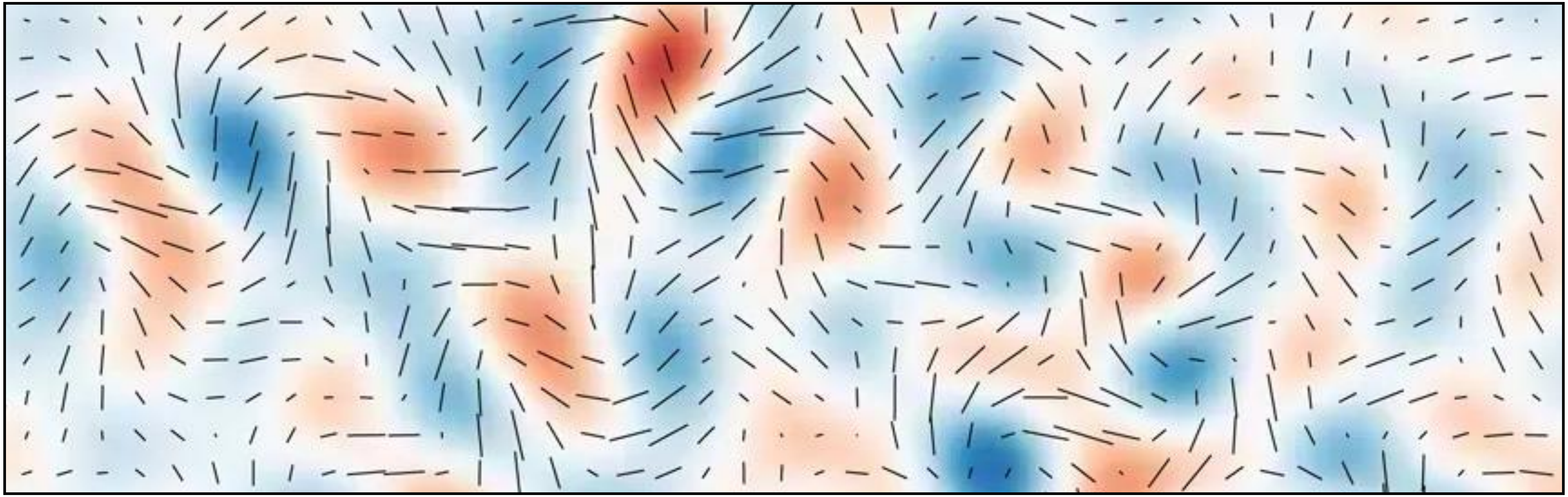
The B-Side of Gravitational Waves:

Imprints of Primordial Tensor Perturbations in CMB B-Modes



Aurora Ireland
Stanford University

UC Berkeley 4D Seminar
Sept. 22, 2025

Based on [2410.23348], [2507.02044], & upcoming!

Observable CMB B-modes from Cosmological Phase Transitions [2410.23348]

Kylar Greene
Seoul National University

Gordan Krnjaic
UChicago & Fermilab

Yuhsin Tsai
Notre Dame

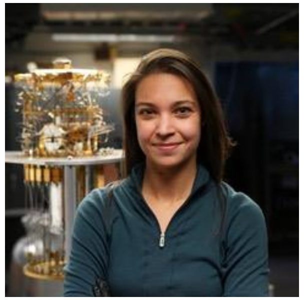
Imprints of $\mathcal{O}(1-10)\ \mathrm{Mpc}^{-1}$ Curvature Perturbations in CMB B-Modes from Scalar-Induced Gravitational Waves [2507.02044]

Kuver Sinha
University of Oklahoma

Tao Xu
Hong Kong University of Science and Technology

First Constraints on Causal Sources of Primordial GWs from BICEP/Keck, SPTpol, SPT-3G, Planck, and WMAP B-Mode Data [Upcoming!]

Christian Reichardt
University of Melbourne



Jessica Zebrowski
UChicago & Fermilab

“Tensor fluctuations are a prime target for future observations of the cosmic microwave background (CMB), because **if detected they can provide a conclusive verification of the theory of inflation**…”

-Flauger & Weinberg,
[astro-ph/0703179]

B-mode polarization in CMB ⇒ **smoking gun** for inflation

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- Inflationary models   Kamionkowski & Kovetz, [1510.06042]
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- Other sources
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  - Foreground contamination (dust, synchrotron radiation)

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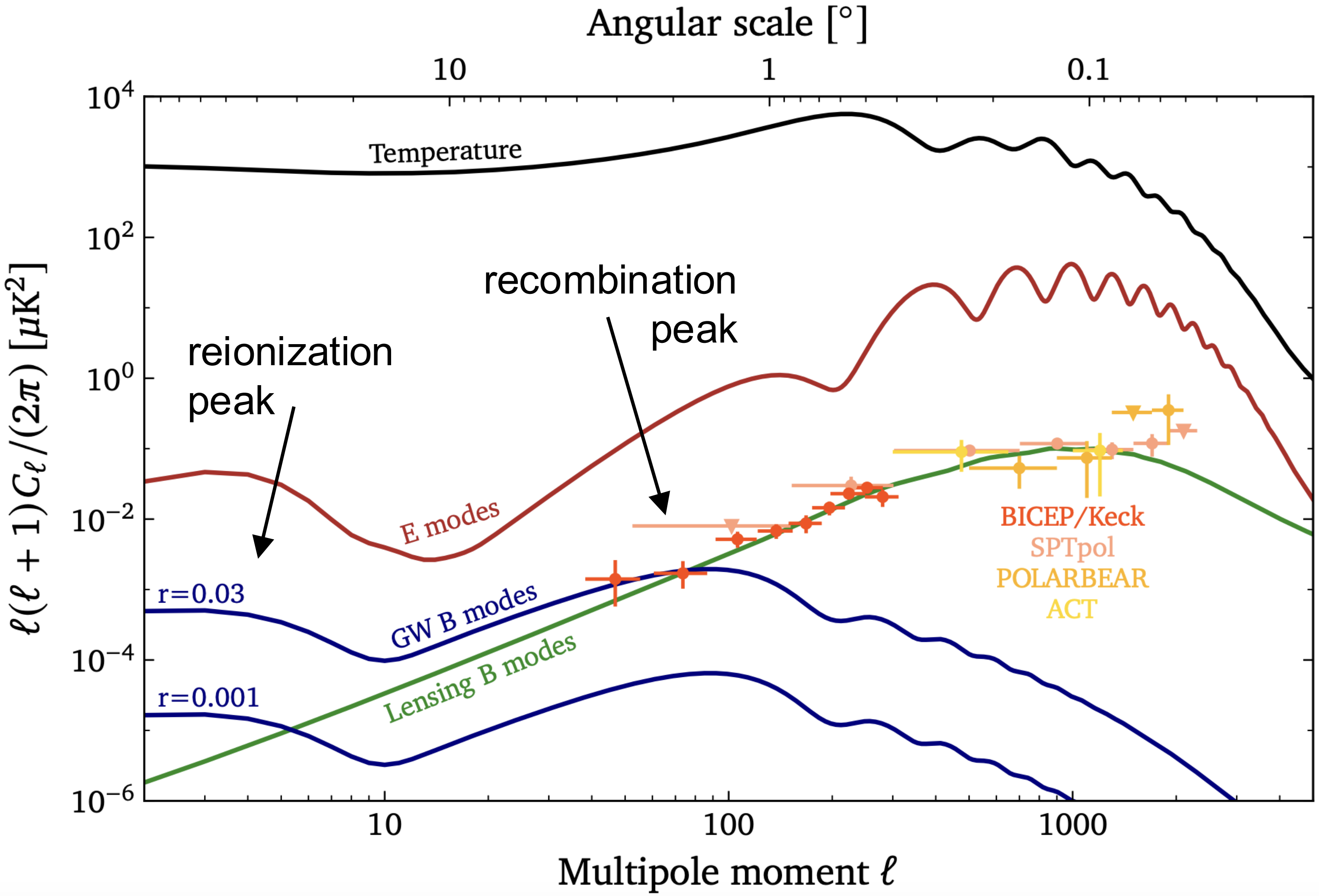
- Standard Lore: Inflation = only **direct, primordial** source

[redacted]



Achúcarro et al., [2203.08128]

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Two sample sources:

1) **First order cosmological phase transition (FOPT)**

   ↪ Source tensor pert. through bubble collisions, sound waves, & turbulence

2) **Scalar-induced GWs**

   ↪ Source tensor pert. from enhanced $\mathcal{R}$ at 2^{nd} order in cosmological pert. theory

[redacted]

- Naive expectation: **Negligible** contribution from both
  - CMB is sensitive to **large scales**
    ↪ Long-wavelength modes $k \lesssim 10^{-2}\ \mathrm{Mpc}^{-1}$
  - Non-inflationary primordial sources predict peak power on **small scales**
    ↪ Power **suppressed** on larger scales as $\mathcal{P}_h \sim (k\tau_*)^3$
    ↪ **Universal*** IR behavior for **causal*** sources

$k\tau_* \ll 1$ for superhorizon modes

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$k\tau_* \ll 1$ for superhorizon modes

- True; however it turns out that there can still be **sufficient power** on CMB scales provided:
  - Strong FOPTs in “late” (but pre-recombination) universe
  - Enhanced scalar power on scales $k \sim \mathcal{O}(1-10)\ \mathrm{Mpc}^{-1}$

i.e. source is late and loud

~~Conclusions~~ Spoilers:

r = tensor-to-scalar ratio

- B-modes from **non-inflationary primordial sources** can be **competitive** with inflation for values of r targeted in upcoming CMB experiments

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- Repackaging tensor perturbations into B-modes gives access to scales **outside sensitivity** of traditional probes
  ↪ FOPT: Stochastic GW background (SGWB)
  ↪ Scalar induced GWs: SGWB, $\mathcal{P}_{\mathcal{R}}$

~~Conclusions~~ **Spoilers:**

r = tensor-to-scalar ratio

- B-modes from **non-inflationary primordial sources** can be **competitive** with inflation for values of r targeted in upcoming CMB experiments

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  ↪ FOPT: Stochastic GW background (SGWB)
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- Existence of primordial B-mode sources can **complicate** an **inflationary interpretation** for future B-mode measurements
  ↪ **Distinct** spectral shapes ⇒ scenarios can be **distinguished**
  ↪ Large degeneracy with lensing spectrum

