Squeezing f_{NL} out of the matter bispectrum with consistency relations

Sam Goldstein

Columbia University

2209.06228*

SG, *Esposito, Philcox, Hui, Hill, Scoccimarro, Abitbol*

***Buchalter Cosmology 1st Prize**

2310.12959

SG, *Philcox, Hill, Esposito, Hui*

Oliver Philcox

J. Colin Hill

Lam Hui

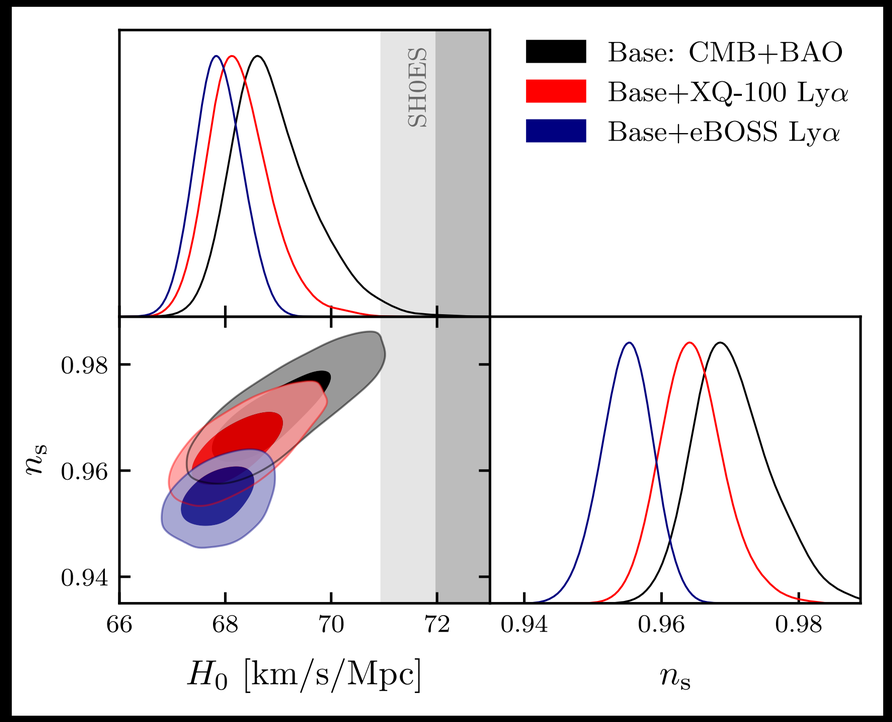
Angelo Esposito

Roman Scoccimarro

Max Abitbol

Early Dark Energy and Lyα Forest

- **SG**, Hill, Irsić, and Sherwin, 2303.00746
  - ***Phys Rev Lett.* 131,** 201001
    - Editors' Suggestion/Featured in Physics

