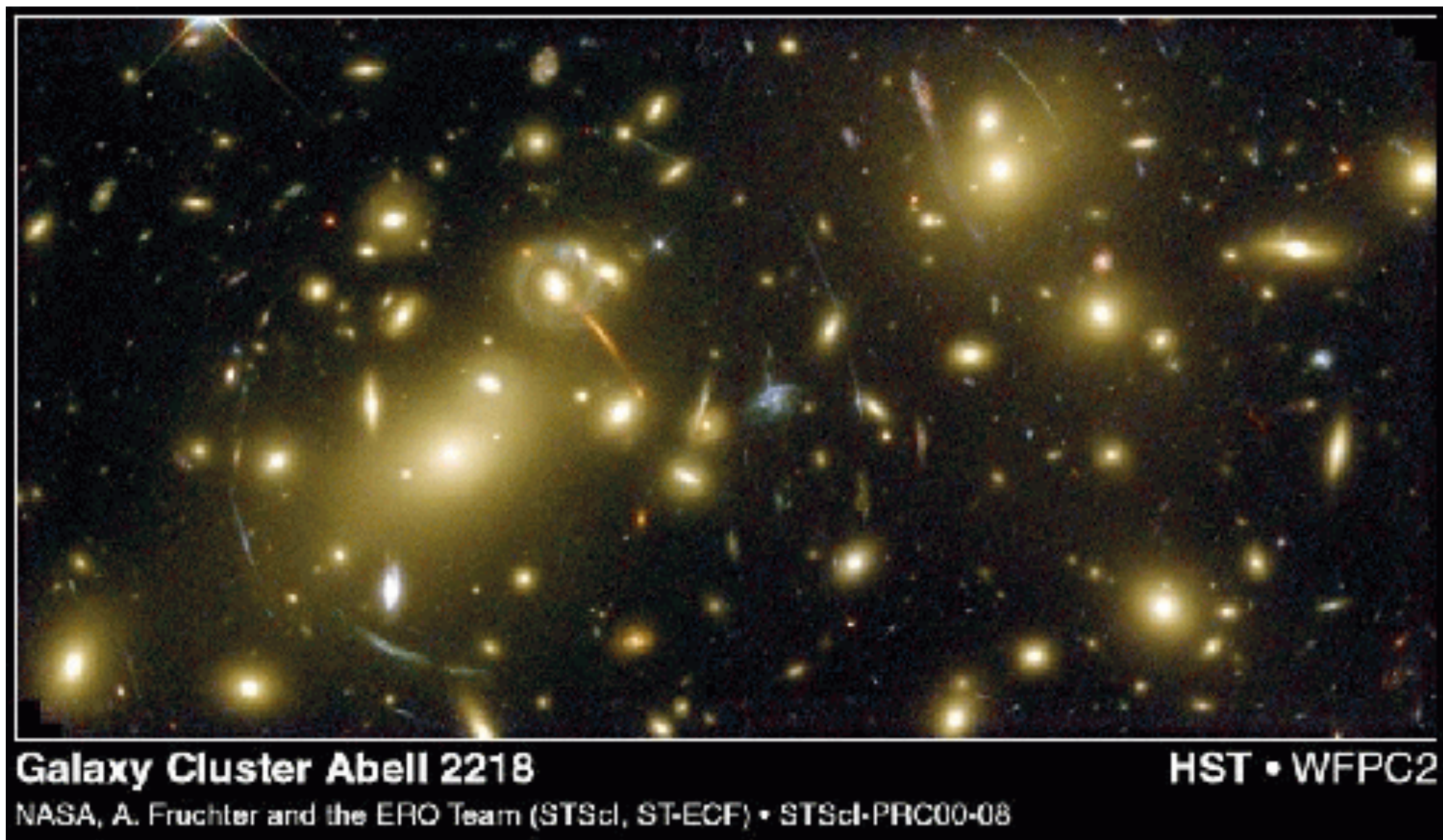


# The Physics of Gravitational Redshifts in Clusters of Galaxies

Nick Kaiser  
Institute for Astronomy, U. Hawaii

LBL, Berkeley 2016/09/15

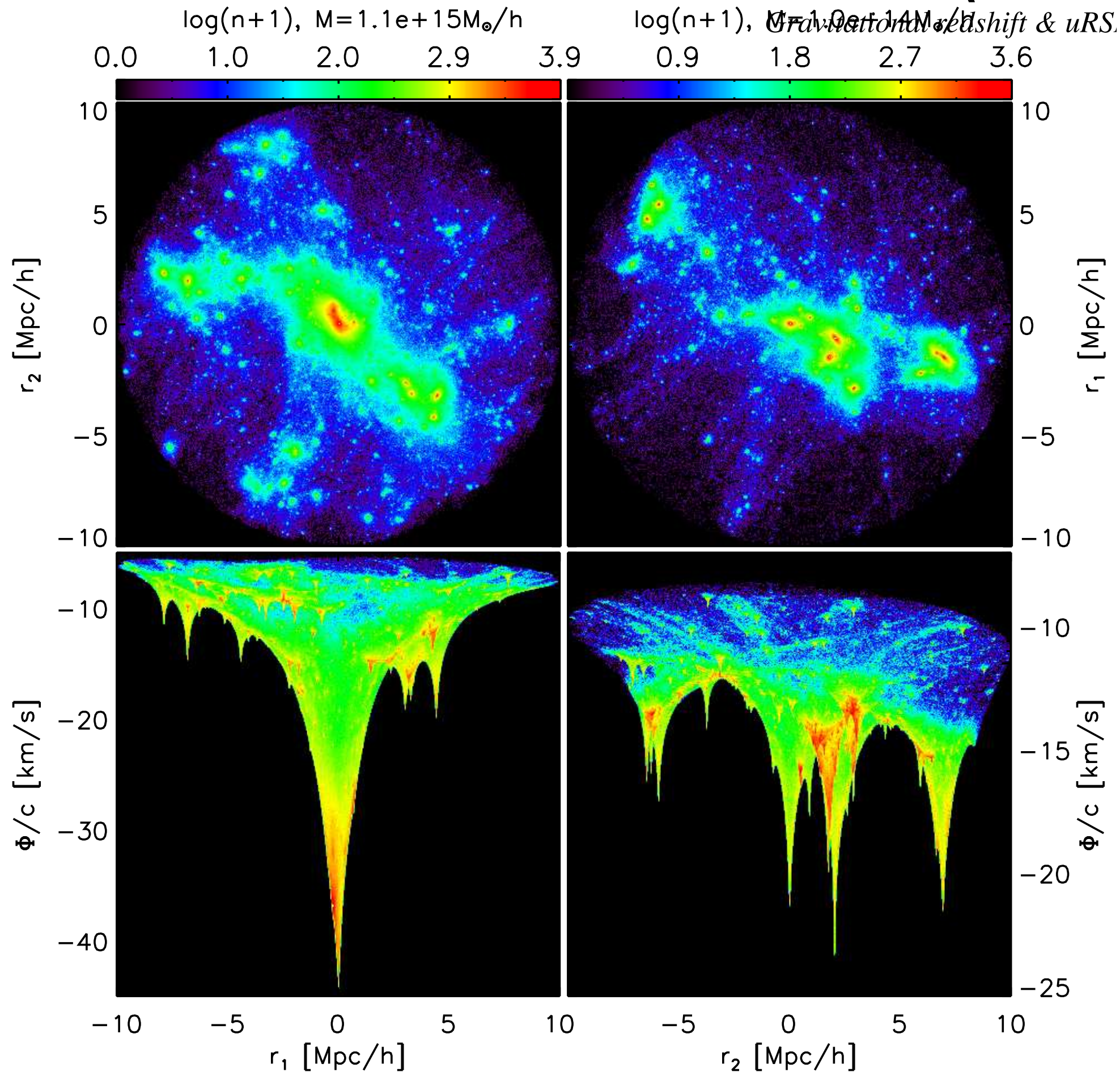
# Clusters of galaxies



- Largest bound virialised systems  $\sim 10^{14}-10^{15} M_{\text{sun}}$
- Velocity dispersion  $\sigma_v \sim 1000 \text{ km/s}$  ( $\sim 0.003c$ )
- Centres - often defined by the brightest galaxy (BCG)
  - Usually very close to peak of light, X-rays, DM
  - $\sigma_{\text{BCG}} \sim 0.4-0.5 \sigma_v$  so much “colder” than general pop<sup>n</sup>

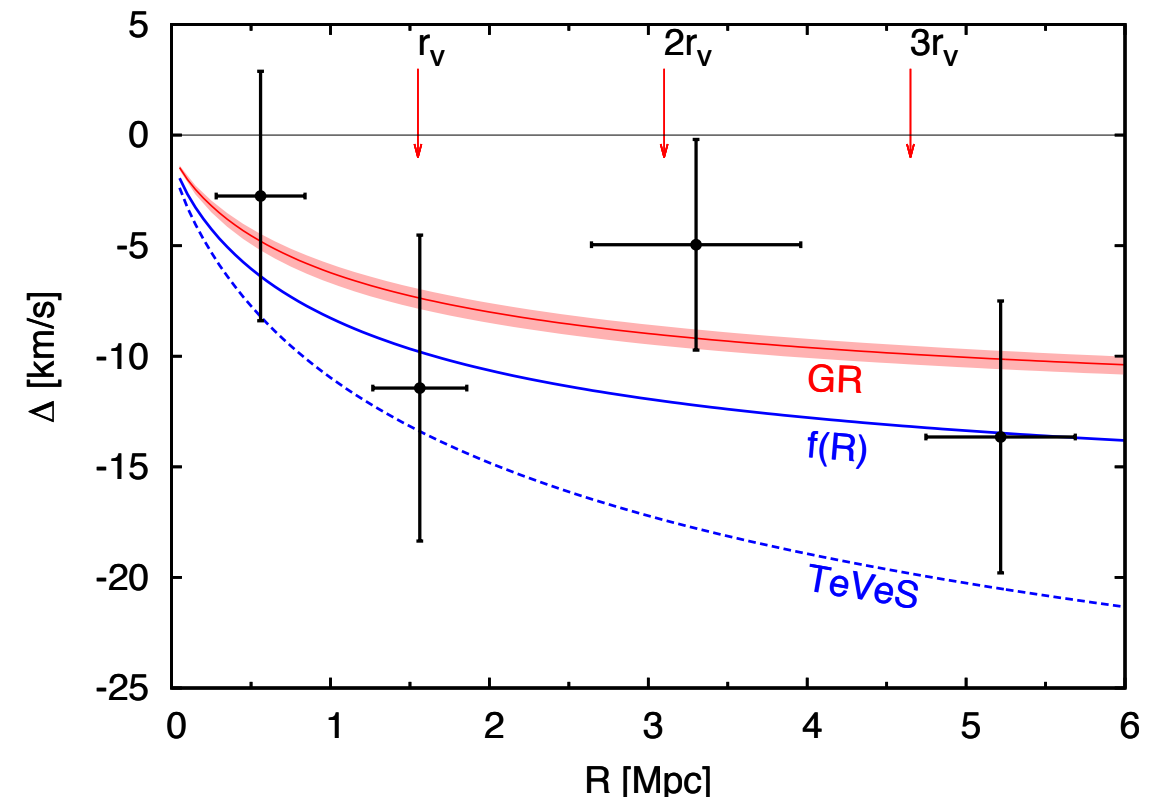
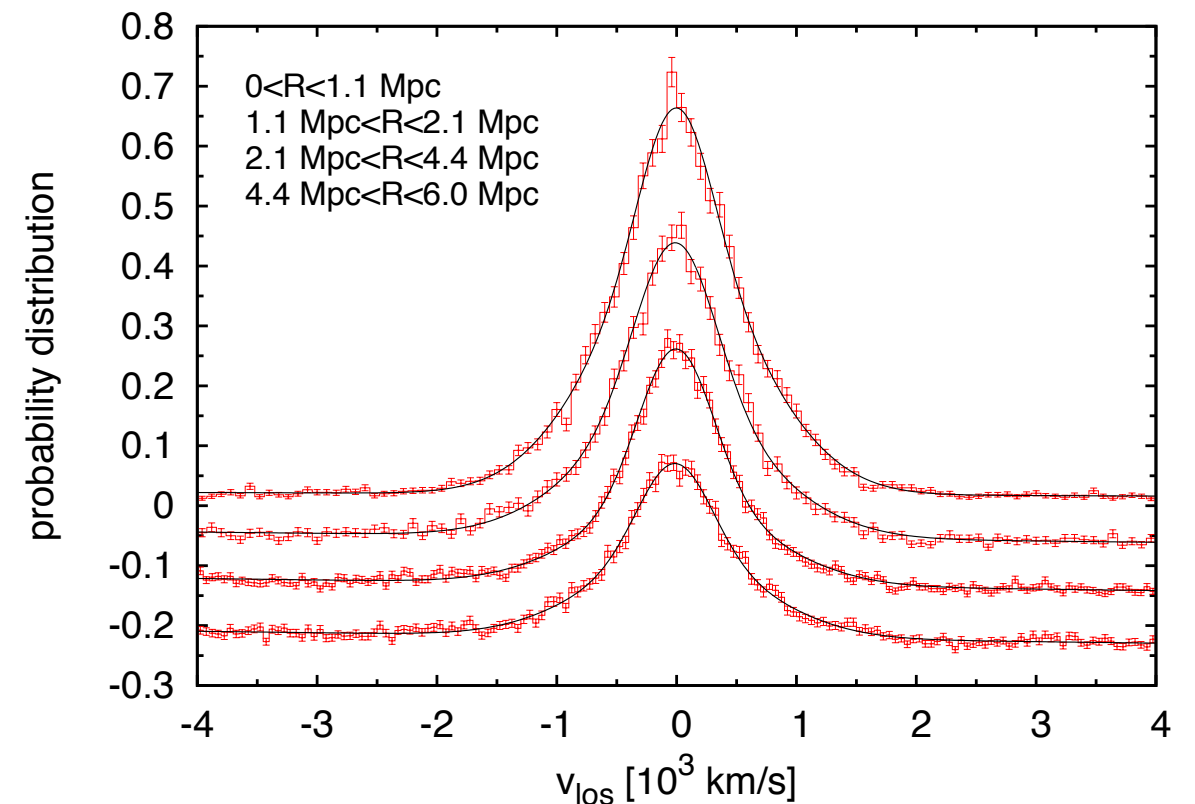


# Clusters in the Millenium Simulation (Y. Cai)



# Wojtak, Hansen & Hjorth (Nature 2011)

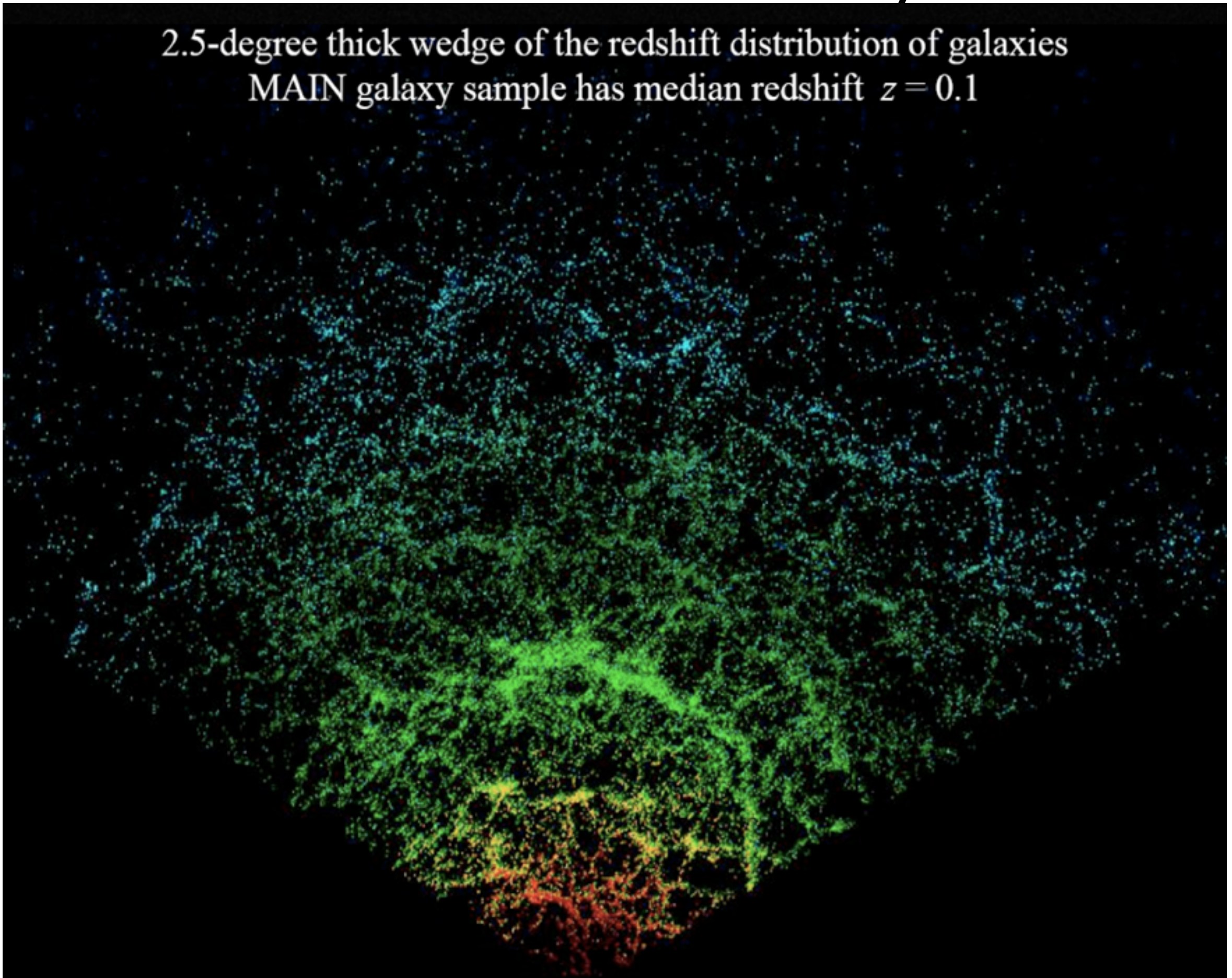
- Wojtek, Hansen & Hjorth stacked 7,800 galaxy clusters from SDSS DR7 in redshift space
- centres defined by the brightest cluster galaxies (BCGs)
- approx 10 redshifts per cluster
- They found a net offset (blue-shift) corresponding to  $v = -10$  km/s
- c.f.  $\sim 600$  km/s l.o.s velocity dispersion
- Interpreted as gravitational redshift effect
- right order of magnitude, sign
- “Confirms GR, rules out TeVeS”
- Had been discussed before (Cappi 1995; Broadhurst+Scannapiaco, ....)



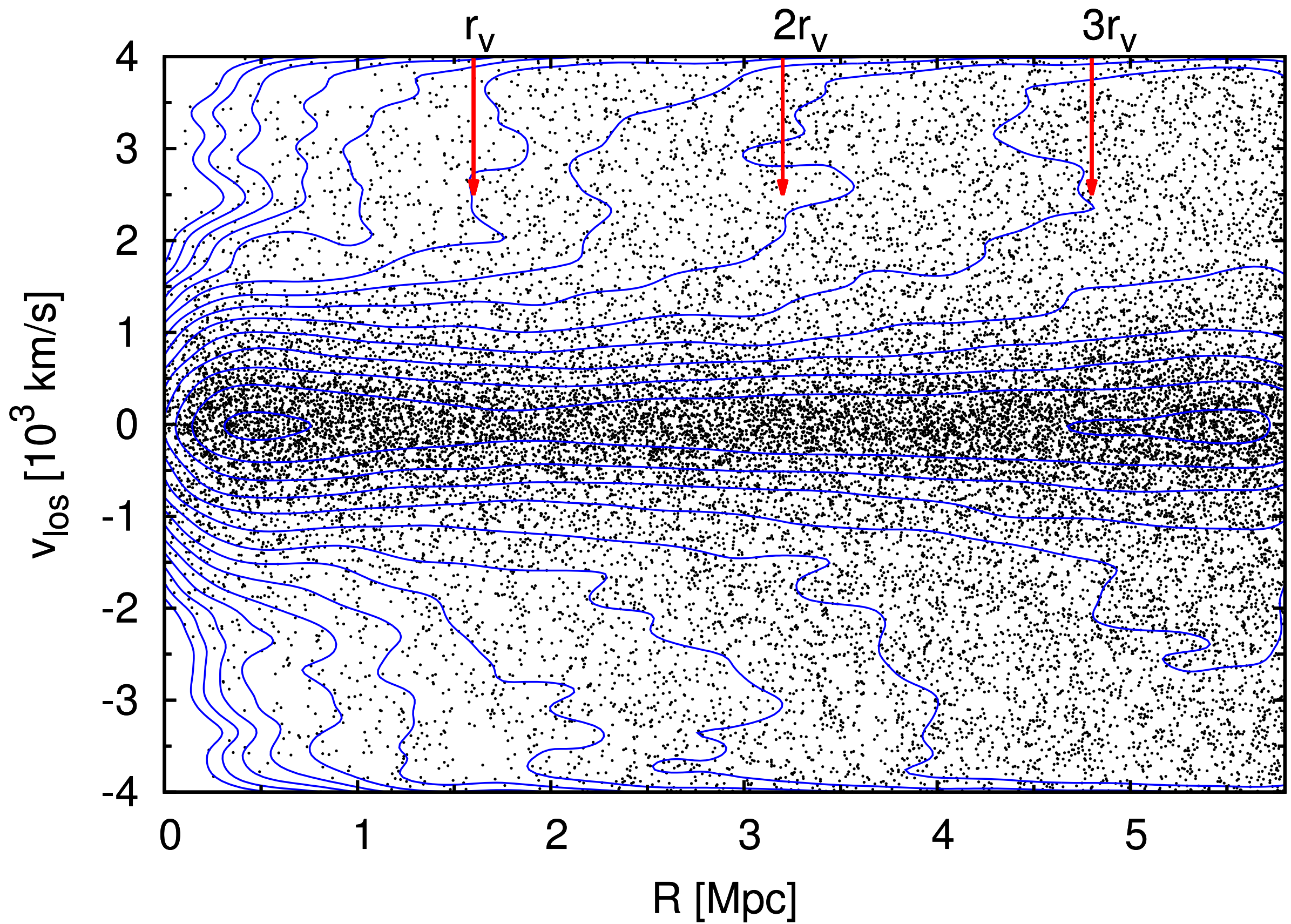


# SDSS Redshift Survey

2.5-degree thick wedge of the redshift distribution of galaxies  
MAIN galaxy sample has median redshift  $z = 0.1$



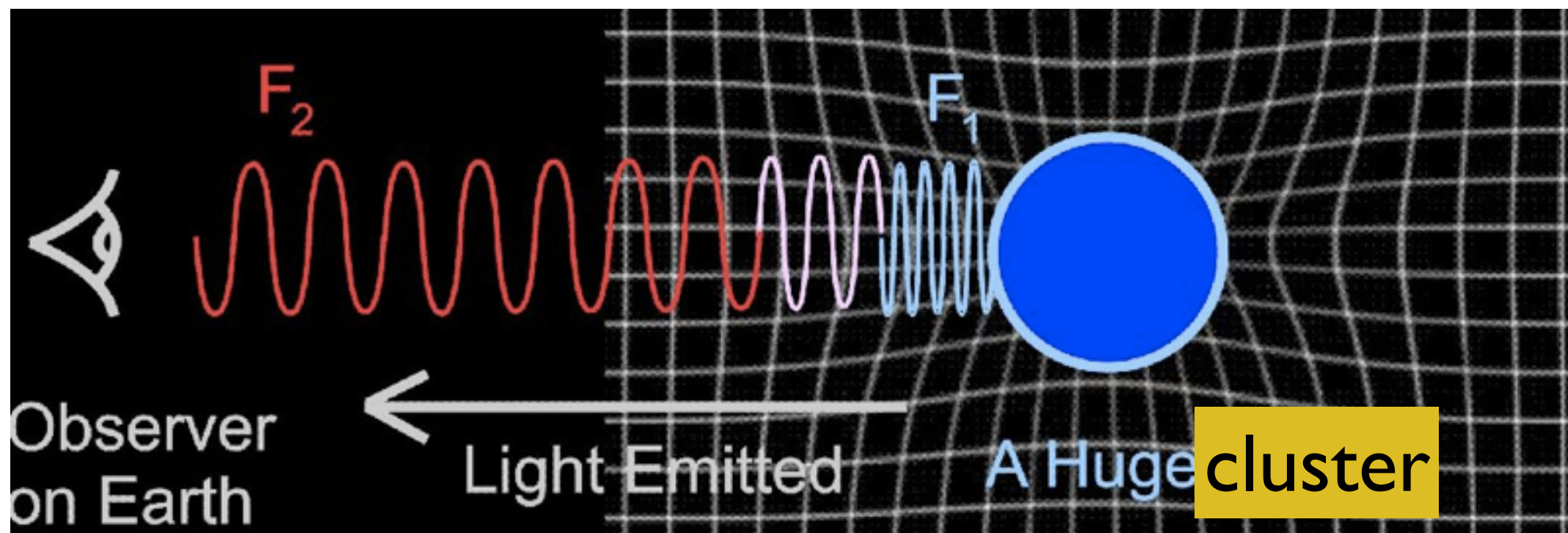
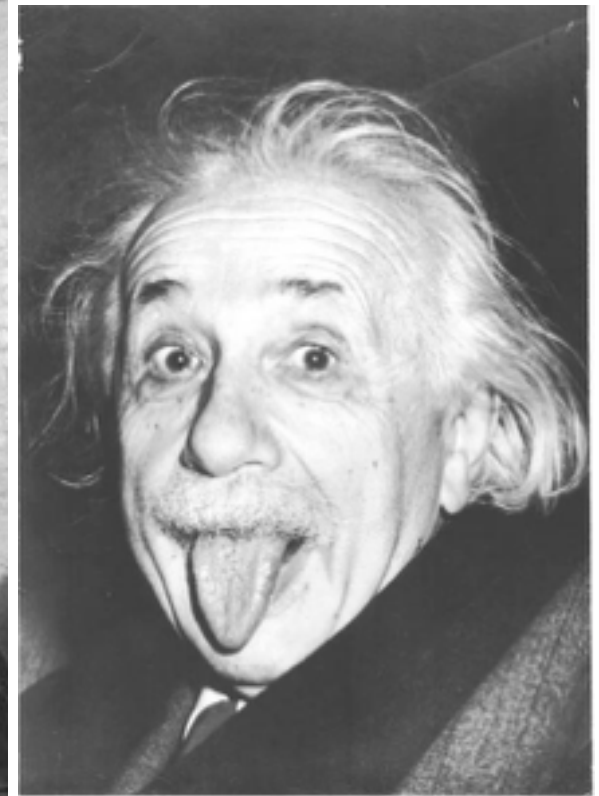






# The physics of cluster gravitational redshifts

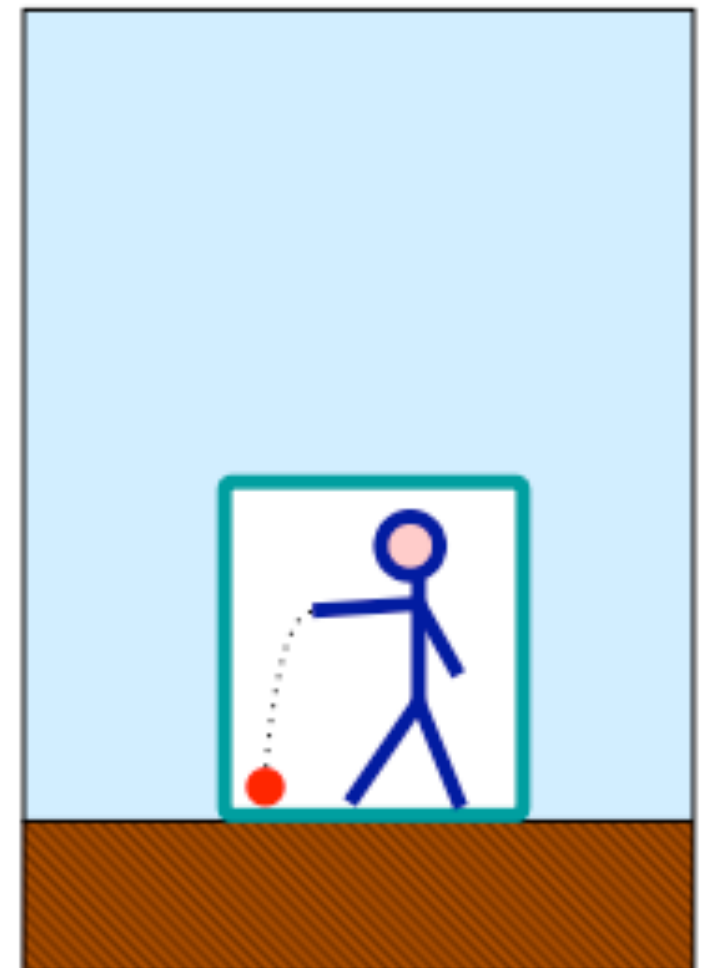
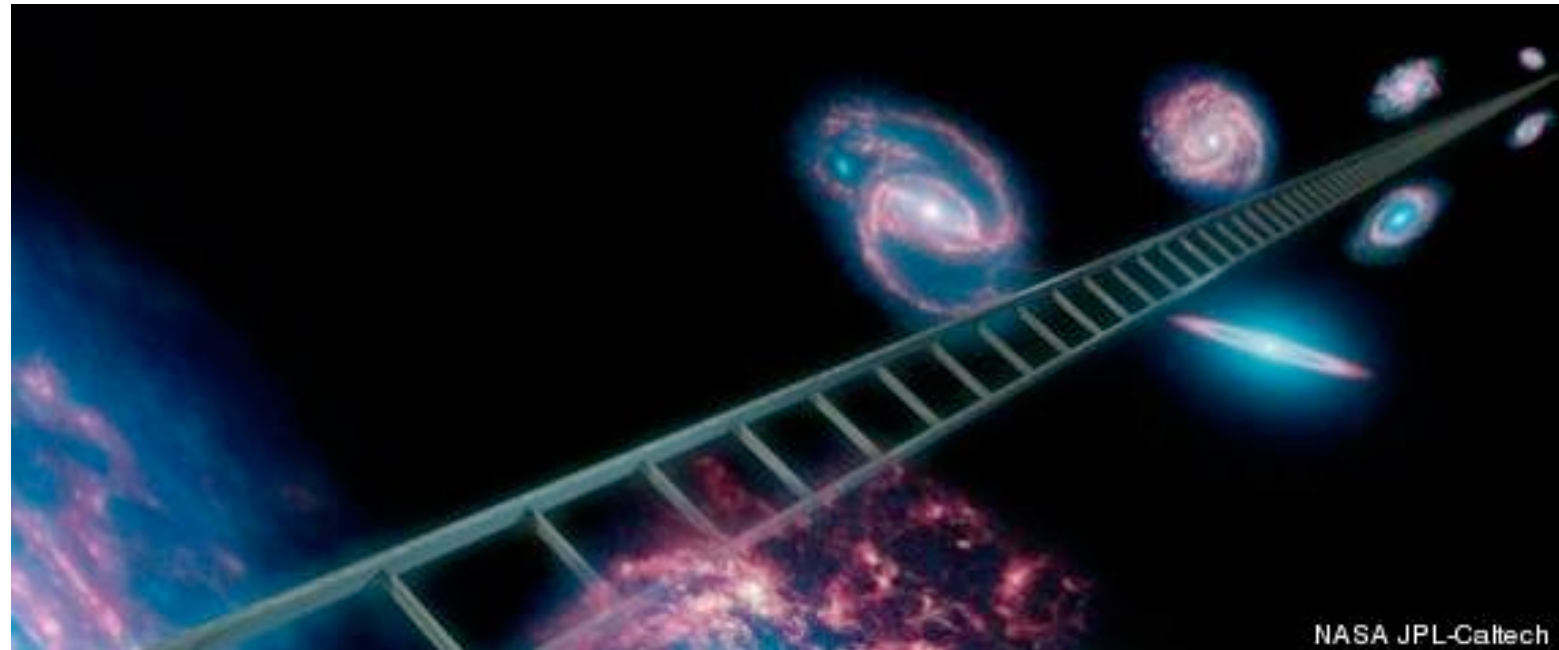
- Einstein gravity
  - gravitational "time dilation"
- Weak field limit
  - $\delta\nu/\nu = -\Phi/c^2$
- Measured by Pound & Rebka



Is that it?

