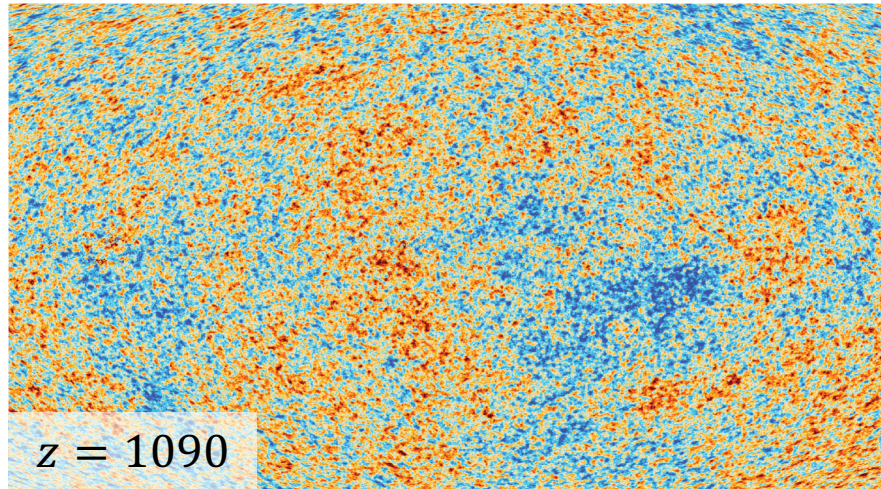


Large Scale Structure Beyond the Two-Point Function

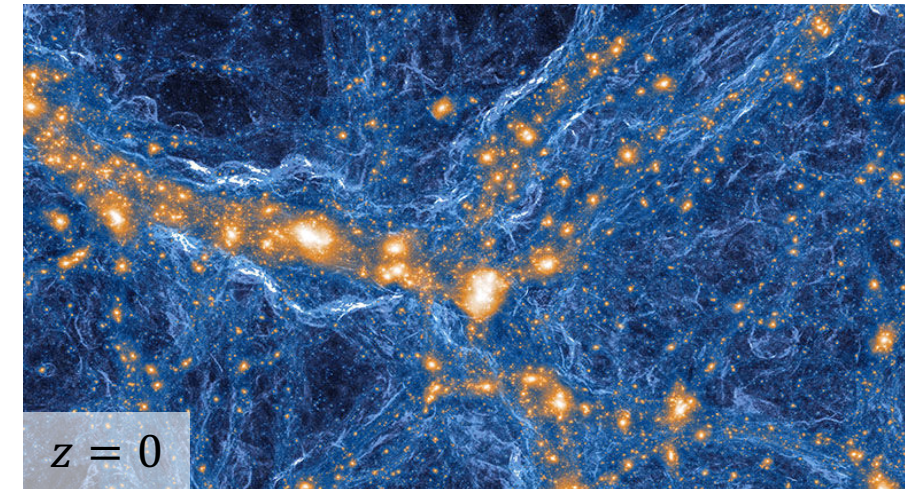
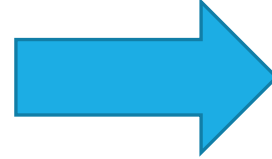
Oliver Philcox (Princeton / IAS)

BCCP Cosmology Seminar 9/21/21

THE LATE UNIVERSE IS NOT GAUSSIAN



Gravitational
Collapse



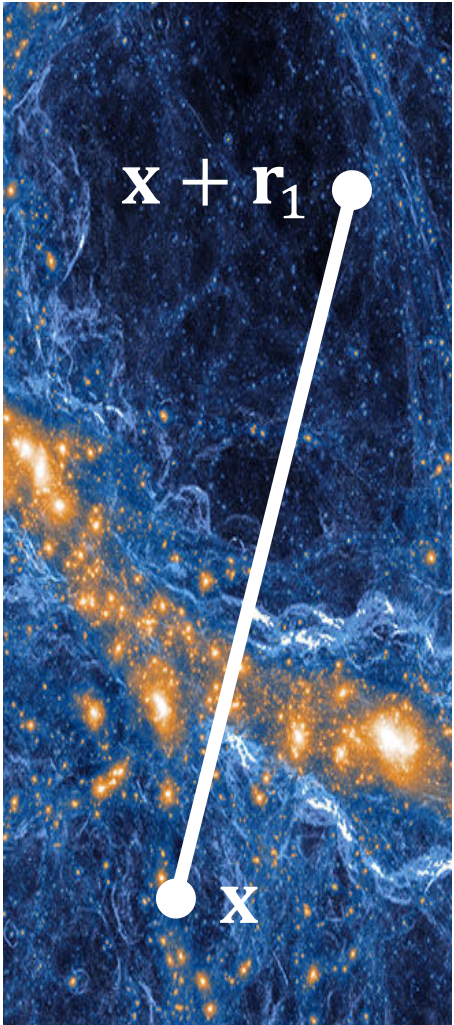
$$\delta(\mathbf{k}) \sim \mathcal{N}(0, P_L(\mathbf{k}))$$

$$\delta(\mathbf{k}) \not\sim \mathcal{N}(0, P_L(\mathbf{k}))$$

- ▶ All information contained in the power spectrum
- ▶ **No** higher order statistics needed!

- ▶ **Not** all information contained in the power spectrum
- ▶ Higher-order statistics needed!

NON-GAUSSIAN DENSITY $\Rightarrow$ NON-GAUSSIAN STATISTICS



Gaussian

1. Power Spectrum:

$$P(\mathbf{k}_1) = \langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \rangle'$$

2. 2-Point Correlation Function:

$$\xi(\mathbf{r}_1) \equiv \langle \delta(\mathbf{x}) \delta(\mathbf{x} + \mathbf{r}_1) \rangle$$

Non-Gaussian

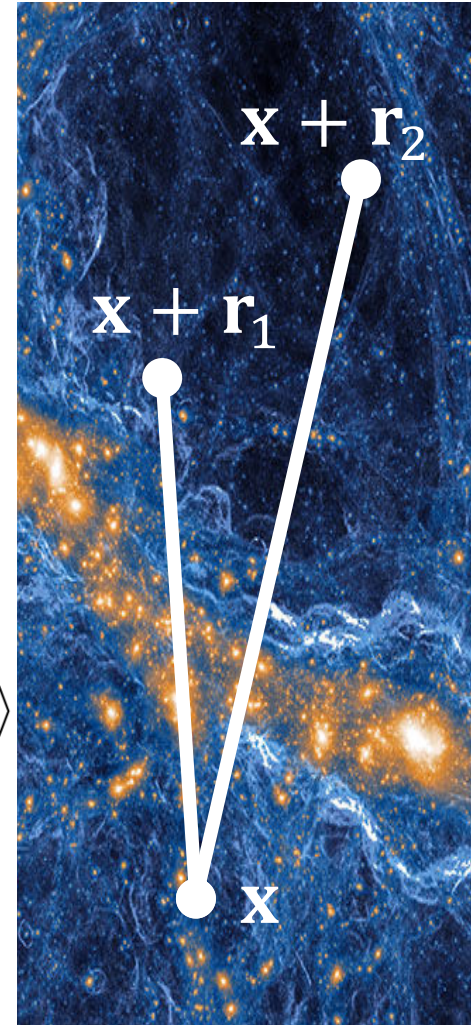
1. Bispectrum:

$$B(\mathbf{k}_1, \mathbf{k}_2) = \langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta(\mathbf{k}_3) \rangle'$$

2. 3-Point Correlation Function:

$$\zeta_3(\mathbf{r}_1, \mathbf{r}_2) \equiv \langle \delta(\mathbf{x}) \delta(\mathbf{x} + \mathbf{r}_1) \delta(\mathbf{x} + \mathbf{r}_2) \rangle$$

And beyond...



INFORMATION LEAKS INTO HIGHER-ORDER STATISTICS

$$B_g(\mathbf{k}_1, \mathbf{k}_2) = \left[2b_1^3 F_2(\mathbf{k}_1, \mathbf{k}_2) + b_2 b_1^2 + 2b_{s^2} b_1^2 \left(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2 - 1/3 \right) \right] P_L(k_1) P_L(k_2) + 2 \text{ perms.}$$

The galaxy bispectrum depends on **galaxy formation physics**, **gravity**, and **early-Universe cosmology**.

▶ To obtain **all** the large-scale information in the initial conditions, we need:*

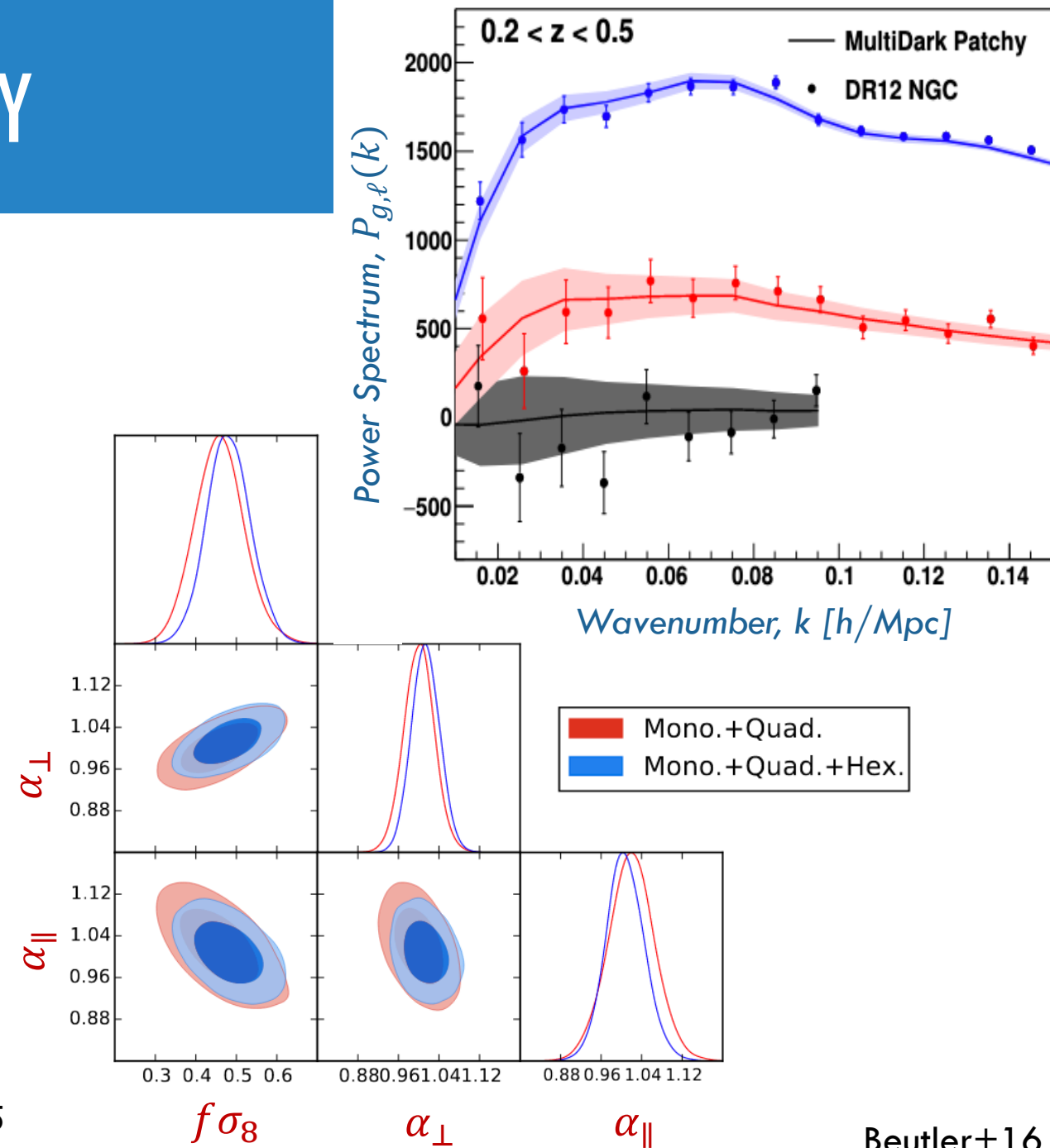
- Power Spectra / 2-Point Functions $\sim P_L(k)$
- Bispectra / 3-Point Functions $\sim P_L^2(k)$
- Trispectra / 4-Point Functions $\sim P_L^3(k)$
- *etc.*

THE CURRENT STATE OF PLAY

- ▷ Only the galaxy **power spectrum** is analyzed
- ▷ Fit to $P_{g,\ell}(k)$ to a simple template searching for **wiggle positions** and **overall amplitudes**, i.e. $[\alpha_{\parallel}, \alpha_{\perp}, f\sigma_8]$
- ▷ 3- and 4-point functions ignored

With a few exceptions...