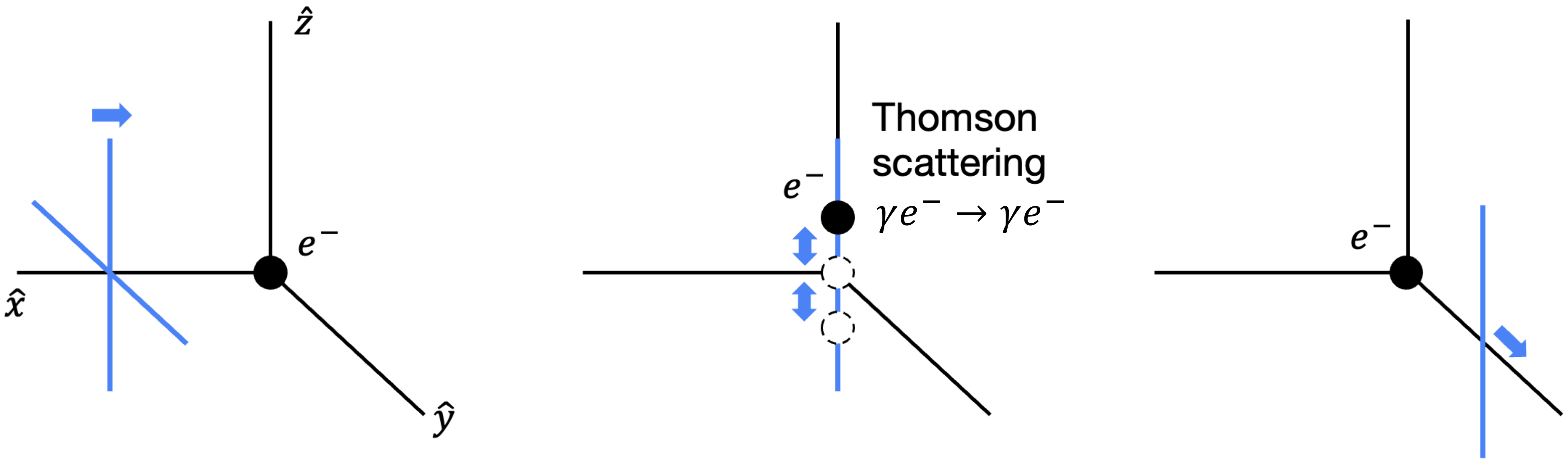
Overview

1) ~~Motivations~~
2) B-Mode Polarization
3) First Order Phase Transition
4) Scalar-Induced GWs
5) Summary & future directions

B-Mode Polarization

Hu & White, [astro-ph/9706147]

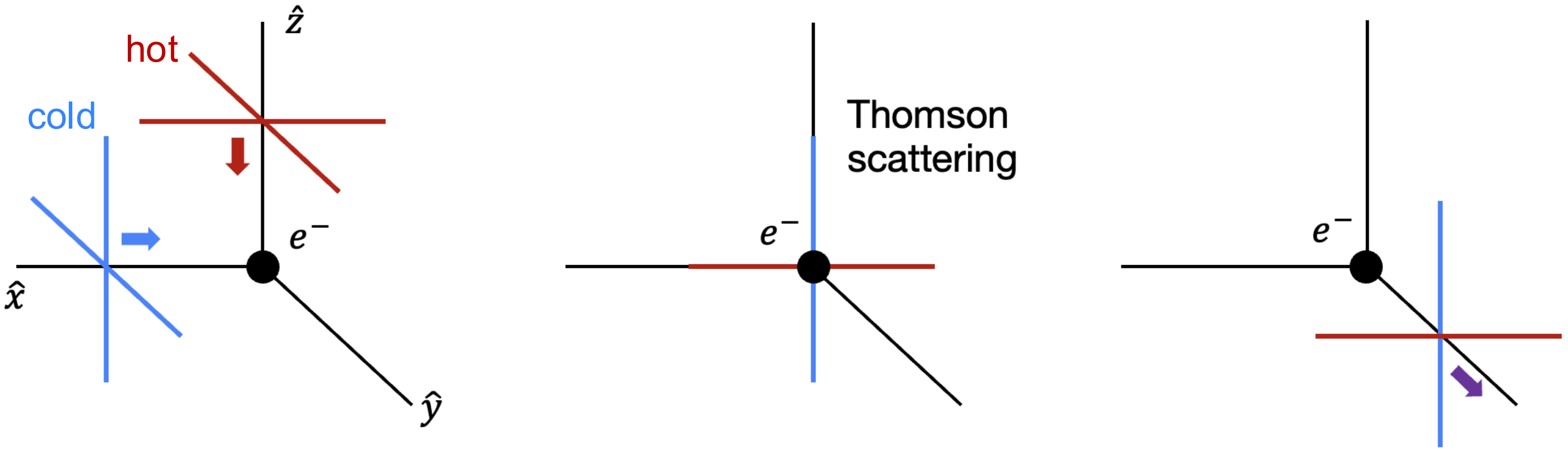
- Scalar & tensor perturbations ⇒ temperature anisotropies
- Anisotropies + Thomson scattering ⇒ polarization



Light cannot be polarized along direction of motion ⇒ only one linear polarization scattered

B-Mode Polarization

Quadrupole anisotropies in temperature distribution



⇒ net linear polarization of scattered photons

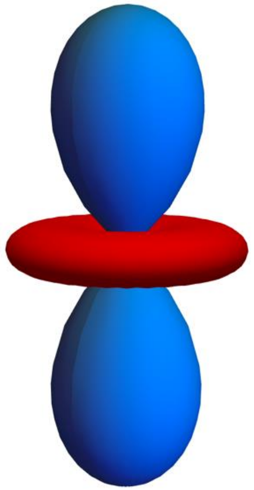
B-Mode Polarization

Quadrupole anisotropies

from vorticity

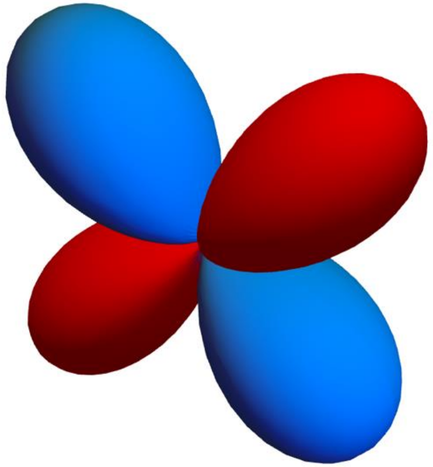
from gravitational waves

from density perturbations



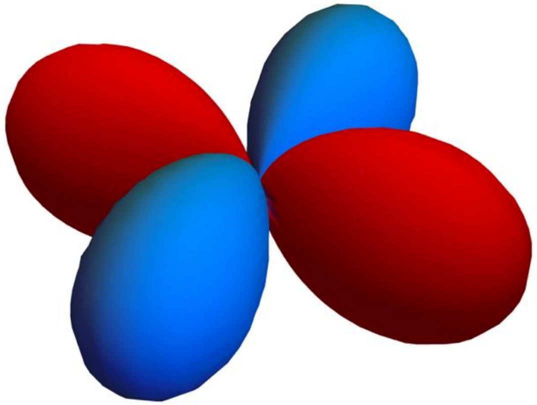
scalar

$\ell = 2,\ m = 0$



vector*

$\ell = 2,\ m = \pm 1$



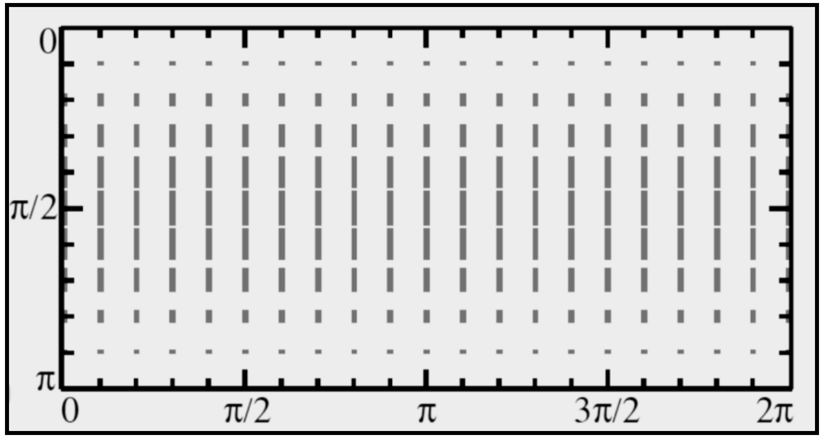
tensor

$\ell = 2,\ m = \pm 2$

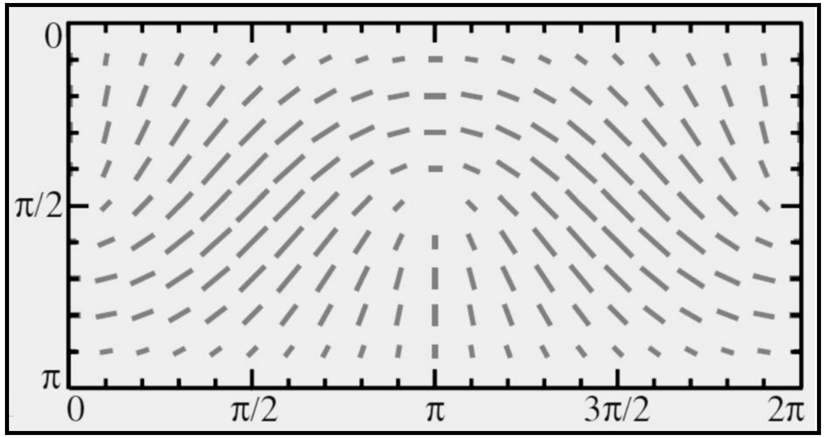
*negligible at recombination ⇒ neglect

B-Mode Polarization

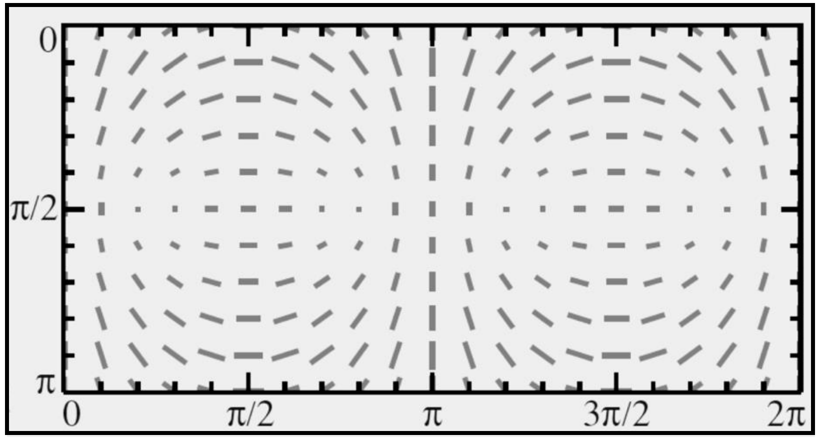
Polarization pattern = projection of quadrupole anisotropies



$\ell = 2,$
$m = 0$



$\ell = 2,$
$m = 1$

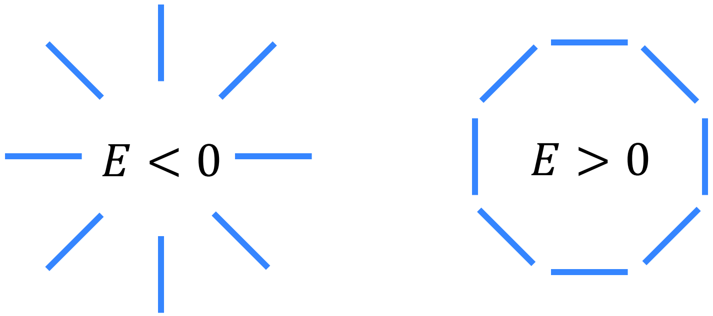


$\ell = 2,$
$m = 2$

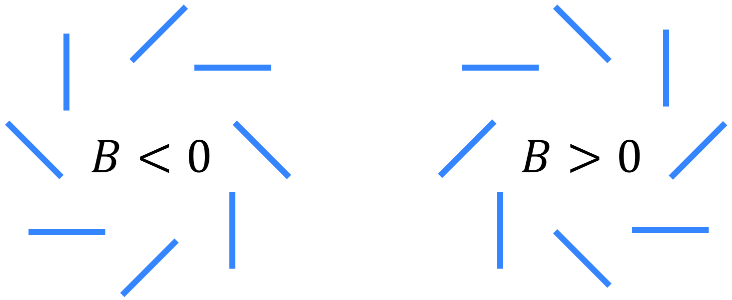
B-Mode Polarization

Convenient to decompose polarization into “divergence” and “curl” parts

- E-mode polarization
  - **Divergence** part
  - Unchanged under reflection (parity even)
  - Sourced by scalar & tensor perturbations