# Is there any more to the physics of cluster grav-z?

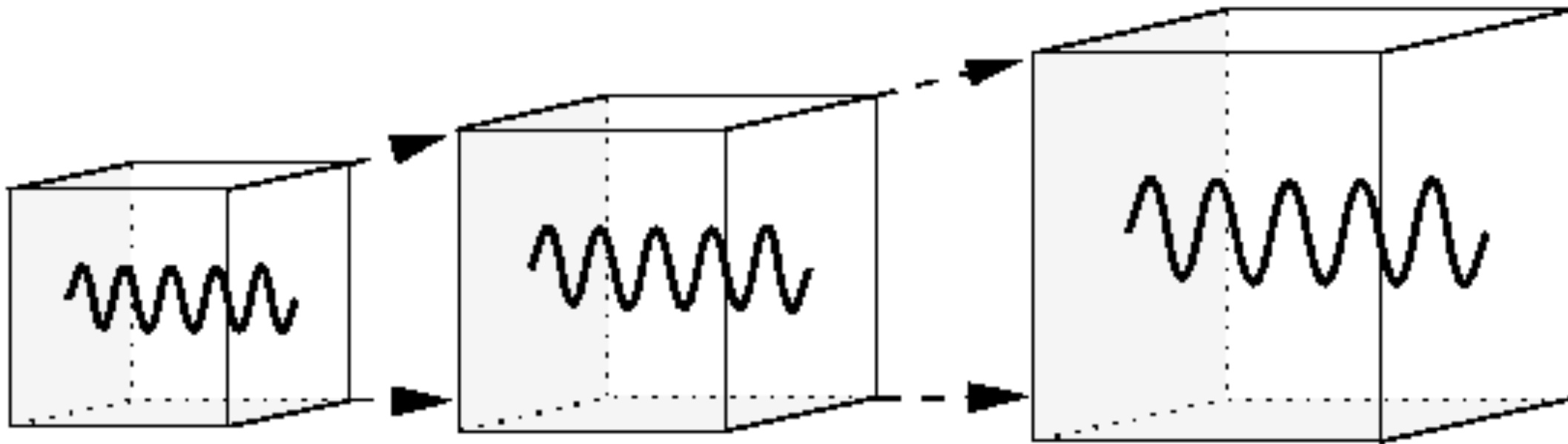
- Is Pound & Rebka relevant?
- equipment lashed to tower in Harvard phys. dept
- where is the tower here?
- What about the principle of equivalence?
- where is the rocket?
- where is the earth?

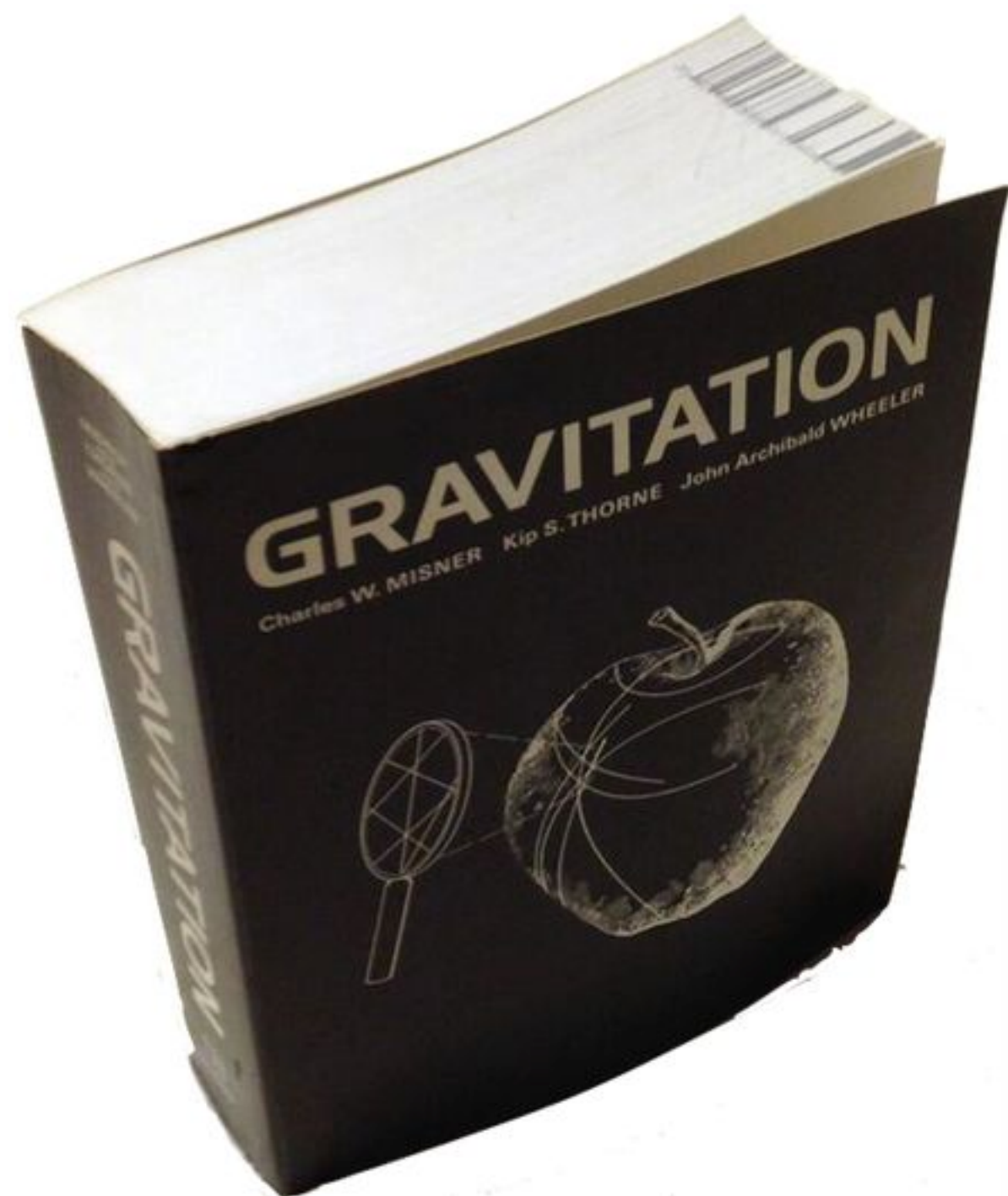




# The nature of astronomical redshifts

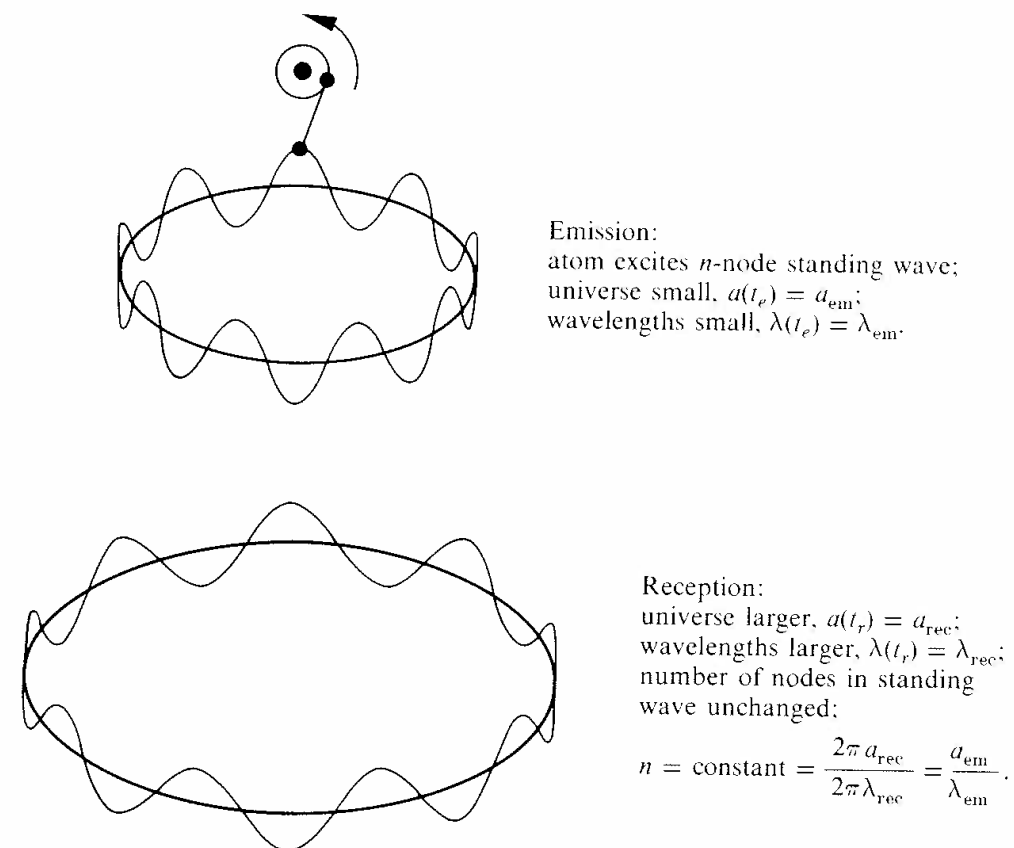
- Homogeneous cosmological models:
- Wavelength scales as  $a(t)$ , or  $\delta\lambda/\lambda = \delta D/D$ , but why?
- Symmetry  $\rightarrow$  standing wave solutions with  $\lambda \sim a(t)$ 
  - Analogy with expanding cavity
    - lots of little redshifts as ‘photons’ bounce off walls
- Maxwell’s equations written in “expanding” coordinates
  - extra cosmic ‘damping’ or ‘friction’ term in EOM
- So expansion of space stretches wavelengths?





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## 29. PRESENT STATE AND FUTURE EVOLUTION OF THE UNIVERSE



**Figure 29.1.**

Redshift as an effect of standing waves. The ratio of wavelengths,  $\lambda_{rec}/\lambda_{em}$ , is identical with the ratio of dimensions,  $a_{rec}/a_{em}$  in any closed spherically symmetrical (Friedmann) model universe. The atom excites an  $n$ -node standing wave in the universe. The number  $n$  stays constant during the expansion.

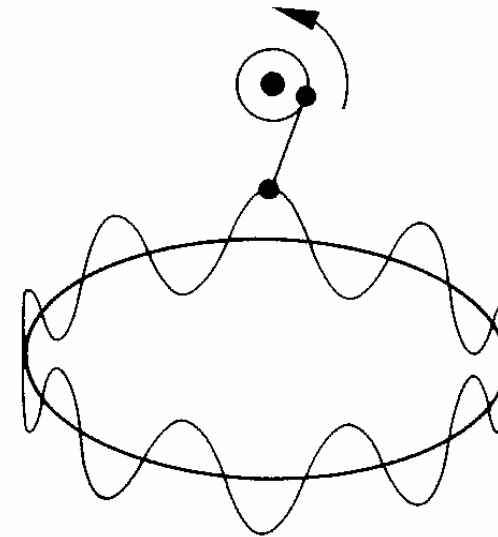
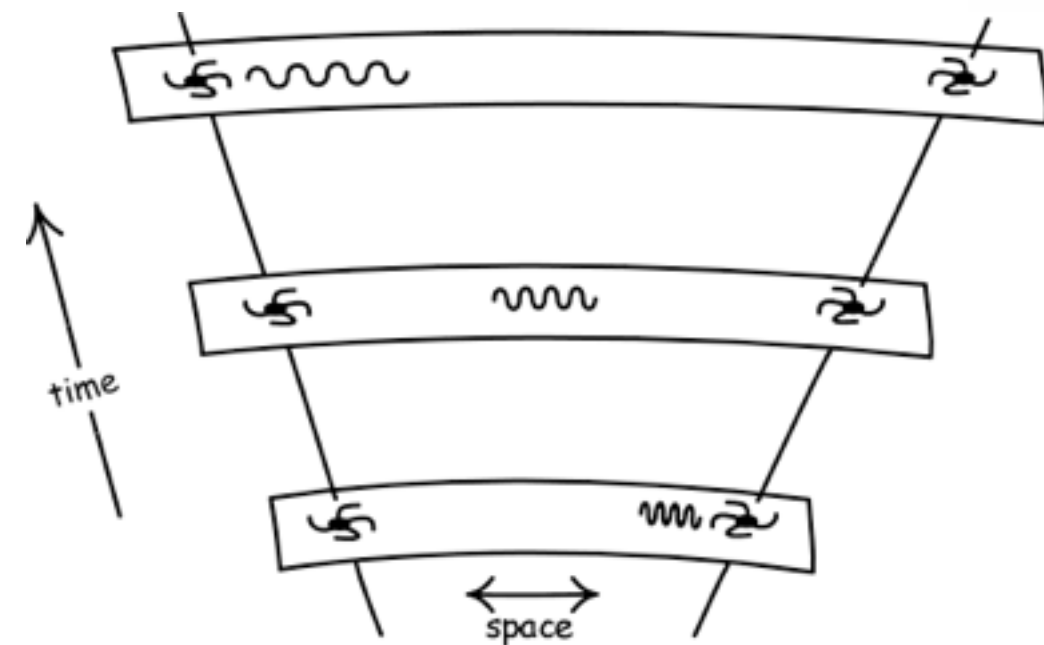


# Misner, Thorne and Wheeler

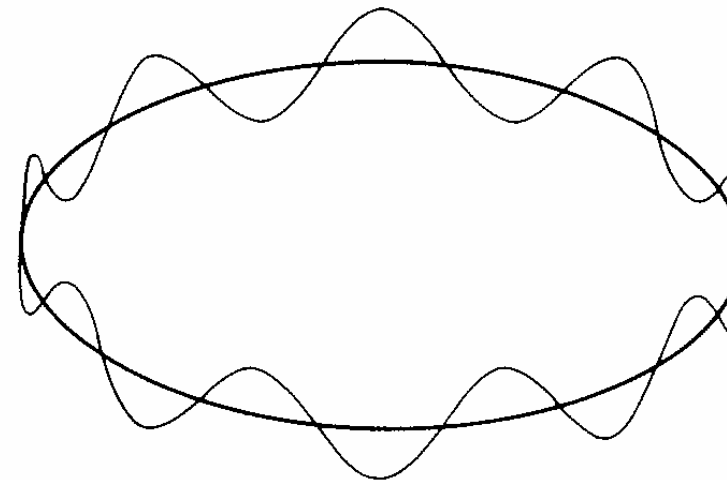
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29. PRESENT STATE AND FUTURE EVOLUTION OF THE UNIVERSE

redshift as an  
effect on standing  
waves....



Emission:  
atom excites  $n$ -node standing wave;  
universe small,  $a(t_e) = a_{em}$ ;  
wavelengths small,  $\lambda(t_e) = \lambda_{em}$ .



Reception:  
universe larger,  $a(t_r) = a_{rec}$ ;  
wavelengths larger,  $\lambda(t_r) = \lambda_{rec}$ ;  
number of nodes in standing  
wave unchanged;

$$n = \text{constant} = \frac{2\pi a_{rec}}{2\pi \lambda_{rec}} = \frac{a_{em}}{\lambda_{em}}.$$

**Figure 29.1.**

Redshift as an effect of standing waves. The ratio of wavelengths,  $\lambda_{rec}/\lambda_{em}$ , is identical with the ratio of dimensions,  $a_{rec}/a_{em}$  in any closed spherically symmetrical (Friedmann) model universe. The atom excites an  $n$ -node standing wave in the universe. The number  $n$  stays constant during the expansion. Therefore wavelengths increase in the same proportion as the dimensions of the universe. One sees immediately in this way that the redshift is independent of all such details as (1) why the expansion came about (spherical symmetry, but arbitrary equation of state); (2) the rate—uniform or nonuniform—at which it came about; and (3) the distance between source and receptor at emission, at reception, or at any time in-between. The reasoning in the diagram appears to depend on the closure of the universe (standing waves;  $k = +1$  rather than 0 or  $-1$ ). That closure is not required for this simple result is seen from the further analysis given in the text.

But is this a standing  
wave?

# Expanding space and redshifts in textbooks.....

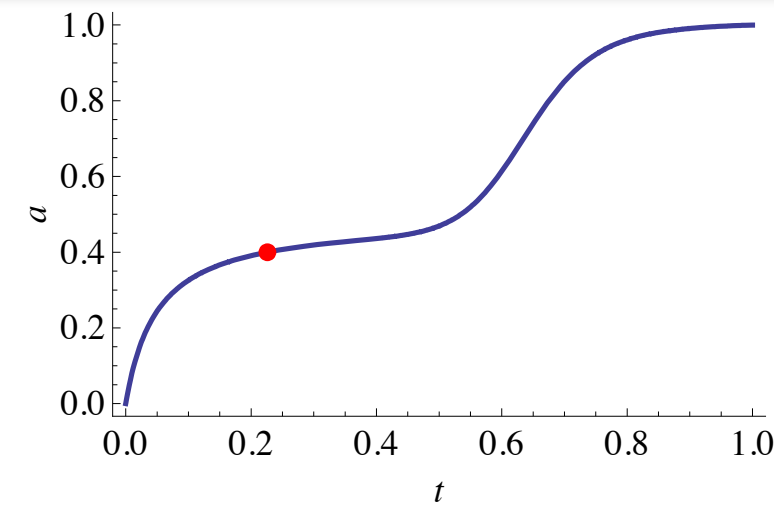
- E.R. Harrison (2000)

We suppose that all galaxies are comoving and that their light is received by observers who are also comoving. Light leaves a galaxy, which is stationary in its own local region of space, and is received by observers who are also stationary in their own local region of space. Between the galaxy and the observers light travels through vast regions of expanding space. What happens is immediately obvious: All wavelengths of the light are stretched by the expansion of space (see Figure 11.1). It is as simple as that.

- Wolfgang Rindler (1970)

Note that the cosmological red shift is really an *expansion* effect rather than a *velocity* effect.

Expansion redshifts are produced by the expansion of space between bodies that are stationary in space: They depend on the increase of distance between the emitter and the receiver during the time of propagation; they are the result of recession velocities and not peculiar velocities; and they are not governed by the rules of special relativity.

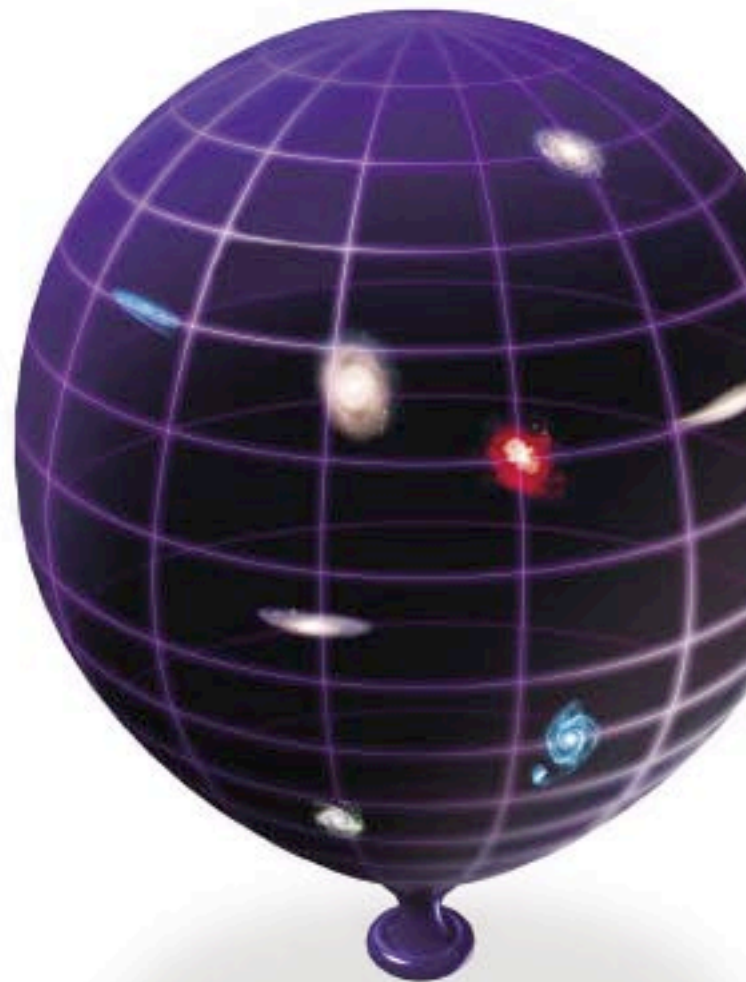




# MISCONCEPTIONS ABOUT THE BIG BANG

Baffled by the expansion of the universe?  
You're not alone. Even astronomers  
frequently get it wrong

By Charles H. Lineweaver and  
Tamara M. Davis



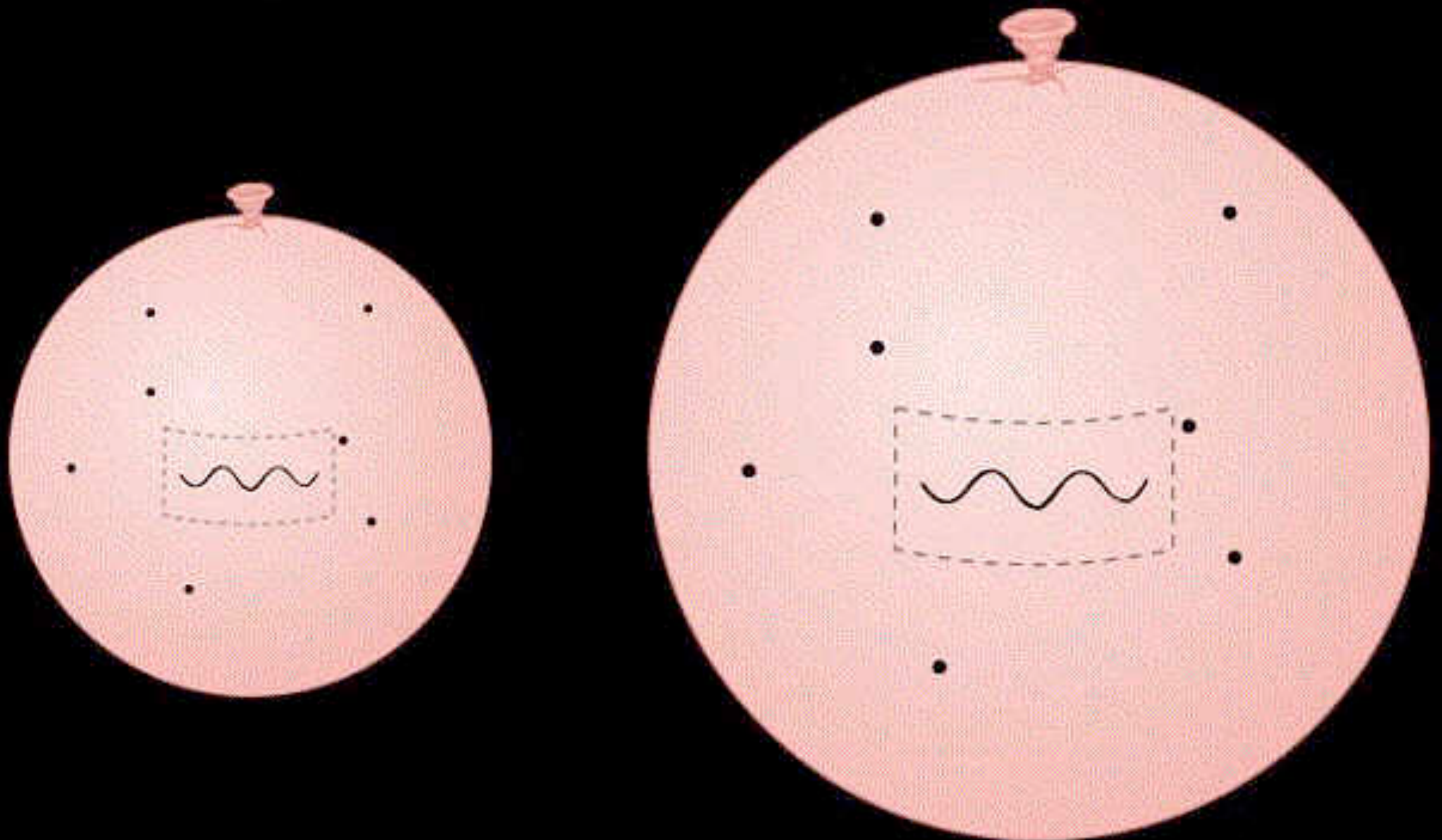
## Stretching and Cooling

THE PRIMARY OBSERVATION that the universe is expanding emerged between 1910 and 1930. Atoms emit and absorb light of specific wavelengths, as measured in laboratory experiments. The same patterns show up in the light from distant galaxies, except that the patterns have been shifted to longer wavelengths. Astronomers say that the galactic light has been redshifted. The explanation is straightforward: As space expands, light waves get stretched. If the universe doubles in size during the waves' journey, their wavelengths double and their energy is halved.

