

CMB LENSING & DELENSING FOR FUNDAMENTAL PHYSICS

THE GOOD, THE BAD & THE SYSTEMATICS

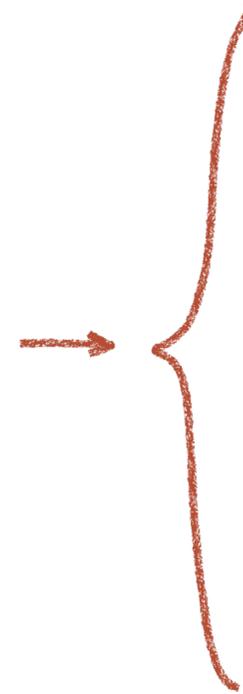
Anton Baleato Lizancos



RPM colloquium & BCCP seminar, LBNL — 2/2/2021

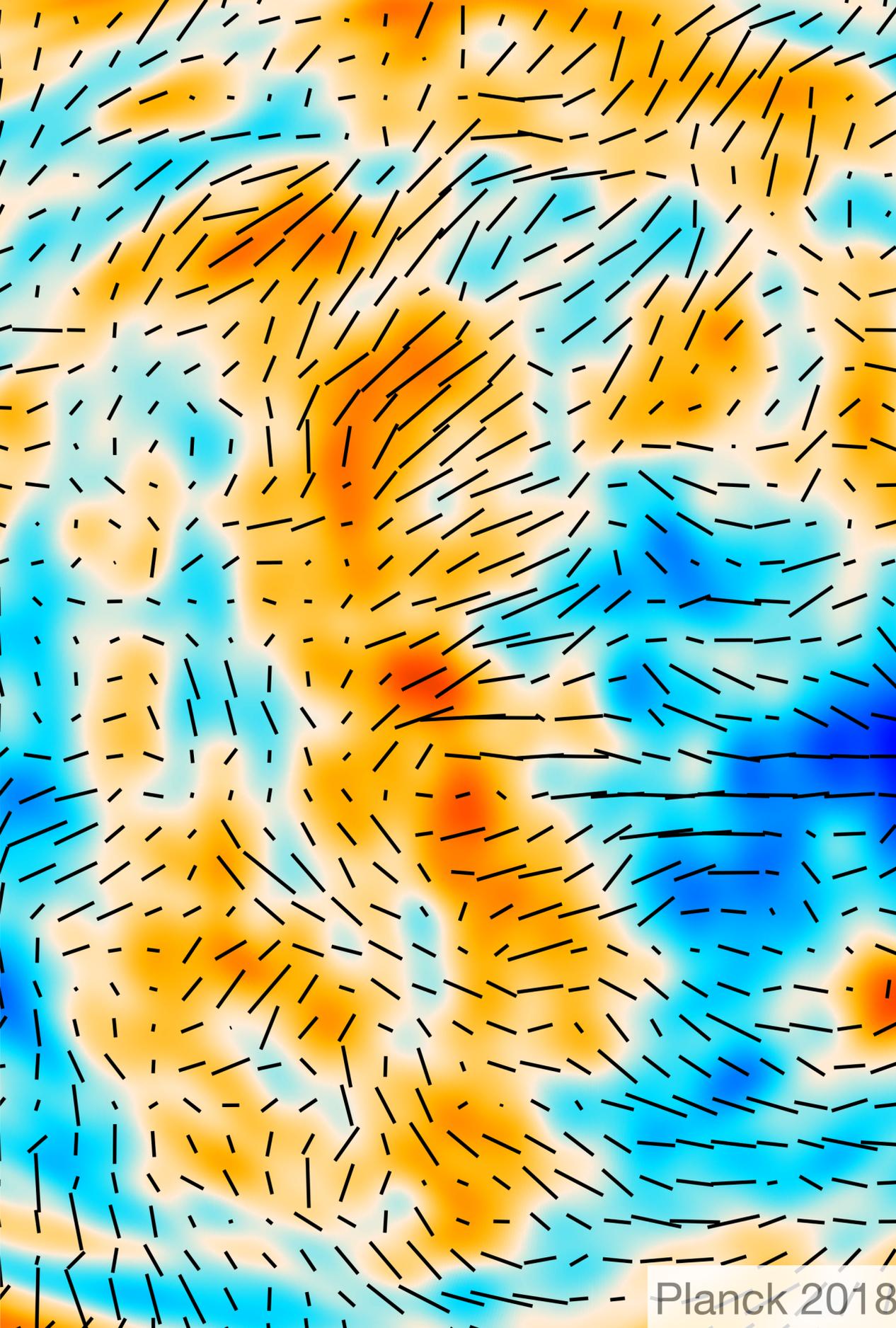
OUTLINE

1. THE PRIMORDIAL & THE LENSED CMB



2. CMB LENSING SPECTRA

3. DELENSING CMB B-MODES

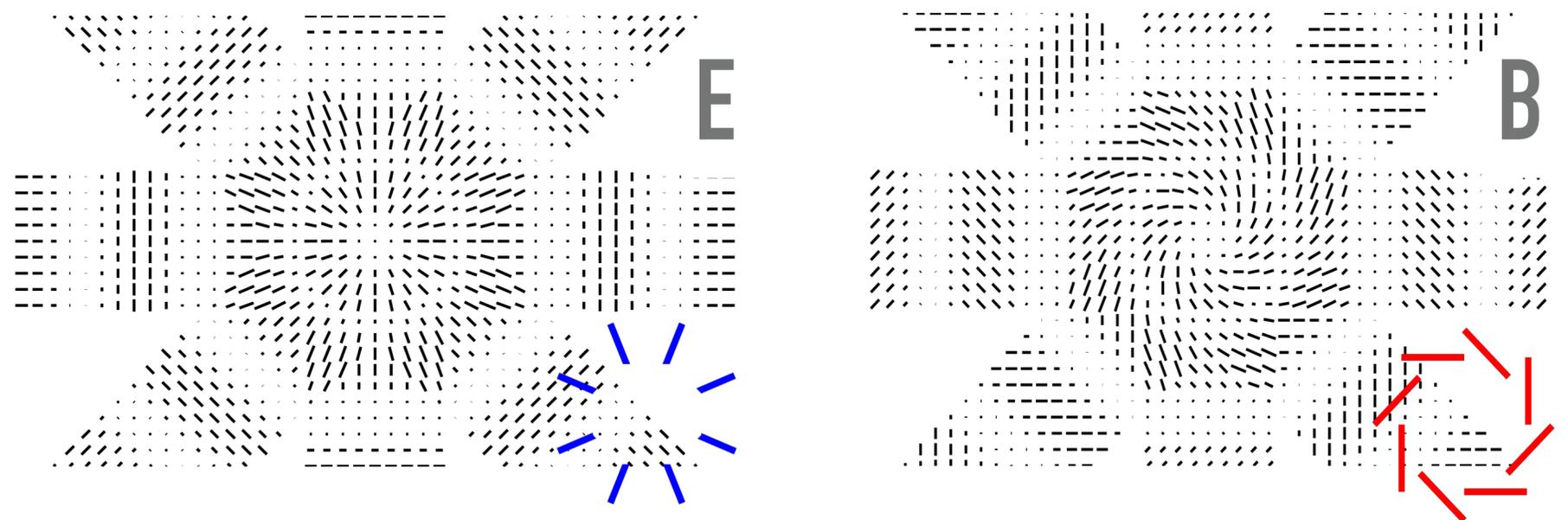


CMB OBSERVABLES — INTENSITY AND POLARISATION

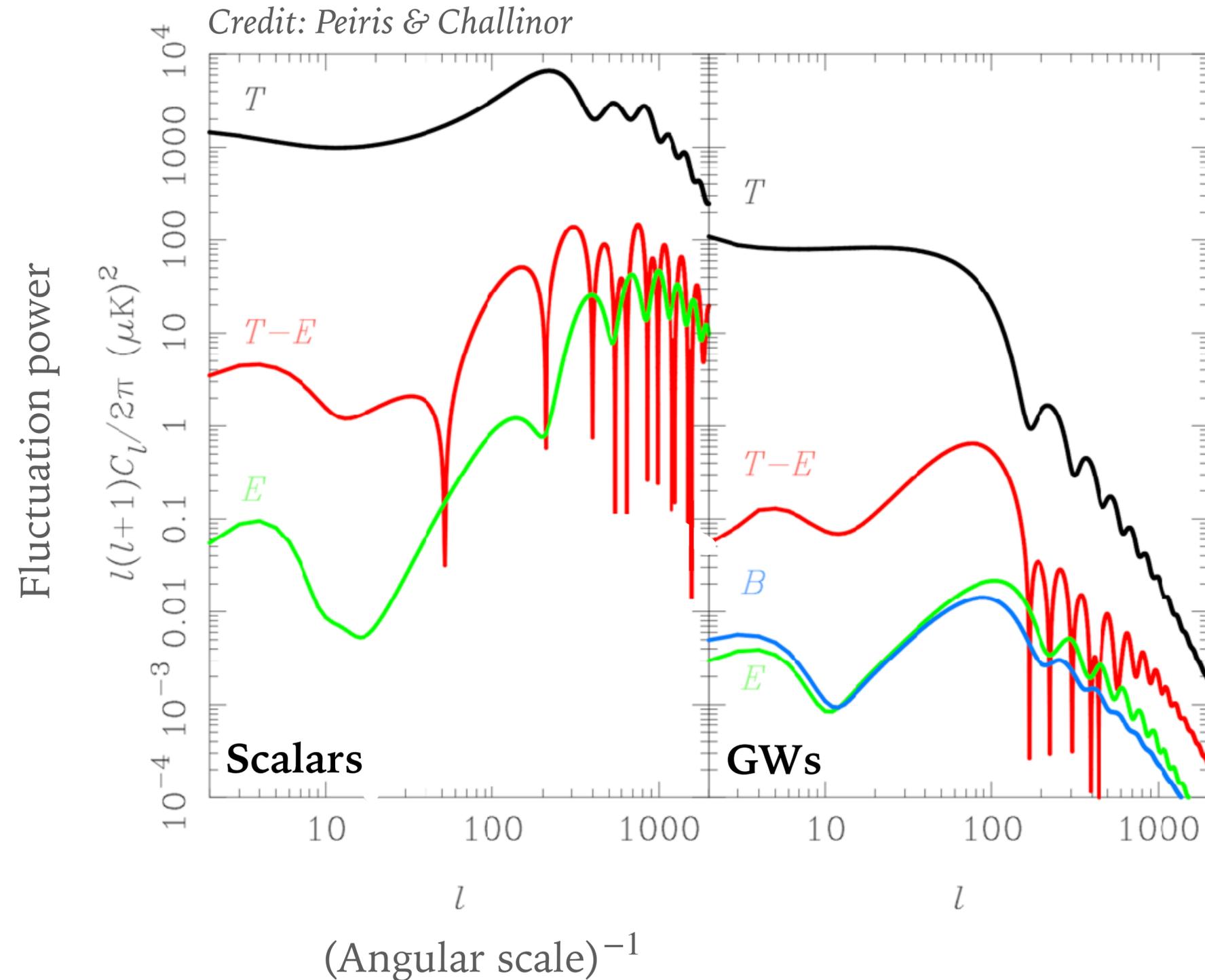
Observe:



Connect with theory:



THE PRIMORDIAL CMB



- To leading order, B-modes sourced only by primordial gravitational waves

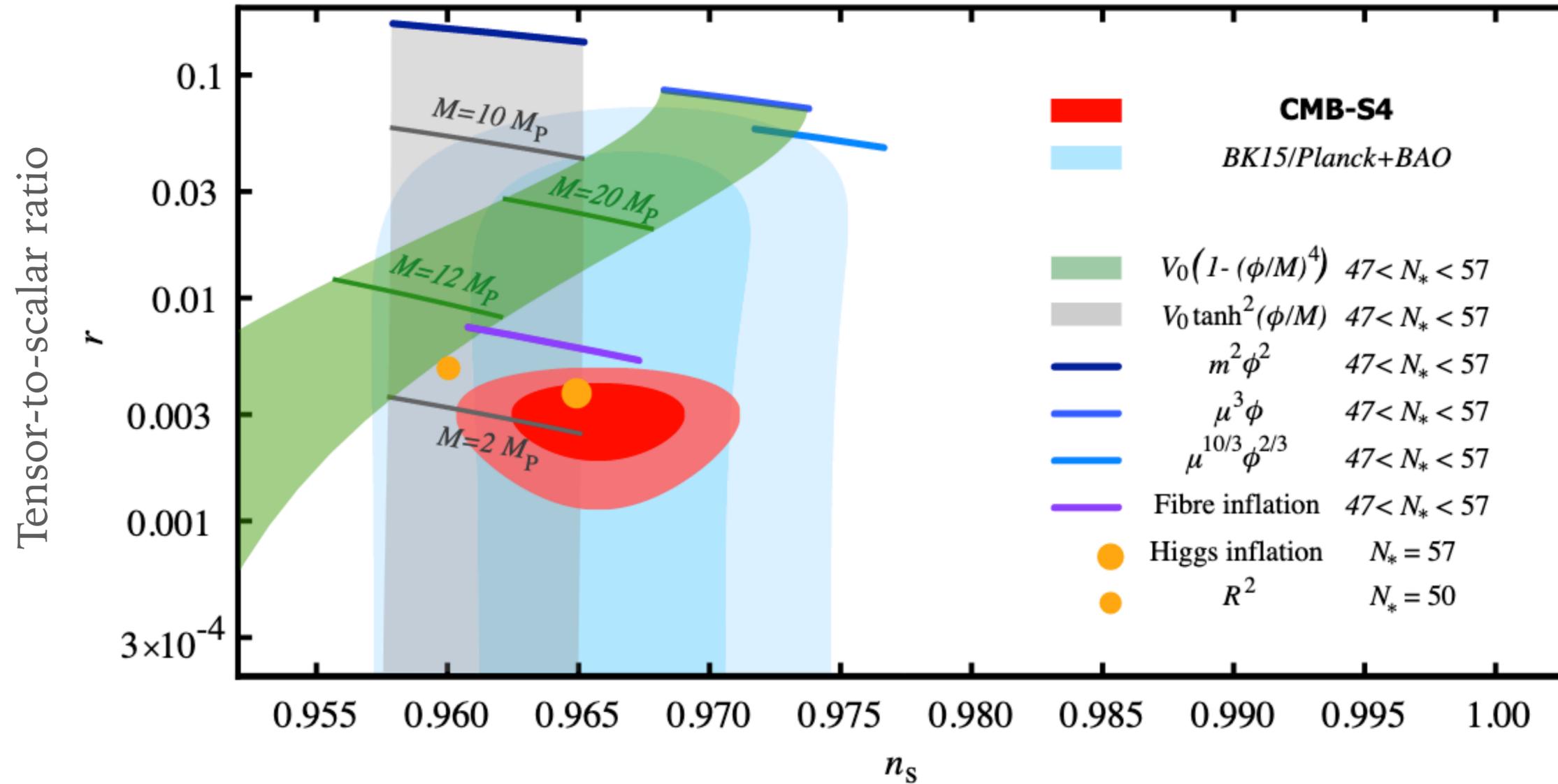
Kamionkowski+ 97 , Seljak & Zaldarriaga 97

$r < 0.044$ *Planck + BICEP/Keck*

- Statistically isotropic, Gaussian random field

CONSTRAINING INFLATION THROUGH THE CMB

CMB-S4, arXiv:1907.04473



Spectral index of primordial (scalar) perturbations

CMB LENSING

Very accurately described as:

$$\tilde{T}(\mathbf{x}) = T(\mathbf{x} + \alpha(\mathbf{x}))$$

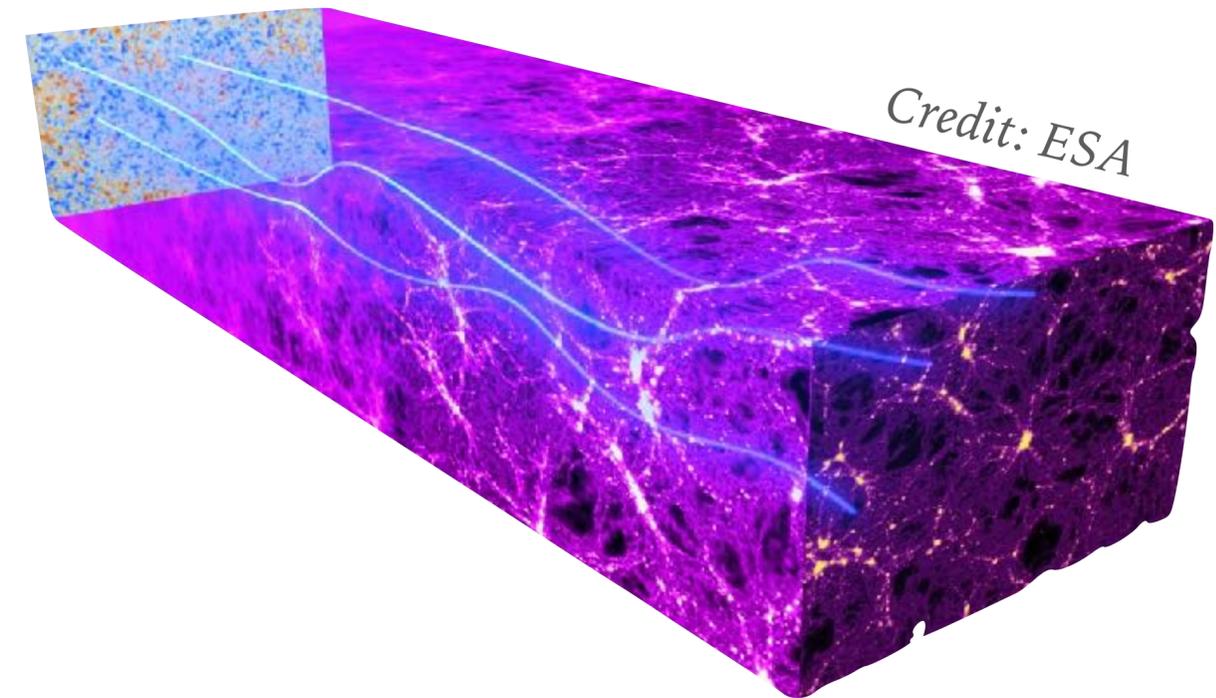
$$\tilde{Q}(\mathbf{x}) = Q(\mathbf{x} + \alpha(\mathbf{x}))$$

$$\tilde{U}(\mathbf{x}) = U(\mathbf{x} + \alpha(\mathbf{x}))$$

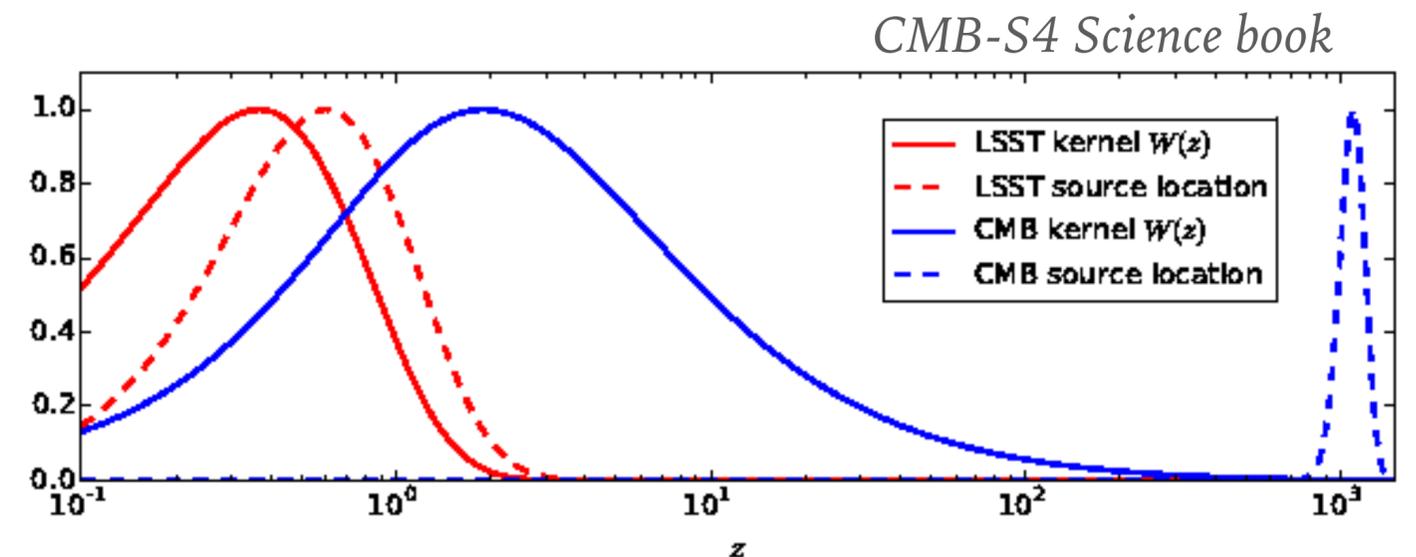
Under the Born approximation, $\alpha(\mathbf{x}) = \nabla \phi(\mathbf{x})$, where

$$\phi(\mathbf{x}) = -2 \int_0^{\chi} d\chi g(\chi, \chi_*) \Psi(\chi \mathbf{x}, \eta_0 - \chi)$$

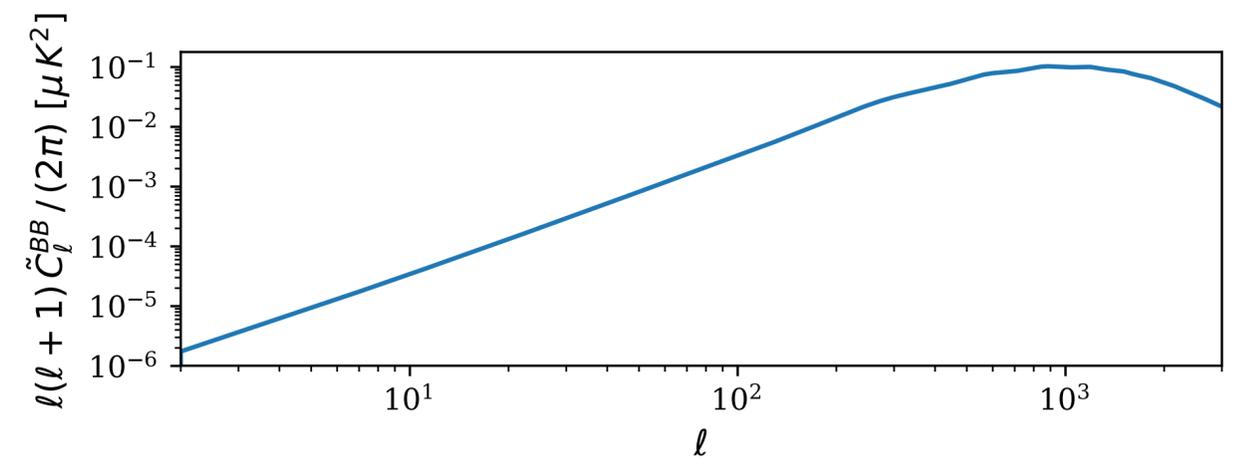
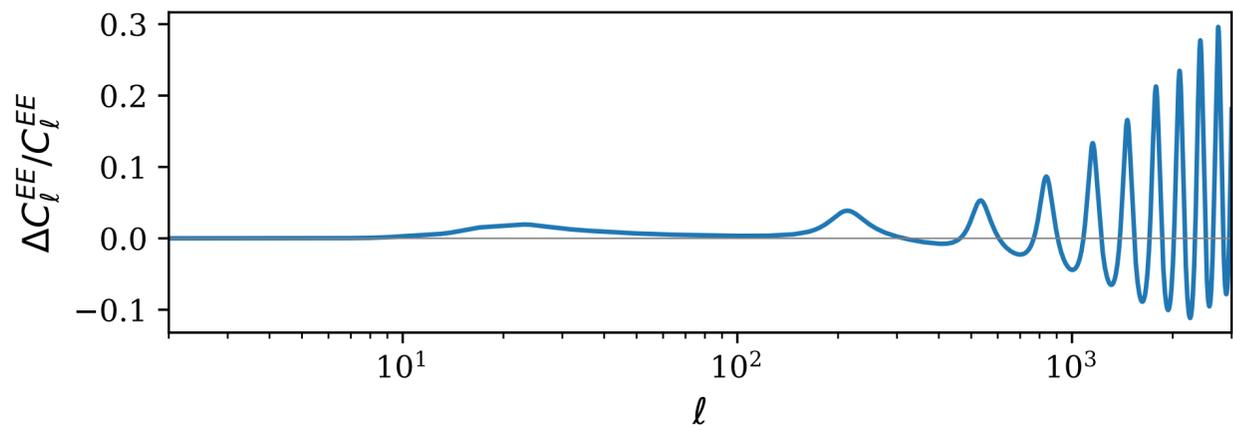
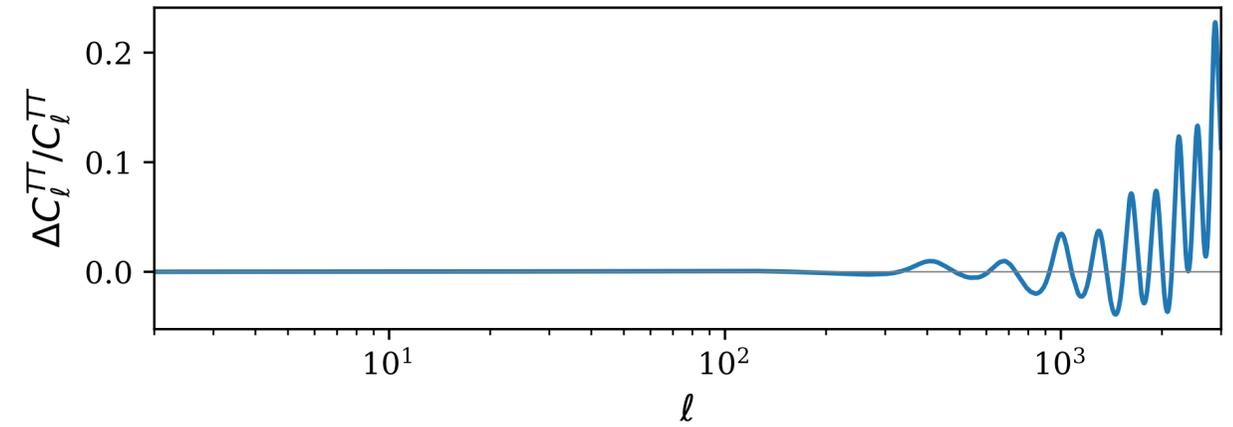
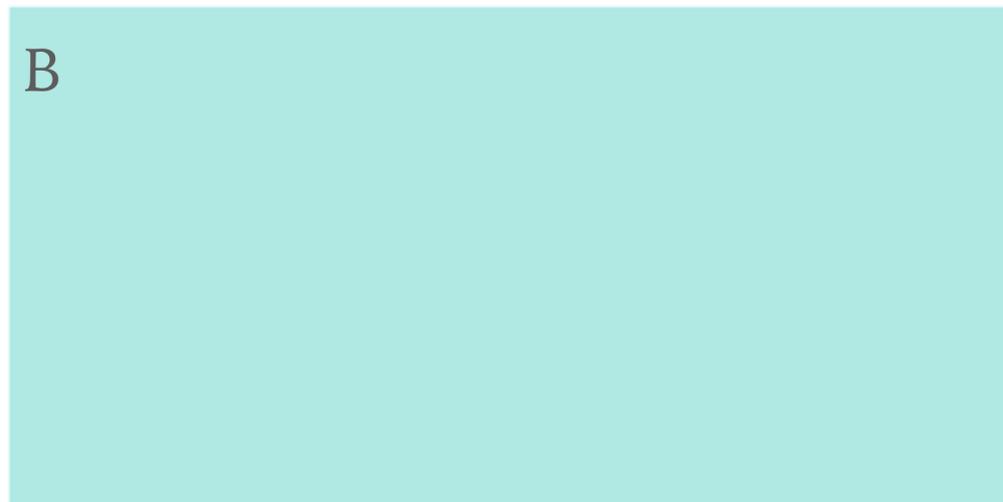
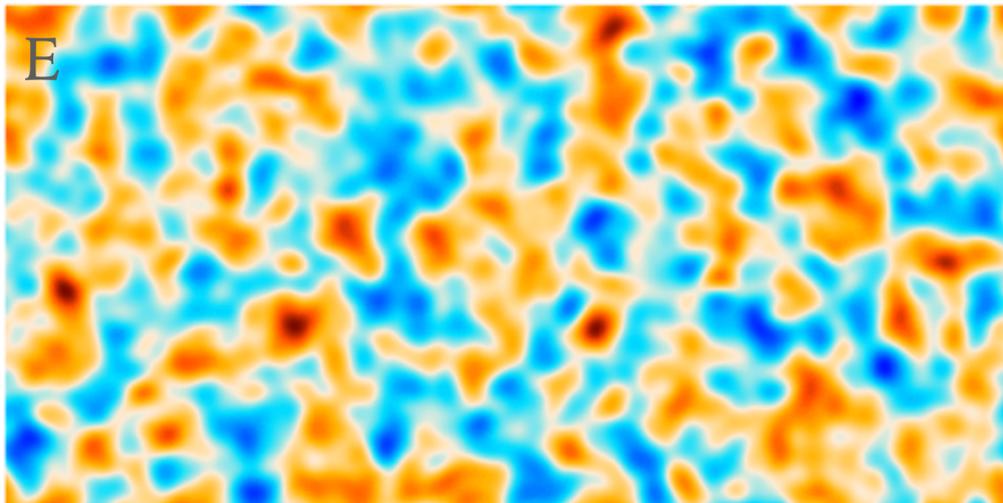
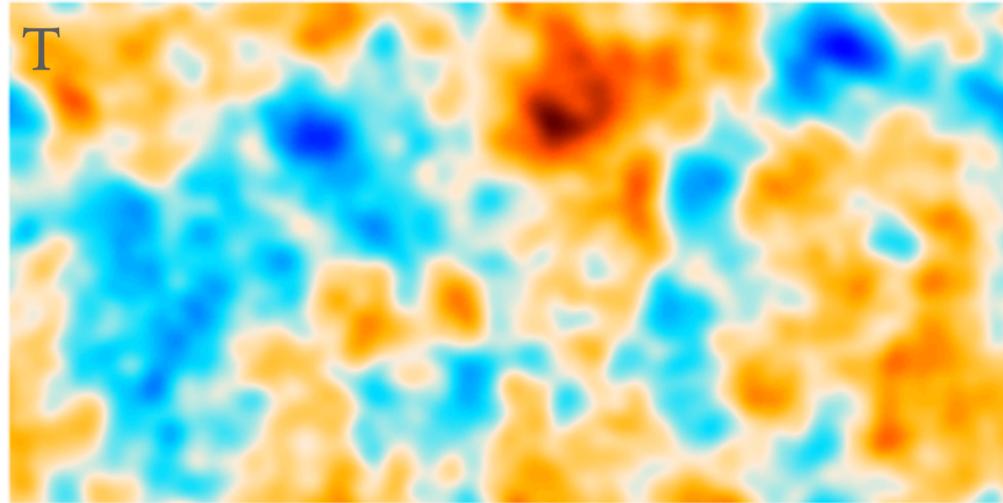
is related to $\kappa = -\frac{1}{2} \nabla^2 \phi$.



