

High-dimensional likelihood-free inference: weak gravitational lensing 2009.08459 2011.05991

Niall Jeffrey (with Justin Alsing, Francois Lanusse, Benjamin Wandelt)



Outline

- 1. Weak lensing map statistics
- 2. Density-estimation likelihood-free inference
- 3. Dark Energy Survey SV results
- 4. High-dimensional inference
- 5. Cosmic Microwave Background foregrounds





Weak lensing mass map statistics

Dark Energy Survey

SV weak lensing data



- I. Ground based 5-band photometric survey
- II. Science Verification (SV) data <5% of final coverage, but final depth



NJ, Lanusse, Lahav, Starck 1908.00543



NJ, Lanusse, Lahav, Starck 1908.00543







NJ, Lanusse, Lahav, Starck 1908.00543





DE L'ÉCOLE NORMALE SUPÉRIEURE

NJ, Lanusse, Lahav, Starck 1908.00543



Growth of structure



Weak lensing map summary statistics

Kaiser-Squires -45° -50° DEC -55° -60° 65° 75° 85° RA







Weak lensing map summary statistics

LABORATOIRE DE PHYSIQUE DE L'ÉCOLE NORMALE SUPÉRIEURE







Parameter inference

1. Observed "data" summary statistic *d*_o



Parameter inference

1. Observed "data" summary statistic do

2. Unknown parameters θ of a given model

 $P(\theta \mid d_o) \propto P(d_o \mid \theta) P(\theta)$ **LIKELIHOOD** PRIOR





Step 1 Forward modelled mock data

 $\{\mathbf{d}_i, \theta_i\}$

- I. \mathbf{d}_i are simulated data vector summary statistics (inc. noise)
- **II.** Draw \mathbf{d}_i from the distribution $P(\mathbf{d} \mid \theta_i)$ by running a simulation
- III. Compress data using fixed compression method to get $\{\mathbf{t}_i, \theta_i\}$



Step 2 Reduce the data dimensionality

$$\mathbf{t} = F(\mathbf{d})$$

- I. For Gaussian likelihoods the compression must be linear
- II. With LFI we can use neural compression



Step 3 Density Estimation



Density estimation





Density estimation

 $P(d,\theta)$ or $P(d|\theta)$





Compression

1. Can compress the data to the dimension of the parameters:

 $\mathbf{t} = F(\mathbf{d})$

3. Poor compression will lead to larger scatter and worse constraints.



 $P(d,\theta)$ or $P(d|\theta)$





DES SV Results

Run simulations with different cosmologies





Run simulations with different cosmologies



LABORATOIRE DE PHYSIQUE DE L'ÉCOLE NORMALE SUPÉRIEURE



Realistic mock data maps





Evaluate posterior with learned likelihood:



Evaluate posterior with learned likelihood:





Density estimation validation:









$$\mathbf{t} = F(\mathrm{map})$$



-0.025





LEPENS LABORATOIRE DE PHYSIQUE DE L'ÉCOLE NORMALE SUPÉRIEURE

$$\mathbf{t} = F(\mathrm{map})$$



$\mathbf{t} = F(\mathrm{map})$

$I(\mathbf{t}, \boldsymbol{\theta}) = D_{\mathrm{KL}}(p(\mathbf{t}, \boldsymbol{\theta}) \parallel p(\mathbf{t})p(\boldsymbol{\theta}))$



 $\mathbf{t} = F(map)$



 $I(\mathbf{t}, \boldsymbol{\theta}) = D_{\mathrm{KL}}(p(\mathbf{t}, \boldsymbol{\theta}) \parallel p(\mathbf{t})p(\boldsymbol{\theta}))$







DES SV results:

- Validation: forward model
- Validation: probability density
- Deep convolutional compression







High-dimensional inference

Problem: curse of dimensionality

I. High dimension D of θ ?

II. Number of mock data scales N^D





Marginal posterior density estimation

$$\boldsymbol{\theta} = [\alpha, \beta]$$

$$p(\alpha, \beta | \boldsymbol{x}_{obs}) = \int p(\alpha, \beta, \boldsymbol{\theta}' | \boldsymbol{x}_{obs}) \, \mathrm{d}\boldsymbol{\theta}'$$



Marginal posterior density estimation

Marginal flow

I. Draw full parameter space from prior $\boldsymbol{\theta}_i \sim p(\boldsymbol{\theta})$

II. Draw simulations $\mathbf{x}_i \sim p(\mathbf{x} \mid \boldsymbol{\theta}_i)$

III. Objective between subsets of parameters, e.g.

$$U(oldsymbol{arphi}) = -\sum_{i=1}^N \log q(lpha_i,eta_i|oldsymbol{x}_i;oldsymbol{arphi})$$

IV. Other parameters are explicitly marginalised away



Marginal flow vs MCMC

- I. 100-parameter model
- II. 10^7 MCMC samples
- III. 8×10^4 simulations for marginal flow





Moment Network: side-step density estimation problem

I. Network takes the data as input: $F(\mathbf{x})$

II. Train with an L₂ loss

$$J_0 = \int ||\boldsymbol{\theta} - \mathcal{F}(\boldsymbol{x})||^2 p(\boldsymbol{x}, \boldsymbol{\theta}) \, \mathrm{d}\boldsymbol{x} \, \mathrm{d}\boldsymbol{\theta}$$

III. The optimal network is the posterior mean $F(\mathbf{x}_o) = \int p(\boldsymbol{\theta} | \mathbf{x}_o) d\boldsymbol{\theta}$



Moment Network: level 2

- I. Network takes the data as input: $G(\mathbf{x})$
- II. Optimization objective

$$J_1 = \int ||(\boldsymbol{\theta} - \mathcal{F}_{\text{fixed}}(\boldsymbol{x}))^2 - \mathcal{G}(\boldsymbol{x})||^2 p(\boldsymbol{x}, \boldsymbol{\theta}) \, \mathrm{d}\boldsymbol{x} \, \mathrm{d}\boldsymbol{\theta}$$

III. Estimate mean, variance and covariance (and higher-moments) of marginal



Marginal Flow & Moment Network vs MCMC





Gravitational wave example





Gravitational wave example



- I. Moment Network posterior mean and variance
- II. Marginal per parameter h_+



Gravitational wave example







Search for primordial B-modes





Search for primordial B-modes



Millea++ 2002.00965



Search for primordial B-modes





Generative models?



Generative models?





Generative models?



- I. Generative adversarial network
- II. Requires large training sets
- III. Data and simulations not available for foregrounds



Wavelet Phase Harmonics $\phi(\mathbf{x})$ (adapted from Allys++ 2006.06298)





Generative model from single training example













Regaldo-Saint Blancard++ (NJ) 2102.03160



-200

-400

map [µK_{CMB.}



Regaldo-Saint Blancard++ (NJ) 2102.03160





Regaldo-Saint Blancard++ (NJ) 2102.03160



Map inference with Moment Network: Foreground simulation





Map inference with Moment Networks: Bayesian high-dimensional inference





Map inference with Moment Networks: Bayesian high-dimensional inference







Merci !