[e.g. Ivanov+19,20, d'Amico+19,20, Philcox+20, Gil-Marín+15,16]



MORE STATISTICS = MORE INFORMATION

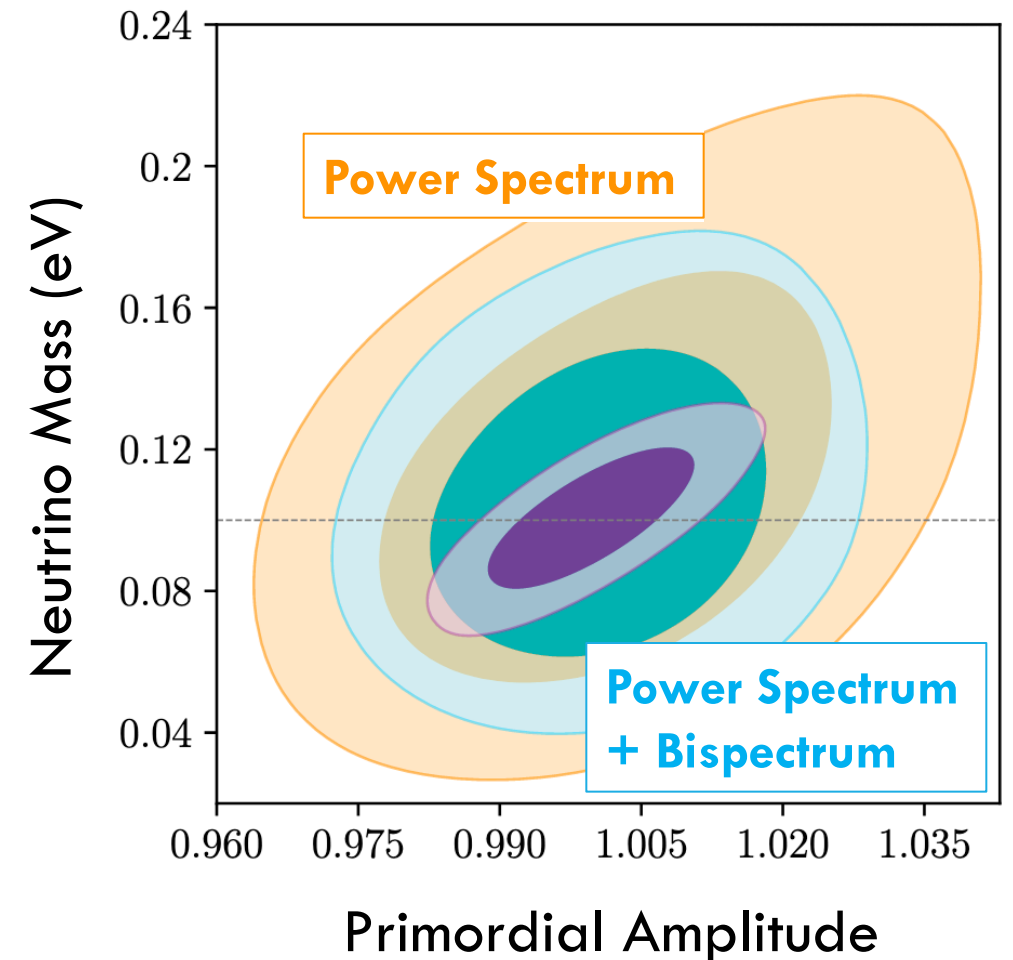
- ▶ **Break** parameter **degeneracies!**

[e.g. $P_g \sim b_1^2 \sigma_8^2$, $B_g \sim b_1^3 \sigma_8^4$]

- ▶ **Sharpen** cosmological **constraints!**

Euclid Forecast [perturbative]

- ▶ Bispectrum improves constraints by $\approx 40\%$
- ▶ 1σ constraint of $\sigma_{M_\nu} = 0.013$ eV [including *Planck*]



MORE STATISTICS = MORE INFORMATION

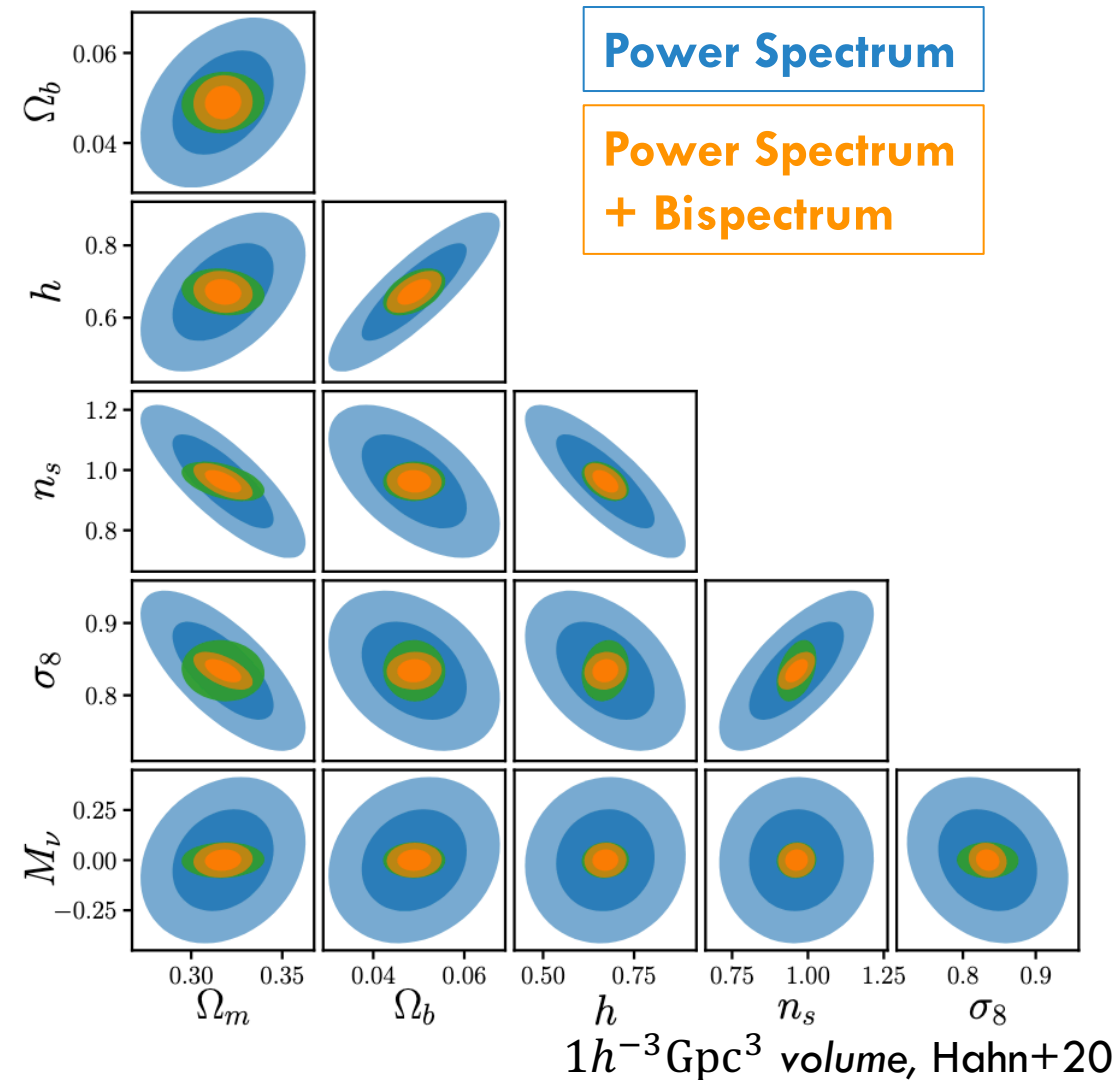
- ▶ **Break parameter degeneracies!**

[e.g. $P_g \sim b_1^2 \sigma_8^2$, $B_g \sim b_1^3 \sigma_8^4$]

- ▶ **Sharpen cosmological constraints!**

Simulation-Based Forecast

- ▶ Bispectrum improves constraints by $> 2\times$
- ▶ Neutrino constraint improves by $5\times$



*Marginalizing over HODs, with $k_{\text{max}} = 0.5h \text{ Mpc}^{-1}$.

NON-GAUSSIAN INFLATION

Are the primordial perturbations **Gaussian** and **adiabatic**?

Standard Model of Inflation:

- ▶ Scalar field ϕ rolling down a potential $V(\phi)$

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) \right]$$

Gravity Kinetic Energy Potential

- ▶ Action, S , encodes **statistics** of the **primordial** curvature perturbations, ζ

Second Order $\Rightarrow$ Power Spectrum

$$S^{(2)} \Rightarrow P_\zeta(k) \approx A_s k^{n_s - 4}$$

Third Order $\Rightarrow$ Bispectrum

$$S^{(3)} \Rightarrow B_\zeta(\mathbf{k}_1, \mathbf{k}_2) \approx \frac{6}{5} f_{\text{NL}} P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perms.}$$

Generates **non-Gaussianity** proportional to f_{NL}

NON-GAUSSIAN INFLATION

$$B_{\zeta}(\mathbf{k}_1, \mathbf{k}_2) \approx \frac{6}{5} f_{\text{NL}} P_{\zeta}(k_1) P_{\zeta}(k_2) + 2 \text{ perms.}$$

The **consistency condition** states that

$$\lim_{k_1 \rightarrow 0} B_{\zeta}(\mathbf{k}_1, \mathbf{k}_2) = (1 - n_s) P_{\zeta}(k_1) P_{\zeta}(k_2)$$



$$f_{\text{NL}} \sim (1 - n_s) \ll 1$$



Non-Gaussianity is too small to be detected!

Non-standard inflation can beat this, e.g.

- ▶ Multifield Inflation [Local Bispectrum]
- ▶ New Kinetic Terms [Equilateral Bispectrum]
- ▶ New Vacuum States [Folded Bispectrum]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} \partial^{\mu} \phi \partial_{\mu} \phi - V(\phi) \right]$$

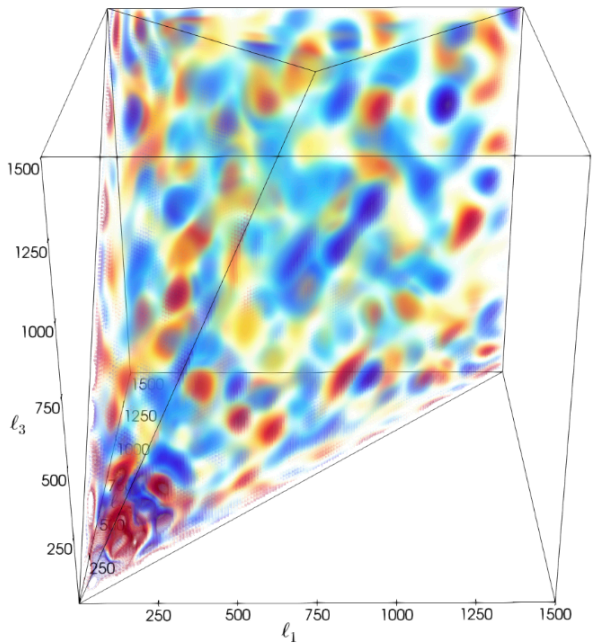
NON-GAUSSIAN INFLATION

$$B_{\zeta}(\mathbf{k}_1, \mathbf{k}_2) \approx \frac{6}{5} f_{\text{NL}} P_{\zeta}(k_1) P_{\zeta}(k_2) + 2 \text{ perms.}$$

How do we measure Primordial non-Gaussianity?