$\alpha \sim \text{arcmin}$, coherent on degree scales (typical lens $O(100\text{Mpc})$)



CMB LENSING



INTERNAL RECONSTRUCTIONS OF CMB LENSING

Unlensed CMB is statistically isotropic:

$$\langle T(\mathbf{l})T(\mathbf{l}') \rangle_{CMB} = (2\pi)^2 \delta^2(\mathbf{l} + \mathbf{l}') \tilde{C}_l^{TT}$$

Lensing induces statistical anisotropy:

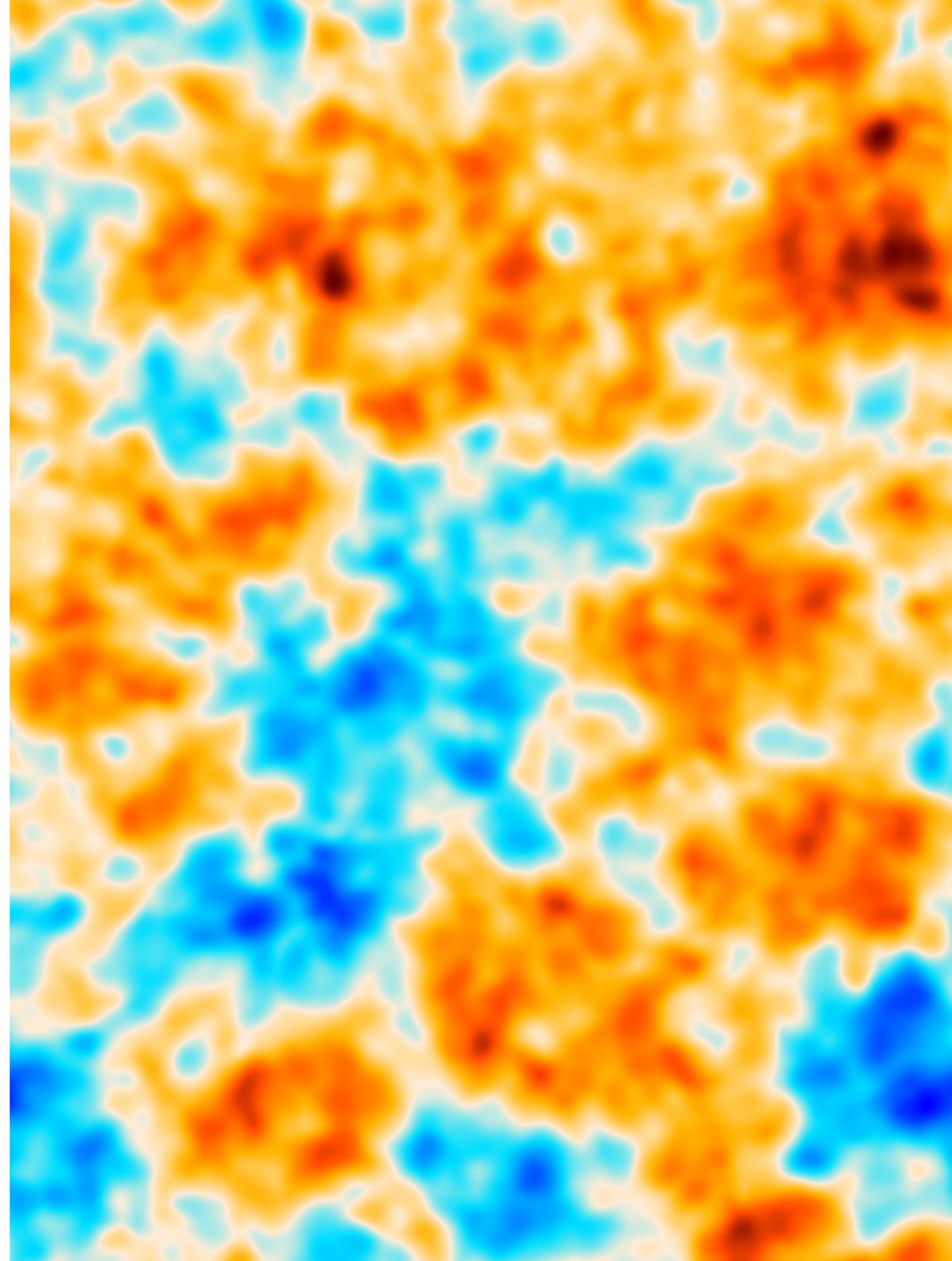
$$\langle \tilde{T}(\mathbf{l})\tilde{T}(\mathbf{l}') \rangle_{CMB} = f^{TT}(\mathbf{l}, \mathbf{l}') \phi(\mathbf{l} + \mathbf{l}')$$

In practice, off-diagonal correlations probe the lensing potential. The **quadratic estimator**:

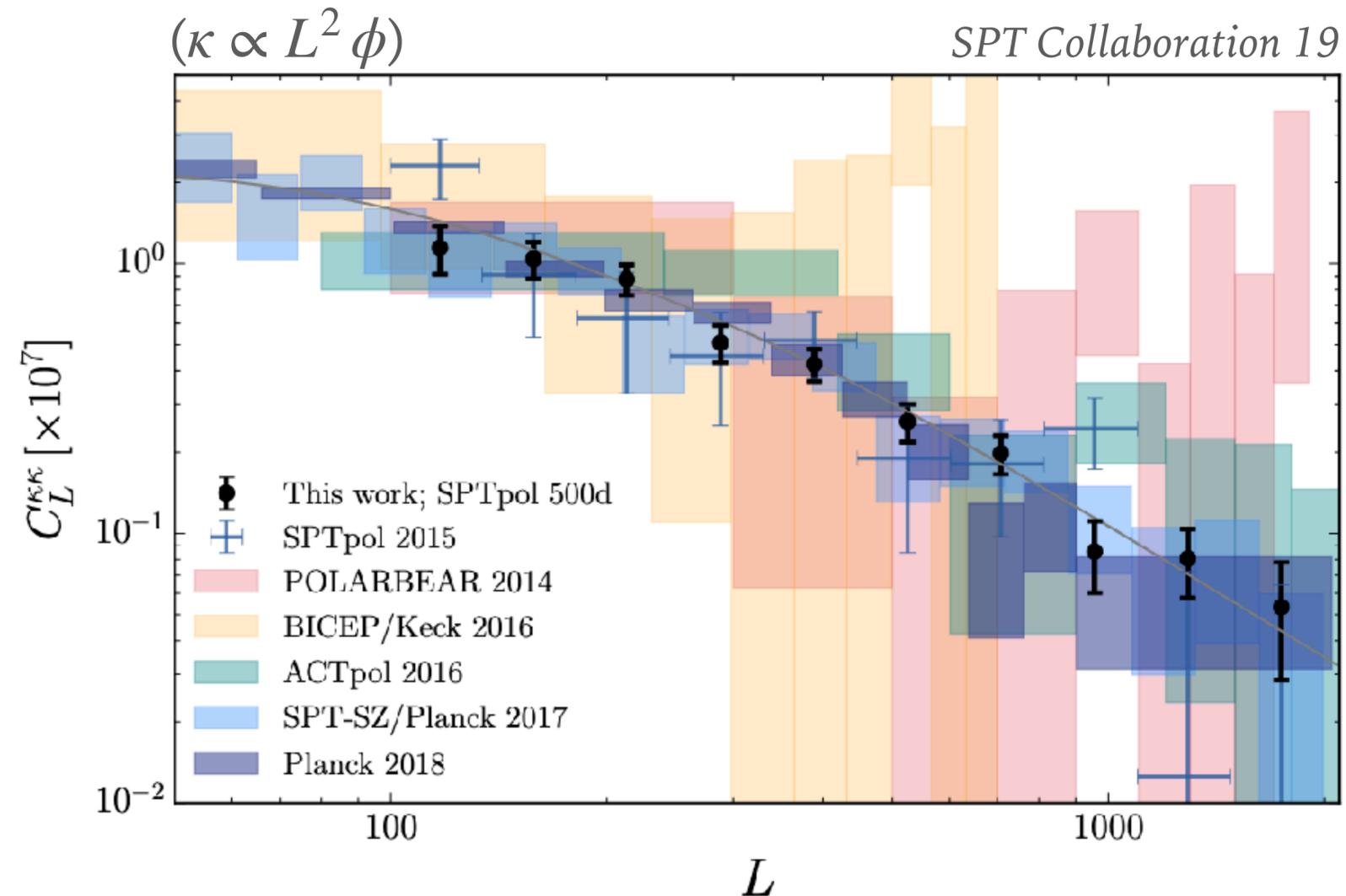
$$\hat{\phi}^{TT}(\mathbf{L}) \equiv N(\mathbf{L}) \int \frac{d^2\mathbf{l}}{2\pi} \tilde{T}(\mathbf{l})\tilde{T}^*(\mathbf{l} - \mathbf{L}) g(\mathbf{l}, \mathbf{L}) \quad \text{Hu \& Okamoto 01}$$

Much more information in the polarization (particularly in small-scale B -modes).

CMB LENSING SPECTRA



CMB LENSING SPECTRA



Breaks degeneracy among late-time Physics in primary CMB

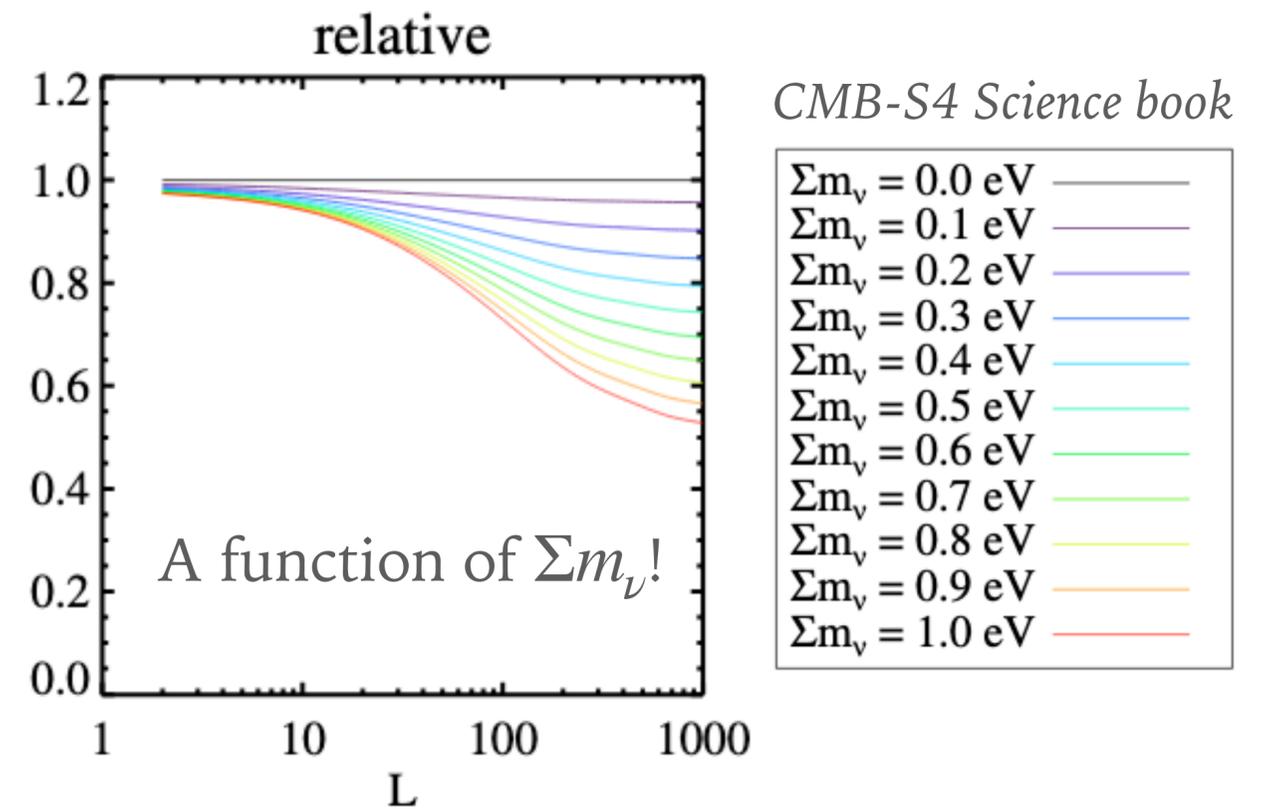
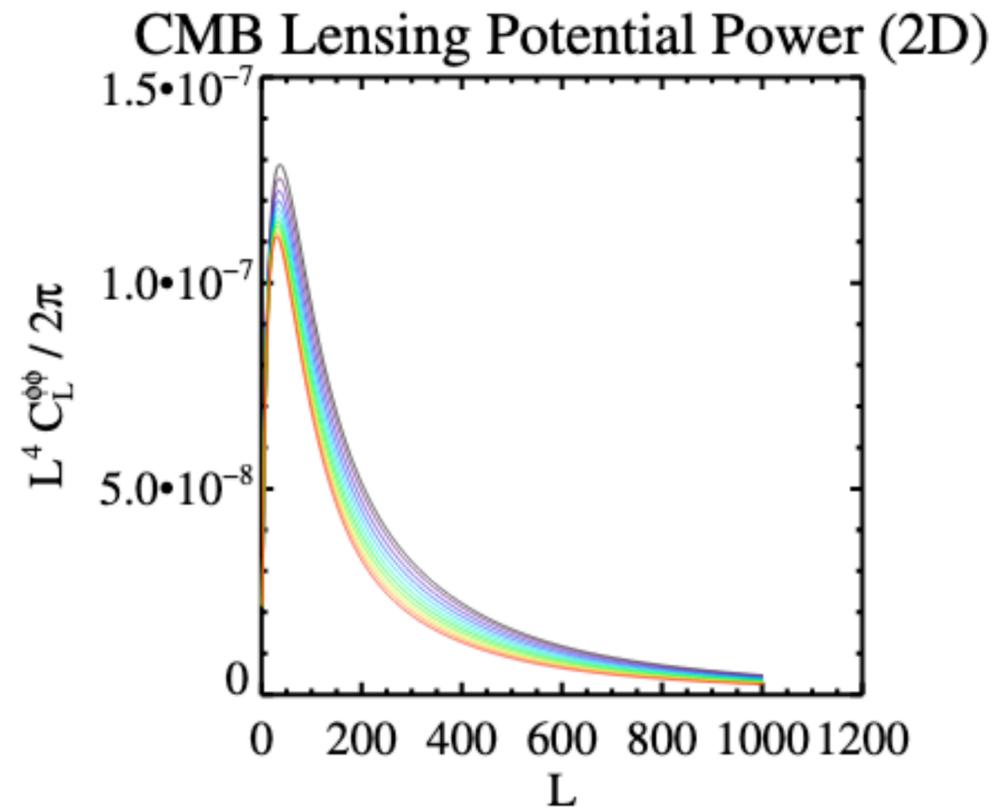
Auto- and cross-correlations probe geometry, neutrino masses, dark energy ++

Cross-correlations useful to calibrate photo-z's and weak lensing shear estimators

Vallinoto 11, Das+14, Schaun+ 17, ...

CMB LENSING CONSTRAINTS OF NEUTRINO MASSES

Massive ν 's cause
suppression of power
on small scales
relative to large scales



SO + DESI
SO forecasts 19

$\left\{ \begin{array}{l} \sigma(\Sigma m_\nu) \approx 31 - 33 \text{ meV (with current } \tau \text{ prior)} \\ \sigma(\Sigma m_\nu) \approx 17 - 22 \text{ meV (with CLASS/LiteBird } \tau \text{ prior)} \end{array} \right. \longrightarrow \text{Guaranteed detection!}$
 $(\Sigma m_\nu > 60 \text{ meV})$

Cross-correlations show great promise (also for $\sigma_8, f_{nl}, +$) — can partially get around τ prior

LENSING POWER SPECTRUM MEASUREMENTS

- ✓ Modeling: very easy — little non-linearities, negligible baryonic effects, no galaxy bias
- ✓ Foregrounds: polarization-based reconstructions very robust

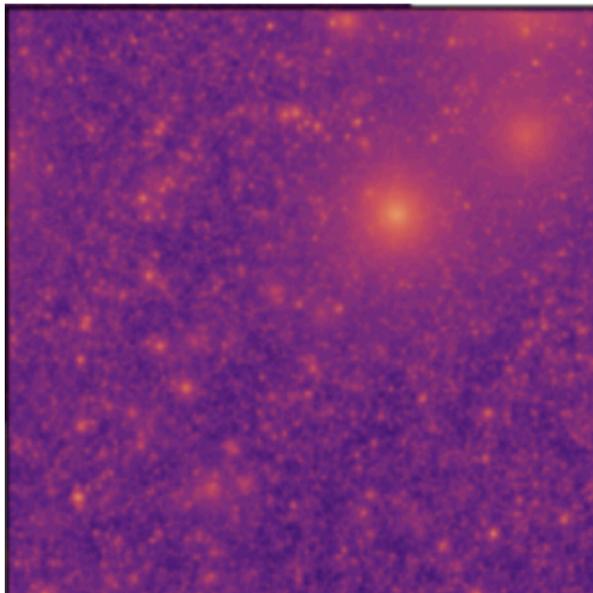


Temperature reconstructions still important for Stage-2 & 3 CMB (AdvACT, Simons Array, SO...)

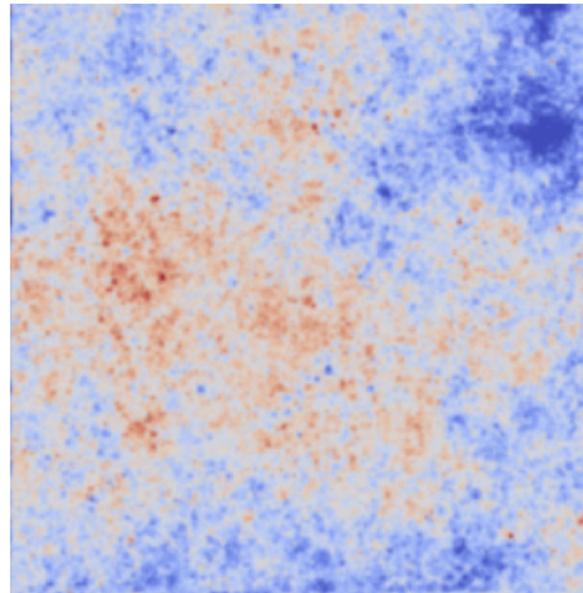
In intensity, microwave sky contaminated by non-Gaussian extragalactic sources: tSZ, kSZ, CIB

Van Engelen + 14, Osborne + 14, Ferraro & Hill 18

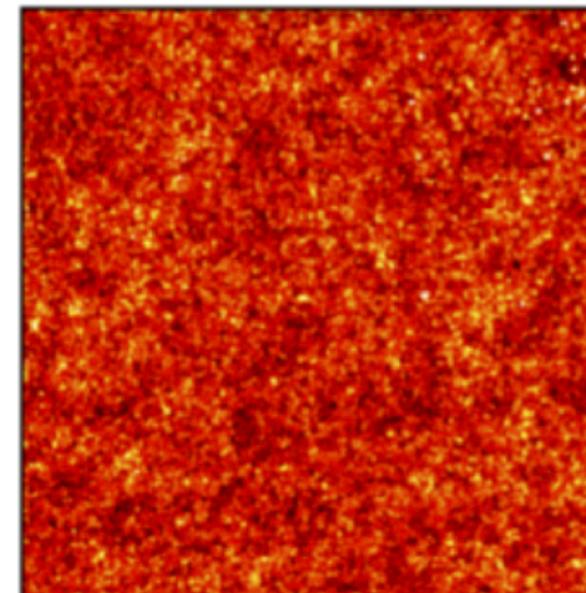
tSZ



kSZ



CIB



Adapted from WebSky

BIASES TO LENSING SPECTRA FROM EXTRAGALACTIC FOREGROUNDS

$$\hat{\phi} = \hat{\phi}[\tilde{T} + s, \tilde{T} + s]$$

Lensing Foreground (correlated with ϕ)

Can bias lensing reconstruction power spectrum

$$\langle \hat{\phi} \hat{\phi} \rangle = \langle \hat{\phi}[\tilde{T}, \tilde{T}] \hat{\phi}[\tilde{T}, \tilde{T}] \rangle + 2 \langle \hat{\phi}[\tilde{T}, \tilde{T}] \hat{\phi}[s, s] \rangle + 4 \langle \hat{\phi}[\tilde{T}, s] \hat{\phi}[\tilde{T}, s] \rangle + \langle \hat{\phi}[s, s] \hat{\phi}[s, s] \rangle$$

“Primary bispectrum bias”

“Secondary bispectrum bias”

“Trispectrum bias”

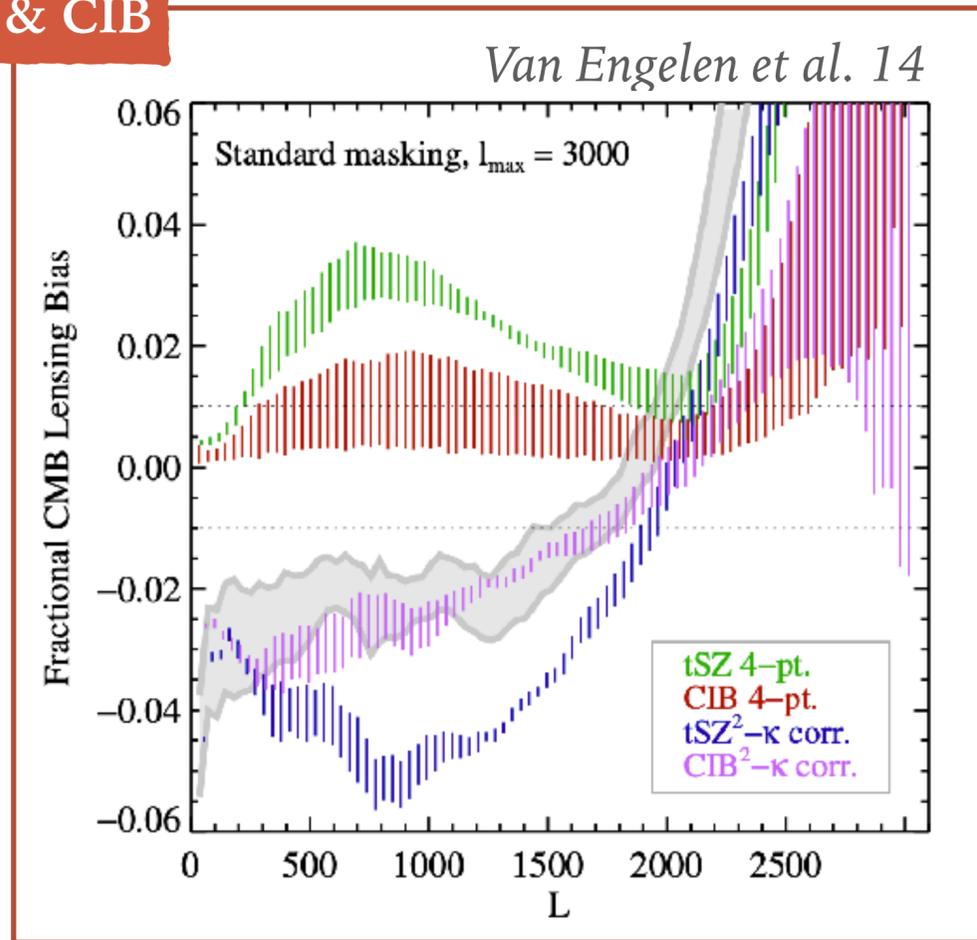
and cross-correlations with low-z matter tracers

$$\langle g[\phi] \hat{\phi} \rangle = \langle g[\phi] \hat{\phi}[\tilde{T}, \tilde{T}] \rangle + \langle g[\phi] \hat{\phi}[s, s] \rangle$$

