UV constraints on IR modifications of gravity

Kurt Hinterbichler (Case Western)

Berkeley, April 16, 2019

Arxiv: 1708.05716, 1804.08686, 1903.09643

w/ James Bonifacio, Austin Joyce, Rachel Rosen

The Cosmological constant:

Given the matter (and the dark matter) we know about, Einstein's equations aren't satisfied:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \neq \frac{1}{M_P^2} T_{\mu\nu}$$

observed expansion history

observed mass/energy
(including dark matter)

Easy to fix:

Alter left hand side:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{m_P^2} T_{\mu\nu}$$

"Cosmological constant"

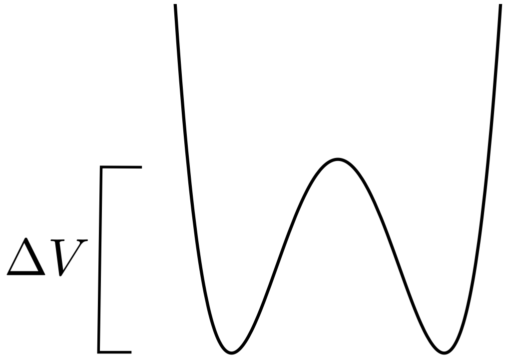
Or alter right hand side:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{m_P^2} \left[T_{\mu\nu} - m_P^2 \Lambda g_{\mu\nu} \right]$$

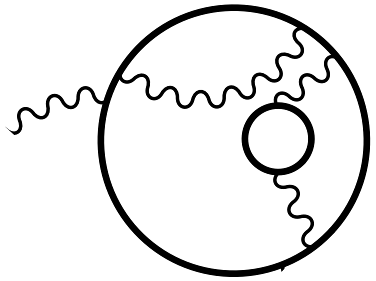
"Mysterious dark energy"

Contributions to Lambda are expected

- Phase transitions: $\Lambda_{\text{phase}} \sim \Delta V / M_P^2$



- Vacuum energy of quantum fields:



$$\Rightarrow \Lambda_{\text{quantum}} \sim \frac{m_{\text{particle}}^4}{M_P^2}$$

calculable contributions

- Bare CC: $\mathcal{L}_{UV} \sim M_P^2 \int d^4x \sqrt{-g}\, \Lambda_{\text{Bare}} + \cdots$

un-known contribution

Observed Cosmological Constant is the sum of everything:

$$\Lambda_{\text{observed}} = \Lambda_{\text{Bare}} + \Lambda_{\text{quantum}} + \Lambda_{\text{phase}} + \cdots$$

The cosmological constant problem

$$\frac{\Lambda}{M_P^2} \sim 10^{-122}$$

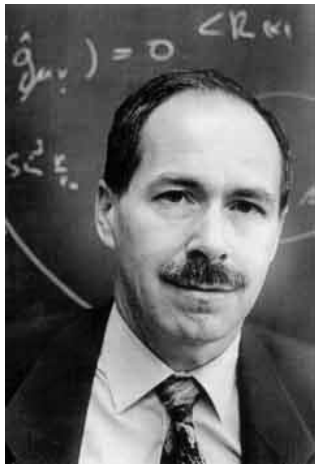
really small

$$\Lambda_{\text{observed}} = \Lambda_{\text{UV}} + \Lambda_{\text{known physics}} + \Lambda_{\text{phase transitions}} + \cdots$$

- The cosmological constant is not calculable (relevant operator, sensitive to unknown UV physics)
- Its observed value is small (not *natural*)
- A small value is unstable under deformations of UV physics (not *technically natural*)

"not natural."

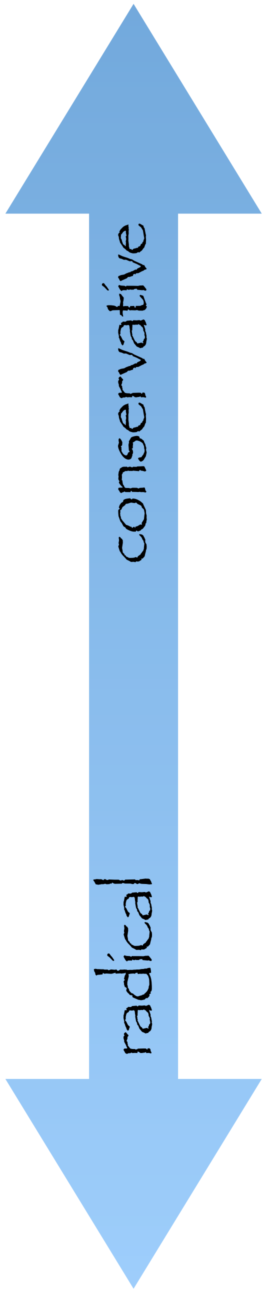
Paul Dirac



"not technically natural."

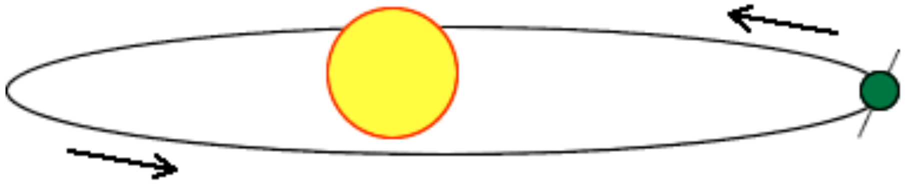
Gerard 't Hooft

Possibilities

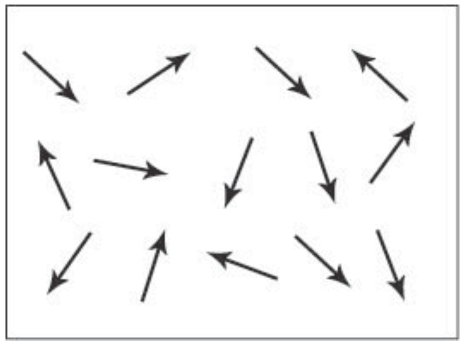


- Sheer luck/no explanation

- Anthropics



- Modified or additional dynamics (new DOF)



- Calculation wrong (the rules of effective quantum field theory are not what we think they are on very large scales)

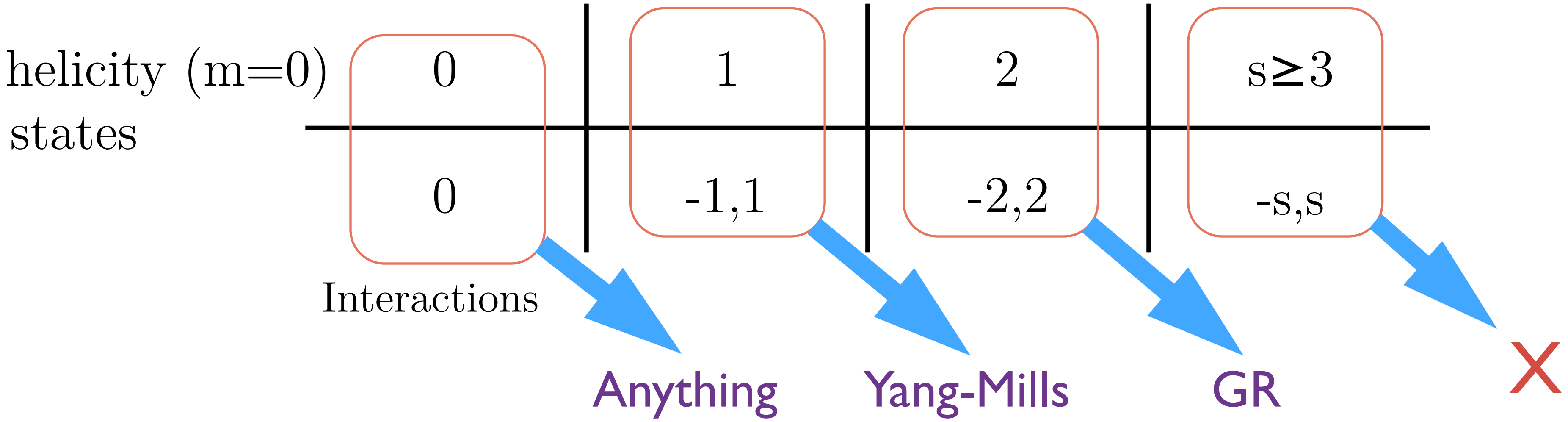
$2+2=5$

Lorentz invariance: degrees of freedom & interactions

- Basic principles: Locality, Lorentz-Invariance ⟹

Eugene Wigner

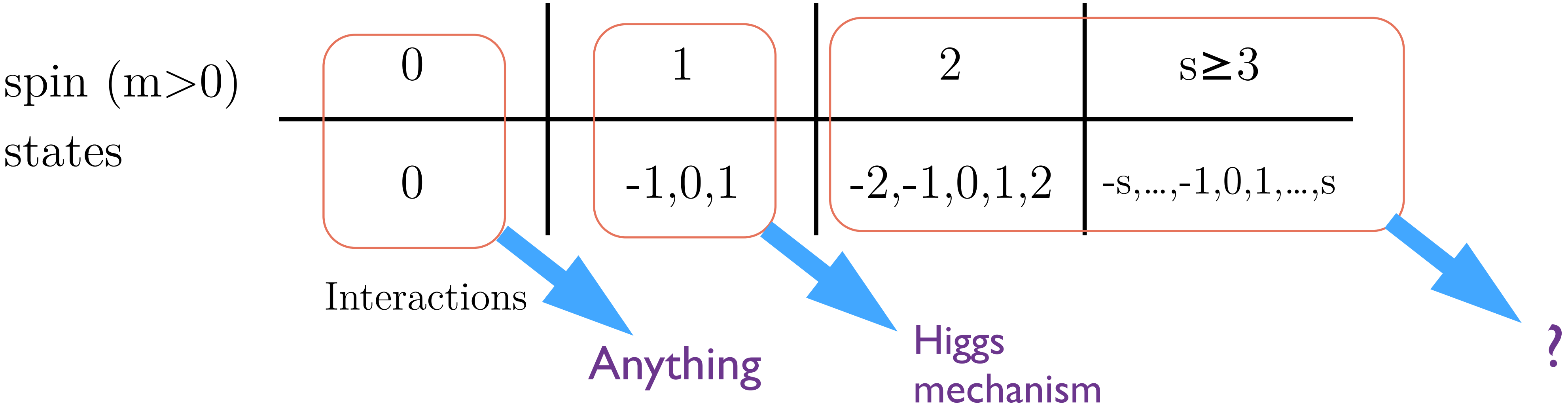
degrees of freedom are classified by mass and spin/helicity



helicity (m=0) states	0	1	2	s≥3
	0	-1,1	-2,2	-s,s
Interactions	Anything	Yang-Mills	GR	X

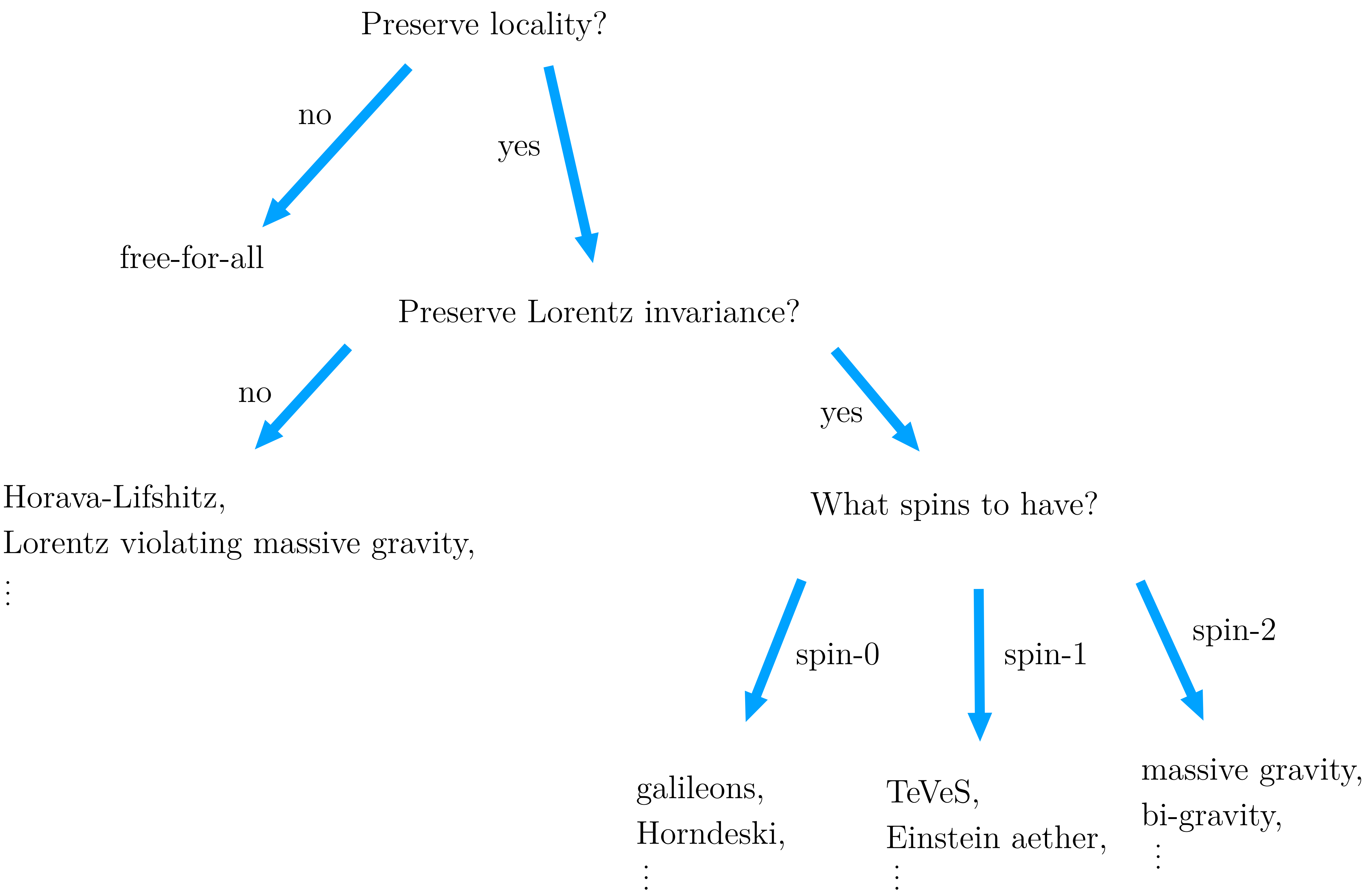
Theorems:

- Yang-mills is the only way for helicity-1's to interact at low energies
- GR is the only way for helicity-2 to self-interact at low energies
- Helicity ≥3 can't interact at low energies



spin (m>0) states	0	1	2	s≥3
	0	-1,0,1	-2,-1,0,1,2	-s,...,-1,0,1,...,s
Interactions	Anything	Higgs mechanism	?	