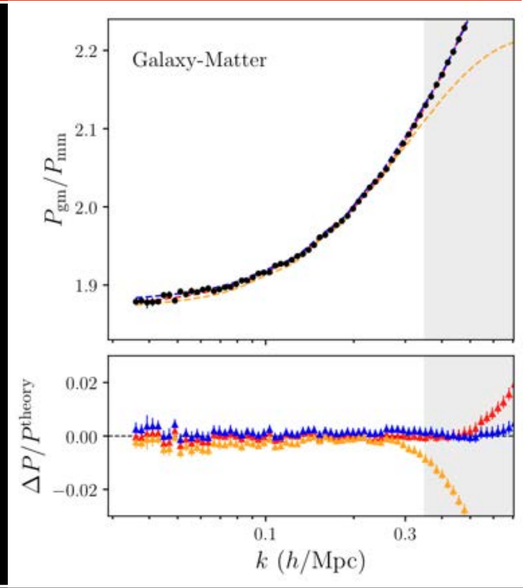


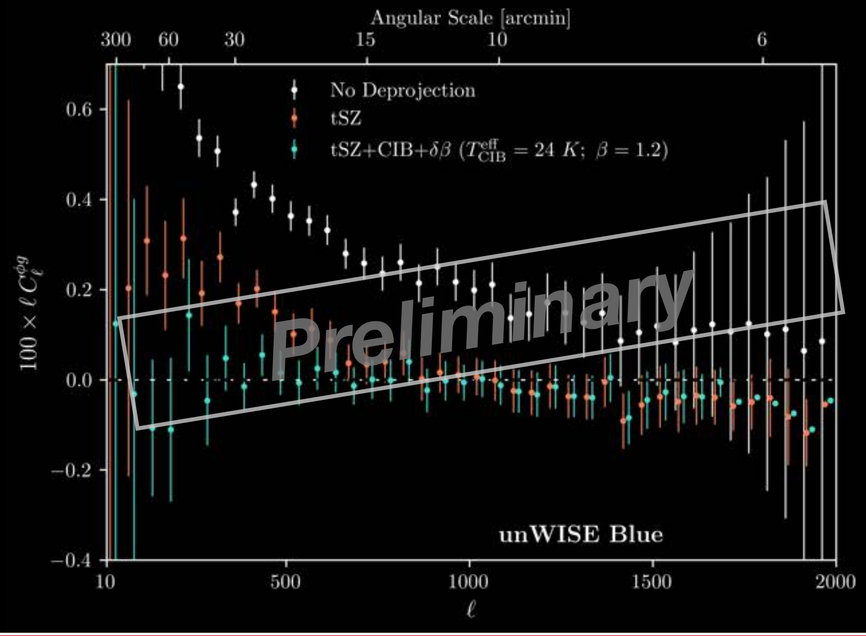
Current Lyα forest data disfavor EDE as a resolution to Hubble tension

What I won't talk about…

Galaxy-Halo Connection

- Comparing simulations with analytic models for LSS
  - ***Galaxy bias models:* SG**, Pandey, Slosar, Blazek, and Jain, 2111.00501
  - ***Splashback radius***: 2111.06499 and 2105.05914





