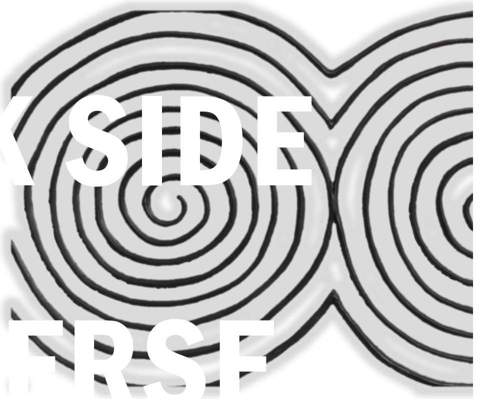
The Energy Minimisation Principle
Linking protohalo counts and shapes to anisotropic infall

Marcello Musso

University of Salamanca

@BCCP - March 2023



Organizing DSU2023 in Kigali, Rwanda.
Hope to see you there!

https://eaifr.ictp.it/events/dsu-2023/

WEBPAGE: https://eaifr.org/events/dsu-2023/

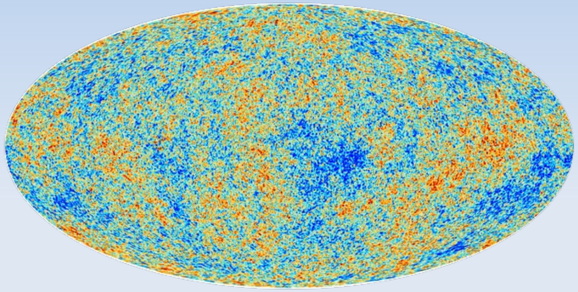
Organisers:

- Paolo Creminelli, ICTP, Trieste
- Joern Kersten, Uni Bergen
- Roy Maartens, UWC, South Africa
- Shoaib Munir, ICTP-EAIFR
- Marcello Musso, Uni Salamanca
- Riccardo Sturani, ICTP-SAIFR
- Filippo Vernizzi, IPhT, Saclay
- Gabrijela Zaharijas, Uni Nova Gorica

ICTP-EAIFR is supported by the University of Rwanda and the Rwandan Ministry of Education

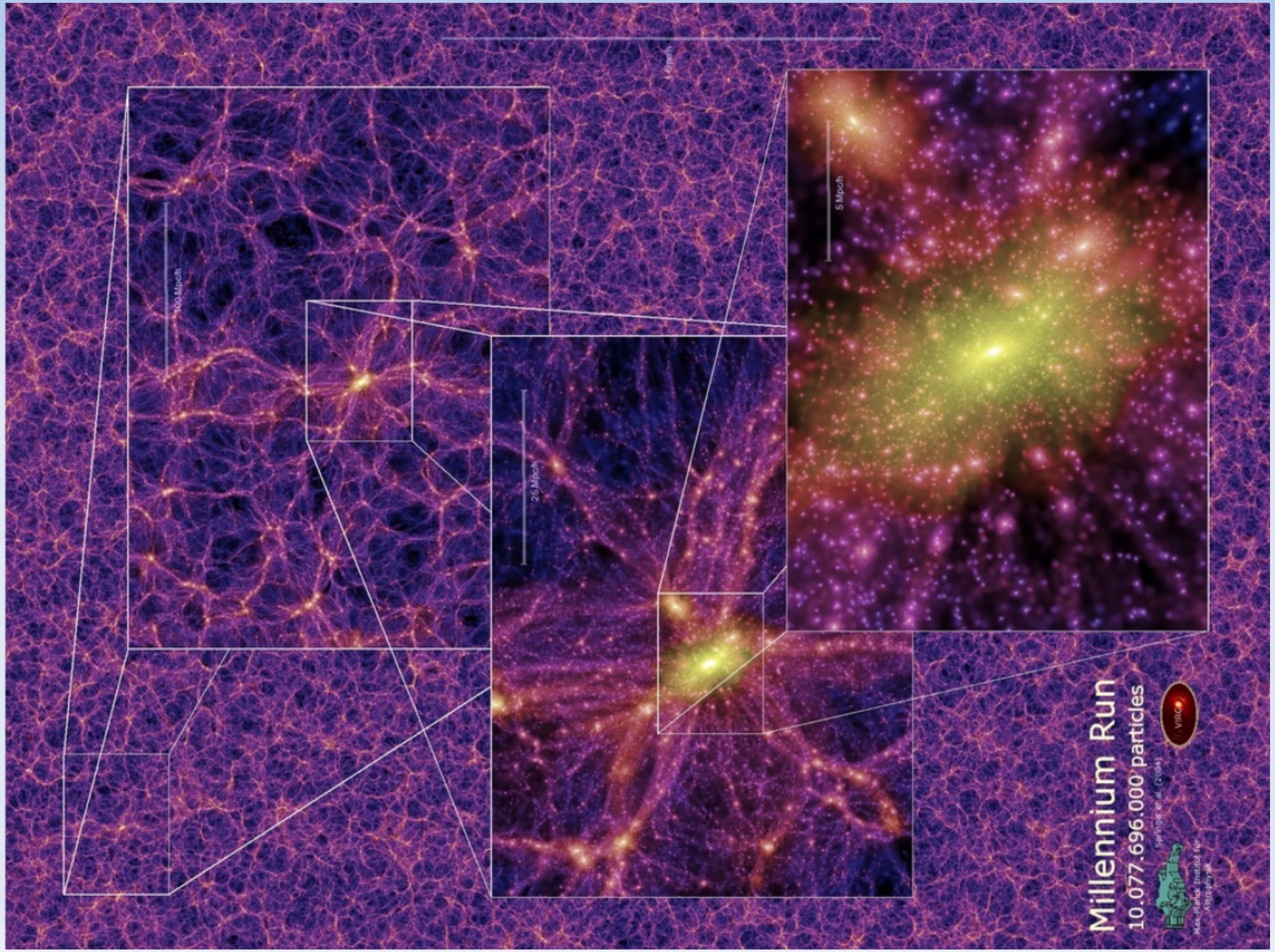
The non-linear Universe

~Gaussian initial conditions



Gravity

Filaments, nodes, voids...

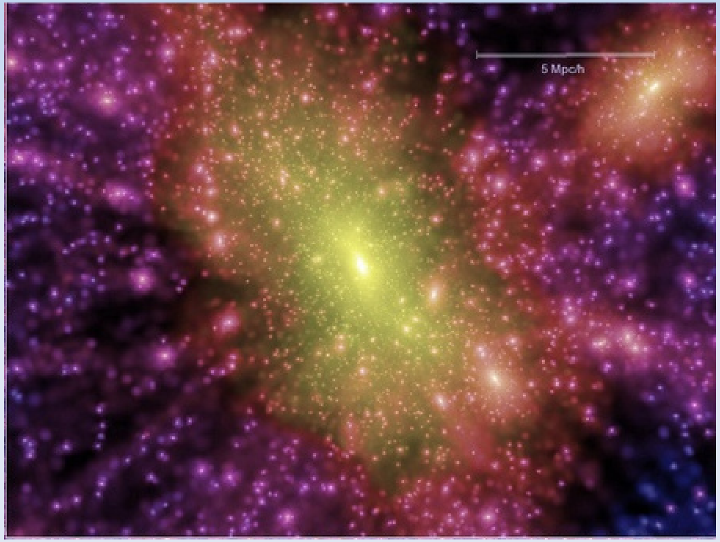


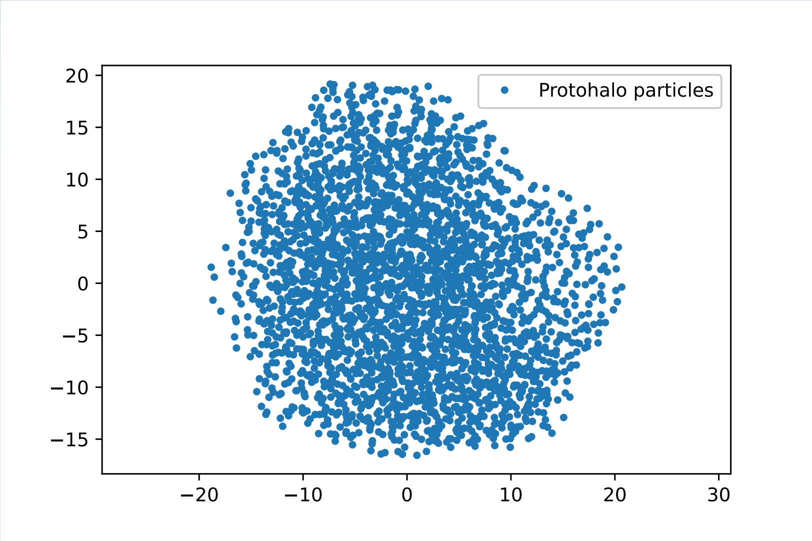
Why analytical models?