Classes of models



Preserve locality?

no

free-for-all

yes

Preserve Lorentz invariance?

no

Horava-Lifshitz,
Lorentz violating massive gravity,
⋮

yes

What spins to have?

spin-0

galileons,
Horndeski,
⋮

spin-1

TeVeS,
Einstein aether,
⋮

spin-2

massive gravity,
bi-gravity,
⋮

Scalars

Minimal new stuff: a single cosmologically relevant scalar

Horndeski theory

Gregory Horndeski (1974)

Deffayet, Deser, Esposito-Farese (2009)

Most general scalar tensor theory with 2nd order equations of motion

Non-linear EFT parametrization of a single degree of freedom

arbitrary functions of ϕ , $X \equiv -\frac{1}{2}(\partial\phi)^2$

$$\mathcal{L}_2 = \sqrt{-g}\, G_2(\phi, X)$$

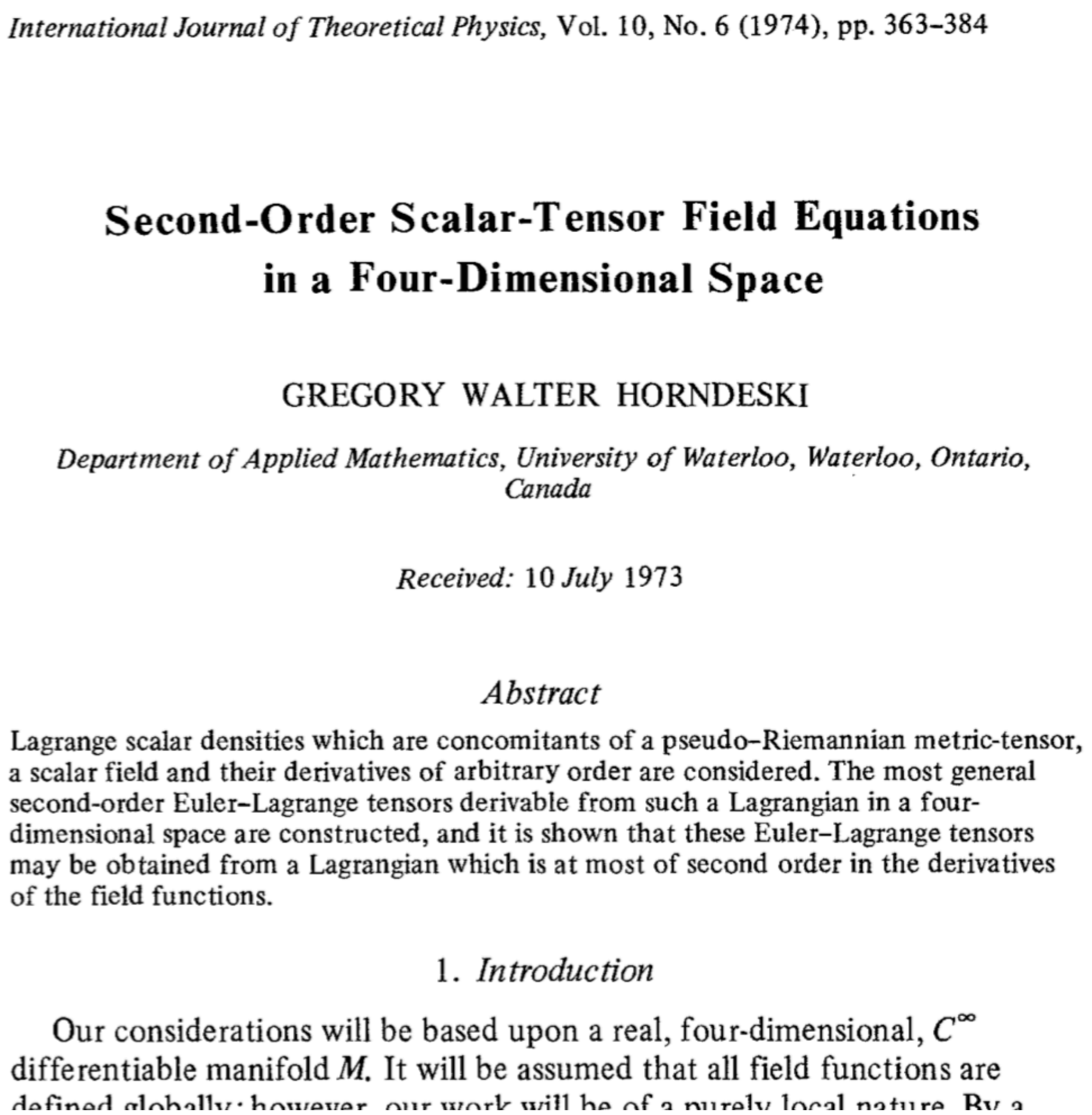
$$\mathcal{L}_3 = \sqrt{-g}\, G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(X)R + G_{4,X}(X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = \sqrt{-g}\left[-\frac{1}{6}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right) + G_5(\phi, X)\nabla^\mu\nabla^\nu\phi\, G_{\mu\nu}\right]$$

Similar Lagrangians exist with multiple scalars, vectors, etc.

Horndeski theory



International Journal of Theoretical Physics, Vol. 10, No. 6 (1974), pp. 363–384

Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

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Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada

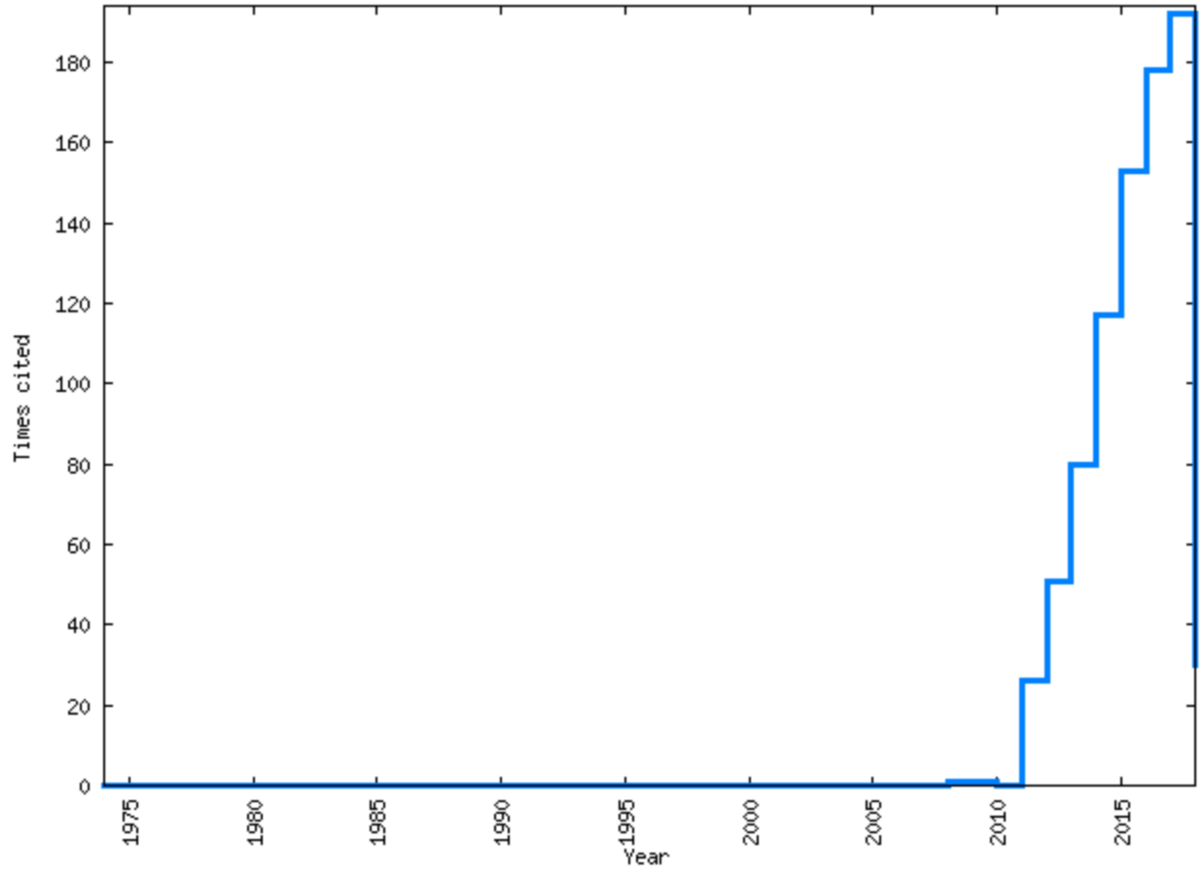
Received: 10 *July* 1973

Abstract

Lagrange scalar densities which are concomitants of a pseudo–Riemannian metric-tensor, a scalar field and their derivatives of arbitrary order are considered. The most general second-order Euler–Lagrange tensors derivable from such a Lagrangian in a four-dimensional space are constructed, and it is shown that these Euler–Lagrange tensors may be obtained from a Lagrangian which is at most of second order in the derivatives of the field functions.