- B-mode polarization
  - **Curl** part
  - Interchange under reflection (parity odd)
  - Sourced by **tensor perturbations only**



B-Mode Polarization

Goal: Angular spectrum of B-mode polarization, C_ℓ^{BB}

- Tensor perturbation h_{ij} contributes to temperature anisotropy $\Delta T/T$
- $\Delta T/T$ enters in CMB polarization tensor P_{ab}
- Decompose P_{ab} into “gradient” (P_E) and “curl” (P_B) parts
- Expand P_B in spherical harmonics, invert to isolate $a_{\ell m}^B$
- Define $C_\ell^{BB} \sim \sum_m \langle a_{\ell m}^B a_{\ell m}^{B\,*} \rangle$

B-Mode Polarization

Angular spectrum of B-mode polarization:

$$C_\ell^{BB} = \frac{1}{2\ell+1} \sum_{m=\pm 2} \int \frac{d^3k}{(2\pi)^3} \left\langle a_{\ell m}^B(\vec{k}) a_{\ell m}^B(\vec{k})^* \right\rangle$$

Evaluating…

$$C_\ell^{BB} = 36\pi \int_0^\infty \frac{dk}{k} \, \mathcal{P}_h(k) \, \mathcal{F}_\ell(k)^2$$

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Evaluating…

tensor power spectrum

$$C_\ell^{BB} = 36\pi \int_0^\infty \frac{dk}{k}\, \mathcal{P}_h(k)\, \mathcal{F}_\ell(k)^2$$

visibility function

transfer function

$$\mathcal{F}_\ell(k) = \int_0^{\tau_0} d\tau\, V(\tau_0,\tau)\, \mathcal{S}_\ell(k,\tau_0,\tau) \int_0^{\tau} d\tau_1\, V(\tau,\tau_1) \int_{\tau_1}^{\tau} d\tau_2 \frac{j_2[k(\tau-\tau_2)]}{k^2(\tau-\tau_2)^2}\frac{\partial\mathcal{T}(\tau_2,k)}{\partial\tau_2}$$

"window function"

$$\mathcal{S}_\ell(k,\tau_0,\tau) = \frac{\ell+2}{2\ell+1}\, j_{\ell-1}[k(\tau_0-\tau)] - \frac{\ell-1}{2\ell+1}\, j_{\ell+1}[k(\tau_0-\tau)]$$

spherical Bessel functions

B-Mode Polarization

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tensor power spectrum

"window function"

B-Mode Polarization

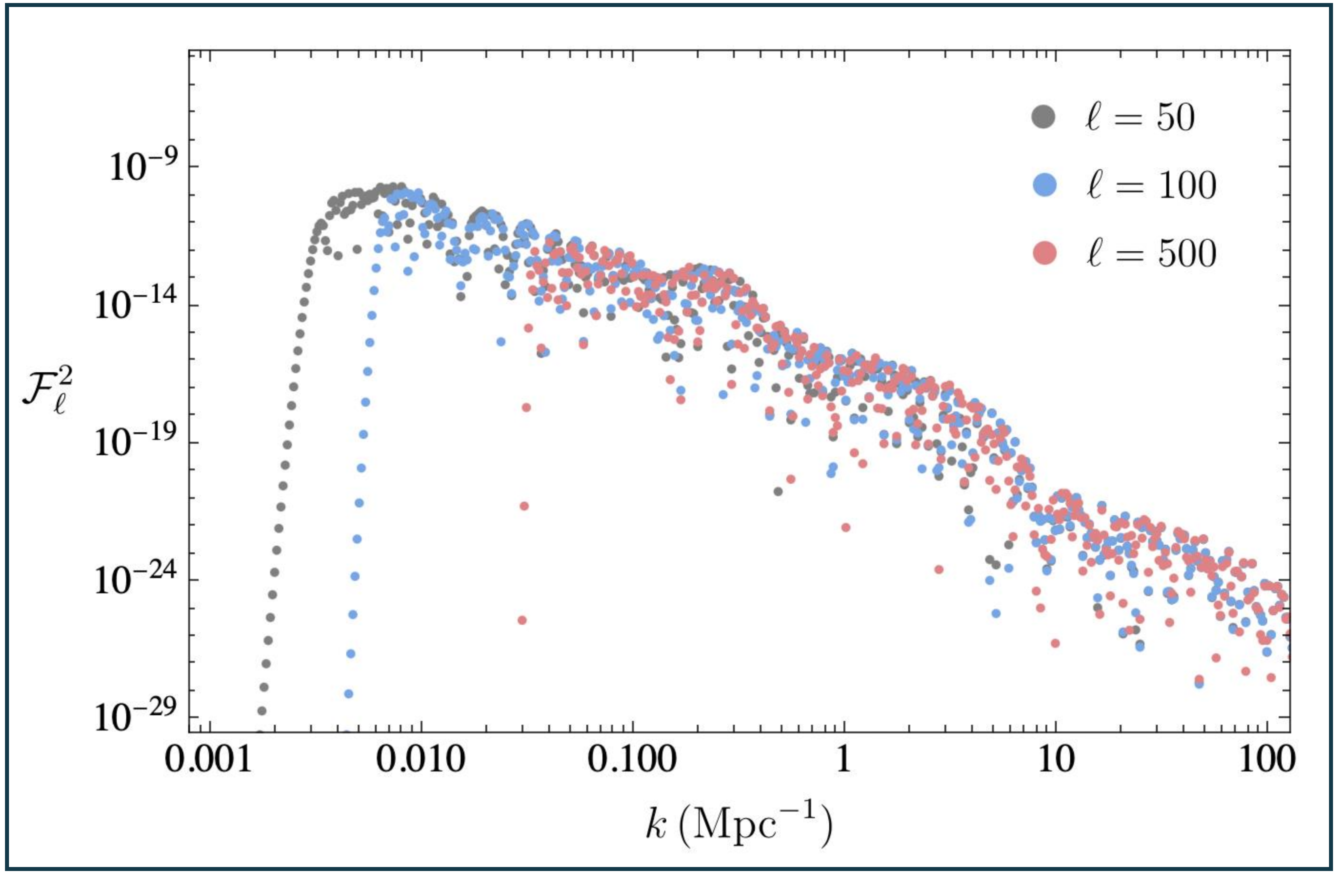
Window function:

- Peaks on large scales
- Shape of B-mode spectrum = result of competition between $\mathcal{F}_\ell(k)^2$ and $\mathcal{P}_h(k)$

power at small k (↑ $\mathcal{F}_\ell(k)^2$)

power at large k (↑ $\mathcal{P}_h(k)$)

$$C_\ell^{BB} \sim \int d\ln k\ \mathcal{P}_h(k)\, \mathcal{F}_\ell(k)^2$$



B-Mode Polarization

Tensor power spectrum:

- Decompose: $h_{ij}(\tau, \vec{k}) = h_{ij}^{\text{ini}}(\vec{k})\, \mathcal{T}(\tau, \vec{k})$ ⟵ $\mathcal{T}$ = transfer function

$$h_{ij}^{\text{ini}}(\vec{k}) = \varepsilon_{ij}^{\lambda}\, h_{\lambda}^{\text{ini}}$$

⟵ $\lambda = +, \times$

- Define:

$$\left\langle h_{\lambda}^{\text{ini}}(\vec{k}) h_{\lambda'}^{\text{ini}}(\vec{k}')^{*} \right\rangle = \frac{2\pi^2}{k^3} \mathcal{P}_h^{\lambda}(k) \delta_{\lambda\lambda'}\, (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}')$$

$$\mathcal{P}_h(k) = \sum_{\lambda=+,\times} \mathcal{P}_h^{\lambda}(k)$$

- $\mathcal{P}_h(k)$ encodes statistical correlations b/w initial amplitudes

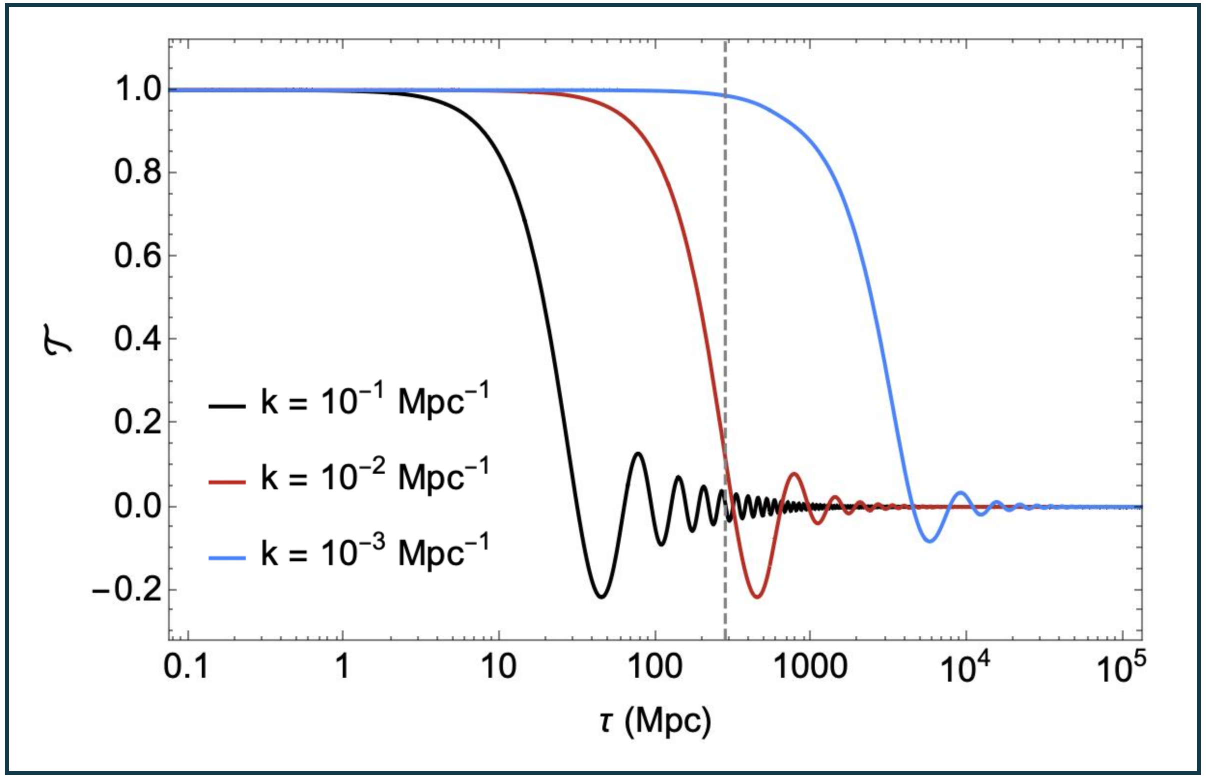
B-Mode Polarization

Transfer function

↪ Captures subsequent evolution

$$\mathcal{T}'' + 2\mathcal{H}\mathcal{T}' + k^2\mathcal{T} \simeq 0$$

$$\Rightarrow \begin{cases} \mathcal{T}_{RD} = A_k\, j_0(k\tau) - B_k\, y_0(k\tau) \\ \mathcal{T}_{MD} = \dfrac{3}{k\tau}\left[C_k\, j_1(k\tau) - D_k\, y_1(k\tau)\right] \end{cases}$$