- Predict/interpret features of the outcome without running the simulation
- Conceptual grasp of the interplay of the many variables involved
- Useful to quickly scan the parameter space. Simulations are still costly!
- Explore beyond-ΛCDM scenarios (DE, ModGrav, axions…) that are difficult to simulate
- Physically motivated fitting formulae

The rules of the game

- Each halo has a protohalo (Lagrangian region occupied by the halo particles)



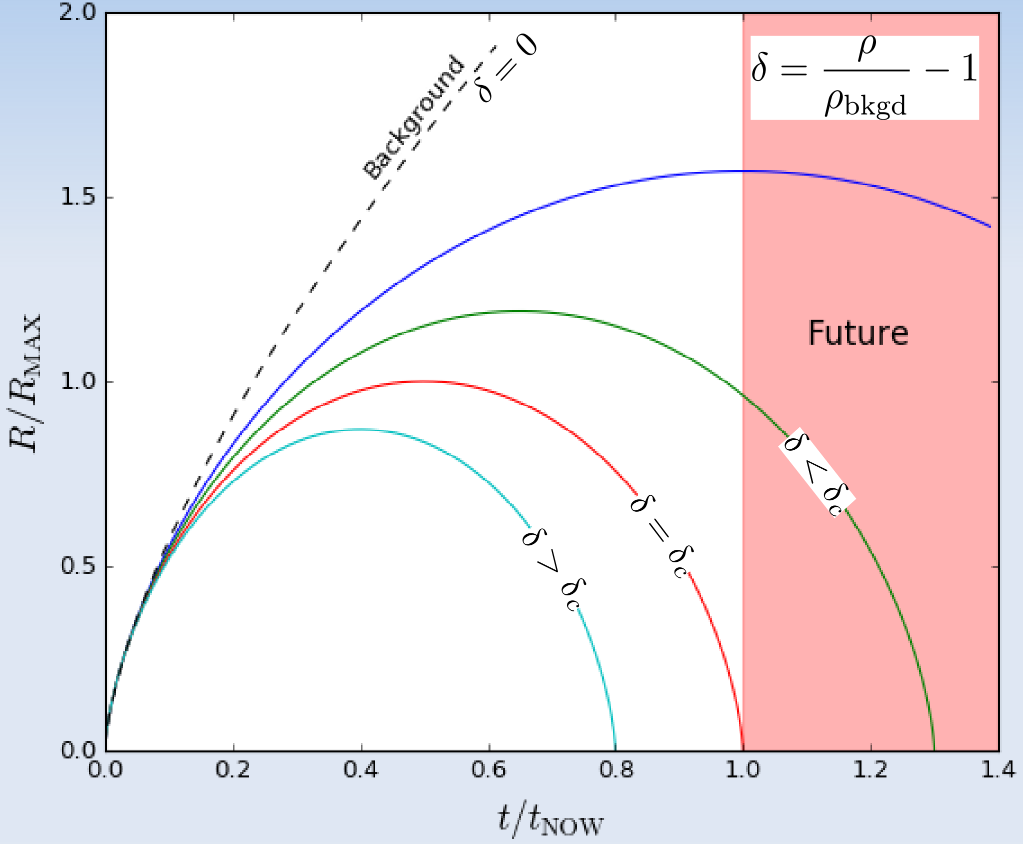


- Protohaloes live in a Gaussian field. Their stats are non-Gaussian, but can be predicted
- Need to identify them!

The critical density

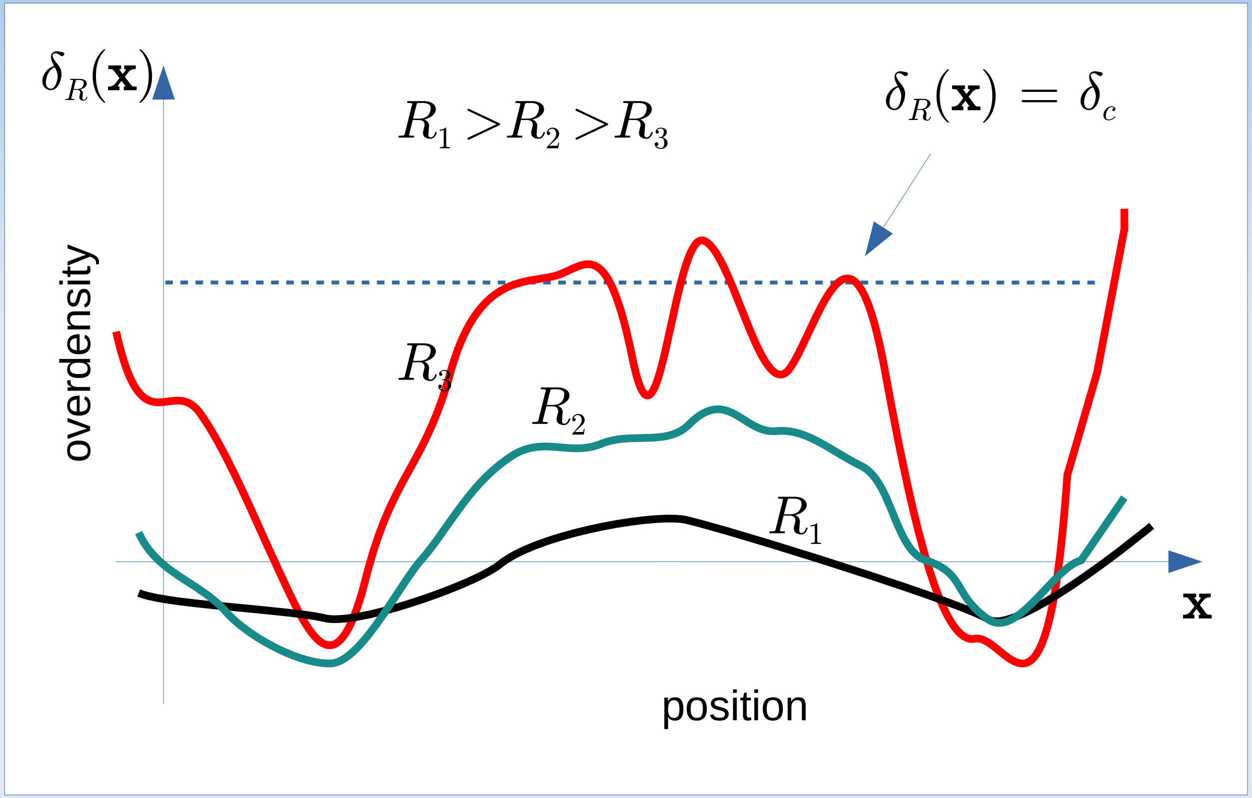
- Protohalos are approximately spherical, follow approx spherical collapse
- Time of collapse depends on mean overdensity: $tH_0 = \pi(3/5\delta_R)^{3/2}$
- Maximum (turnaround) radius: $r_{max} = 3R/5\delta_R$
- A shell of Lagrangian radius R collapses today if $\delta_R = \delta_c$