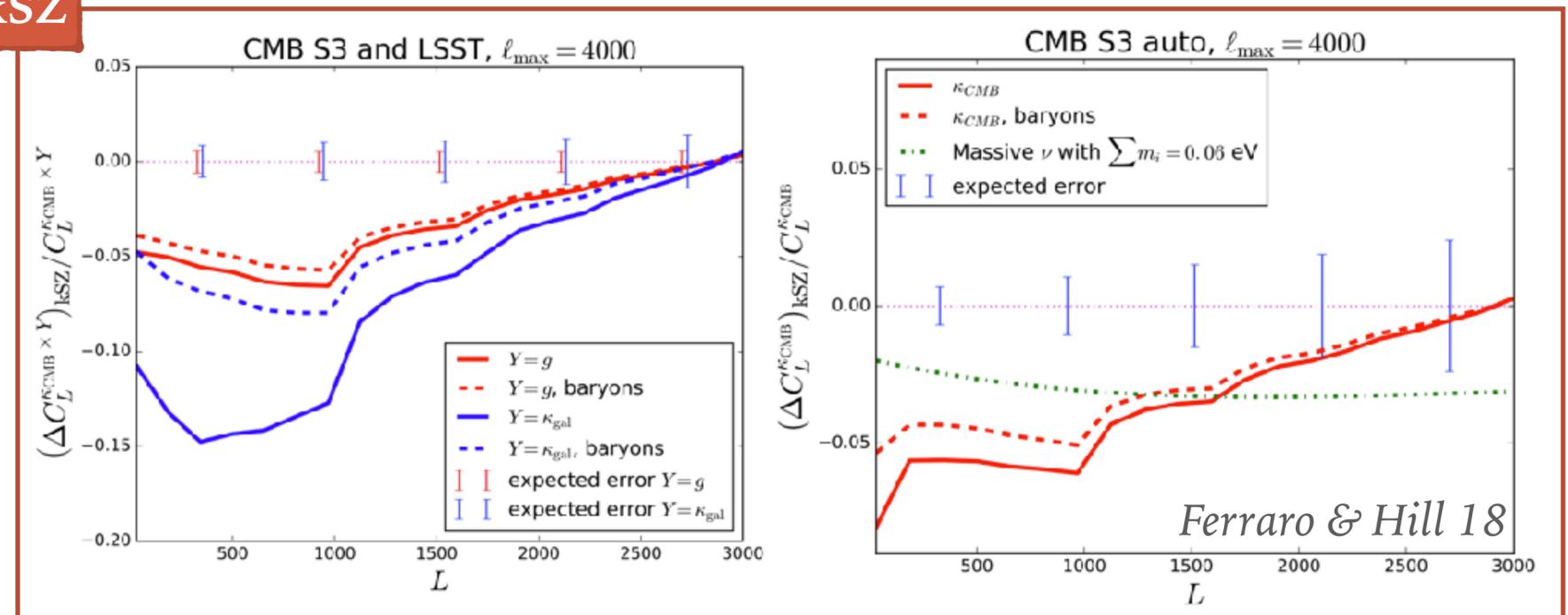
“Bispectrum bias”

BIASES TO LENSING SPECTRA FROM EXTRAGALACTIC FOREGROUNDS

tSZ & CIB



kSZ



➤ Impedes reconstruction of lensing from small scales, i.e., sets l_{\max}

➤ Multi-frequency cleaning helps to some extent

➤ Mitigation techniques:

➤ Simulations *Van Engelen+ 14*

➤ Cleaning of gradient leg in QE *Madhavacheril & Hill 18*

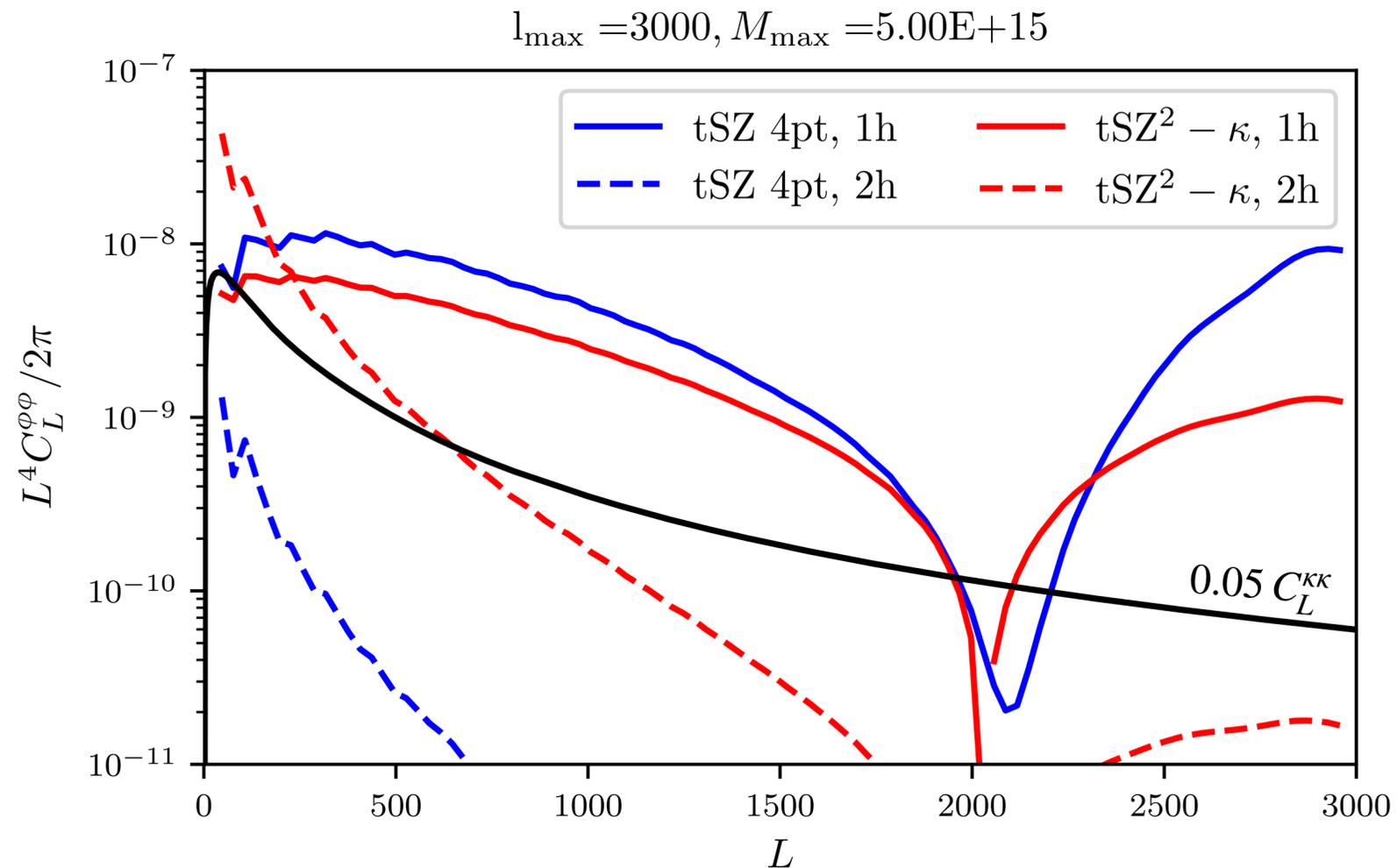
➤ Bias hardening *Namikawa+13, Osborne+14 Sailer+ 20*

➤ Shear-only estimators

Schaan & Ferraro 18

MODELING LENSING BIASES FROM EXTRAGALACTIC SOURCES

We calculate these biases analytically as a function of experimental sensitivity, resolution, point-source masking, etc, using a halo model prescription

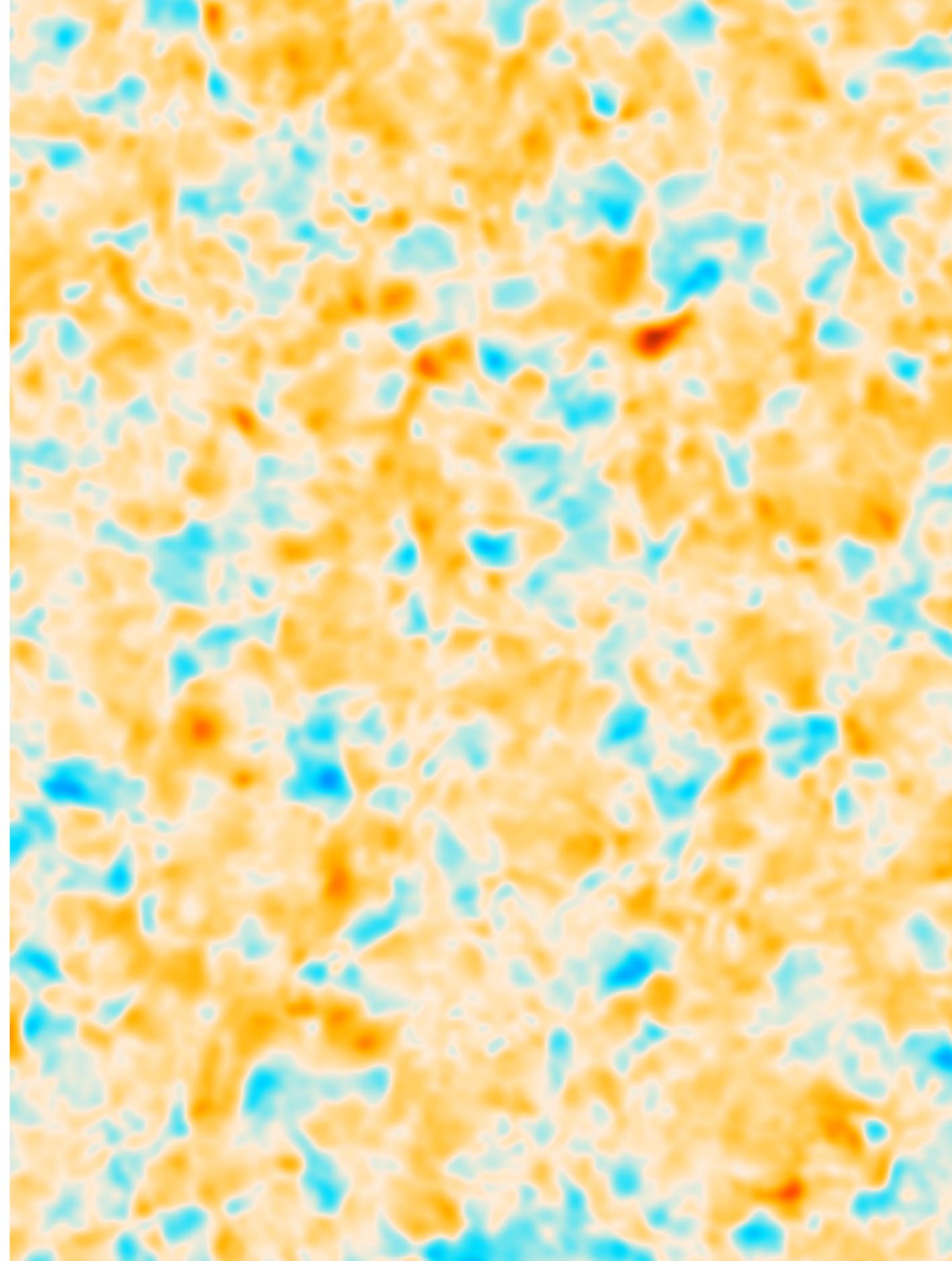


(Example: subset of tSZ biases for an SPT-like experiment)

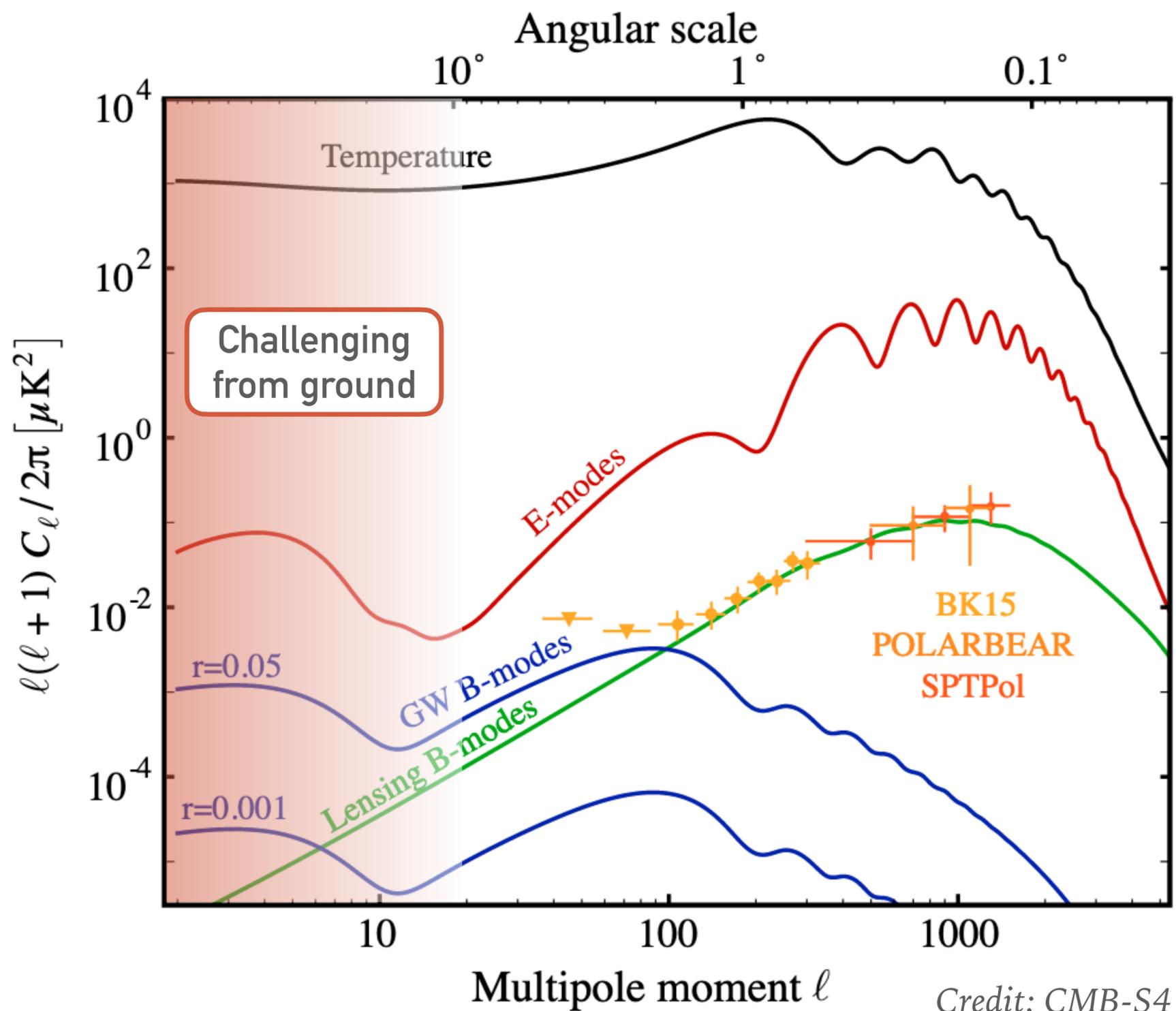
Preliminary

- **Very fast, 1D method.** Lensing reconstructions done using FFTlog (for fast, discrete Hankel transforms), taking O(10ms) per lensing reconstruction on a single laptop core.
- Can calculate biases and associated uncertainties
- Perhaps we could use QE down to smaller scales and correct bias at PS level?

DELENSING B-MODES



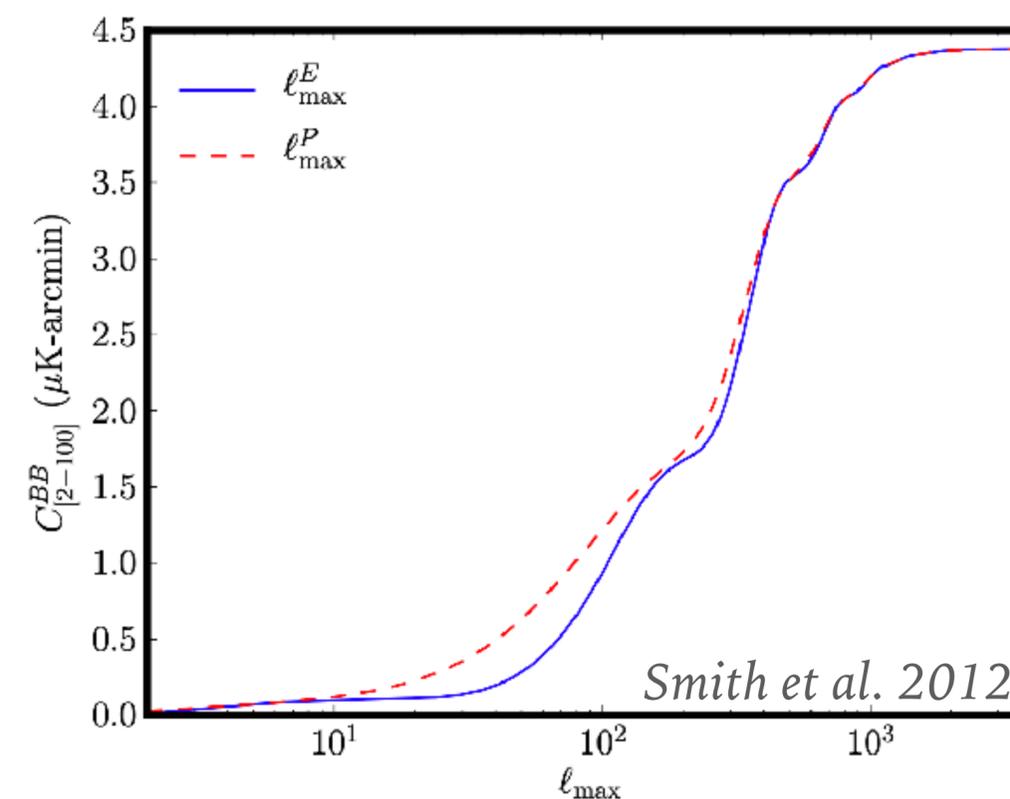
THE LENSING B-MODE



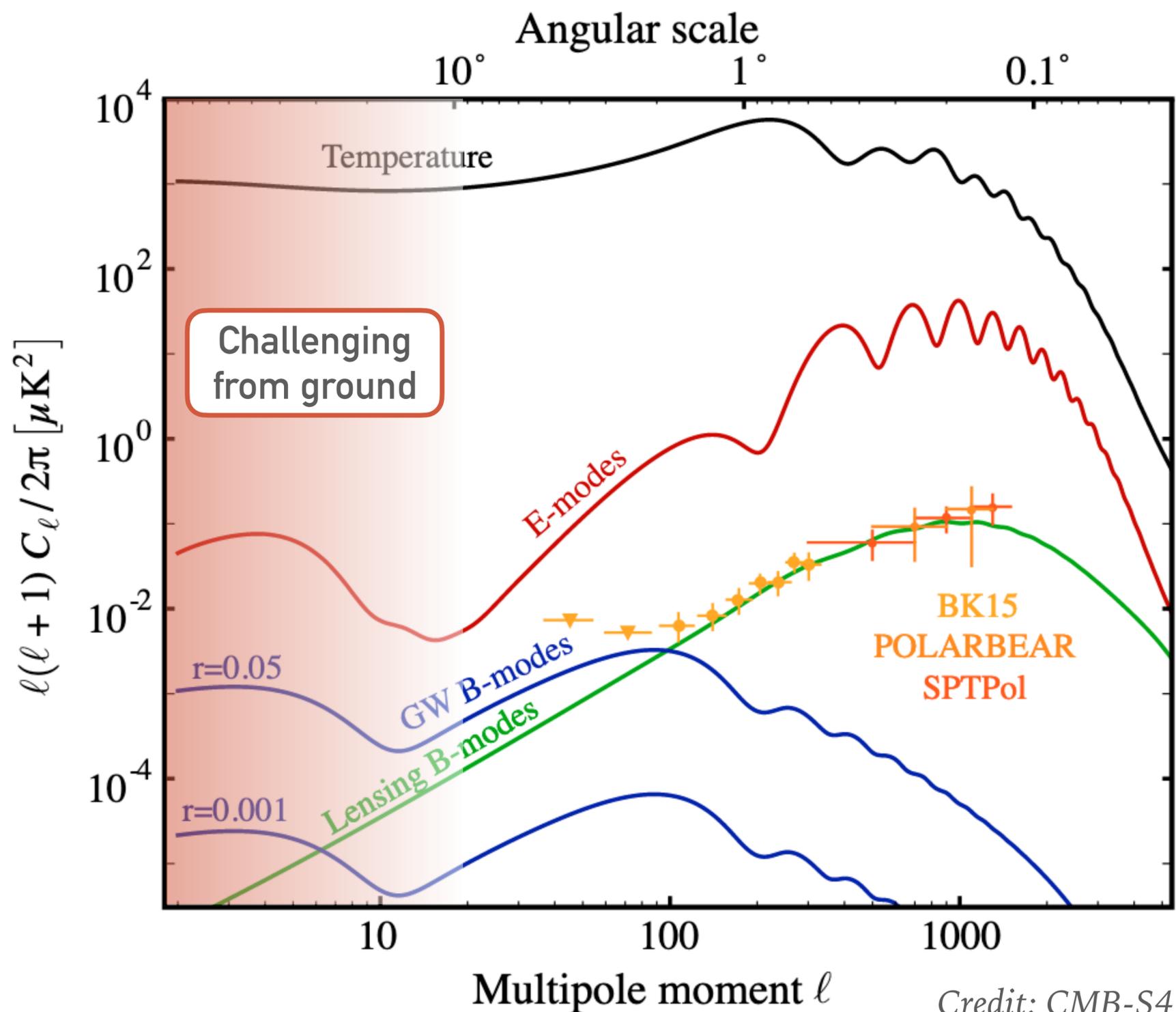
Lensing converts E- into B-modes, inducing noise with $\Delta_P \approx 5 \mu K \text{ arcmin}$

Zaldarriaga & Seljak 98

Why does it look like white noise?



THE LENSING B-MODE



Credit: CMB-S4

Lensing converts E- into B-modes, inducing noise with $\Delta_P \approx 5\mu\text{K arcmin}$

Zaldarriaga & Seljak 98

$$\sigma(r) \propto C_l^{BB} + N_l^{BB}$$

$\left. \begin{array}{l} \uparrow \\ \left. \begin{array}{l} - \text{ Primordial} \\ - \text{ Foregrounds} \\ - \text{ Lensing} \end{array} \right\} \end{array} \right\}$

For CMB-S4, delensing improves $\sigma(r)$ by $\times 5$ or more

B-MODE DELENSING

Lensing: $\tilde{P}(\mathbf{x}) = P(\mathbf{x} + \alpha(\mathbf{x}))$

\implies Delensing: $P^{\text{del}}(\mathbf{x}) = \tilde{P}(\mathbf{x} + \alpha^{-1}(\mathbf{x}))$ (often $\alpha^{-1} \approx -\alpha$)

Challenge: r goals require measuring both

- large (degree) angular scale B -modes, where primordial signal peaks
- intermediate & small scale lenses and E -modes, to delens those B -modes

Small
Aperture
Telescope
(SAT)



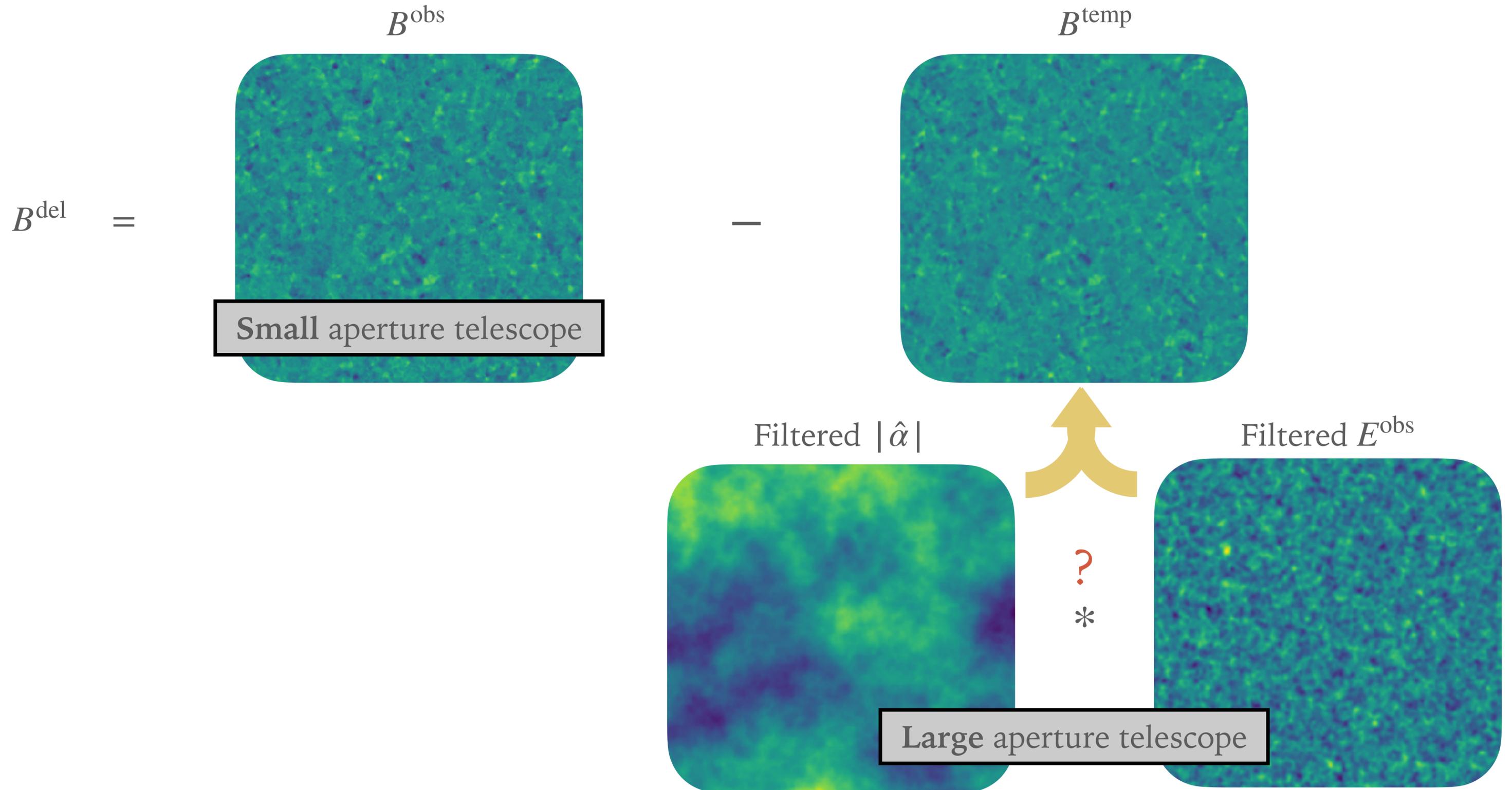
e.g., BICEP/Keck, SO SATs, CLASS, ABS, QUBIC...

Large
Aperture
Telescope
(LAT)



e.g., SPT, ACT, SO LAT, POLARBEAR/SA

B-MODE TEMPLATE DELENSING



Patches of 8° on a side. Colour scales differ across panels.

HOW EXACTLY IS THE TEMPLATE BUILT?

The lensing B-mode is

$$\tilde{B} = E \circledast \phi + O(\phi^2) + \dots$$

So the template is often built to leading (“gradient”) order

$$B^{\text{temp}} = \bar{E}^{\text{obs}} \circledast \hat{\phi}. \quad \text{e.g., SPT 17}$$

Why?