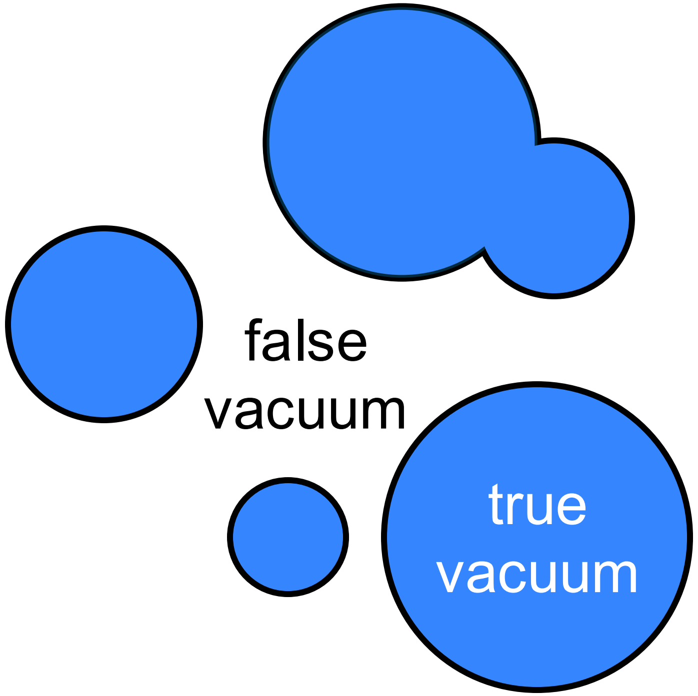
Overview

1) ~~Motivations~~
2) ~~B-Mode Polarization~~
3) First Order Phase Transition
4) Scalar-Induced GWs
5) Summary & future directions

Ex. 1) FOPT

- First-order cosmological phase transitions (FOPTs)
  - Proceed through bubble nucleation
  - Source tensor perturbations in 3 stages

1) Bubble collision phase
   - Collisions break spherical symmetry
   - Gradient energy of scalar sources anisotropic stress
2) Acoustic phase
   - Shells of fluid kinetic energy continue to propagate & collide
3) Turbulent phase
   - Sound wave collisions produce vorticity, turbulence, & shocks



Kamionkowski, Kosowsky, & Turner [astro-ph/9310044]

Ex. 1) FOPT

Caprini, Durrer, & Servant, [0711.2593]

Jinno & Takimoto, [1605.01403]

- Case study: **Supercooled** FOPT in dark sector
  - Negligible plasma friction ⇒ “runaway” bubble wall, $v_w \to 1$
  - Dominant contribution from **bubble collision** stage

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  - Negligible plasma friction ⇒ "runaway" bubble wall, $v_w \to 1$
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- Solve wave eq for $h_{ij}(\tau, \vec{k})$

source

$$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = 8\pi G a^2 \Pi_{ij}$$

conformal Hubble rate, $\mathcal{H} = 1/\tau$

- Source $\Pi_{ij}(\tau, \vec{k})$
  - Transverse, traceless stress tensor
  - Statistically homogeneous, isotropic random variable
  - Short-lived; FOPT completes "fast", $\beta/H_* > 1$ ← PT duration = β^{-1}

Ex. 1) FOPT

$x \equiv k\tau,\ ' = \frac{d}{dx}$

During PT, $x_i \leq x \leq x_f$:

$$h''_{ij} + h_{ij} \approx \frac{8\pi G a_*^2}{k^2} \Pi_{ij} \quad \Rightarrow \quad \boxed{h_{ij}(x) = \frac{8\pi G a_*^2}{k^2} \int_{x_i}^{x} dy\ \sin(x-y)\ \Pi_{ij}(y)}$$

After PT, $x \geq x_f$:

$$h''_{ij} + \frac{2}{x} h'_{ij} + h_{ij} \approx 0 \quad \Rightarrow \quad \boxed{h_{ij}(x) = A_{ij} \frac{\sin(x - x_f)}{x} + B_{ij} \frac{\cos(x - x_f)}{x}}$$

$$A_{ij} = \frac{B_{ij}}{x_f} + \frac{8\pi G a_*^2}{k^2} x_f \int_{x_i}^{x_f} dy \cos(x_f - y)\, \Pi_{ij}(y)$$

$$B_{ij} = \frac{8\pi G a_*^2}{k^2} x_f \int_{x_i}^{x_f} dy \sin(x_f - y)\, \Pi_{ij}(y)$$

Ex. 1) FOPT

Sub-horizon regime, $x_f > 1$:

efficiency factor κ

transition strength $\alpha = \rho_0/\rho_{\rm rad}$

- $h_{ij}^{\rm ini} \equiv h_{ij}(x_f)$: $\mathcal{P}_h^{\rm sub}(k) = 3\kappa^2 \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{H_*}{\beta}\right)^2 \left(\frac{a_* H_*}{k}\right)^2 \Delta\left(\frac{k}{a_*\beta}\right)$

- Define: $\Delta\left(\frac{k}{a_*\beta}\right) \equiv \frac{3}{4\pi^2}\frac{a_*^2\beta^2k^3}{\kappa^2\rho_0^2}\int_{\tau_i}^{\tau_f} d\tau_1 \int_{\tau_i}^{\tau_f} d\tau_2 \cos[k(\tau_1-\tau_2)]\,\Pi(\tau_1,\tau_2,\vec{k})$

unequal time correlator $\Pi \sim \langle \Pi_{ij}(\tau_1)\Pi_{ij}(\tau_2)\rangle$

- $\Delta(x \gg 1) \simeq \frac{0.11}{x}$ Jinno & Takimoto, [1605.01403]

$$\Rightarrow \boxed{\mathcal{P}_h^{\rm sub}(k) = 0.33\kappa^2\left(\frac{\alpha}{1+\alpha}\right)^2\left(\frac{H_*}{\beta}\right)(k\tau_*)^{-3}}$$

$\longleftarrow$ $\tau_* = 1/a_*H_*$

Ex. 1) FOPT

Super-horizon regime, $x_f \ll 1$:

- Activating a super-horizon mode ~ exciting an **overdamped oscillator**
  ↪ takes time $\epsilon \ll 1$ to reach max amplitude
- $h_{ij}^{\rm ini} \equiv h_{ij}(x_f + \epsilon)$: $\mathcal{P}_h^{\rm super}(k) = 3\kappa^2 \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{H_*}{\beta}\right)^2 \Delta\left(\frac{k}{a_*\beta}\right)$
- $\Delta(x \ll 1) \simeq 0.35\, x^3$ Jinno & Takimoto, [1605.01403]

$$\Rightarrow \boxed{\mathcal{P}_h^{\rm super}(k) \simeq 1.1\kappa^2 \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{H_*}{\beta}\right)^5 (k\tau_*)^3}$$

- Expected k^3 causality-limited scaling ⇒ **white noise**!
- **Suppressed** since $(k\tau_*) \ll 1$ for super-horizon modes

Aside: White Noise and Causality Cai, Pi, & Sasaki [1909.13728]

- Real space 2-point correlator: $\langle \Pi_\lambda(t,\vec{x})\, \Pi_{\lambda'}(t,\vec{x}')\rangle \equiv \delta_{\lambda\lambda'}\, \xi(r)$

- Relation to power spectrum: $P_\Pi(k) = 4\pi \int_0^\infty dr\, r^2\, \frac{\sin kr}{kr}\, \xi(r)$

- Causal source: $\xi(r) = \begin{cases} \xi(r) & r \le R \\ 0 & r > R \end{cases}$ ⟵ correlation scale $R < H^{-1}$

$$\Rightarrow \quad P_\Pi(k) = 4\pi \int_0^R dr\, r^2\, \frac{\sin kr}{kr}\, \xi(r)$$

Aside: White Noise and Causality

Cai, Pi, & Sasaki [1909.13728]

- For $k \to 0$: $\frac{\sin kr}{kr} \simeq 1 - \frac{(kr)^2}{3!} + \frac{(kr)^4}{5!} - \cdots$
- Power spectrum: $P_\Pi(k) = A_0 + A_2 k^2 + A_4 k^4 + \cdots$

$$A_0 = 4\pi \int_0^R dr\, r^2\, \xi(r),\ A_2 = 4\pi \int_0^R dr\, r^4\, \xi(r),\ \text{etc.}$$

- Dimensionless power spectrum:

"white noise" scaling!

$$\mathcal{P}_\Pi(k) = \frac{k^3}{2\pi^2} P_\Pi(k) \quad \Rightarrow \quad \mathcal{P}_\Pi(k) \sim k^3$$

Ex. 1) FOPT

Full spectrum:

- Peak set by max bubble size, $k\tau_* \sim \beta/H_*$ $\Rightarrow$ $k_p \simeq 1.24\left(\frac{\beta}{H_*}\right)\tau_*^{-1}$

- Thus:

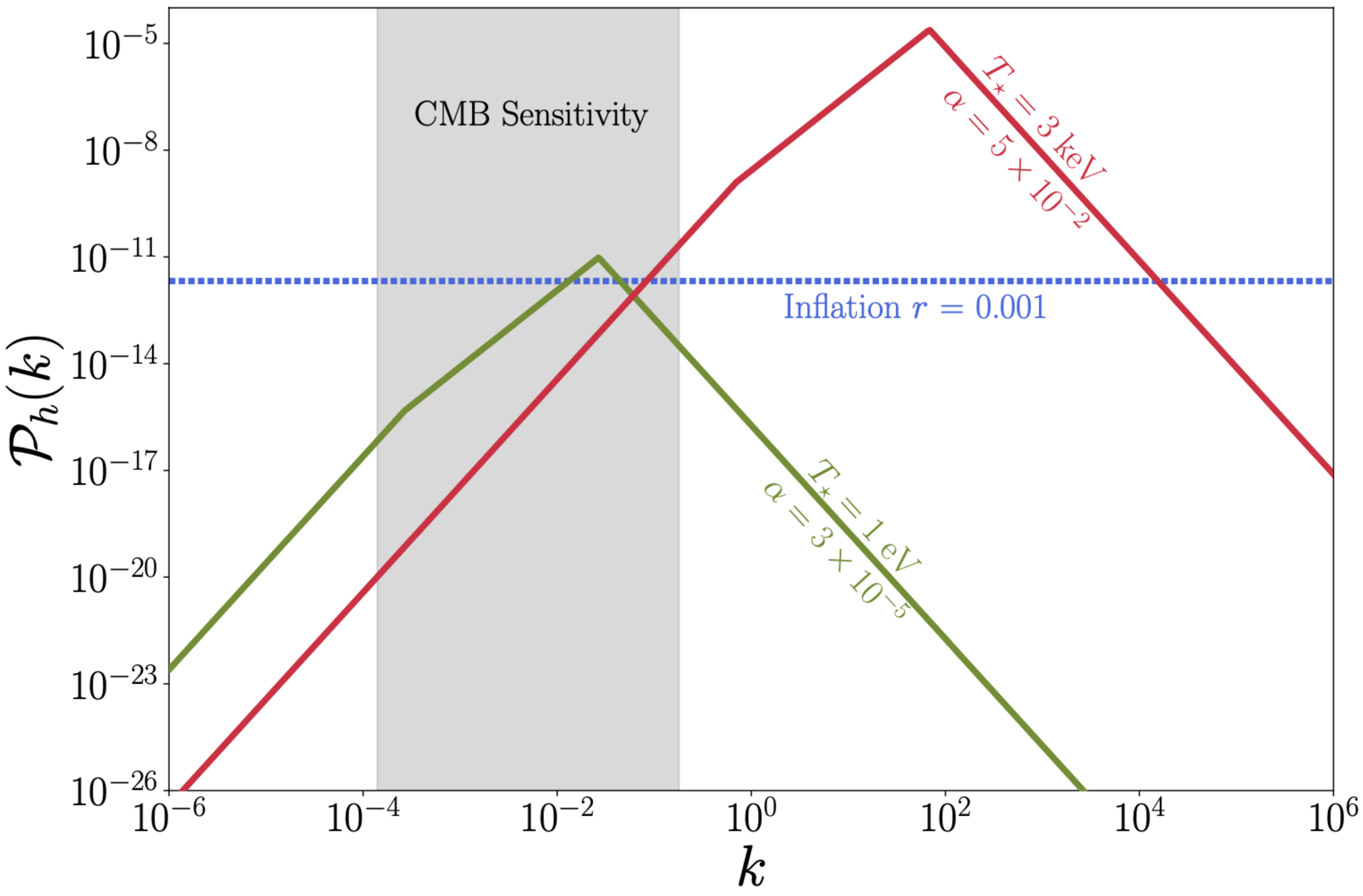
$$\mathcal{P}_h = \begin{cases} 0.33\kappa^2\left(\frac{\alpha}{1+\alpha}\right)^2\left(\frac{H_*}{\beta}\right)(k\tau_*)^{-3} & k \geq k_p \\ \mathcal{P}_h^{\text{int}} & k_b \leq k \leq k_p \\ 1.1\kappa^2\left(\frac{\alpha}{1+\alpha}\right)^2\left(\frac{H_*}{\beta}\right)^5(k\tau_*)^3 & k \leq k_b \end{cases}$$

