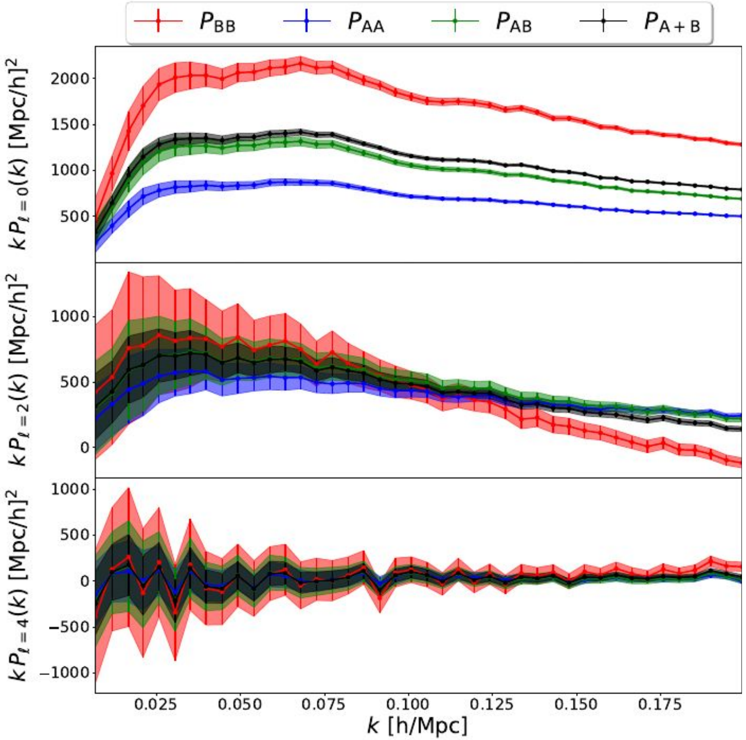
ArXiv: 2108.11363, 2306.05474

With Francesco Conteddu

ArXiv: 2504.18245

Why?

$$
\begin{aligned}
P^{AB}(k) =\ & b_1^A b_1^B \left[P_{\rm lin}(k) + P_{\rm 1L}(k)\right] + \frac{1}{2}\left(b_1^A b_2^B + b_1^B b_2^A\right)\mathcal{I}_{\delta^2}(k) \\
& + \left(b_1^A b_{\mathcal{G}_2}^B + b_1^B b_{\mathcal{G}_2}^A\right)\mathcal{I}_{\mathcal{G}_2}(k) + \left[\left(b_1^A b_{\mathcal{G}_2}^B + b_1^B b_{\mathcal{G}_2}^A\right) + \frac{2}{5}\left(b_1^A b_{\Gamma_3}^B + b_1^B b_{\Gamma_3}^A\right)\right]\mathcal{F}_{\mathcal{G}_2} \\
& + \frac{1}{4} b_2^A b_2^B \mathcal{I}_{\delta^2\delta^2}(k) + b_{\mathcal{G}_2}^B b_{\mathcal{G}_2}^A \mathcal{I}_{\mathcal{G}_2\mathcal{G}_2}(k) + \frac{1}{2}\left(b_2^A b_{\mathcal{G}_2}^B + b_2^B b_{\mathcal{G}_2}^A\right)\mathcal{I}_{\delta^2\mathcal{G}_2}(k) \\
& + P_{\rm ct}^{AB}(k) + P_{\varepsilon^A\varepsilon^B}(k)\,,
\end{aligned}
$$