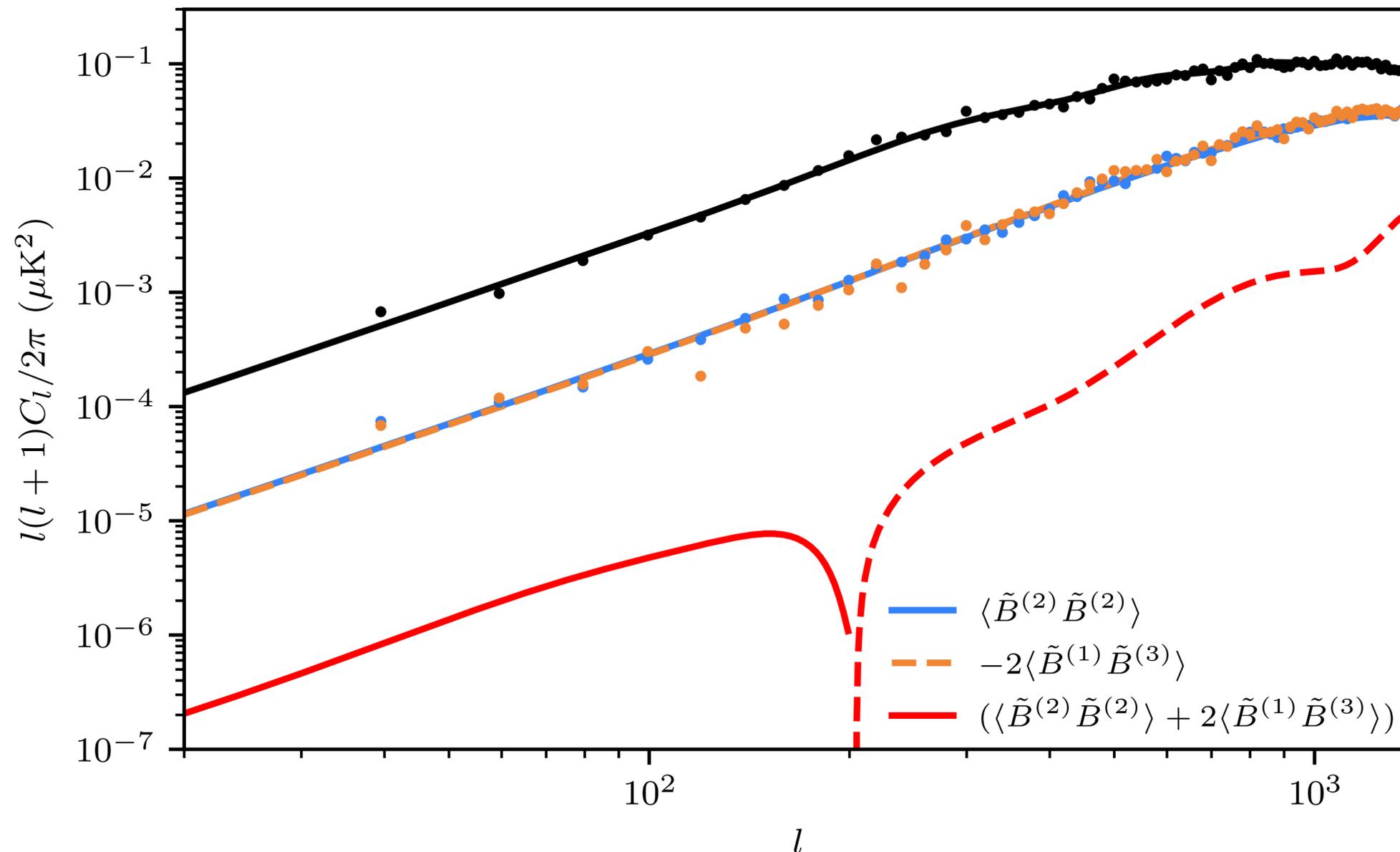
- Optimises strengths of LAT & SAT
- Analytically transparent (clear understanding of systematics)
- Template is assumed to track true B-modes very accurately

Non-perturbative template can also be built by deflecting observed E-modes directly:

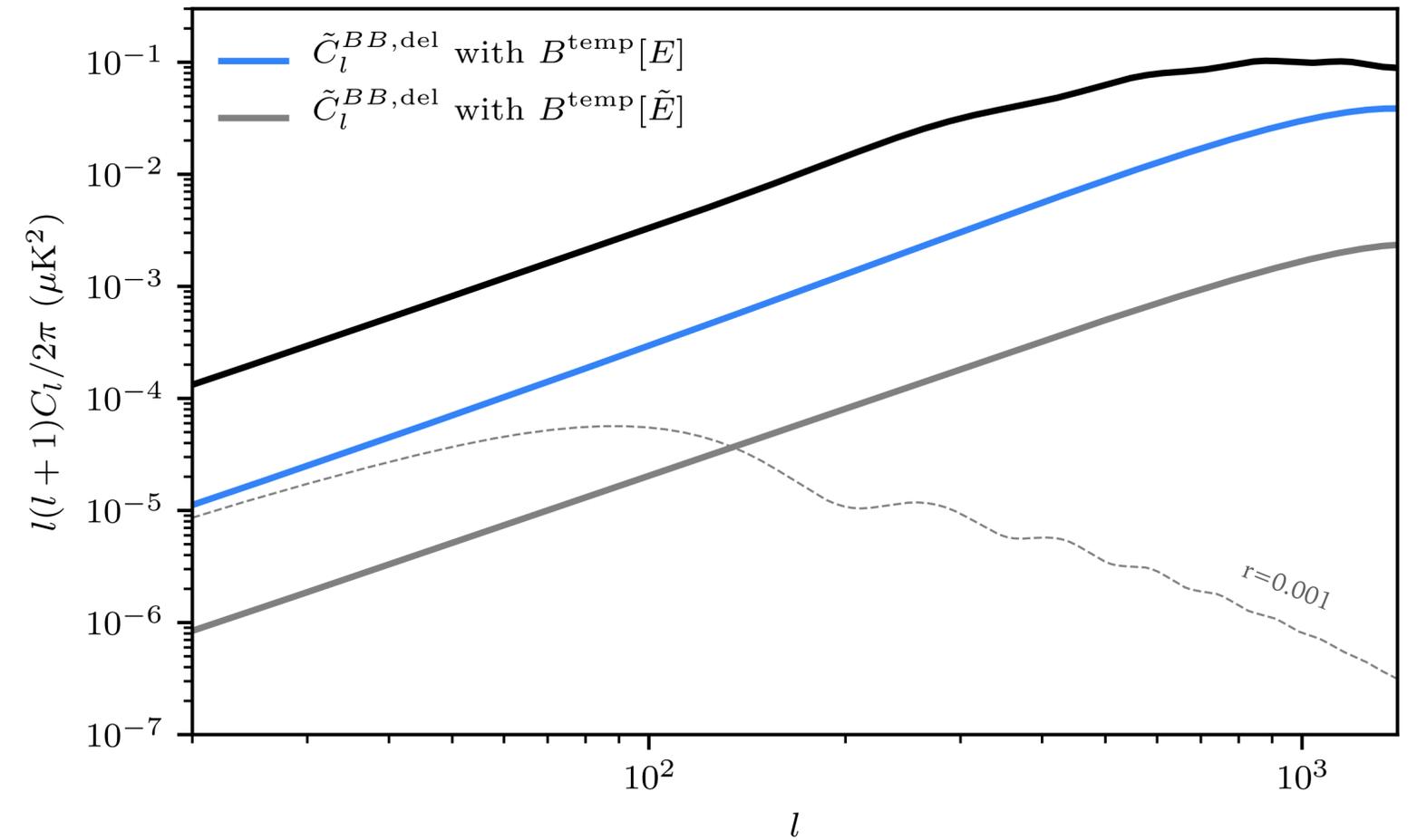
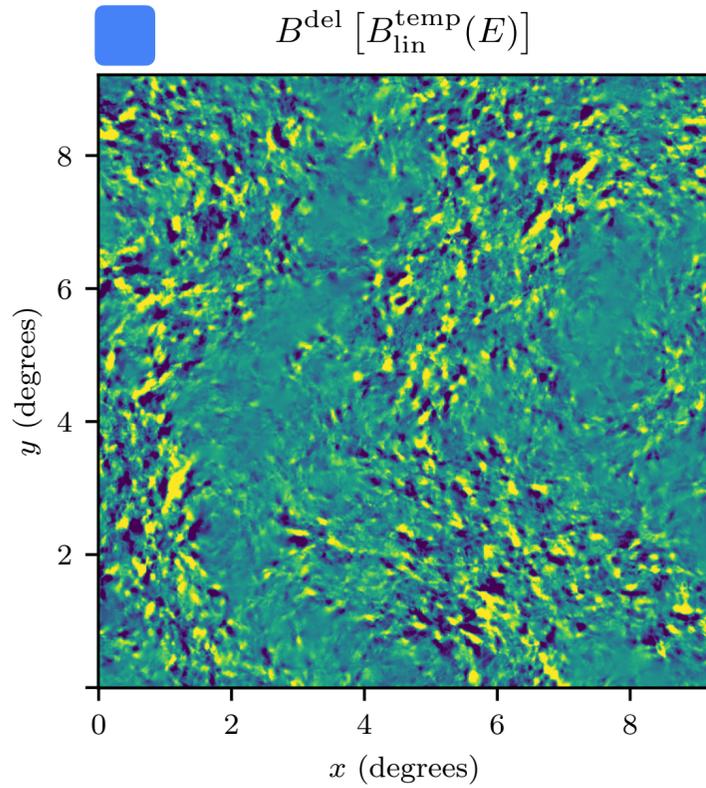
$$B_{\text{non-pert}}^{\text{temp}}(\mathbf{l}) = \mathcal{B}_1 \left[P^{E^{\text{obs}}}(\mathbf{x} + \nabla \hat{\phi}) \right]. \quad \text{e.g., Planck 18, POLARBEAR 19}$$

LIMITATIONS OF B-MODE DELENSING USING A TEMPLATE

- Corrections to the leading-order calculation of lensing B-mode power smaller than O(1)% because of extensive cancelations between large terms at higher orders.

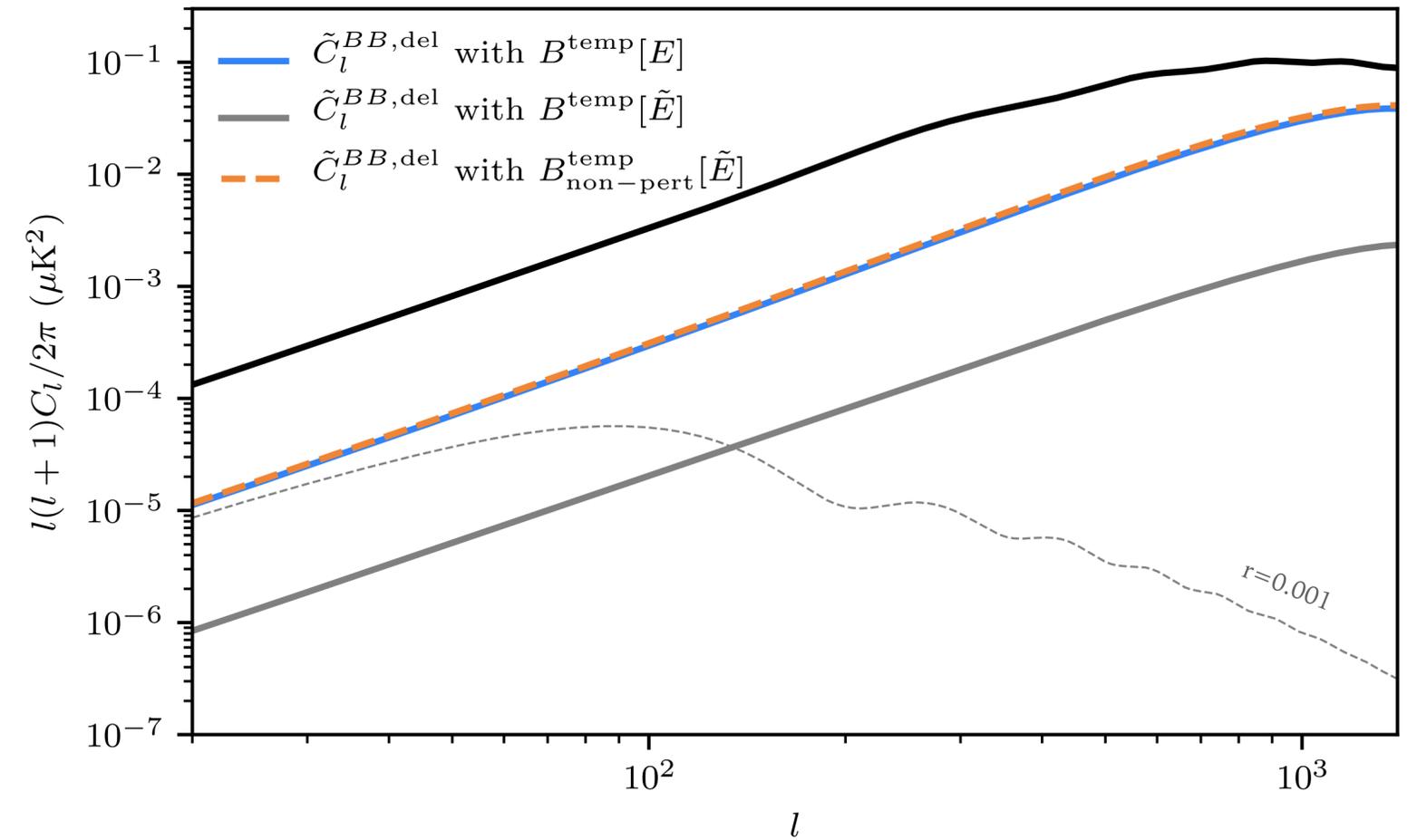
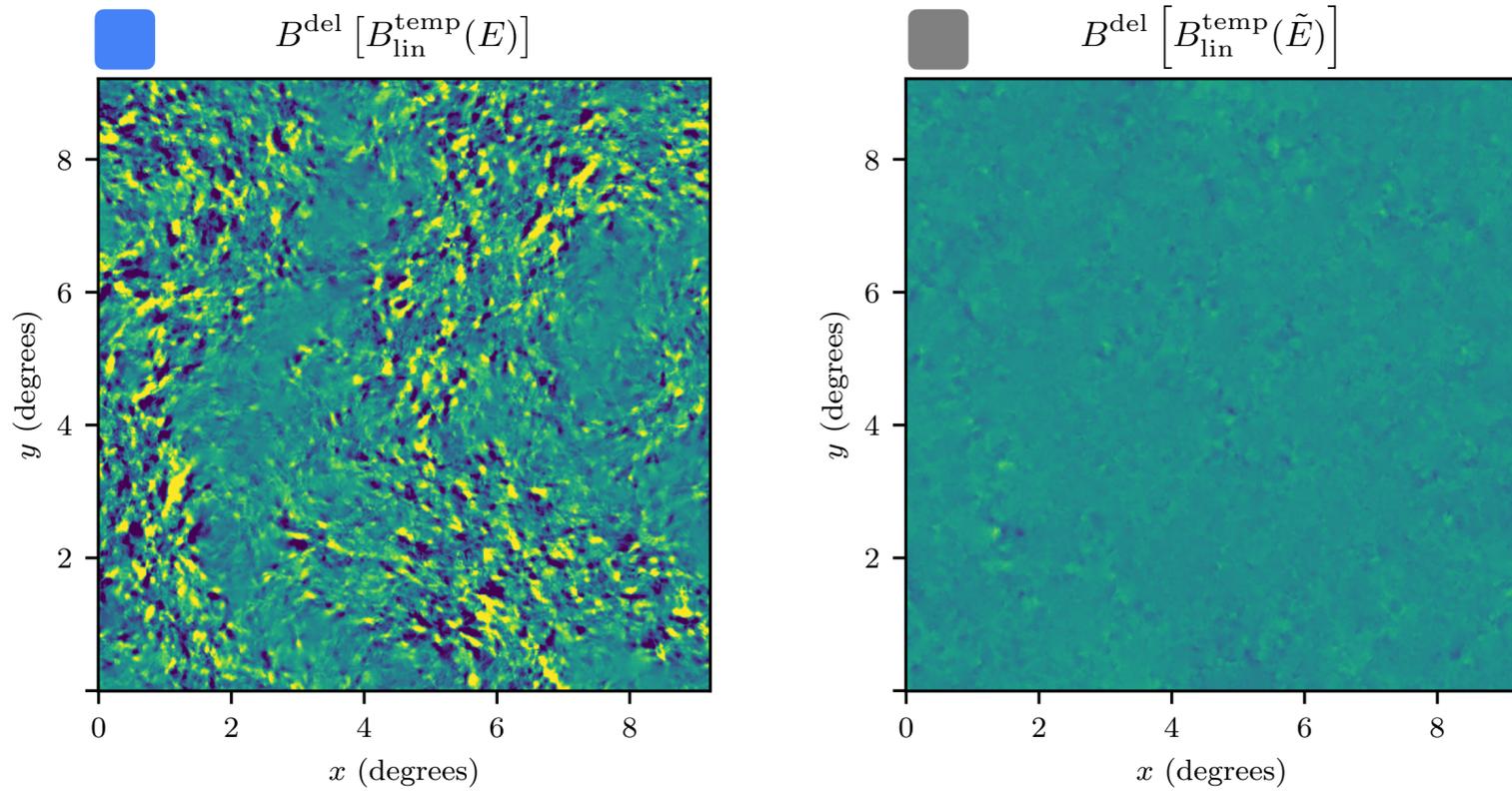


LIMITATIONS OF B-MODE TEMPLATE DELENSING: NOISELESS SCENARIO



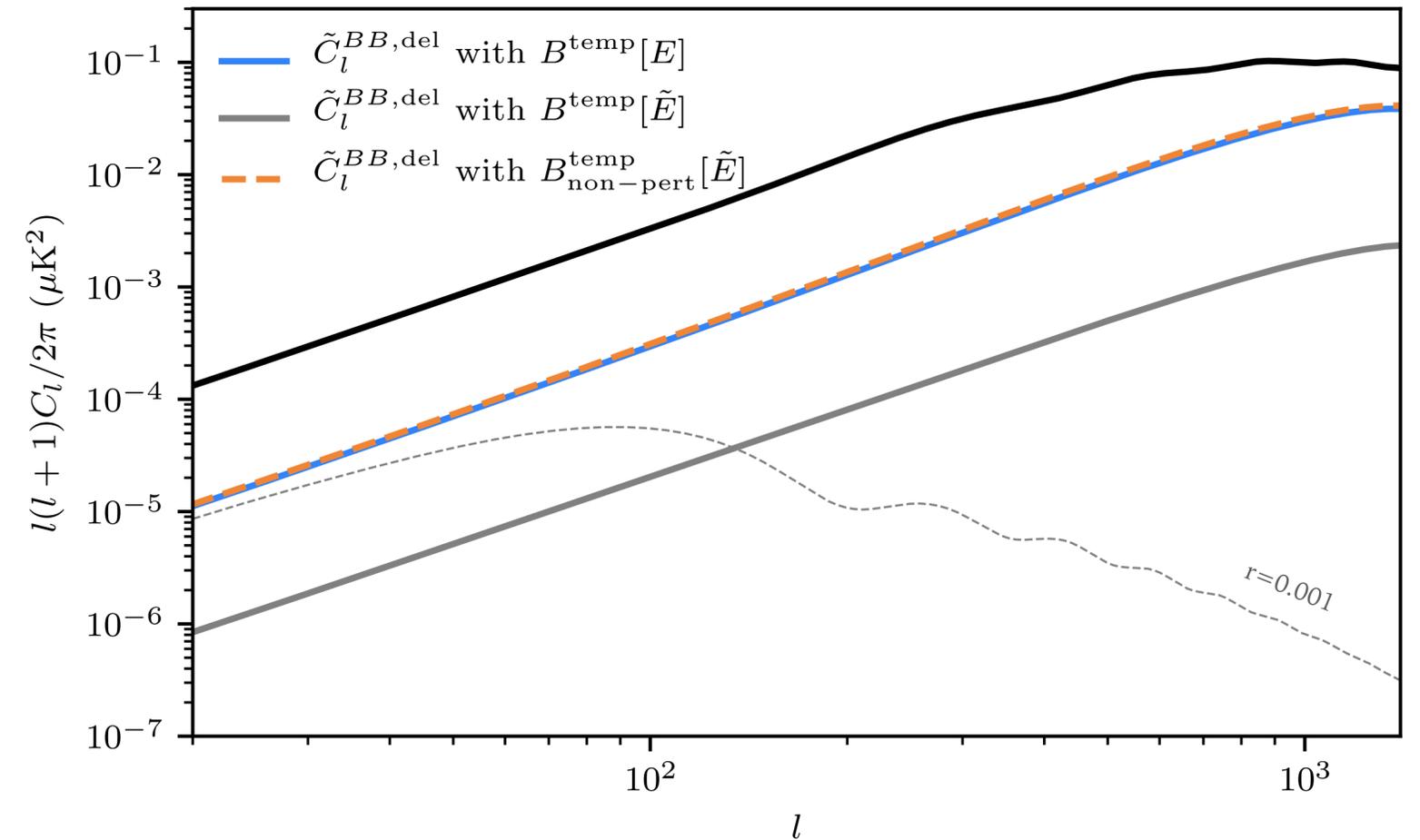
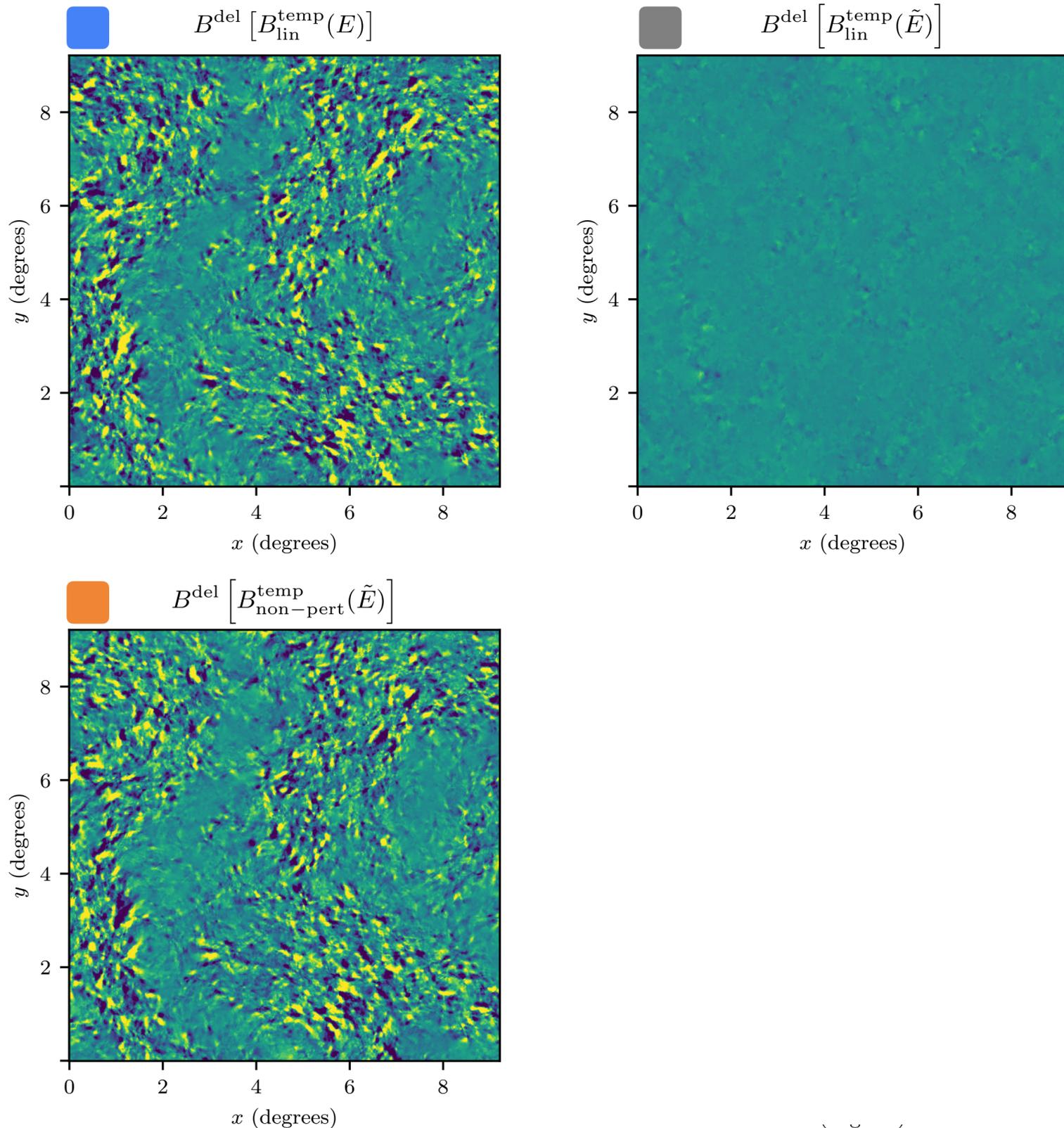
- Cancellations disappear if linear template is built from unlensed or delensed E-modes, hit delensing floor of $O(10)\%$

LIMITATIONS OF B-MODE TEMPLATE DELENSING: NOISELESS SCENARIO



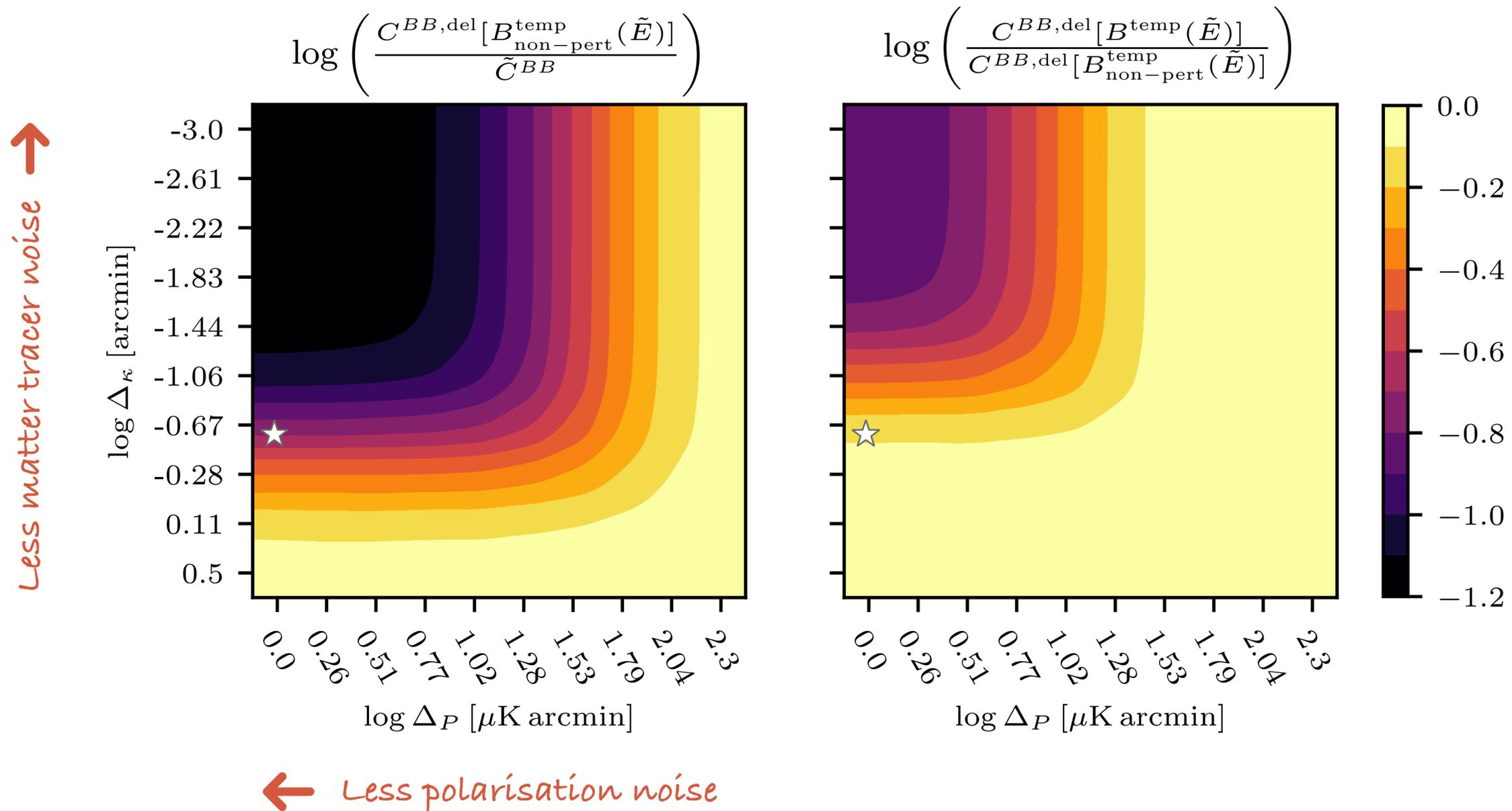
- Cancellations disappear if linear template is built from unlensed or delensed E-modes, hit delensing floor of $O(10)\%$
- New cancellations arise when the lensed E-modes are used in the linear template, so delensing floor is $O(1)\%$