corresponds to modes that are **sub-horizon** but **super-bubble**

"Breaking scale": $k_b \equiv \epsilon_b k_p,\ \epsilon_b \ll 1$

We will assume power law: $\mathcal{P}_h^{\text{int}} = Ak^m$

Ex. 1) FOPT

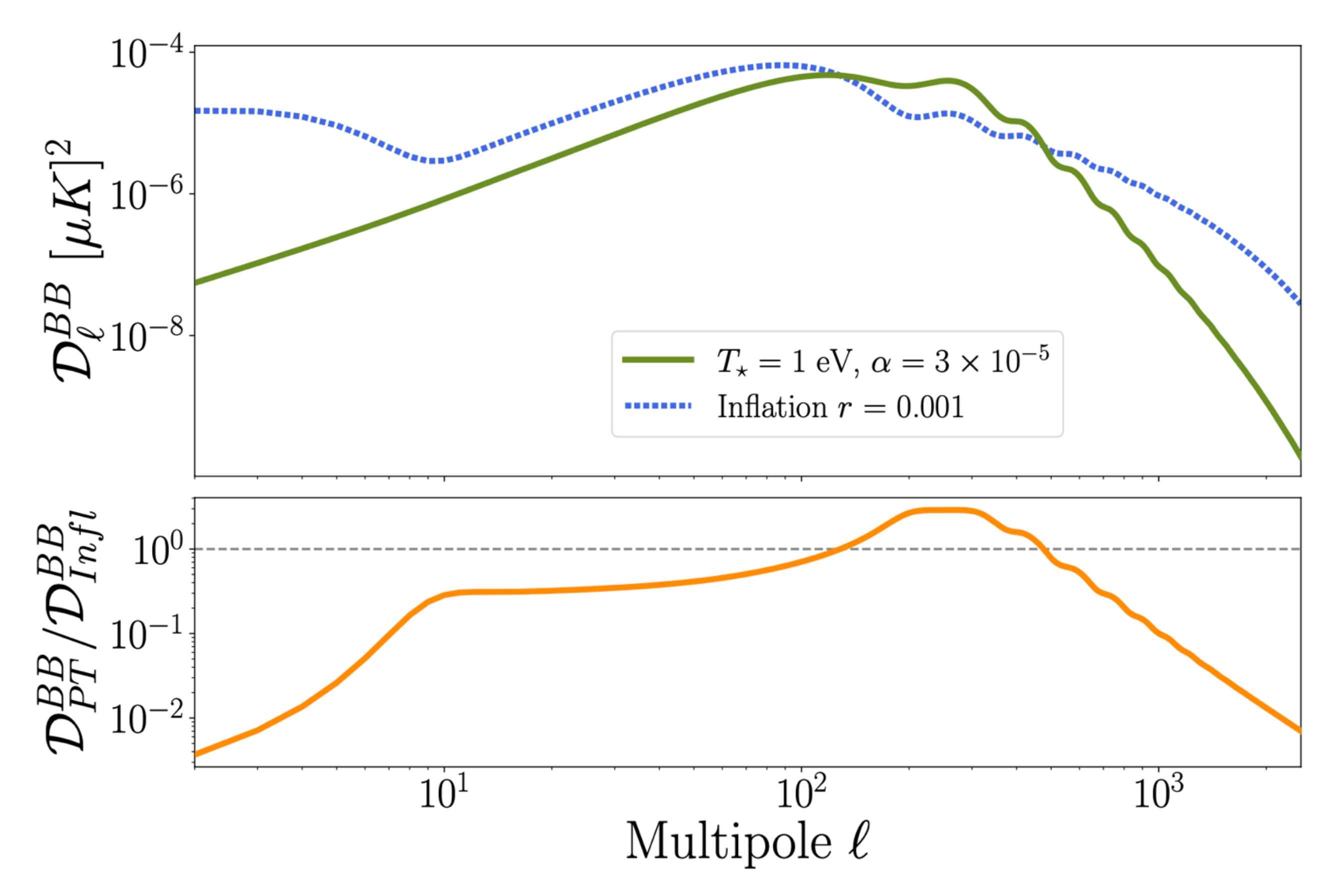


Solid green: PT with $\kappa = 1$, $\beta/H_* = 2$, $\alpha = 3 \times 10^{-5}$, $T_* = 1$ eV

Solid red: PT with $\kappa = 1$, $\beta/H_* = 2$, $\alpha = 5 \times 10^{-2}$, $T_* = 3$ keV

Dotted blue: Inflation with tensor-to-scalar ratio $r = 0.001$

Ex. 1) FOPT

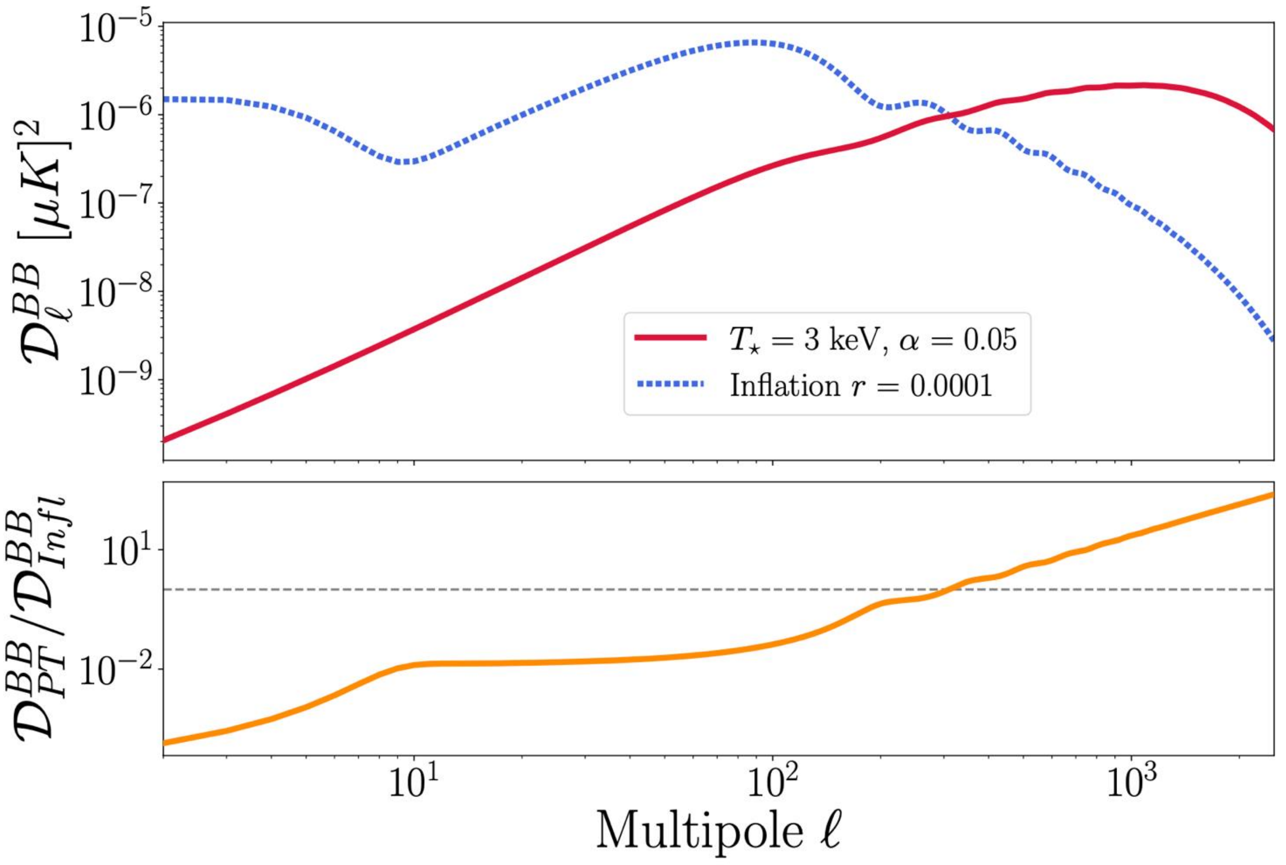


Top: B-mode polarization spectrum* for PT with $\kappa = 1$, $\alpha = 3 \times 10^{-5}$, $\beta/H_* = 2$, $T_* = 1$ eV (**solid green**) compared with inflationary prediction with $r = 0.001$ (**dotted blue**).

Bottom: Ratio of B-mode signals (**solid orange**) showing support at different multipoles.

*plotted in terms of $\mathcal{D}_\ell^{BB} = \frac{\ell(\ell+1)}{2\pi} T_0^2 C_\ell^{BB}$; lensing removed

Ex. 1) FOPT



Top: B-mode polarization spectrum* for PT with $\kappa = 1$, $\alpha = 5 \times 10^{-2}$, $\beta/H_* = 2$, $T_* = 3$ keV (**solid red**) compared with inflationary prediction with $r = 0.0001$ (**dotted blue**).

Bottom: Ratio of B-mode signals (**solid orange**) showing support at different multipoles.