This process can be described in terms of temperature. The photons emitted by a body collectively have a temperature—a certain distribution of energy that reflects how hot the body is. As the photons travel through expanding space, they lose energy and their temperature decreases. In this way, the universe cools as it expands, much as compressed air in a scuba tank cools when it is released and allowed to expand. For example, the microwave background radiation currently has a temperature of about three kelvins, whereas the process that released the radiation occurred at a temperature of about 3,000 kelvins. Since the time of the emission of this radiation, the universe has increased in size by a factor of 1,000, so the temperature of the photons has decreased by the same factor. By observing the gas in distant galaxies, astronomers have directly measured the temperature of the radiation in the distant past. These measurements confirm that the universe has been cooling with time.

Misunderstandings about the relation between redshift and velocity abound. The redshift caused by the expansion is often confused with the more familiar redshift generated by the Doppler effect. The normal Doppler effect causes sound

## An Expanding Balloon and the Redshift





# Mathematics of radiation in expanding space

- In non-expanding space, extremizing the action
  - $S = \int dt d^3x L(\partial_t \varphi, \nabla_x \varphi)$
  - with e.g.  $L = ((\partial_t \varphi)^2 - (\nabla_x \varphi)^2)/2$  (i.e. massless scalar)
- Gives the wave equation with propagation speed  $c$  (unity here)
  - $\partial_t^2 \varphi - \nabla_x^2 \varphi = 0$
- In expanding universe, physical coordinate  $x = a(t) r$ , where  $r$  is 'comoving' coordinate, so
  - $\delta S = \delta \int dt d^3r L'(\partial_t \varphi, \nabla_r \varphi) = 0$ 
    - with  $L' = a(t)^3 L$  (and  $\nabla_r \varphi = a(t) \nabla_x \varphi$ )
  - Gives  $\partial_t^2 \varphi + 3 H \partial_t \varphi - a^{-2} \nabla_r^2 \varphi = 0$ 
    - expansion gives extra 'damping', or 'friction', term  $3 H \partial_t \varphi$  involving the expansion rate  $H = (da/dt)/a$ .
  - energy *not* conserved (Lagrangian formally time dependent)
  - admits damped standing wave (in  $r$ ) solutions  $(\partial_t \varphi)^2 \sim a^{-4}$
- And this does give the right answer for photons between FOs



# Expanding Space: the Root of all Evil?\*

*Matthew J. Francis<sup>1,4</sup>, Luke A. Barnes<sup>1,2</sup>, J. Berian James<sup>1,3</sup> & Geraint F. Lewis<sup>1</sup>*

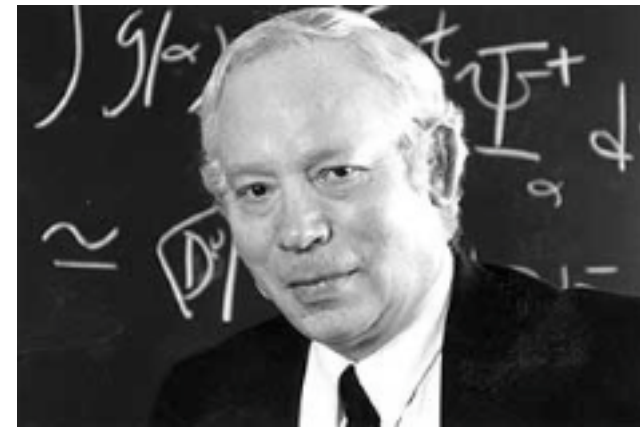
However, the academic argument surrounding the expansion of space is not as clear as standard explanations suggest; an interested student and reader of New Scientist may have seen Martin Rees & Steven Weinberg (1993) state

...how is it possible for space, which is utterly empty, to expand? How can nothing expand? The answer is: space does not expand. Cosmologists sometimes talk about expanding space, but they should know better.

while being told by Harrison (2000) that

expansion redshifts are produced by the expansion of space between bodies that are stationary in space

What is a lay-person or proto-cosmologist to make of this apparently contradictory situation?

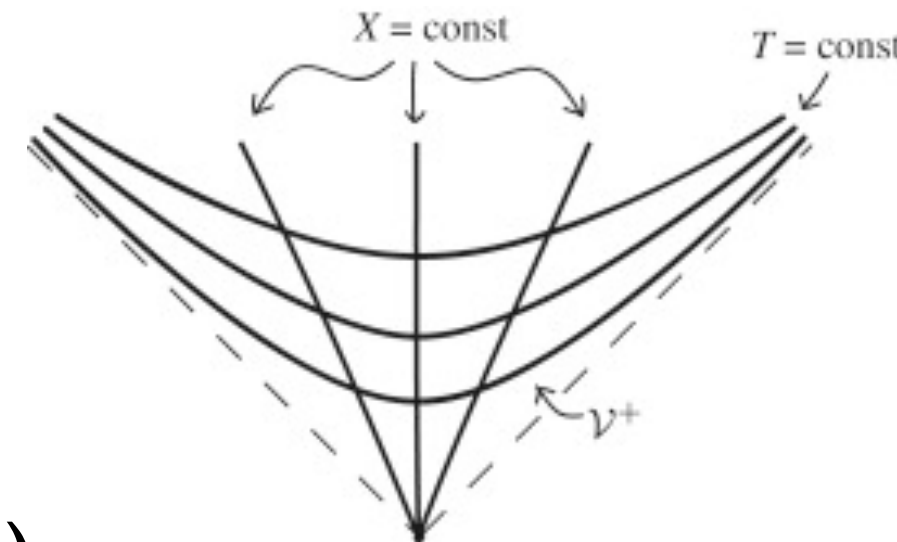


But see also Weinberg, 1st 3 Minutes, p31: "One can think of the wave crests being pulled farther and farther apart by the expansion of the universe."

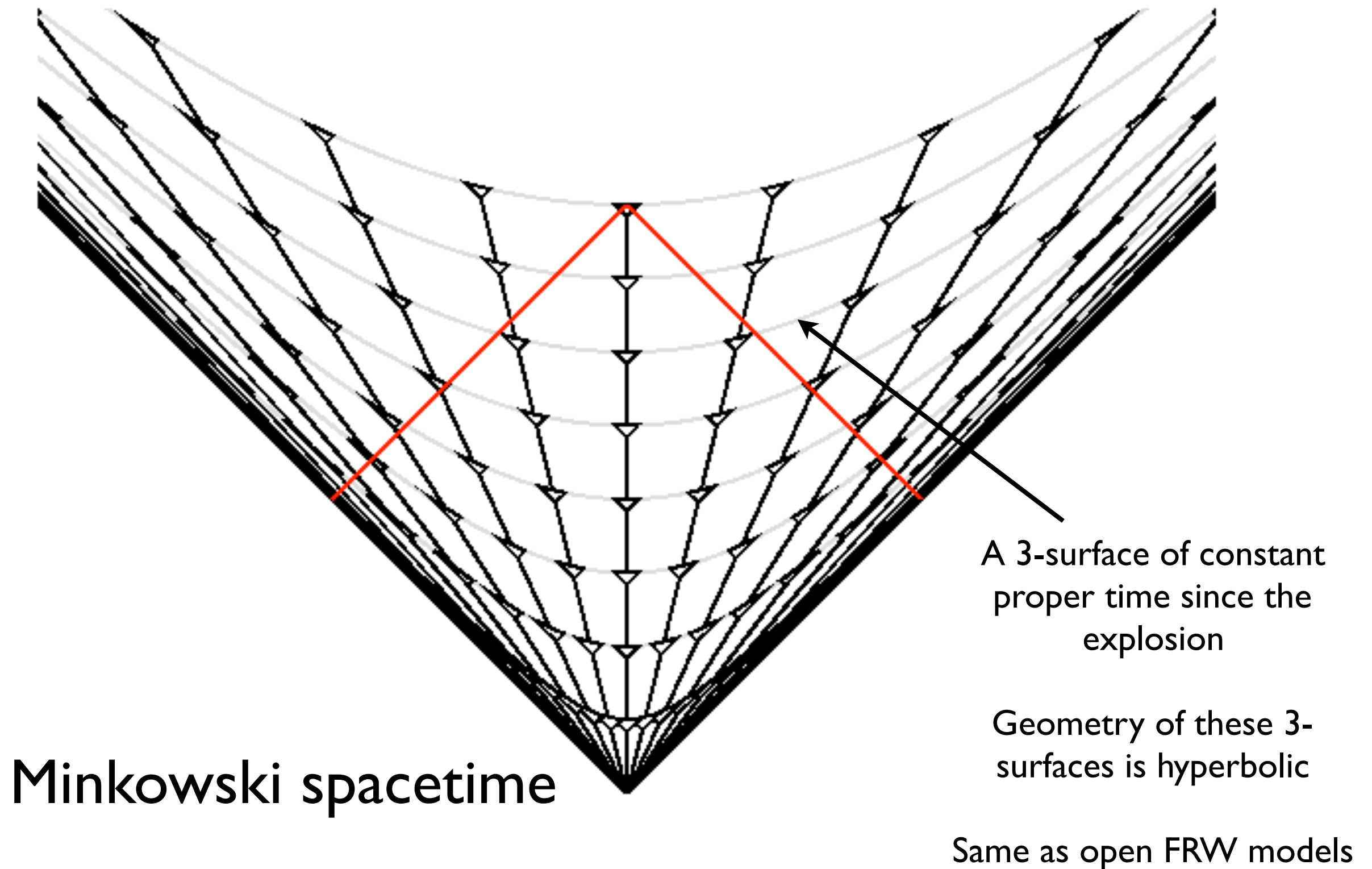


# Arguments against “expanding space”

- Space-time is locally flat (a.k.a. ‘Minkowskian’)
  - Whiting, 2004; Peacock, 2009; Bunn & Hogg 2009, Chodorowski, 2007, 2011, Rees & Weinberg....
- Writing  $\partial_t^2 \varphi - \nabla_x^2 \varphi = 0$  in re-scaled coordinates does not change the local physics.
  - despite apparent damping term
- Example: the Milne model
  - Limiting case of  $\Omega \rightarrow 0$  FRW model
  - Different families of FOs  $\rightarrow$  *different*  $a(t)$
  - $\rightarrow$  Red-shifting or blue-shifting fireball solutions
  - multiple families of uniformly expanding (or contracting) observers and radiation fields - even in same region of space
- The expansion rate  $H$  is defined by the radiation itself!
- Determined by the initial conditions.



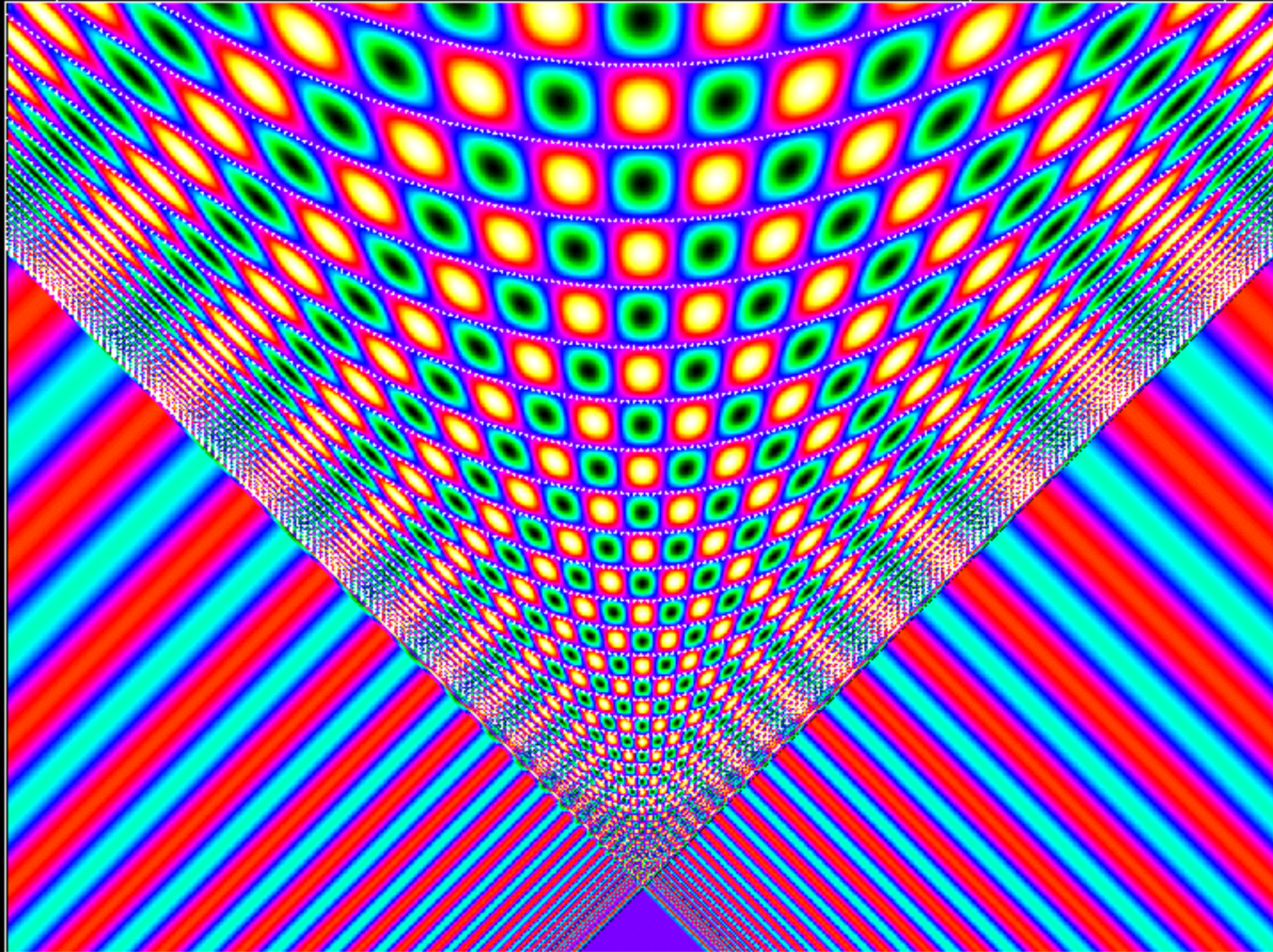
# Observers and photon paths in the Milne Model





# Standing waves (in expanding coords) in Milne's model

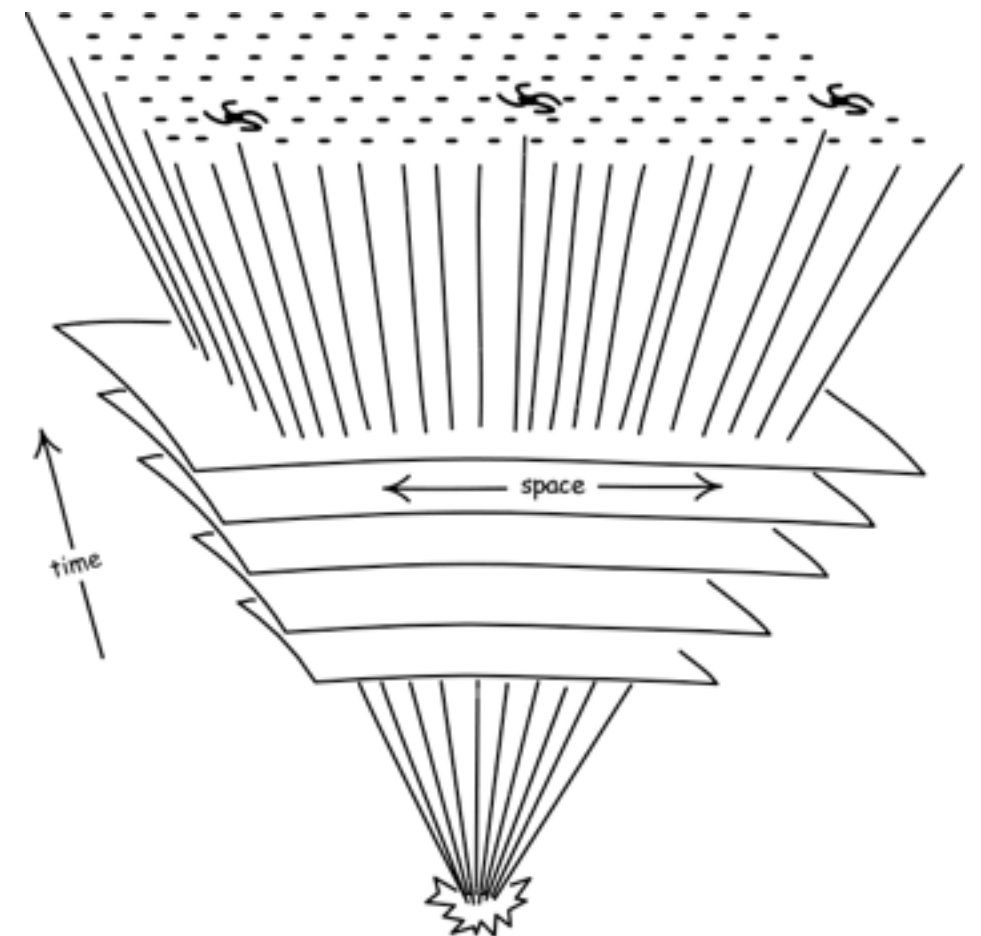
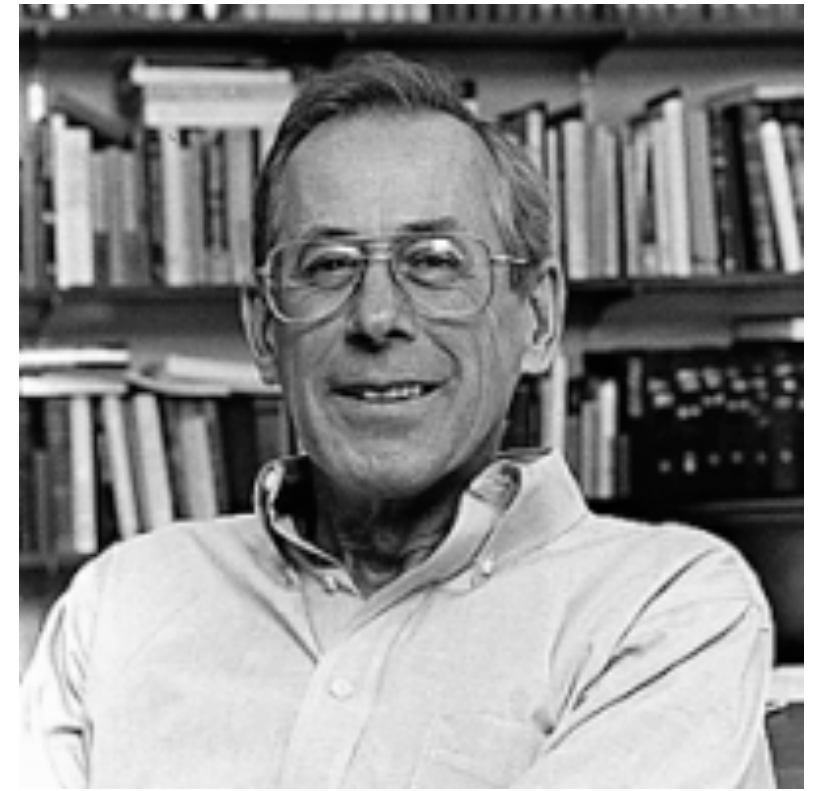
$$\cos(k \ln(t+x)) + \cos(k \ln(t-x)) = 2 \cos(k \ln(t^2-x^2)^{1/2}) \cos(k \ln((1+x/t)/(1-x/t))^{1/2})$$





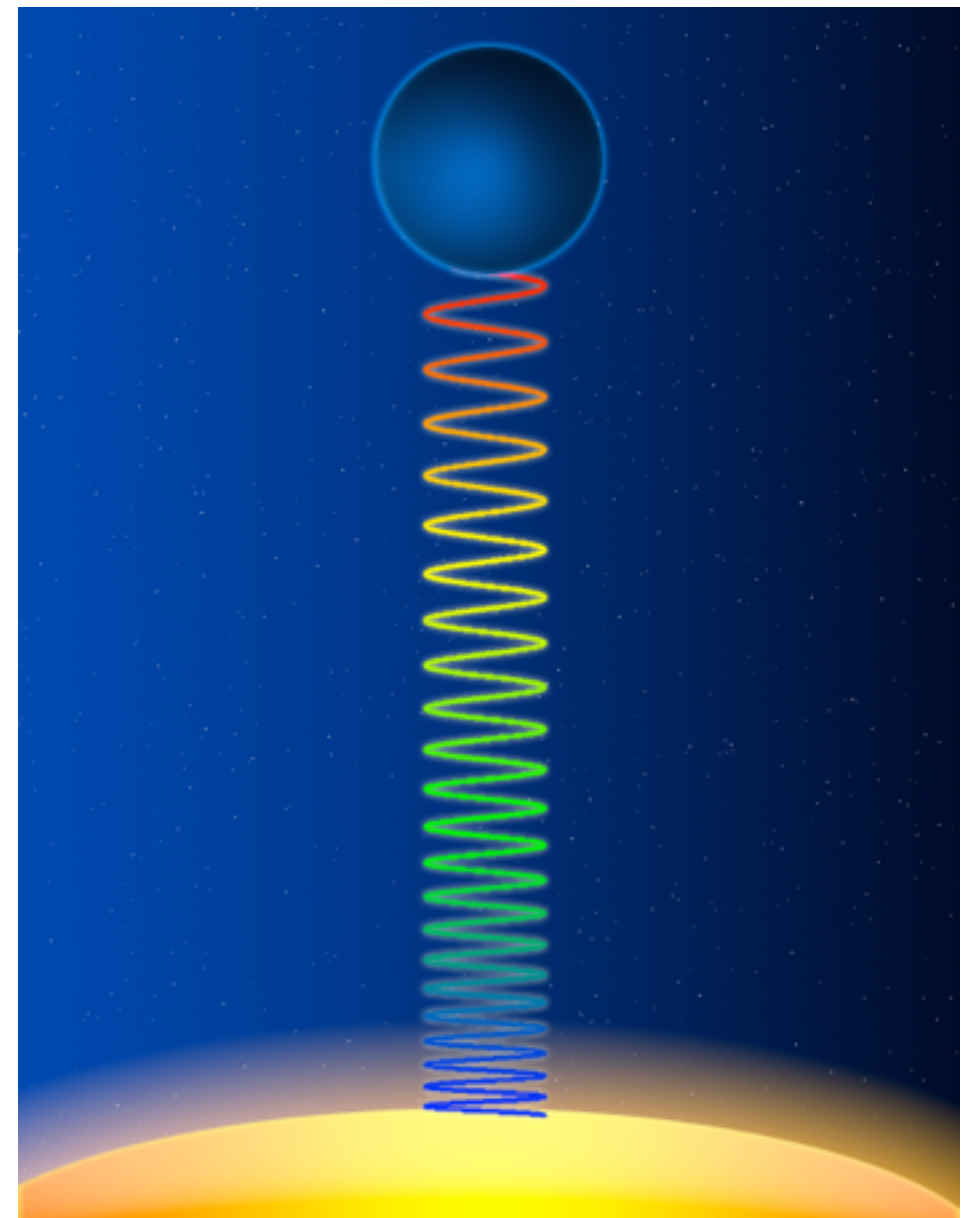
# Peebles ('71) explanation of cosmological redshift

- The redshift  $\lambda_{\text{rec}}/\lambda_{\text{em}}$  is the product of a lot of small shifts between a set of FOs along the look-back path
- In the vicinity of a neighbouring pair of FOs
  - space-time is locally flat, so
  - incremental redshifts are Doppler shifts
- Yields differential equation
  - $d\lambda/\lambda = da/a$  with solution  $\lambda \propto a(t)$
- So fractional change in proper separation is the same as the fractional change in  $\lambda$
- i.e.  $\delta \log(\lambda/D) = 0$



# What about our *lumpy* universe?

- Bondi (1947): Spherical models:
  - for low- $Z$ , redshift is product of Doppler and gravitational redshift
  - also in homogeneous case
- But Synge (1960) shows that *all* redshifts are given by Doppler formula
- “In attributing a *cause* to this spectral shift, one would say .... that the spectral shift was caused by the relative velocity of the source and the observer”.





# Synge, 1960; General Relativity

- Observed (or emitted) energy is dot product of observer 4-velocity and the photon 4-momentum.
- → wavelength shift is given by Doppler's formula with “relative velocity” being the l.o.s. component of the difference of the receiver 4-velocity and a parallel transported version of the emitter 4-velocity
- “Not a gravitational redshift as the Riemann tensor does not appear in formula”
- But parallel transport does know about gravity

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CHRONOMETRY IN SPACE-TIME

[CH. III, § 7

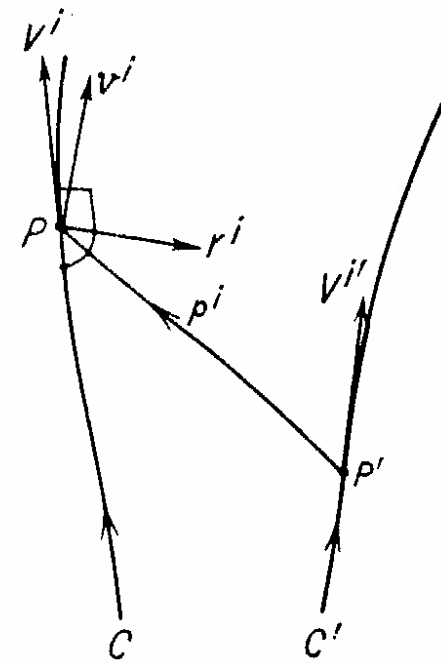


Fig. 10—Relative velocity and Doppler effect (mechanical)

some luminous object, such as a star or planet. We connect them with null geodesics such as  $P'P$ . We cannot immediately compare the 4-velocity  $V^i$  of  $C$  at  $P$  with the 4-velocity  $V'^i$  of  $C'$  at  $P'$ , because they are vectors at different events. The obvious plan is to bring them to a common event by subjecting  $V'^i$  to parallel transport along  $P'P$ ; this gives us at  $P$  the vector

$$v_i = g_{ij} V'^j, \quad (32)$$

where  $g_{ij}$  is the parallel propagator [II-(71)]. Let  $\lambda^i_{(\alpha)}$  be a frame of reference on  $C$  with  $\lambda^i_{(4)} = V^i$ . This might be a Fermi frame, but the question does not arise at the moment, because we are

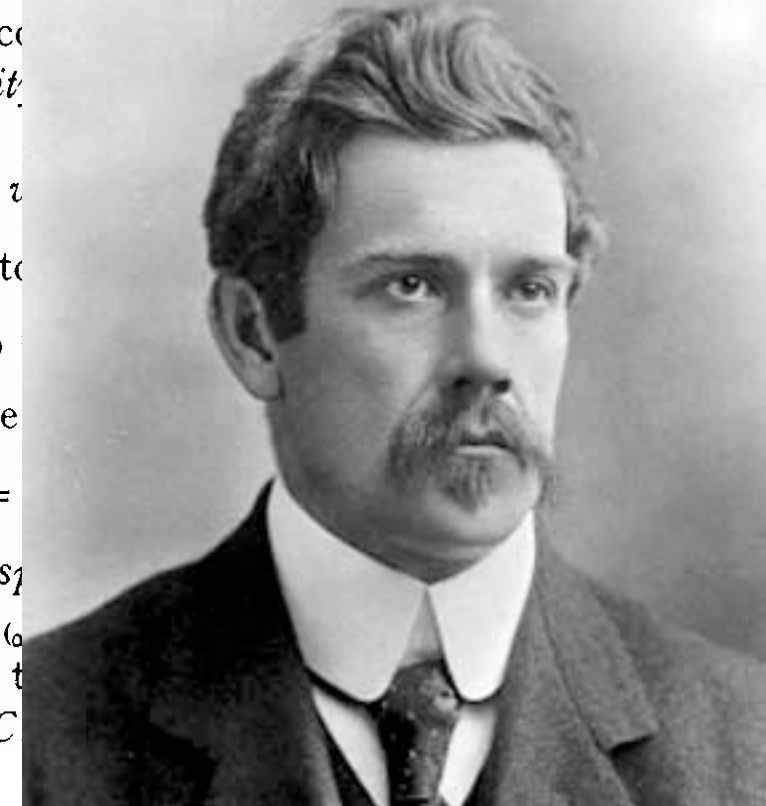
We now define the 3-velocity components

Since  $v^i$  and  $V^i$  are unit vectors

is expressible in terms of the

$$v^{(4)} = -v_{(4)} =$$

We may call  $v$  the *relative speed* all the three components  $v_{(a)}$  parallel for transport along  $P'P$  that  $C'$  is *at rest* relative to  $C$



# Bunn & Hogg, 2009

- Like Peebles they break photon path into a set of intervals
  - set of intervening observers along line of sight
  - local flatness  $\rightarrow$  product of Doppler shifts
- Intervening observers need not be freely falling
- Any incremental shift can be *considered* to be either *Doppler* or *gravitational*
- gravitational redshifts are just Doppler shifts viewed from an unnatural coordinate system - so an enlightened cosmologist would never try to draw any distinction
- All redshifts can (and should!) be considered to be Doppler, or 'kinematic' in nature. (much like Synge)
- Does this mean  $\Delta \ln(\lambda/D) = 0$  is *universal*?