LIMITATIONS OF B-MODE TEMPLATE DELENSING: NOISELESS SCENARIO



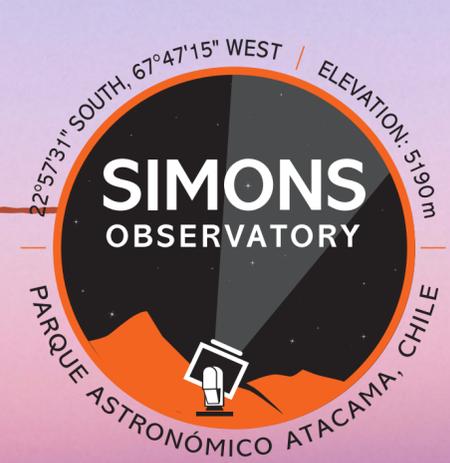
- Cancellations disappear if linear template is built from unlensed or delensed E-modes, hit delensing floor of $\text{O}(10)\%$
- New cancellations arise when the lensed E-modes are used in the linear template, so delensing floor is $\text{O}(1)\%$
- Advantage is lost when a non-perturbative template is built from lensed E-modes, so the delensing floor is also $\text{O}(10)\%$

LIMITATIONS OF B-MODE TEMPLATE DELENSING: REALISTIC CASE



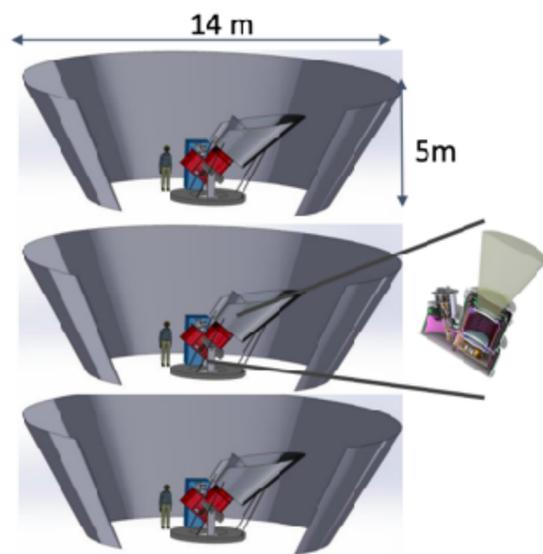
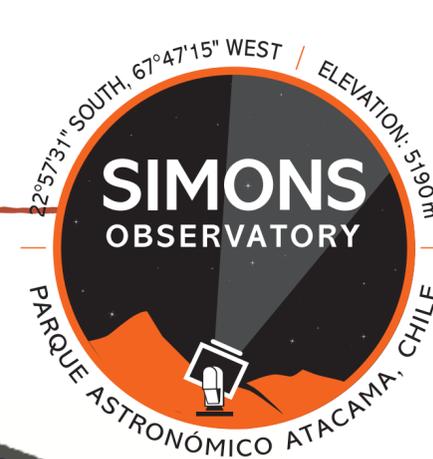
- With lensed E, advantageous to build gradient-order template even in realistic scenarios
- For CMB-S4, this removes $\sim 5\%$ more lensing power. Most transparent to systematics.

THE SIMONS OBSERVATORY (SO)

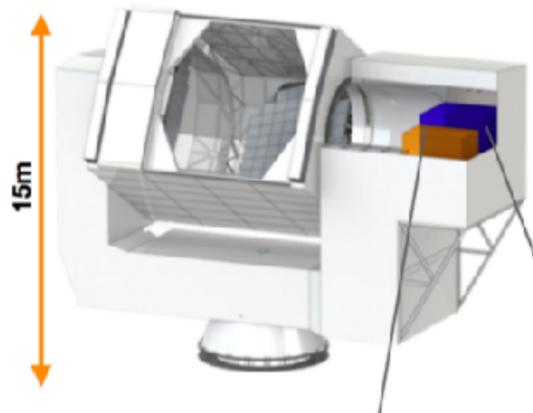
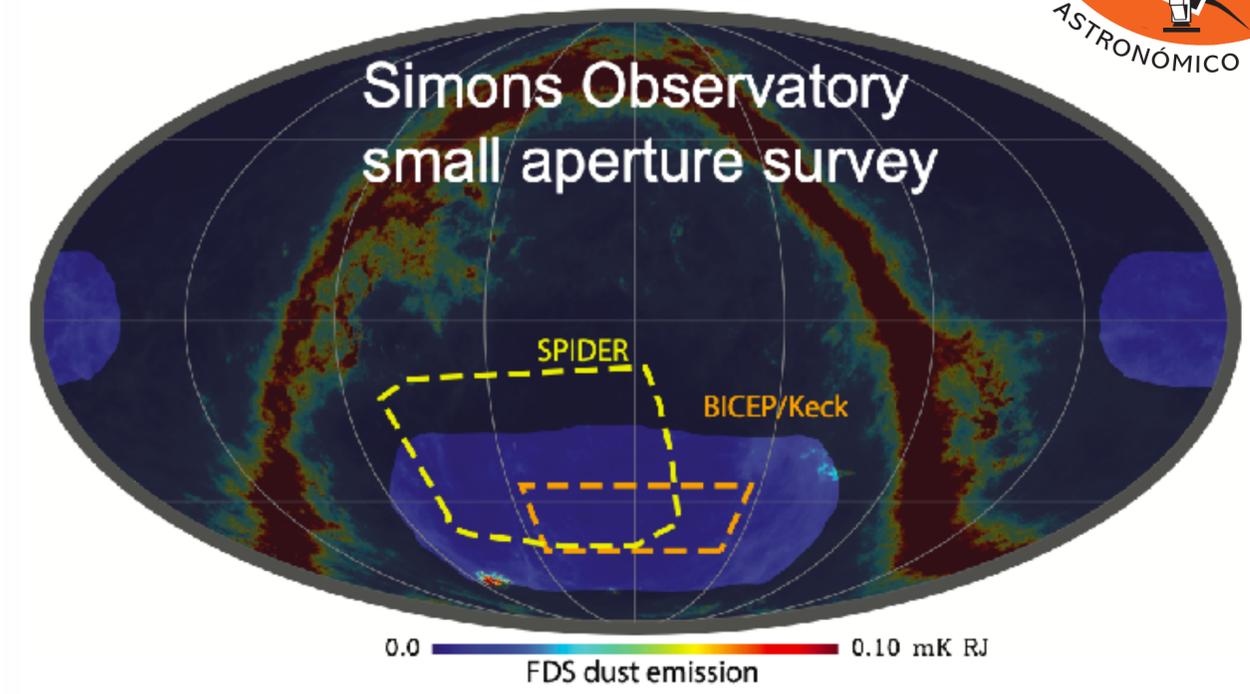


Credit: Deborah Kellner

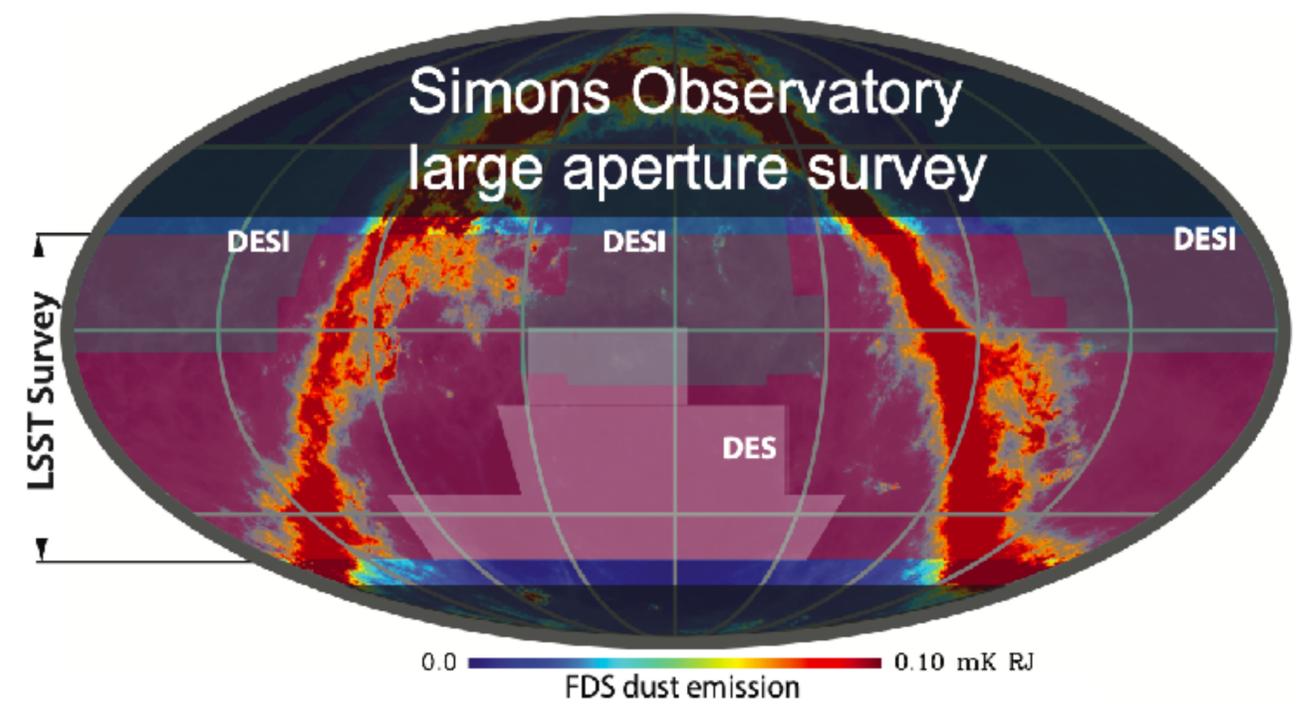
THE SIMONS OBSERVATORY (SO)



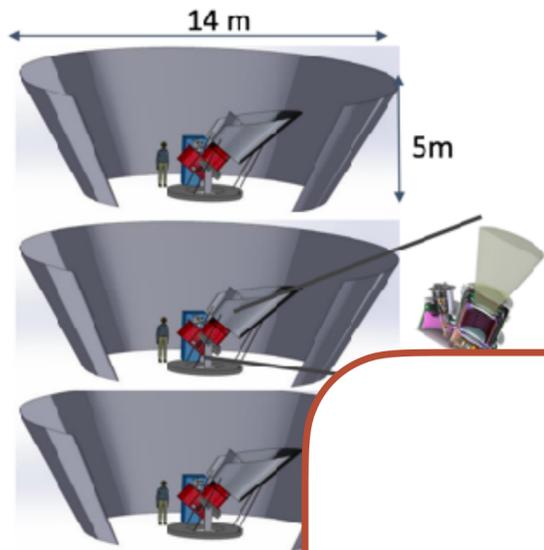
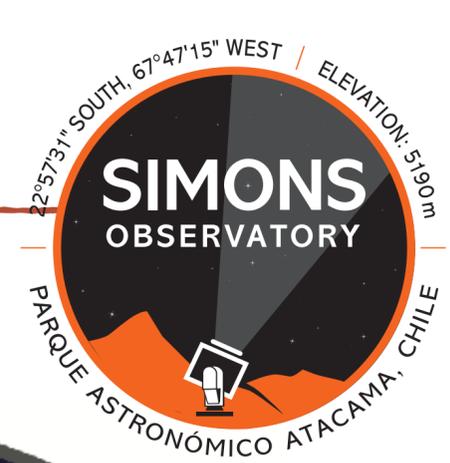
Freq. [GHz]	SATs ($f_{\text{sky}} = 0.1$)		
	FWHM (')	Noise (baseline) [$\mu\text{K-arcmin}$]	Noise (goal) [$\mu\text{K-arcmin}$]
27	91	35	25
39	63	21	17
93	30	2.6	1.9
145	17	3.3	2.1
225	11	6.3	4.2
280	9	16	10



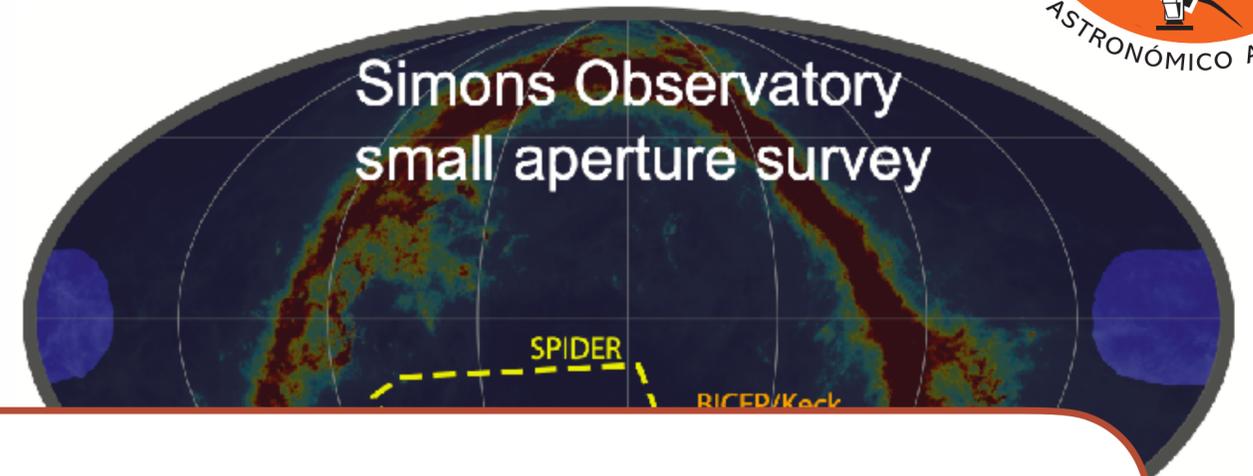
Freq. [GHz]	LAT ($f_{\text{sky}} = 0.4$)		
	FWHM (')	Noise (baseline) [$\mu\text{K-arcmin}$]	Noise (goal) [$\mu\text{K-arcmin}$]
27	7.4	71	52
39	5.1	36	27
93	2.2	8.0	5.8
145	1.4	10	6.3
225	1.0	22	15
280	0.9	54	37



THE SIMONS OBSERVATORY (SO)

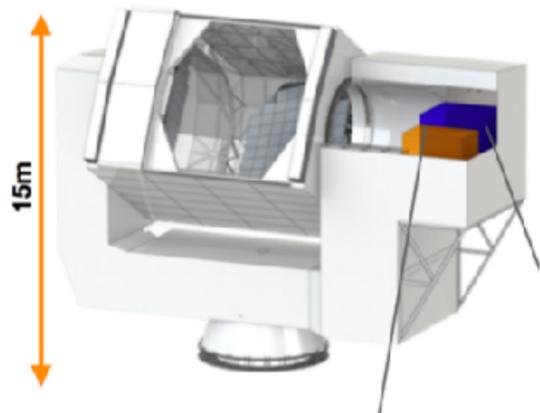


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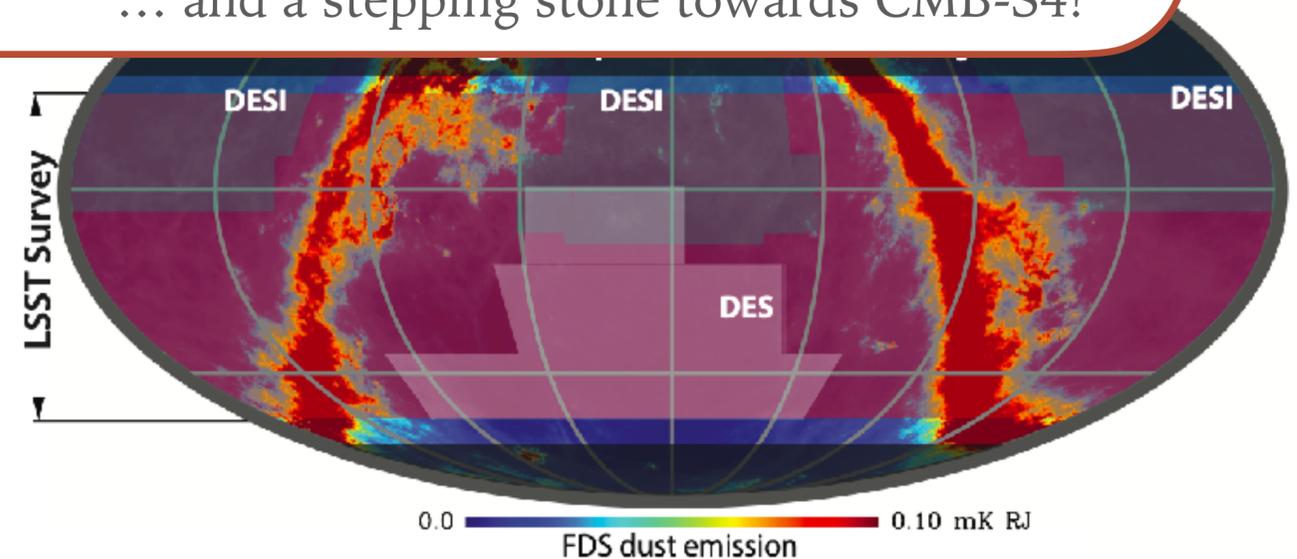


Early '21	Mid '22	Early '23	Mid '24
Testing and integration, optical validation	First light for both SAT + LAT	First science observations expected	Full science observations expected

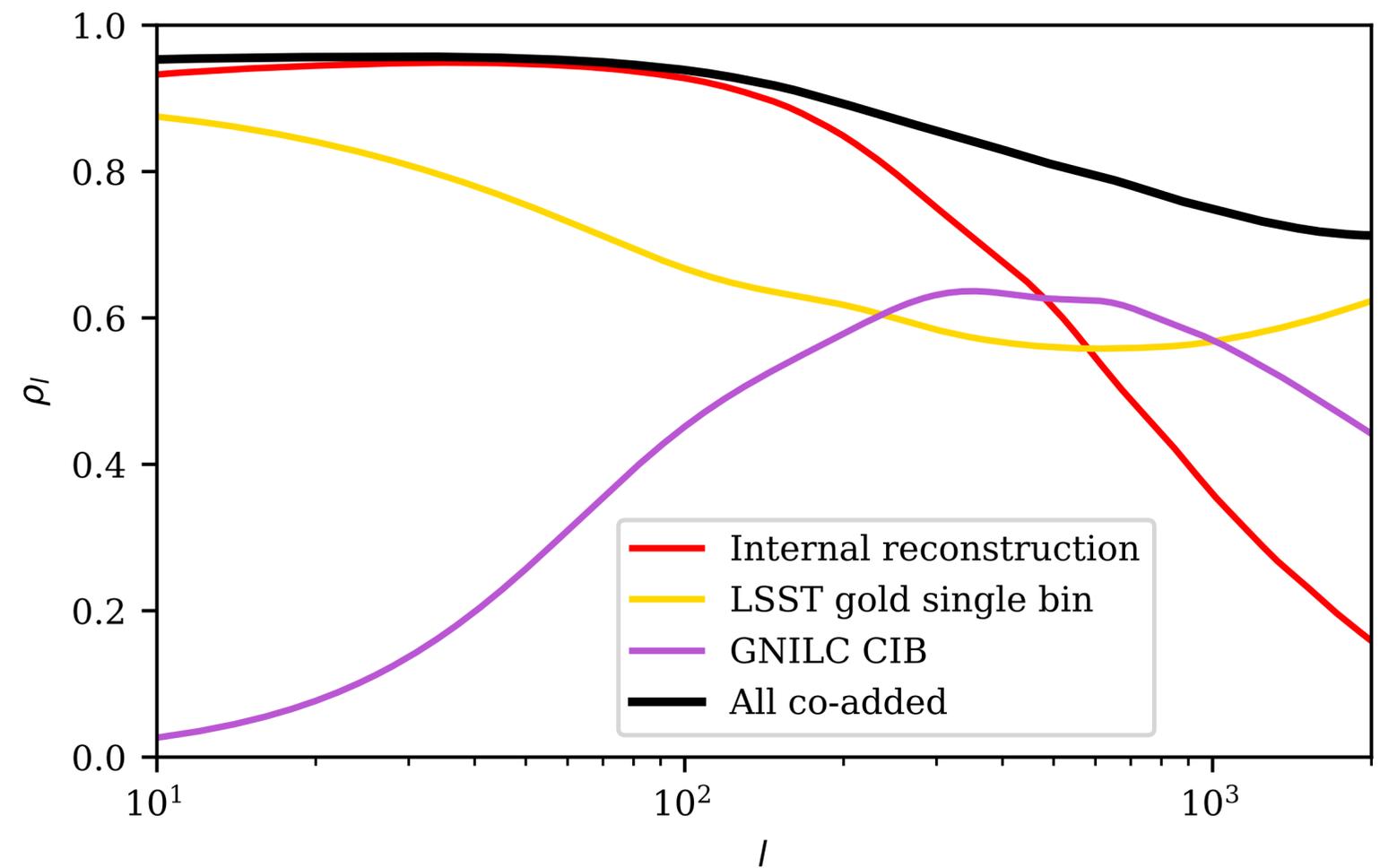
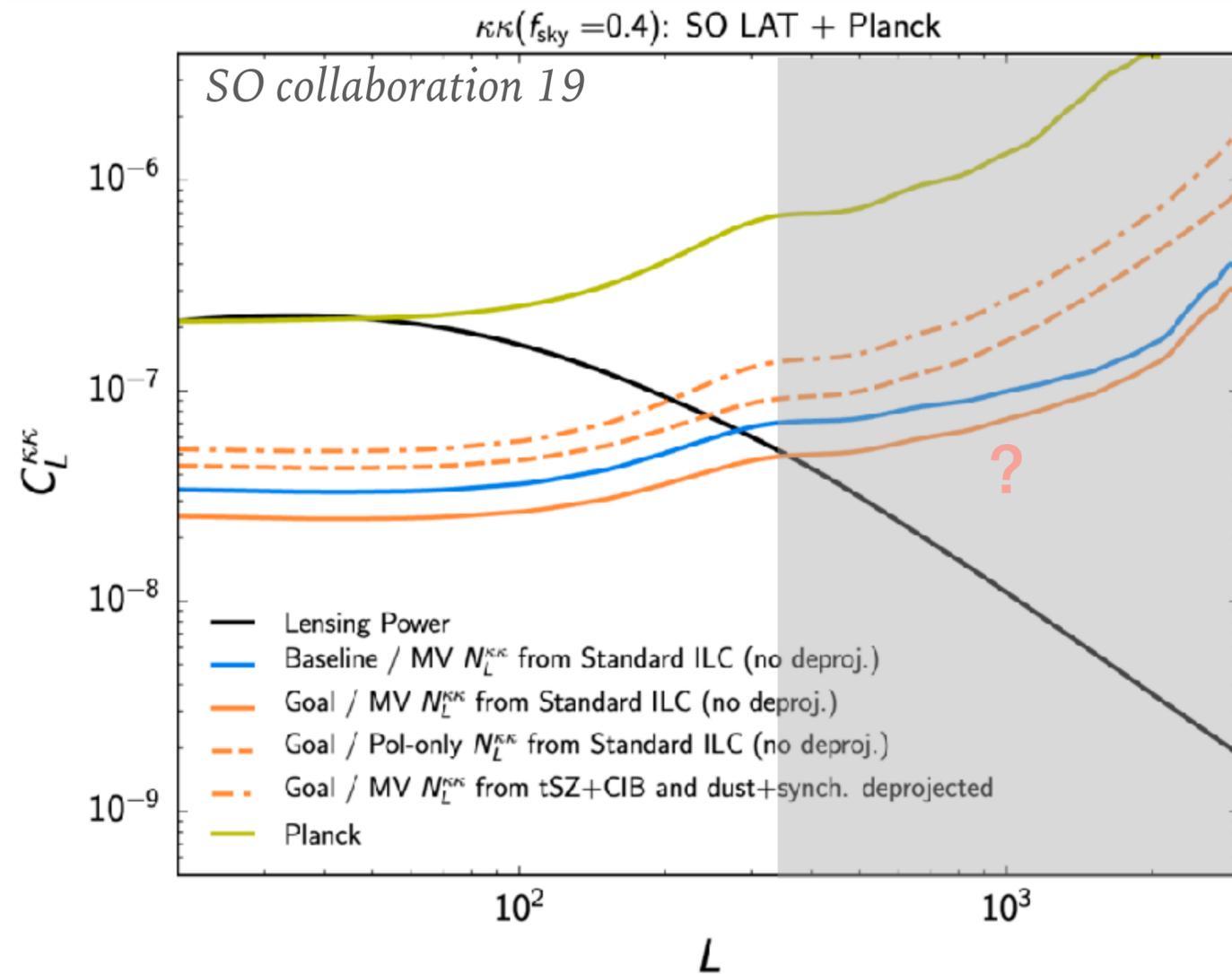
... and a stepping stone towards CMB-S4!