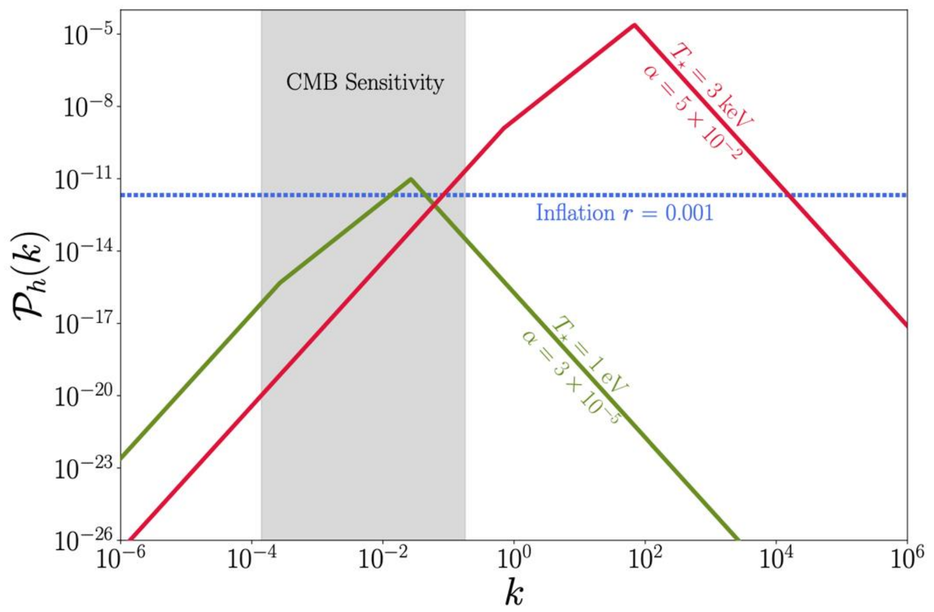
*plotted in terms of $\mathcal{D}_\ell^{BB} = \frac{\ell(\ell+1)}{2\pi} T_0^2 C_\ell^{BB}$; lensing removed

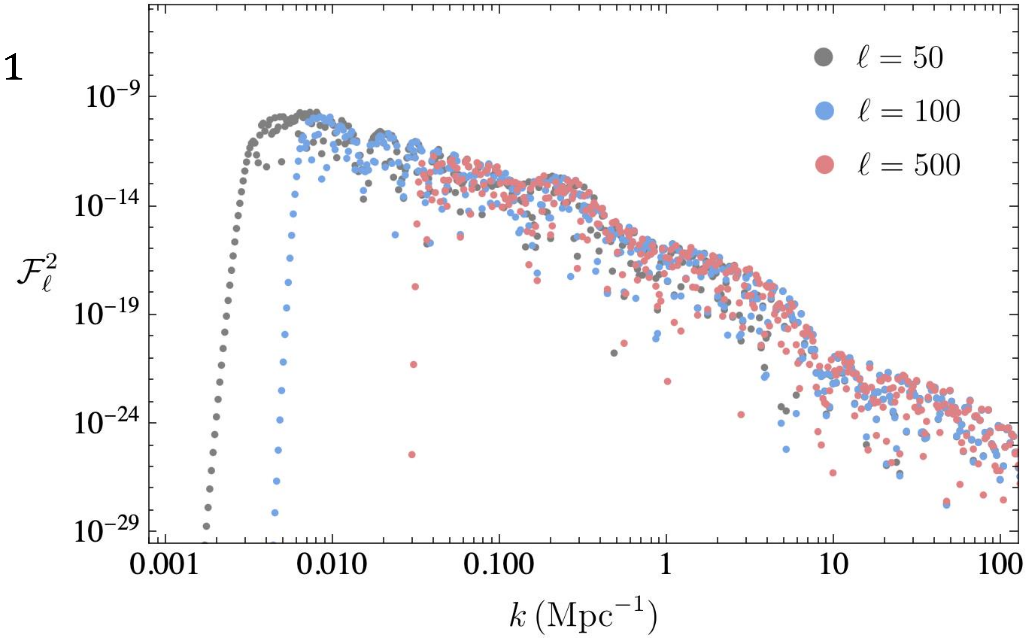
Ex. 1) FOPT

How to understand spectral shape?

$$C_\ell^{BB} \sim \int_0^\infty \frac{dk}{k} \mathcal{P}_h(k)\, \mathcal{F}_\ell(k)^2$$

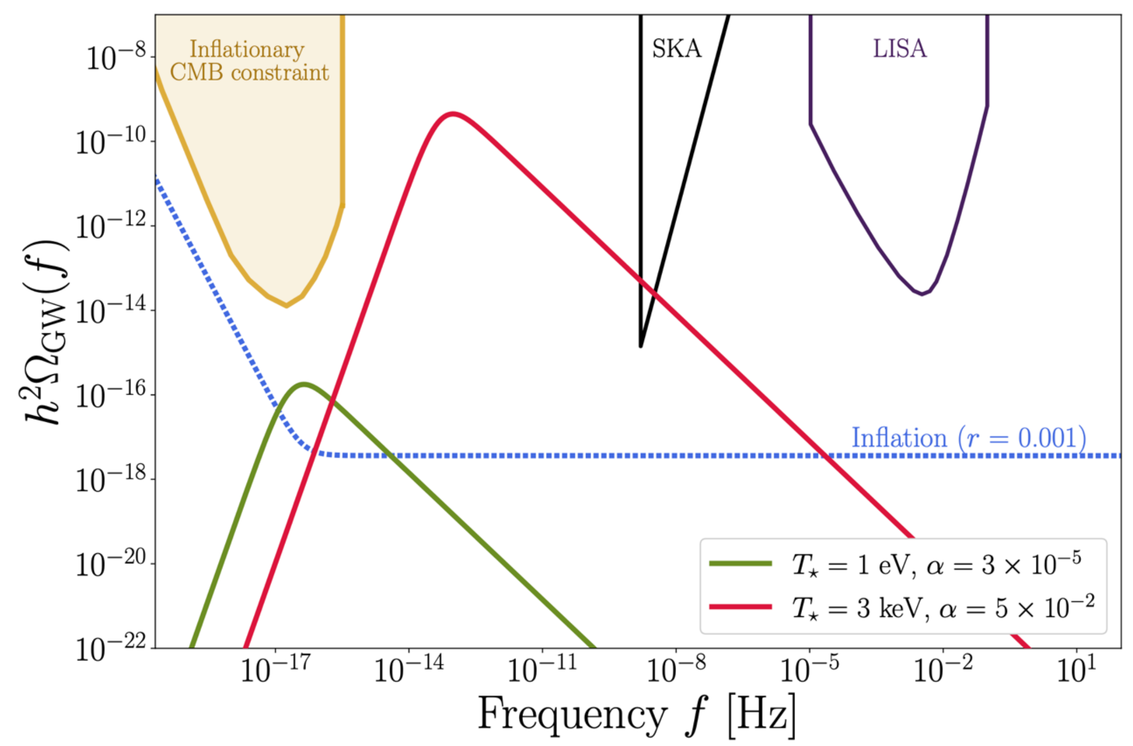
- Inflation: $\mathcal{P}_h \sim k^0$
  ↪ dominant contribution from $k \sim 0.01\ \mathrm{Mpc}^{-1}$
- PT: $\mathcal{P}_h \sim (k\tau_*)^3$
  ↪ dominant contribution from small scales (despite suppression in $\mathcal{F}_\ell^2$)
  ↪ spectrum peaks at **larger ℓ**





Ex. 1) FOPT

The same tensor perturbations also contribute to the SGWB
↪ **complimentary signature** of this scenario



Solid green: PT with
$\kappa = 1, \beta/H_* = 2,$
$\alpha = 3 \times 10^{-5}, T_* = 1\text{ eV}$

Solid red: PT with
$\kappa = 1, \beta/H_* = 2,$
$\alpha = 5 \times 10^{-2}, T_* = 3\text{ keV}$

Dotted blue: Inflation
with tensor-to-scalar
ratio $r = 0.001$

Overview

1) ~~Motivations~~
2) ~~B-Mode Polarization~~
3) ~~First Order Phase Transition~~
4) Scalar-Induced GWs
5) Summary & future directions

Ex. 2) Scalar-Induced GWs

- Scalar-induced GWs   Domènech, [2109.01398]
  - **Curvature perturbations** $\mathcal{R}$ source tensor perturbations at **2nd order** in cosmological perturbation theory
  - Enhanced small-scale $\mathcal{P}_\mathcal{R} \Rightarrow$ non-trivial $\mathcal{P}_h$

- First, solve wave eq for $h_\lambda(\tau, \vec{k})$

source

$$h_\lambda'' + 2\mathcal{H} h_\lambda' + k^2 h_\lambda = \mathcal{S}_\lambda(\tau, \vec{k})$$

$$\mathcal{S}_\lambda(\tau, \vec{k}) = \int \frac{d^3 q}{(2\pi)^{3/2}} \underbrace{\varepsilon_\lambda^{ij} q_i q_j}_{\text{projection factor}} \underbrace{f(\tau, |\vec{k} - \vec{q}|, q)}_{\text{time evolution/ transfer function}} \underbrace{\mathcal{R}(\vec{k} - \vec{q}) \mathcal{R}(\vec{q})}_{\text{curvature perturbations}}$$

Ex. 2) Scalar-Induced GWs

- Solution:

$$h_\lambda(\tau,\vec{k}) = 4\int \frac{d^3q}{(2\pi)^{3/2}}\, \varepsilon_\lambda^{ij} q_i q_j\, I(\tau,|\vec{k}-\vec{q}|,q)\, \mathcal{R}(\vec{k}-\vec{q})\mathcal{R}(\vec{q})$$

$$I(\tau,|\vec{k}-\vec{q}|,q) = \frac{1}{a}\int_0^\tau d\tilde{\tau}\, \frac{\sin[k(\tau-\tilde{\tau})]}{k\tilde{\tau}} f(\tau,|\vec{k}-\vec{q}|,q)$$

- Power spectrum:

$$\left\langle h_\lambda^{\mathrm{ini}}(\vec{k}) h_{\lambda'}^{\mathrm{ini}}(\vec{k}')^* \right\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_h^\lambda(k)\delta_{\lambda\lambda'}\,(2\pi)^3\delta^{(3)}(\vec{k}-\vec{k}')$$

h_λ peaks at horizon crossing; define $h_\lambda^{\mathrm{ini}} = h_\lambda|_{k\tau=1}$

Ex. 2) Scalar-Induced GWs

- In general:

$$\left\langle h_\lambda^{\rm ini}(\vec{k}_1) h_{\lambda'}^{\rm ini}(\vec{k}_2)^* \right\rangle \sim \left\langle \mathcal{R}(\vec{k}_1 - \vec{q}_1)\mathcal{R}(\vec{q}_1)\mathcal{R}(\vec{k}_2 - \vec{q}_2)^* \mathcal{R}(\vec{q}_2\ )^* \right\rangle$$

- Decompose into connected (trispectrum) and disconnected contributions
- Assumption: **Gaussian** statistics