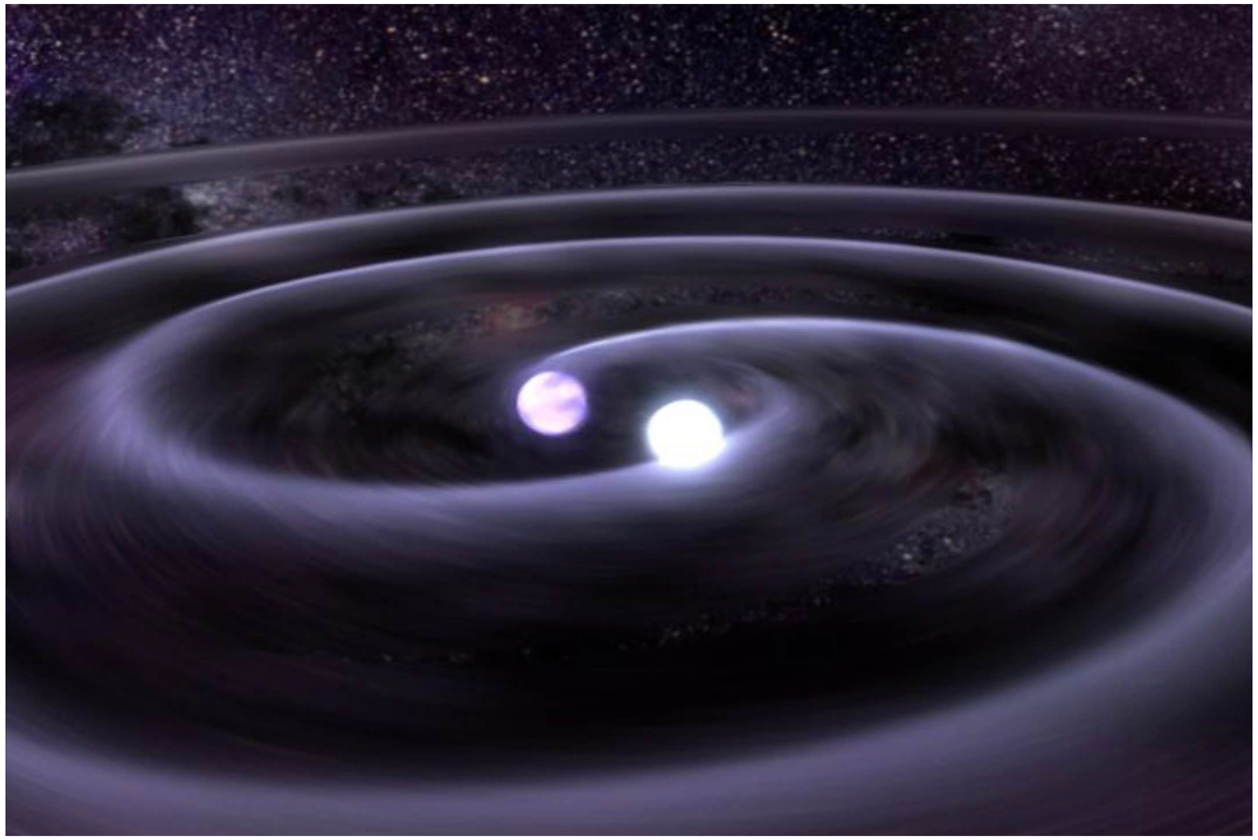
1. *Introduction*

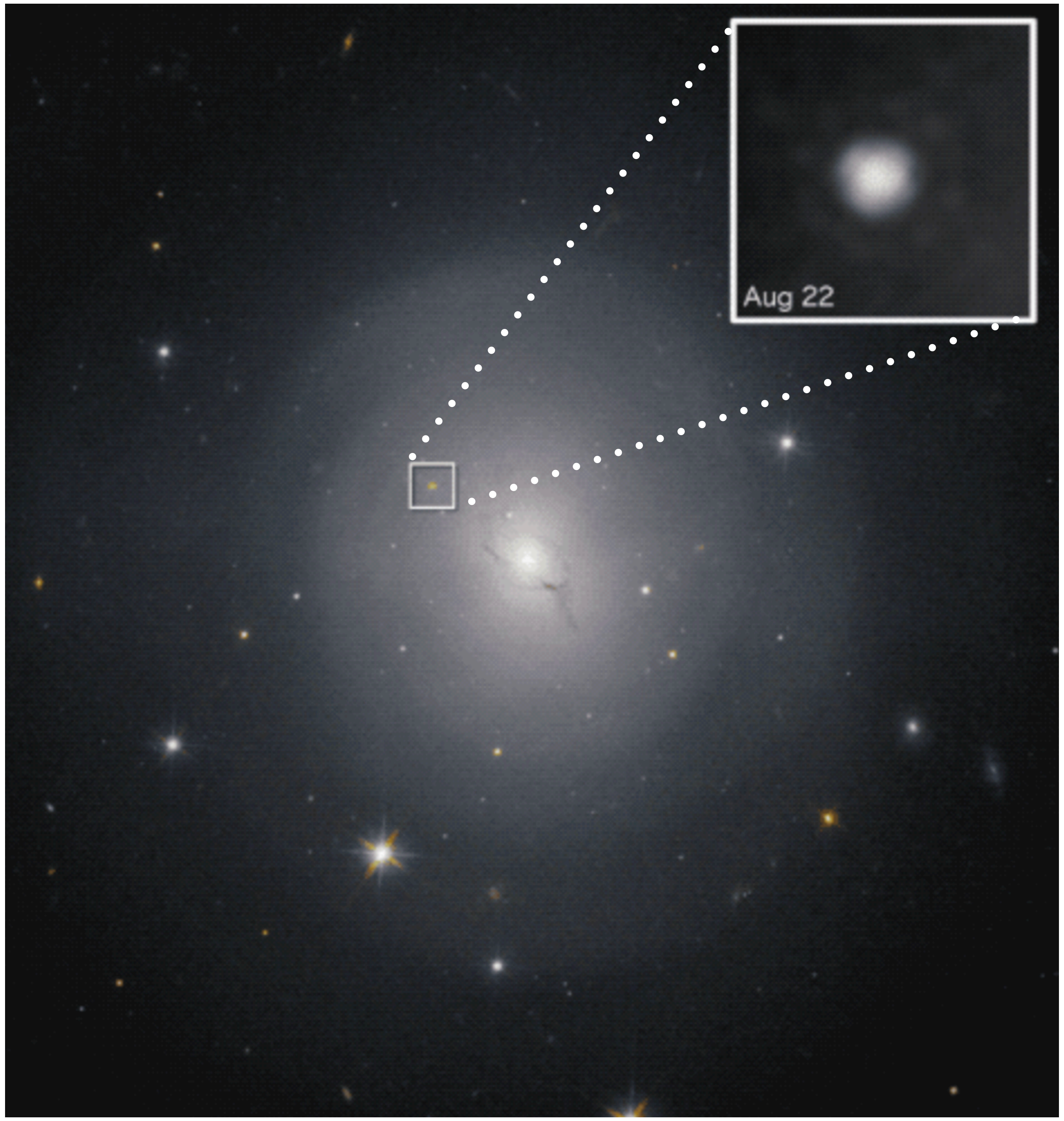
Our considerations will be based upon a real, four-dimensional, C^∞ differentiable manifold *M.* It will be assumed that all field functions are defined globally; however, our work will be of a purely local nature. By a



Graviton propagation speed: GW170817/GRB170817A

Binary neutron star merger





galaxy NGC 4993

~ 40 Mpc distance

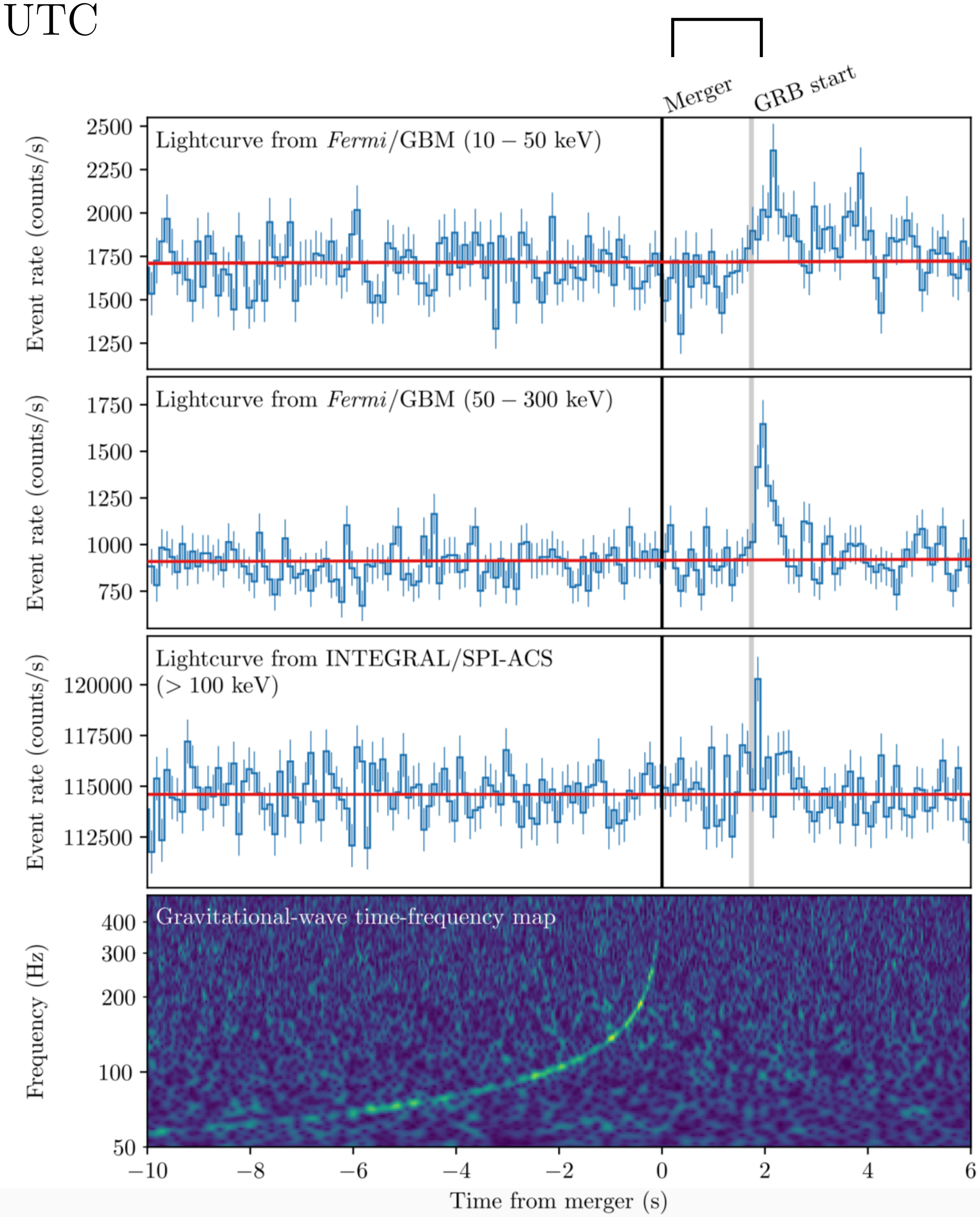
GW170817/GRB170817A

August 17, 2017 12:41:04 UTC

$\Delta t \sim 1.7$ s

light →

Gravity waves →



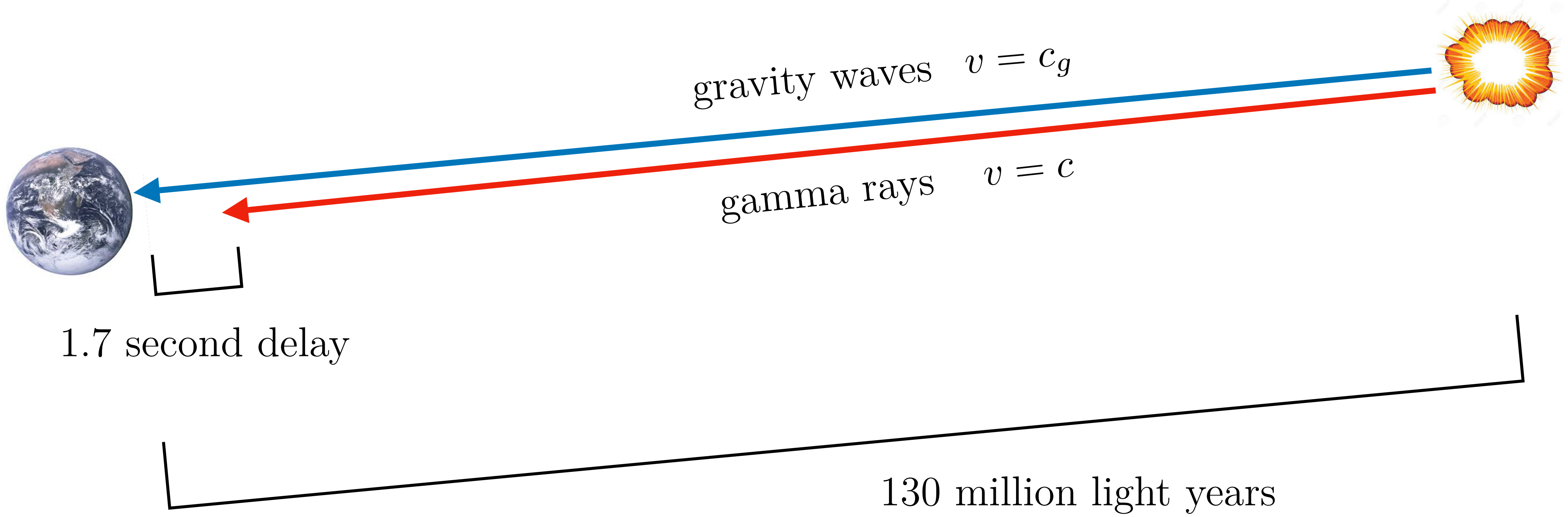
Abbott et. al. Astrophysical Journal Letters, 848:L13

Constraint on the speed of gravity

Bettoni, Ezquiaga, KH, Zumalacárregui 1608.01982

Ezquiaga, Zumalacarregui 1710.05901

Creminelli, Vernizzi 1710.05877



$$\Delta t \lesssim 1.7\ \text{s} \quad \text{over a distance} \quad 40\ \text{Mpc} \quad \Longrightarrow \quad \boxed{\left|\frac{c_g}{c} - 1\right| \lesssim 10^{-15}}$$

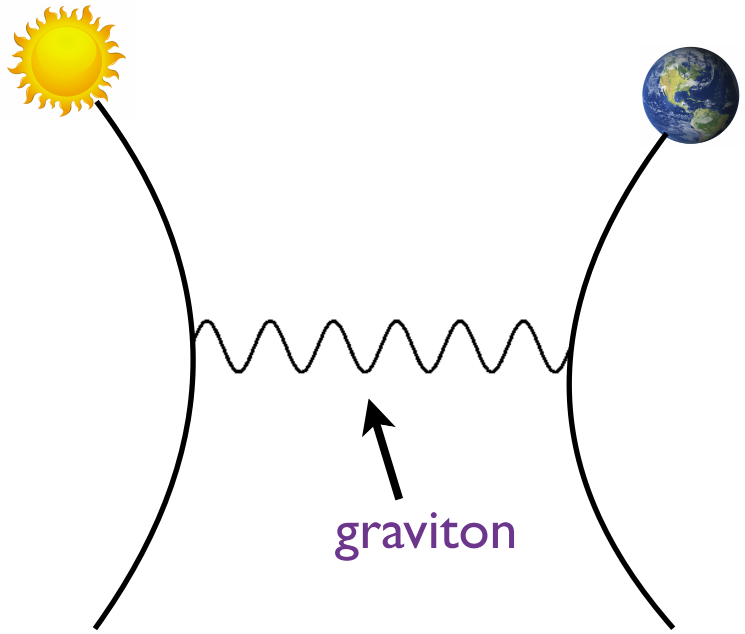
Constraints:

certain parameters and VEVs $\lesssim 10^{-15}$ (caveats: de Rham, Melville 1806.09417)

single scalar not completely ruled out

Spin-2: massive gravity

Massive gravity: what about the massive spin-2 representation?