Freq. [GHz]	LAT ($f_{\text{sky}} = 0.4$)		
	FWHM (')	Noise (baseline) [$\mu\text{K-arcmin}$]	Noise (goal) [$\mu\text{K-arcmin}$]
27	7.4	71	52
39	5.1	36	27
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145	1.4	10	6.3
225	1.0	22	15
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MULTI-TRACER DELENSING WITH SO



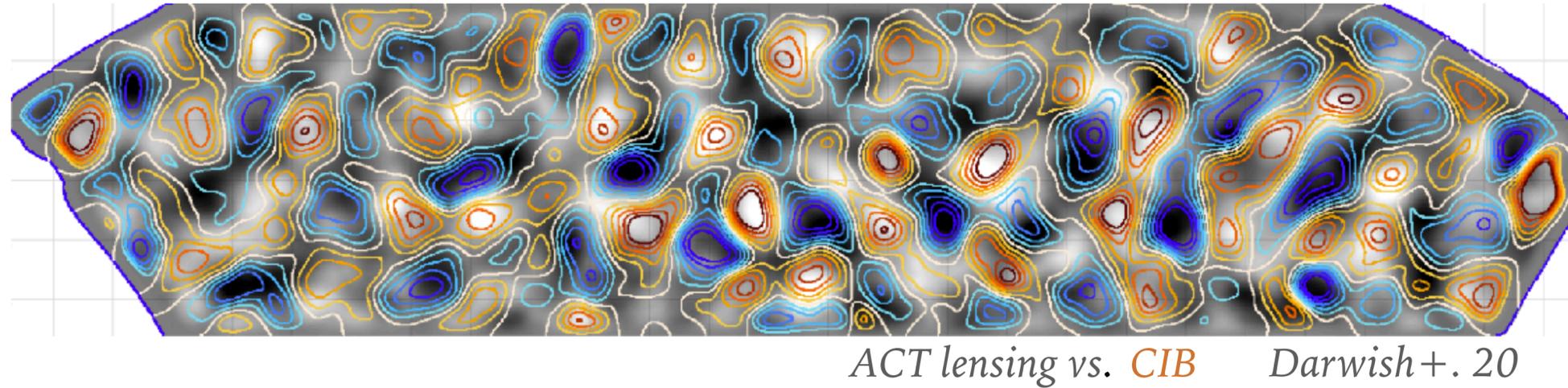
Combine internal reconstructions with external tracers of the LSS to get small-scale lenses at high z

Sherwin & Schmittfull 15, Manzotti 18 ...

ASIDE – THE COSMIC INFRARED BACKGROUND (CIB)

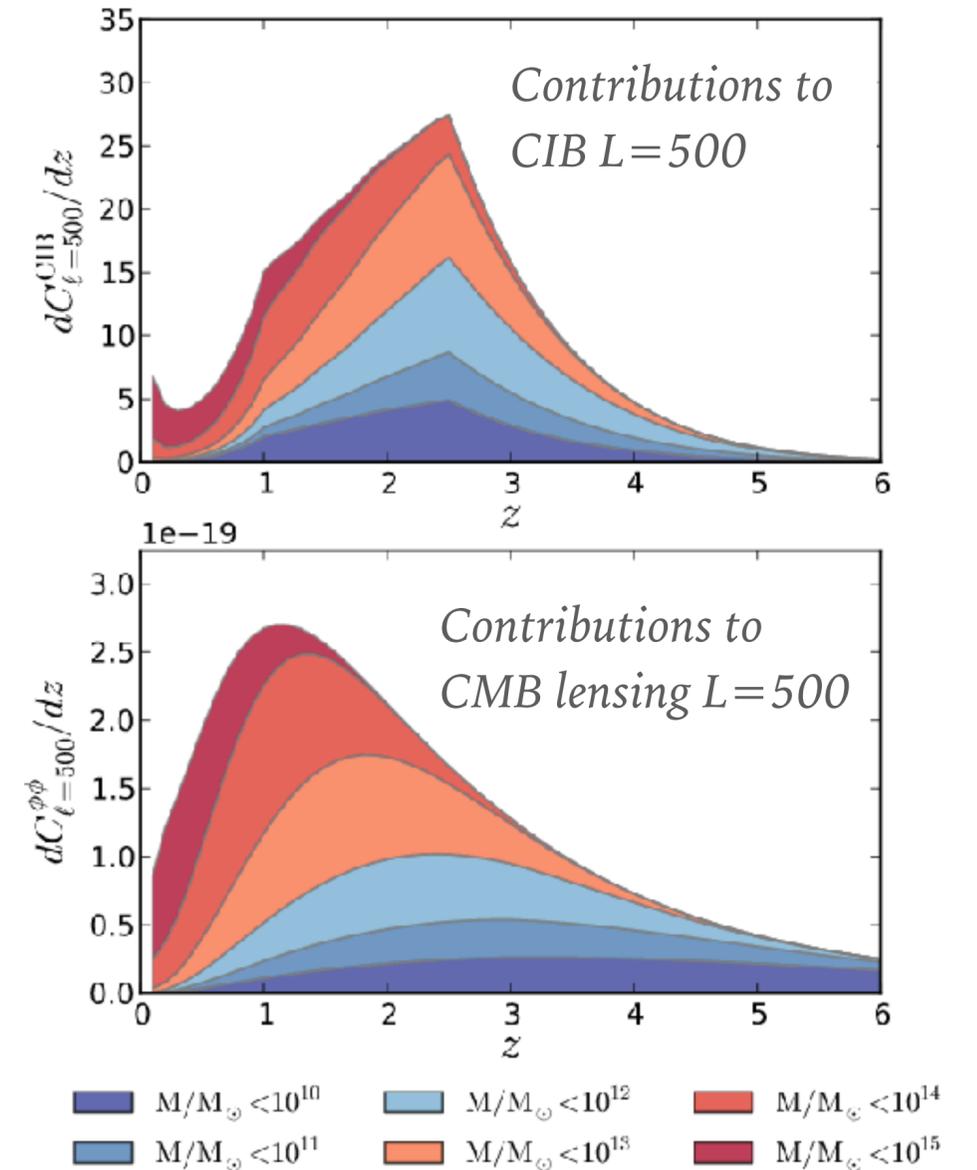
The CIB: emission from UV-heated dust in star-forming galaxies

Highly correlated with CMB lensing on the (arcminute) scales we need
Sherwin & Schmittfull 15



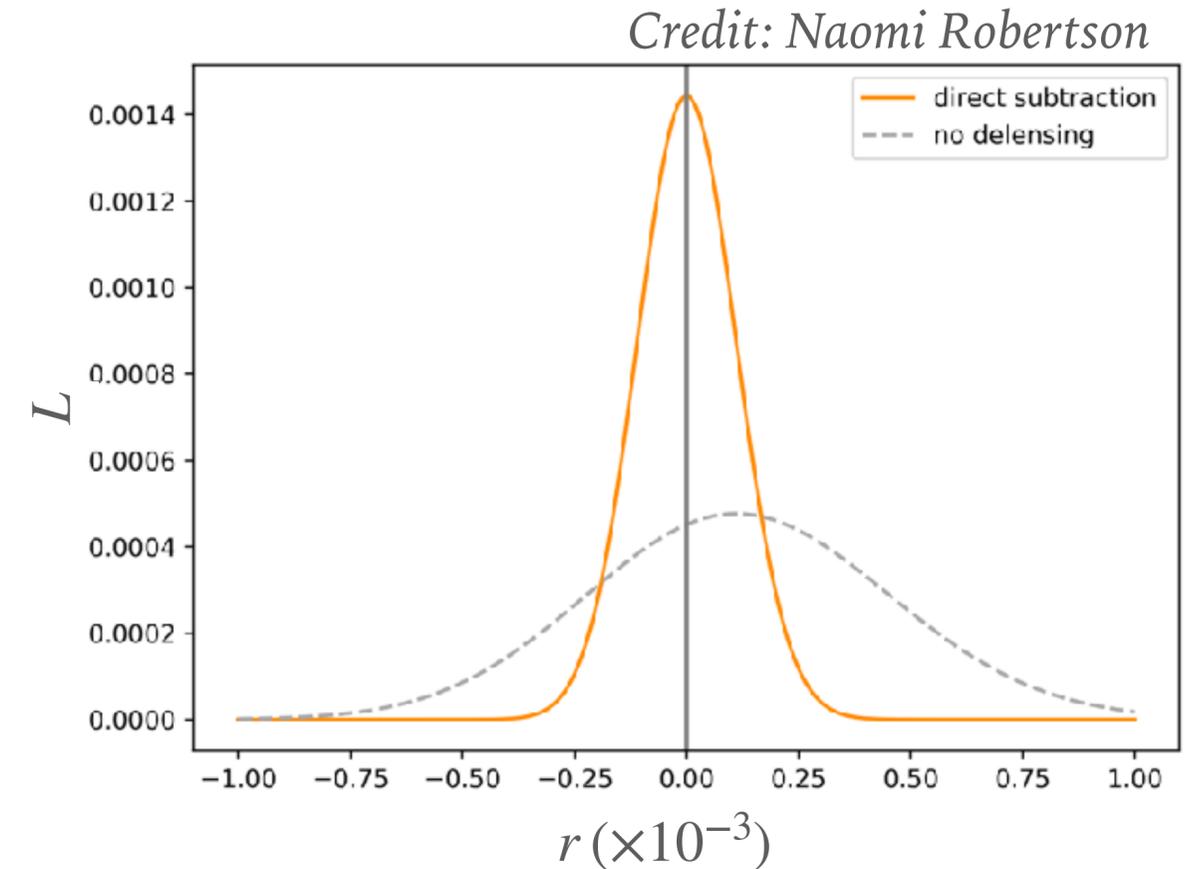
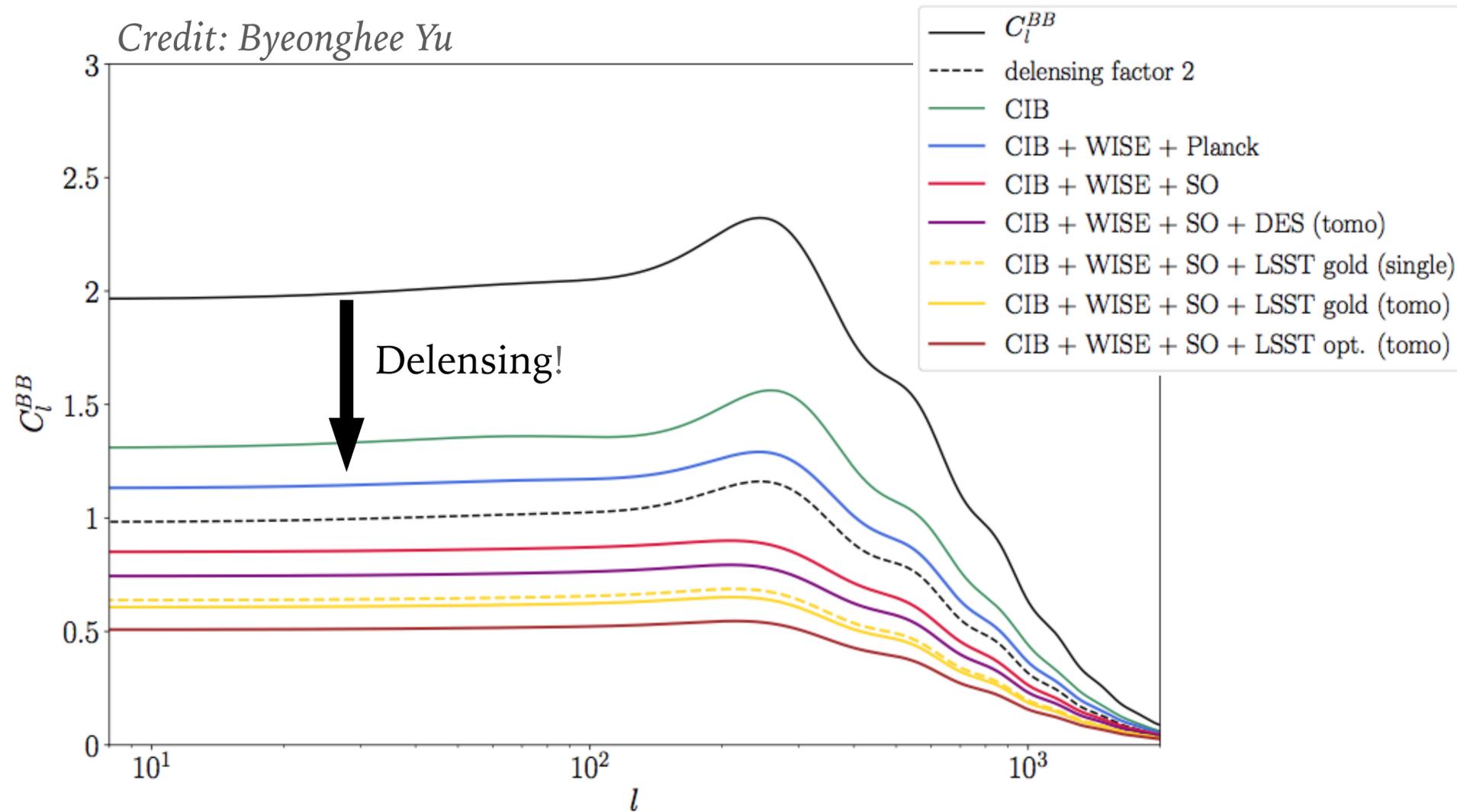
Used in:

- First demonstration of delensing in general *Larsen+ 16*
- First demonstration of B-mode delensing *SPT 17*
- First improvement on $\sigma(r)$ from delensing *SPT + BICEP/Keck 20*



Planck 13 XVIII

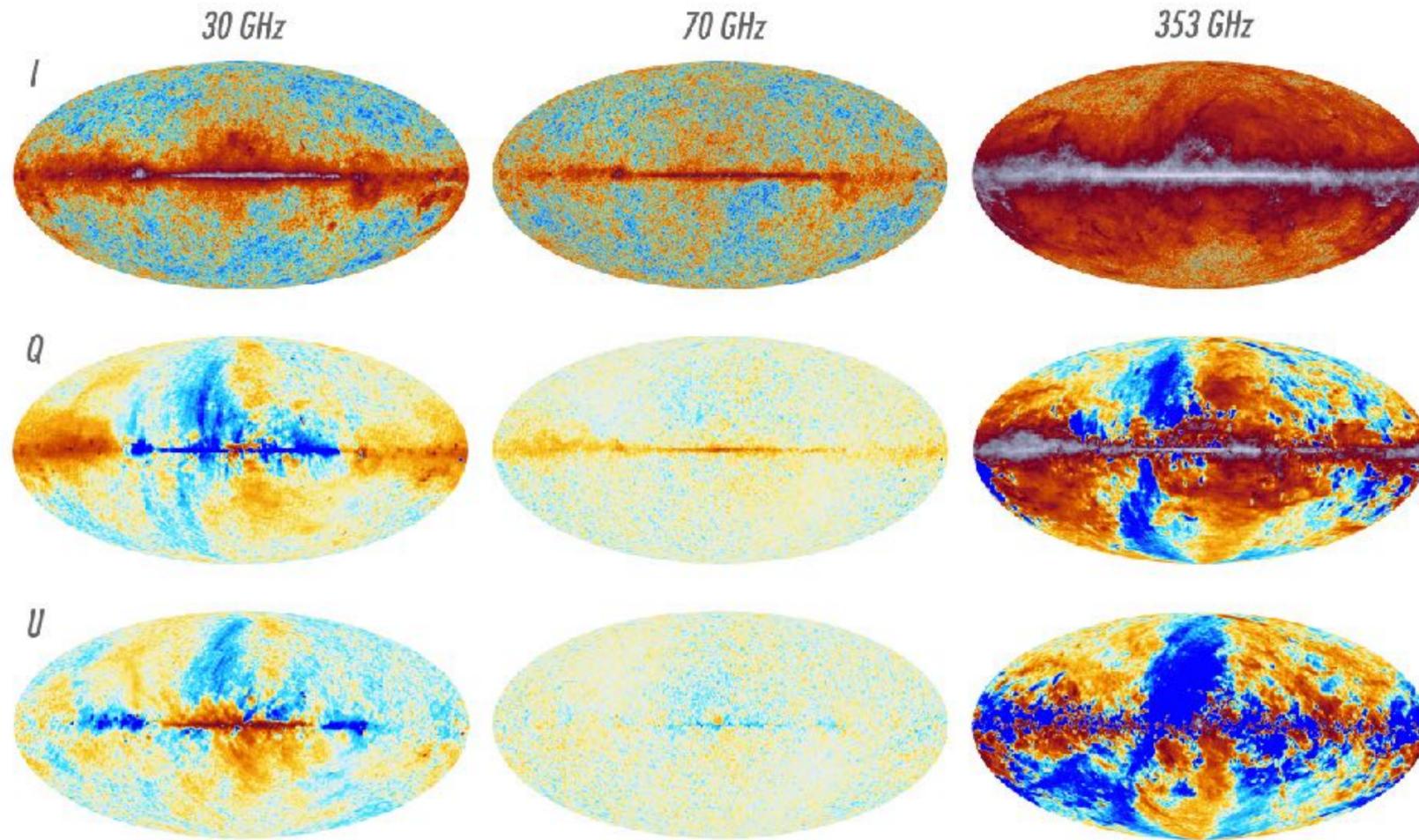
DELENSING WITH SO



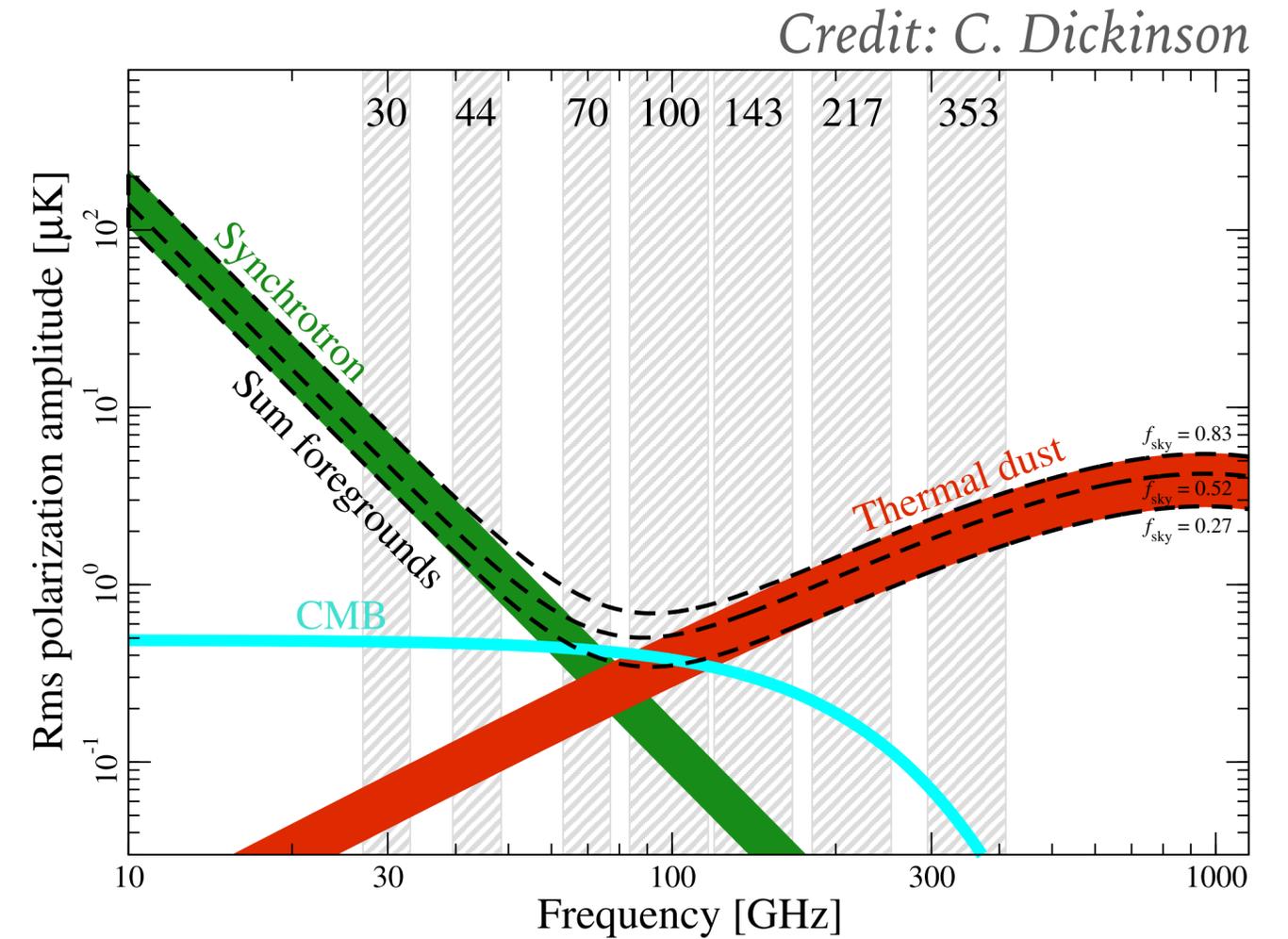
Close to idealised performance on realistic simulations including inhomogeneous noise and masking

Currently $\sigma(r) = 0.044$ (Planck + BICEP/Keck), SO forecast after delensing $\sigma(r) = 0.003$

BIASES TO DELENSING FROM FOREGROUNDS



Credit: Planck

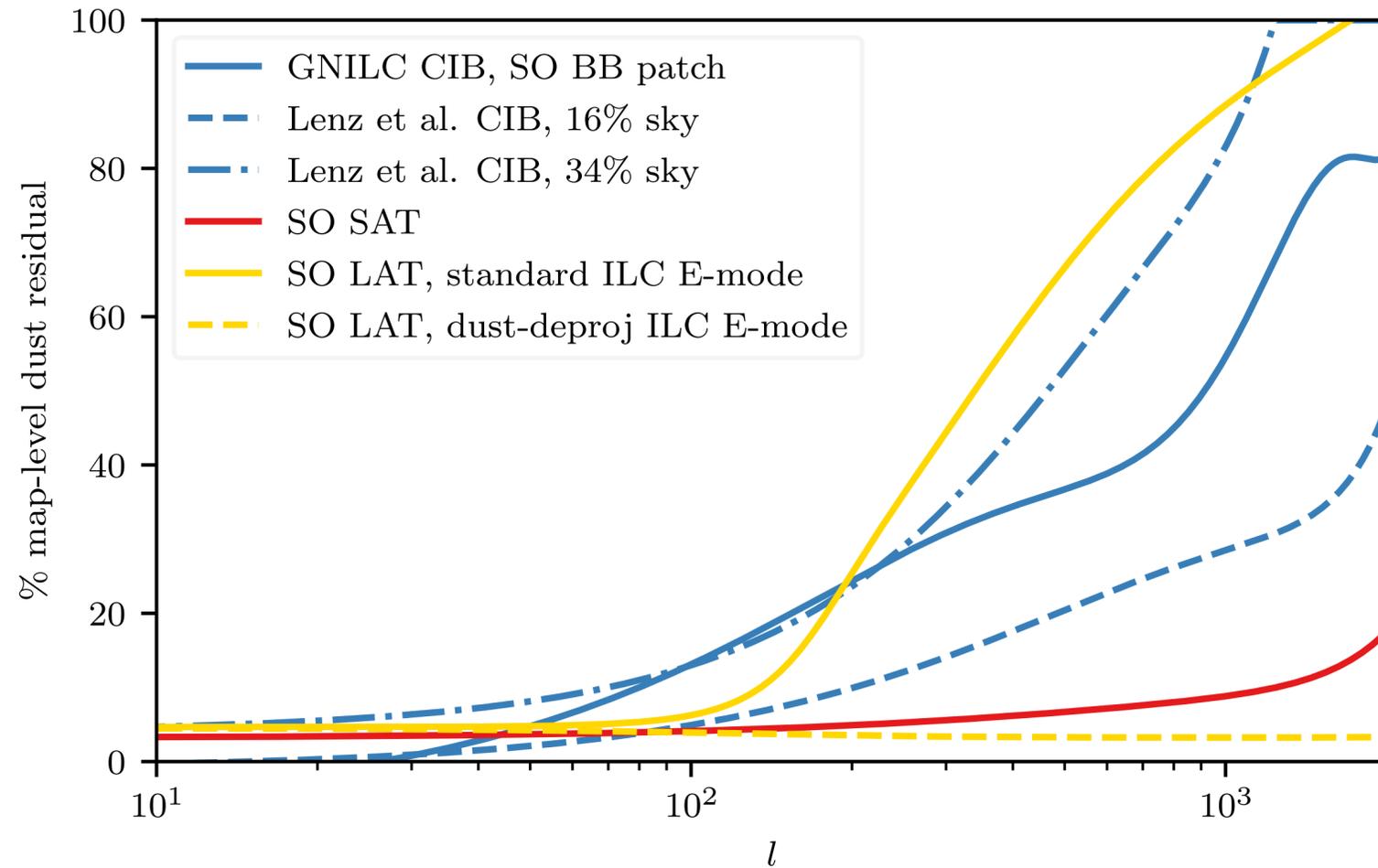


Credit: C. Dickinson

Negligible bias from internal delensing of CMB-S4 after foreground cleaning *D. Beck + 20*

... but what about CIB-delensing?