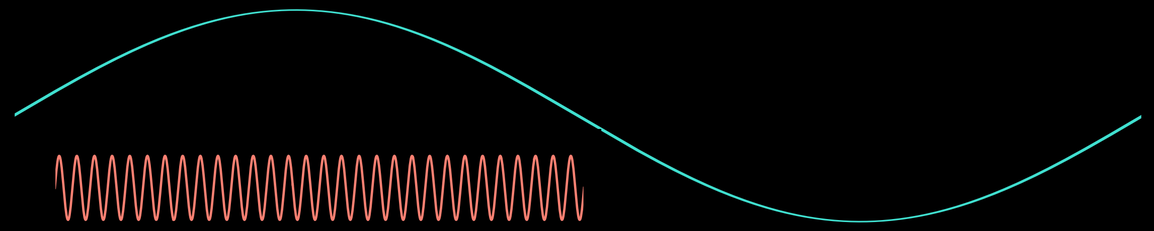
New physics from CMBxLSS

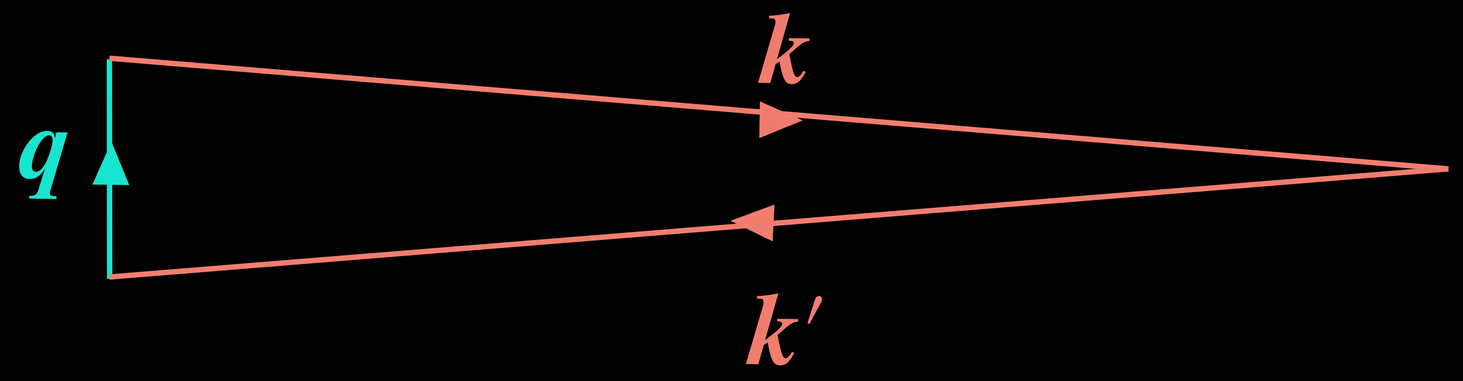
Note
Notation

Correlations between long and short wavelength cosmological perturbations are highly constrained by symmetries

Soft/long wavelength mode:

Hard/short wavelength mode:





"Squeezed" bispectrum ($q \ll k \approx k'$)

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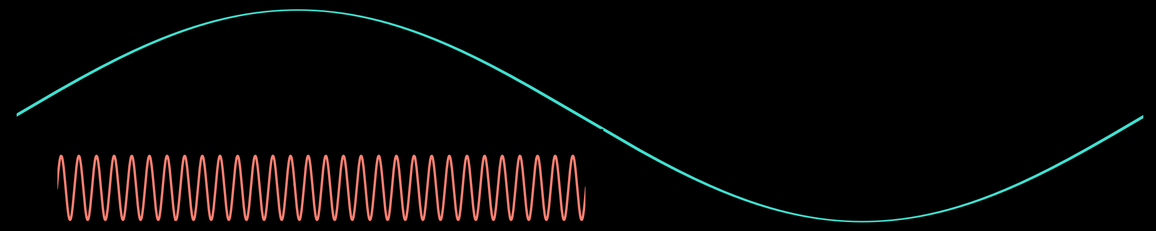