1. CMB Bispectrum

Planck TTT Bispectrum



f_{NL} Constraints

Local	6.7 ± 5.6
Equilateral	6 ± 66
Orthogonal	-38 ± 36

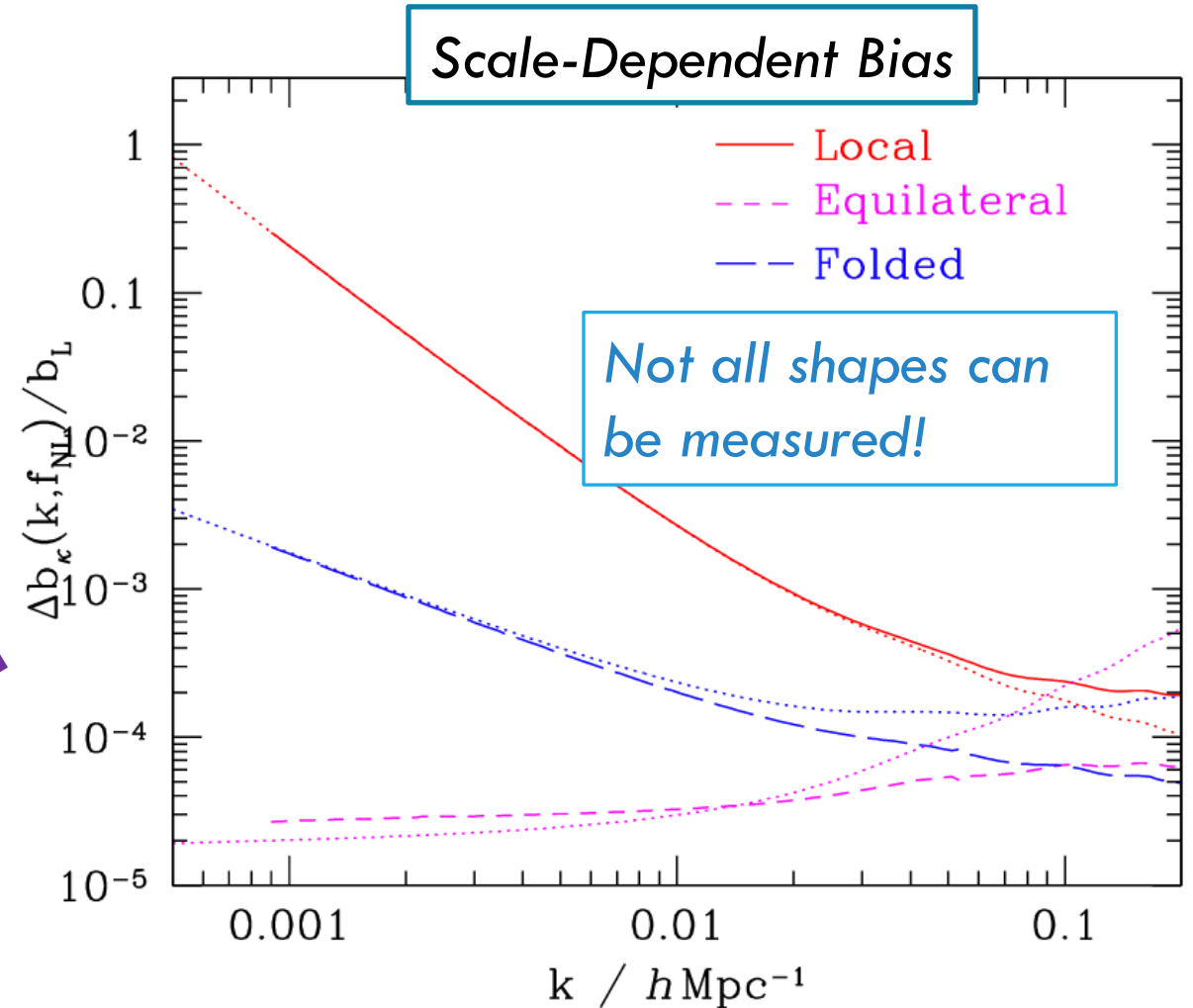
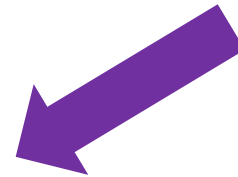
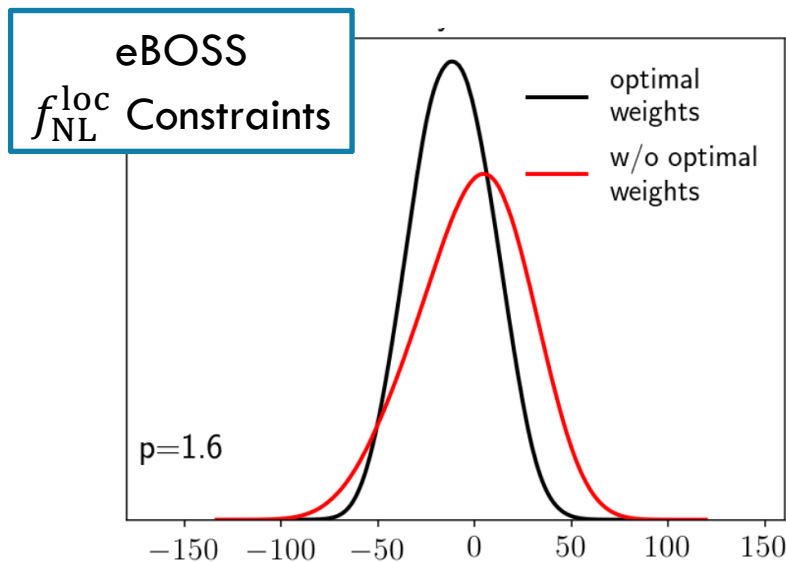
NON-GAUSSIAN INFLATION

$$B_{\zeta}(\mathbf{k}_1, \mathbf{k}_2) \approx \frac{6}{5} f_{\text{NL}} P_{\zeta}(k_1) P_{\zeta}(k_2) + 2 \text{ perms.}$$

How do we measure this?

1. CMB Bispectrum

2. Galaxy Power Spectrum

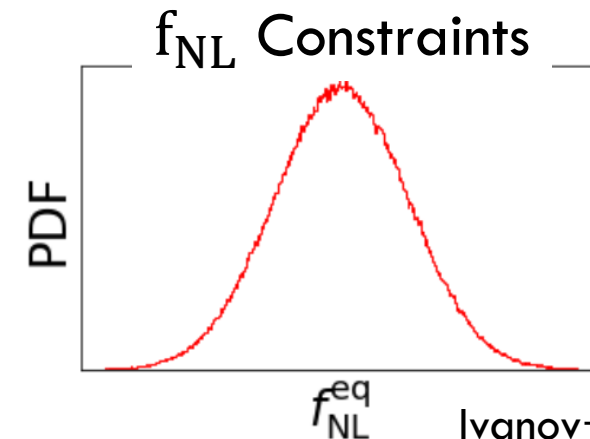
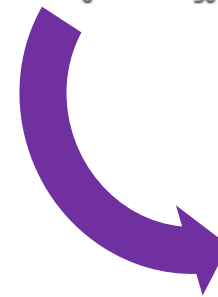
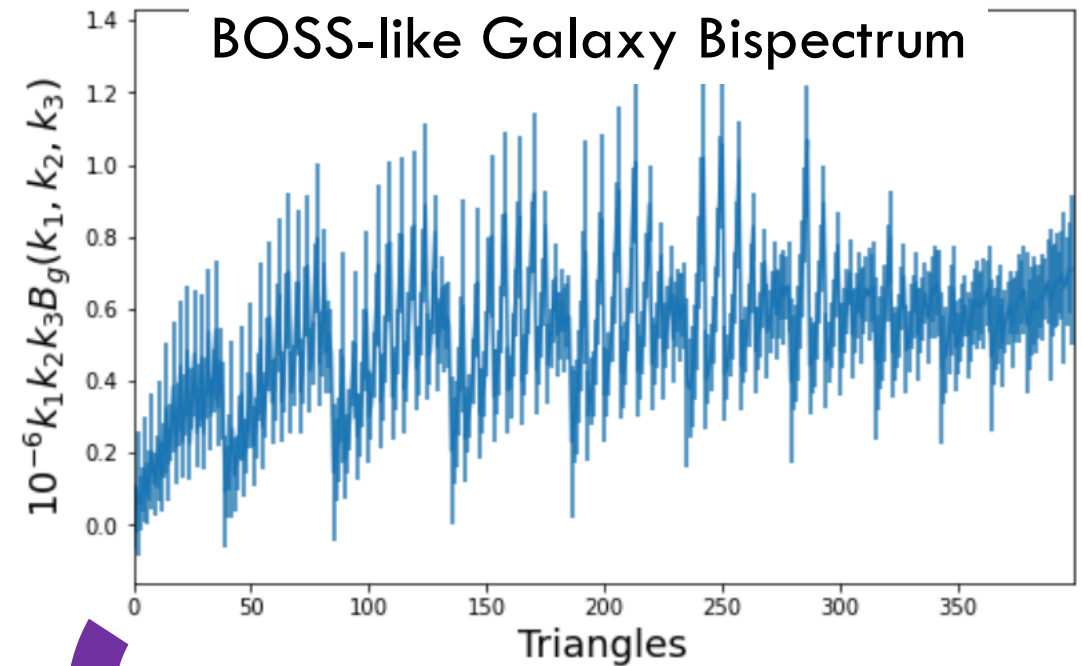


NON-GAUSSIAN INFLATION

$$B_{\zeta}(\mathbf{k}_1, \mathbf{k}_2) \approx \frac{6}{5} f_{\text{NL}} P_{\zeta}(k_1) P_{\zeta}(k_2) + 2 \text{ perms.}$$

How do we measure this?

1. CMB Bispectrum
2. Galaxy Power Spectrum
3. **Galaxy Bispectrum**



Ivanov+, Philcox+ (in prep.)

CHERN-SIMONS INTERACTIONS VIOLATE PARITY

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) - \frac{1}{4} f(\phi) F_{\mu\nu} F^{\mu\nu} + \frac{\gamma}{4} f(\phi) F_{\mu\nu} \tilde{F}^{\mu\nu} \right]$$

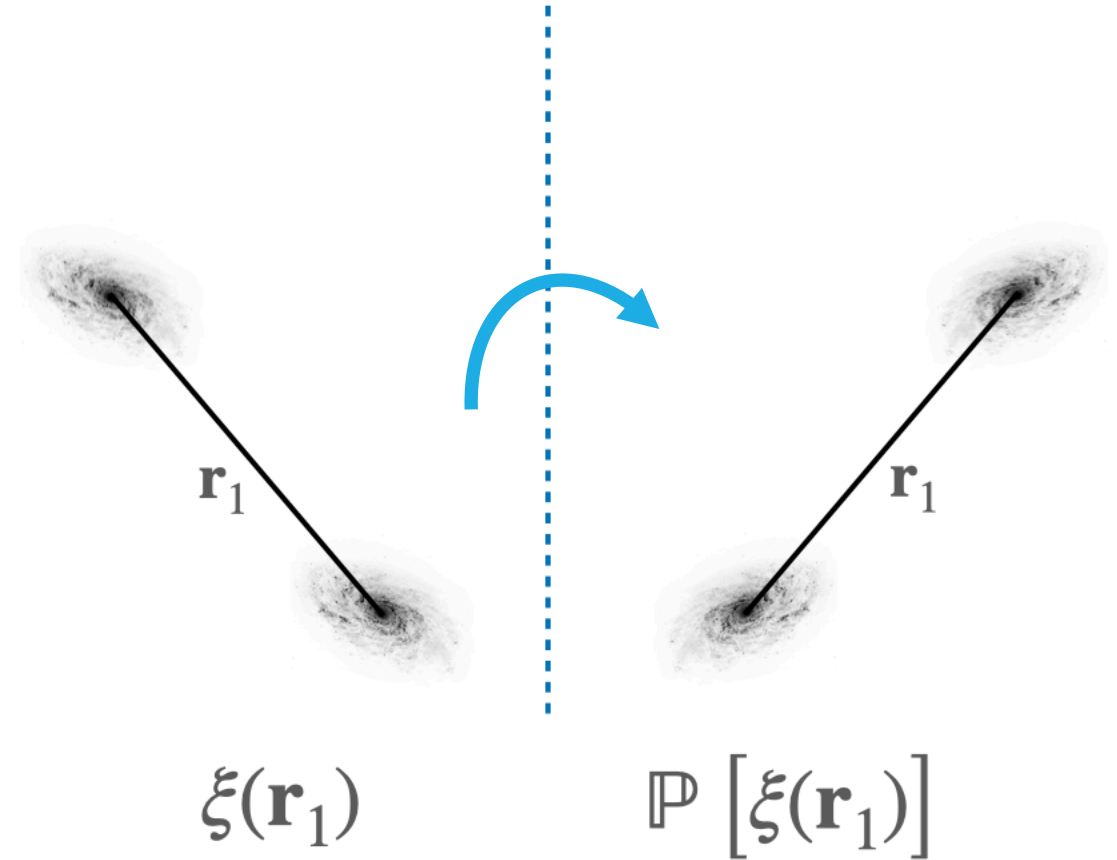
- ▶ Add a **gauge field** A_μ to the inflationary action, via $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$
- ▶ This can include a **Chern-Simons coupling** to the (pseudo-)scalar ϕ [motivated by baryogenesis]
- ▶ $f(\phi) F_{\mu\nu} \tilde{F}^{\mu\nu}$ violates **parity symmetry** $\Rightarrow$ parity-violating correlators!

Where should we look for these signatures?