DELENSING WITH THE CIB — POSSIBLE BIASES

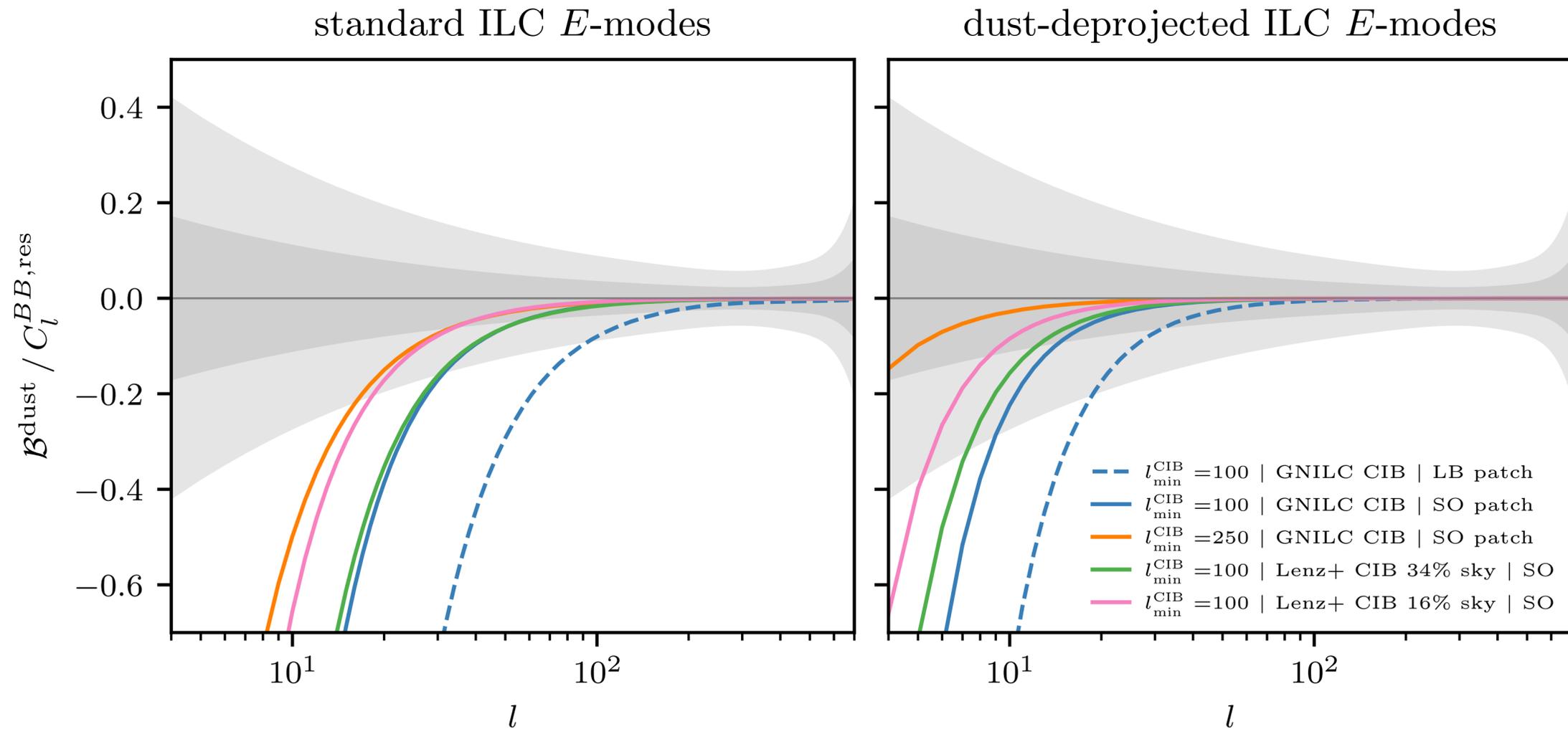


$$\begin{array}{c}
 \tilde{E} + E^{CIB} + \delta E^{dust} \\
 \underbrace{\hspace{10em}} \\
 B^{del} = B^{obs} - E^{obs} \otimes I \\
 \underbrace{\hspace{10em}} \quad \underbrace{\hspace{10em}} \\
 \tilde{B} + B^{CIB} + \delta B^{dust} \quad \underbrace{I^{CIB} + \delta I^{dust}} \\
 \text{Difficult to disentangle!}
 \end{array}$$

The power spectrum of delensed B-modes is then biased:

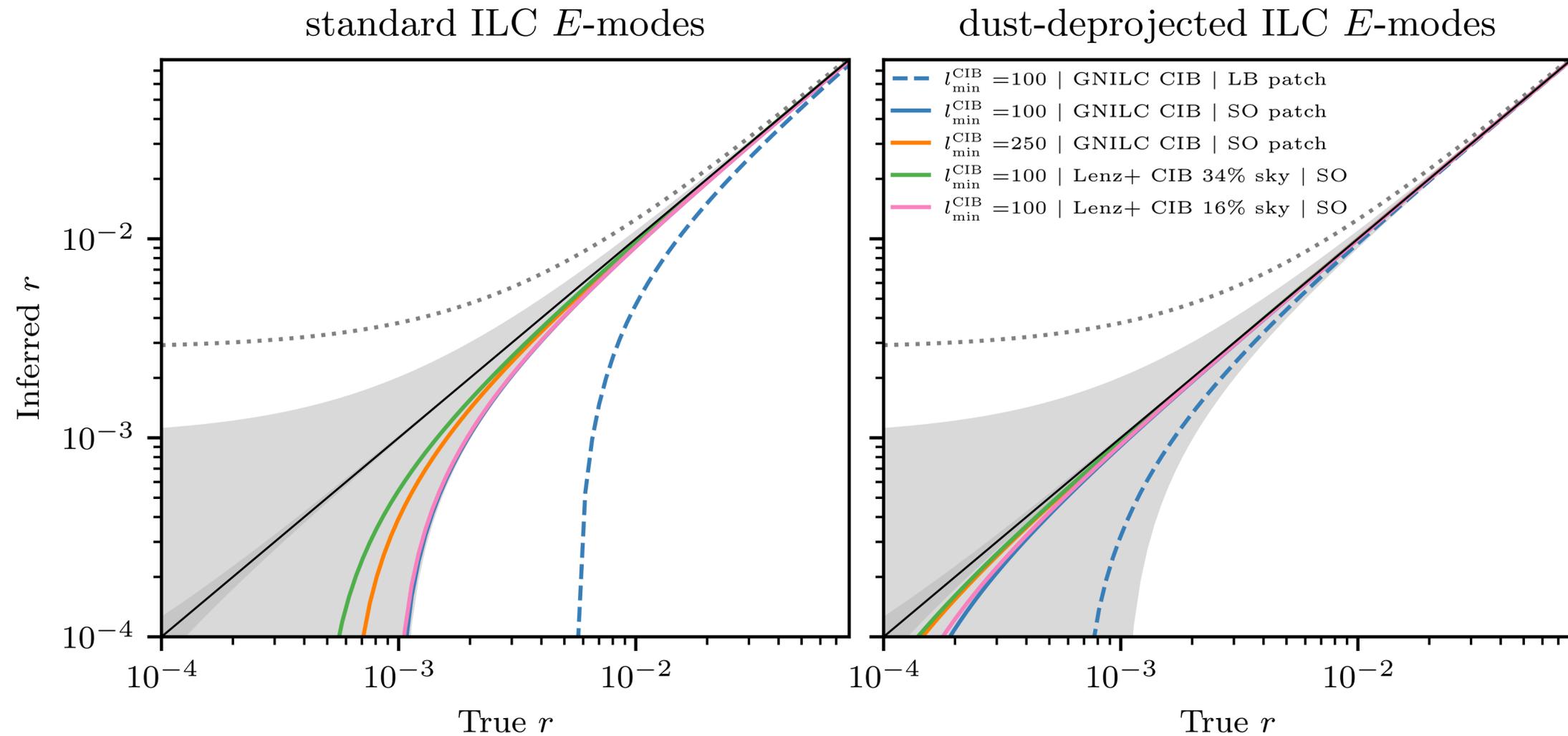
$$C_{\ell}^{BB,del} \supset \langle B_{100GHz}^{dust} E_{100GHz}^{dust} I_{353GHz}^{dust} \rangle, \langle B_{100GHz}^{CIB} E_{100GHz}^{CIB} I_{353GHz}^{CIB} \rangle \dots$$

DELENSING SO WITH THE CIB — BIAS FROM RESIDUAL GALACTIC DUST



- Negative bias dominated by $\langle B^{\text{dust}} E^{\text{dust}} I^{\text{dust}} \rangle$
- Can be mitigated by nulling dust contribution to E-modes, with only small penalty in delensing efficiency
- Small on scales probed from the ground

DELENSING SO WITH THE CIB — BIAS FROM RESIDUAL GALACTIC DUST

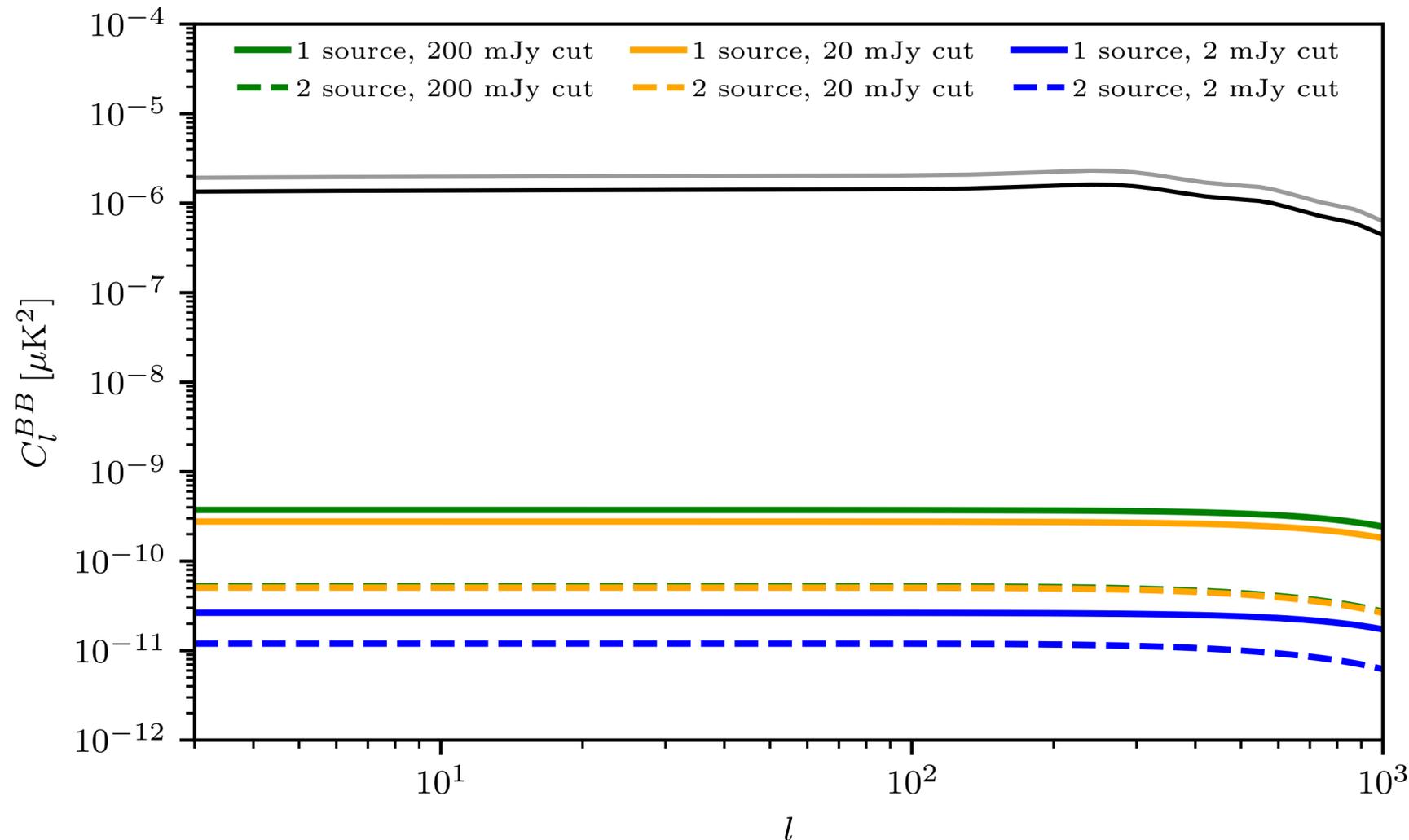


- Bias leads to underestimation of r
- Can be mitigated by nulling dust contribution to E-modes, with only small penalty in delensing efficiency
- Small on scales probed from the ground

DELENSING WITH THE CIB — BIAS FROM CIB BI- & TRISPECTRUM

In a minimal analytic model with uncorrelated source polarisation angles,

$$\mathcal{B}_l^{BEI} = 2p^2 G_{353 \text{ GHz}} G_{145 \text{ GHz}}^2 \int \frac{d^2 l'}{(2\pi)^2} \frac{l' \cdot (l - l')}{|l - l'|^2} \mathcal{W}^E(l') \mathcal{W}^I(|l - l'|) \sin^2 2(\psi_{l'} - \psi_l) \\ \times \int dz \left(\frac{I^{\text{CIB}}[145(1+z) \text{ GHz}]}{I^{\text{CIB}}[353(1+z) \text{ GHz}]} \right)^2 \left[S_{353 \text{ GHz}}^{(3)}(z) + \frac{H(z)}{cr^2(z)} S_{353 \text{ GHz}}^{(2)}(z) S_{353 \text{ GHz}}^{(1)}(z) P_g(|l - l'|/r(z); z) \right]$$

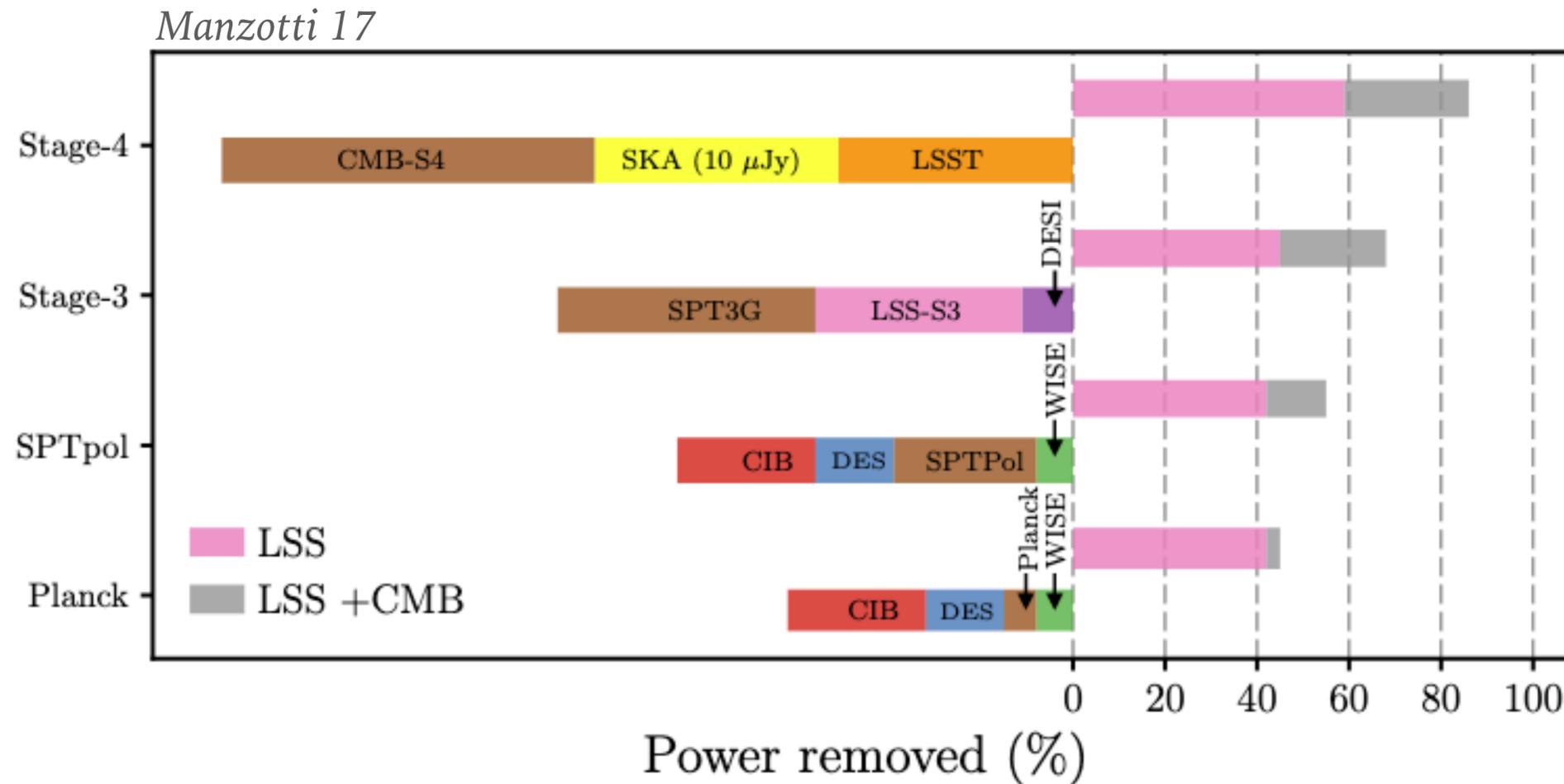


where $S_\nu^{(n)}(z) \equiv \int_{s_{\min}}^{s_{\max}} dS S^n \frac{d^2 N}{dS dz d\Omega}(z, \nu)$

and $\langle B^{\text{CIB}} E^{\text{CIB}} T^{\text{CIB}} \rangle \gg \langle E^{\text{CIB}} T^{\text{CIB}} E^{\text{CIB}} T^{\text{CIB}} \rangle$
for expected flux cuts.

Hence, negligible for any implementation of CIB-delensing

INTERNAL DELENSING



Soon dominated by internal lensing reconstructions based on CMB polarisation (mainly EB correlations)

Maximum-a-posteriori (MAP) $\hat{\phi}$ reconstruction will supersede QEs *Hirata & Seljak 03, Carron & Lewis 17*

Delensing with MAP $\hat{\phi}$ and Bayesian lensing reconstruction both recently demonstrated on data!

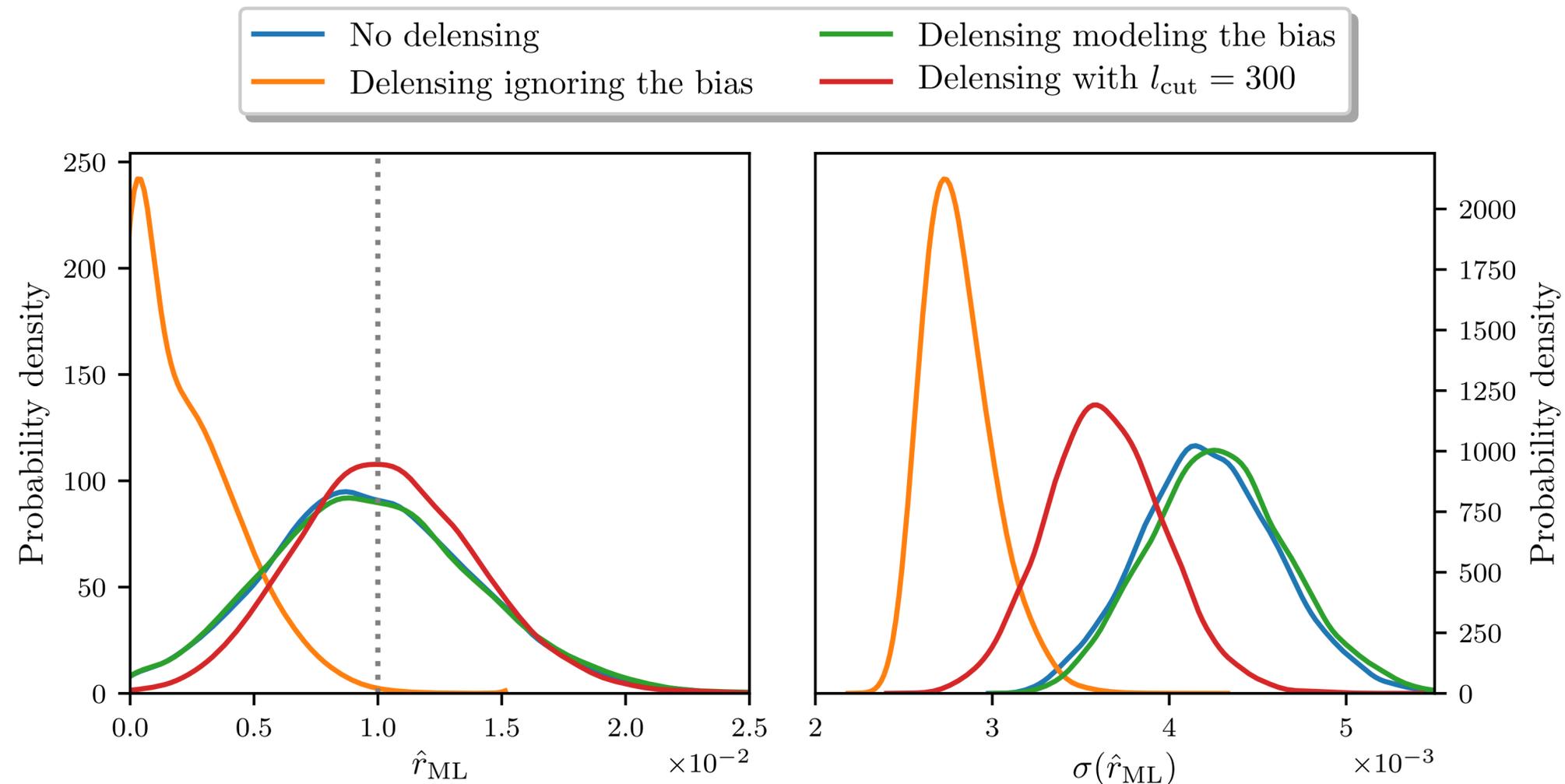
Millea+20, POLARBEAR 20

INTERNAL DELENSING BIAS

- Overlap in modes between B in EB estimator and B -modes to be delensed causes spurious suppression of delensed power spectrum and its **variance**

Teng et al. 11, Namikawa & Nagata 14

- Could this lower variance translate to improved constraints on r ? **NO**



- Primordial signal is suppressed by a larger fraction than the “noise”
- Preferable to avoid by masking overlapping modes (small impact on S/N) than renormalising/modeling

CONCLUSIONS

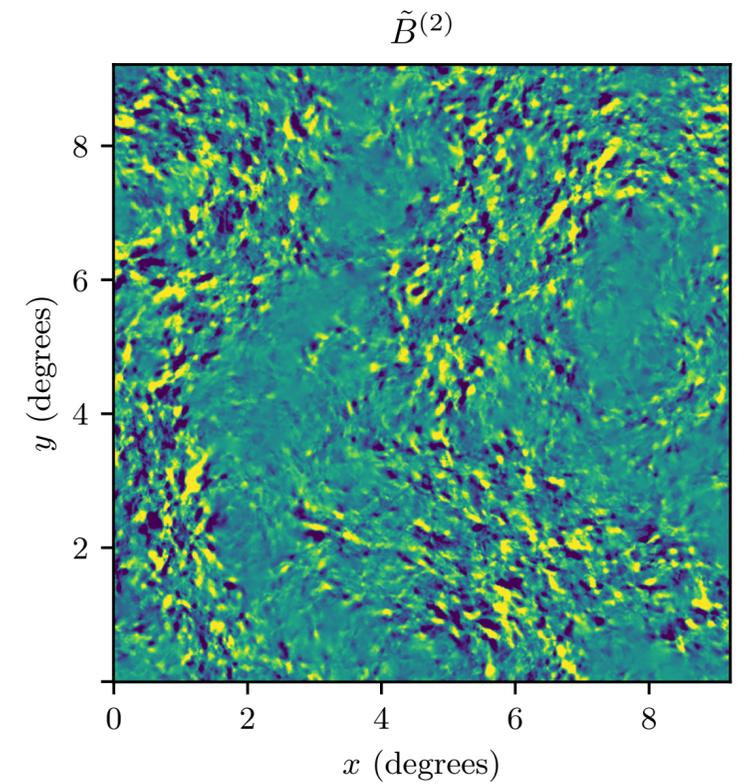
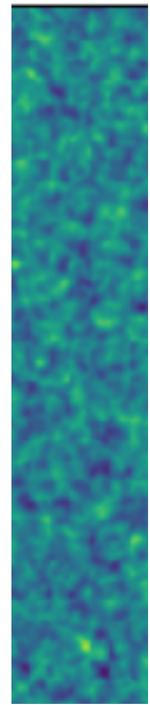
- ▶ **CMB lensing spectra** probes anything affecting growth of structure (including Σm_ν). Great synergy with Stage-4 LSS surveys
- ▶ Polarization-based reconstructions very clean. Temperature can be biased by extragalactic foregrounds, but can be mitigated
 - ▶ We calculate these biases analytically — good for understanding, informing masking & marginalising over modeling uncertainties.
 - ▶ Upcoming data (CCATp & SO) will improve models. Next: extend to polarisation & iterative reconstructions.
- ▶ Primordial B-modes probe inflation/alternatives via the tensor-to-scalar ratio, r
- ▶ **Delensing B-modes** is essential for CMB-S3 & CMB-S4 r targets
 - ▶ First-order lensing B-mode template built from lensed E is effectively optimal beyond CMB-S4 & transparent to systematics
 - ▶ For SO, multi-tracer delensing removes $\sim 70\%$ of power, halves $\sigma(r)$
 - ▶ LSS tracers great for delensing (even CMB-S3). CIB particularly useful, but deproject galactic dust to avoid bias
 - ▶ Internal delensing biases, with us for the foreseeable future — better to mask overlapping modes than to model

Thank you very much for having me!

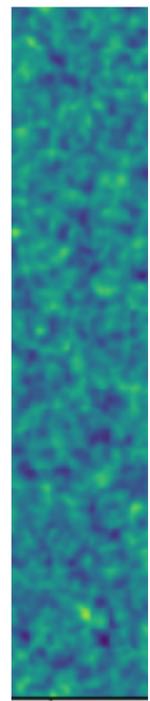
a.baleatolizancos@ast.cam.ac.uk

BACK-UP SLIDES

WHY THE CANCELLATIONS IN $B^{\text{del}}[B_{\text{lin}}^{\text{temp}}(\tilde{E})]$?



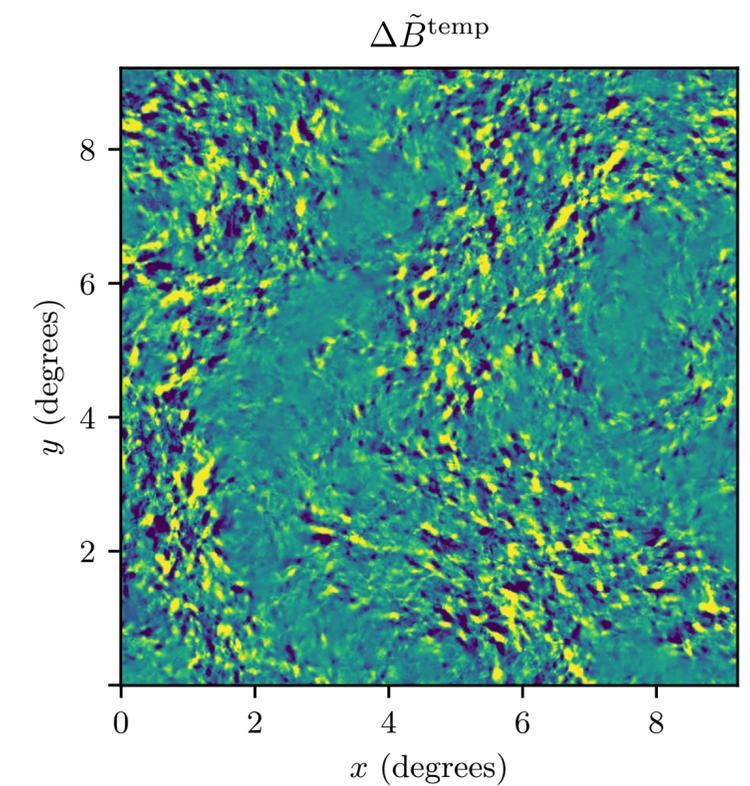
E-mode at emission



$\tilde{E}^{(1)}$



Template-making



WHY THE LARGE FLOOR IN $B^{\text{del}}[B^{\text{temp}}_{\text{non-pert}}(\tilde{E})]$?

$$\tilde{B}^{\text{temp}}_{\text{non-pert}}(\mathbf{l}) = \tilde{B}^{(1)}(\mathbf{l}) + \Delta\tilde{B}^{\text{temp}}(\mathbf{l}) + \tilde{B}^{(2)}(\mathbf{l}) + O(\phi^3).$$