# ideas about redshifts in astronomy - summary

- The redshift of light in cosmology
  - redshift is caused by the expansion of space?
  - standing waves in a cavity
  - Maxwell's equations in expanding space: "Hubble damping"
  - counter arguments (e.g. "how can space expand?")
  - Peebles' picture - lots of little Doppler shifts
- The redshift of light in general
  - Bondi ('47): combination of gravity & Doppler
  - Synge ('60) redshifts "caused by the relative velocity..."
  - Bunn & Hogg ('09): "gravitational redshifts are just Doppler shifts viewed from an unnatural coordinate system"

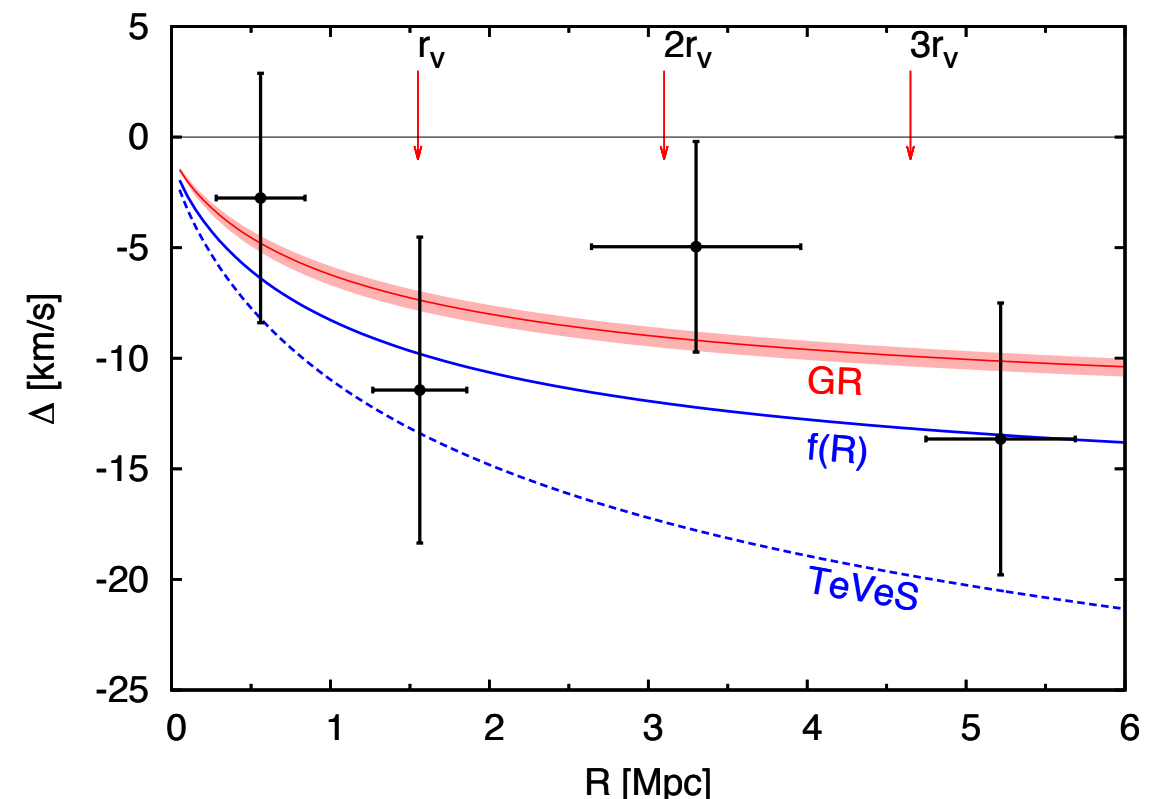
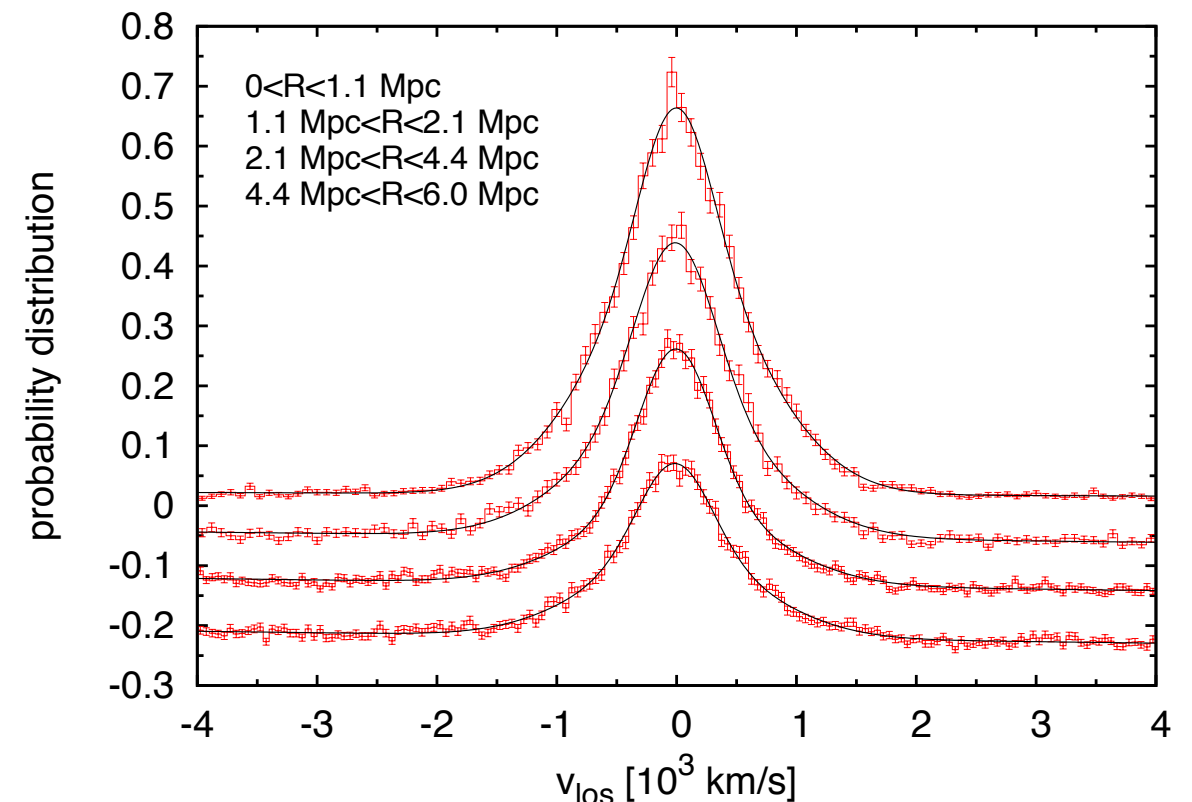


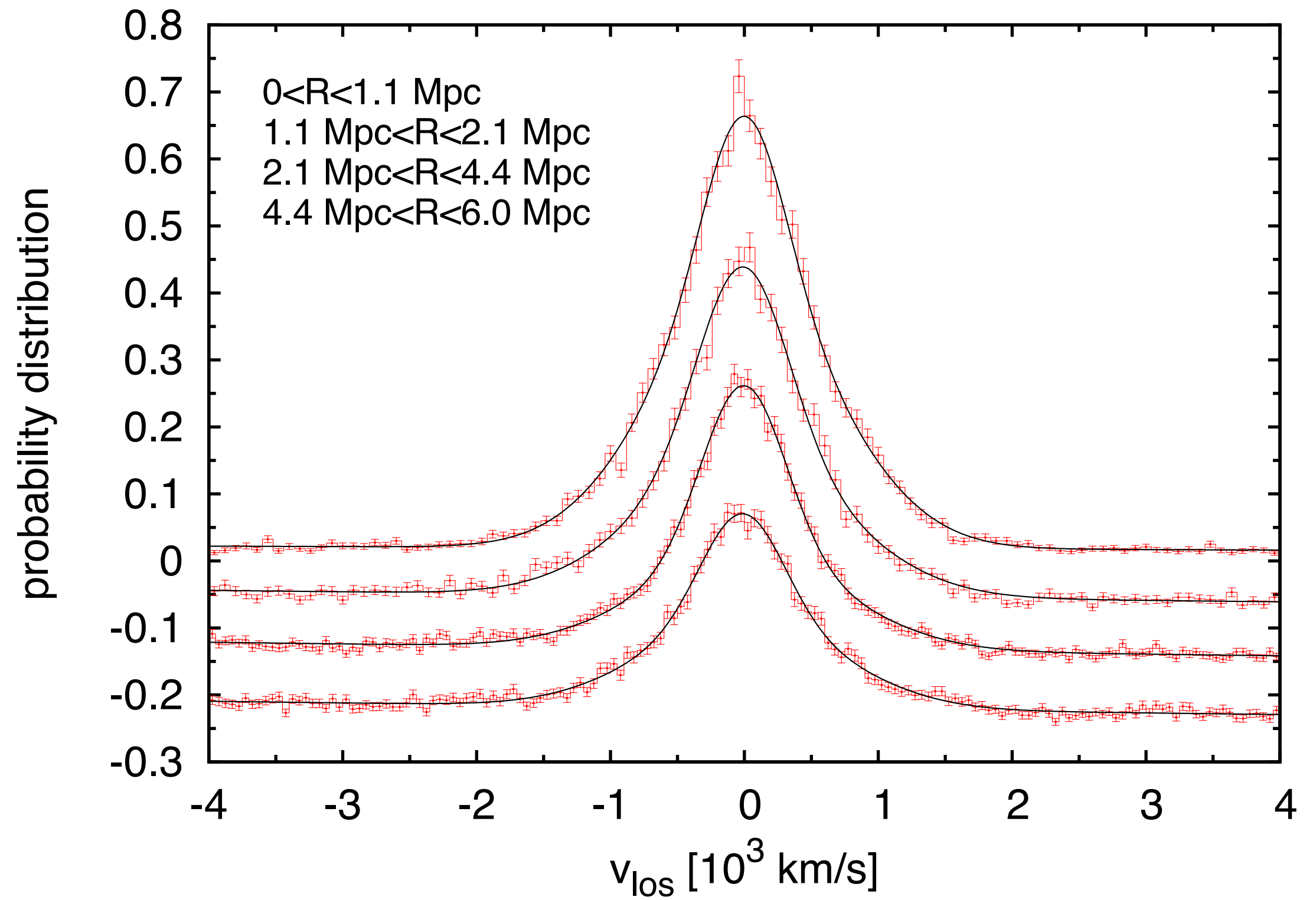
# Interpretation of redshifts: why do we care?

- The equivalence principle:
  - locally gravity does not exist
  - "transformed away" for freely falling observers
  - special relativity rules
- Lesson from cosmology:
  - wavelength is tied to the expansion of space
  - we don't seem to see the effect of gravity
  - redshift appears to be purely kinematic
- The "kinematic picture" may suggest this is general
- But clusters are not expanding
  - why would we see any gravitational redshift?
  - is Einstein/Newton/Pound+Rebka complete?

# Back to the Wojtak *et al.* measurement

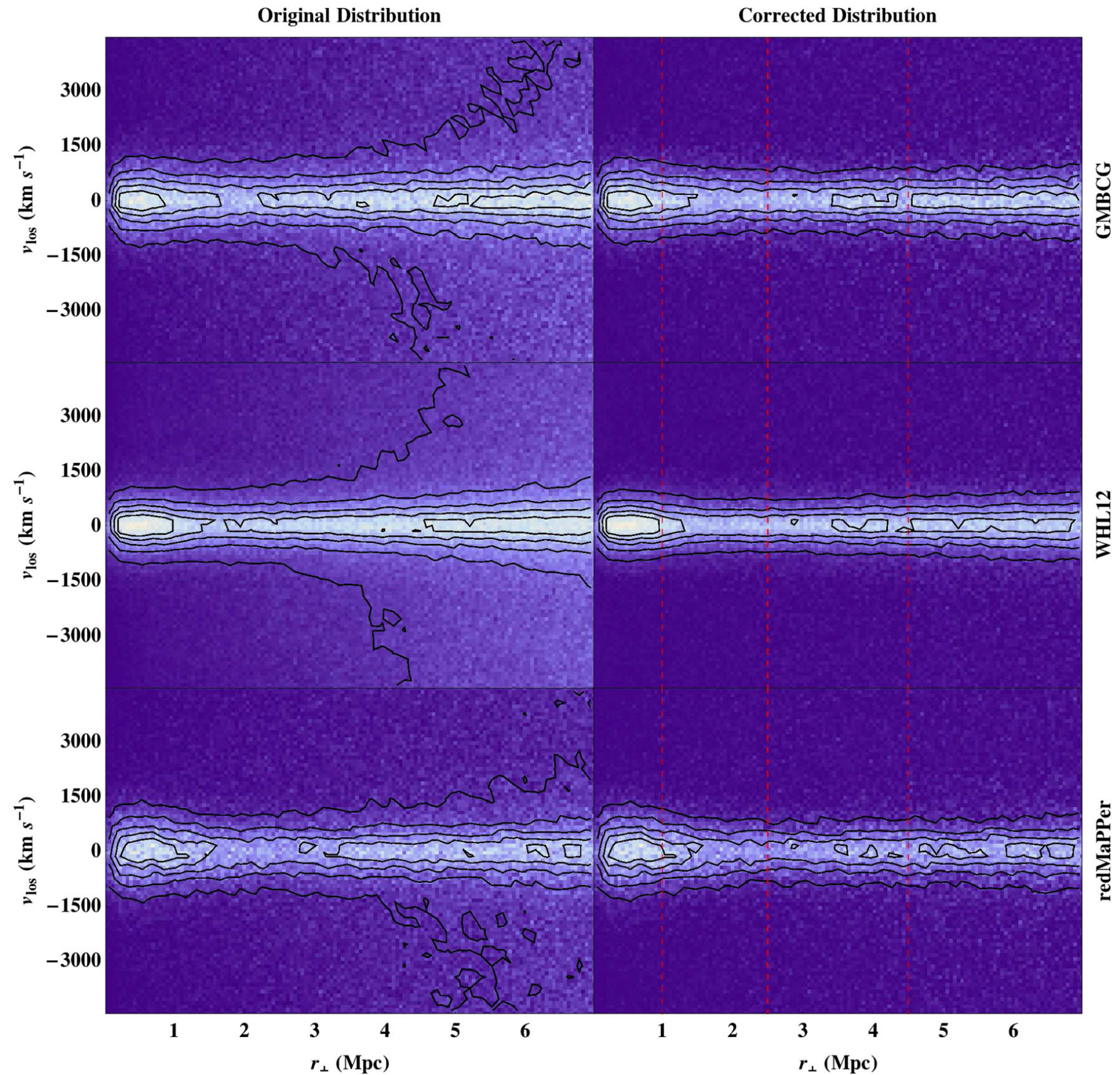
- Gravitational redshift for light climbing out of potential wells of clusters of galaxies
  - Long predicted by theorists
  - perhaps a bit oversimplified
- Now finally measured
  - at  $\sim 2.5$  sigma level
- Claimed to conflict with TeVeS modified gravity
  - descendent of Milgrom theory
- But OK with GR or e.g.  $f(R)$  modifications



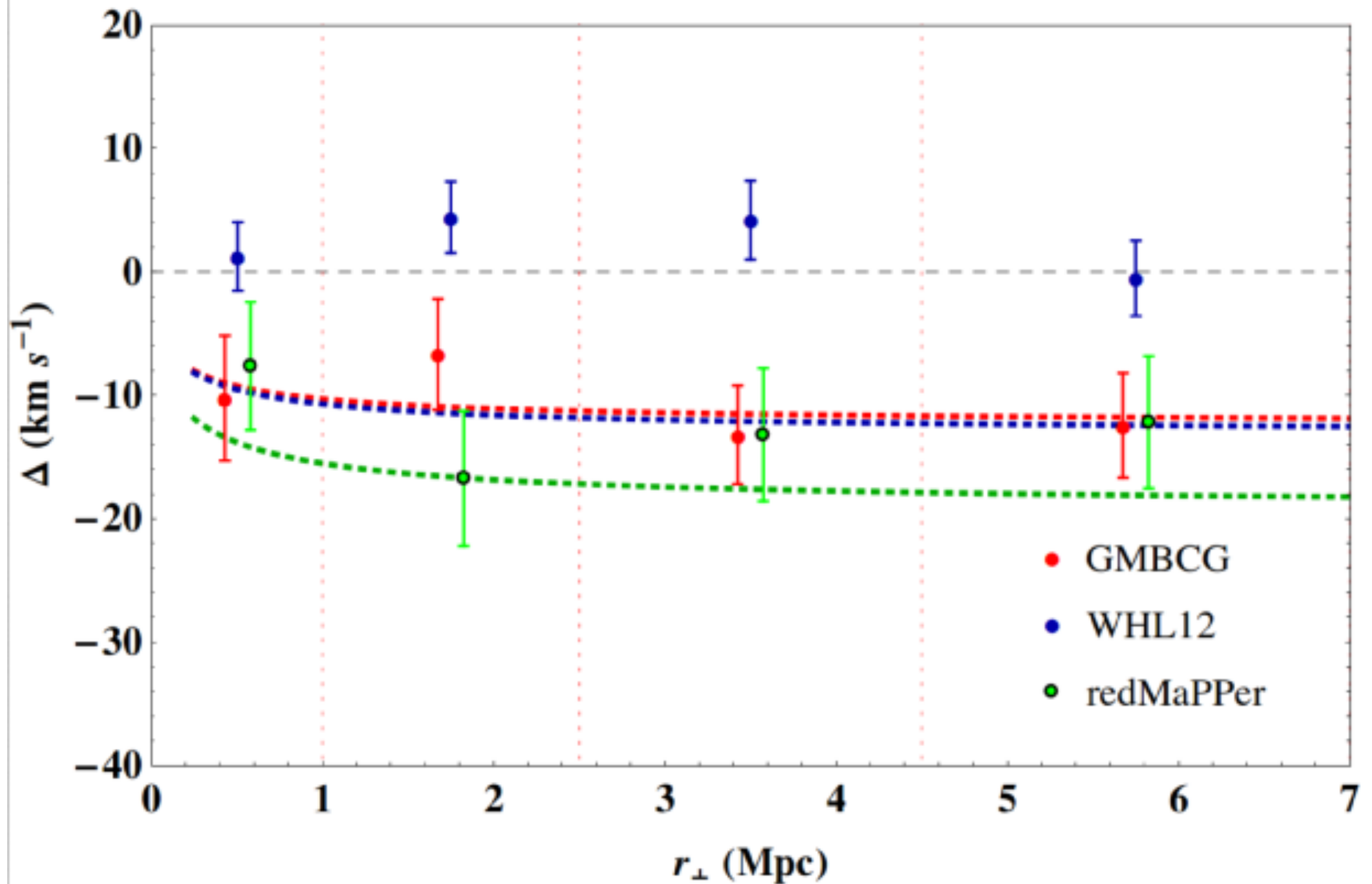




# Jimeno, Broadhurst, Coupon, Imetzu, Lazkos 2015

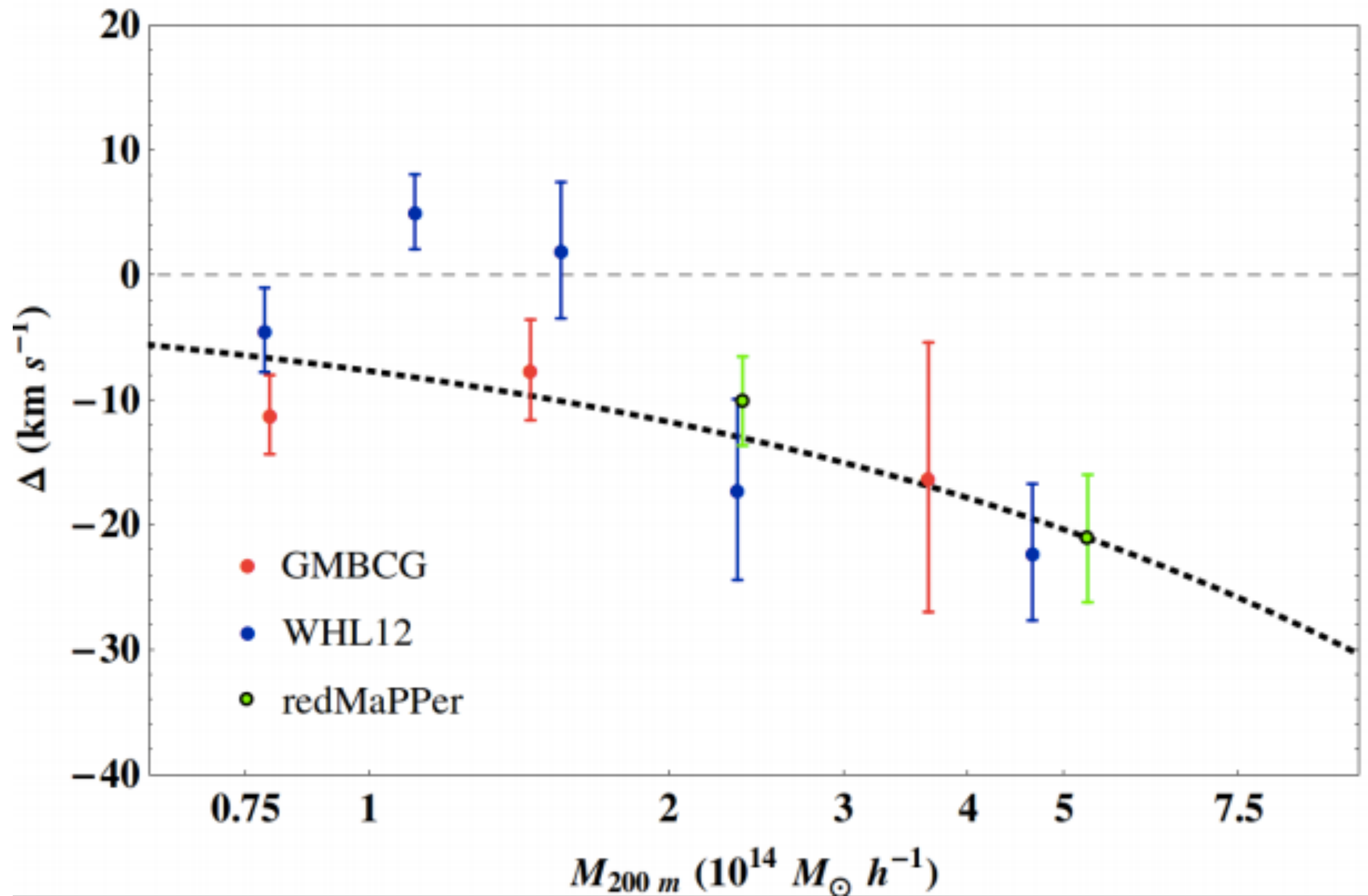


# Jimeno, Broadhurst, Coupon, Imetzu, Lazkos 2015





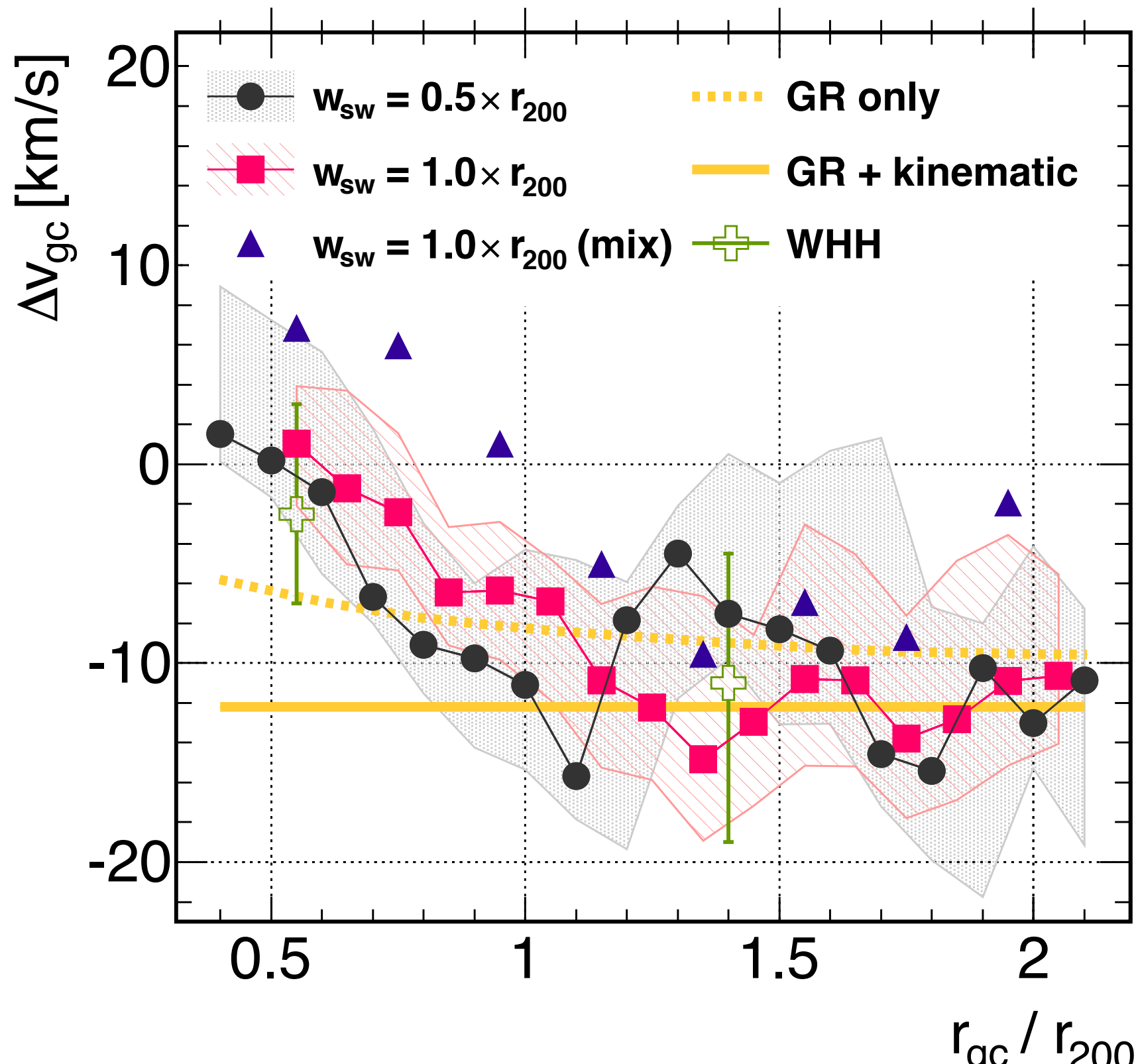
# Jimeno, Broadhurst, Coupon, Imetzu, Lazkos 2015