$$\delta_c \equiv \frac{3}{10}(18\pi^2)^{1/3} \simeq 1.686$$



Density peaks

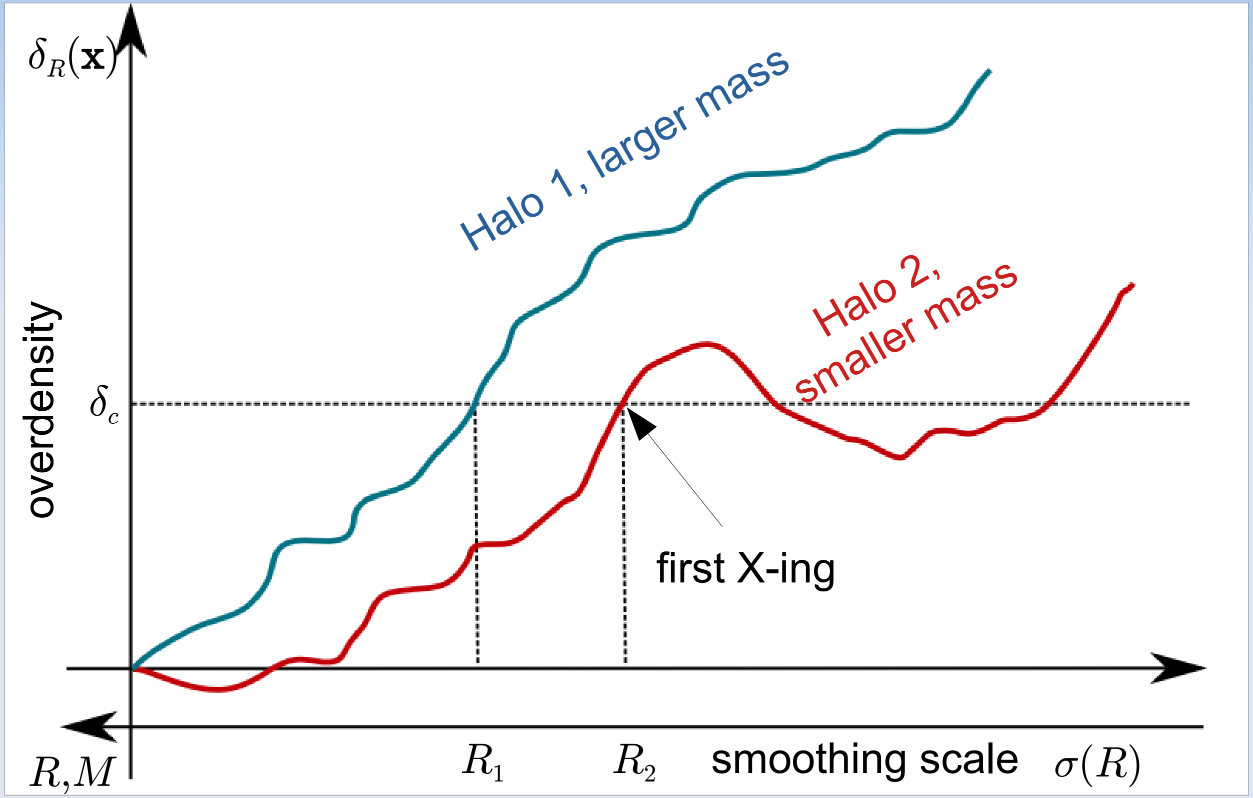
- Protohaloes collapse faster than neighboring regions
- They must be local maxima of the mean (=smoothed) density field
- Which smoothing scale?



BBKS86

Excursion sets

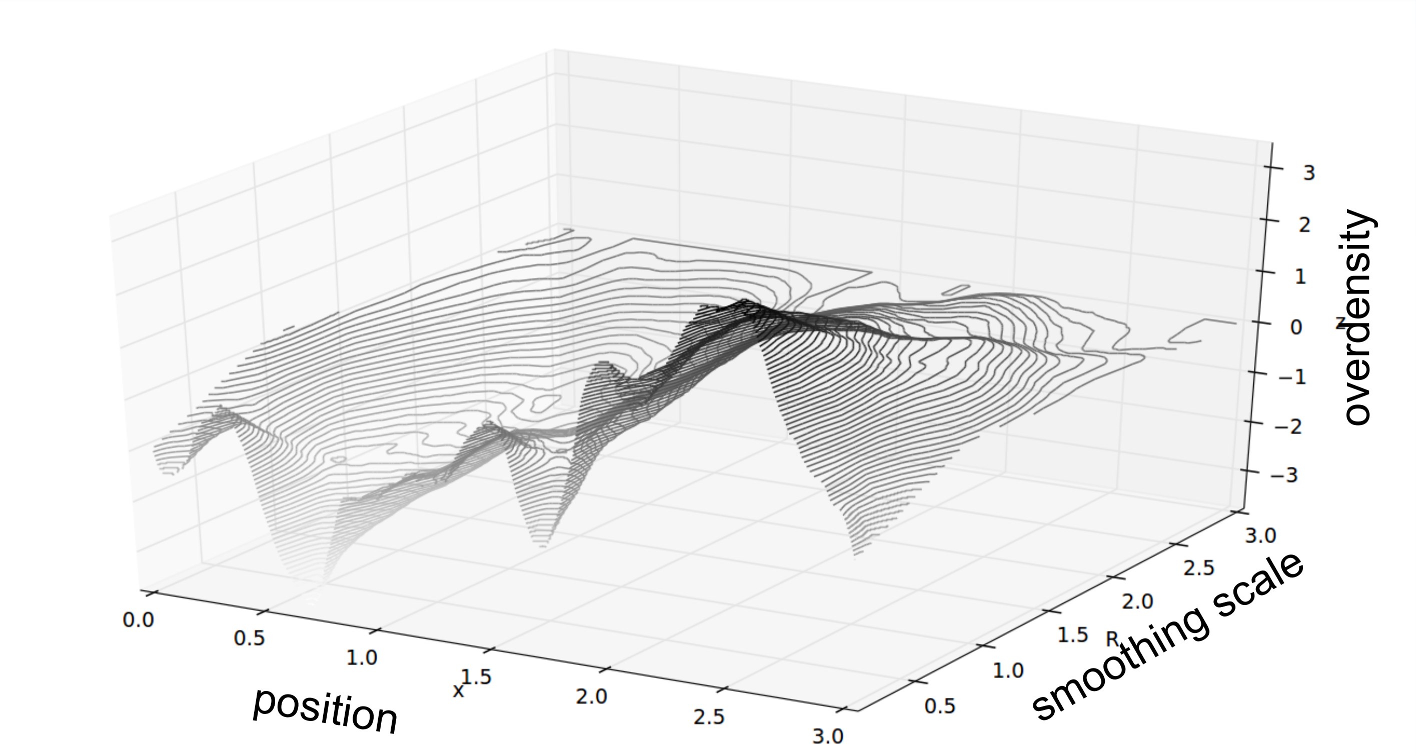
- At fixed $\mathbf{x}$, the value of $\delta_R(\mathbf{x})$ gives the collapse time of the patch
- Imposing $\delta_R(\mathbf{x}) = \delta_c$ fixes the smoothing scale R
- Look for the largest such R ("first crossing")



Press & Schechter 74, Bond et al 91, ….