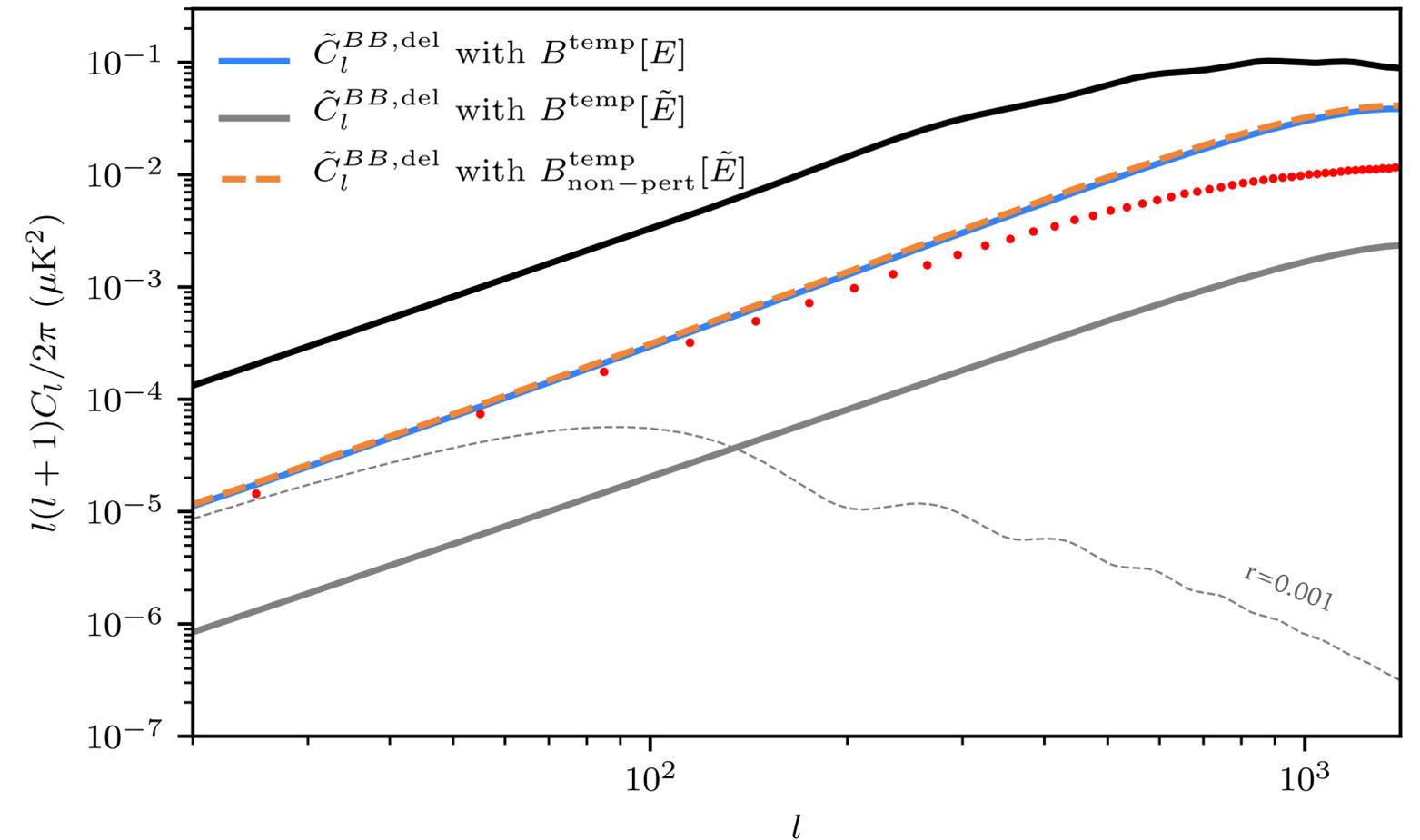
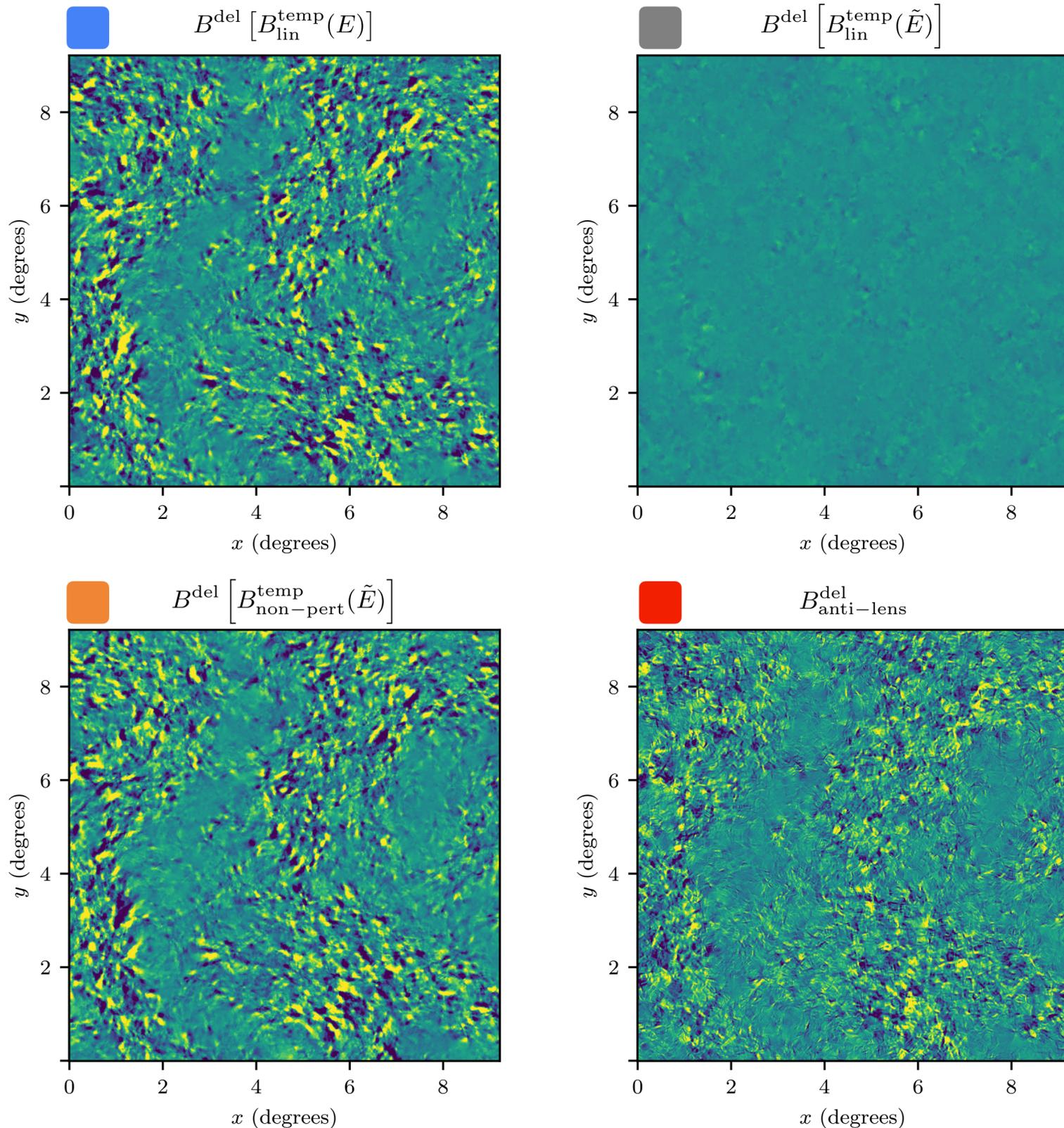
When subtracting this off of observations,

$$B^{\text{del}} = B^{\text{obs}} - \tilde{B}^{\text{temp}}_{\text{non-pert}} = (\cancel{\tilde{B}^{(1)}(\mathbf{l})} + \cancel{\tilde{B}^{(2)}(\mathbf{l})} + O(\phi^3)) - (\cancel{\tilde{B}^{(1)}(\mathbf{l})} + \underline{\Delta\tilde{B}^{\text{temp}}(\mathbf{l})} + \cancel{\tilde{B}^{(2)}(\mathbf{l})} + O(\phi^3)).$$

So the power spectrum of delensed B-modes is has a large floor:

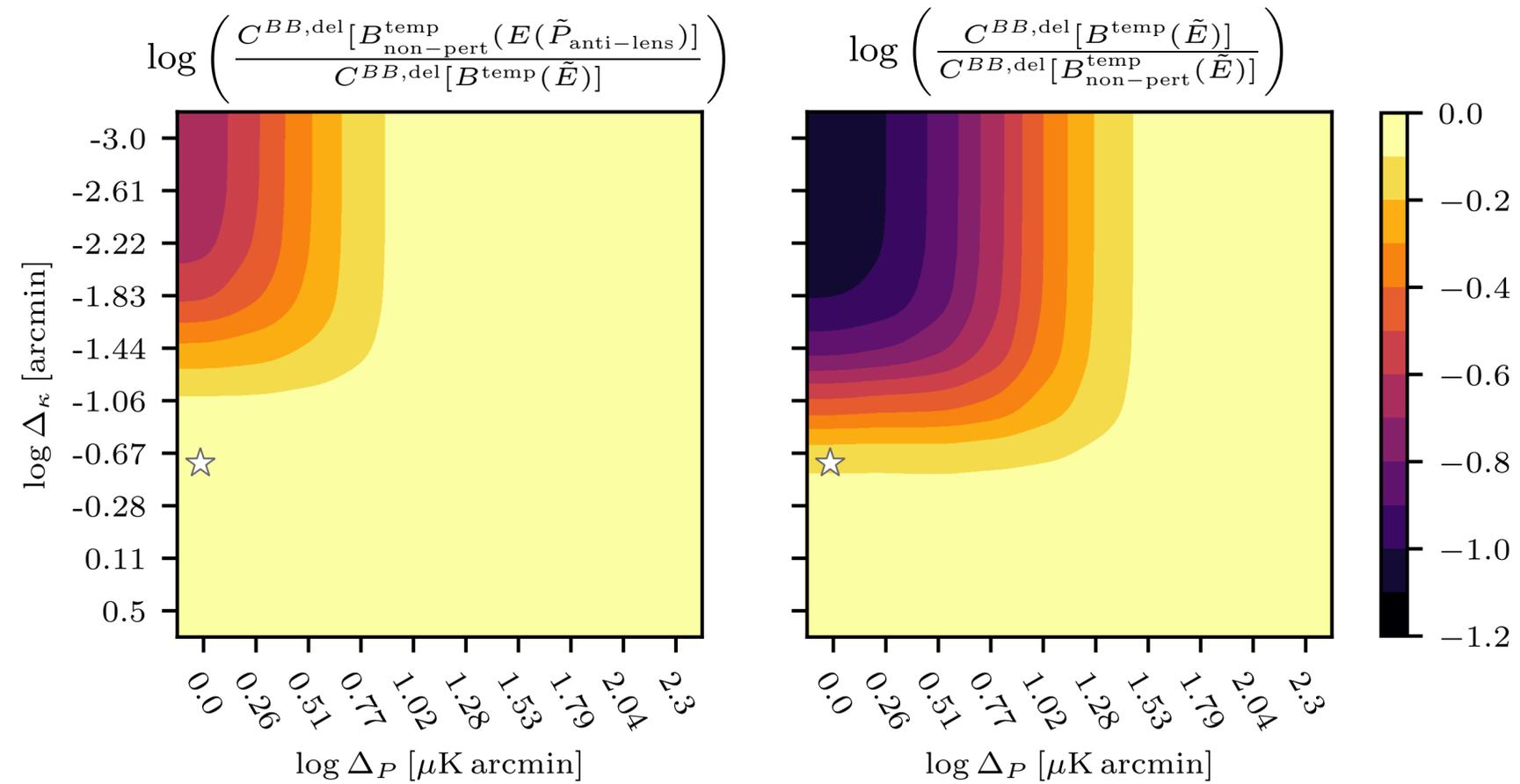
$$\langle |B^{\text{del}}|^2 \rangle \approx \langle |\Delta\tilde{B}^{\text{temp}}|^2 \rangle \sim O(0.1 \tilde{C}^{BB}).$$

LIMITATIONS OF B-MODE TEMPLATE DELENSING: NOISELESS SCENARIO



- Cancellations disappear if linear template is built from unlensed or delensed E-modes, hit delensing floor of $\mathcal{O}(10)\%$
- New cancellations arise when the lensed E-modes are used in the linear template, so delensing floor is $\mathcal{O}(1)\%$
- Advantage is lost when a non-perturbative template is built from lensed E-modes, so the delensing floor is also $\mathcal{O}(10)\%$

ADDITIONAL SLIDES



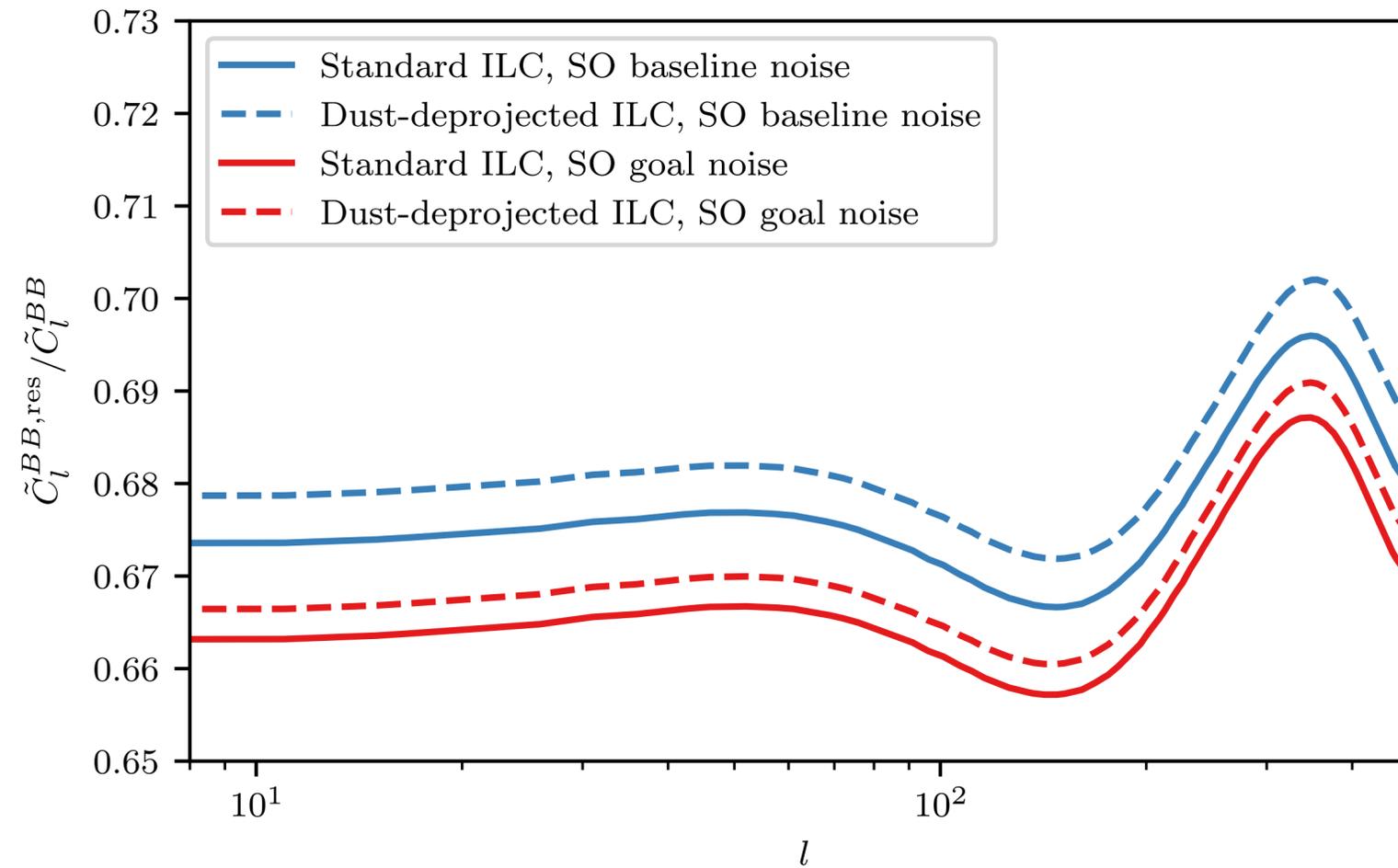
Left panel numerator:

1. Observe QU, extract E
2. Anti-lens these observations
3. Extract E -modes
4. Form non-perturbative template
5. Delens

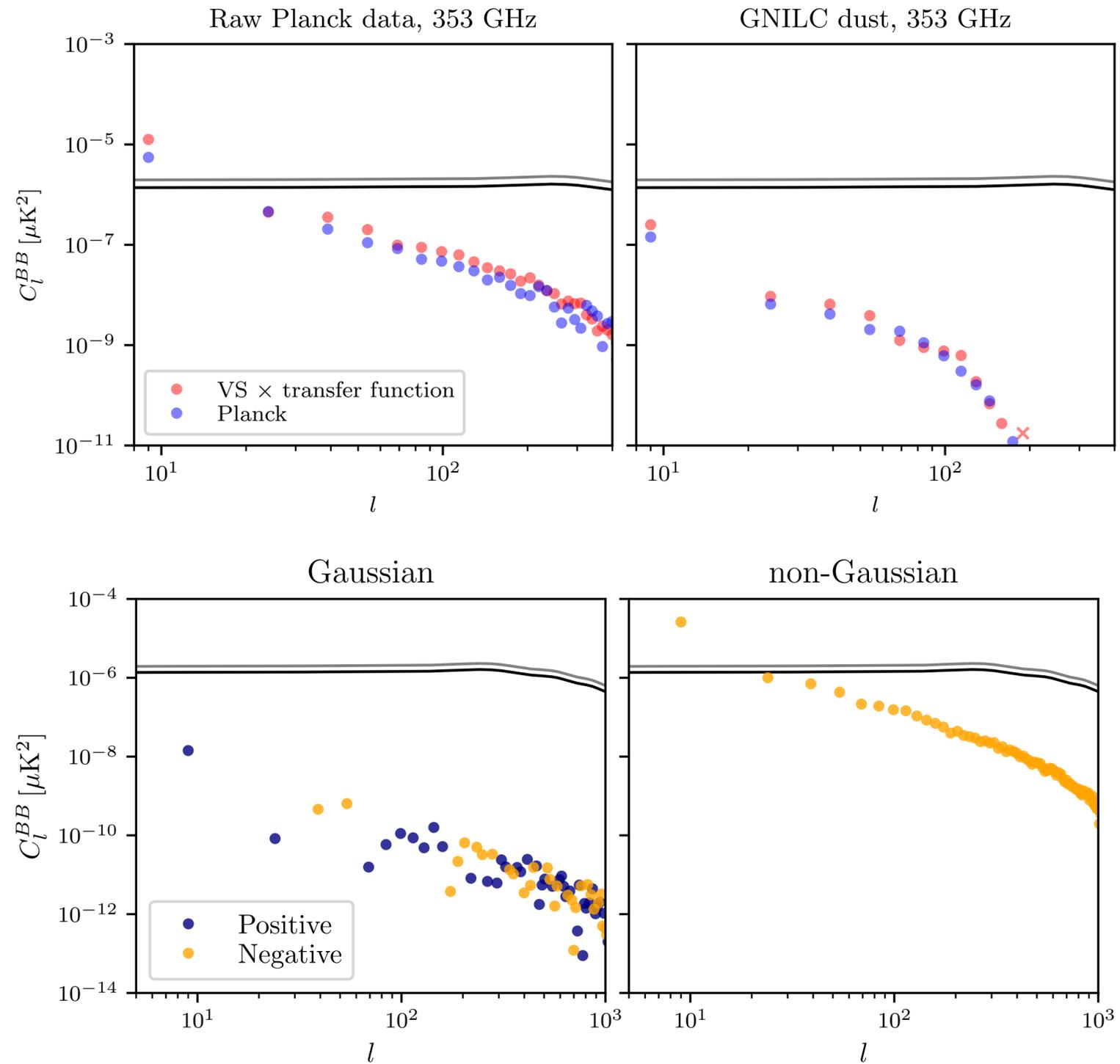
... but less clear what's happening to primordial B -modes.

ADDITIONAL SLIDES

Small degradation in delensing efficiency from deprojecting dust from LAT E-modes

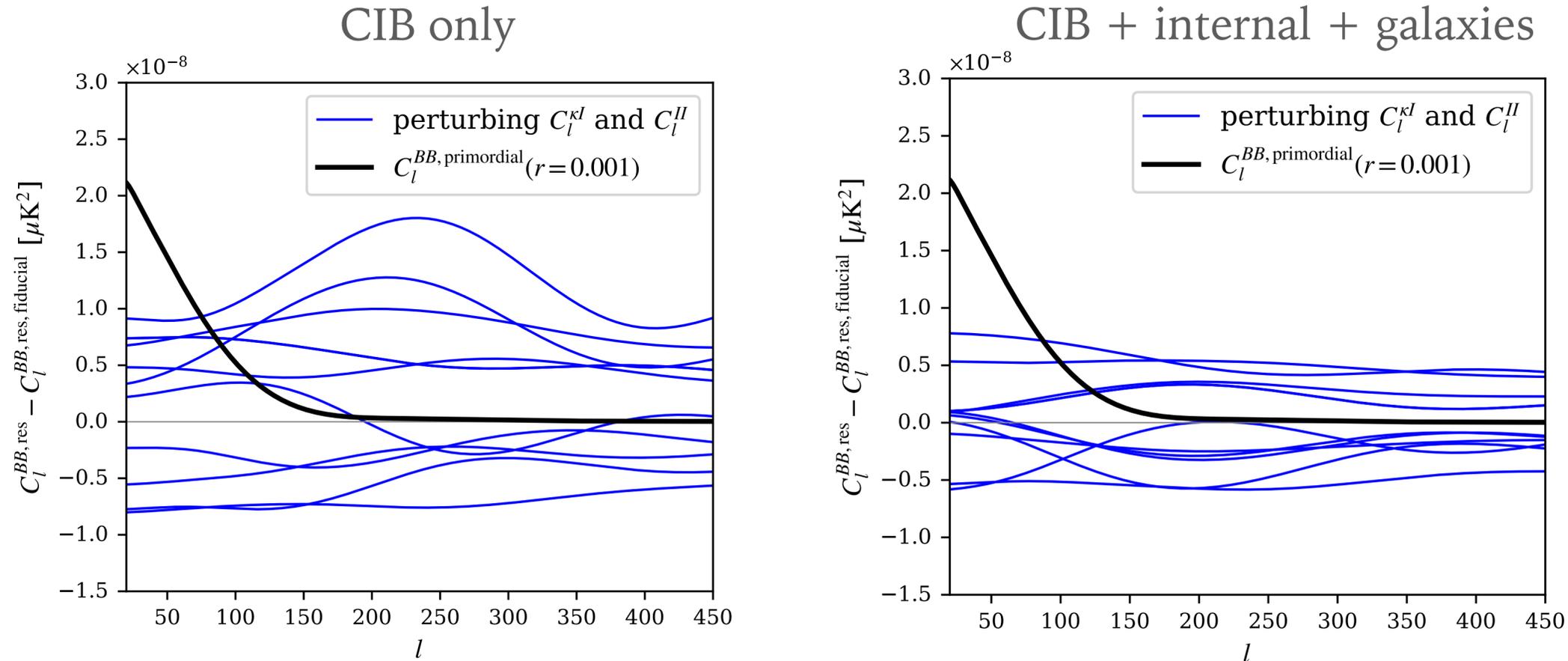


ADDITIONAL SLIDES



CHALLENGES TO MULTI-TRACER DELENSING

- Uncertainties in measurements of tracer auto- and cross- spectra:



Flat enough to marginalise over amplitude parameter!

T. Namikawa, ABL, N. Robertson, A Challinor, B. Sherwin & B. Yu 21 (to be submitted)

- The impact of foregrounds

RECAP OF B-MODE DELENSING

Schematically:

$$\langle |B^{obs} - E^{obs} \hat{\phi}|^2 \rangle = \langle |B^{obs}|^2 \rangle - 2 \langle B^{obs} E^{obs} \hat{\phi} \rangle + \langle E^{obs} \hat{\phi} E^{obs} \hat{\phi} \rangle:$$



delensing: $\langle \tilde{B}(E, \phi) \tilde{E} \phi \rangle_c$



= residual lensing + experimental noise

INTERNAL DELENSING BIAS

Schematically:

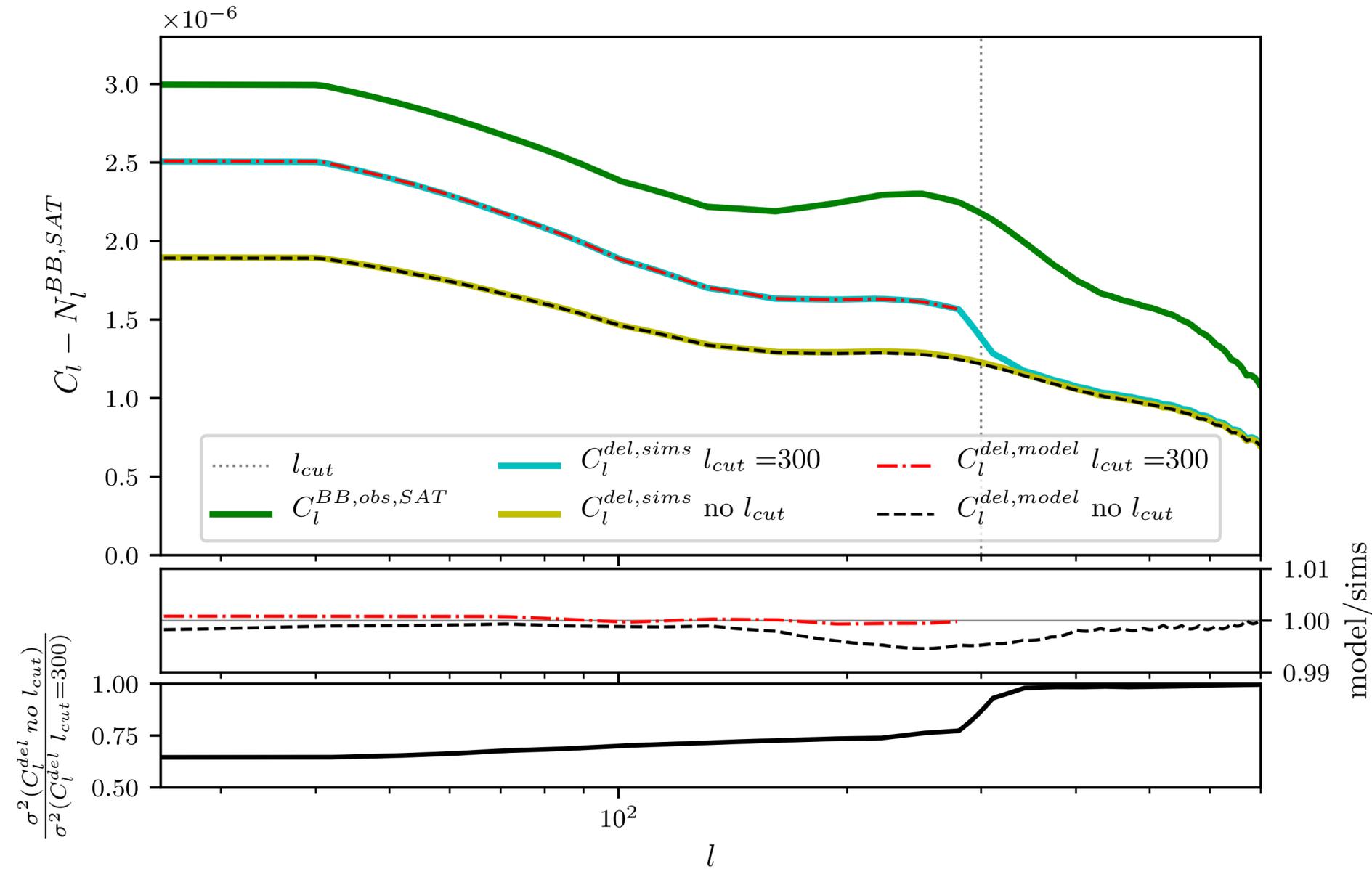
$$\langle |B^{obs} - E^{obs} \hat{\phi}|^2 \rangle = \langle |B^{obs}|^2 \rangle - 2 \langle B^{obs} E^{obs} \hat{\phi} \rangle + \langle E^{obs} \hat{\phi} E^{obs} \hat{\phi} \rangle:$$

$$\hat{\phi} = \hat{\phi}^{EB}(E, B)$$

$$\begin{array}{c} \uparrow \\ \langle B^{obs} E^{obs} E^{obs} B^{obs} \rangle \\ \uparrow \\ \langle E^{obs} E^{obs} B^{obs} E^{obs} E^{obs} B^{obs} \rangle \end{array}$$

- Suppression of power beyond a simple removal of lensing
- Bias is *local* — avoid by removing overlapping modes

INTERNAL DELENSING BIAS



Variance of delensed spectrum is reduced!

$$\text{Model: } \frac{C_l^{BB,del,biasd}}{(D_l - 1)^2} = C_l^{BB,del,unbiased} + \left(\frac{D_l}{D_l - 1} \right)^2 \left[C_l^W + N_l^{BB,LAT} + N_l^{BB,SAT} \left(\frac{2}{D_l} - 1 \right) - \frac{2}{D_l} N_l^X \right], \quad 0 < D_l < 1$$