Maybe gravity is mediated by a *massive* particle:

Massive particle obeys the Klein Gordon equation:

$$(\Box - m^2) = 0$$

Solution gives a Yukawa potential:

IR modification scale

$$V(r) \sim \frac{M}{M_P^2} \frac{1}{r} e^{-mr}, \quad m \sim H$$

Extra DOF: 5 massive spin states as opposed to 2 helicity states

Massive graviton: linear theory

Massive spin 2 particle: 5 degrees of freedom (as opposed to 2 for massless helicity 2)

Fierz-Pauli action: Fierz, Pauli (1939)

$$\mathcal{L} = \boxed{-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h} \boxed{-\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)} + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

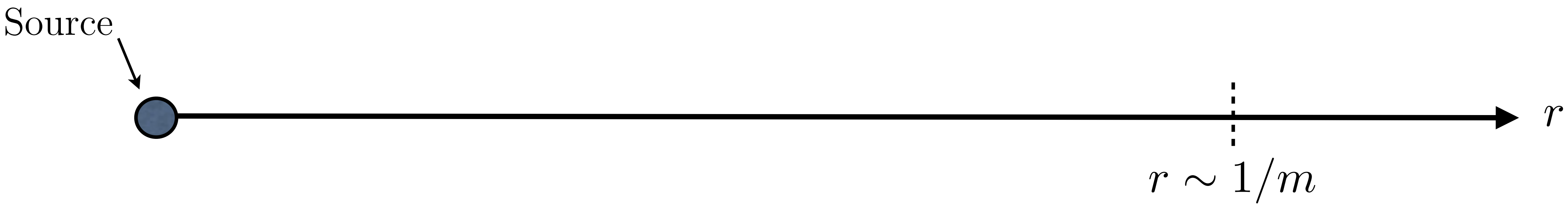
Einstein-Hilbert (massless) part.
Gauge symmetry: $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

Mass term breaks gauge symmetry.
Fierz-Pauli tuning ensures 5 D.O.F.

Equations of motion: $(\Box - m^2)h_{\mu\nu} = 0, \quad \partial^\mu h_{\mu\nu} = 0, \quad h = 0$

Linear solutions around sources

Newtonian Potential: $\phi_N = -\frac{4}{3}\frac{GM}{r}e^{-mr}$



Massless gravity vs. massless limit of massive gravity: the *vDVZ discontinuity*

van Dam, Veltman, Zakharov (1970)

	$m \to 0$	$m = 0$
Newtonian potential	$\phi_N = -\frac{4}{3}\frac{GM}{r}$	$\phi_N = -\frac{GM}{r}$
Light bending angle (at impact parameter b)	$\alpha = \frac{4GM}{b}$	$\alpha = \frac{4GM}{b}$

Helicity components

Introduce fields for each helicity component:

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\,\partial_\mu\partial_\nu\phi$$

$$\underset{\text{5 DOF}}{h_{\mu\nu}} \quad \xrightarrow[\text{relativistic limit } m \to 0]{} \quad \begin{cases} h_{\mu\nu} \sim \text{helicity } \pm 2 & \text{2 DOF} \\ A_\mu \sim \text{helicity } \pm 1 & \text{2 DOF} \\ \phi \sim \text{helicity } 0 & \text{1 DOF} \end{cases}$$

This is the vDVZ discontinuity:
scalar fifth force

$$\underset{\substack{\uparrow \\ \text{helicity-2}}}{\mathcal{L}_{m=0}(h')} - \frac{1}{2}\underset{\substack{\uparrow \\ \text{helicity-1}}}{\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}} - 3\,\underset{\substack{\uparrow \\ \text{helicity-0}}}{\partial_\mu\hat{\phi}\,\partial^\mu\hat{\phi}} + \frac{1}{M_P}h'_{\mu\nu}T^{\mu\nu} + \frac{1}{M_P}\overset{\swarrow}{\hat{\phi}T}$$

Interaction terms

Effective field theory philosophy: write every possible term parametrized by arbitrary coefficients

$$\frac{M_P^2}{2}\int d^4x\ \left[(\sqrt{-g}R)-\sqrt{-g}\frac{1}{4}m^2V(g,h)\right],$$

$$V(g,h)=V_2(g,h)+V_3(g,h)+V_4(g,h)+V_5(g,h)+\cdots\ ,$$

$$\begin{aligned}
V_2(g,h) &= \langle h^2\rangle-\langle h\rangle^2,\\
V_3(g,h) &= +c_1\langle h^3\rangle+c_2\langle h^2\rangle\langle h\rangle+c_3\langle h\rangle^3,\\
V_4(g,h) &= +d_1\langle h^4\rangle+d_2\langle h^3\rangle\langle h\rangle+d_3\langle h^2\rangle^2+d_4\langle h^2\rangle\langle h\rangle^2+d_5\langle h\rangle^4,\\
V_5(g,h) &= +f_1\langle h^5\rangle+f_2\langle h^4\rangle\langle h\rangle+f_3\langle h^3\rangle\langle h\rangle^2+f_4\langle h^3\rangle\langle h^2\rangle+f_5\langle h^2\rangle^2\langle h\rangle\\
&\quad +f_6\langle h^2\rangle\langle h\rangle^3+f_7\langle h\rangle^5,\\
&\ \vdots
\end{aligned}$$

The effective field theory

Arkani-Hamed, Georgi and Schwartz (2003)
Creminelli, Nicolis, Pappuchi, Trincherini (2005)
de Rham, Gabadadze (2010)

After replacement $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\,\partial_\mu \partial_\nu \phi + \cdots$ there are interaction terms:

$$m^2 M_P^2 h^{n_h} (\partial A)^{n_A} (\partial^2 \phi)^{n_\phi} \sim \Lambda_\lambda^{4-n_h-2n_A-3n_\phi} \hat{h}^{n_h} (\partial \hat{A})^{n_A} (\partial^2 \hat{\phi})^{n_\phi}$$

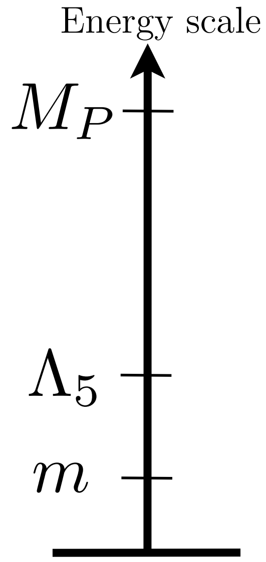
Various strong coupling scales:
The larger **λ**, the smaller the scale

$$\Lambda_\lambda = \left(M_P m^{\lambda-1}\right)^{1/\lambda}, \quad \lambda = \frac{3n_\phi + 2n_A + n_h - 4}{n_\phi + n_A + n_h - 2}$$

The smallest scale is carried by a cubic scalar interaction:

$$\sim \frac{(\partial^2 \hat{\phi})^3}{\Lambda_5^5}, \qquad \Lambda_5 = (M_P m^4)^{1/5}$$

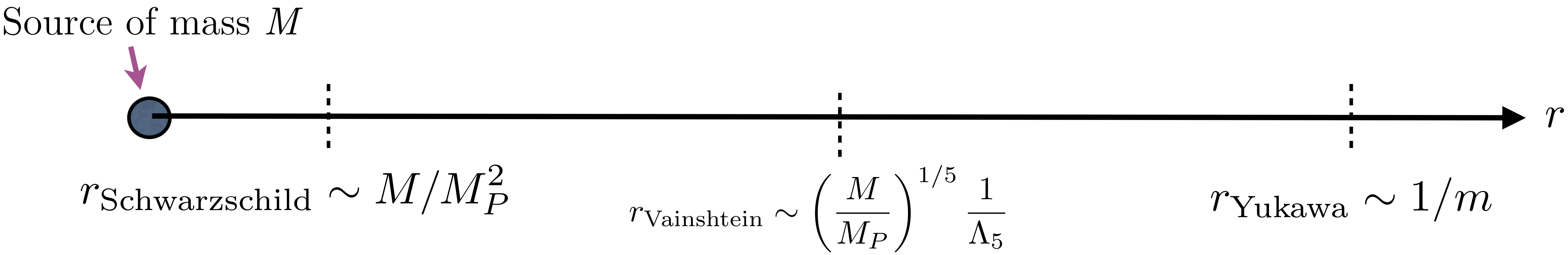
This is the (UV) strong coupling scale of the theory



The effective field theory

$$\sim \frac{(\partial^2 \hat{\phi})^3}{\Lambda_5^5}, \qquad \Lambda_5 = (M_P m^4)^{1/5}$$

- Scalar self-interactions responsible for Vainshtein radius:



Source of mass M

r

$r_{\text{Schwarzschild}} \sim M/M_P^2$

$r_{\text{Vainshtein}} \sim \left(\frac{M}{M_P}\right)^{1/5} \frac{1}{\Lambda_5}$

$r_{\text{Yukawa}} \sim 1/m$

- Non-linearities restore continuity with GR (Vainshtein mechanism) Vainshtein (1972)

- Scalar self-interactions display the *Boulware-Deser ghost* Boulware, Deser (1972)
  Deffayet, Rombouts (2005)

The Λ_3 theory

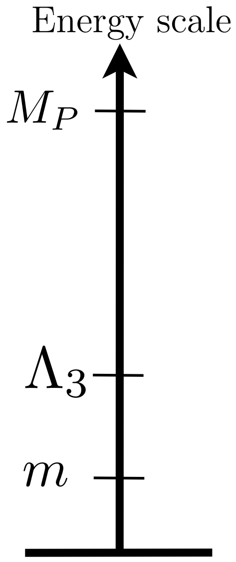
Can choose the interactions, order by order in h, so that the scalar self-interactions appear in total derivative combinations.

Arkani-Hamed, Georgi and Schwartz (2003)
Creminelli, Nicolis, Pappuchi, Trincherini (2005)
de Rham, Gabadadze (2010)

- Two parameter family of ways to do this

Cutoff raised to: $\Lambda_3 \sim \left(M_P m^2\right)^{1/3}$

Longitudinal mode is described by Galileon interactions:



$$
\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2}(\partial\phi)^2 \ , \\
\mathcal{L}_3 &= -\frac{1}{2}(\partial\phi)^2[\Pi] \ , \\
\mathcal{L}_4 &= -\frac{1}{2}(\partial\phi)^2\left([\Pi]^2-[\Pi^2]\right) \ , \\
\mathcal{L}_5 &= -\frac{1}{2}(\partial\phi)^2\left([\Pi]^3-3[\Pi][\Pi^2]+2[\Pi^3]\right)
\end{aligned}
$$

$(\Pi_{\mu\nu} \equiv \partial_\mu\partial_\nu\phi)$

dRGT massive gravity

de Rham, Gabadadze, Tolley (2011)

The theory can be re-summed:

$$\frac{M_P^{D-2}}{2}\int d^Dx\,\sqrt{-g}\left[R-\frac{m^2}{4}\sum_{n=0}^{D}\beta_n S_n(\sqrt{g^{-1}\eta})\right]$$

Characteristic Polynomials

$$\begin{aligned}
S_0(M) &= 1\,,\\
S_1(M) &= [M]\,,\\
S_2(M) &= \frac{1}{2!}\left([M]^2-[M^2]\right)\,,\\
S_3(M) &= \frac{1}{3!}\left([M]^3-3[M][M^2]+2[M^3]\right)\,,\\
&\;\vdots
\end{aligned}$$

$$S_D(M)=\det M\,,$$

- Full theory has no Boulware-Deser ghost (propagates 5 DOF non-linearly)

Hassan, Rosen (2011)
de Rham, Gabadadze, Tolley (2011)
KH, Rosen (2012)
+ many others

Vielbein formulation of ghost-free massive gravity

KH, Rachel Rosen (arXiv:1203.5783)

Or in terms of vierbeins $g_{\mu\nu} = e_\mu{}^A e_\nu{}^B \eta_{AB}$

$$\frac{M_P^{D-2}}{2}\int d^Dx\ |e|R[e] - m^2\sum_n a_n \int \epsilon_{A_1\cdots A_D} e^{A_1}\wedge\cdots\wedge e^{A_n}\wedge 1^{A_{n+1}}\wedge\cdots\wedge 1^{A_n}$$