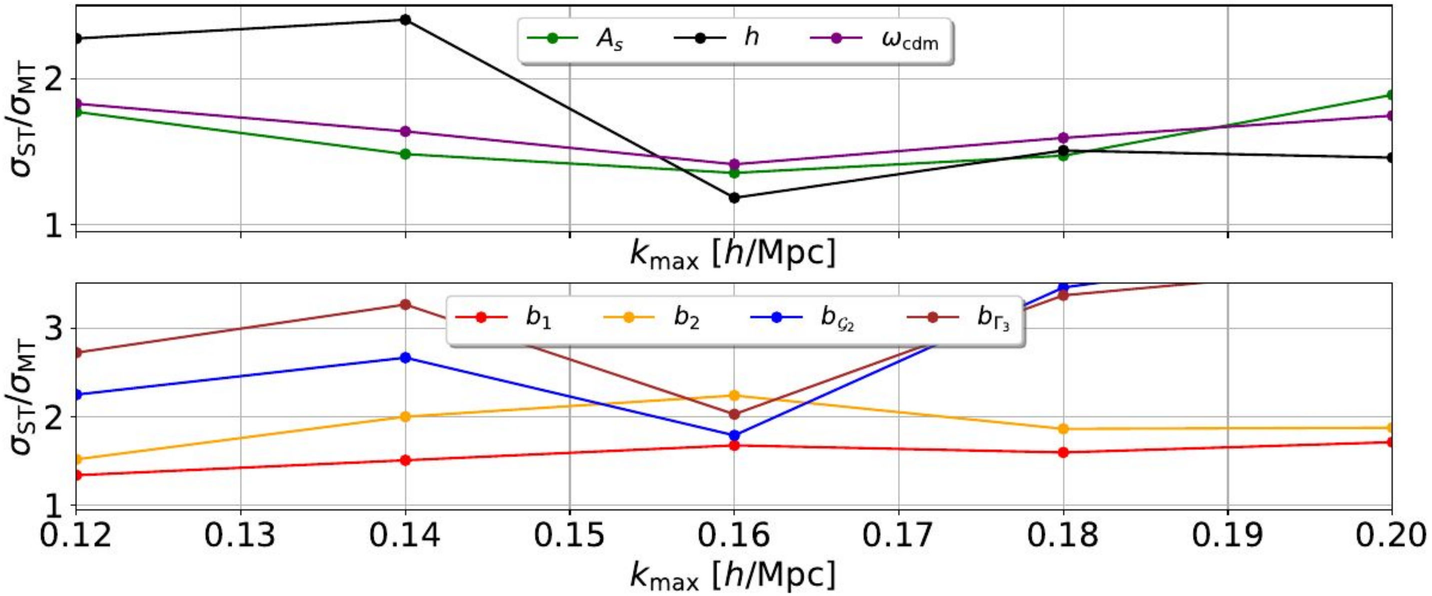
Bias + counter-terms + stochastic

$b^A_{\mathcal{O}} b^B_{\mathcal{O}}$ products break degens

Notice a few things:

- linear bias change
- non-linear scales change
- FoG sample selection
  (see also Lizancos, Seljak + 25)

Results

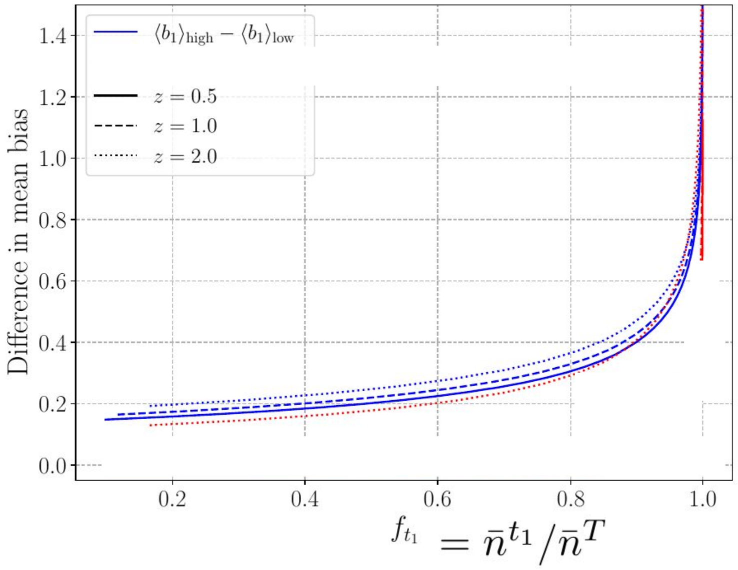


A_s
h
ω_{cdm}
σ_{ST}/σ_{MT}
1
2
k_{max} [h/Mpc]
b_1
b_2
$b_{\mathcal{G}_2}$
b_{Γ_3}
σ_{ST}/σ_{MT}
1
2
3
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.20
k_{max} [h/Mpc]

Three problems (with linear MT)

1) Finding tracers with different linear bias;

$$|b_1^A - b_1^B|$$



Three problems (with linear MT)