THE 2PCF + 3PCF ARE NOT SENSITIVE TO PARITY*

2-Point Correlation Function (2PCF):

Parity Inversion = Rotation



*Except for the polarized CMB, and redshift-space effects

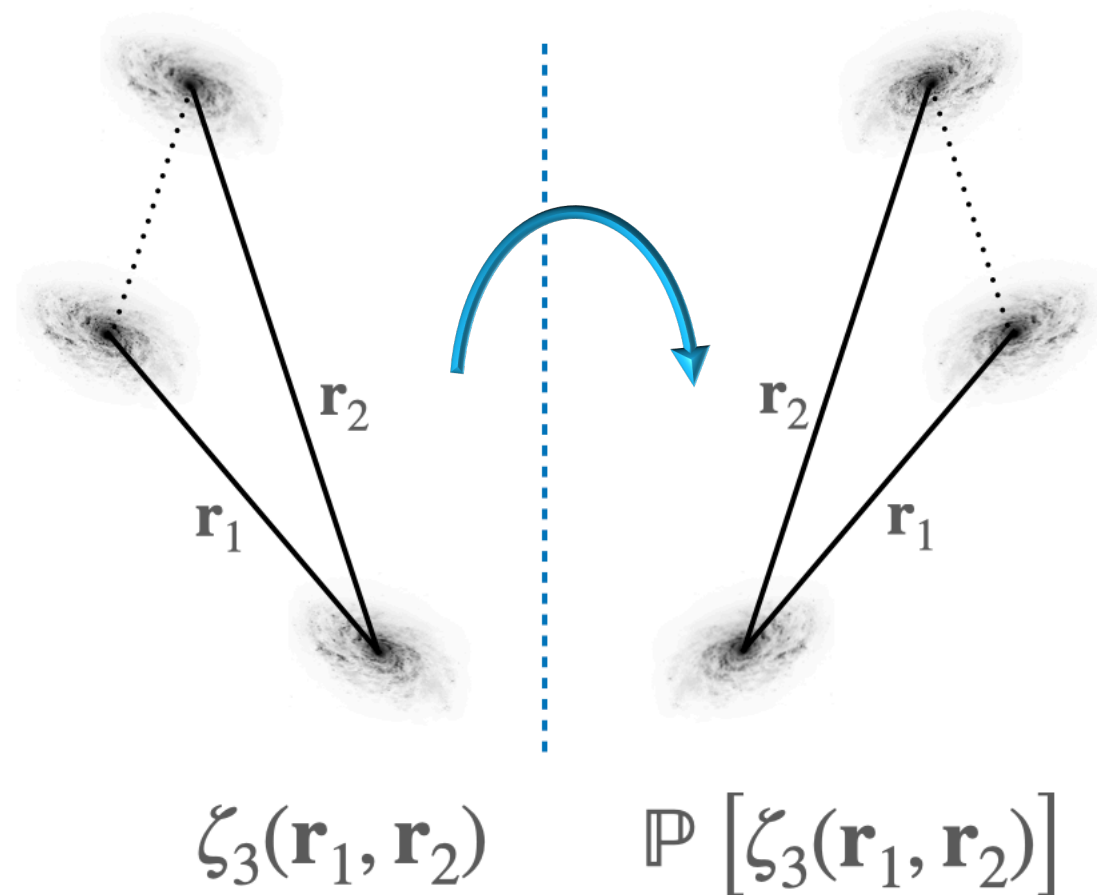
THE 2PCF + 3PCF ARE NOT SENSITIVE TO PARITY

2-Point Correlation Function (2PCF):

Parity Inversion = Rotation

3-Point Correlation Function (3PCF):

Parity Inversion = Rotation



THE 2PCF + 3PCF ARE NOT SENSITIVE TO PARITY

2-Point Correlation Function (2PCF):

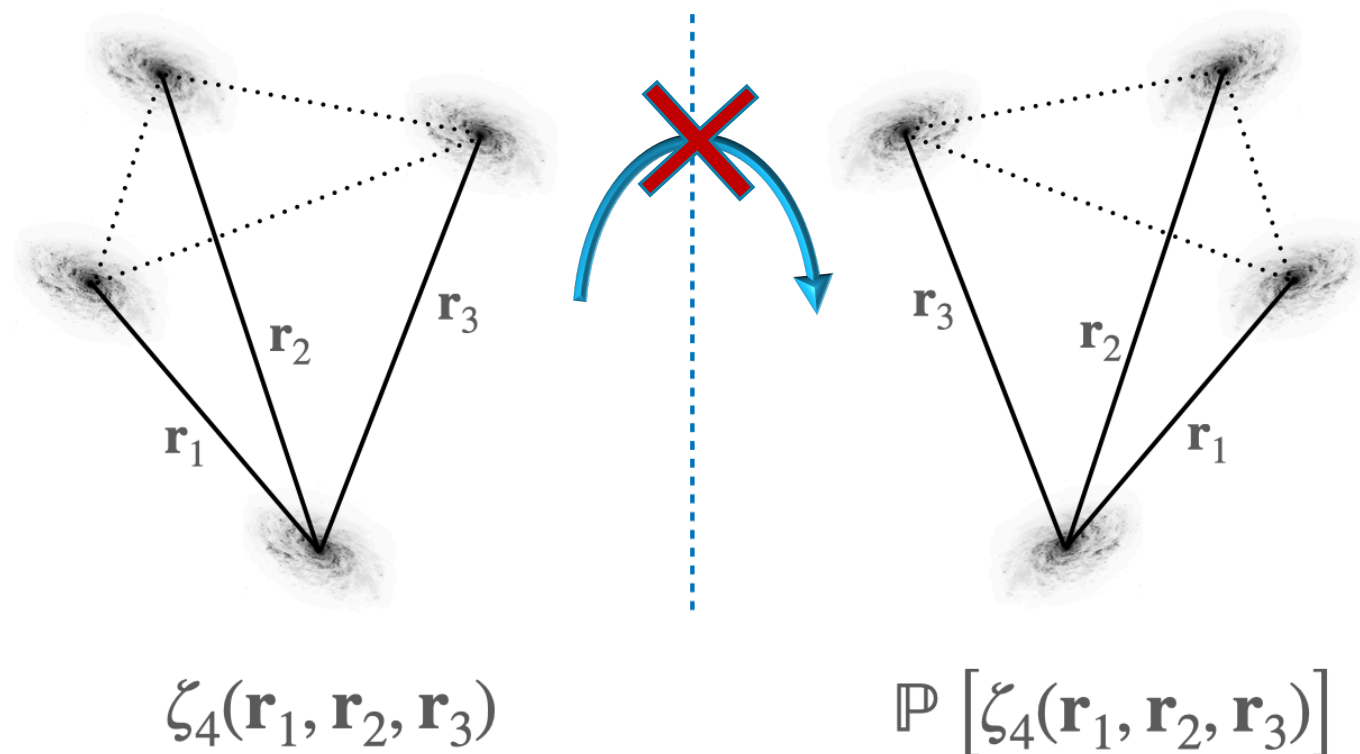
Parity Inversion = Rotation

3-Point Correlation Function (3PCF):

Parity Inversion = Rotation

4-Point Correlation Function (4PCF):

Parity Inversion $\neq$ Rotation



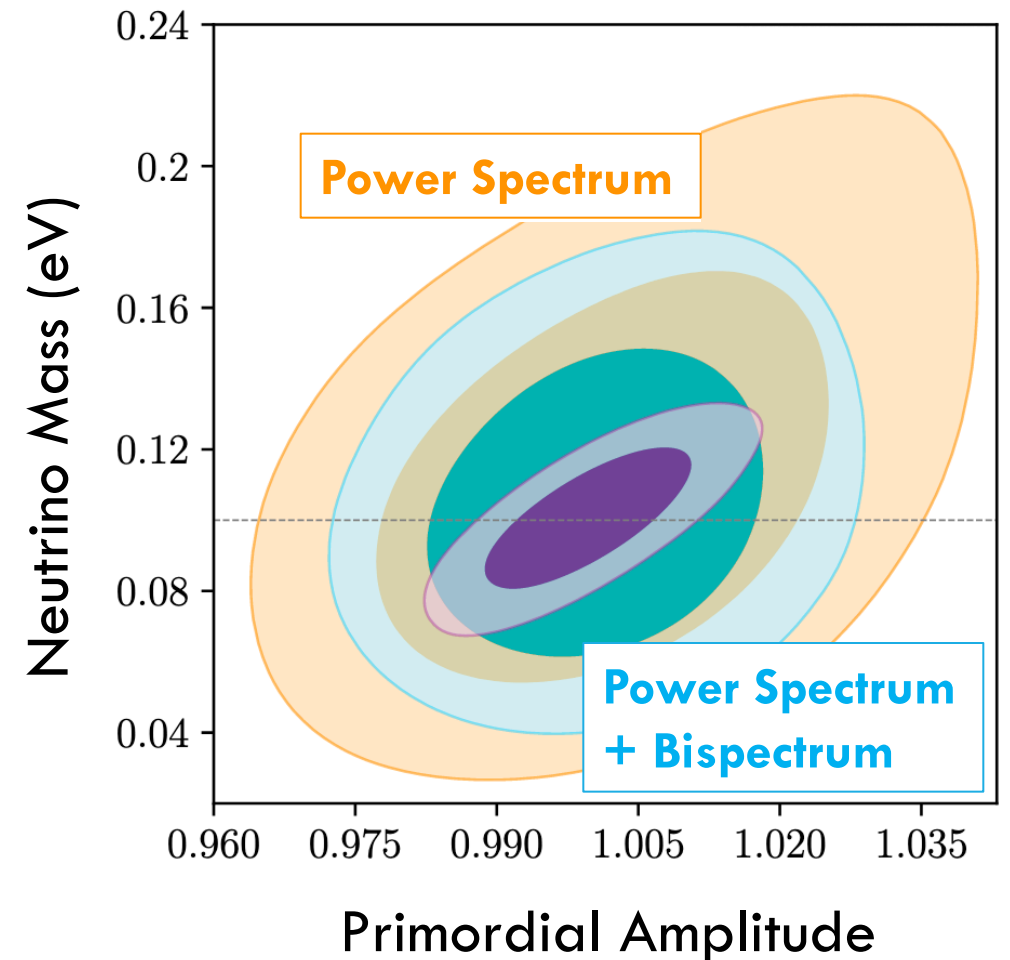
Use the 4PCF to search for Chern Simons terms!

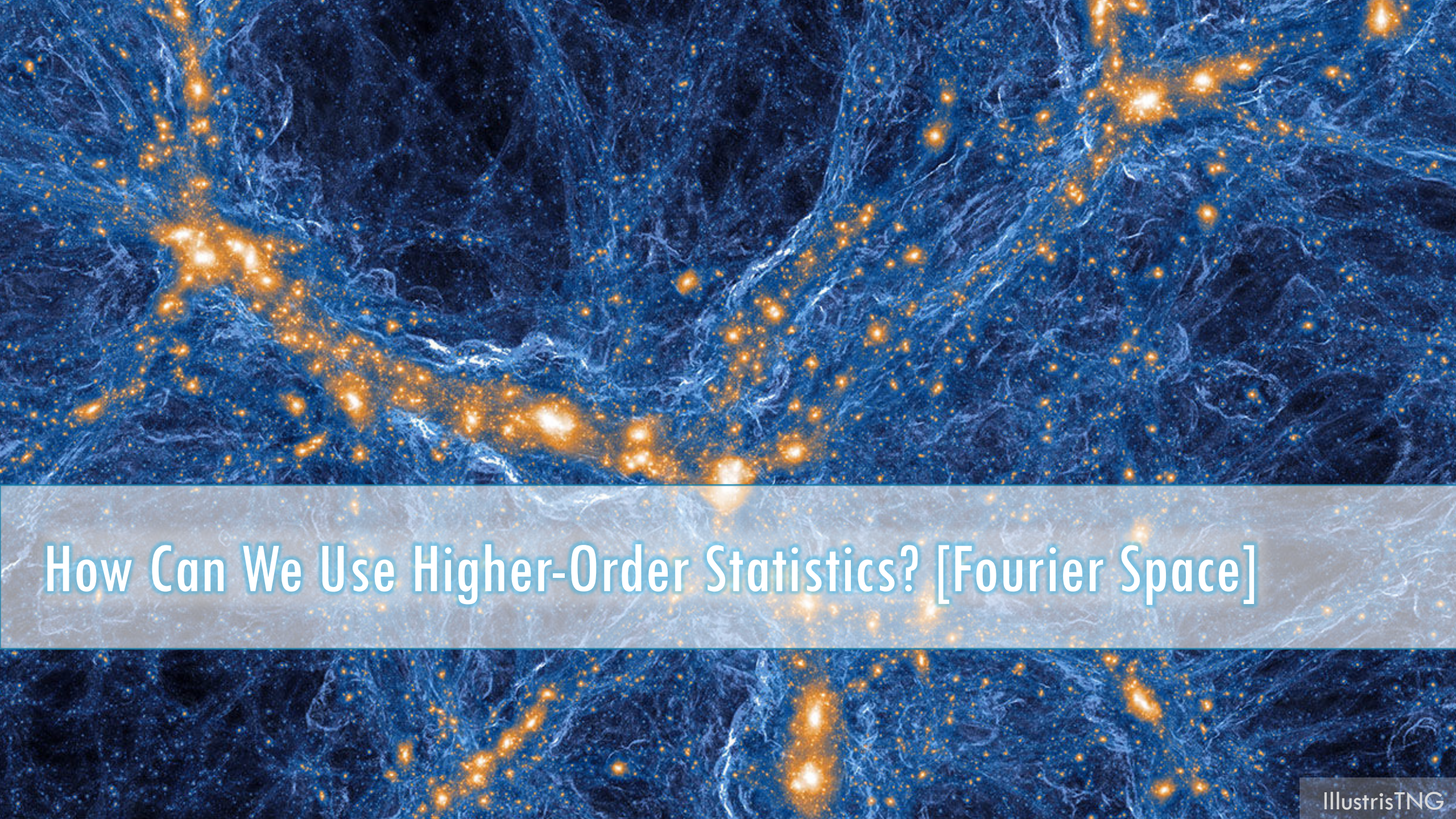
WHY USE HIGHER-ORDER STATISTICS?