↪ Connected contribution vanishes

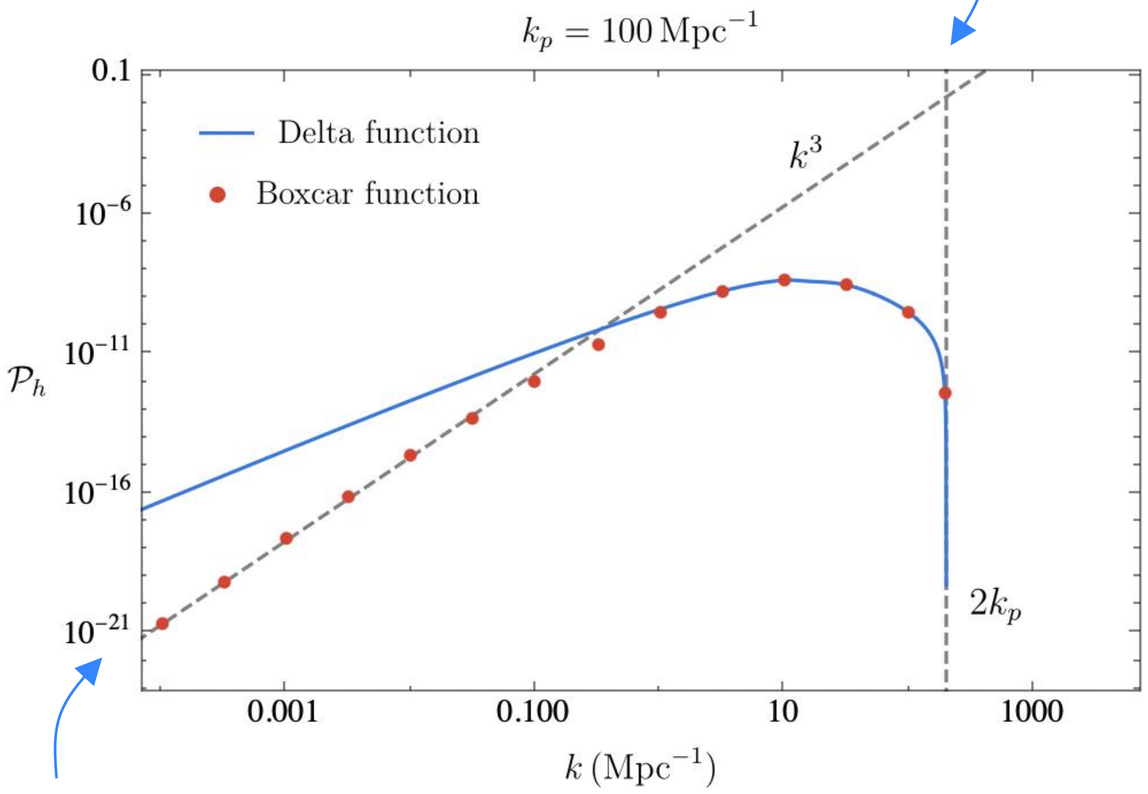
↪ $\langle hh \rangle \propto P_\mathcal{R}(|\vec{k} - \vec{q}|) P_\mathcal{R}(q)$

$$\mathcal{P}_h^\lambda(k) = \left(\frac{k^3}{2\pi^2}\right) 32 \int \frac{d^3q}{(2\pi)^{3/2}} \left(\varepsilon_\lambda^{ij} q_i q_j\right)^2 I(\tau, |\vec{k} - \vec{q}|, q)^2\, P_\mathcal{R}(|\vec{k} - \vec{q}|) P_\mathcal{R}(q)$$

Ex. 2) Scalar-Induced GWs

- Precise shape of $\mathcal{P}_h$ depends on choice of $\mathcal{P}_\mathcal{R}$
- Delta function? $\mathcal{P}_\mathcal{R} = \mathcal{A}\delta\left(\ln\left(\frac{k}{k_p}\right)\right)$
  ↪ Unphysical k^2 IR scaling
  ↪ Don't use ✖
- Narrow-peaked boxcar or lognormal
  ↪ Expected k^3 IR scaling 🙂
  ↪ Peaks at different scale than $\mathcal{P}_\mathcal{R}$

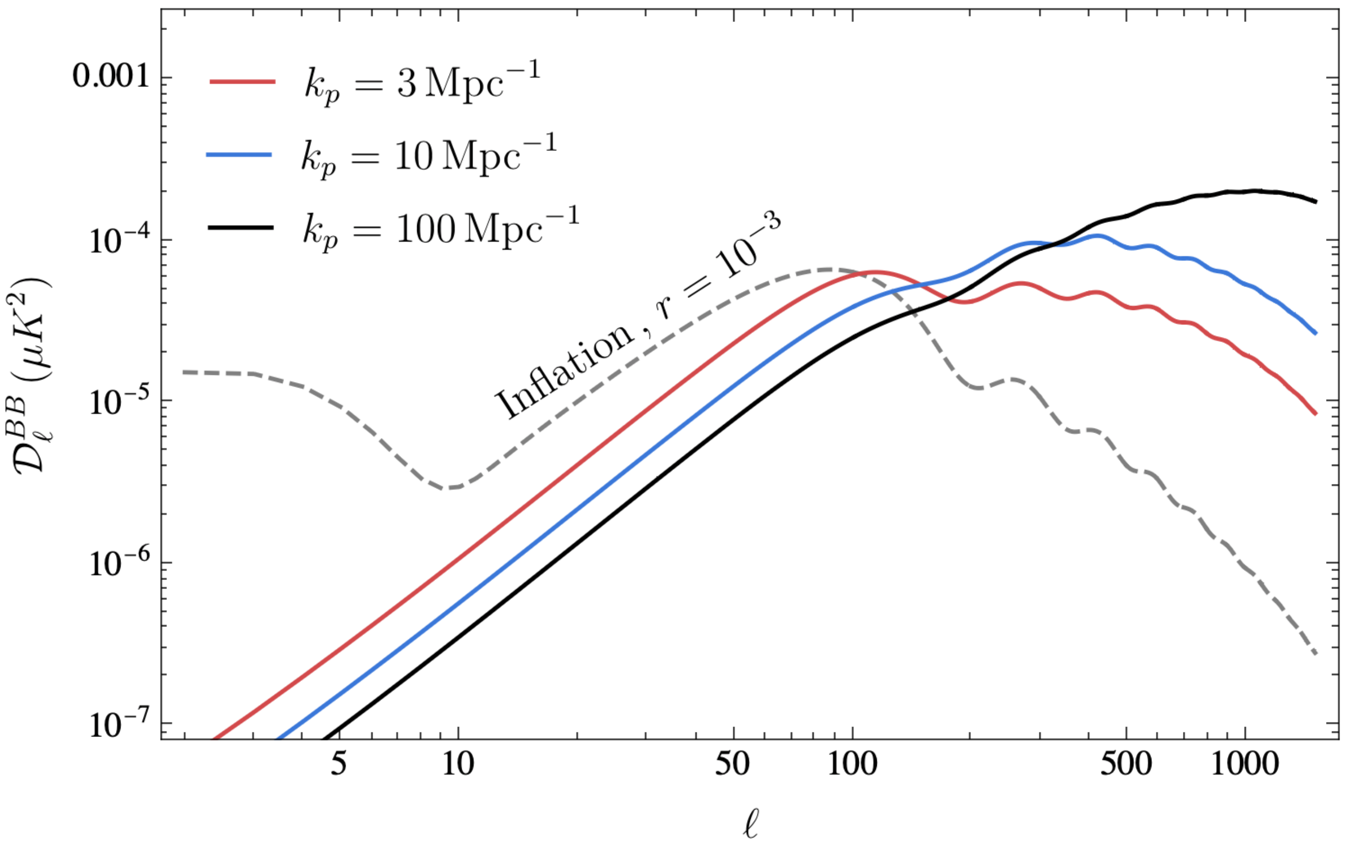
cutoff at $k = 2k_p$ from momentum conservation



expected $\sim k^3$ causality-limited scaling

Ex. 2) Scalar-Induced GWs

$$B(k, k_p, \Delta) = \begin{cases} 1/\Delta & k_p e^{-\Delta/2} \leq k \leq k_p e^{\Delta/2} \\ 0 & \text{otherwise} \end{cases}$$



Scalar power spectrum:

$$\mathcal{P}_\mathcal{R} = \mathcal{A}\, B(k, k_p, \Delta)$$

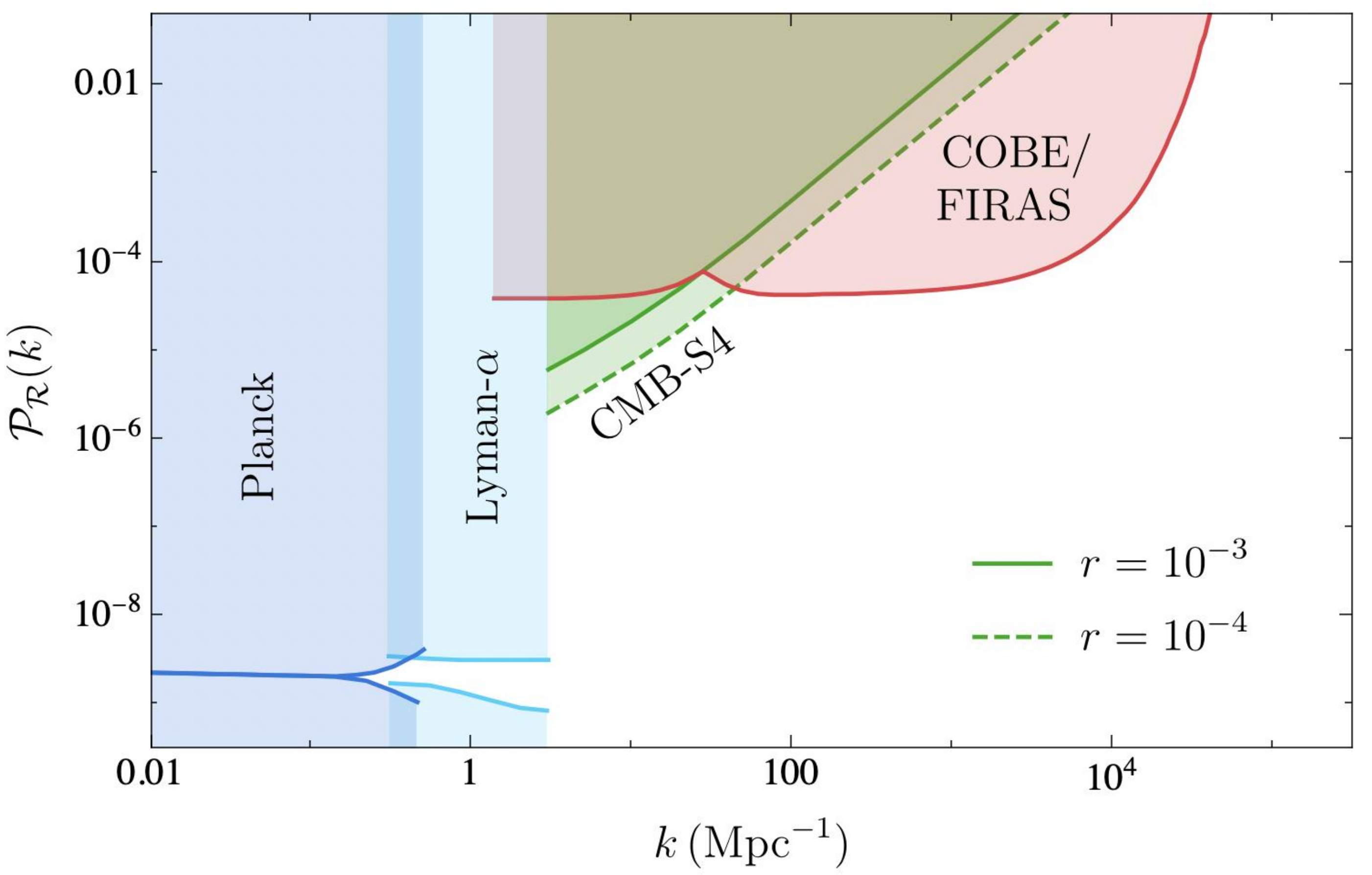
with $\mathcal{A}$ chosen such that SNR matches inflationary signal with $r = 10^{-3}$

(presuming noise spectra for CMB-S4 like experiment)

*plotted in terms of $\mathcal{D}_\ell^{BB} = \frac{\ell(\ell+1)}{2\pi} T_0^2 C_\ell^{BB}$; lensing removed

Ex. 2) Scalar-Induced GWs

Can also map this future sensitivity range to scalar power spectrum



Non-observation of CMB B-modes (beyond lensing contribution) in future experiments would allow us to **constrain $\mathcal{P}_{\mathcal{R}}$**

Overview

1) ~~Motivations~~
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Summary

- B-modes from **non-inflationary primordial sources** can be **competitive** with inflation for values of r targeted in upcoming CMB experiments

- Repackaging tensor perturbations into B-modes gives access to scales **outside sensitivity** of traditional probes
  ↪ FOPT: Stochastic GW background (SGWB)
  ↪ Scalar induced GWs: SGWB, $\mathcal{P}_{\mathcal{R}}$

- Existence of primordial B-modes can **complicate** an **inflationary interpretation** for future B-mode measurements
  ↪ **Distinct** spectral shapes ⇒ scenarios can be **distinguished**
  ↪ Large degeneracy with lensing spectrum

Sneak Peak

Are there any constraints we can place now?