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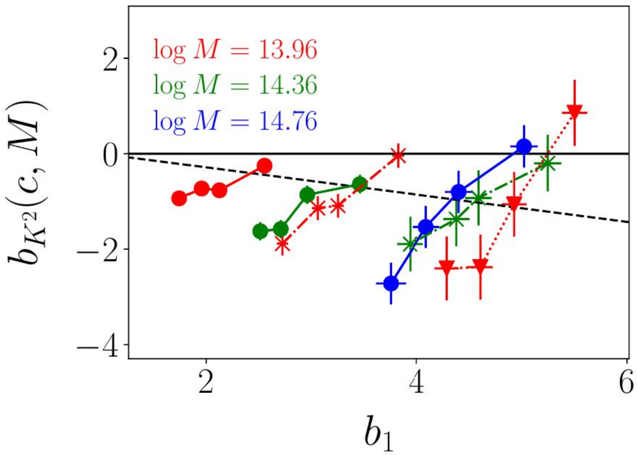
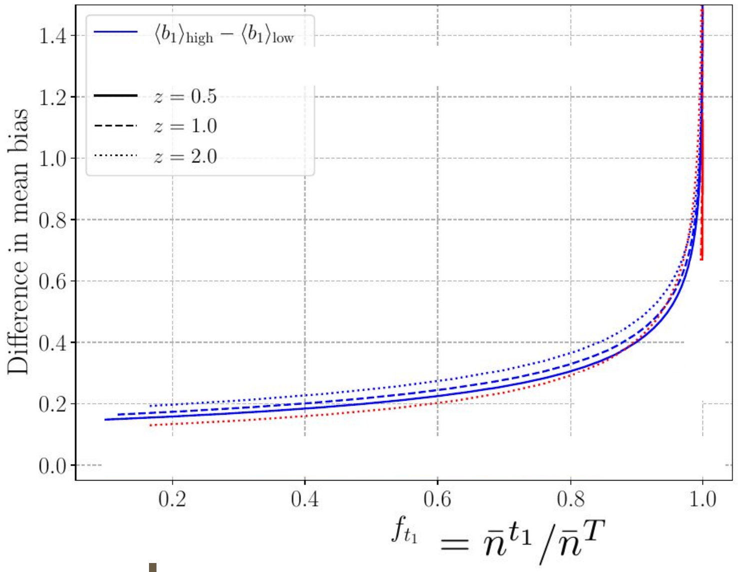
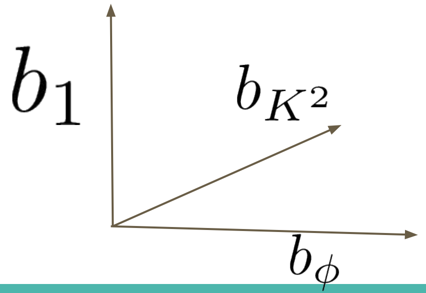




Lazeyras, Barreira, Schmidt 2021

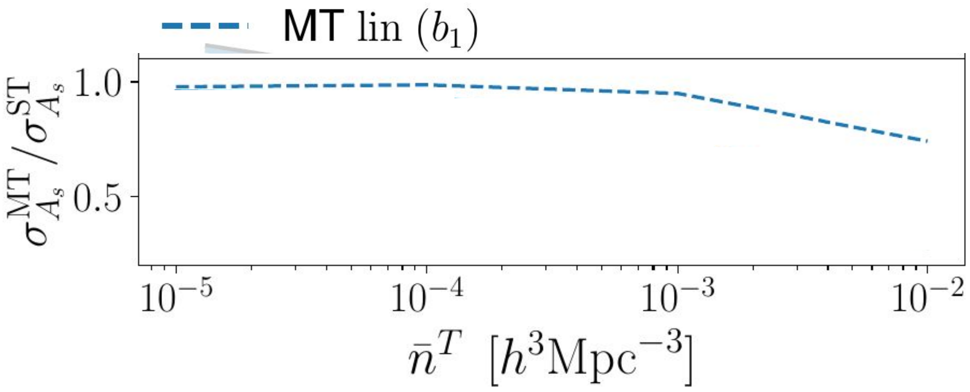
We should find samples with different tidal bias!

Conclusion: Assembly bias can help a lot!



