Dark Matter Halos, Mass Functions, and Cosmology: a Theorist’s View

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Special thanks to Sergei Bashinsky, Salman Habib, Steve Haroz, Zarija Lukic, Kwan-Liu Ma, Pat McCormick, Darren Reed, Paul Ricker

Background: A small section of the first light image obtained by the Sloan Digital Sky Survey
Dark Energy Probes:
- Supernovae
- Baryon Acoustic Oscillations
- Weak Lensing
- **Clusters of Galaxies**
Clusters of Galaxies

- Chandra X-ray images of clusters from Jeltema et al., ApJ, 2005
- Clusters appear in all forms! Not always nice and friendly round blobs
- Cosmology from clusters? In this talk:
  - Count them! Mass function
  - Merger statistics
- Theorist’s Approach: Simulations
  - Precision cosmology
  - What can go wrong?
  - Halo definition?
A Theorist’s Universe (Dark Matter only)
The Mass Function

- Statistics describing the halo mass distribution in the Universe
- \( n(M) \): number density of clusters/halos with mass > \( M \) in comoving volume element
- Evolution of mass function is highly sensitive to cosmology because matter density controls rate at which structure grows
- After Press/Schechter: semi-analytic fits by Sheth & Tormen (1999), Jenkins et al. (2001), and Warren et al. (2006) (and many more...) using simulations
- Fits and their evolution are controlled by growth function \( D(z) \), which itself is a function of \( \Omega_m \), \( \Omega_\Lambda \), and \( \omega \)
Evolution of the Mass Function for Different Cosmologies

from G.M. Voit, astro-ph/0410173, SCDM: Standard-CDM, $\Omega_m=1.0$, OCDM: Open-CDM, $\Omega_m=0.3, \Omega_\Lambda=0$
$t\text{CDM}$: ad hoc power spectrum, adjust shape
$\omega\text{CDM}$: $\Lambda\text{CDM}$ with $\omega=-0.8$
What is a Halo?


- How can we find a halo in a simulation?
  (i) group finder        (ii) density finder
- How can we compare the results?
- How do we compare to observations?
- Until further notice: FOF with linking length $b=0.2$

Friends-of-Friends halo finder

Overdensity finder
Challenges for the Simulation of the Mass Function and its Evolution


- First halos at high redshift very small: need very high mass resolution (difficult because lots of particles are needed) or small boxes (not very good statistics)

- Very high starting redshift: small halos form early, initial conditions in simulation don’t have halos, enough time for halo formation to take place is required

- Number of time steps: high starting redshift and enough time for halo formation lead to time step criteria

- Force resolution: in order to be able to resolve small halos, force resolution needs to be sufficient
Contradictory Results on the Form of the Mass Function at High Redshift: Press-Schechter or Sheth-Tormen like?

Particle Mesh Code PMM
50Mpc/h box,
initial redshift $z=60$

Tree Code PKDGRAV
50Mpc/h box,
initial redshift $z=130$

Both code participated in code comparison, good agreement!
Halo Growth Function

Time for halo to form and virialize, give particle in a halo 10 crossing times ("violent relaxation")

If simulation starts too late, initial rise will be missed and simulation may never catch up

8 Mpc/h box
Mass function as a function of redshift leads to information about halo formation time

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Haroz & Heitmann 2008, invited paper

Early Start

Late Start
Force Resolution Criteria

- Force resolution $\delta_f$ should be smaller than $R_{200}$
- We do not need to resolve the inner part of the halo!

\[ \frac{\delta_f}{\Delta_p} < 0.62 \left( \frac{n_h \Omega(z)}{\Delta} \right)^{1/3} \]

Interparticle spacing  # of particles in halo

Overdensity

Predicted resolution limit
Mass Function Summary I

- 64 simulations of varying box size, covering large mass range
- Good agreement with Warren et al. (2006) down to $z=20$
- Press-Schechter is off by an order of magnitude at high $z$
- Careful study of resolution and time step criteria, starting $z$, box size effects
Mass Function Summary II: Universality

- Mass function in terms of $\sigma(M)$
- Universality holds at the 10% level over wide redshift range
- Advantage: no need to run a simulation for each cosmology
- Can we accurately connect FOF halo mass to SO and give simple translation between mass functions? (earlier work: White 2001)
Halo Masses

- Green: particles in sphere with radius 1.1 x farthest FOF particle
- Blue: FOF particles, $m \approx 7 \cdot 10^{13}$
- Red: SO boundary, $r \approx 0.6 \text{ Mpc}/h$, $m \approx 5 \cdot 10^{13}$
- Black: 2 dim density contour

~3.15 Mpc/h

- FOF mass: follows iso-density contours, hence tracks shapes of bound objects faithfully
- SO mass: builds spheres around density peaks, easier to relate to e.g. X-ray gas, which is measured in 2-d projection
- Not all halos are round! Neither in observations nor in simulations!
- Choose halo definition with observational probe in mind, none is prefect...
- Easy translation between them would be very helpful!
Assume density profile for halo, here: Navarro-Frenk-White (NFW, 1996) profile, empirically found to fit dark matter halo profiles in simulation.