- ▷ **Sharpen** parameter constraints!
- ▷ **Break** parameter **degeneracies**!
- ▷ Test **non-standard** physics models!

Why Use Large Scale Structure?

- Signal-to-Noise $\sim k_{\text{max}}^3$, unlike $\sim \ell_{\text{max}}^2$ for CMB
- New physics constraints **don't** dilute with redshift



A visualization of the cosmic web, showing a complex network of blue filaments and nodes of orange-yellow galaxies against a dark background. The filaments represent the large-scale structure of the universe, while the orange-yellow spots represent individual galaxies or clusters of galaxies.

How Can We Use Higher-Order Statistics? [Fourier Space]

HOW TO MEASURE A BISPECTRUM

Problem: We don't measure the density field directly.

$$\delta_g(\mathbf{r}) \rightarrow W(\mathbf{r})\delta_g(\mathbf{r}) \quad \delta(\mathbf{k}) \rightarrow \int \frac{d\mathbf{p}}{(2\pi)^3} W(\mathbf{k} - \mathbf{p})\delta(\mathbf{p}).$$

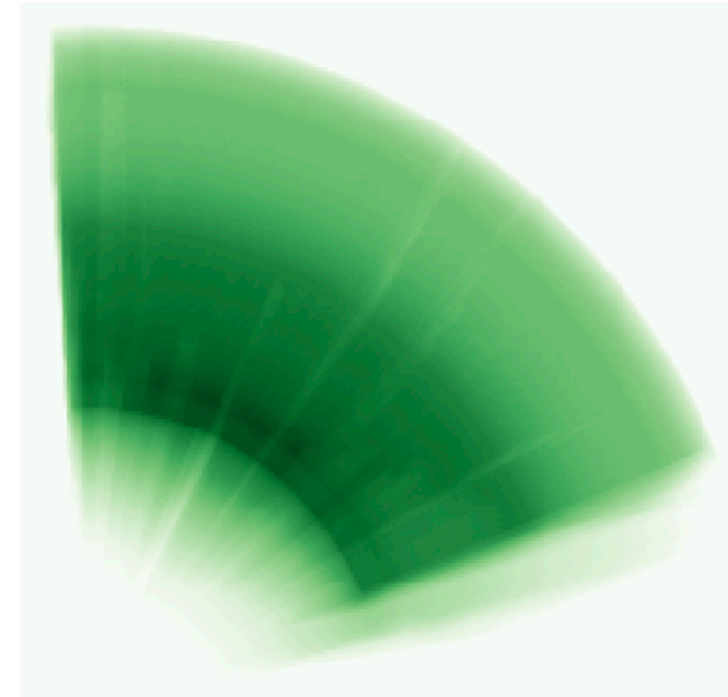

Window Function

The measured bispectrum is a triple **convolution**

$$B_g(\mathbf{k}_1, \mathbf{k}_2) \rightarrow \int_{\mathbf{p}_1 \mathbf{p}_2} W(\mathbf{k}_1 - \mathbf{p}_1) W(\mathbf{k}_2 - \mathbf{p}_2) W(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{k}_1 - \mathbf{k}_2) B_g(\mathbf{p}_1, \mathbf{p}_2)$$

Solution: Convolve the **theory model** too

$$\hat{B}_g(k_1, k_2, k_3) = \int_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3 \in \text{bins}} \delta_g(\mathbf{k}_1) \delta_g(\mathbf{k}_2) \delta_g(\mathbf{k}_3) (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$



Survey Window Function

CONVOLUTION IS EXPENSIVE

$$B_g^{\text{win}}(\mathbf{k}_1, \mathbf{k}_2) = \int_{\mathbf{p}_1 \mathbf{p}_2} W(\mathbf{k}_1 - \mathbf{p}_1) W(\mathbf{k}_2 - \mathbf{p}_2) W(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{k}_1 - \mathbf{k}_2) B_g(\mathbf{p}_1, \mathbf{p}_2)$$

- ▶ Window convolution is too costly to do repeatedly!
- ▶ Common approximation: apply the window **only** to the power spectrum

$$B_g(\mathbf{k}_1, \mathbf{k}_2) \supset P_L(k_1) P_L(k_2)$$

But:

- This gives **systematic shifts** on large scales
- Spectra cannot be used to search for new physics!

BISPECTRA WITHOUT WINDOWS

New Approach

- ▶ Start from the **likelihood** for data $\mathbf{d}$, using an Edgeworth expansion

$$L[\mathbf{d}](\mathbf{b}) = L_G[\mathbf{d}](\mathbf{b}) \left[1 + \frac{1}{3!} \mathbf{B}^{ijk} \left\{ [\mathbf{C}^{-1}\mathbf{d}]_i [\mathbf{C}^{-1}\mathbf{d}]_j [\mathbf{C}^{-1}\mathbf{d}]_k - (\mathbf{C}_{ij}^{-1} d_k + 2 \text{ perms.}) \right\} + \dots \right]$$

Gaussian Piece Three-Point Function, $\mathbf{B}^{ijk} \equiv \langle d^i d^j d^k \rangle$ Covariance, $\mathbf{C}^{ij} = \langle d^i d^j \rangle$

- ▶ This depends on **survey geometry** through $\mathbf{C}^{ij}$ and **bispectrum** through $\mathbf{B}^{ijk}$

$$\nabla_{\mathbf{b}} \log L[\mathbf{d}](\mathbf{b}) = \mathbf{0}$$

- ▶ Optimize for true bispectrum, $\mathbf{b}$:

$$\hat{b}_{\alpha}^{\text{ML}} = \sum_{\beta} F_{\alpha\beta}^{-1, \text{ML}} \hat{q}_{\beta}^{\text{ML}},$$

$$\hat{q}_{\alpha}^{\text{ML}} = \frac{1}{6} \mathbf{B}_{,\alpha}^{ijk} [\mathbf{C}^{-1}\mathbf{d}]_i \left([\mathbf{C}^{-1}\mathbf{d}]_j [\mathbf{C}^{-1}\mathbf{d}]_k - 3\mathbf{C}_{jk}^{-1} \right)$$

Cubic Estimator

$$F_{\alpha\beta}^{\text{ML}} = \frac{1}{6} \mathbf{B}_{,\alpha}^{ijk} \mathbf{B}_{,\beta}^{lmn} \mathbf{C}_{il}^{-1} \mathbf{C}_{jm}^{-1} \mathbf{C}_{kn}^{-1},$$

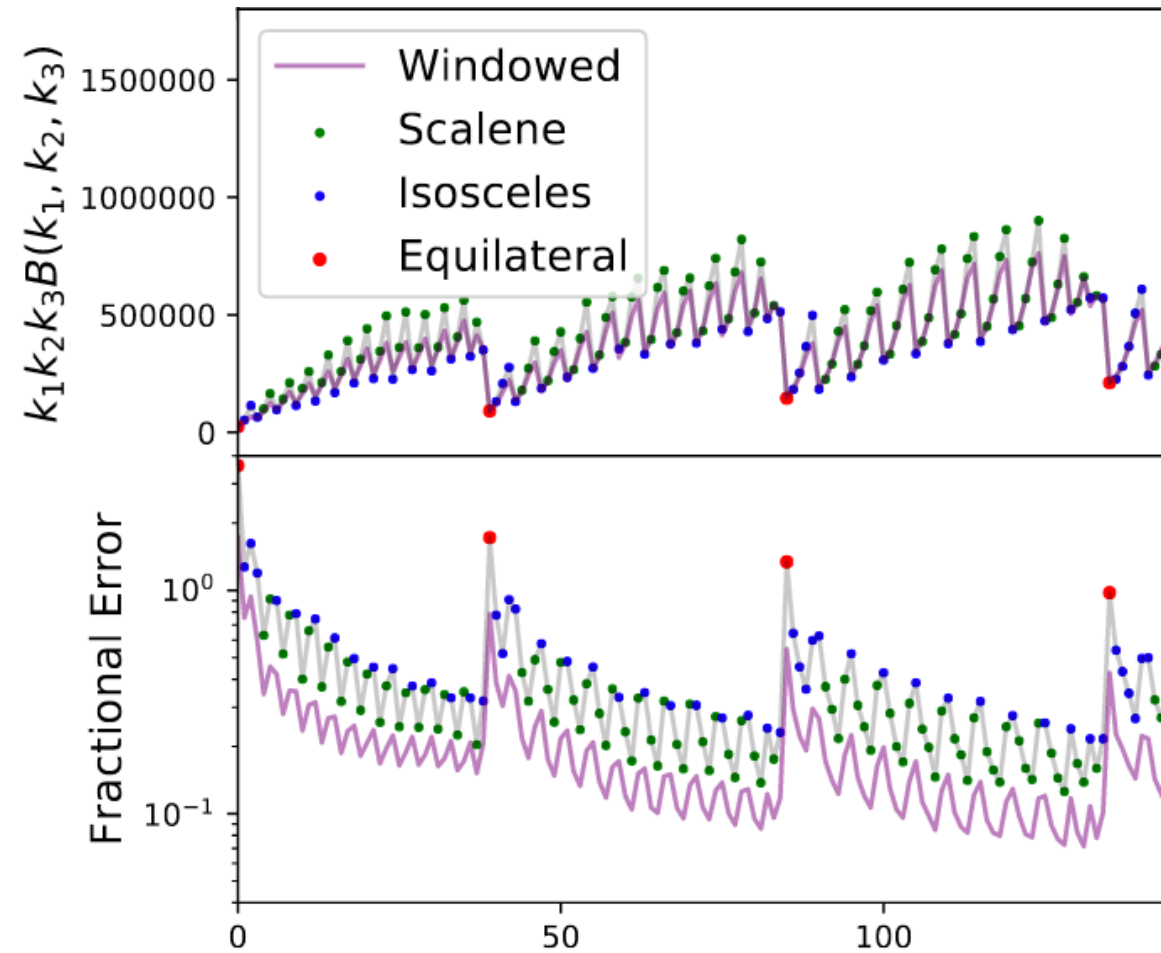
Fisher Matrix

BISPECTRA WITHOUT WINDOWS

$$\hat{b}_{\alpha}^{\text{ML}} = \sum_{\beta} F_{\alpha\beta}^{-1,\text{ML}} \hat{q}_{\beta}^{\text{ML}},$$

Properties of the **cubic estimator**:

1. Unbiased
 2. Minimum variance [as $B(k_1, k_2, k_3) \rightarrow 0$]
 3. Window-free [effectively a deconvolution]
- Requires various tricks for dealing with high-dimensional data [e.g. conjugate gradient descent, Monte Carlo estimation etc.]