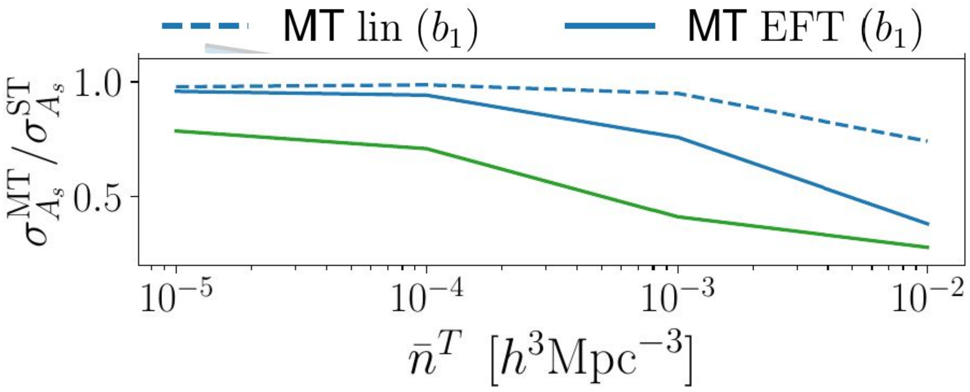
Three problems (with linear MT)

2) Very high number density (low shot noise)



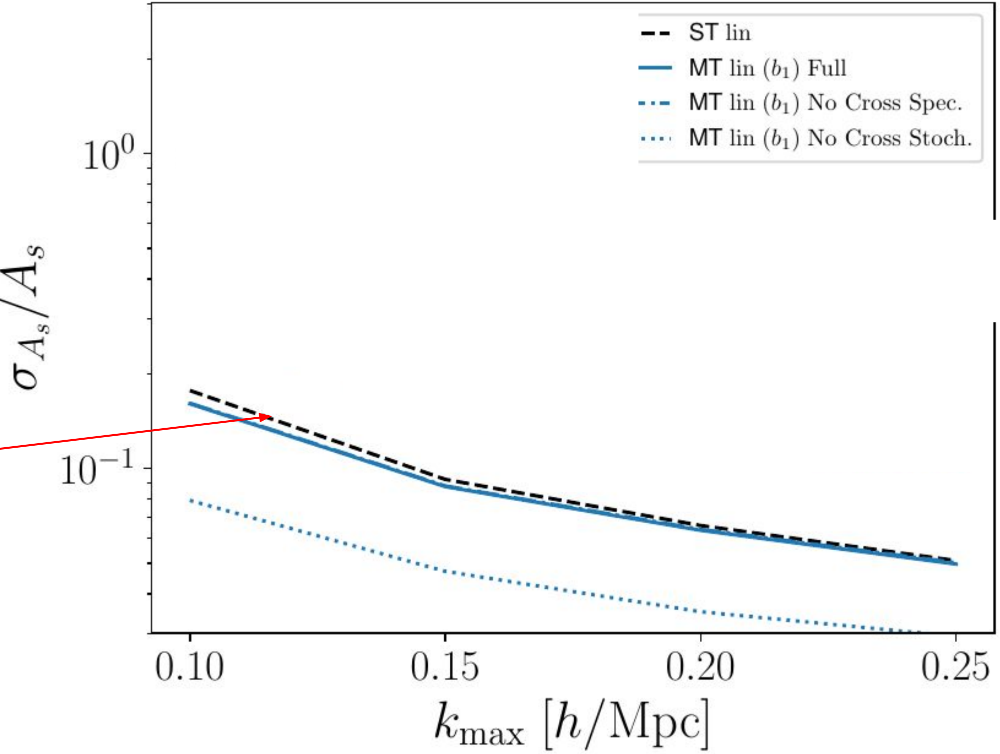
Three problems (with linear MT)

2) Very high number density (low shot noise)



Three problems (with linear MT)