Excursion sets + peaks

- Combine the two ideas
- After smoothing, 4-dimensional landscape in $\mathbf{x}$ and R
- Look for peaks that first cross the critical height
- Peak constraint fixes position $\mathbf{x}$



- Threshold fixes the smoothing scale R

The standard game

- Protohalos are peaks of the (smoothed) initial density field
- Smooth the initial overdensity field $\delta_R(\mathbf{x})$ on ALL possible scales R
- Peak constraint: $\nabla\delta_R = 0 \quad \rightarrow$ fixes the position $\mathbf{x}$
- Peaks of different height collapse at different times
- Critical peak height: $\delta_R = \delta_c \quad \rightarrow$ fixes the smoothing scale R
- Mass is conserved. The mass of the final halo is $M = 4\pi\bar{\rho}R^3/3$

The good, the bad and the ugly

- The good. Simple and intuitive.
  An ad-hoc stochastic threshold gives good (~5-10%) mass function AND bias (success for bias is non-trivial)

Paranjape, Sheth & Desjacques (2013); Castorina et al. (2016)

- The bad. Scatter in δ_c agrees with N-body only up to 30%
  Only 75% of small-mass protohalos are peaks

e.g. Ludlow, Borszykowski, Porciani (2014)

- The ugly. Top-Hat filter gives divergences in ΛCDM
  No connection with perturbation theory
  No information on the shape
  Why is δ_c stochastic?

1. Excursion set peaks in energy

Or: getting the position right

(based on 1907.09147 with Ravi Sheth)

Matter vs energy peaks

- Geometrical radius: $R^3 = 3V/4\pi$
- $$\frac{\dot{R}^2}{2} - \frac{GM}{R} - \frac{\Lambda}{6}R^2 \simeq -\frac{5}{3}\frac{GM}{R_{\rm in}}\delta_{R,{\rm in}}$$
- Mass: $M = \frac{4\pi}{3}\bar{\rho}(1+\delta_R)R^3$

vs

- Inertial radius: $\frac{R_I^2}{5} \equiv \int_V \frac{{\rm d}^3 r}{3M}\rho\,|\mathbf{r}-\mathbf{r}_{\rm cm}|^2$
- $$\frac{\dot{R}_I^2}{2} - \frac{GM_I}{R} - \frac{\Lambda}{6}R_I^2 \simeq -\frac{5}{3}\frac{GM_I}{R_{I,{\rm in}}}\epsilon_{\rm in}$$
- Inertial mass: $M_I = \frac{4\pi}{3}\bar{\rho}(1+\epsilon_R)R_I^3$

- Governed by **matter** overdensity

$$\delta_R \equiv \frac{1}{V}\int_V {\rm d}^3 r\,\delta(\mathbf{r})$$

- Governed by **energy** overdensity

$$\epsilon_R \equiv 5\int_V \frac{{\rm d}^3 r}{MR_I^2}\rho(\mathbf{r})\,\mathbf{r}\cdot(\nabla\phi - \nabla\phi_{\rm cm})$$

Matter vs energy peaks

- Characteristic time ~ $(1/\delta_R)^{3/2}$
- Halos of mass M are peaks of $\delta_R(\mathbf{x})$
- In Fourier space:

$$\delta_R = \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \delta(\mathbf{k})\, \frac{3 j_1(kR)}{kR}$$

vs

- Characteristic time ~ $(1/\epsilon_R)^{3/2}$
- Halos of mass M are peaks of $\epsilon(\mathbf{x})$
- In Fourier space:

$$\epsilon_R = \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \delta(\mathbf{k})\, \frac{15 j_2(kR)}{(kR)^2}$$

(extra power of $1/k$)

What is the advantage?

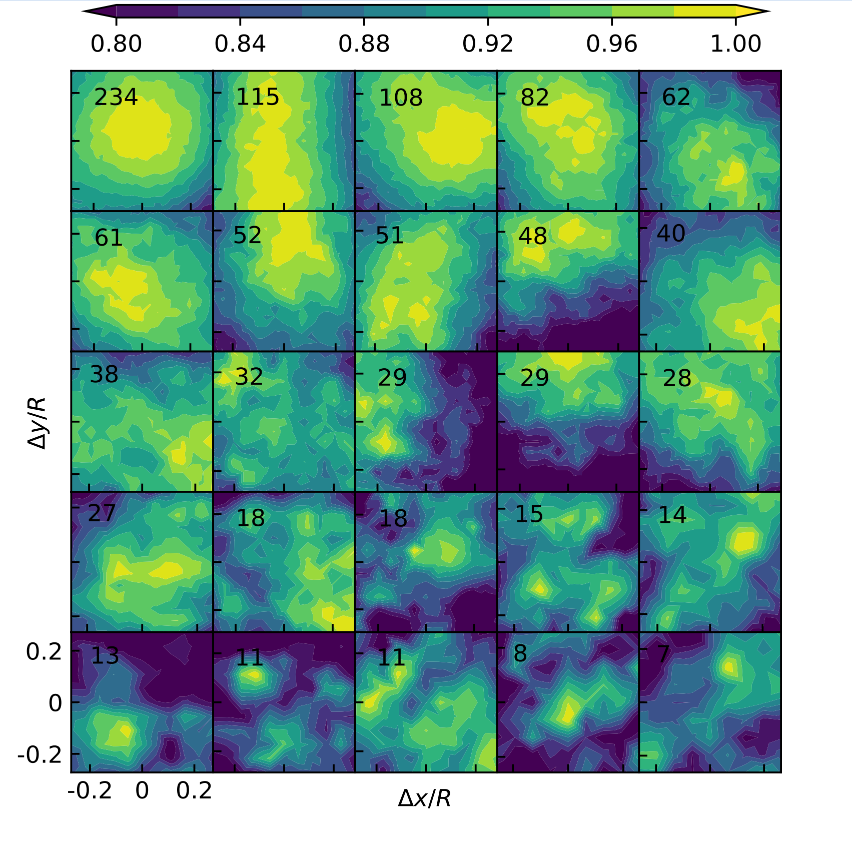
- R is very sensitive to the halo boundary
- No dynamical meaning in $\nabla\delta_R = 0$
- More small-scale power. $\langle(\nabla^2\delta_R)^2\rangle$ diverges in ΛCDM.
- Usually resort to Gaussian filter. Blurred physical interpretation

vs

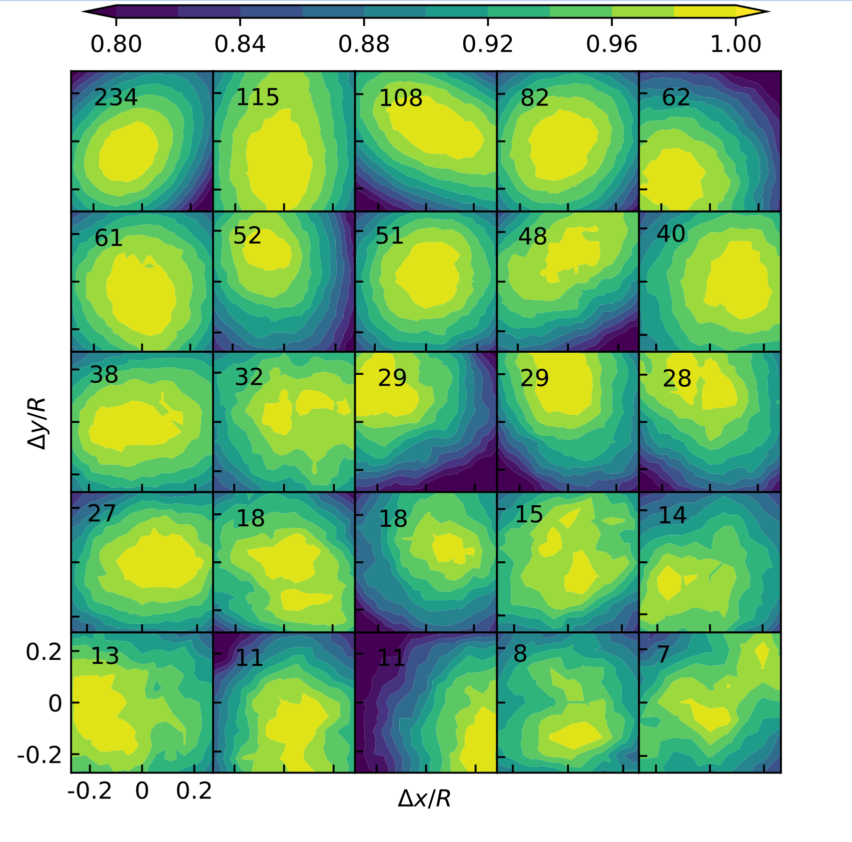
- R_I is density weighted, less sensitive to halo boundary at late times
- $\nabla\epsilon_R$ ~ dipole moment. $\nabla\epsilon_R = 0$ implies radial infall
- Less small-scale power. $\langle(\nabla^2\epsilon_R)^2\rangle$ remains finite.
- No need to “tweak” the filter. Clearly rooted in the EoM