TOWARDS A ROBUST BISPECTRUM ANALYSIS

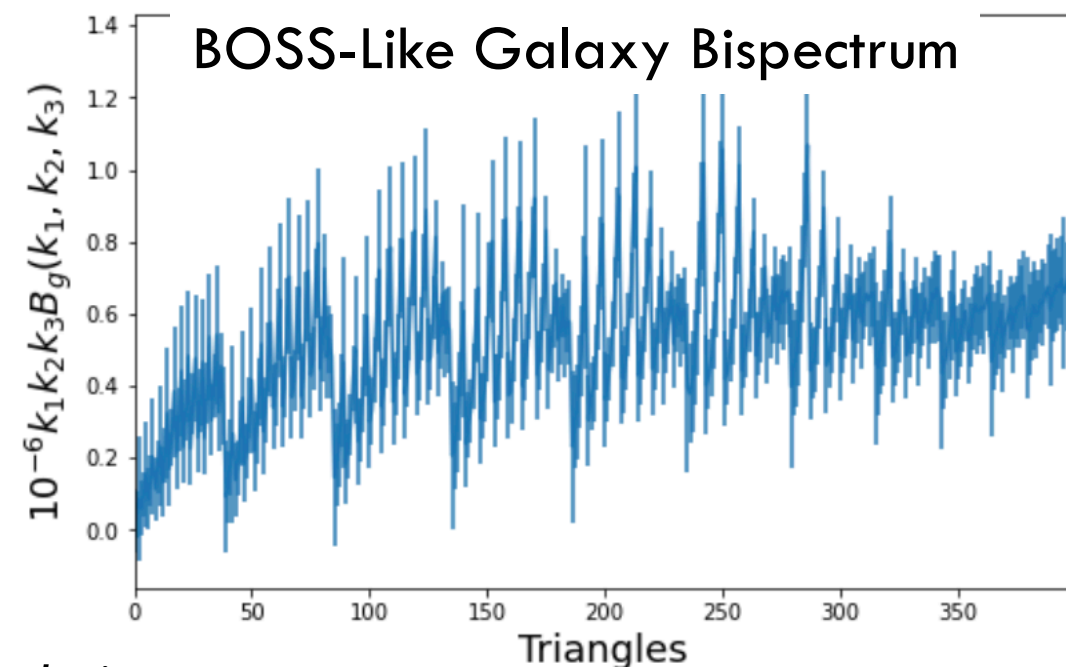
Bispectrum Model Ingredients:

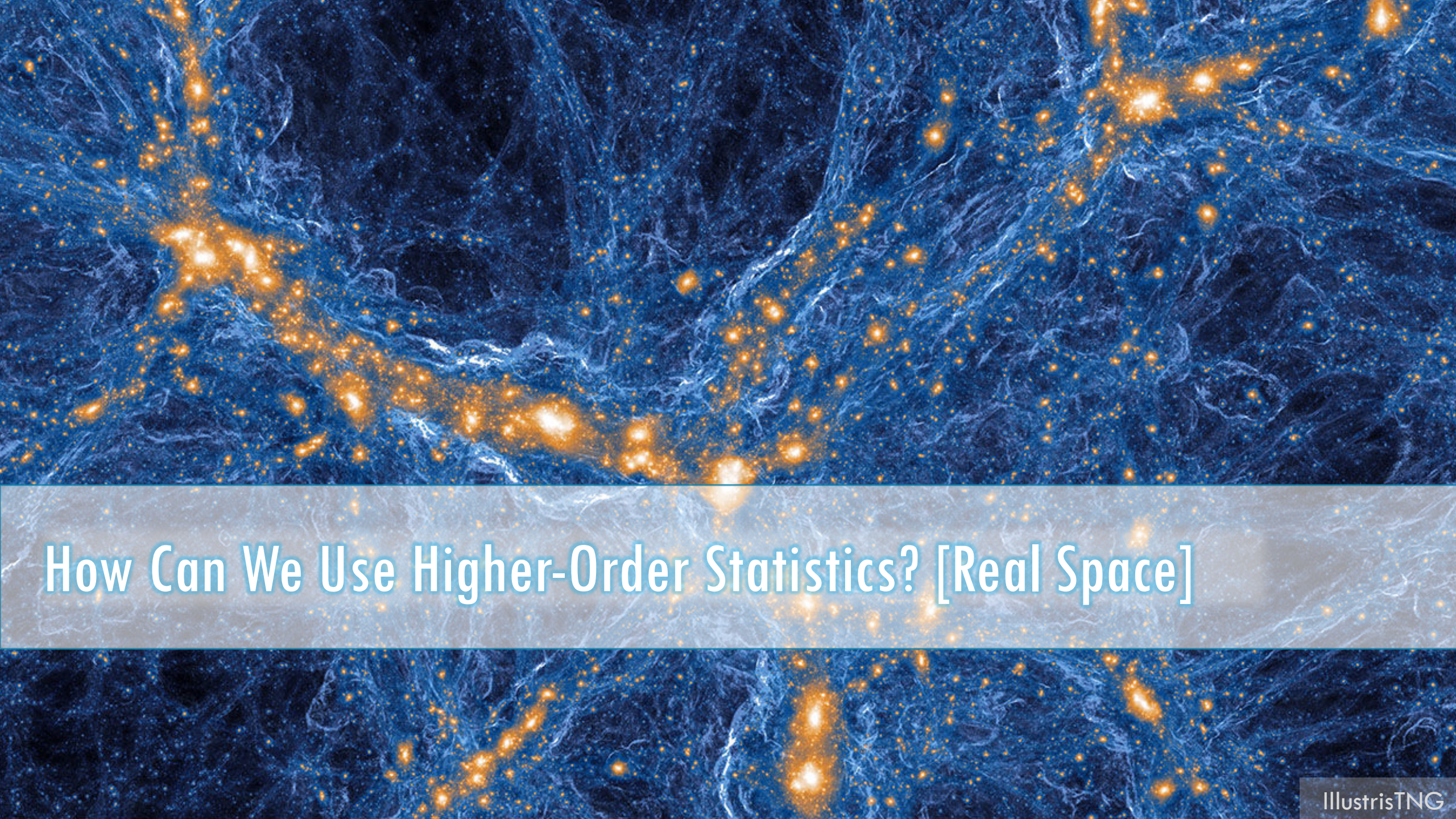
- ▷ ~~Window Function~~
- ▷ Tree-Level Perturbation Theory
- ▷ Redshift-Space Distortions
- ▷ Alcock-Paczynski Effects
- ▷ Infrared Resummation
- ▷ Discreteness Effects

Theory model under development using high-resolution simulations

Applications:

1. Sharper constraints on cosmological parameters ($\approx 20\%$)
2. Primordial Non-Gaussianity Measurements, e.g. $\sigma_{f_{\text{NL}}^{\text{eq}}} \approx 500$



A visualization of the cosmic web, showing a complex network of blue filaments and nodes of orange-yellow galaxies against a dark background. The filaments represent the large-scale structure of the universe, with galaxies clustered along them.

How Can We Use Higher-Order Statistics? [Real Space]

HOW TO MEASURE A CORRELATION FUNCTION

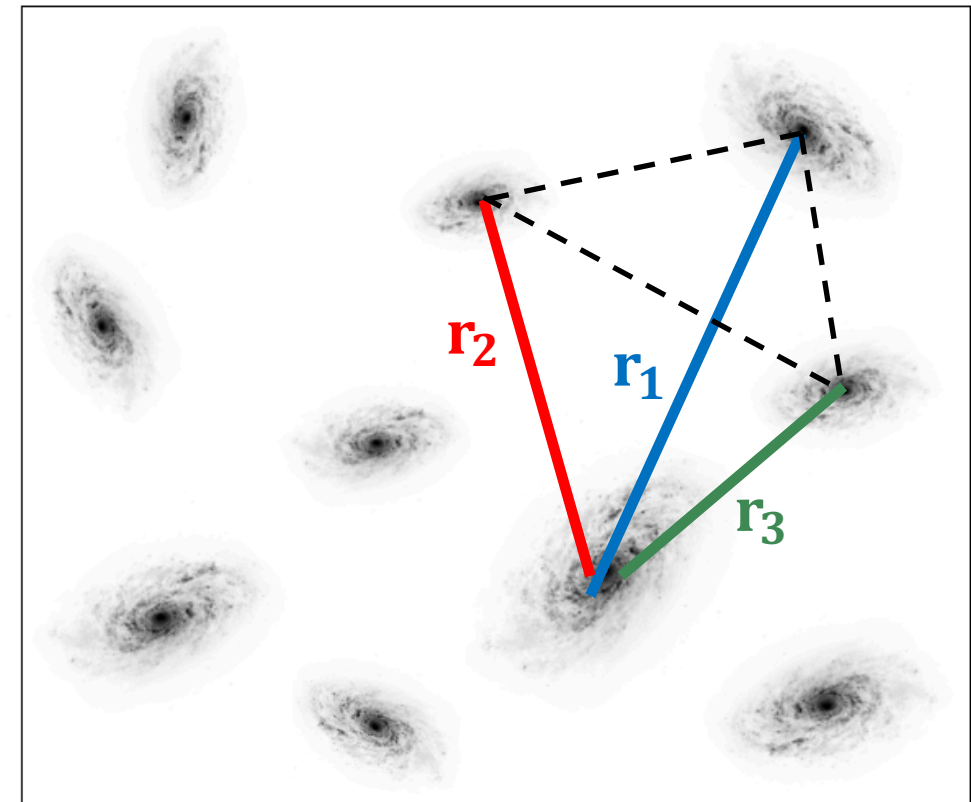
$$\hat{\zeta}_{g,4}(r_1, r_2, r_3, \hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_2, \hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_3, \hat{\mathbf{r}}_2 \cdot \hat{\mathbf{r}}_3) = \int \frac{d\mathbf{x}}{V} \int_{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3 \in \text{bins}} \delta_g(\mathbf{x}) \delta_g(\mathbf{x} + \mathbf{r}_1) \delta_g(\mathbf{x} + \mathbf{r}_2) \delta_g(\mathbf{x} + \mathbf{r}_3)$$

Measuring the 4PCF involves counting **quadruplets** of galaxies

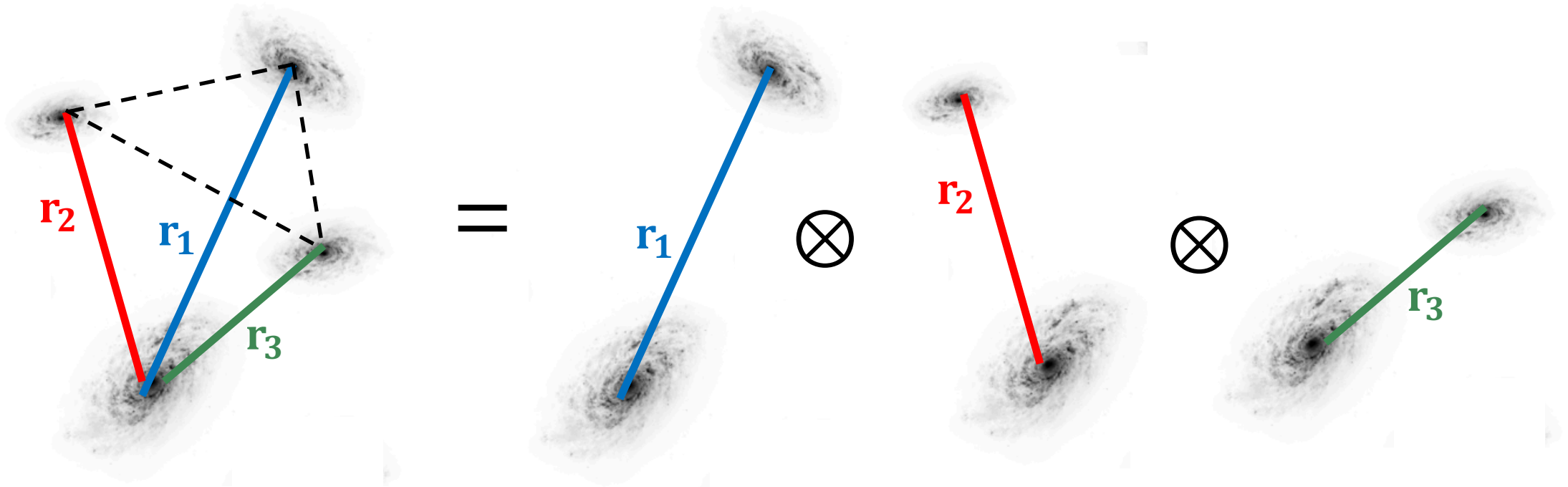
Total number of quadruplets:

$$\mathcal{O}(N_{\text{gal}}^4)$$

This is too many to count...



ONE TETRAHEDRON = THREE VECTORS



3 lengths + 3 angles

$(r_1, r_2, r_3, \hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_2, \hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_3, \hat{\mathbf{r}}_2 \cdot \hat{\mathbf{r}}_3)$