3) Cross-stochasticity



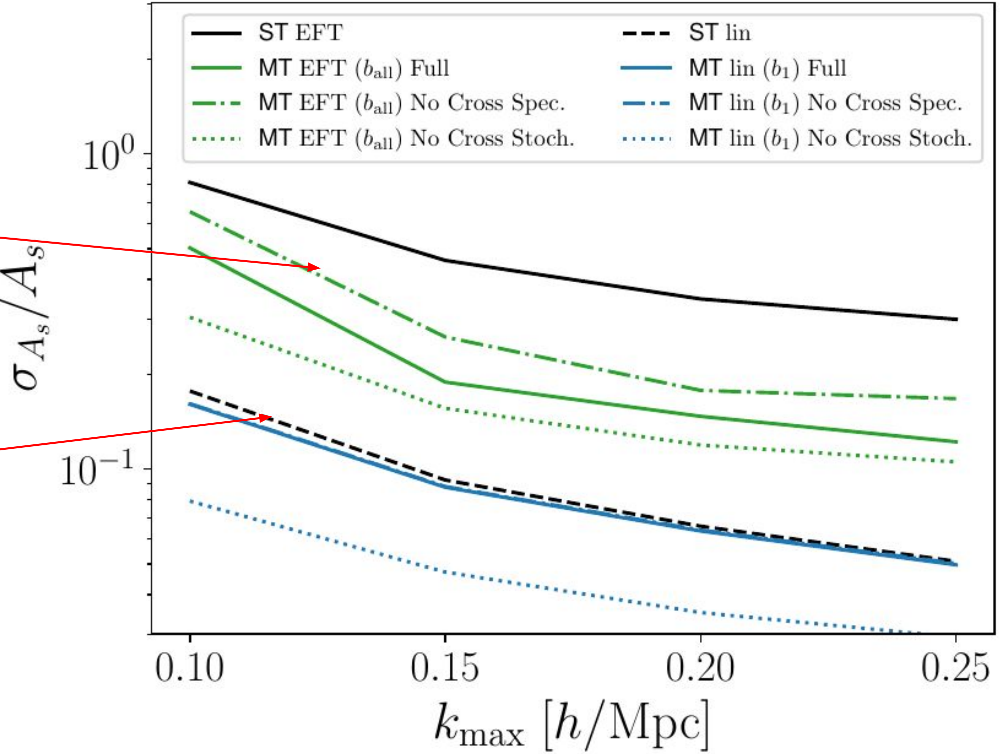
Linear:
Gains when neglecting cross stochastic!

Three problems (with linear MT)

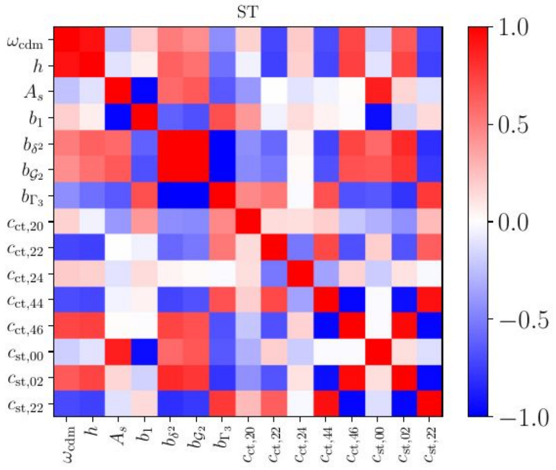
3) Cross-stochasticity

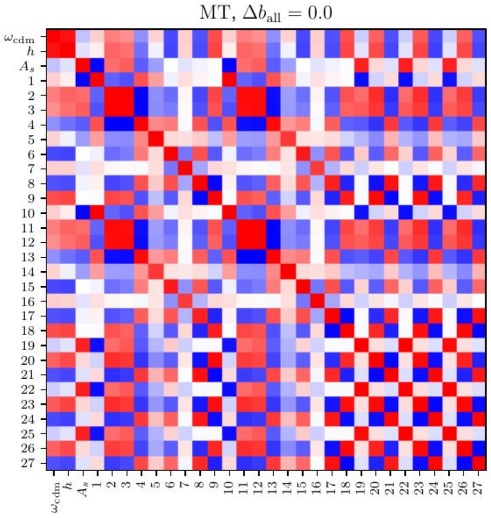
non-Linear:
Gains even with cross stoch!

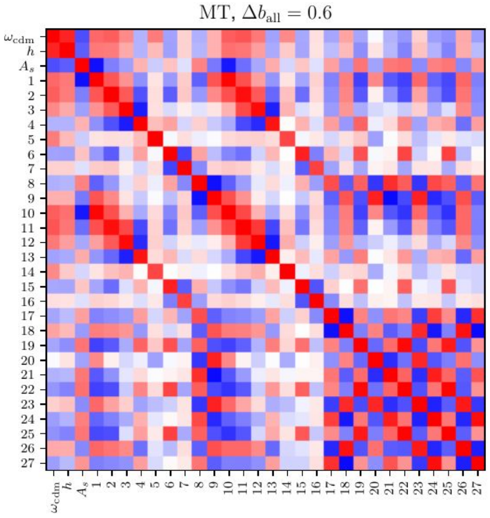
Linear:
Gains when neglecting cross stochastic!