Ghost-free mass terms are simply all possible ways of wedging a vierbein and background vierbein:

$$\epsilon_{A_1A_2A_3A_4}e^{A_1}\wedge e^{A_2}\wedge e^{A_3}\wedge e^{A_4}$$

$$\epsilon_{A_1A_2A_3A_4}e^{A_1}\wedge e^{A_2}\wedge e^{A_3}\wedge 1^{A_4}$$

$$\epsilon_{A_1A_2A_3A_4}e^{A_1}\wedge e^{A_2}\wedge 1^{A_3}\wedge 1^{A_4}$$

$$\epsilon_{A_1A_2A_3A_4}e^{A_1}\wedge 1^{A_2}\wedge 1^{A_3}\wedge 1^{A_4}$$

$$\epsilon_{A_1A_2A_3A_4}1^{A_1}\wedge 1^{A_2}\wedge 1^{A_3}\wedge 1^{A_4}$$

dRGT theory

CC problem:

New CC problem

- Exist self accelerating solutions, in the absence of a CC (acceleration is caused by graviton mass $m \sim H$)

de Rham, Gabadadze, Heisenberg, Pirtskhalava (2010)
Gumrukcuoglu, Lin, Mukohyama (2011)

- A small graviton mass is protected from large quantum corrections (diff invariance restored as $m \to 0$)

de Rham, Heisenberg, Ribeiro (2013)

Old CC problem

- More difficult (need screening of a large CC, or a new symmetry)

Dvali, Gabadadze, Shifman (2002)
Arkani-Hamed, Dimopolous, Dvali Gabadadze (2002)
Dvali, Hoffman, Khoury (2007)
de Rham, KH, Rosen, Tolley (2013)

Phenomenology/new signals:

- Vainshtein mechanism hides 5-th force from experiments, so residual screening effects might be observable

Graviton mass constraints:

No real constraint from speed of gravity

LIGO	$m_g < 1.2 \times 10^{-22}$ eV
Clusters	10^{-29}
Lunar laser ranging	10^{-32}

de Rham, Deskins, Tolley, Zhou 1606.08462

UV completion

These theories are *effective field theories* (EFT) with a strong coupling scale parametrically larger than the mass:

$$\Lambda \gg m$$

Something has to happen before the scale Λ to complete them:

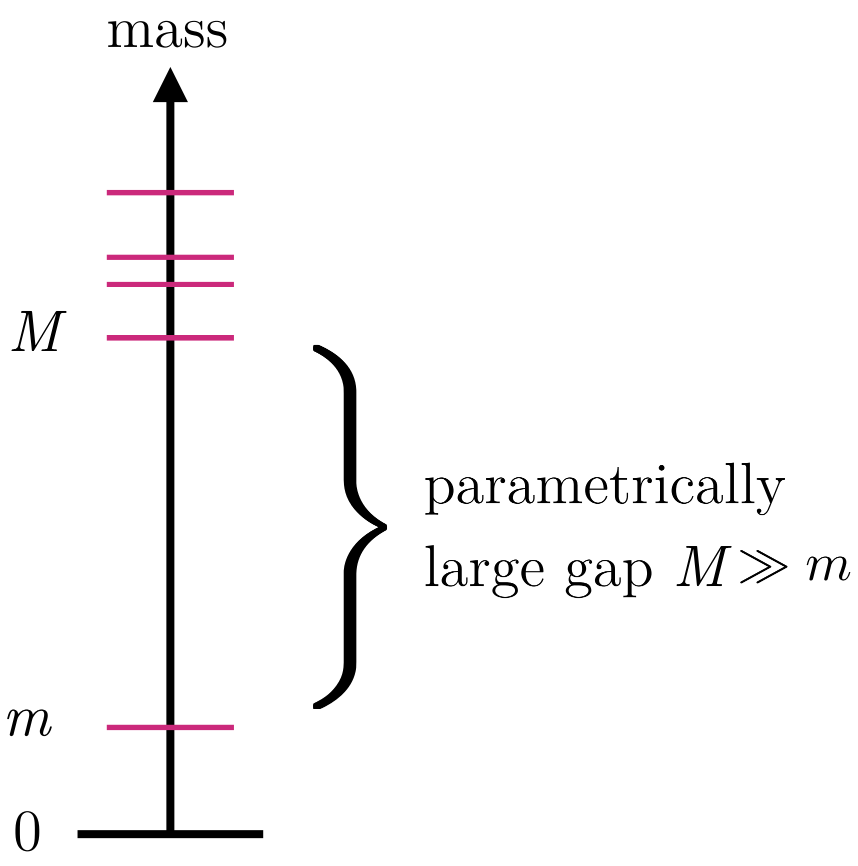
- New particles/degrees of freedom $\begin{cases} \text{weakly coupled} \\ \text{strongly coupled} \end{cases}$
- Strong coupling effects

Theory that includes these new effects is valid to a higher scale (UV extension or UV completion)

Do such completions exist? (landscape vs. swampland)

Isolated massive spinning particles?

Is it possible to have a complete theory with a spectrum like this:



Spin 0, 1/2: Yes (pseudo Goldstones)

Spin 1, 3/2: Yes (spontaneously broken weakly coupled gauge theory/SUGRA)

Spin ≥ 2: ?

Isolated massive spinning particles?

Can there be "elementary"
particles with spin ≥ 2 ?

Are there high spin hadrons with
Compton wavelength $\gg$ intrinsic size ?

Can the graviton have a small
Hubble-scale mass?

Common lore says No: a massive higher spin always comes
with more states at parametrically the same mass

Isolated massive spinning particles?

Examples:

Kaluza Klein theory:

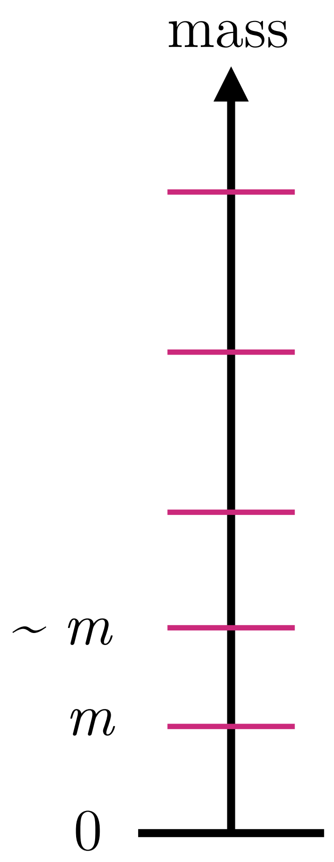
towers of spin ≤ 2 $m^2 \sim \lambda_{\text{laplacian}}$

Confining gauge theory

towers of all spins $m^2 \sim \Lambda^2_{\text{QCD}}$

String theory

towers of all spins $m^2 \sim \frac{1}{\alpha'}$



no large gap $\gg m$

Can we do any better with the cutoff?

KH, James Bonifacio (1804.08686)

Look at observables: amplitudes

strong coupling scale $\Lambda_3 \sim \left(M_P m^2\right)^{1/3}$ ↔ E^6 growth of amplitude

Is there any way to do better than dRGT's E^6 growth of amplitudes
(i.e. raise strong coupling scale further within the EFT)

$$
\begin{aligned}
\mathcal{L} \sim (\partial h)^2 &+ h^2 \\
&+h^3 + \partial^2 h^3 + \partial^4 h^3 + \cdots \\
&+h^4 + \partial^2 h^4 + \partial^4 h^4 + \cdots \\
&\vdots
\end{aligned}
$$

Field redefinitions → put fields on shell: transverse, traceless, $\Box \to -m^2$

Classify all on-shell cubic and quartic vertices

cubic vertices

Polarization tensors:

$$\epsilon_{\mu_1 \dots \mu_s} \to z_{\mu_1} z_{\mu_2} \dots z_{\mu_s} \quad , \qquad z^2 = 0$$

No on-shell non-trivial functions of momenta:

$$p_1^\mu + p_2^\mu + p_3^\mu = 0 \qquad \Rightarrow \qquad p_1 \cdot p_2 = \frac{1}{2}\left(m_1^2 + m_2^2 - m_3^2\right) \ , \ \text{etc.}$$

$$\mathcal{A}_3 \sim \ z_{12}^{n_{12}} z_{13}^{n_{13}} z_{23}^{n_{23}} zp_{12}^{m_{12}} zp_{23}^{m_{23}} zp_{31}^{m_{31}}$$

$$\begin{aligned} n_{12} + n_{13} + m_{12} &= s_1, \\ n_{12} + n_{23} + m_{23} &= s_2, \\ n_{13} + n_{23} + m_{31} &= s_3. \end{aligned}$$

Finite number of solutions → On-shell cubic amplitudes nailed down by Lorentz invariance.

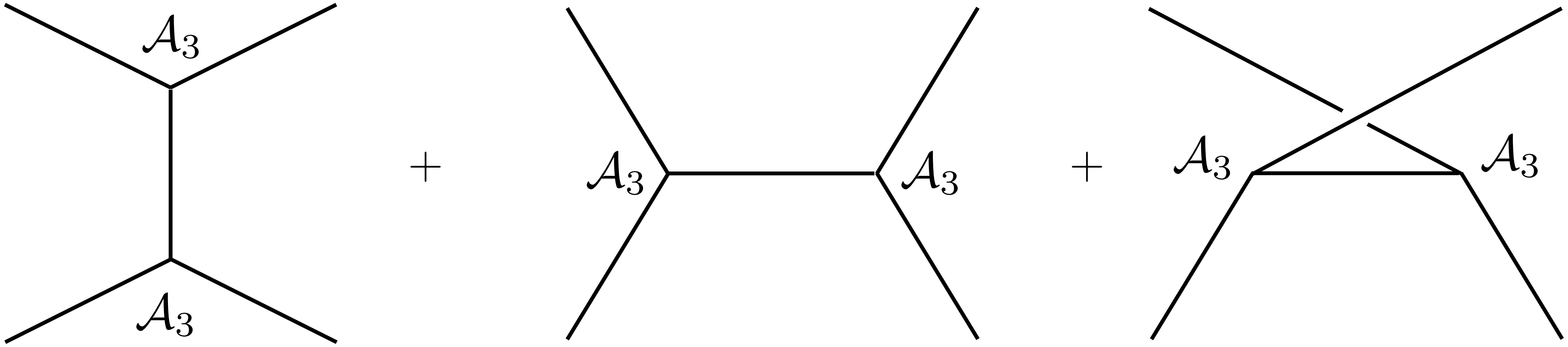
Cubic massive spin-2 vertices

$\mathcal{A}_1$	$z_1 \cdot z_2 \;\; z_2 \cdot z_3 \;\; z_3 \cdot z_1$	$h_{\mu\nu}^3$
$\mathcal{A}_2$	$(p_1 \cdot z_3 \;\; z_1 \cdot z_2 + p_3 \cdot z_2 \;\; z_1 \cdot z_3 + p_2 \cdot z_1 \;\; z_2 \cdot z_3)^2$	$\sqrt{-g} R \big\|_{(3)}$
$\mathcal{A}_3$	$(p_1 \cdot z_3)^2 (z_1 \cdot z_2)^2 + (p_3 \cdot z_2)^2 (z_1 \cdot z_3)^2 + (p_2 \cdot z_1)^2 (z_2 \cdot z_3)^2$	$\delta^{[\mu_1}_{\nu_1} \delta^{\mu_2}_{\nu_2} \delta^{\mu_3}_{\nu_3} \delta^{\mu_4]}_{\nu_4} \partial_{\mu_1} \partial^{\nu_1} h_{\mu_2}{}^{\nu_2} h_{\mu_3}{}^{\nu_3} h_{\mu_4}{}^{\nu_4}$
$\mathcal{A}_4$	$p_1 \cdot z_3 \; p_2 \cdot z_1 \; p_3 \cdot z_2 \;\; (p_1 \cdot z_3 \;\; z_1 \cdot z_2 + p_3 \cdot z_2 \;\; z_1 \cdot z_3 + p_2 \cdot z_1 \;\; z_2 \cdot z_3)$	$\sqrt{-g} \left(R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2 \right) \Big\|_{(3)}$
$\mathcal{A}_5$	$(p_1 \cdot z_3)^2 (p_2 \cdot z_1)^2 (p_3 \cdot z_2)^2$	$\sqrt{-g} \; R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta} R^{\alpha\beta}{}_{\mu\nu} \Big\|_{(3)}$

D=4: no $\mathcal{A}_4$, 2 additional parity violating amplitudes

Best possible scaling

Build the exchange Feynman diagrams:



Finite number of cubic vertices → finite number of exchange diagrams → bounded growth with energy

$$\mathcal{A}_{\text{exchange}} \sim E^{\#}$$

Best possible scaling

KH, James Bonifacio (1804.08686)

Classify all analytic quartic amplitudes (contact terms):

2 independent invariants made of momenta (2 Mandelstams)