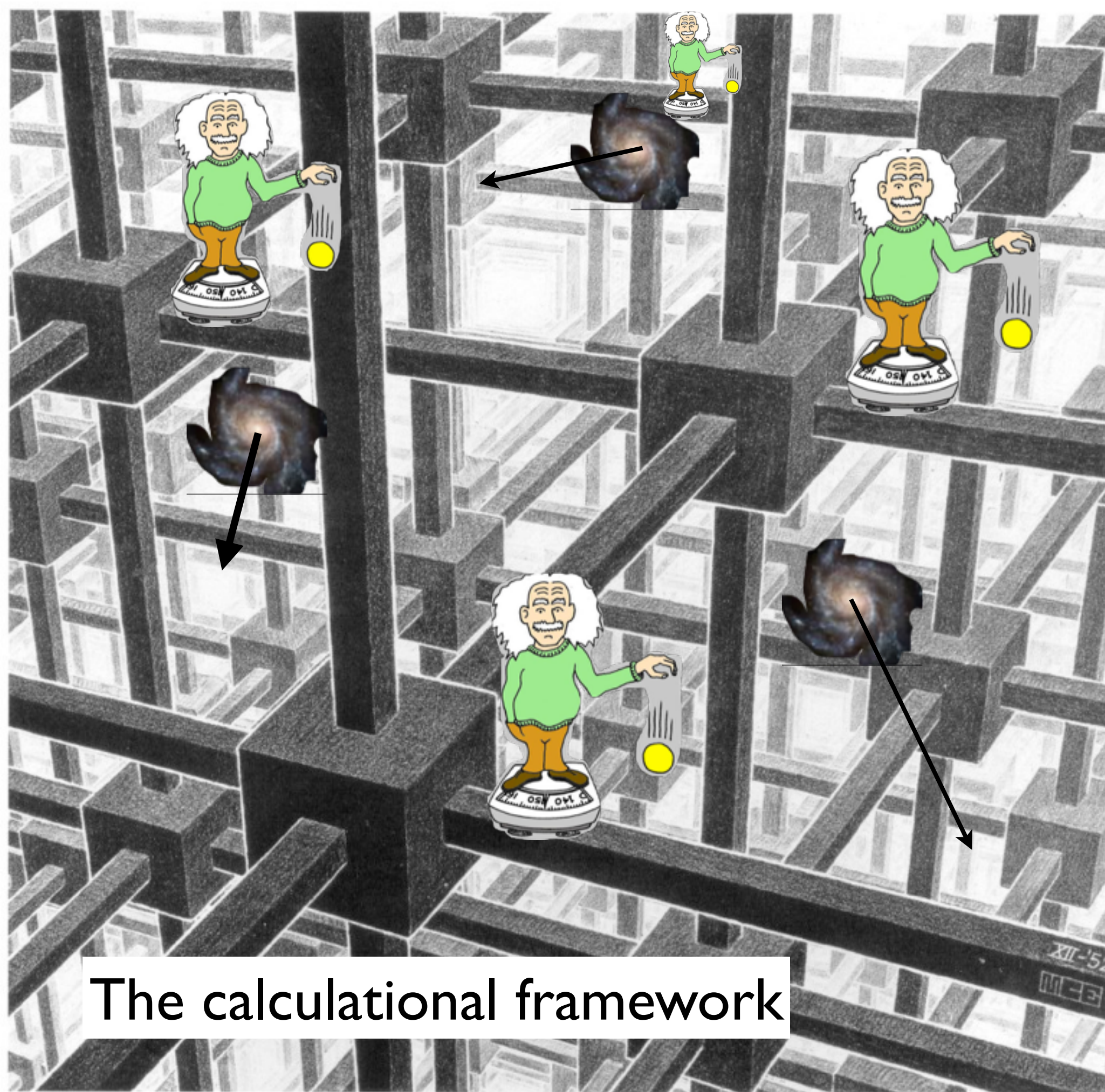




# Sadeh et al 2014

$r_{gc}$  [Mpc] :  $0.5^{+0.2}_{-0.2}$   $1.0^{+0.3}_{-0.2}$   $1.6^{+0.4}_{-0.3}$   $2.1^{+0.5}_{-0.3}$





The calculational framework



- $\delta z$  is not just gravitational redshift
- Sources are moving, so we also see
  - *transverse Doppler effect*:
    - 1st order Doppler effect averages to zero, but....
    - to 2nd order  $\langle \delta z \rangle = \langle v^2/c^2 \rangle / 2$
    - can be understood as time dilation
- Generally of same order of magnitude as gravitational redshift from virial theorem, Jeans eq...
- (And this doesn't really test GR
  - see also Bekenstein & Sanders, 2012
  - more later....)
- Is that the full story?





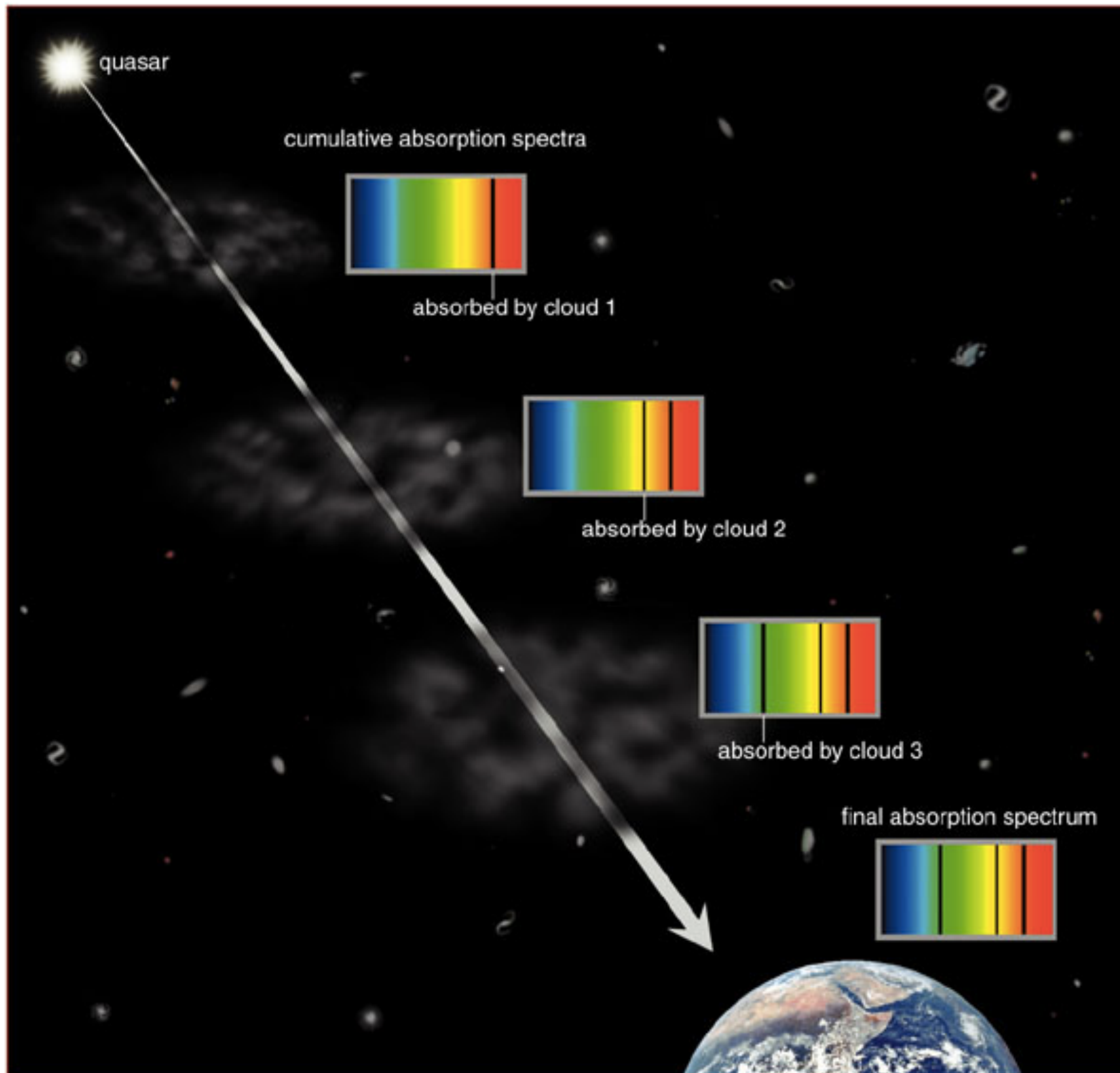
# No - there is another effect of same order

- Light cone effect
  - we will naturally tend to see more objects moving away from us than towards us in any observation made using light as a messenger
  - this gives an extra red-shift effect
  - again of the same order of magnitude as the gravitational redshift

# Light-cone effect

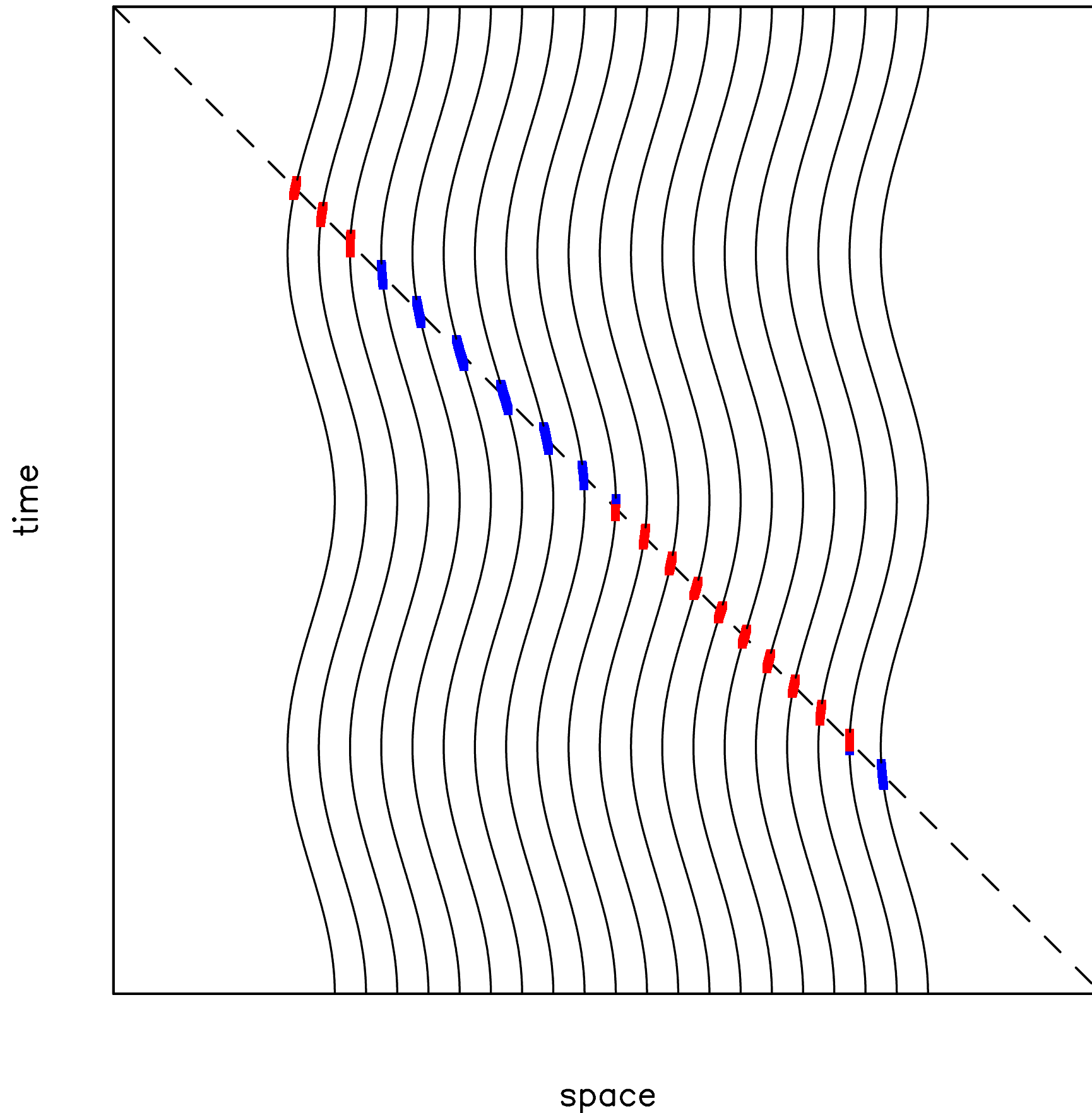
- Light cone effect
  - we will see more galaxies moving away from us in a photograph of a swarm of particles
    - past light cone of event of our observation overtakes more galaxies moving away than coming towards us
    - just as a runner on a trail sees more hikers going the other way...
    - So *not* Lorentz-Fitzgerald contraction effect
  - phase space density contains a factor  $(1-v/c)$
  - $\langle \delta z \rangle = \langle (v_{los}/c)^2 \rangle$
  - same sign as TD effect
    - 2/3 magnitude (for isotropic orbits)

# Quasar absorption lines



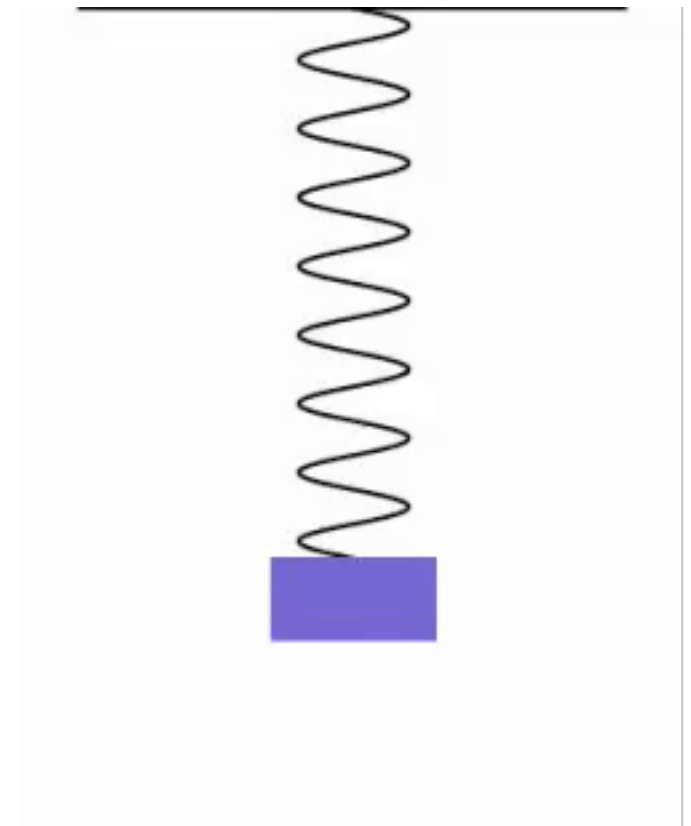


# Light-cone effect - more particles moving away!



# Another way to look at LC effect

- Particle oscillating in a pig-trough
  - $r(t) = a \cos(\omega t + \varphi)$
  - $v(t)/c = -(a\omega/c) \sin(\omega t + \varphi)$
  - $v(t)$  averages to zero
    - average could be over phase or time
- but  $v_{\text{obs}} = v + (r/c) dv/dt + \dots$ 
  - where  $r/c$  is the look-back time
  - and the extra term does *not* average to zero
- ~ same as Einstein prediction for Pound & Rebka
  - $\delta z \approx \langle r dv/dt \rangle / c^2$ .



# Why is the transverse Doppler effect a redshift?

- Transverse Doppler redshift effect:
  - first order Doppler shift  $\sim v/c$  is large but averages to zero
  - residual is a quadratic  $\sim (v/c)^2$  effect which caused randomly moving objects appear redshifted on average
  - can also be understood as a time dilation effect
- But moving objects have more energy per unit mass (in the observer frame)
- So if they convert their rest mass to photons we should see a blue-shift of photons on average

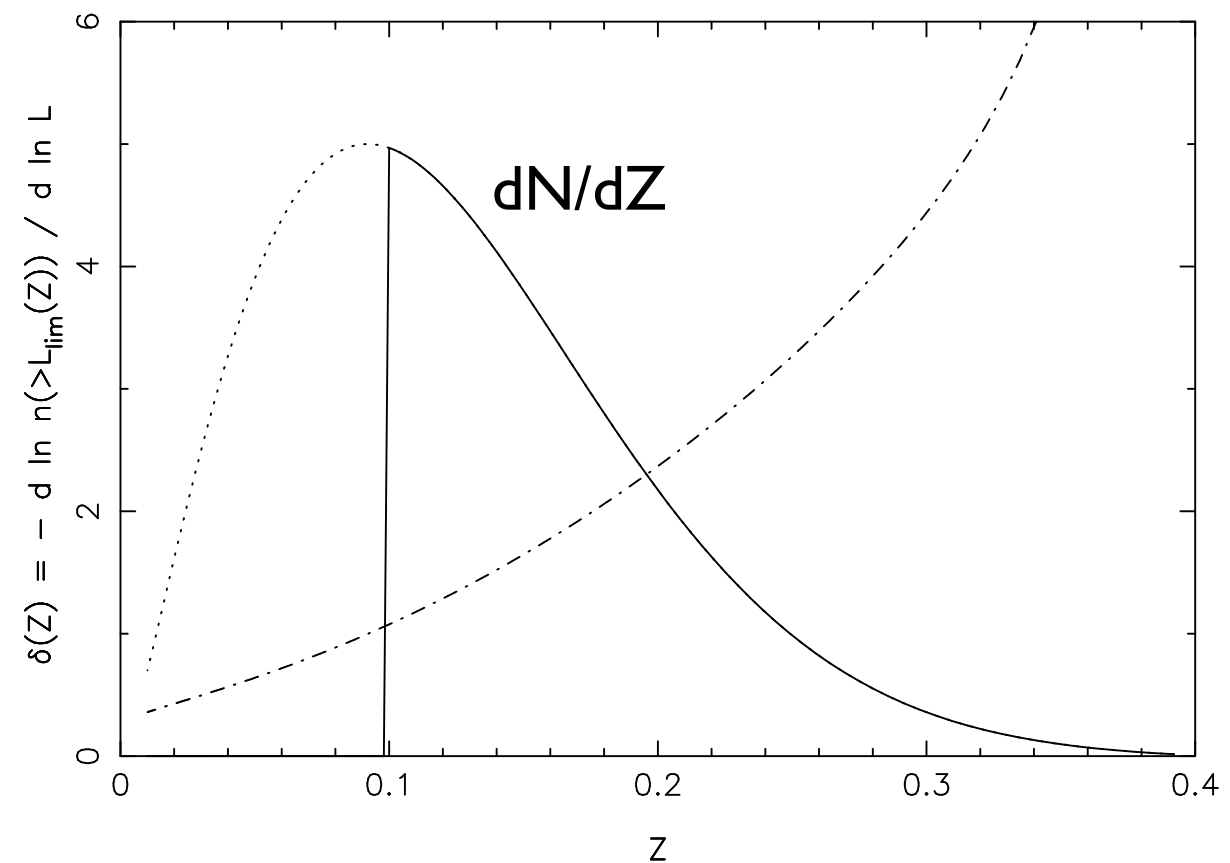
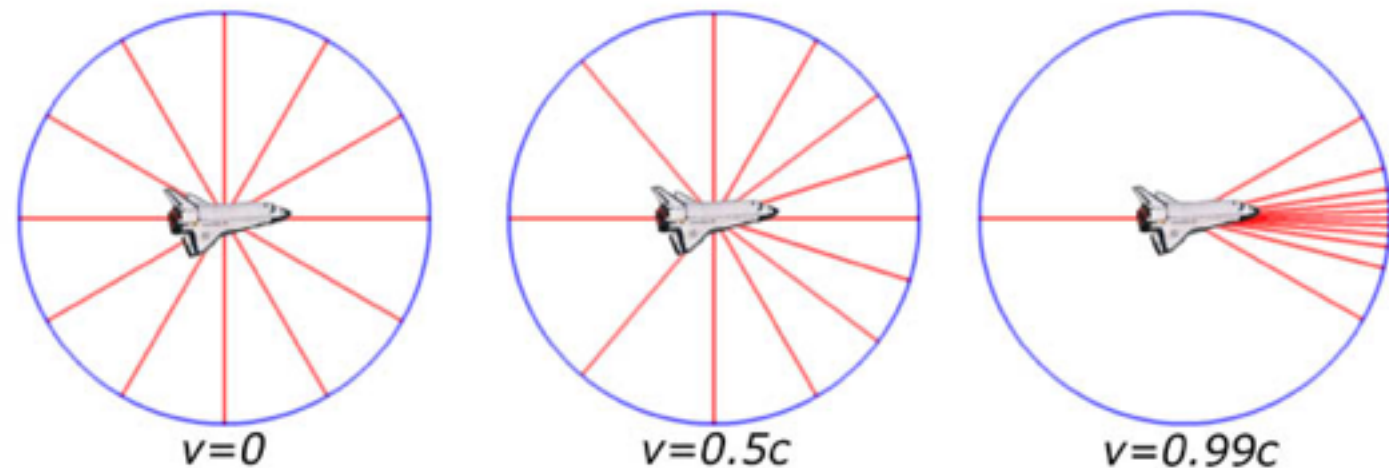


# Transverse Doppler Effect: Blue is the new Red

- Moving objects appear redshifted on average
- But moving objects have more energy per unit mass (in the observer frame)
- So photons should be blue-shifted on average
- Averaging over objects vs averaging over photons
  - e.g. unresolved objects => blue shift (e.g. BCG or low resolution HI of total cluster  $z$ )
  - here we have a hybrid situation:
    - redshifts measured for objects
    - but objects are selected according to flux density
    - surface brightness modulation => net blueshift

# Surface brightness modulation

- Line of sight velocity changes surface brightness
- relativistic beaming (aberration) plus change of frequency
- but doesn't change the surface area
- so velocities modulate luminosity
  - depends on SED:  $\delta L/L = (3 + \alpha)v/c$
  - $\alpha \approx 2$ , so big amplification
  - spectroscopic sample is flux limited at  $r=17.8$
  - $\Delta n/n = -d \ln n(>L_{\text{lim}}(Z))/d \ln L * \Delta L/L$
- opposite sign to LC,TD effects, but larger because the sample here is limited to bright end of the luminosity function



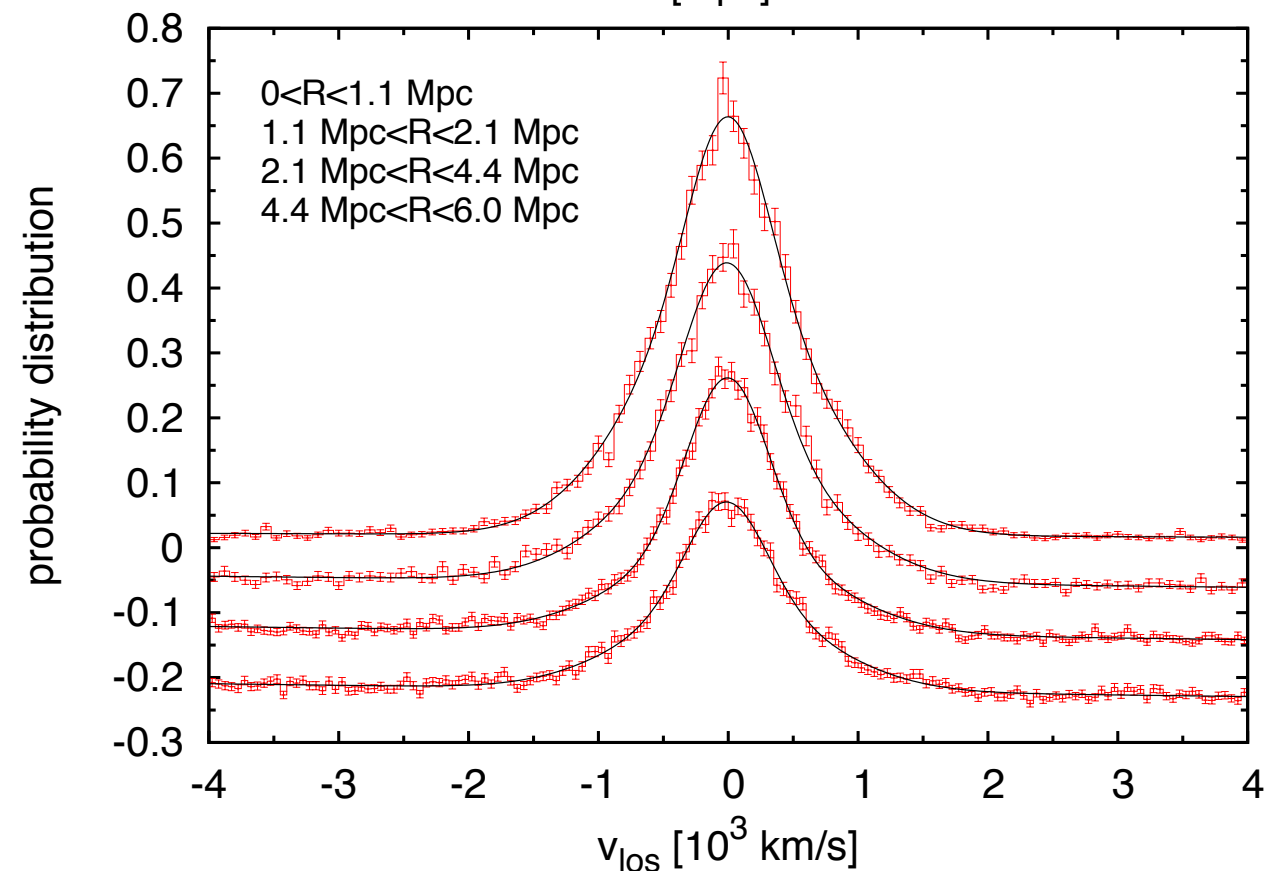
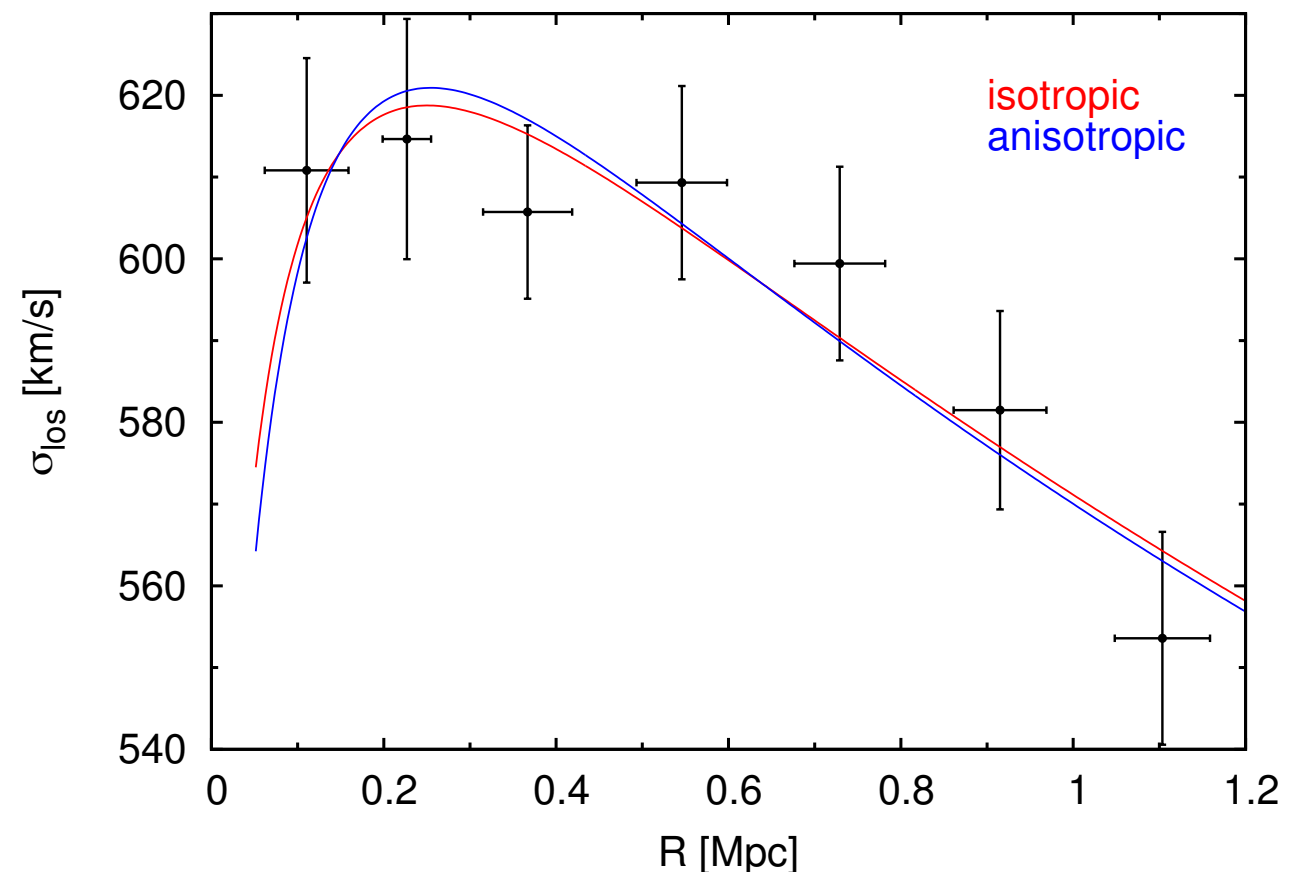
# More implications of the transverse Doppler red/blue-shift dichotomy