Testing the energy peak ansatz

Mean matter overdensity field δ_R



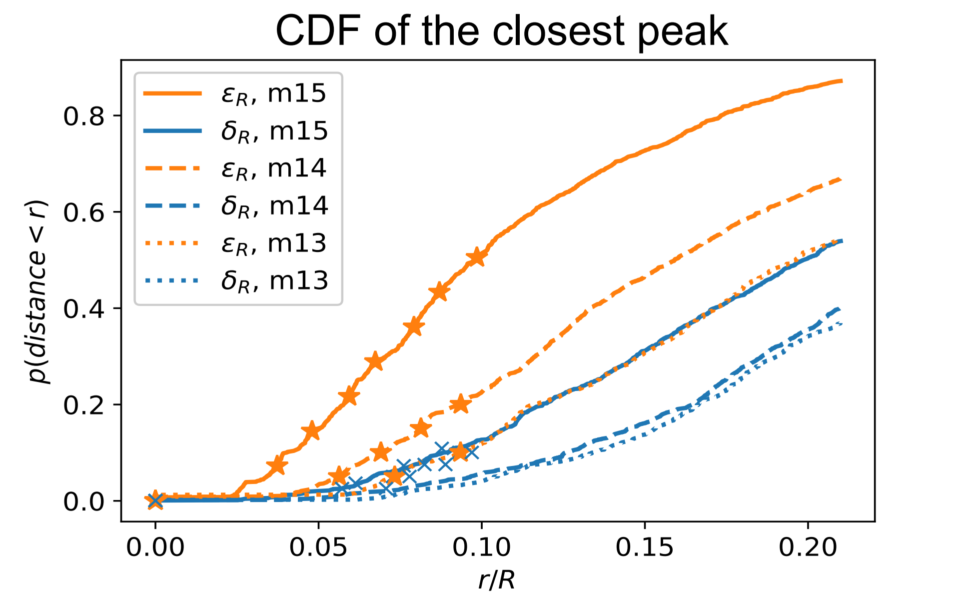
Mean energy overdensity field ϵ_R

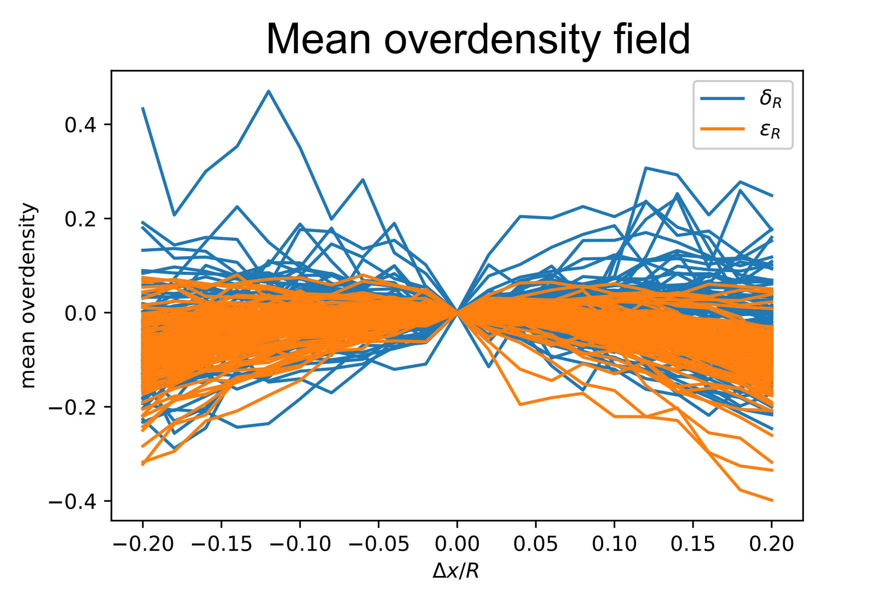


- Energy peaks are a better proxy for protohalo centers!

Testing the energy peak ansatz

- Regions around protohalo centers are more likely to be energy peaks than matter density ones:

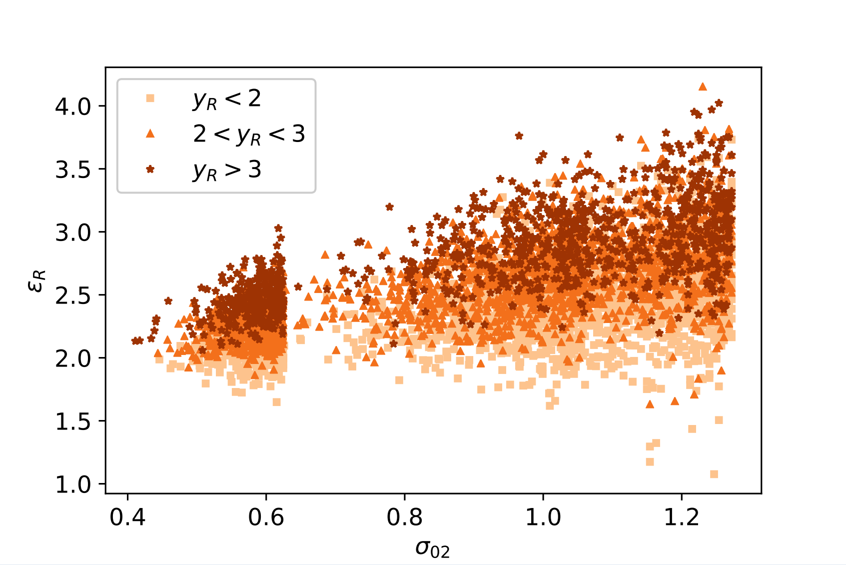


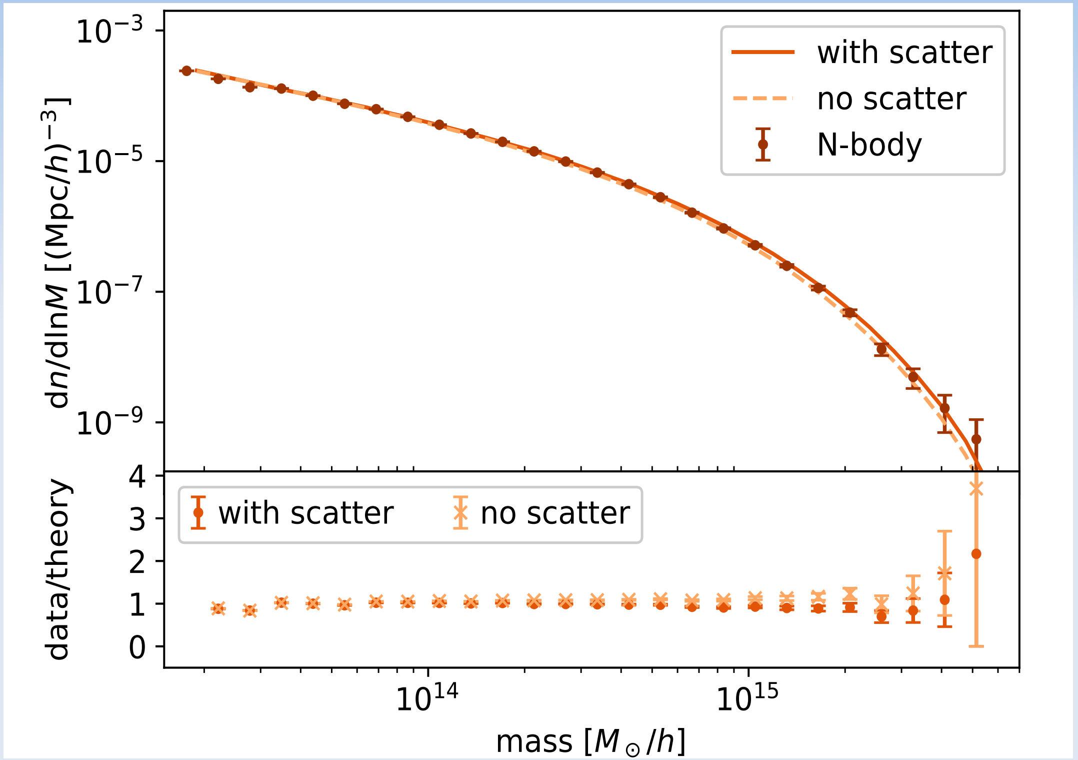


- Protohaloes are more likely to be close to energy peaks

Halo mass function

- Predicted, fitting the scatter of ϵ_R





- Scatter can describe assembly bias.

Other filters?

- There are other popular filter choices (Gaussian, sharp-k…)
- They have interesting mathematical properties that can make calculations easier
- However, they are not obviously connected to physical quantities
- They don’t have a clear dynamical meaning (to me...)

2. The Minimum Energy Principle

Or: getting the shape right

(based on 2303.02142 with Ravi Sheth)

Shape of maximal ϵ

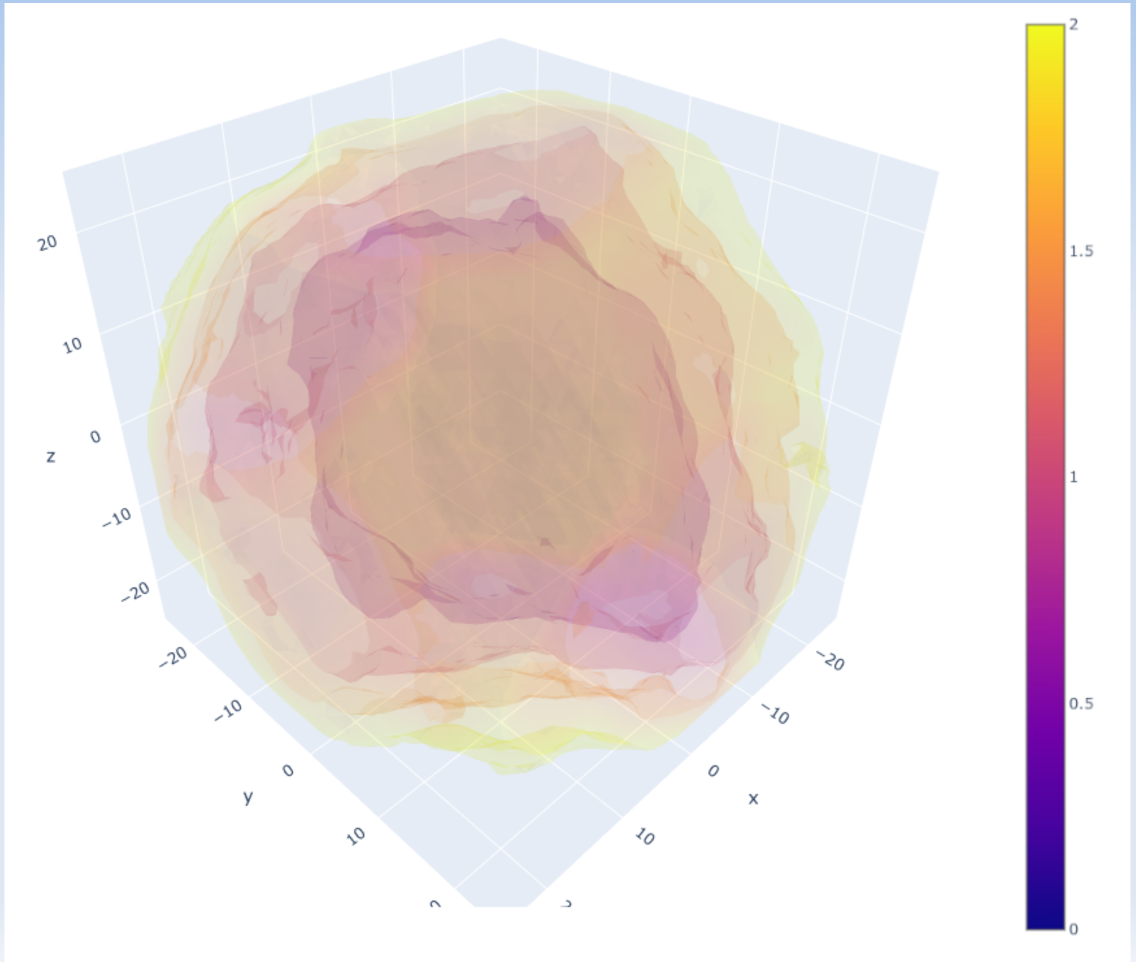
- Once a spherical peak is found, one can further increase ϵ (decrease E) by deforming the sphere at fixed volume.
- The inertial radius R_I of the deformed region collapses even faster
- The boundary of the region of maximal ϵ (minimal E) must be an isocontour of

$$\mathcal{V}(\mathbf{r}) \equiv (\mathbf{r} - \mathbf{r}_{\rm cm}) \cdot \left[\nabla\phi - \nabla\phi_{\rm cm} - \frac{\epsilon}{3}(\mathbf{r} - \mathbf{r}_{\rm cm}) \right]$$