Populate halo with particles according to profile and given overdensity mass, add additional particles in the tail.

Generate millions of FOF mock halos and measure SO mass (here: $M_{200}$ wrt. critical).

Relation between SO and FOF mass depends on concentration $c$, particle sampling, linking length, code resolution (changes $c$).

Mock Halos

- $N_{200} = 10,000, c = 10$
- $N_{200} = 1,000$
- $N_{200} = 100$

$M_{\text{fof}} / M_{\text{200}}$

$N_{200}$

$N_{200} = 100$
$N_{200} = 1,000$
$N_{200} = 10,000$

Fraction of Halos [%]

Mock Halos

- $b = 0.2$
- $c = 2$
- $c = 3$
- $c = 5$
- $c = 10$
- $c = 20$

Graph showing the relation between $M_{\text{fof}} / M_{\text{200}}$ and $N_{200}$ for different values of $c$. The graph demonstrates how the fraction of halos changes with the overdensity mass.
Comparison with Simulated Halos

- Measure FOF and SO from simulation, SO is found from FOF centers
- Halos with more than 1,000 particles
- For some halos: mass ratio much too high, for some halos concentration very low
- Take a “look” at halos from above the prediction, below the prediction, following the prediction

\[
\frac{M_{\text{FOF}}}{M_{200}} = \frac{a_1}{c^2} + \frac{a_2}{c} + a_3
\]

4 Gadget-2 runs: 512³ particles each, two 174Mpc/h and 512Mpc/h boxes each, two cosmologies, main difference: \(\sigma_8=0.75\) and \(\sigma_8=1.0\)
c=9, $M_{\text{fof}}/M_{\text{200}}=1.15$
NFW profile fits well,
Clear center around
which SO is determined,
Prediction works well

$c=8.1, M_{\text{fof}}/M_{\text{200}}=1.8$
“Bridged halo”, center
is not well defined,
NFW profile fits well
for main profile halo, but mass
prediction ignores
second component

$c=1.4, M_{\text{fof}}/M_{\text{200}}=1.37$
Lots of substructure,
NFW profile off,
concentration wrong
The “good” and the “bad” Halos

• Three different methods explored to identify “bad” halos:

1. Exclude every halo with ratio > 2
2. Measure potential minimum and center of mass for FOF halo, if their distance > 0.4/R₂₀₀, exclude
3. Find deepest potential minimum and get SO halo, look for second deepest minimum and grow next SO halo, if second mass is > 20% of first, exclude

• Consistent results for all methods, ~15% exclusion
Results for the Mass Function

Black: “good” FOF halos
Red: measured SO halos
Blue: predicted SO mass

- Consider only “good” halos, 85% of all halos
- Measure FOF mass and SO mass
- Predict SO mass from FOF mass and concentration halo by halo
- Agreement better than 5%!
- Next step: replace halo-by-halo mapping by using M-c relation and scatter (in prep.)
- Got sidetracked on that because....
Perhaps the “bad” ones are also good??

- Measure fraction of excluded halos as a function of their mass: clear cosmology dependence!
- As function of $M_\ast$ (characteristic collapse mass): universal (?)
- For $M_{200} \geq 10^{14}$ Msun: fraction of halos with major satellite as function of satellite mass fraction cosmology dependent
- Advantage over mass function: relative measure

**WMAP-3, $\sigma_8 = 0.75$**

$\sigma_8 = 1.0$
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Conclusion and Summary

- FOF mass function carefully characterized out to $z=20$
- Extremely robust: we derived conditions for starting redshift, force resolution, number of time to get mass function at 5% accuracy
- Universality holds at 5% level for $b=0.2$
- SO mass: much more difficult to measure, sufficient mass resolution important
- Derived connection between FOF mass and SO mass for “nice” halos, depending on $c$ and $N$
- Fraction of halos in merging state contains information about cosmology, advantage: relative measure, volume selection is mitigated