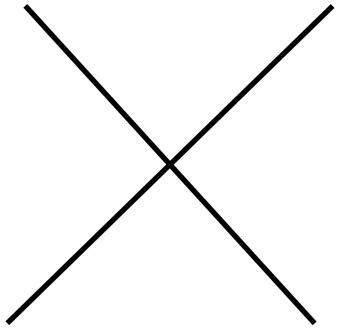
$$\boxed{p_{12}^{k_{12}}\, p_{13}^{k_{13}}}\, z_{12}^{n_{12}}\, z_{13}^{n_{13}}\, z_{14}^{n_{14}}\, z_{23}^{n_{23}}\, z_{24}^{n_{24}}\, z_{34}^{n_{34}}\, zp_{13}^{m_{13}}\, zp_{14}^{m_{14}}\, zp_{21}^{m_{21}}\, zp_{24}^{m_{24}}\, zp_{31}^{m_{31}}\, zp_{32}^{m_{32}}\, zp_{42}^{m_{42}}\, zp_{43}^{m_{43}}$$

unconstrained

$$\begin{aligned}
n_{12}+n_{13}+n_{14}+m_{13}+m_{14} &= s_1,\\
n_{12}+n_{23}+n_{24}+m_{21}+m_{24} &= s_2,\\
n_{13}+n_{23}+n_{34}+m_{31}+m_{32} &= s_3,\\
n_{14}+n_{24}+n_{34}+m_{42}+m_{43} &= s_4.
\end{aligned}$$

This is the contact Feynman diagram:

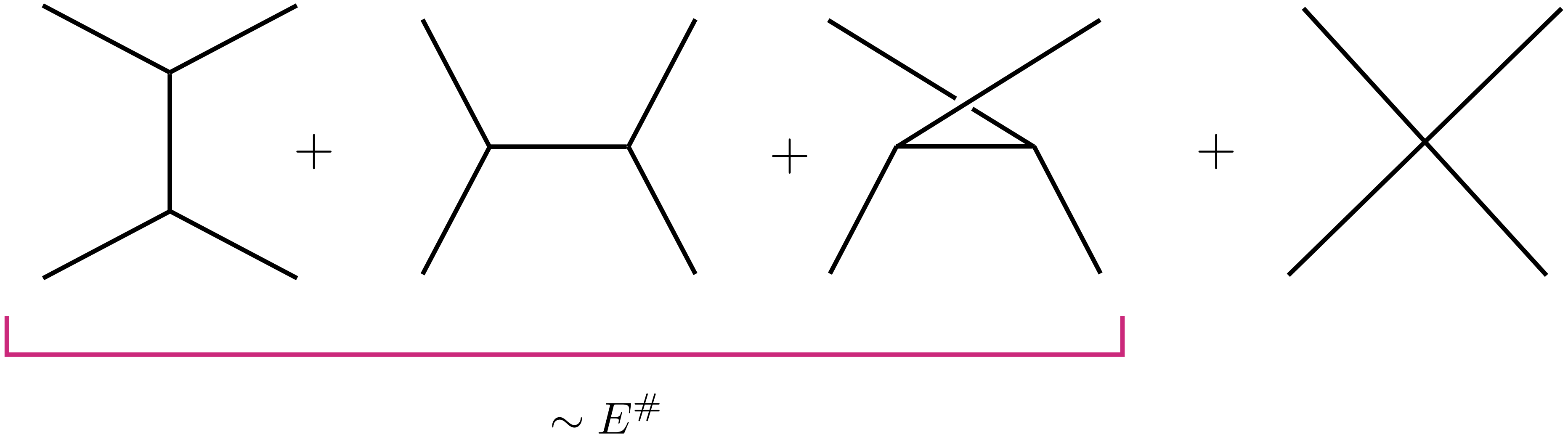
$$\mathcal{A}_{\text{contact}} \sim$$



Best possible scaling

Try to cancel off highest energy scaling of exchange diagrams, working down:

$$\mathcal{A}_4 = \mathcal{A}_{\text{exchange}} + \mathcal{A}_{\text{contact}}$$



Result: Best possible scaling is E^6

Only theories that achieve this are dRGT theory and pseudo-linear

KH (1305.7227)

Same is true of bi-gravity: KH, James Bonifacio (1806.10607)

Best possible scalings for all spins

KH, James Bonifacio (1804.08686)

Best scaling for spin-1: E^4 $\Lambda_2 \sim (M_P m)^{1/2}$

Best scaling for spin-2: E^6 $\Lambda_3 \sim \left(M_P m^2\right)^{1/3}$

Conjecture for higher spins:

$$\mathcal{A}_4 \sim \begin{cases} E^{3s} & s \text{ even}, \\ E^{3s+1} & s \text{ odd}. \end{cases}$$

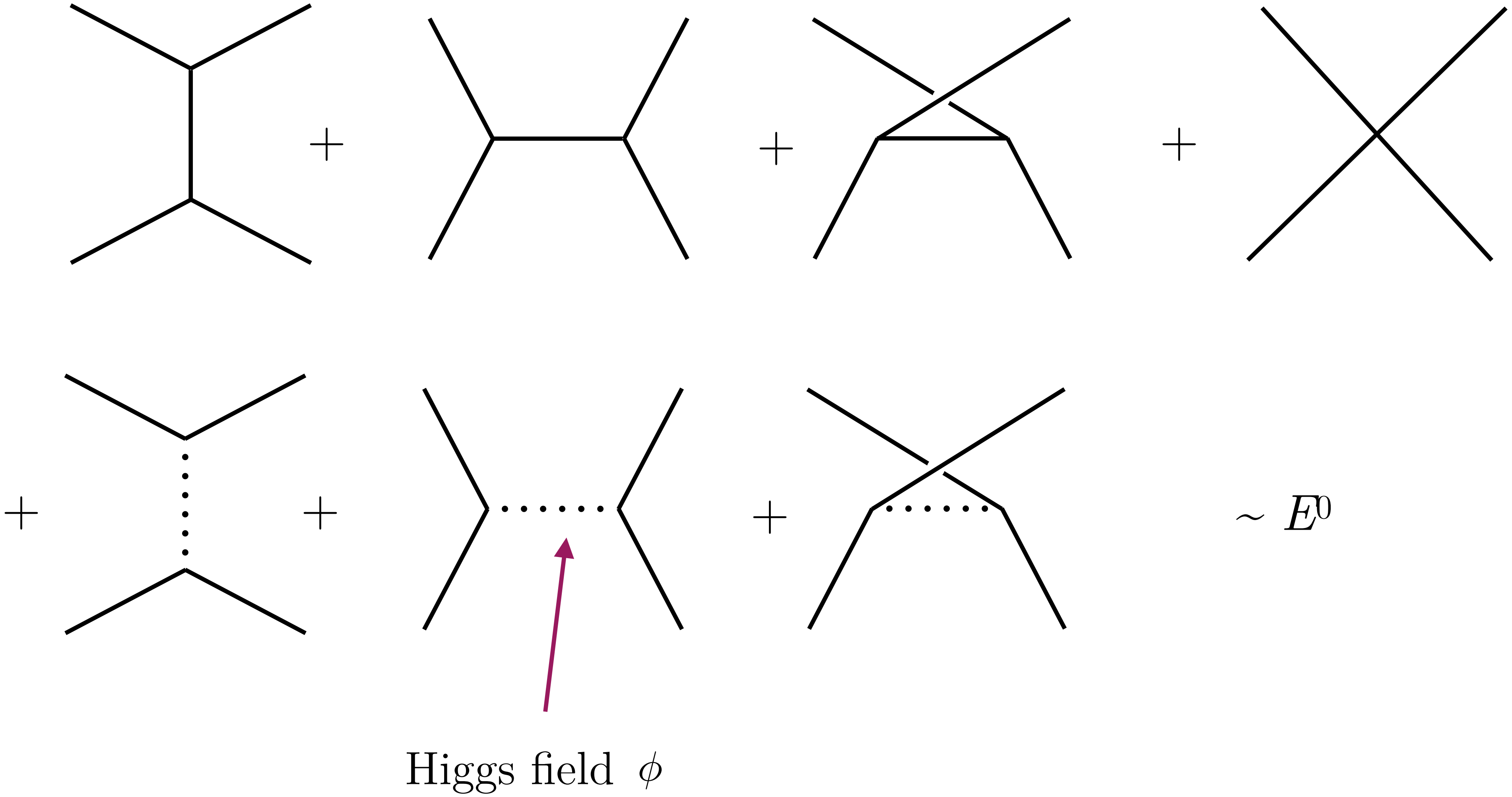
$$\Lambda_{\max} = \begin{cases} \Lambda_{\frac{3s}{2}} & s \text{ even}, \\ \Lambda_{\frac{3s+1}{2}} & s \text{ odd}. \end{cases}$$

$$\Lambda_n \equiv (M_p m^{n-1})^{1/n}$$

Higgs mechanism

Best scaling for spin-1: E^4 $\Lambda_2 \sim (M_P m)^{1/2}$

Can this be improved by adding new fields?



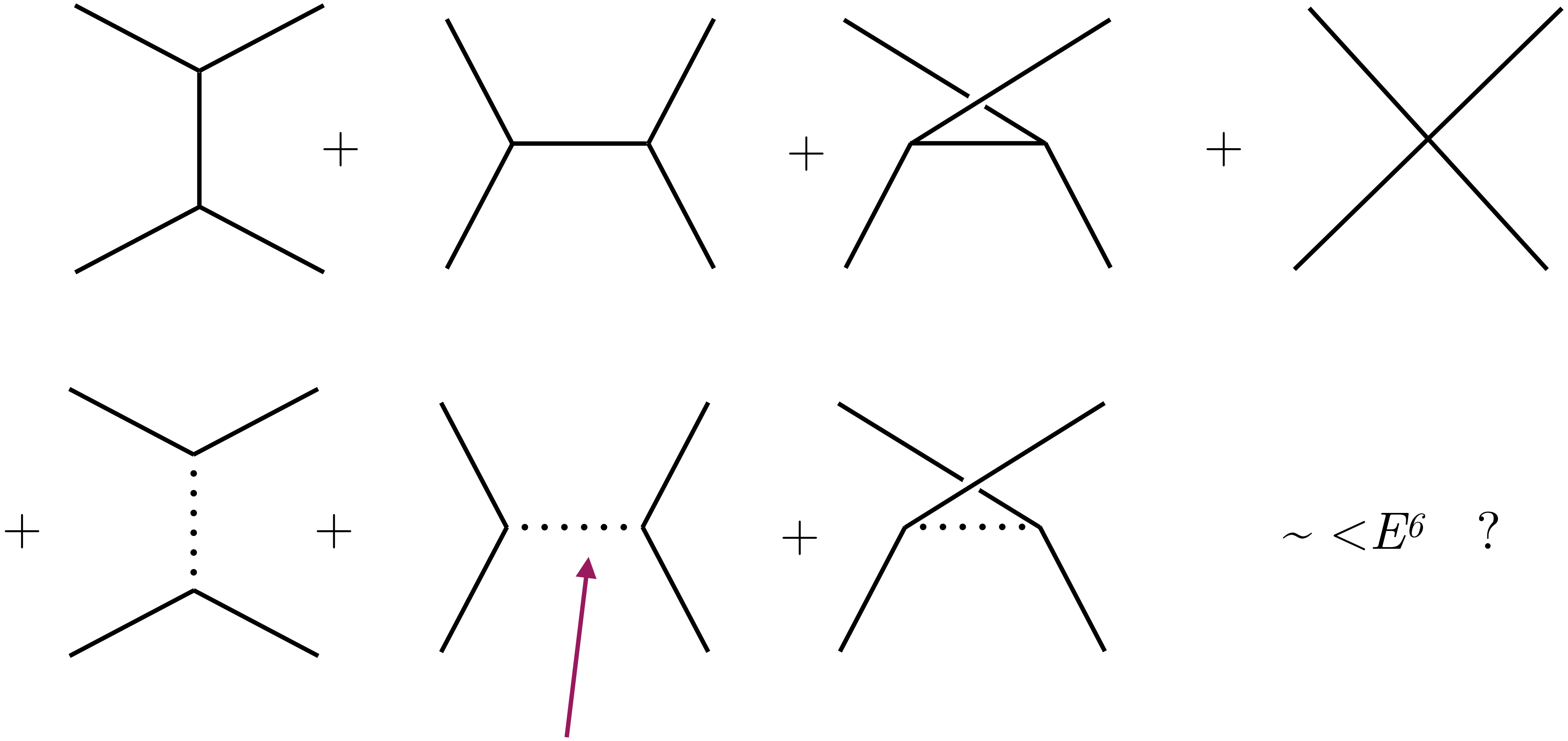
allow additional scalar → can achieve E^0 → Higgs mechanism

Higgs mechanism for gravity?

KH, Rachel Rosen, James Bonifacio (1903.09643)

Best scaling for spin-2: E^6 $\Lambda_3 \sim \left(M_P m^2\right)^{1/3}$

allow any number of additional spins < 2



Higgs fields ϕ, A_μ

$\rightarrow$ no gravitational Higgs mechanism

UV completion?

no gravitational Higgs mechanism means:

No tree level UV extension with a finite number of spins <2

There exist UV extensions with infinite numbers of spin 2's (Kaluza Klein) and infinite numbers of higher spins (string theory)

There may still be strong coupling/loop effects that kick in and UV complete the theory, with or without new degrees of freedom

Are there other, more general UV completion constraints we can impose?