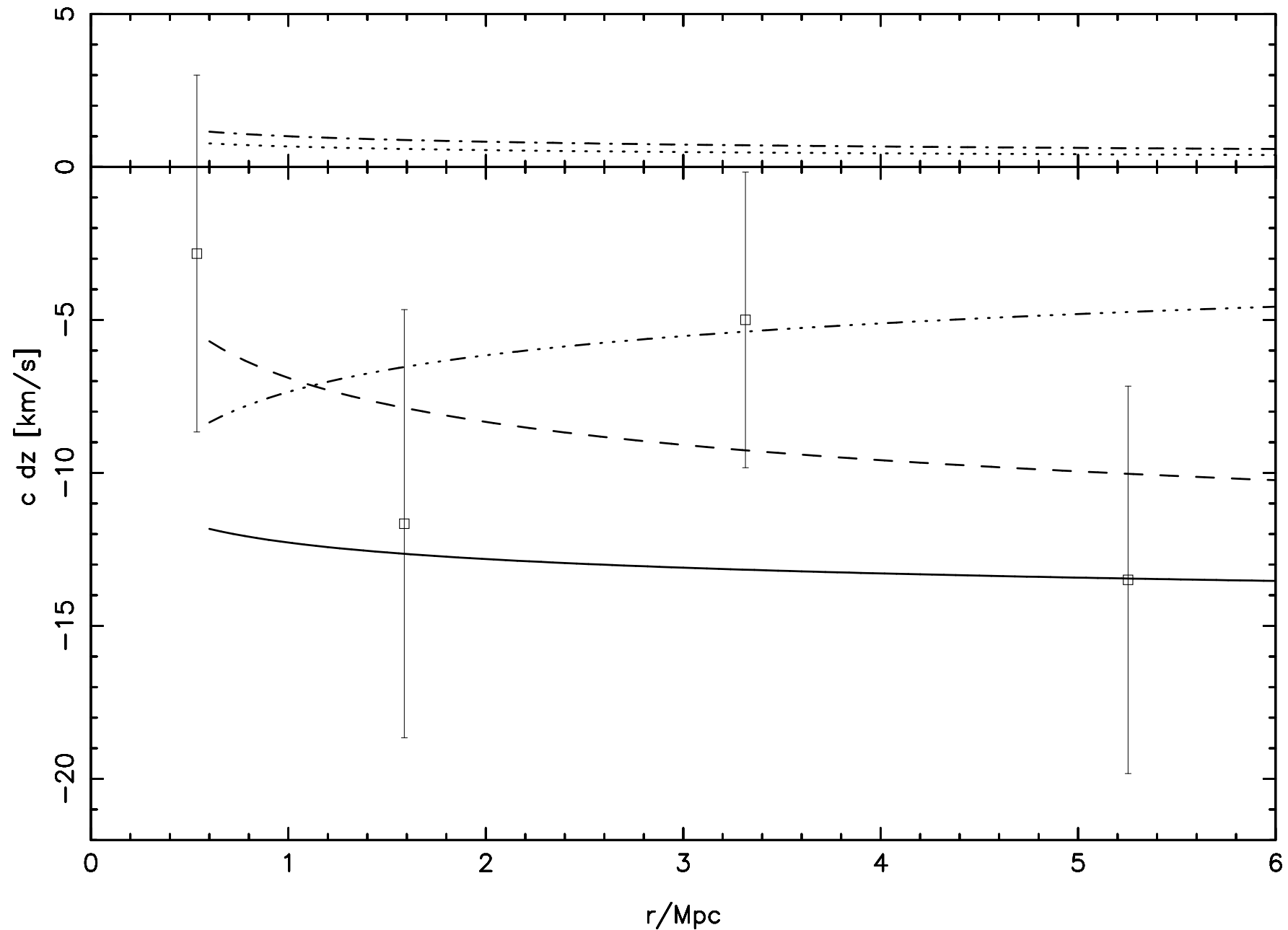
- Contribution to cluster grav- $Z$  from motions of stars in the BCG
  - velocity dispersions are smaller than in cluster, but not negligible
  - stars are unresolved so we get a transverse Doppler blue-shift
- 21cm radio observations of galaxies
  - sees mostly galaxies falling into cluster for first time as gas is stripped within virial region
  - should have a large potential difference relative to BCG
  - but the prediction for  $\delta Z$  is highly dependent on whether one makes unresolved single dish (e.g. Aricebo) measurements or resolved (e.g. Westerbork, ASKAP)



# Corrected grav-z measurement

- Fairly easy to correct for TD +LC+SB effects
- TD depends on vel. disp. anisotropy
- LC+SB directly measured
- net effect is a blue-shift
  - $\sim -9\text{km/s}$  in centre, falling to  $\sim -6\text{km/s}$  at larger  $r$
- minor effects from infall/outflow velocity
- Substantial change in measured grav-z term
  - but still consistent with dynamical mass estimate





**Figure 3.** Data points from figure 2 of WHH and prediction based on mass-traces-light cluster halo profile and measured velocity dispersions as described in the main text. The dashed line is the gravitational redshift prediction, which is similar to the WHH model prediction. The dot-dash line is the transverse Doppler effect. The dotted line is the LC effect. The triple dot-dash line is the surface brightness effect. The solid curve is the combined effect.

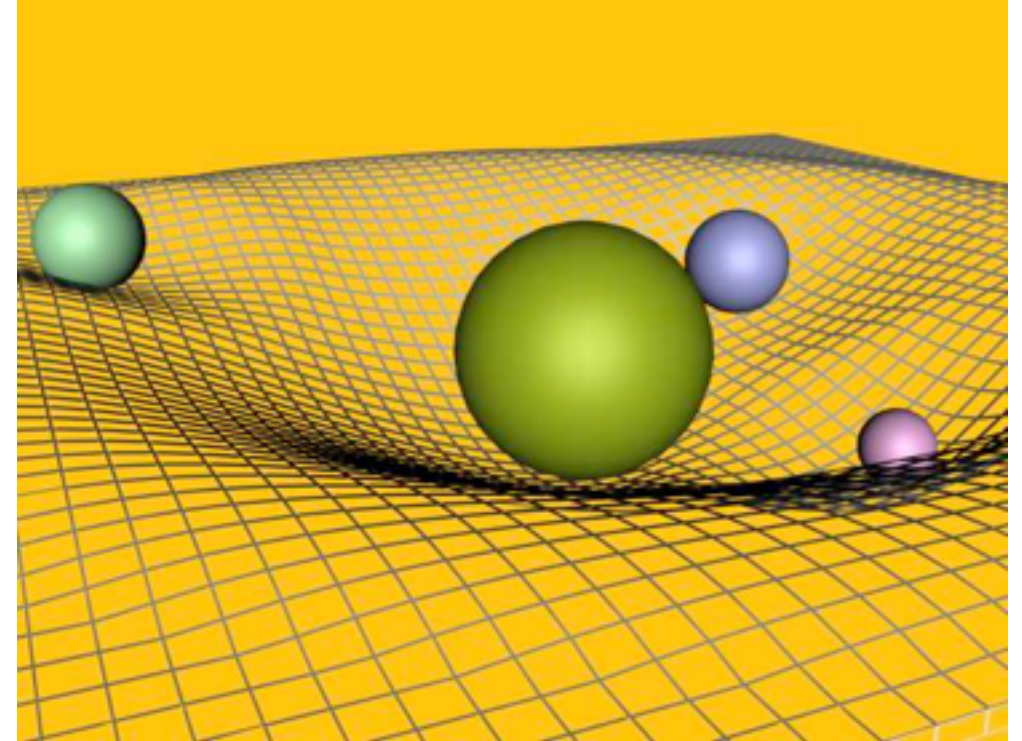
# What does it all mean?

- Effect is very small - and hard to measure
  - measuring 10km/s mean offset with 600km/s velocity dispersion is truly impressive
  - requires careful modeling of background & cluster velocity distribution function  $f(v)$
  - and predicting potential from kinematics is not trivial
    - rather sensitive to assumed velocity distribution for the brightest cluster galaxies (BCGs) used as centres
- but it probably is a real measurement of gravitational redshift (+ special relativistic nuisance factors)



# What else does it mean?

- Probe of curvature of space in GR?
  - matter tells space how to curve
  - space tells matter how to move....
- Sadly no....
- Effect does not rule out any most metric theories of gravity
  - motion non-relativistic matter & gravitational redshifts are determined only by ' $g_{tt}$ '; the spatial metric is irrelevant
- It is really a test of the equivalence principle
- Provides a test of theories that invoke long-range non-gravitational forces in the “dark sector”
  - e.g. Gradwohl & Frieman 1992; Farrar & Peebles 2004; Farrar & Rosen 2007; Keselman, Nusser & Peebles 2010; and many, many more.... and (maybe)  $f(R)$  gravity.
  - though such theories are already constrained by X-ray temp. vs galaxy motions in clusters....



# Future prospects...

- Can expect immediate improvements in measurement
  - 3x increase in number of redshifts available (BOSS)
  - and more to come:
    - optical: big-BOSS
    - radio: FAST, ASKAP-Wallaby+WNSHS
    - interesting to compare unresolved radio and optical
- Extension to larger scales....
- Lots of rich material in the front-back asymmetry of the galaxy correlation function.

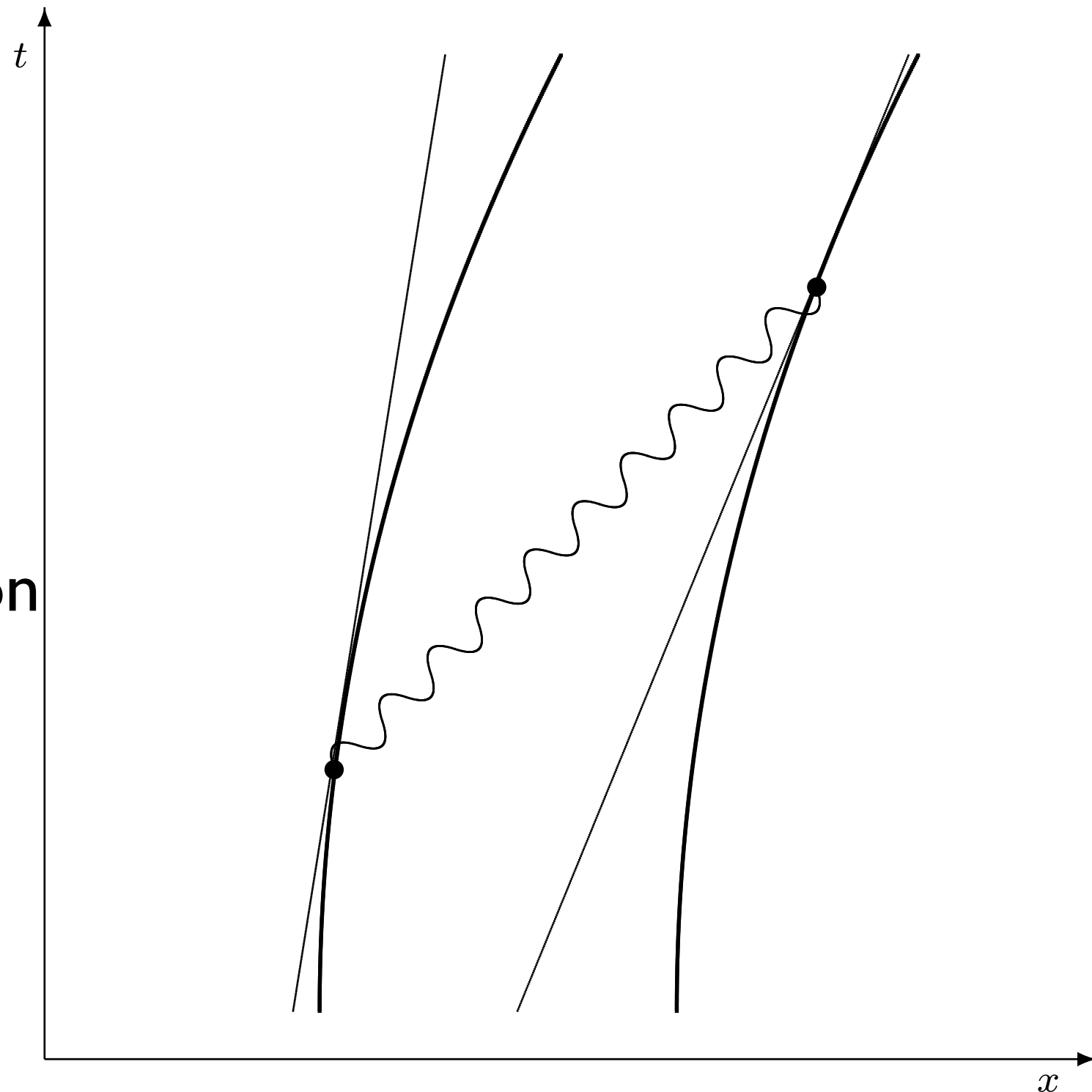
# What's wrong with the "kinematic picture"?

- Recall Bunn & Hogg: "A gravitational redshift is just a Doppler shift viewed from an unnatural coordinate system"
- Confusion of gravity and acceleration
- Einstein rocket thought experiment:
  - the redshift caused by rocket motor is just a Doppler shift
- From this perspective Pound & Rebka did not measure a gravitational redshift; they measured the non-gravitational redshift caused by the laboratory being accelerated
- In GR the gravitational field is the Riemann tensor
  - or the tidal field in the Newtonian limit
- So is there a truly gravitational component to the redshift?



# Are gravitational and Doppler shifts the same?

- Heavy lines are pair of accelerated observers
- with same constant acceleration
- Light lines are a pair of freely falling observers
- These pairs perceive the same redshift for the photon (wiggly line)
- Redshift only depends on instantaneous velocity, not on the path before or after the interaction event.
- But is this gravity?

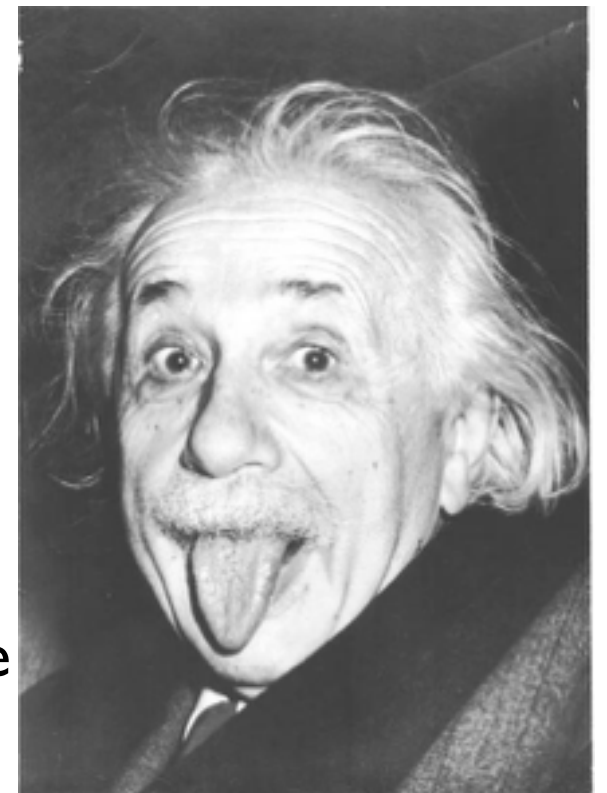


# Equivalence principle & the Pound + Rebka experiment



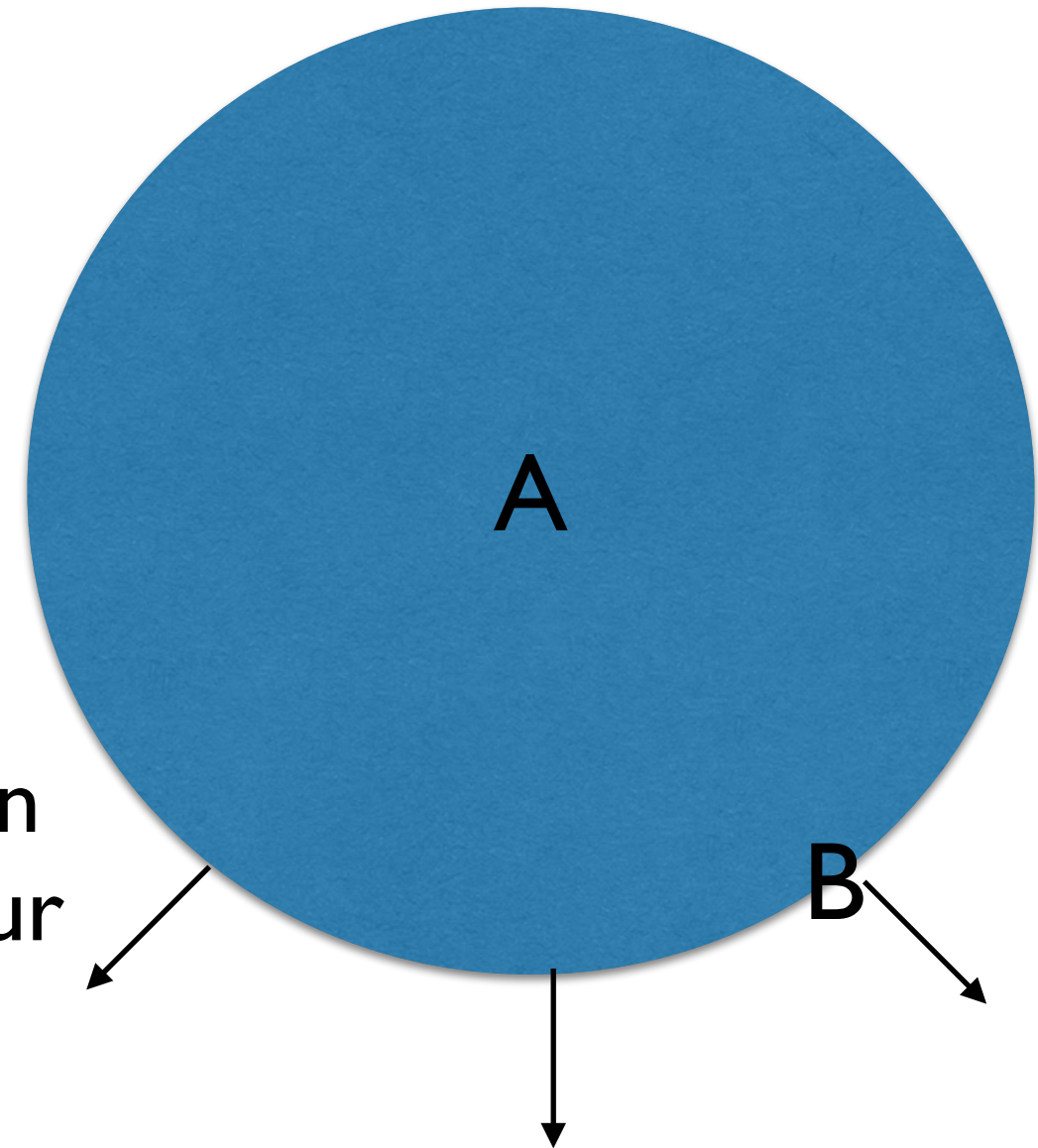
$$\left(\frac{\Delta E}{E}\right)_{\text{down}} - \left(\frac{\Delta E}{E}\right)_{\text{up}} = (5.1 \pm 0.5) \times 10^{-15}$$

- Einstein's Equivalence Principle: Observers on earth (being accelerated by the stress in the ground under them imparting momentum to them) will see light being red-shifted (and all other local physics being modified) exactly as would a pair of astronauts in empty space being accelerated by a rocket motor.
- Pound and Rebka (1959, 1960): He was right.
- But if you replace non-inertial apparatus by freely falling kit with same instantaneous velocities then B&H say Doppler formula will apply. They are right too - almost exactly....



# Why is the gravitational-z hidden in cosmology?

- Cosmological redshift a la Bondi
- Consider expanding sphere of dust and source A sending photon to receiver B
- Photon suffers gravitational red-shift climbing up the potential and then a Doppler red-shift on reception
- For source B sending to A the photon has a Doppler red-shift (as seen in our frame) then enjoys a gravitational blue-shift
- But the net effect is the same.
- The opposite gravitational shifts are cancelled by the Doppler shift change





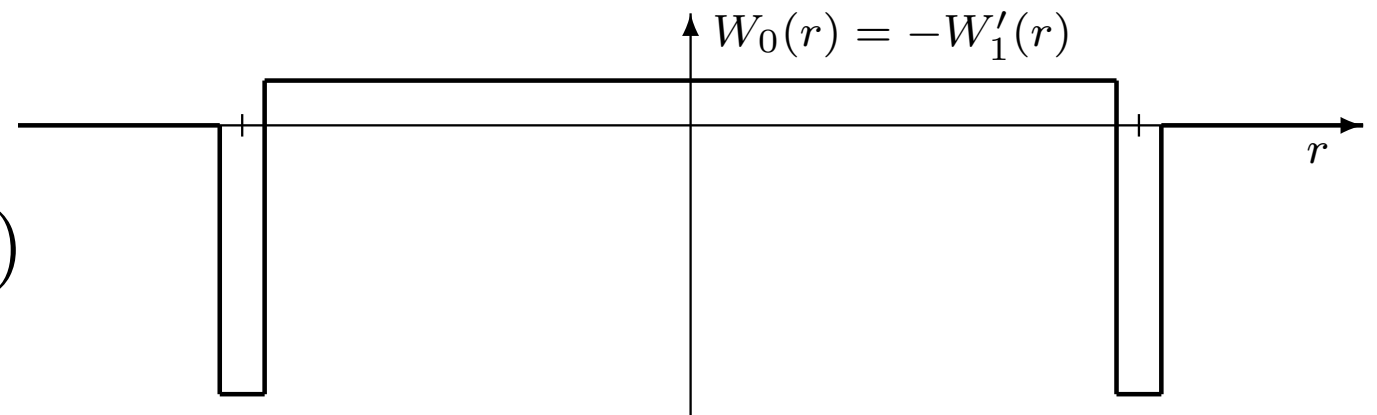
# Whereas in general.....

- Consider pair of freely-falling observers 1,2 in arbitrary gravitational field who exchange a photon.
- Use rigid, non-rotating lattice picture to calculate changes in wavelength and proper separation (work in CoM frame)
  - work to 2nd order in  $v/c$  and 1st order in  $\varphi/c^2$
- $\Delta\lambda/\lambda = \mathbf{n} \cdot (\mathbf{v}_1 - \mathbf{v}_2)_{t1} / c + \int d\mathbf{r} \cdot (\mathbf{g}_2 - \mathbf{g}(\mathbf{r})) / c^2 \quad (1)$
- $\Delta D/D = \mathbf{n} \cdot (\mathbf{v}_1 - \mathbf{v}_2)_{t1} / c + \Delta\mathbf{r} \cdot (\mathbf{g}_2 - \mathbf{g}_1) / 2c^2 \quad (2)$
- Both are 1st order Doppler (with initial  $\Delta v$ ) plus 'tidal' term
- Spatially constant tidal field stretches  $\lambda$  just like  $D$ 
  - includes Minkowski spacetime and FRW
    - but that's because of special *symmetry* of FRW
  - does not apply for a galaxy cluster
- extra intrinsically gravitational term (gradient of tide)

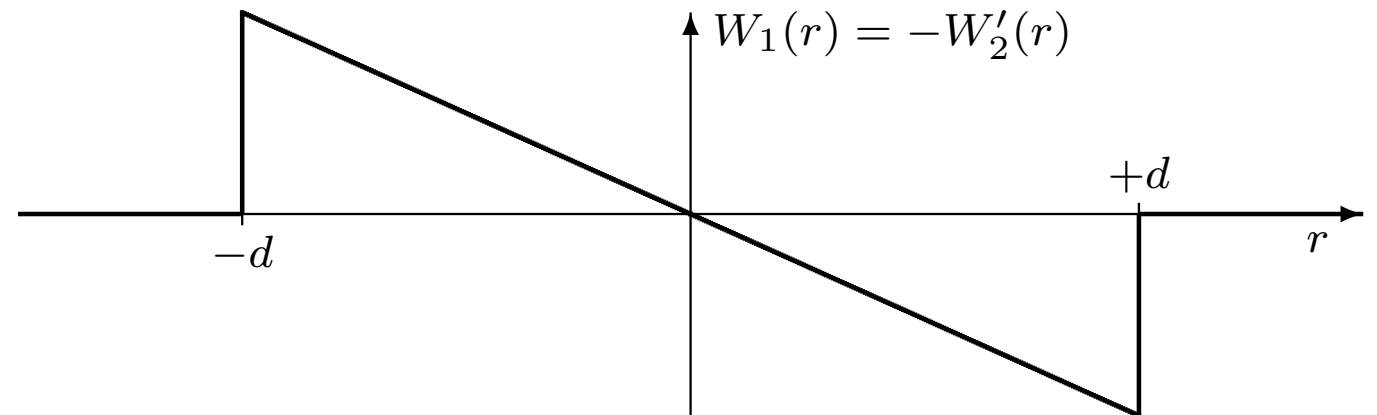
Subtracting (1) - (2) gives

$$\Delta \log(\lambda/D) = \frac{1}{c^2} \left( d \mathbf{n} \cdot (\mathbf{g}_1 + \mathbf{g}_2) - \int d\mathbf{r} \cdot \mathbf{g} \right) \quad \text{or}$$

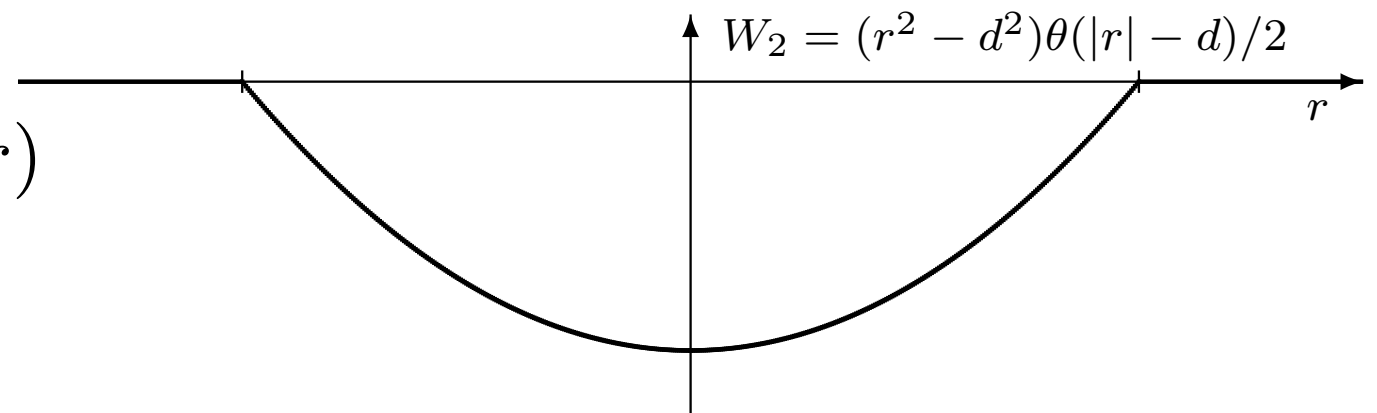
$$\Delta \log(\lambda/D) = \frac{1}{c^2} \int dr W_0(r) \phi'(r)$$



$$\Delta \log(\lambda/D) = \frac{1}{c^2} \int dr W_1(r) \phi''(r)$$



$$\Delta \log(\lambda/D) = \frac{1}{c^2} \int dr W_2(r) \phi'''(r)$$



There is a non-kinematic component of the redshift: it is a measurement of the gradient of the tide

# Conclusions

- Gravitational redshifts in clusters of galaxies have been measured!
- Technically challenging but apparently real and prospects for better measurements and extension to larger scales is promising.
- Potentially useful for testing some alternatives to GR
- But mainly interesting as a "sand-box" that illustrates some subtleties of simple special relativity + Newtonian gravity
- Effect raises some questions of principle about how to think about redshifts in cosmology and astronomy in general.
- Redshifts are not entirely kinematic - there is an truly gravitational component - but it is hidden in cosmology





# A strange incident in the history of physics (C. Moller, 1967)

- 1905 - Einstein establishes SR
- By 1909, Planck, Einstein, Pauli all concluded that temperature of a moving body is  $T(\text{rest frame}) / \gamma$
- Enshrined in text books (e.g. Tolman) and there it rested
- until '60s, when Ott (1963) and Arzelies (1965) turned it all around  $T = \gamma T(\text{rest frame})$
- much confusion ensued
  - P.T. Landsberg (2 Nature articles, '66, 67) "Does a moving body appear cool" (ans: no!)
  - largely clarified by Kibble, '66: Ott, Arzelies were right!
- issue reverberates to this day:
  - Dunkel, Haenggi, & Hilbert 2009 - light-cone effect
- But now seems anachronistic....