1 length + 1 direction

$(r_1, \hat{\mathbf{r}}_1)$

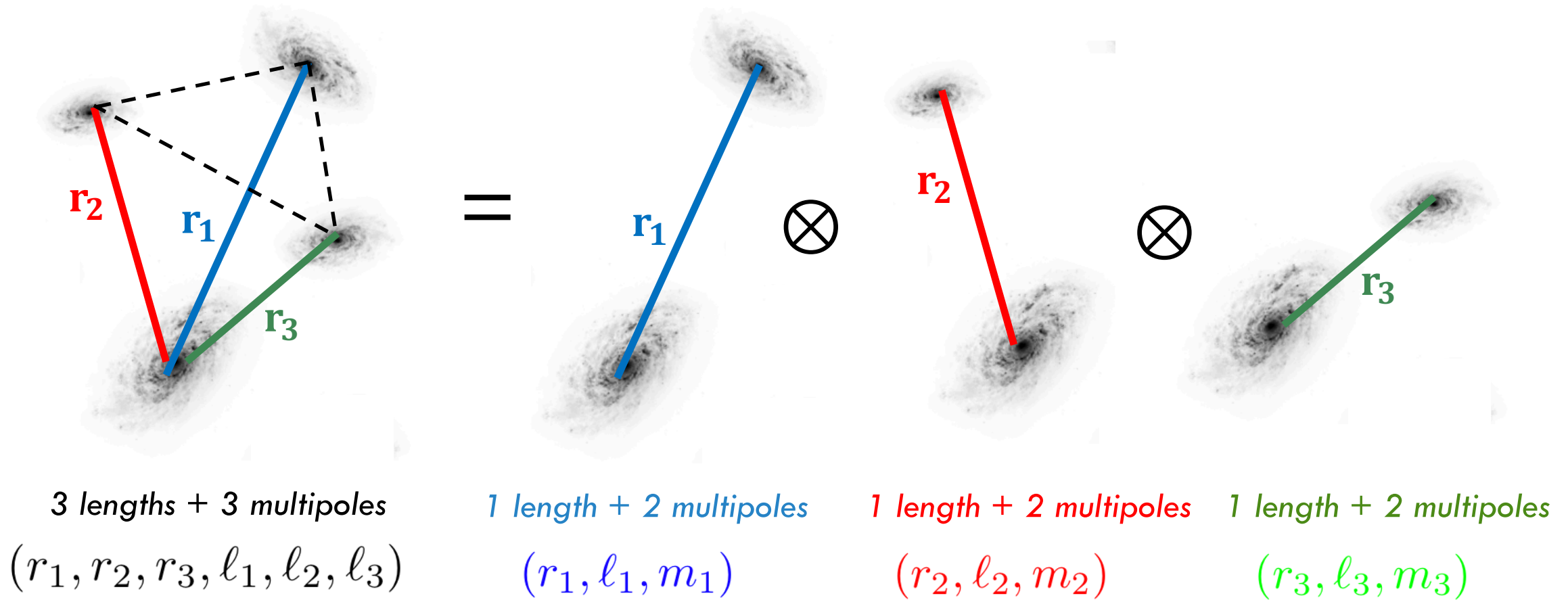
1 length + 1 direction

$(r_2, \hat{\mathbf{r}}_2)$

1 length + 1 direction

$(r_3, \hat{\mathbf{r}}_3)$

ONE TETRAHEDRON = THREE VECTORS



ANGULAR MOMENTUM BASIS

Expand (isotropic) 4PCF in basis of **isotropic functions**

$$\zeta_4(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \sum_{\ell_1 \ell_2 \ell_3} \zeta_{\ell_1 \ell_2 \ell_3}(r_1, r_2, r_3) \mathcal{P}_{\ell_1 \ell_2 \ell_3}(\hat{\mathbf{r}}_1, \hat{\mathbf{r}}_2, \hat{\mathbf{r}}_3)$$

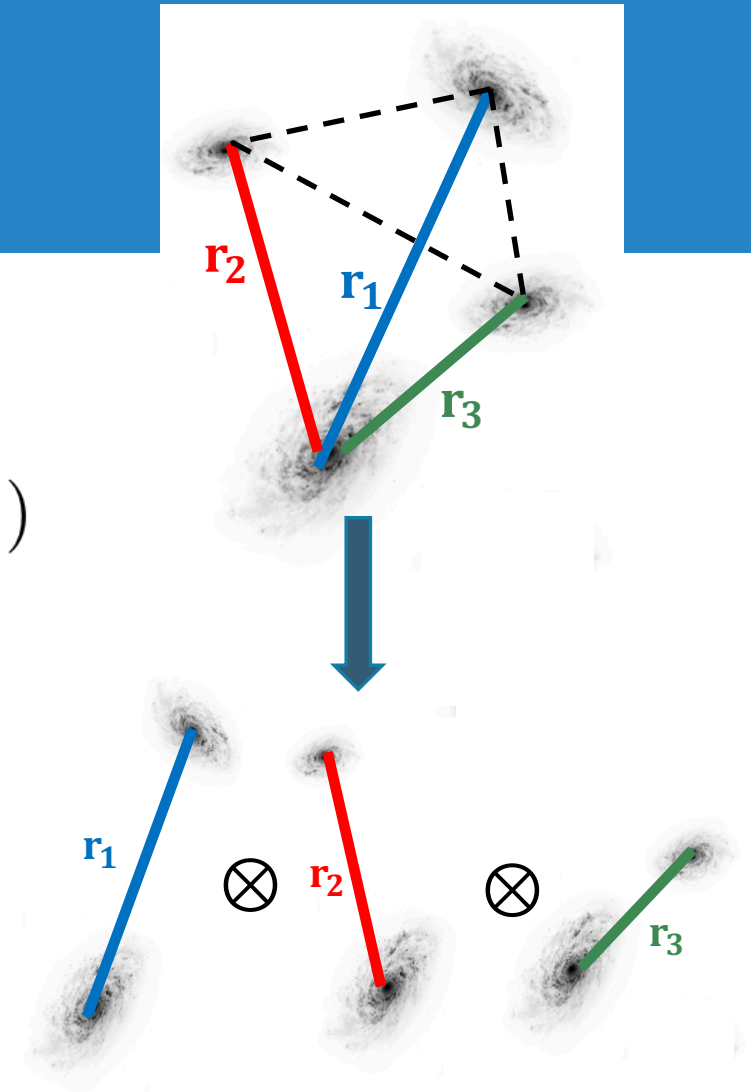
Coefficients

Basis Functions

Basis formed from **angular momentum addition** in 3D

$$\mathcal{P}_{\ell_1 \ell_2 \ell_3}(\hat{\mathbf{r}}_1, \hat{\mathbf{r}}_2, \hat{\mathbf{r}}_3) = \sum_{m_1 m_2 m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} Y_{\ell_1 m_1}^*(\hat{\mathbf{r}}_1) Y_{\ell_2 m_2}^*(\hat{\mathbf{r}}_2) Y_{\ell_3 m_3}^*(\hat{\mathbf{r}}_3)$$

This is **separable** in $\hat{\mathbf{r}}_1, \hat{\mathbf{r}}_2, \hat{\mathbf{r}}_3$



A SEPARABLE BASIS $\Rightarrow$ A QUADRATIC ESTIMATOR

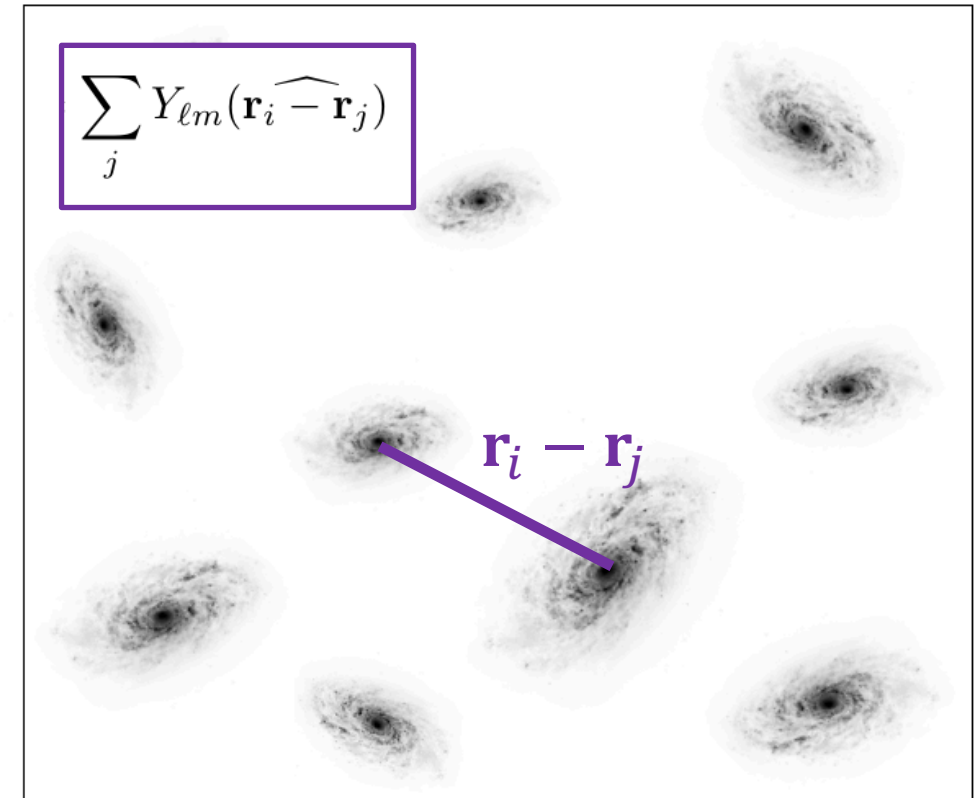
$$\hat{\zeta}_{\ell_1 \ell_2 \ell_3}(r_1, r_2, r_3) = \sum_{m_1 m_2 m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \int d\mathbf{x} \delta_g(\mathbf{x}) \left[\int_{\mathbf{r}_1} \delta_g(\mathbf{x} + \mathbf{r}_1) Y_{\ell_1 m_1}(\hat{\mathbf{r}}_1) \right] \left[\int_{\mathbf{r}_2} \delta_g(\mathbf{x} + \mathbf{r}_2) Y_{\ell_2 m_2}(\hat{\mathbf{r}}_2) \right] \left[\int_{\mathbf{r}_3} \delta_g(\mathbf{x} + \mathbf{r}_3) Y_{\ell_3 m_3}(\hat{\mathbf{r}}_3) \right]$$

The estimator **factorizes** into **independent** pieces

To compute the 4PCF: count *pairs* of galaxies

Total number of pairs: $\mathcal{O}(N_{\text{gal}}^2)$

This can be computed!



ENCORE: ULTRA-FAST N-POINT FUNCTIONS

- ▶ Public C++/CUDA code
- ▶ Computes isotropic 2-, 3-, 4-, 5- and 6-point correlation functions
- ▶ Corrects for **survey geometry**
- ▶ Requires ~ 10 CPU-hours to compute 4PCF of current data

oliverphilcox/
encore



encore: Efficient isotropic 2-, 3-, 4-, 5- and 6-point correlation functions in C++ and CUDA

 2
Contributors

 0
Issues

 4
Stars

 1
Fork



oliverphilcox/encore

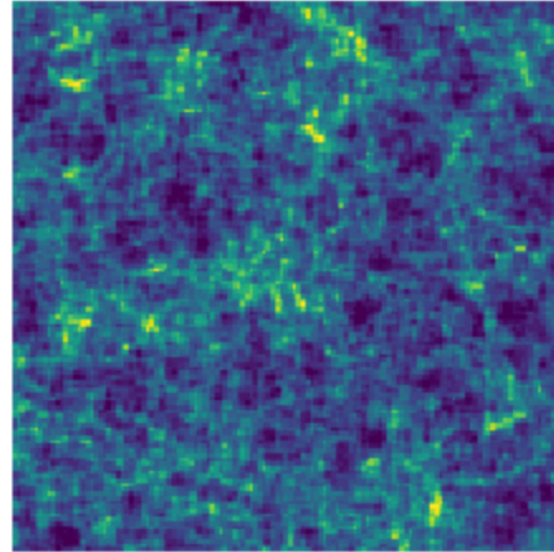
encore: Efficient isotropic 2-, 3-, 4-, 5- and 6-point correlation functions in C++ and CUDA - oliverphilcox/encore

 [github.com](https://github.com/oliverphilcox/encore)

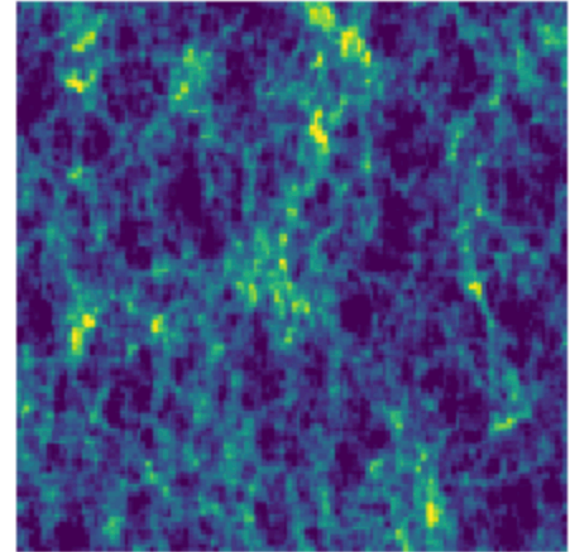
BEYOND THE 4-POINT FUNCTION

This generalizes **beyond** the 4PCF

- ▷ 5PCF, 6PCF, ...
- ▷ **Anisotropic** correlation functions
- ▷ Non-Flat Universes
- ▷ Two, Three, Four, ... Dimensions



Real Space



Redshift Space

Requires the addition of N angular momenta in D dimensions [i.e. $\mathfrak{so}(D)$ Lie algebra]