Dispersion relations

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi (2006)

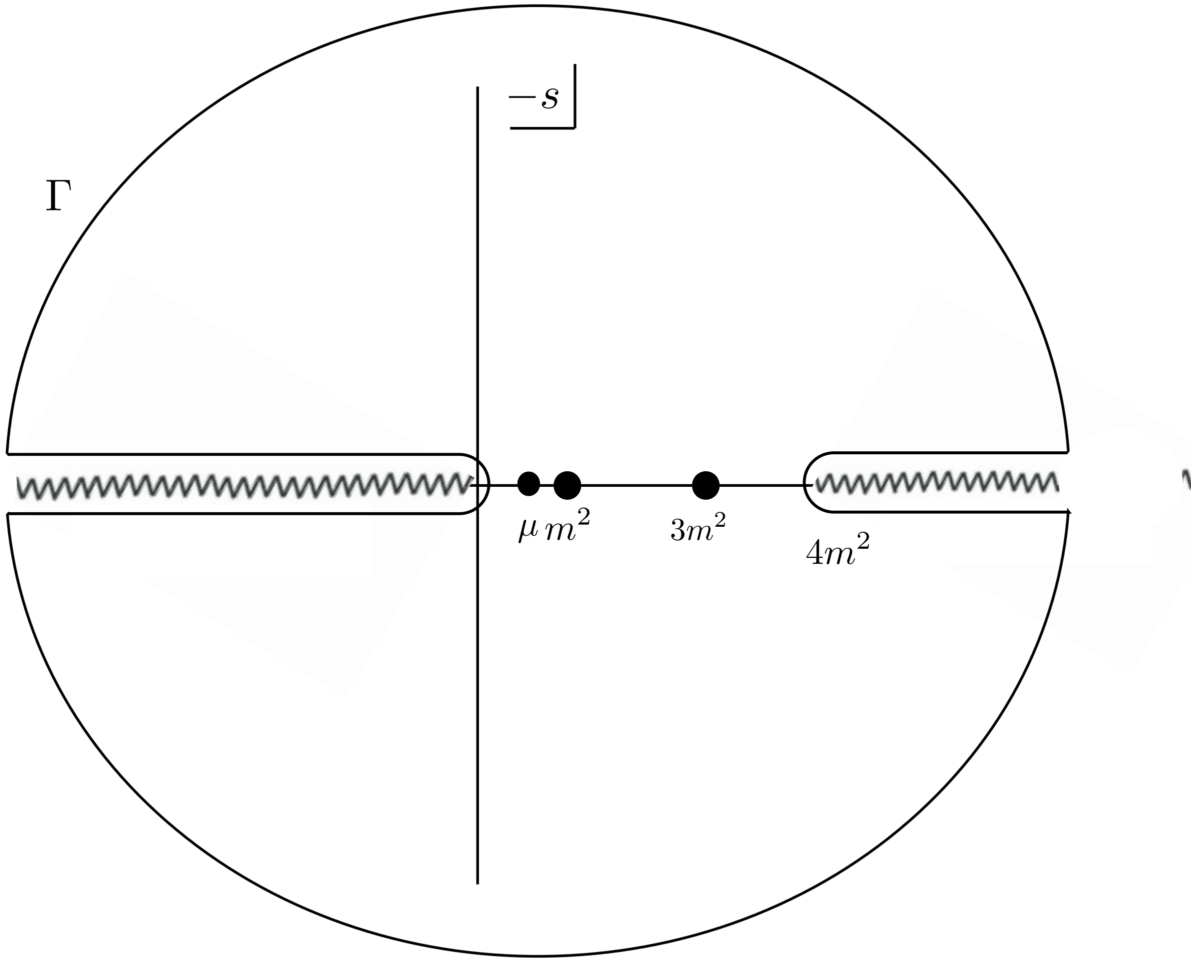
Forward amplitude: $\mathcal{A}(s) \equiv \mathcal{A}(s, t=0)$

$$f \equiv \frac{1}{2\pi i} \oint_{\Gamma} ds \frac{\mathcal{A}(s)}{(s-\mu^2)^{n+1}}$$

$$= -\left(\underset{s\to\infty}{\text{res}} \frac{\mathcal{A}(s)}{(s-\mu^2)^{n+1}} \right)_{\text{EFT,tree}} \quad > 0$$

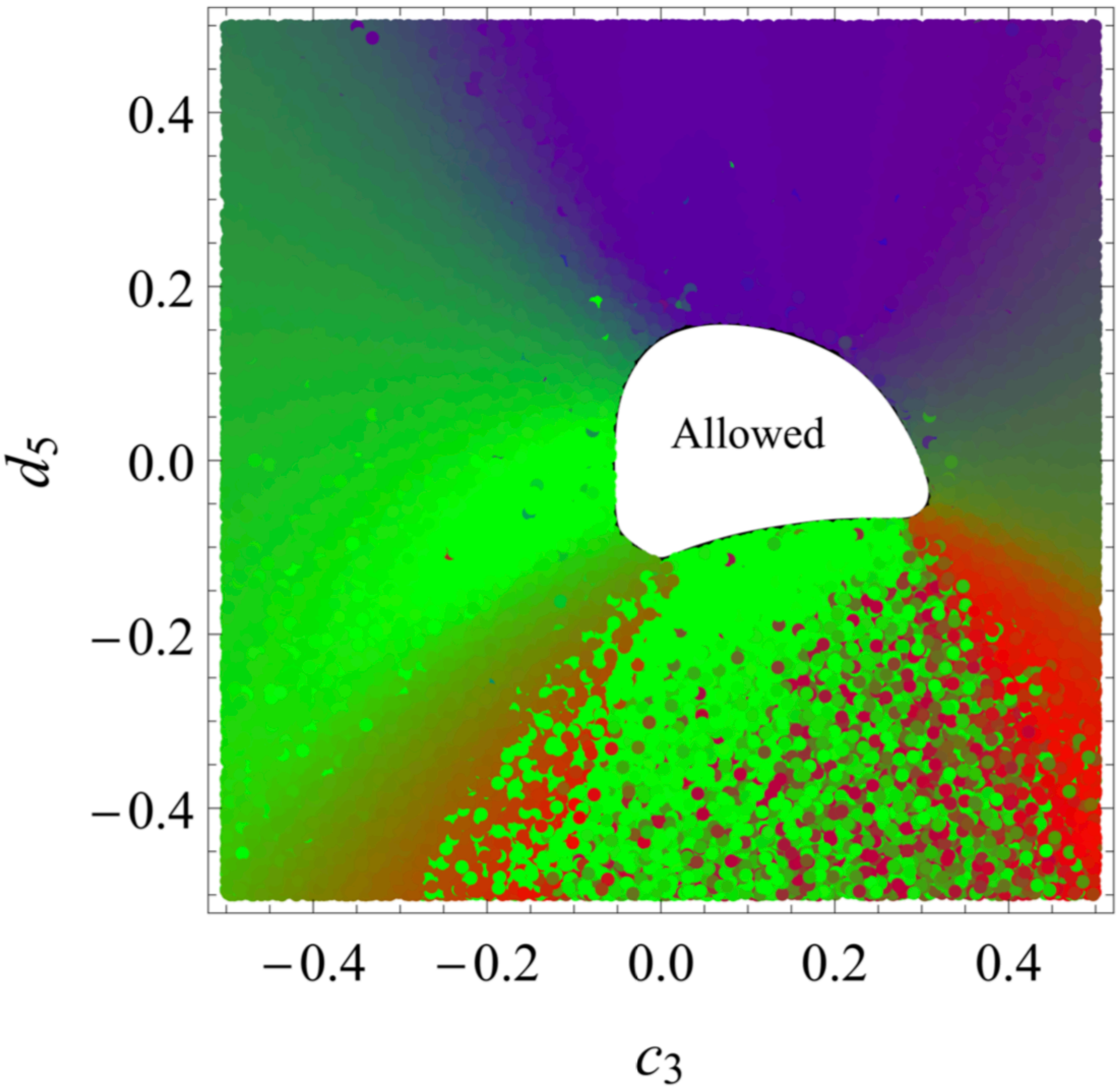
$\uparrow$

Constrains coefficients of leading terms in tree-level of EFT E^{2n}



dRGT theory allowed island

Cheung, Remmen (2016)



Superluminality constraints

Another traditional constraint on EFTs:
Superluminality of small fluctuations on non-trivial Lorentz-violating backgrounds (e.g. Velo-Zwanziger problem)

Less problematic: superluminality in the S-matrix

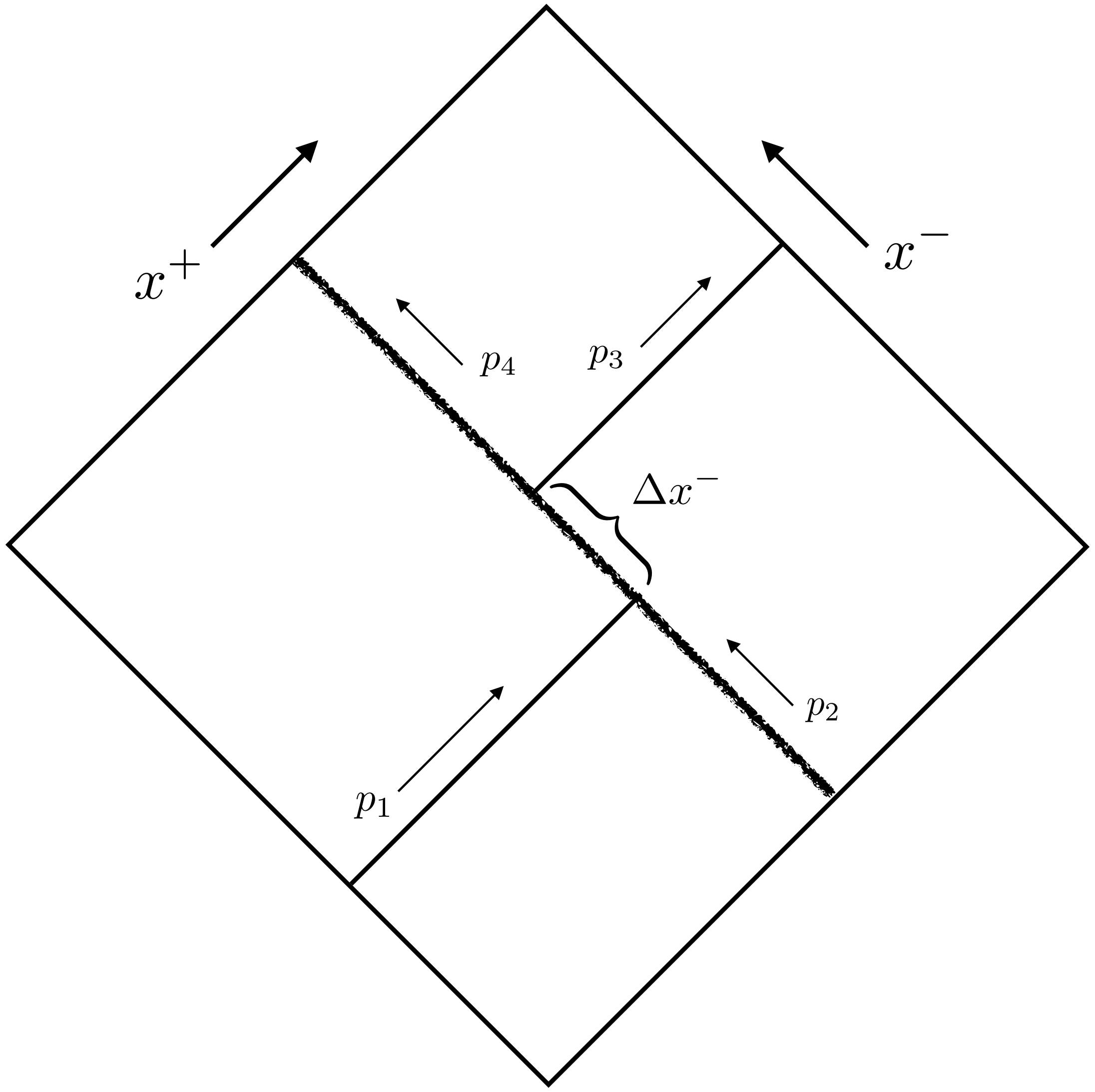
Camanho, Edelstein, Maldacena, Zhiboedov (2016)

Eikonal scattering:

high-energy, fixed impact parameter:

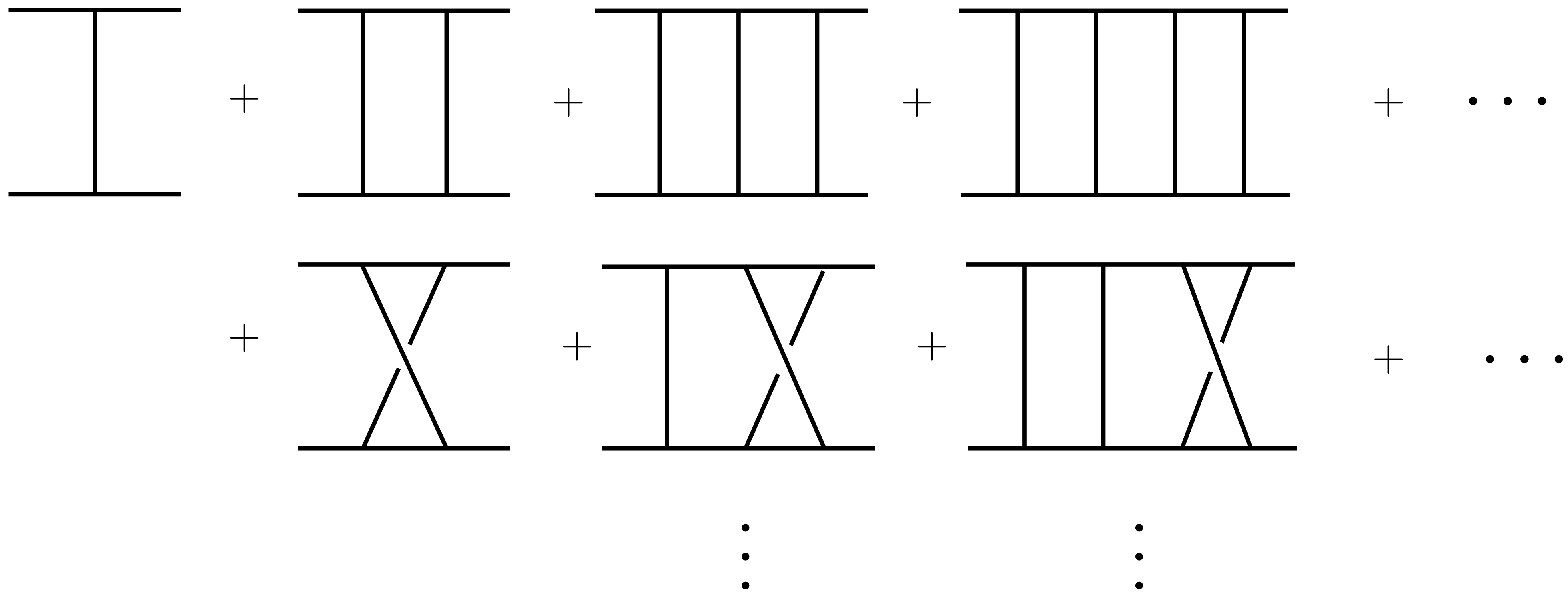
$$s/t \to \infty$$

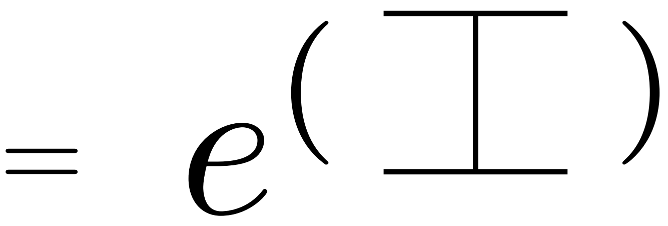
Time delay in scattering



Time delay: eikonal limit

$\mathcal{A}_{\text{eikonal}} =$



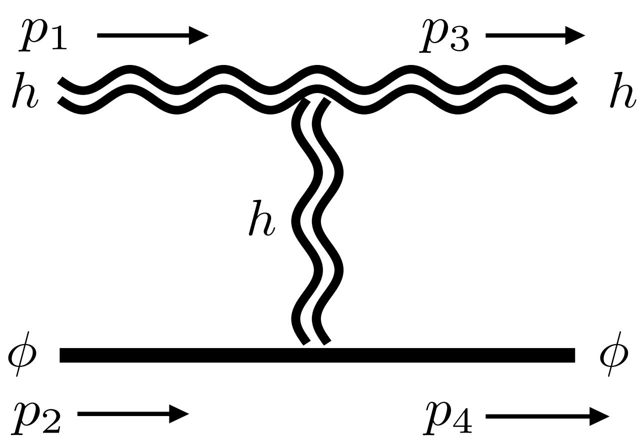


$$i\mathcal{A}_{\text{eikonal}} = 4p^-p^+ \int \mathrm{d}^2\mathbf{b}\, e^{i\mathbf{b}\cdot\mathbf{q}} \left(e^{i\delta(\mathbf{b})} - 1\right) \quad , \qquad \delta(\mathbf{b}) = \frac{1}{4p^-p^+} \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\, e^{-i\mathbf{b}\cdot\mathbf{q}} \mathcal{A}_0(\mathbf{q})$$

Time delay: $\Delta x^- = \dfrac{1}{p^-}\delta$

Massive spin-2 eikonal

Time delay depends only on *on-shell* three point amplitudes:



$$\frac{1}{s}\delta^{\lambda,\lambda'} = \begin{array}{c|ccccc} & T & T & V & V & S \\ \hline T & \frac{48\left(b^2\ m^2+6\right)K_1(b\,m)\,\alpha_5+K_0(b\,m)\left(b^3\ (\alpha_2-\alpha_3+18\,\alpha_5)\,m^3+144\,b\,\alpha_5\ m\right)}{2\,b^3\ m^3\ \text{Mp}^2\ \pi} & 0 & \frac{K_1(b\,m)\left(b^2\ m^2\ (\alpha_3-24\,\alpha_5)-96\,\alpha_5\right)-48\,b\,m\,K_0(b\,m)\,\alpha_5}{4\,b^2\ m^2\ \text{Mp}^2\ \pi} & 0 & \frac{K_2(b\,m)\,(\alpha_3-18\,\alpha_5)}{2\sqrt{3}\ \text{Mp}^2\ \pi} \\ T & 0 & \frac{K_0(b\,m)\left(b^3\ m^3\ (\alpha_2-\alpha_3+6\,\alpha_5)-144\,b\,m\,\alpha_5\right)-48\left(b^2\ m^2+6\right)K_1(b\,m)\,\alpha_5}{2\,b^3\ m^3\ \text{Mp}^2\ \pi} & 0 & \frac{b\,m\,K_1(b\,m)\,\alpha_3+48\,K_2(b\,m)\,\alpha_5}{4\,b\,m\,\text{Mp}^2\ \pi} & 0 \\ V & \frac{K_1(b\,m)\left(b^2\ m^2\ (\alpha_3-24\,\alpha_5)-96\,\alpha_5\right)-48\,b\,m\,K_0(b\,m)\,\alpha_5}{4\,b^2\ m^2\ \text{Mp}^2\ \pi} & 0 & \frac{K_0(b\,m)\,(\alpha_1-2\,\alpha_2+24\,\alpha_5)-\frac{K_1\,(b\,m)\,(\alpha_3-24\,\alpha_5)}{b\,m}}{4\,\text{Mp}^2\ \pi} & 0 & \frac{K_1(b\,m)\,(2\,\alpha_1-6\,\alpha_2+\alpha_3+24\,\alpha_5)}{4\sqrt{3}\ \text{Mp}^2\ \pi} \\ V & 0 & \frac{b\,m\,K_1(b\,m)\,\alpha_3+48\,K_2(b\,m)\,\alpha_5}{4\,b\,m\,\text{Mp}^2\ \pi} & 0 & \frac{K_0(b\,m)\,(\alpha_1-2\,\alpha_2+\alpha_3)+\frac{K_1\,(b\,m)\,(\alpha_3-24\,\alpha_5)}{b\,m}}{4\,\text{Mp}^2\ \pi} & 0 \\ S & \frac{K_2(b\,m)\,(\alpha_3-18\,\alpha_5)}{2\sqrt{3}\ \text{Mp}^2\ \pi} & 0 & \frac{K_1(b\,m)\,(2\,\alpha_1-6\,\alpha_2+\alpha_3+24\,\alpha_5)}{4\sqrt{3}\ \text{Mp}^2\ \pi} & 0 & \frac{K_0(b\,m)\,(2\,\alpha_1-5\,\alpha_2+2\,(\alpha_3+6\,\alpha_5))}{4\,\text{Mp}^2\ \pi} \end{array}$$

$\alpha_1 \leftrightarrow h^3_{\mu\nu}$

$\alpha_2 \leftrightarrow$ Einstein-Hilbert

$\alpha_3 \leftrightarrow$ Pseudo-linear

$\alpha_4 \leftrightarrow$ Gauss-Bonnet

$\alpha_5 \leftrightarrow$ Riemann^3

Massive spin-2 time delay constraints

KH, Austin Joyce, Rachel A. Rosen (1708.05716)

diagonalize phase shift matrix $\delta^{\lambda_1\lambda_2,\lambda_3\lambda_4}$ → constraints:

$$\alpha_3 = \alpha_5 = 0,\ \ \alpha_3 = 3\alpha_2$$

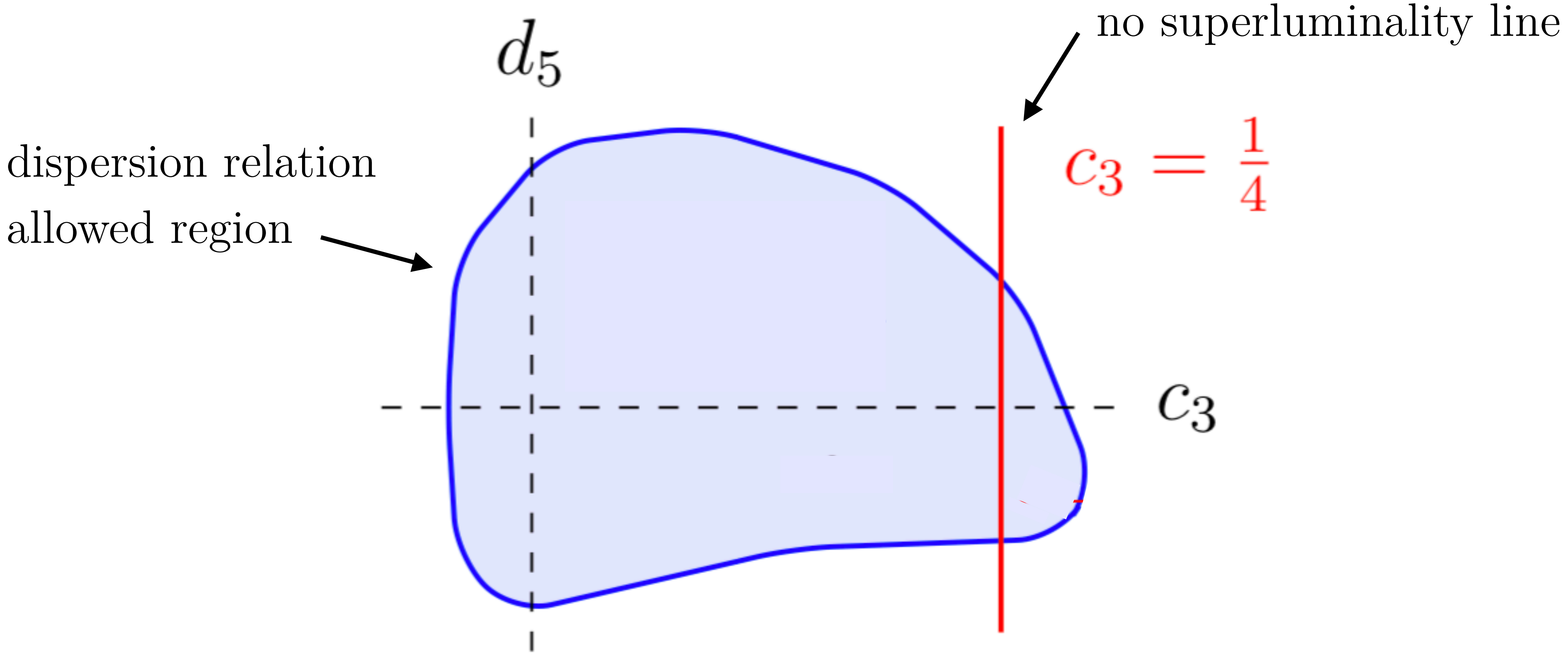
Allowed cubic vertex: $\mathcal{L}_3 \propto \frac{1}{2M_{\rm Pl}} R^{(3)}_{\rm EH} + \frac{m^2}{2M_{\rm Pl}} h^3_{\mu\nu}$

Vertex not of this form $\rightarrow$ new physics at m

Massive spin-2 time delay constraints

Allowed cubic vertex: $\mathcal{L}_3 \propto \frac{1}{2M_{\rm Pl}} R^{(3)}_{\rm EH} + \frac{m^2}{2M_{\rm Pl}} h^3_{\mu\nu}\,,$

Constraints on dRGT theory:



Similar conclusions for bi-gravity

James Bonifacio, KH, Austin Joyce, Rachel A. Rosen (1712.10020)

Summary

- Dark energy models are typically effective field theories with a very low UV cutoff (by particle physics standards)

- They parametrize the possible interactions of a certain choice of new long range degrees of freedom

- Central theoretical issue is UV completion: can any of these models be completed into full theories valid at smaller scales?

- An isolated massive spin-2 (e.g. massive gravity) is constrained but not completely ruled out

- Need to explore more exotic possibilities for UV completion