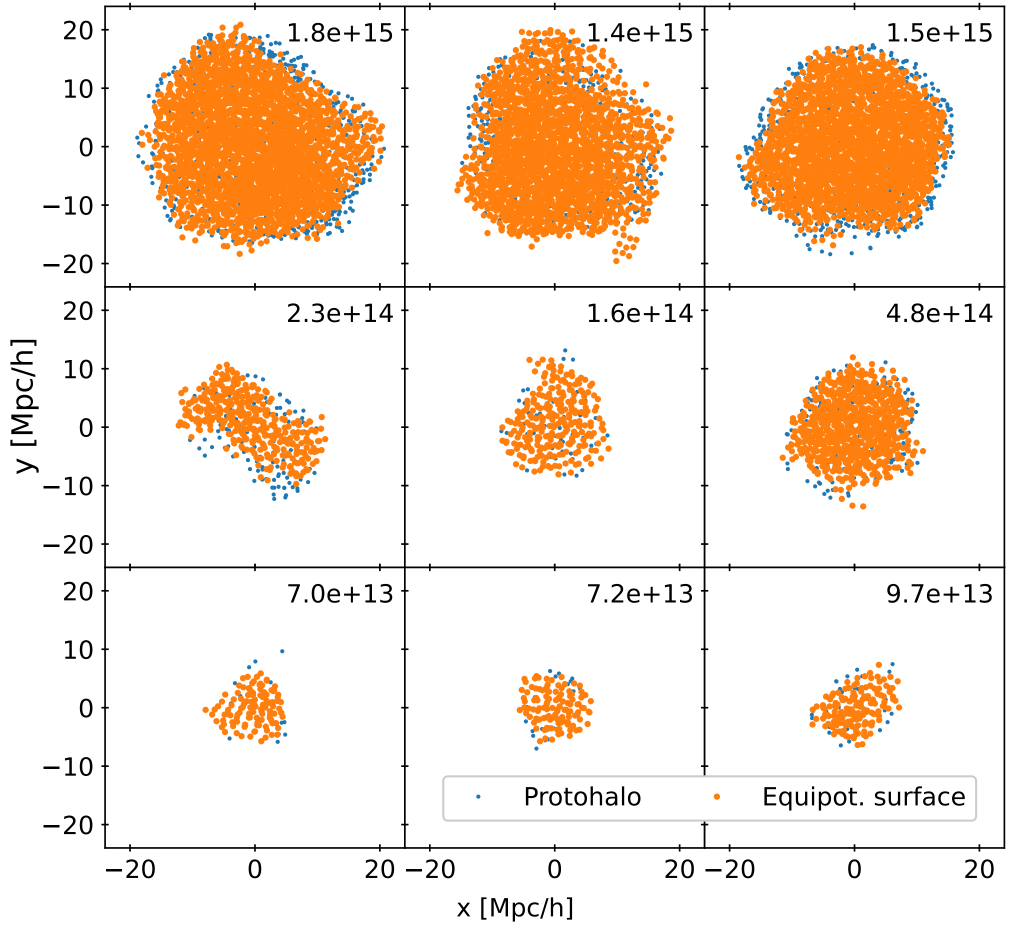
- Proxy for protohalo shape and boundary!
- Longest axis in the direction of maximum compression (orthogonal to the filament)
- Can predict initial torques

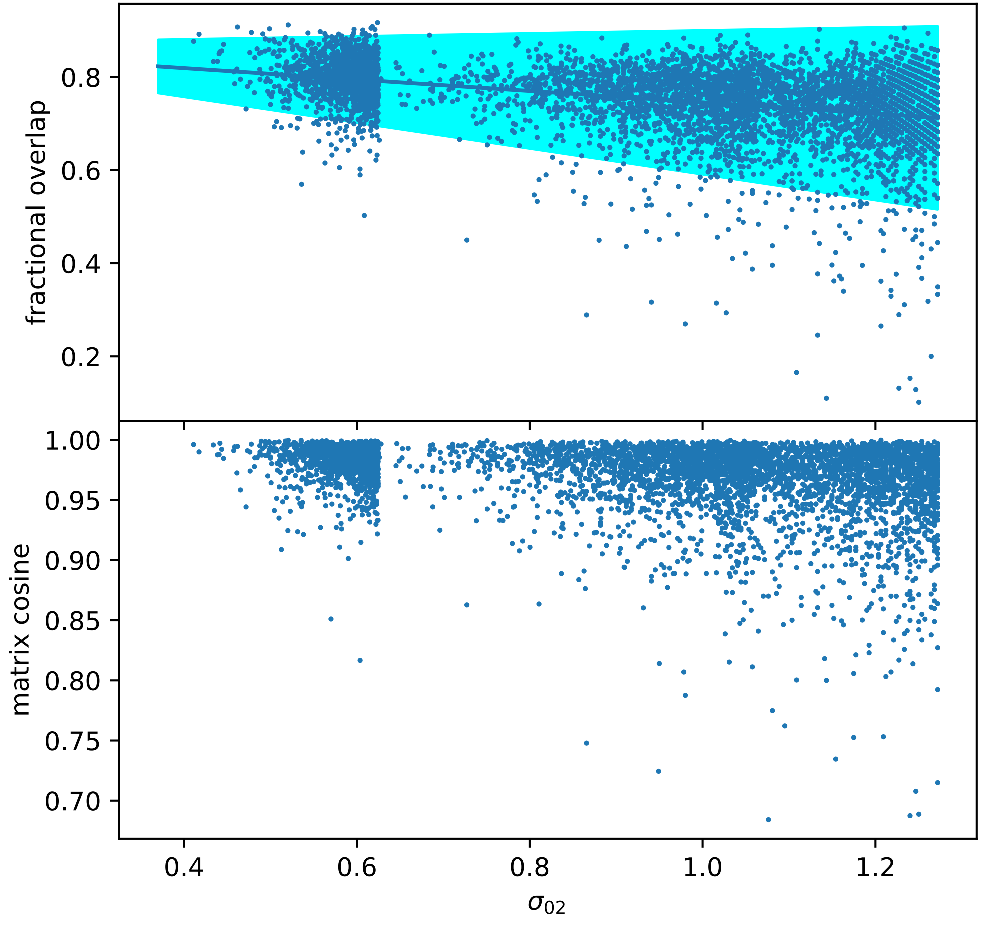
Equipotential surfaces

- Nested equipotential surfaces with different overdensity ϵ and volume V describe the mass accretion history
- Excursion sets of peaks of arbitrary shape!