Why is MT non-linear better?

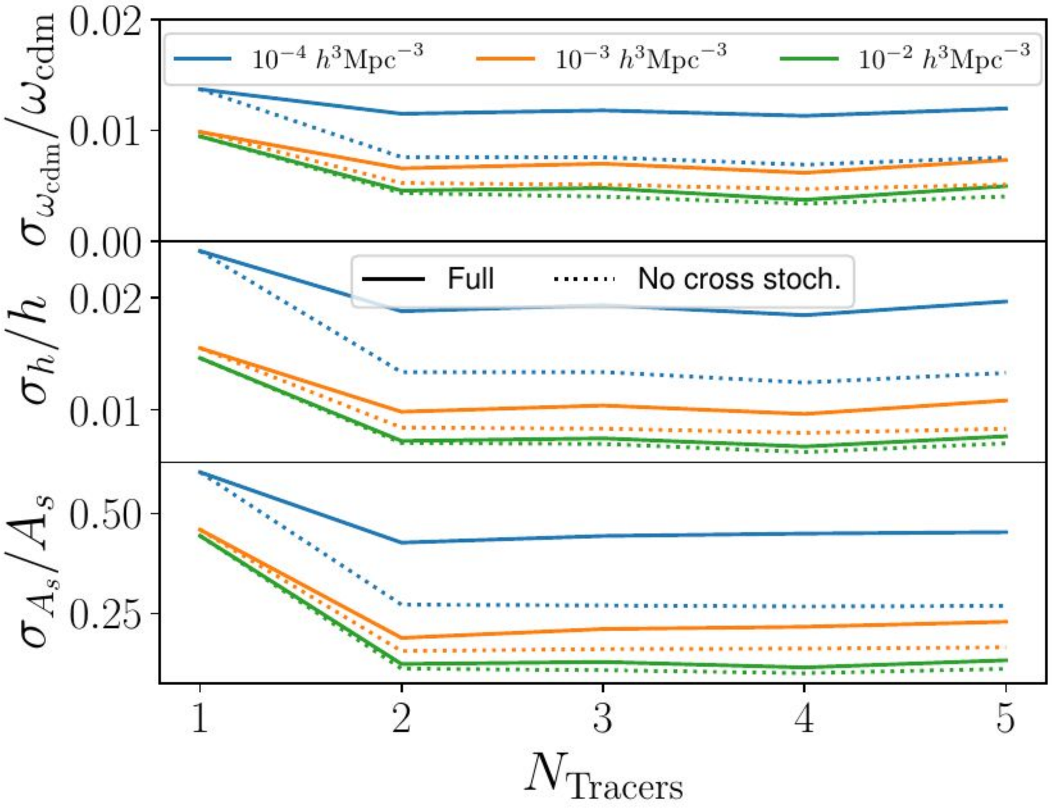






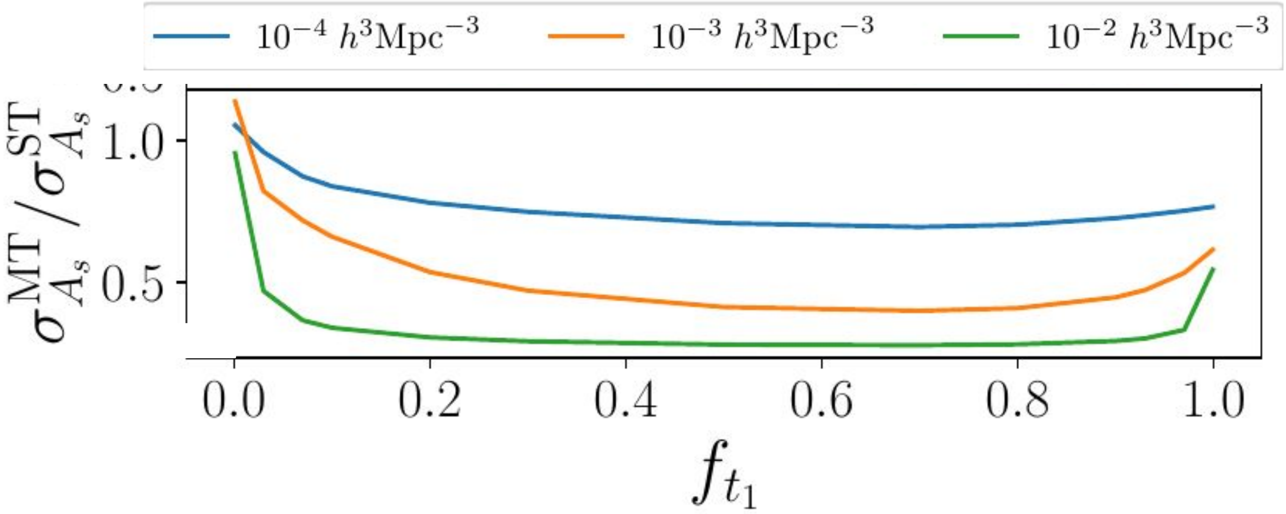
More tracers

2 tracers is the sweet spot



Unbalanced split