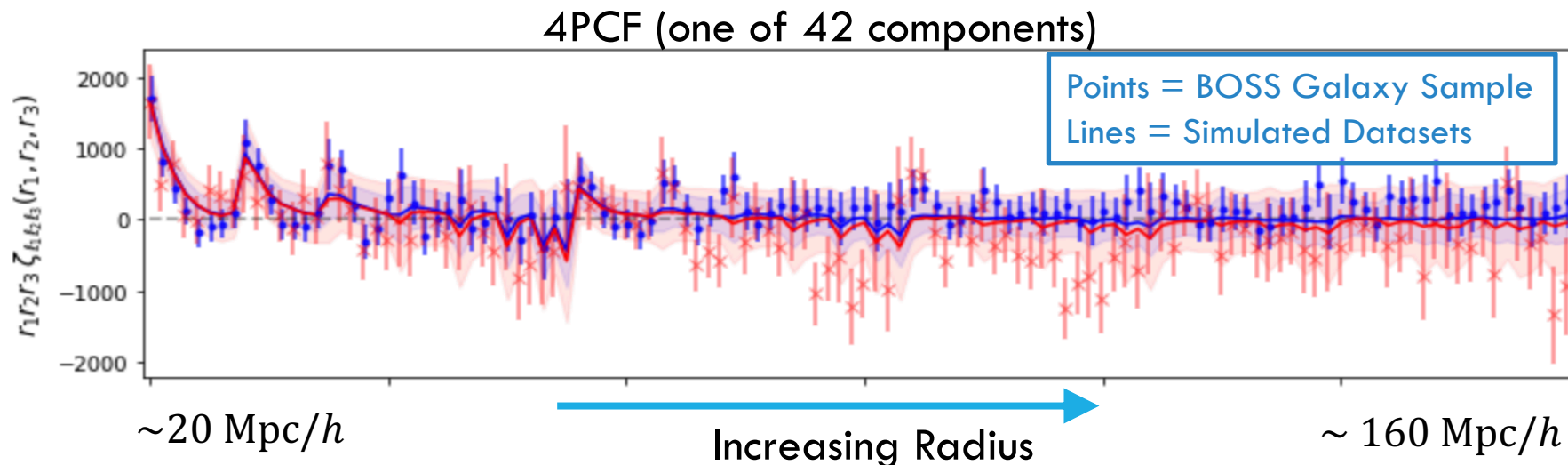
MEASURING THE 4-POINT FUNCTION

▷ Compute the connected 4PCF from $\sim 10^6$ **BOSS CMASS** galaxies

▷ Must remove **Gaussian** contribution 4PCF at the estimator level:

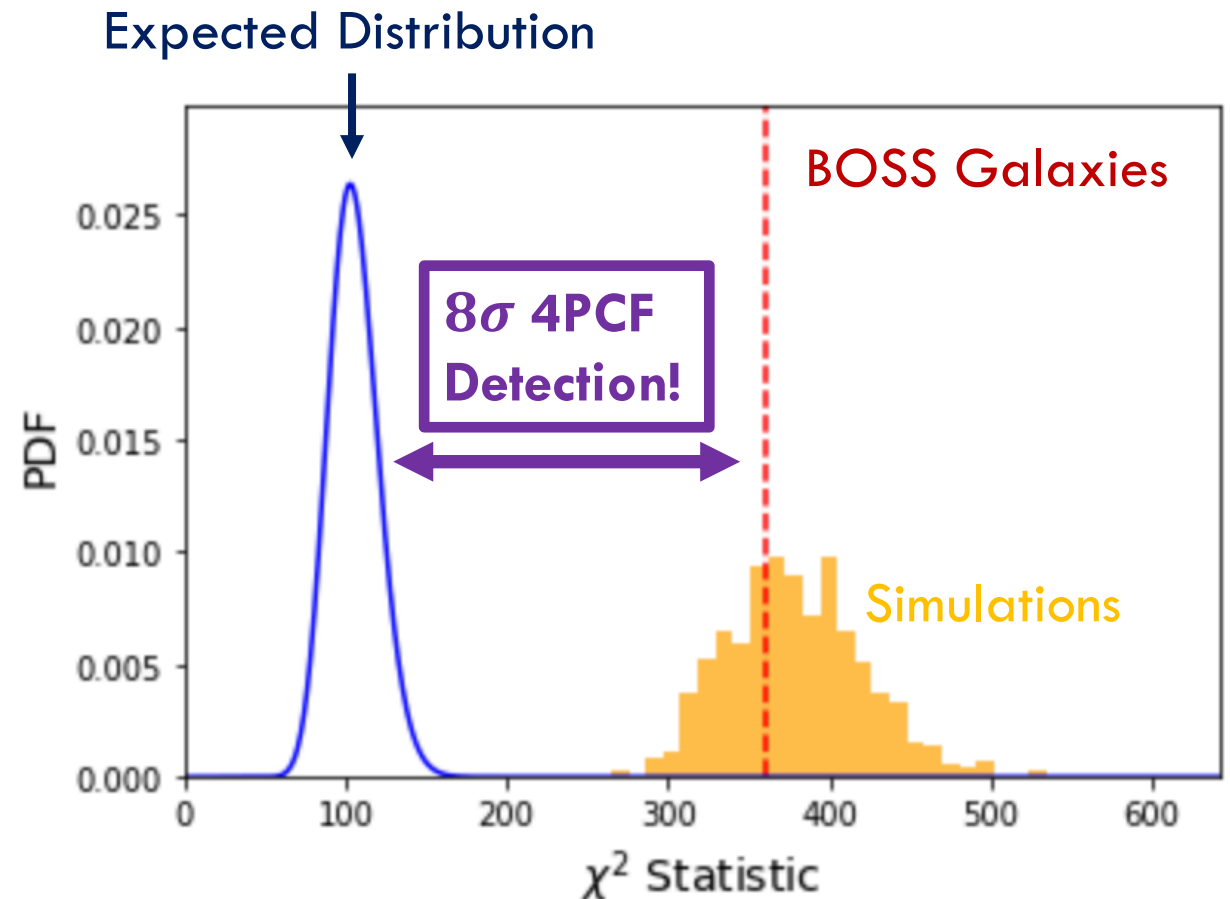
$$\zeta_{g,4}^{(c)} \sim \langle \delta_g \delta_g \delta_g \delta_g \rangle - [\langle \delta_g \delta_g \rangle \langle \delta_g \delta_g \rangle + 2 \text{ perms.}]$$

Do we detect a signal?



CAN WE DETECT THE GRAVITATIONAL 4PCF?

- ▶ Perform a χ^2 -test to search for a **gravitational 4PCF**
- ▶ Null Hypothesis: **4PCF = 0**.
- ▶ **Strong** detection of non-Gaussianity!
- ▶ Most information is on short scales:
 5σ detection for $r > 14h^{-1}$ Mpc



EXTRACTING INFORMATION FROM THE 4PCF

In configuration space, **windows** are easy,
but **perturbation theory** is hard

▷ Formulate theory in Fourier-space then FT

$$\zeta_{g,4}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \int_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3} e^{i(\mathbf{k}_1 \cdot \mathbf{r}_1 + \mathbf{k}_2 \cdot \mathbf{r}_2 + \mathbf{k}_3 \cdot \mathbf{r}_3)} T_{g,4}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

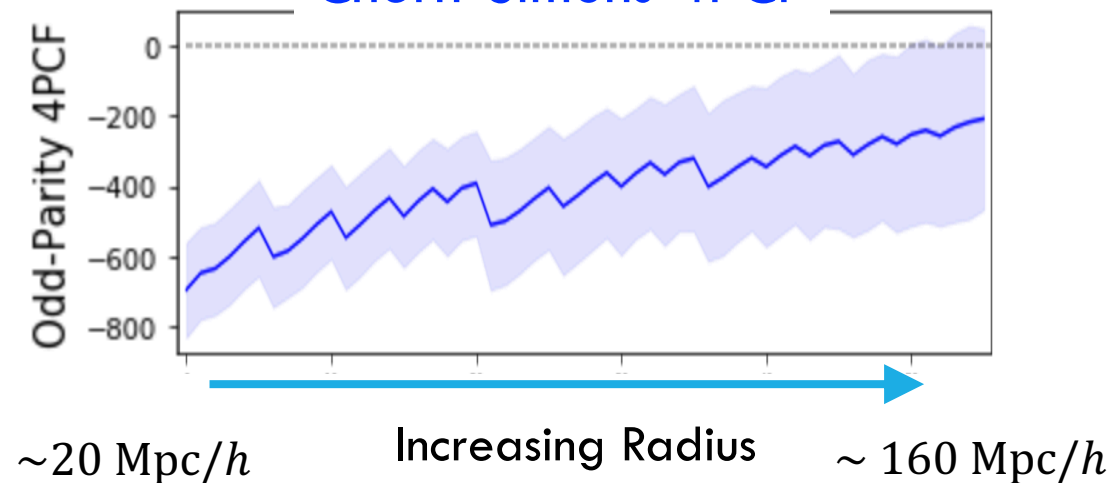
▷ Requires *many* coupled integrals, even at
tree-level

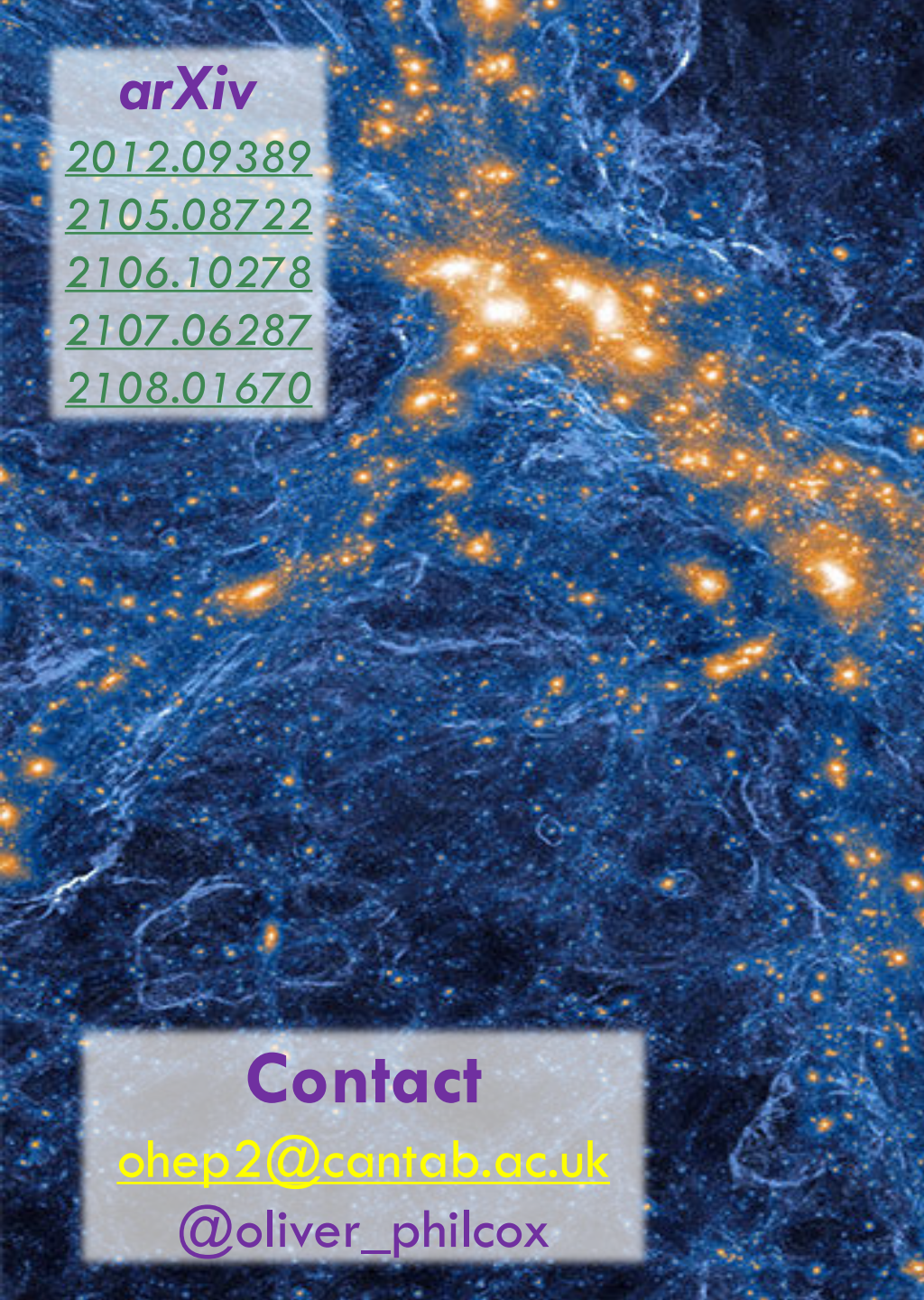
Example: Chern-Simons 4PCF

Odd-Parity 4PCF

$$\zeta_4(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) - \mathbb{P}[\zeta_4(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)]$$

Chern-Simons 4PCF



A visualization of the cosmic web, showing a complex network of blue filaments and orange galaxy clusters against a dark background.

arXiv

[2012.09389](#)

[2105.08722](#)

[2106.10278](#)

[2107.06287](#)

[2108.01670](#)

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CONCLUSIONS

- Non-Gaussian statistics:
 1. **Sharpen** cosmological constraints
 2. Probe **new physics** in the early Universe, e.g. f_{NL} , Chern-Simons couplings
- New methods allow statistics to be used in practice:
 1. **Bispectra** computed without **windows**
 2. **Correlation** functions compute at $\mathcal{O}(N_{\text{gal}}^2)$