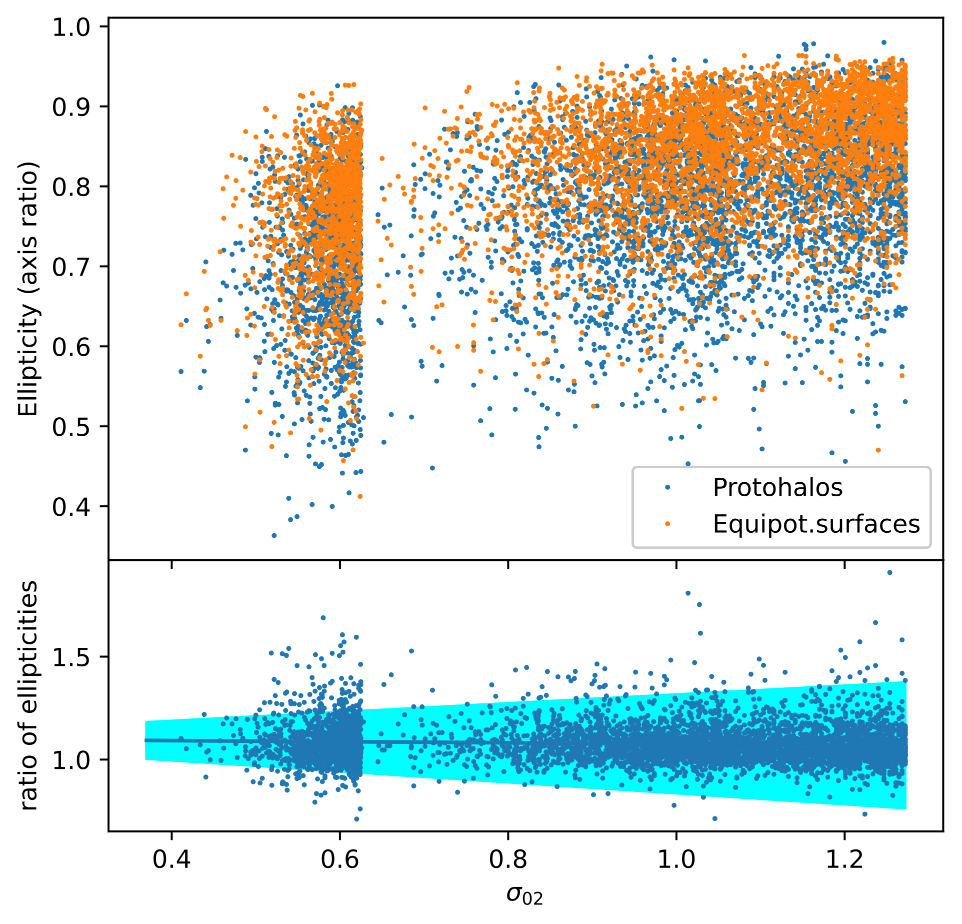


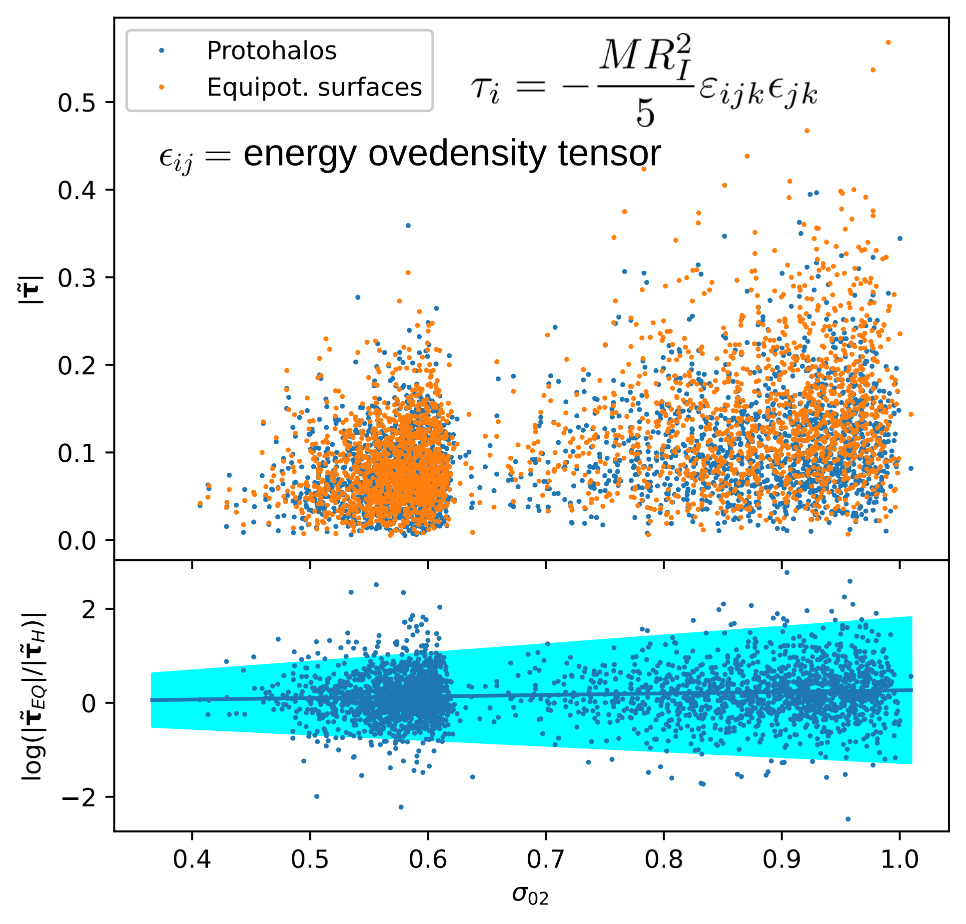
Protohaloes vs equipotential surfaces





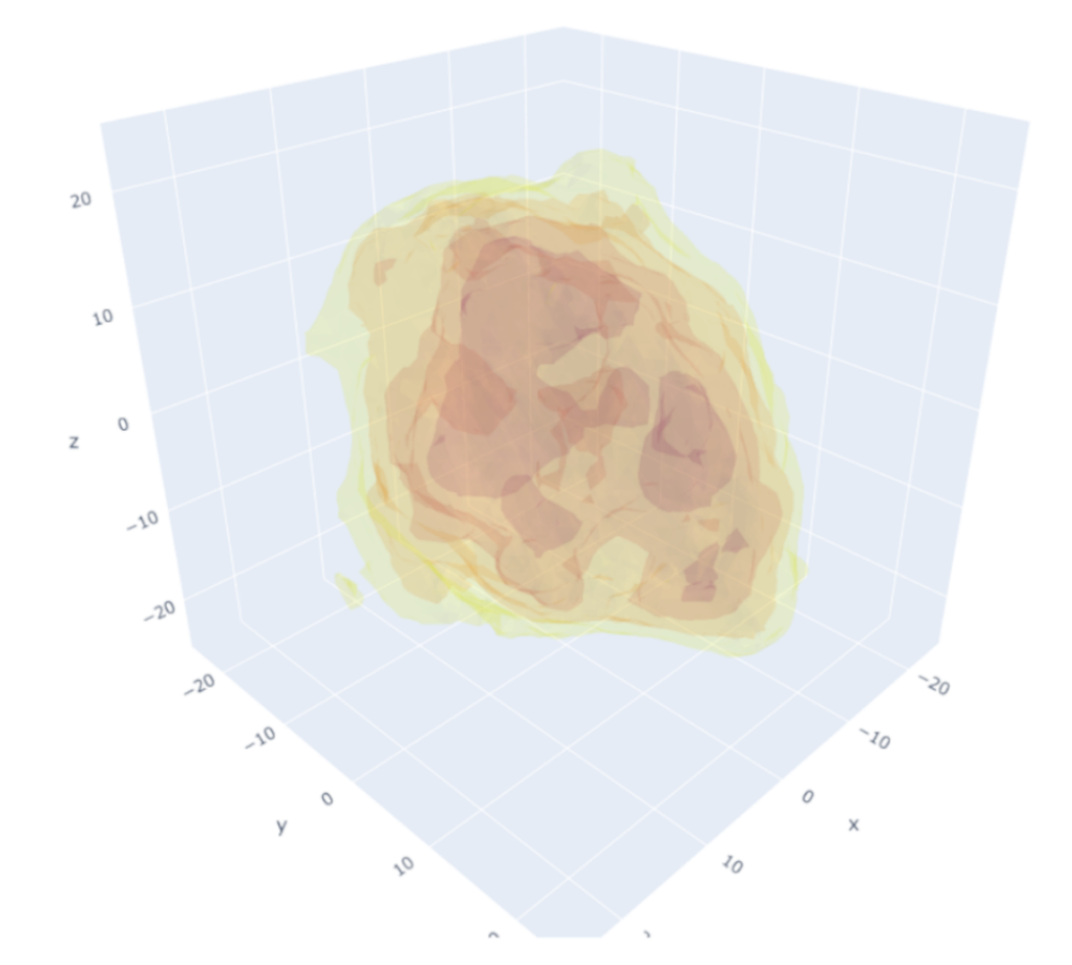
Ellipticities and torques





Mergers

- Zooming in, the surfaces of constant infall potential $\mathcal{V}$ may fragment
- Natural prediction of merger events!



Conclusions

- Protohaloes are peaks of the initial energy overdensity field. Not densest but most energetically bound initial regions, having fastest collapse times.
- Peaks in ϵ_R are convergent matter flows. Initial evolution matches perturbation theory. Final high mean density results dynamically, not put in "by hand".
- Using ϵ_R instead of δ_R simply means changing the filter (to a more convergent one)
- Energy density peaks are better behaved, and better proxies for protohalo centers
- Protohalo shapes are very well described by equipotential surfaces
- Excellent prediction of ellipticities, shear-shape alignments and torques

Open questions and outlook

- Can we predict critical value ϵ_c ? Must model virialization (in progress)
- Relation with halo finder? Ellipsoidal? FOF? Energy-based?
- Angular momentum? (in progress)
- How to improve even more? Account for non-conservation of energy?
- Final shear/shape alignments?
- (Assembly) bias? Voids? Skeleton/cosmic web?
- Primordial BHs?
- …

Thank you!!