Sneak Peak

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- k^3 IR scaling is a generic prediction of any* non-inflationary primordial causal sources of tensor perturbations
  ↪ Consequence of causality and finite correlation lengths
  ↪ Universally valid for k^{-1} larger than physical scales associated with the source

*Cai, Pi, & Sasaki [1909.13728]

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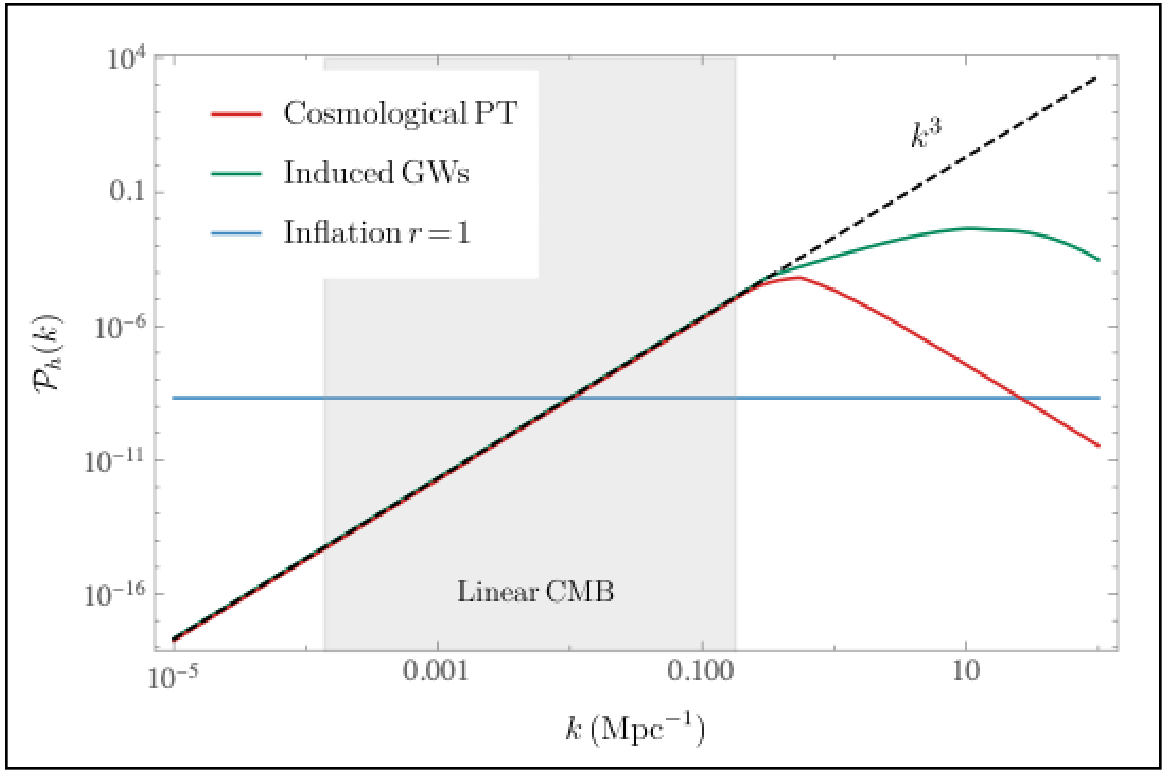
- Universal behavior ⇒ **universal shape** for B-mode spectrum

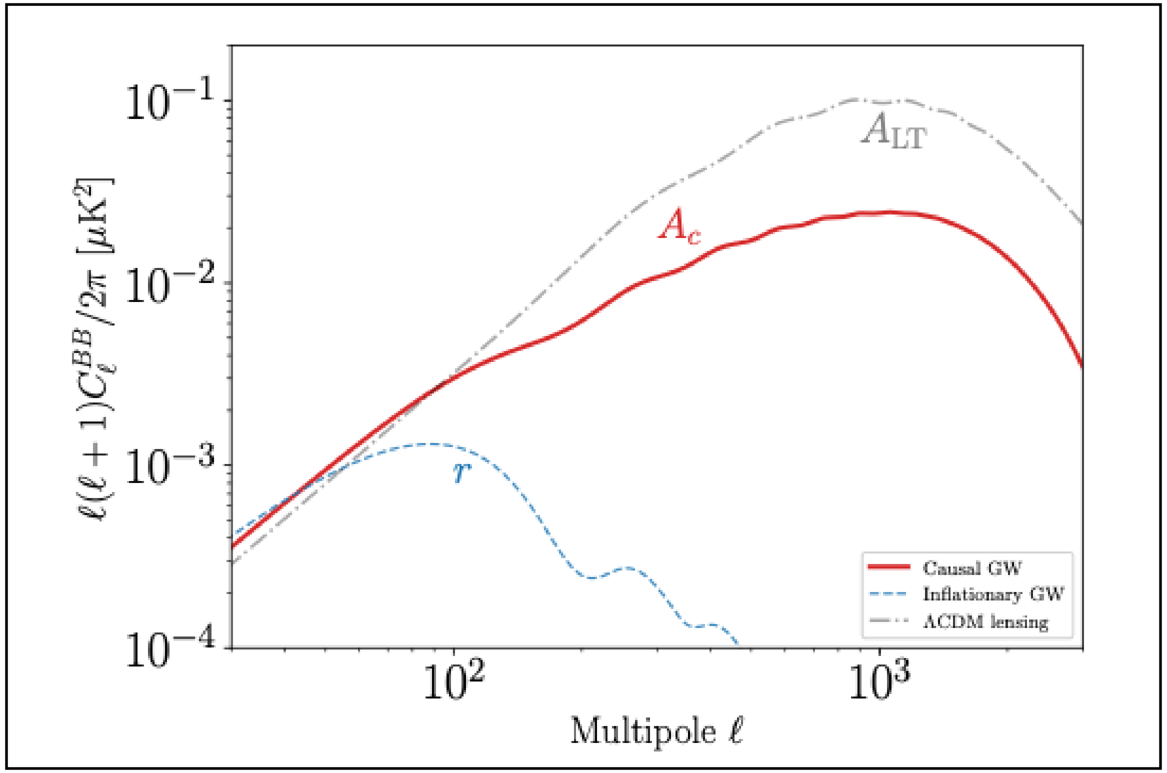
- Idea: Set **generic constraint** on amplitude of this causal tail
  ↪ Can easily translate to constraint on your primordial source of choice

*Cai, Pi, & Sasaki [1909.13728]

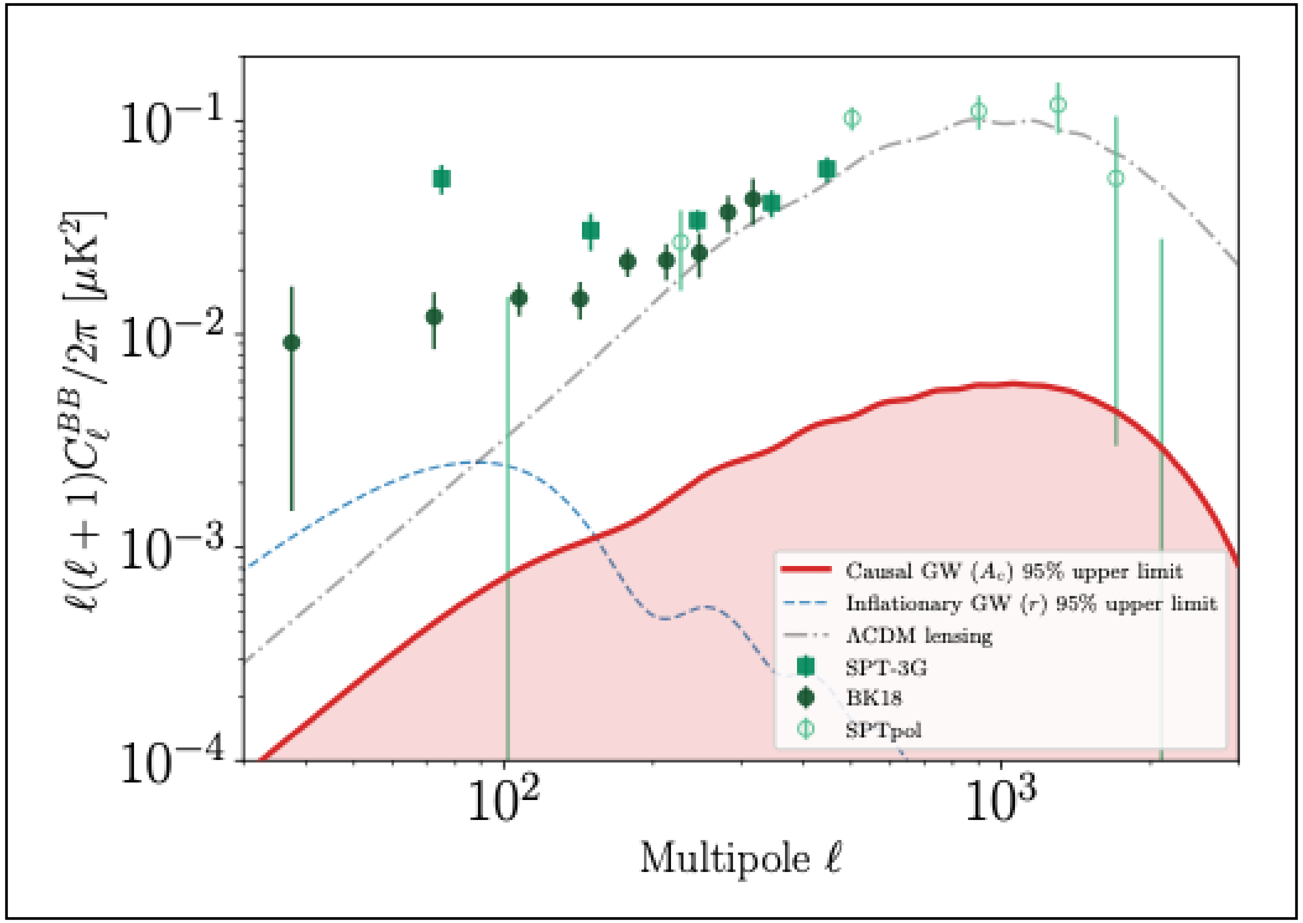
Sneak Peak

- Parameterize as: $\mathcal{P}_h = A_c A_s \left(\frac{k}{k_{\rm ref}}\right)^3$





Sneak Peak



$A_c < 0.0048$ at 95% CL

$\sigma(A_c) = 0.0015$

Future Directions

- Constraints on generic causal sources
- Predictions for realistic particle physics models
- Other sources of tensor perturbations
  - ↪ FOPTs: sound waves, turbulence
  - ↪ Cosmic strings
  - ↪ Domain walls

Back-Up Slide: Parameter Space