# Relativistic Transformation Laws for Thermodynamic Variables (\*).

T. W. B. KIBBLE

*Department of Physics, Imperial College - London*

(ricevuto il 18 Ottobre 1965)

## 1. — Introduction.

ARZELIÈS <sup>(1)</sup> and GAMBÀ <sup>(2)</sup> have recently suggested that the hitherto generally accepted transformation laws of relativistic thermodynamics <sup>(3)</sup> are incorrect, and in need of revision. Superficially their results resemble those of OTT <sup>(4)</sup>, inasmuch as all three authors advocate the same transformation laws for temperature and heat transfer, namely

$$(1) \quad T = T_0(1 - \beta^2)^{-\frac{1}{2}}, \quad dQ = dQ_0(1 - \beta^2)^{-\frac{1}{2}}.$$

However, they do not agree about the transformation laws for certain other thermodynamic variables, notably the internal energy of the system. It is the purpose of this note to point out that, although the formulae (1) are unexceptionable, the arguments presented by ARZELIÈS and GAMBÀ are wrong, particularly in respect of the transformation laws of energy and work. Our results are in complete agreement with those of Ott. We shall also show that for very similar reasons the treatment of the problem of the stressed lever given by ARZELIÈS <sup>(5)</sup> is erroneous, and that the conventional solution to this problem <sup>(6)</sup> is in fact perfectly correct.

We begin, in Sect. 2, with some general remarks on the relativistic transformation laws of thermodynamic variables. Then in Sect. 3 we discuss the specific case of the total energy, illustrating our remarks with the example of cavity radiation discussed by GAMBÀ. In Sect. 4 we consider the work done in changing the volume or pressure of the system. Finally, Sect. 5 is devoted to a demonstration of the necessity for, and physical reality of, the kind of energy flux which plays a crucial

(\*) The Research reported in this document has been sponsored in part by the Air Force Office of Scientific Research OAR through the European Office Aerospace Research, United States Air Force.

<sup>(1)</sup> H. ARZELIÈS: *Nuovo Cimento*, **35**, 792 (1965).

<sup>(2)</sup> A. GAMBÀ: *Nuovo Cimento*, **37**, 1792 (1965).

<sup>(3)</sup> See for example, W. PAULI: *Theory of Relativity* (London, 1958), p. 154.

<sup>(4)</sup> H. OTT: *Zeits. f. Phys.*, **175**, 70 (1963).

<sup>(5)</sup> H. ARZELIÈS: *Nuovo Cimento*, **35**, 783 (1965).

<sup>(6)</sup> See for example R. C. TOLMAN: *Relativity, Thermodynamics and Cosmology* (Oxford, 1950), p. 79.

# Biro & Van 2010

## ABOUT THE TEMPERATURE OF MOVING BODIES

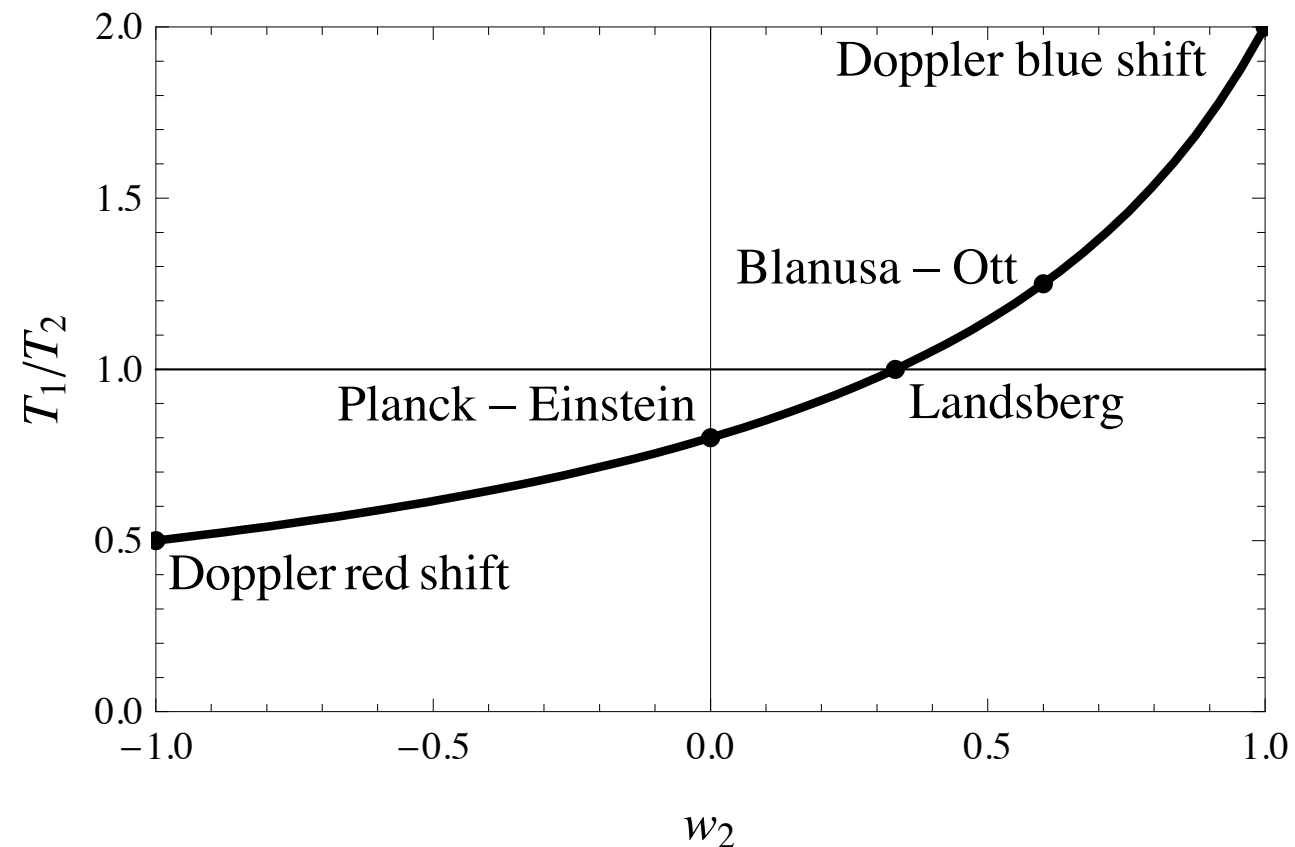


FIGURE 1. Ratio of the temperatures of the observed body in its rest frame,  $T_2$  to that shown by an ideal thermometer,  $T_1$  as a function of the the speed of the heat current in the body,  $w_2$  while approaching with the relative velocity  $v = -0.6$ .

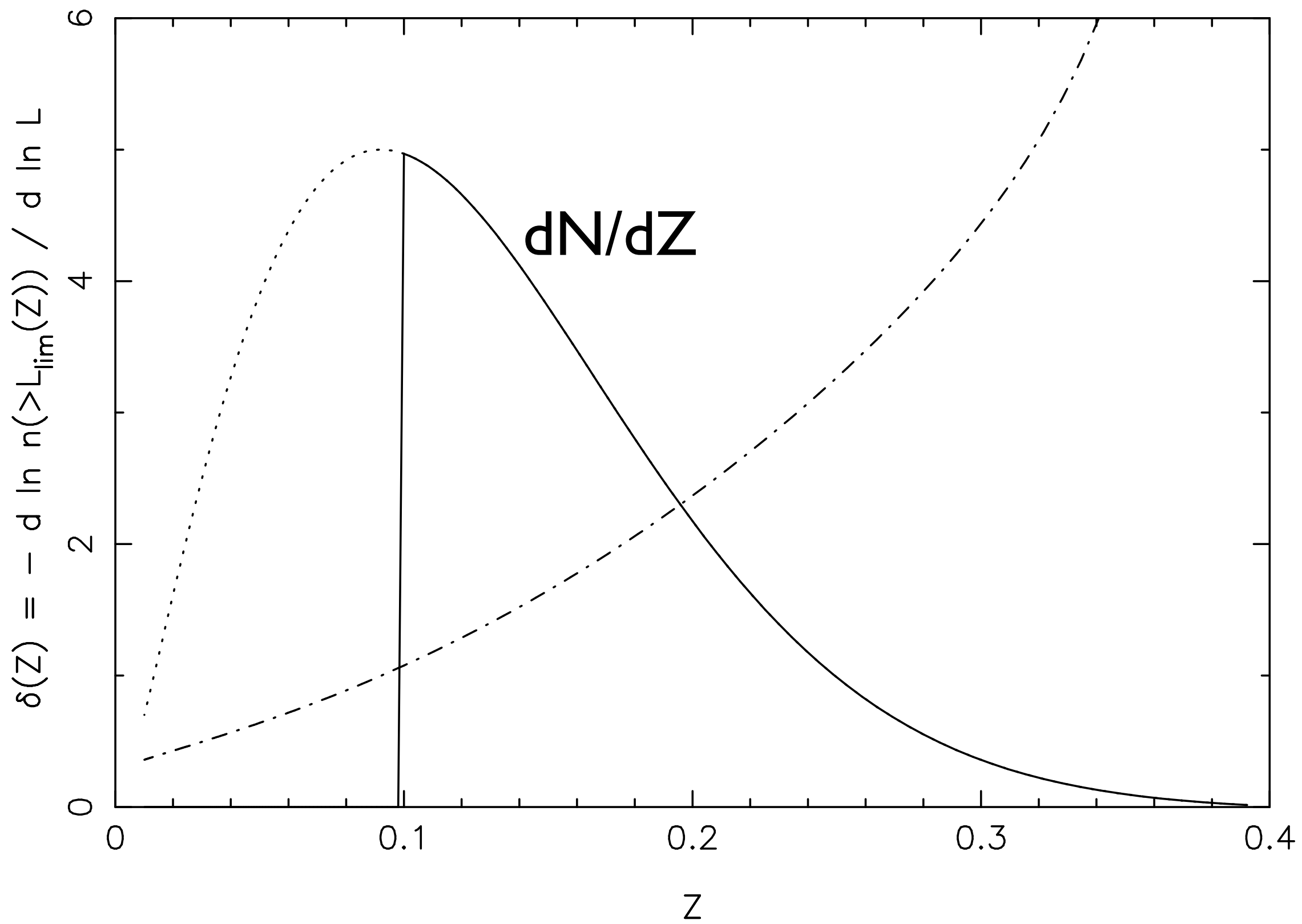


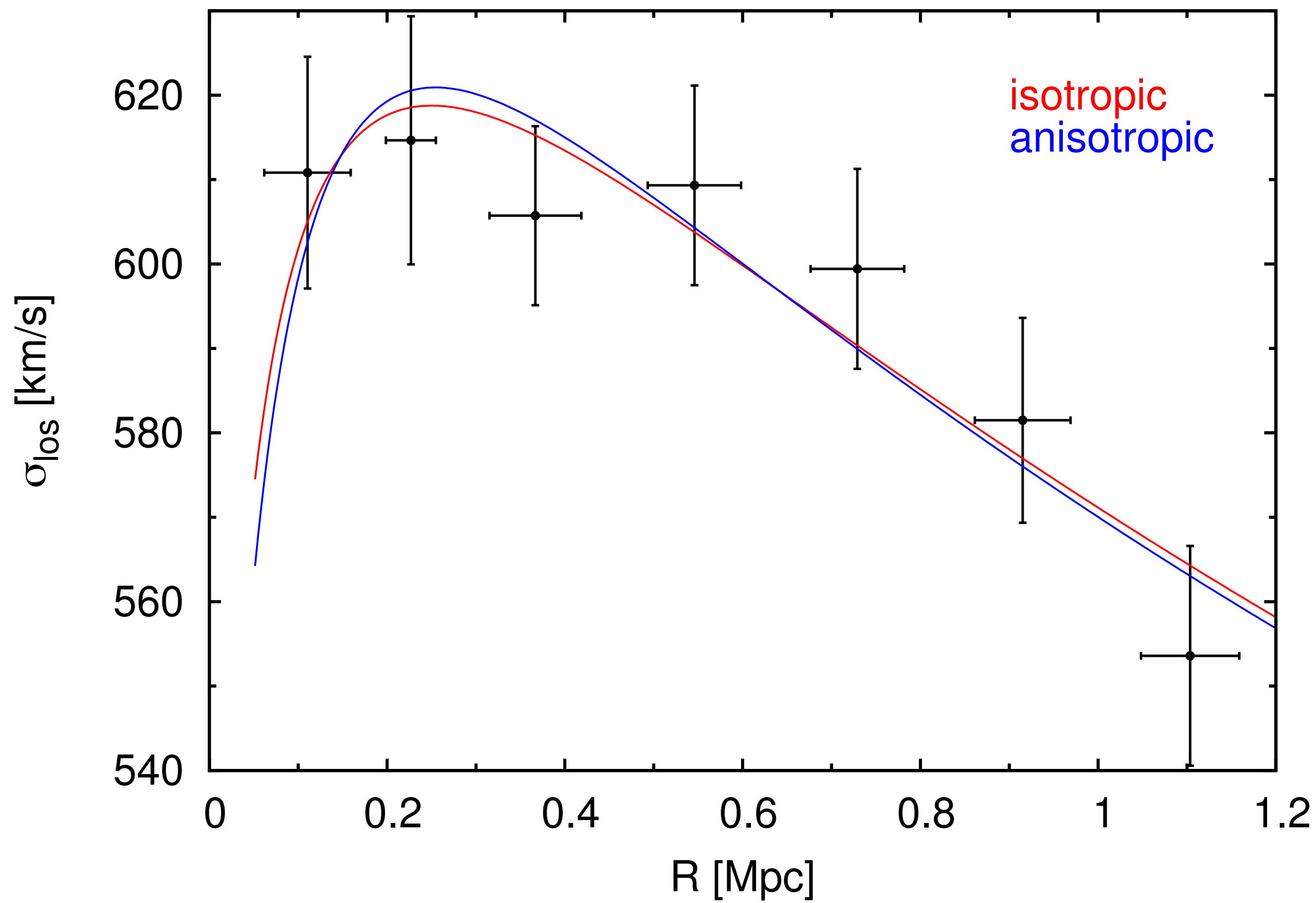
# Conclusions: 1) redshifts in general

- Redshifts in FRW are not caused by the expansion of space
- both changes in wavelength and changes in the space between observers are 'caused' by the initial relative velocities and the tidal field in which the observers and photon move.
- In homogeneous models they happen to be equal because a constant tide stretches  $\lambda$  the same way it stretches  $D$
- But neither can they be thought of as being essentially kinematic, or Doppler, in nature in presence of inhomogeneity
- that gives the wrong answer for the gravitational redshift component of the redshift
- which is an integral of the *gradient of the tide* along the photon path
- Allowance for this largely reconciles conventional view with direct calculation (modulu kinematic TD, LC, SB effects)

# Conclusions: 2) cluster gravitational-Z

- Gravitational redshifts in clusters have been measured!
- but the interpretation is considerably more complicated than originally thought
- The static gravitational redshift is augmented by 3 other kinematic effects that are generally of the similar magnitude
  - time dilation + light-cone effect + relativistic beaming
- These measurements essentially provide a test of the equivalence principle
  - i.e. whether light and galaxies “fall” the same way in clusters
  - constrains “5th force” theories
  - and we can expect more precise measurements in the near future

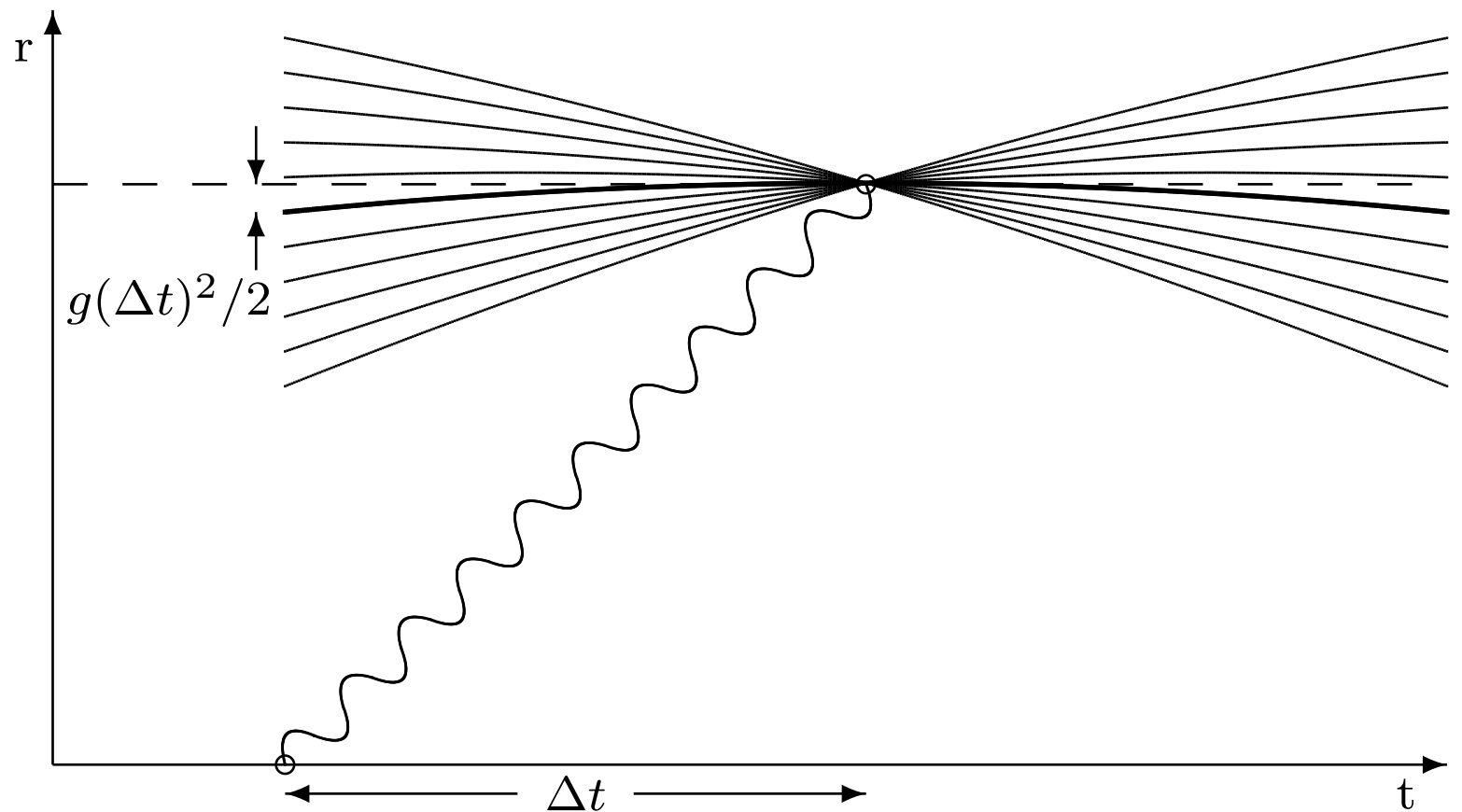






# Possible resolution of static and “kinematic” view?

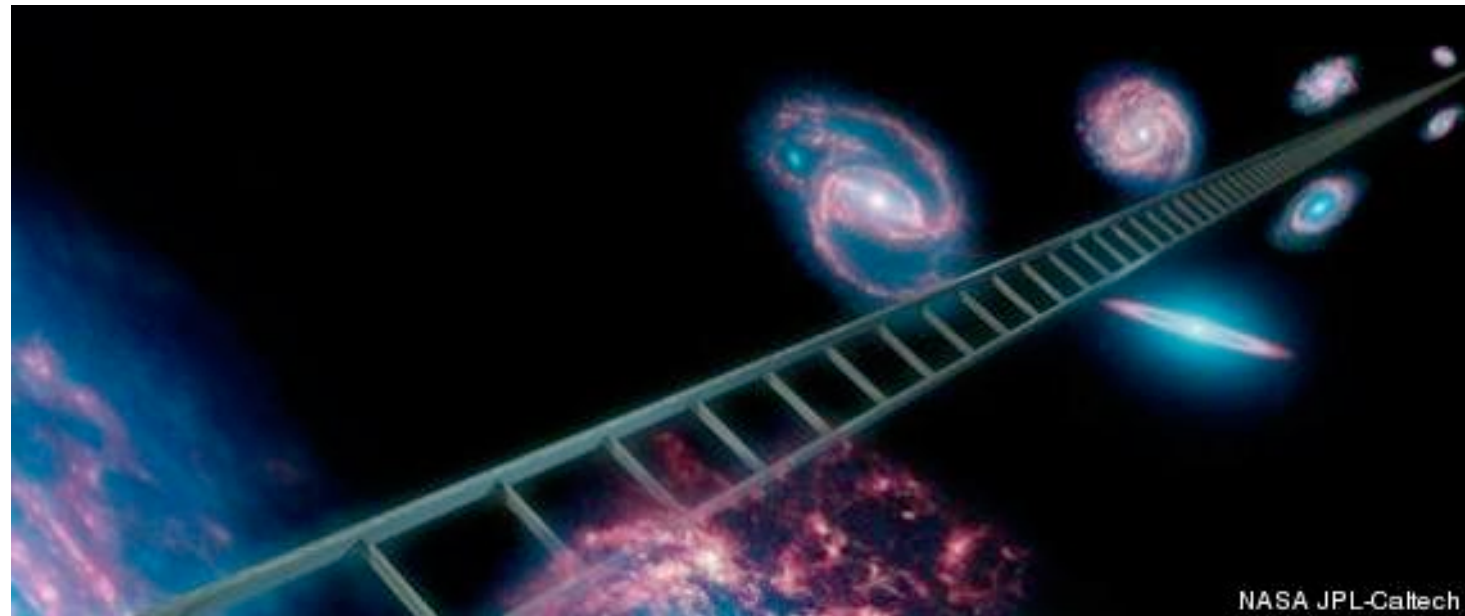
- Consider particles in equilibrium in potential well
- photon emitted by “cold” particle at bottom of potential well and received by randomly chosen “hot” particle at larger radius
- For quadratic potential, Synge’s velocity is the average velocity over the time of flight of the photon
- Reconciles “kinematic” view with  $\Delta\lambda/\lambda = \Delta\varphi$
- But is this generally true?



**Figure 2.** Illustration of situation described in text where a photon is emitted by a ‘cold’ particles near the centre of a smooth potential well and is received by a randomly chosen ‘hot’ particle at large distance. A sample of orbits from the distribution of velocities – here a simple box-car – is shown as curves. The orbit for the average velocity particle is shown as the heavy curve. The average radial velocity is zero at the time of reception, but the average velocity over the time-of-flight of the photon is  $-g\Delta t/2$  and, for a parabolic potential, this velocity, in units of  $c$  is equal to the gravitational redshift. It is tantalising to think that a generalisation of this reconciles the GR kinematic view of redshifts with more conventional view of gravitational redshifts.

# Is the 'kinematic' picture correct?

- B&H (and many others) say that the only way to compare velocities of separated objects is *via* parallel transport
- But that would mean the only way to measure the relative velocity is via redshifts. But then saying 'redshift is a velocity effect' is circular
- But there are other ways:
  - this is how we think about measuring geodesic deviation (and hence the gravitational field)
- It *not* vacuous to say that redshifts in flat space-time are 'Doppler' because the velocity that gives the redshift really is the same as the rate of change of proper separation.
- Same is true for FOs in FRW, and many other situations
- Key question: is this a general law? Seems plausible, right?



# Application to gravitational redshifts in clusters

- WHH experiment measures the redshift of the general galaxy population relative to the BCG
  - In equilibrium, their mean separation  $D$  is unchanging
  - So the 1st order Doppler effect averages to zero.
  - What is the residual (2nd order effect)?
- Conventional view: The relative redshift is just the mean gravitational potential difference.
  - Just as in Pound and Rebka. But doesn't it matter that the galaxies are in free fall?
- Kinematic picture:  $D$  is unchanging, so redshift vanishes?
- Or alternatively, one might imagine that the redshift between a pair of galaxies would not be sensitive to the first order potential difference  $\delta\varphi \sim D \cdot g$  (EP: gravity  $g$  is “transformed away” in free fall) but would see higher order effects (i.e. tidal field to lowest order).
- Several plausible pictures - which is right?