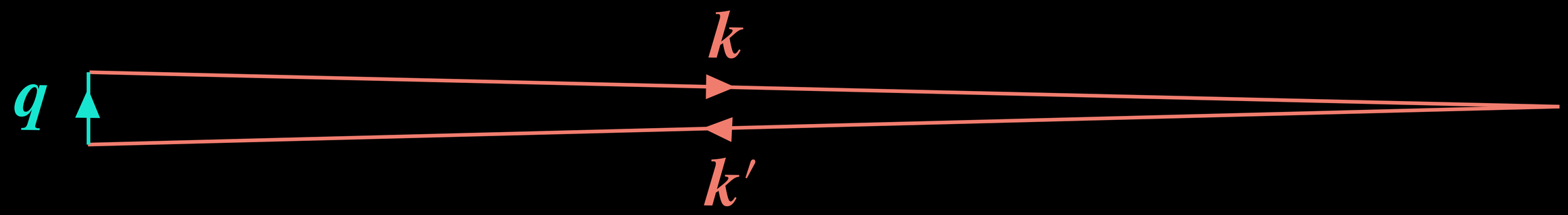

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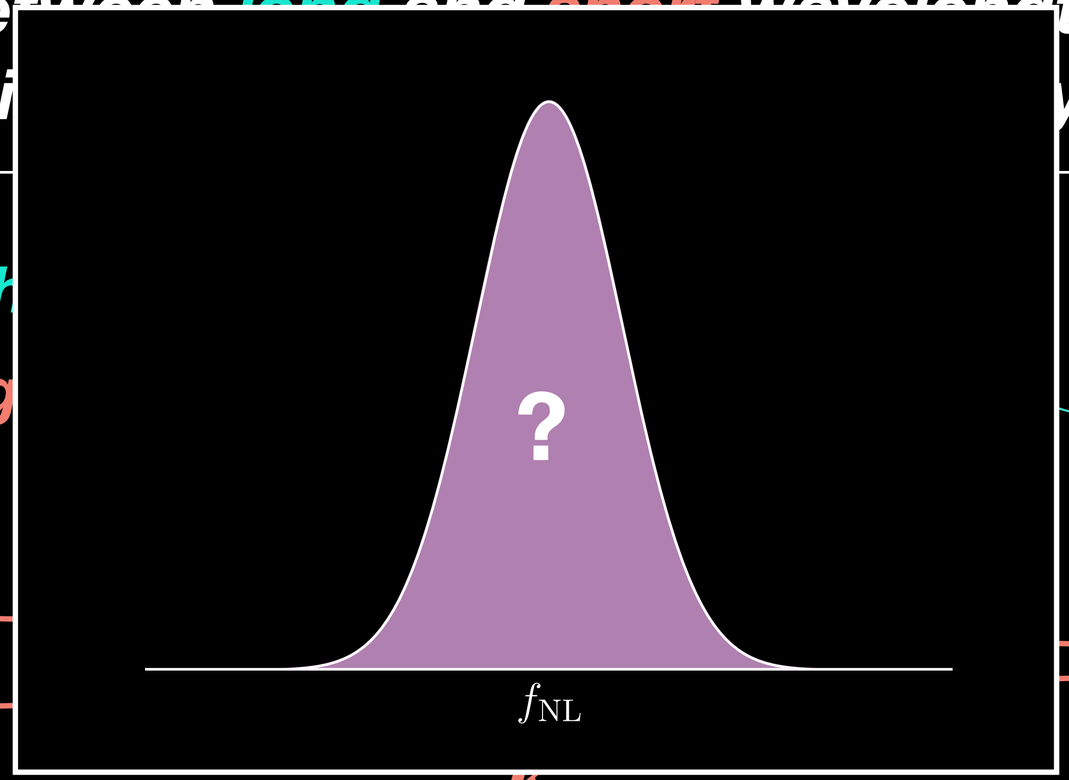