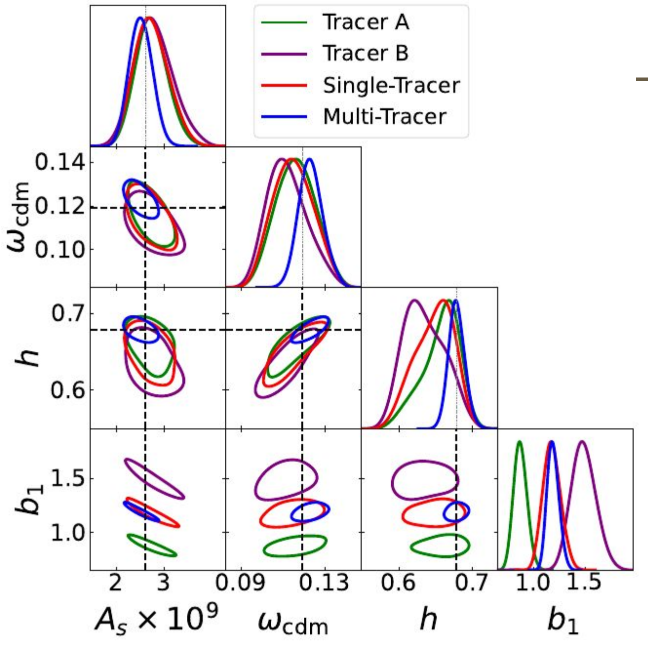
$$f_{t_1} = \frac{\bar{n}^{t_1}}{\bar{n}^T}$$

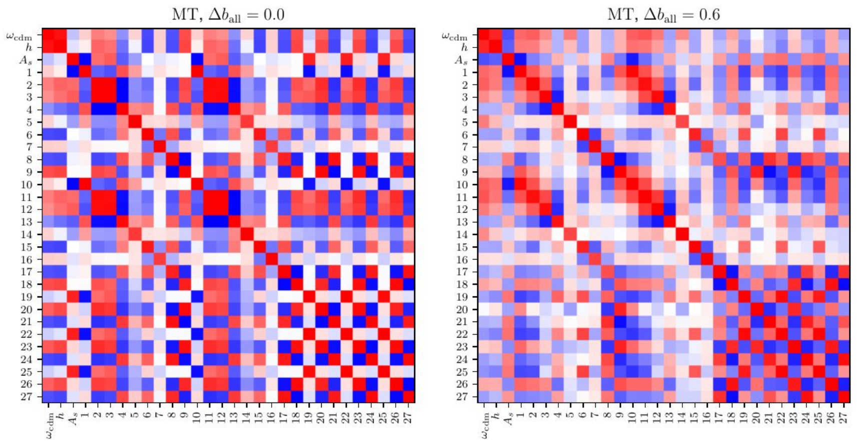


Unbalanced split already favours MT!