# But wait! There's something fishy here...

- Why is the transverse Doppler effect a *red*-shift?
  - Take a birthday cake; light the candles and put it on a turntable and spin it.
  - Detect all the photons and measure their frequency
  - Compare with non-rotating experiment.
- Shouldn't we see *blue*-shift  $\lambda_{\text{obs}} = \lambda_{\text{em}}/\gamma$ ? As moving candles have more energy than candles at rest
- Or what if we have a swarm of moving astrophysical sources destroying rest mass and turning it into light and we catch all the photons and measure their energy?
- Does their motion induce a red-shift? If so, how is can that be compatible with energy conservation?
  - Note: this is SR, so unlike in cosmology, energy *is* supposed to be conserved

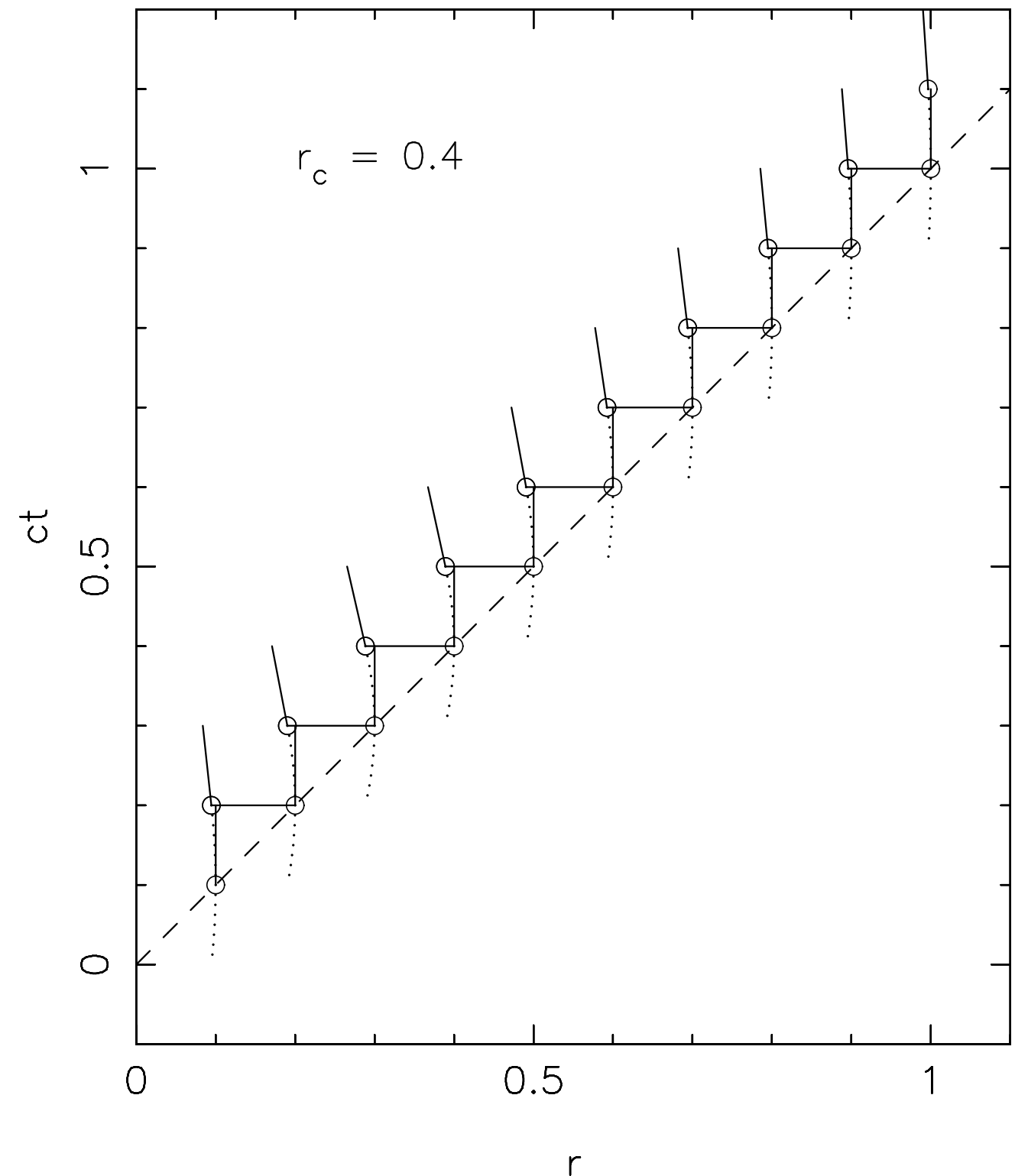


# Unresolved sources composed of moving sources have a net transverse Doppler *blue*-shift

- Objects will appear red-shifted (on average at least)
  - And a swarm of objects will have an additional red-shift from their motions (light-cone effect)
- But photons from an object composed of moving sources must, on average, be blue-shifted
  - if not, energy conservation would be violated
- The apparent contradiction is resolved once you appreciate that a source that radiates isotropically in its rest frame is not radiating isotropically in the observer (or lab) frame
- It is a mild relativistic beaming effect:
  - slightly more photons emerge in the forward direction
  - and these pick up a 1st order Doppler blue-shift
  - which leads to a 4th effect:

# What went wrong with the argument about little Doppler shifts?

- We break the null path into a set of segments with a family of observers
- Each can be assumed to be turning around as photon passes
- But  $\Delta v$  is in rest-frame of the pairs
- Generally these don't add up to the rate of change of separation of end-points
- but in constant tidal field they do



**Figure 1.** Illustration of the example described in the text. Here we have a potential with gravity  $g \sim r/(r^3 + r_c^3)$  which is like that for a uniform density sphere at  $r \ll r_c$  and is Keplerian at  $r \gg r_c$ . The redshift between an emitter at  $r = 0$  and a distant

# Bunn & Hogg

## II. REDSHIFTS OF NEARBY GALAXIES ARE DOPPLER SHIFTS

We begin by returning to the parable of the speeding ticket, mentioned in Sec. I.

“A driver is pulled over for speeding. The police officer says to the driver, ‘According to the Doppler shift of the radar signal I bounced off your car, you were traveling faster than the speed limit.’

“The driver replies, ‘In certain coordinate systems, the distance between us remained constant during the time the radar signal was propagating. In such a coordinate system, our relative velocity is zero, and the observed wavelength shift was not a Doppler shift. So you can’t give me a ticket.’ ”

If you believe that the driver has a legitimate argument, then you have our permission to believe that cosmological redshifts are not really Doppler shifts. If, on the other hand, you think that the officer is right, and the redshift can legitimately be interpreted as a Doppler shift, then you should believe the same thing about redshifts of nearby galaxies in the expanding universe.

Why is the police officer right and the driver wrong? Assuming the officer majored in physics, he might explain the situation like this: “Spacetime in my neighborhood is very close to flat. That means that I can lay down space and time coordinates in my neighborhood such that, to an excellent approximation, the rules of special relativity hold. Using those coordinates, I can interpret the observed redshift as a Doppler shift (because there is no gravitational redshift in flat spacetime) and calculate your coordinate velocity relative to me. The errors in this method are of the same order as the departures from flatness in the spacetime in a neighborhood containing both me and you. As long as I’m willing to put up with that very small level of inaccuracy, I can interpret that coordinate velocity as your actual velocity relative to me.”

The principle underlying the officer’s reasoning is uncontroversial. It is no different from the principle that lets football referees ignore the curvature of Earth and use a flat coordinate grid in describing a football field.

# The gravitational redshift according to wikipedia

## Gravitational redshift

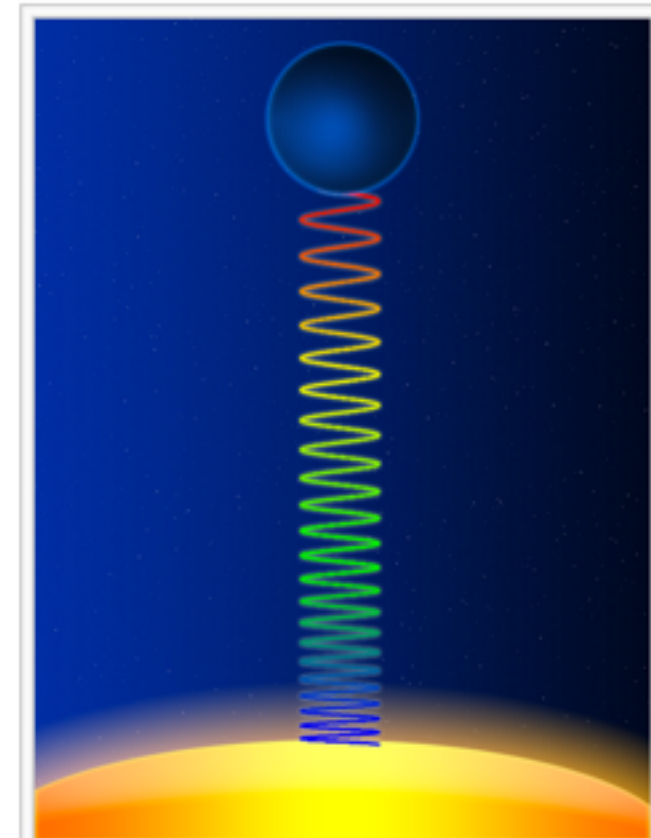
From Wikipedia, the free encyclopedia

In [astrophysics](#), **gravitational redshift** or **Einstein shift** is the process by which [electromagnetic radiation](#) originating from a source that is in a [gravitational field](#) is reduced in [frequency](#), or [redshifted](#), when observed in a region of a weaker gravitational field. This is a direct result of [gravitational time dilation](#) - as one moves away from a source of gravitational field, the rate at which time passes is increased relative to the case when one is near the source. As frequency is inverse of time (specifically, time required for completing one wave oscillation), frequency of the electromagnetic radiation is reduced in an area of a lower gravitational field (i.e., a higher gravitational potential). There is a corresponding reduction in energy when electromagnetic radiation is red-shifted, as given by [Planck's relation](#), due to the electromagnetic radiation propagating in opposition to the gravitational gradient. There also exists a corresponding [blueshift](#) when electromagnetic radiation propagates from an area of a weaker gravitational field to an area of a stronger gravitational field.

If applied to optical wavelengths, this manifests itself as a change in the colour of visible light as the wavelength of the light is increased toward the red part of the [light spectrum](#). Since frequency and wavelength are inversely proportional, this is equivalent to saying that the frequency of the light is reduced towards the red part of the light spectrum, giving this phenomenon the name [redshift](#).

### Contents [\[hide\]](#)

- [1 Definition](#)
- [2 History](#)



The gravitational redshift of a light wave as it moves upwards against a gravitational field (produced by the yellow star below). The effect is greatly exaggerated in this diagram.



# The nature of astronomical redshifts - prologue

- For photons exchanged between fundamental observers in FRW  $\lambda$  increases with the scale-factor  $a(t)$ 
  - Equivalently, wavelength changes in proportion to the emitter-receiver proper separation  $D$  so  $\Delta \ln(\lambda/D) = 0$
  - Just as for a pair of observers in flat space-time
- Widely accepted, but there has been much debate as to *why*?
- There has been a shift away from describing redshifts as being *caused* by the 'expansion of space' and towards a 'kinematic' description in which redshifts --- and perhaps all redshifts --- are thought of as essentially Doppler-like, or 'kinematic' in nature.
- Here I will review the old picture and the arguments against it, and the arguments for the kinematic picture
  - but both lead to wrong expectations for cluster grav-Z
- Calculation: the intrinsically gravitational component of Z

# The kinematic origin of the cosmological redshift

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*Department of Physics, University of Richmond, Richmond, Virginia 23173*

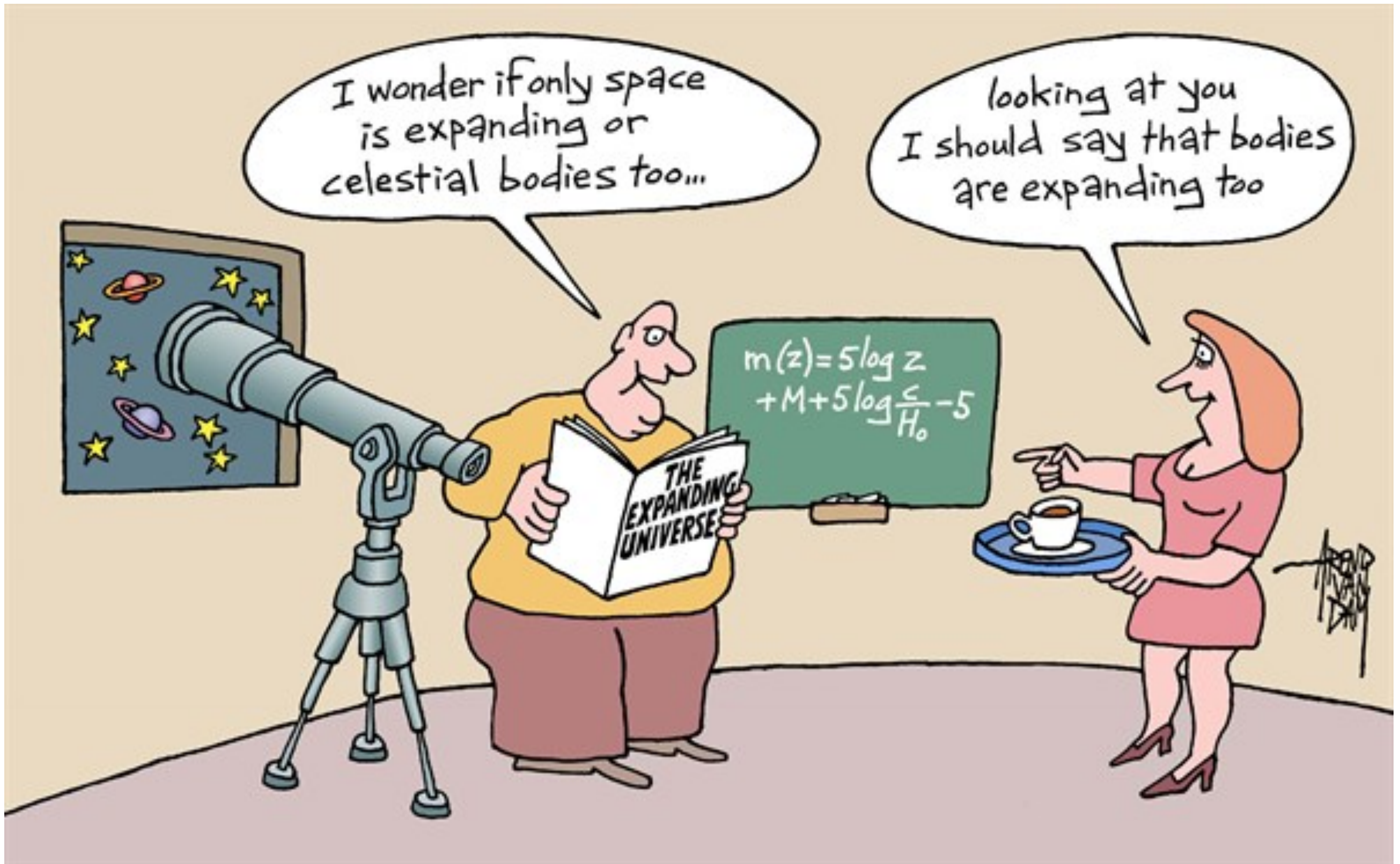
David W. Hogg

*Center for Cosmology and Particle Physics and Department of Physics,  
New York University, 4 Washington Place, New York, New York 10003*

A common belief about big-bang cosmology is that the cosmological redshift cannot be properly viewed as a Doppler shift (that is, as evidence for a recession velocity), but must be viewed in terms of the stretching of space. We argue that, contrary to this view, the most natural interpretation of the redshift is as a Doppler shift, or rather as the accumulation of many infinitesimal Doppler shifts. The stretching-of-space interpretation obscures a central idea of relativity, namely that it is always valid to choose a coordinate system that is locally Minkowskian. We show that an observed frequency shift in any spacetime can be interpreted either as a kinematic (Doppler) shift or a gravitational shift by imagining a suitable family of observers along the photon's path. In the context of the expanding universe the kinematic interpretation corresponds to a family of comoving observers and hence is more natural.

either the observations or the general-relativistic equations that successfully explain them. Rather, our focus is on the interpretation: given that a photon does not arrive at the observer conveniently labeled “Doppler shift,” “gravitational shift,” or “stretching of space,” when can or should we apply these labels?

Arguably an enlightened cosmologist never asks this question. In the curved spacetime of general relativity, there is no unique way to compare vectors at widely separated spacetime points, and hence the notion of the relative velocity of a distant galaxy is almost meaningless. Indeed, the inability to compare vectors at different points is the definition of a curved spacetime.<sup>1,2,3,4</sup> In practice, however, the enlightened view is far from universal. The view presented



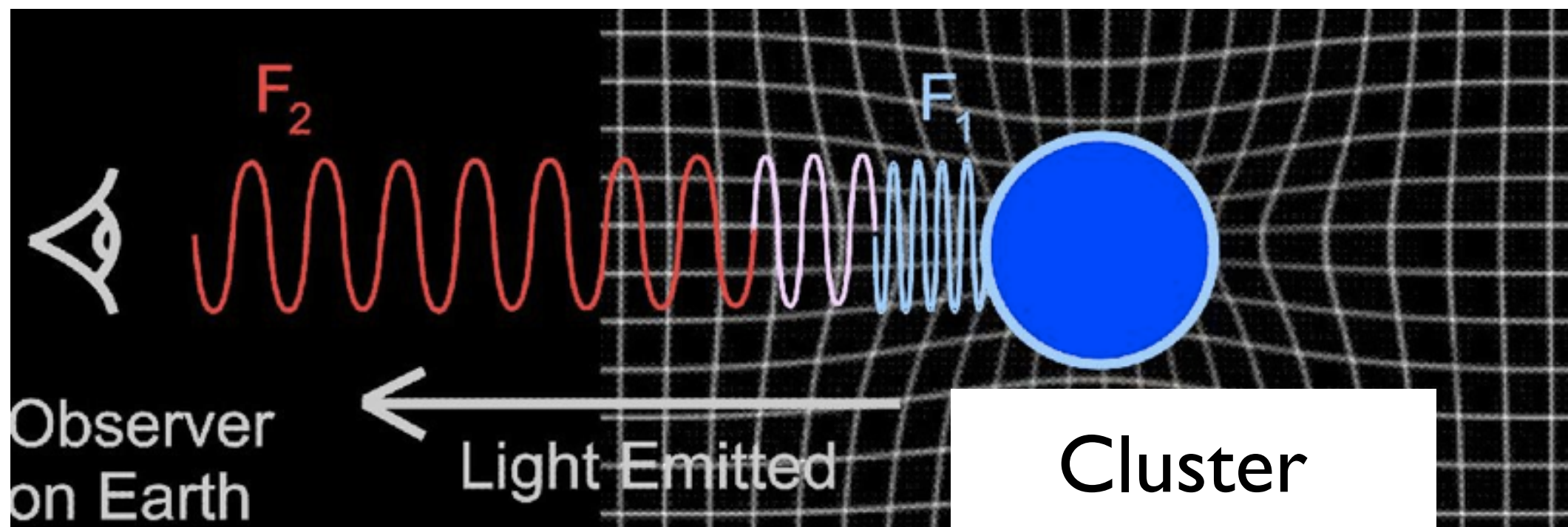
# My thoughts on P&R, B&H, Synge, Rindler

- B&H argument that gravitational and Doppler redshifts are in some way equivalent is misleading.
- Accelerated observers *know* that they are accelerated
  - So when they allow for this they agree that the redshift measured by the cop was a Doppler shift caused by  $\delta v$
- An acceleration *does not* create gravity ( $R_{\alpha\beta\gamma\delta} = 0$ )
- Pound and Rebka *did not measure* a gravitational redshift
  - they measured a redshift caused by the *non-gravitational* acceleration that their apparatus experienced
- Synge is right: gravity = *curvature* (or *tidal* field)
  - Pound & Rebka too insensitive to measure tidal effects
- But Synge is wrong to say that redshift is just Doppler
  - parallel transport of photon involved the *connection*
- Most redshifts are mostly Doppler - but there is an intrinsically gravitational component - it is the *gradient* of the tide

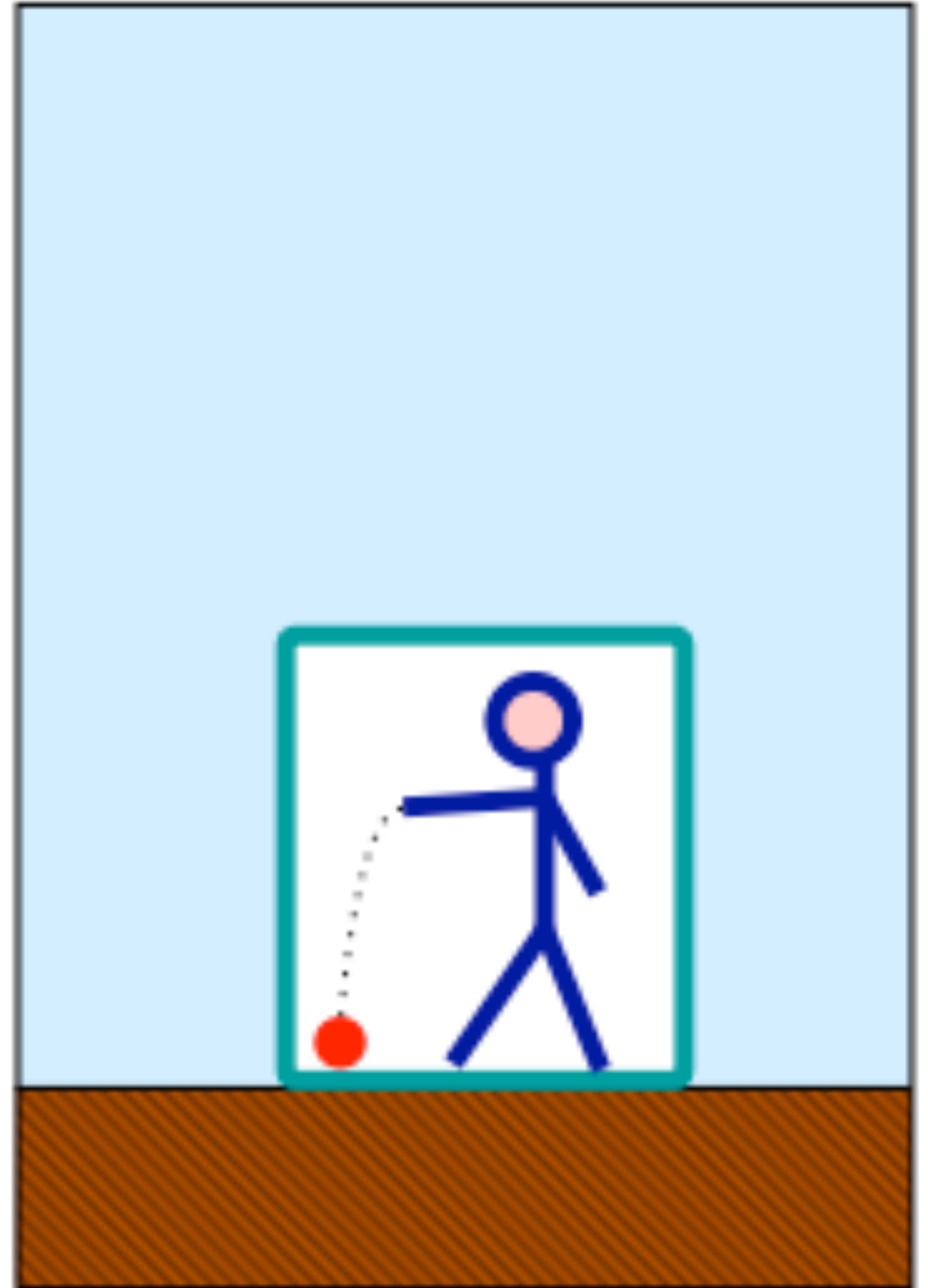


So why does the 'kinematic' law give the wrong answer?

- Recap: kinematic law  $\Delta \log(\lambda/D) = 0$  holds in a wide range of circumstances
- any observers in flat space-time, FOs in FRW, freely falling observers co-moving with Pound & Rebka apparatus
- Bunn & Hogg's argument  $\Rightarrow$  universal
- But consider emitter at rest in centre of a cluster and distant receiver turning around at large radius
- calculation  $\Rightarrow \Delta\lambda/\lambda \gg \Delta D/D$



# equivalence principle



# Principle of Equivalence (from [dummies.com](https://www.dummies.com))

## Einstein's General Relativity Theory: Gravity as Acceleration

By **Andrew Zimmerman Jones** and **Daniel Robbins** from **String Theory For Dummies**

*General relativity* was Einstein's theory of gravity, published in 1915, which extended special relativity to take into account *non-inertial frames of reference* — areas that are accelerating with respect to each other. General relativity takes the form of field equations, describing the curvature of space-time and the distribution of matter throughout space-time. The effects of matter and space-time on each other are what we perceive as gravity.

Einstein immediately realized that his theory of special relativity worked only when an object moved in a straight line at a constant speed. What about when one of the spaceships accelerated or traveled in a curve?

Einstein came to realize the *principle of equivalence*, and it states that an accelerated system is completely physically equivalent to a system inside a gravitational field.

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# THE PHYSICS OF THE UNIVERSE

DIFFICULT TOPICS MADE UNDERSTANDABLE

QUANTUM THEORY

$$E = h\nu$$

BEGINNINGS OF LIFE

SPACE-TIME

THE BIG BANG

$$E = mc^2$$

RELATIVITY

BLACK HOLES

DARK MATTER

UNCERTAINTY PRINCIPLE

$$F = \frac{Gm_1m_2}{d^2}$$

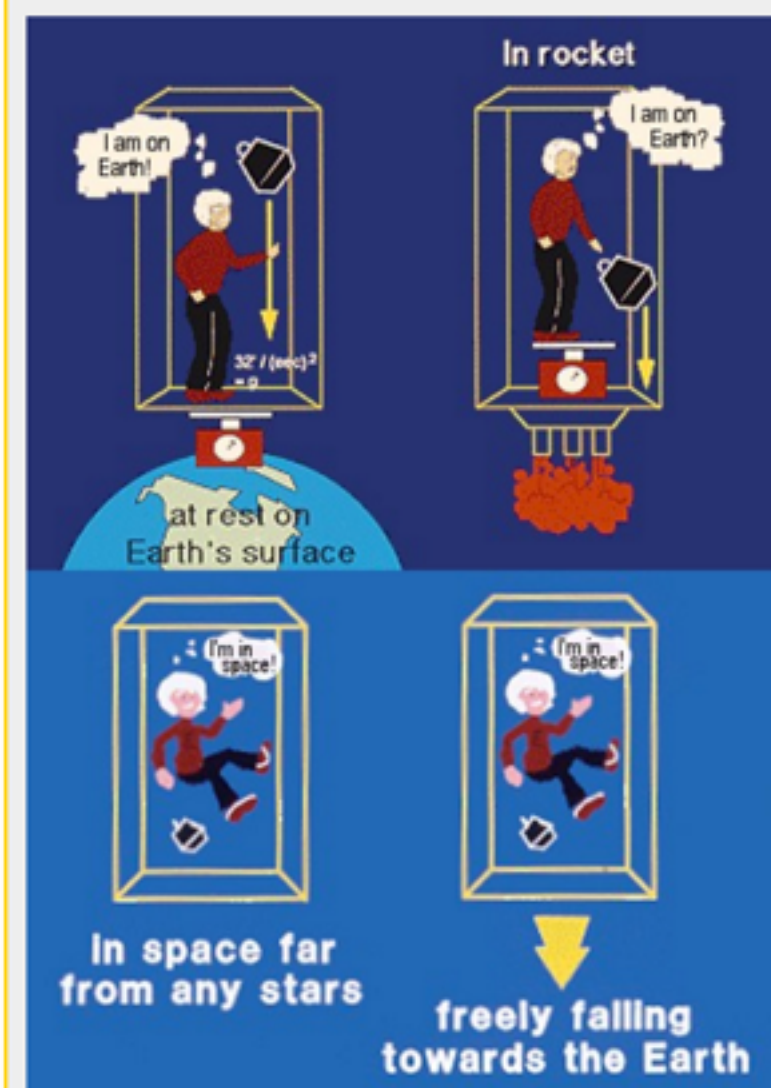
THE BIG CRUNCH

COSMOLOGICAL THEORIES

$$\Delta x \Delta p \geq \frac{1}{2}\hbar$$

WORMHOLES

[Einstein](#)'s ground-breaking realization (which he called "the happiest thought of my life") was that [gravity](#) is in reality not a force at all, but is indistinguishable from, and in fact the same thing as, [acceleration](#), an idea he called the "[principle of equivalence](#)". He realized that if he were to fall freely in a [gravitational field](#) (such as a skydiver before opening his parachute, or a person in an elevator when its cable breaks), he would be unable to feel his own weight, a rather remarkable insight in 1907, many years before the idea of freefall of astronauts in space became commonplace.



([Click for a larger version](#))

The principle of equivalence says that gravity is not a force at all, but is in fact the same thing as acceleration

(Source: Time Travel Research Center:  
<http://www.zamandayolculuk.com/cetinbal/HTMLdosya1/RelativityFile.htm>)



# What is the equivalence principle?

- Synge:

Perhaps they speak of the Principle of Equivalence. If so, it is my turn to have a blank mind, for I have never been able to understand this Principle. Does it mean that the signature of the space-time metric is  $+2$  (or  $-2$  if you prefer the other convention)? If so, it is important, but hardly a Principle. Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does not or does vanish. This is an absolute property; it has nothing to do with any observer's world-line. Space-time is either flat or curved, and in several places in the book I have been at considerable pains to separate truly gravitational effects due to curvature of space-time from those due to curvature of the observer's world-line (in most ordinary cases the latter predominate).

- Rindler:

As a consequence of the equivalence principle, not only can we *eliminate* gravity by free fall, we can also create it by acceleration.

version of the EP is directly supported by Mach's principle. The rocket in outer space sees the universe accelerate past it. The accelerating universe creates a gravitational field inside the rocket.

# Redshifts in homogeneous FRW models

- $\lambda$  scales with proper separation
- *analogous* to EM waves in an expanding cavity
- stretching of  $\lambda$  is *caused* by the expansion of space
- that this should be so is *obvious*
- expansion of space *causes* damping in Maxwell's equations
- cosmological redshifts do not obey special relativity
- $z$  is a combination of SR Doppler shift + gravitational  $z$ 
  - at low- $z$  at least (Bondi 1947)
- overall frequency shift is the product of little Doppler shifts
  - Peebles

# Redshifts in general

- all redshifts are Doppler shifts
  - Riemann tensor  $R_{\alpha\beta\psi\delta}$  does not appear in the formula
- acceleration and gravity are the same thing
  - principle of equivalence
- acceleration *creates* gravity
- Pound and Rebka measured a *gravitational* redshift
- redshifts can be considered as either gravitational or Doppler
  - simply a difference of coordinate systems
- all redshifts can (and should) be considered to be Doppler
- all redshifts are kinematic in nature
  - the only way to measure velocity is through redshift