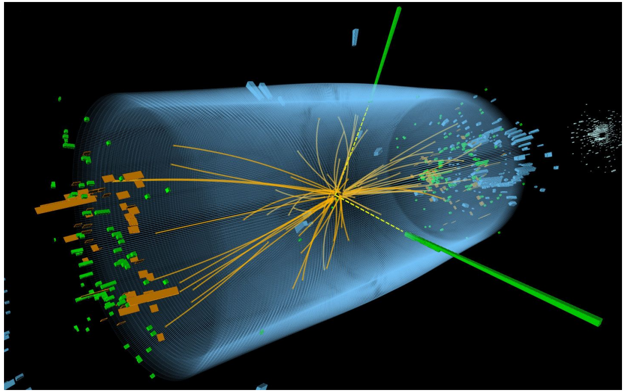
Conclusions

- Linear MT relies on low shot noise and large b1 difference, this is not the case for non-linear MT
- Using information about galaxies bias adds a lot!
- Assembly bias can play an important role.
- We only have to find the 10/15% sample with different bias

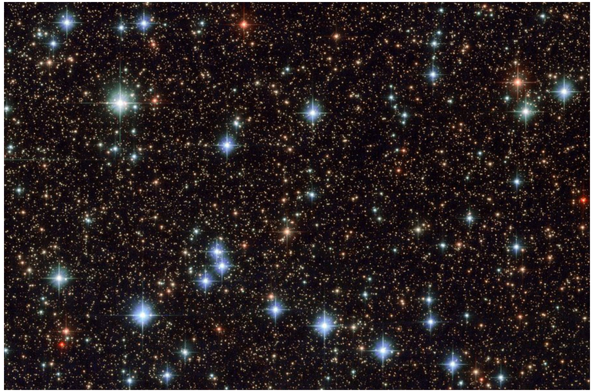




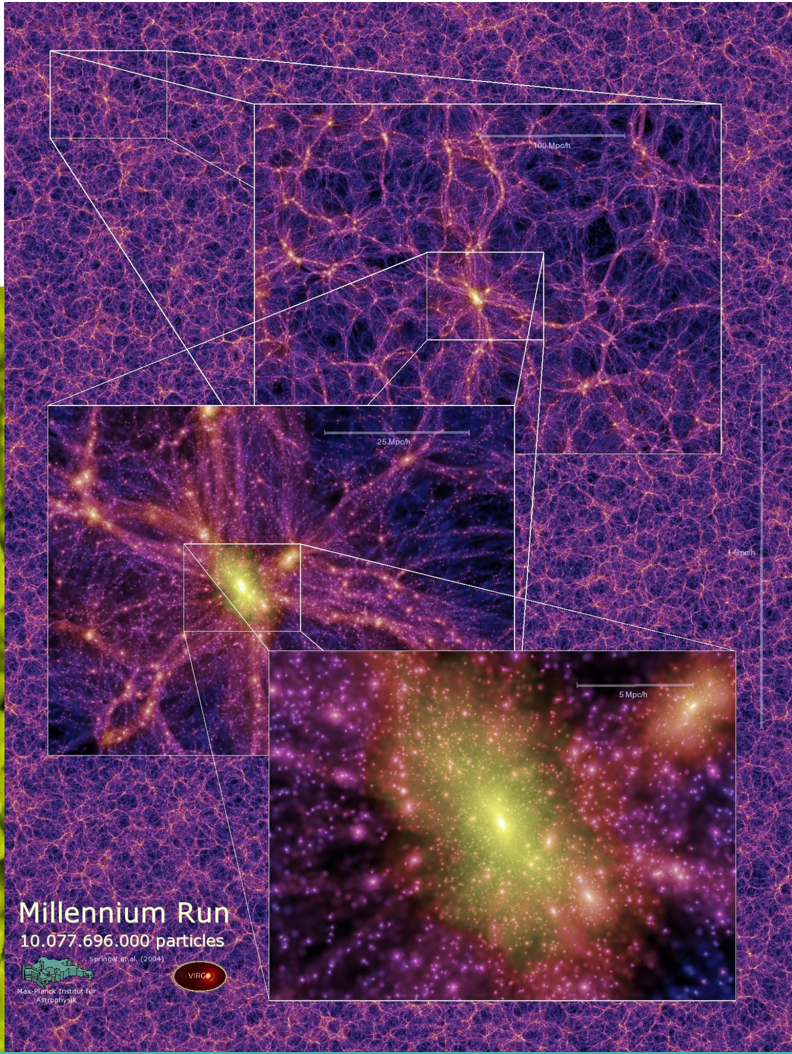
Wrapping up and concluding



$\sim$



Thanks a lot!


100 Mpc/h
25 Mpc/h
5 Mpc/h
Millennium Run
10.077.696.000 particles
Springel et al. (2004)
Max-Planck Institut für Astrophysik