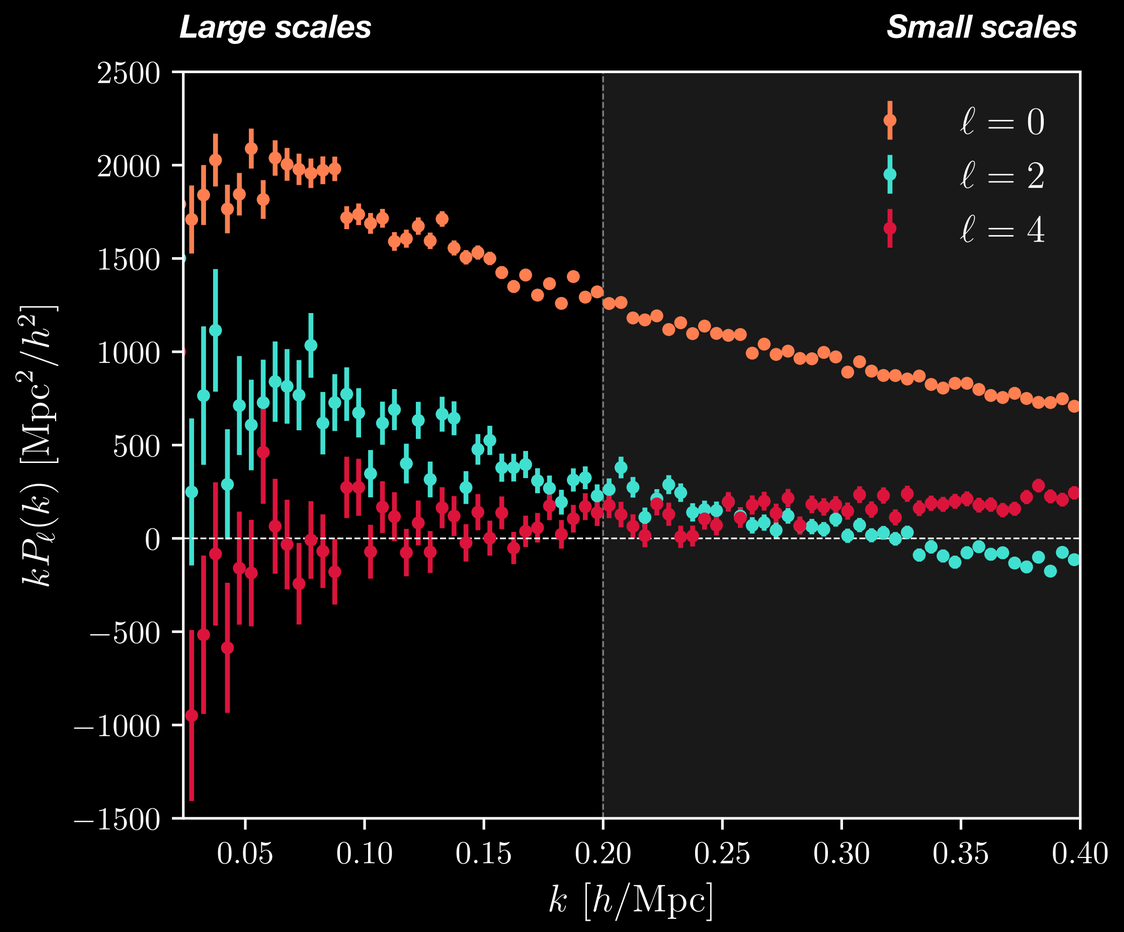
q

k

"Squeezed" bispectrum ($q \ll k \approx k'$)

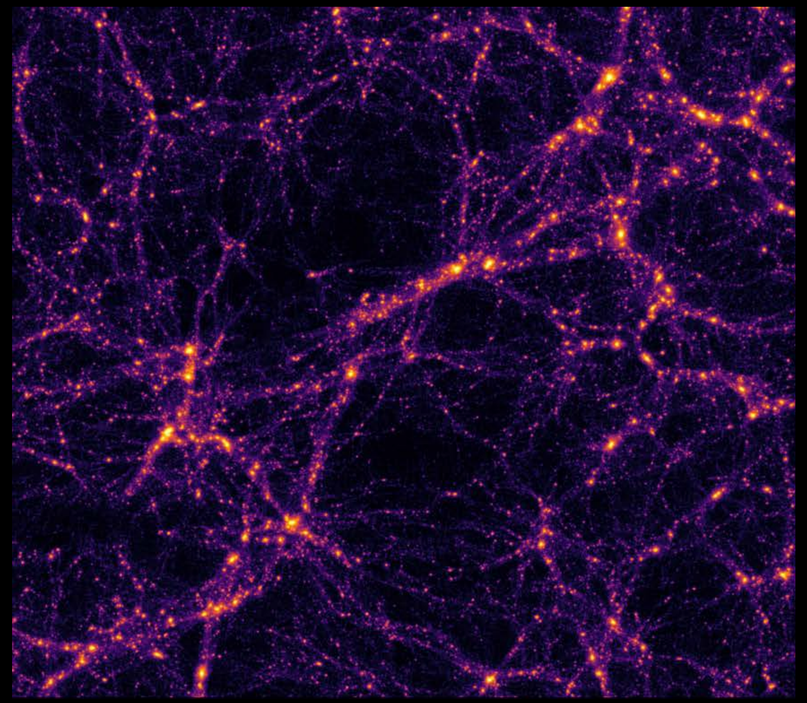
Introduction

- Upcoming large-scale structure (LSS) surveys will measure many modes
  - Stress test ΛCDM
  - Provide insight into **initial conditions**
- Theoretical challenge: **non-linearities**
  - Impose scale cuts
  - Particularly challenging for non-Gaussian/ higher-order statistics



BOSS galaxy power spectrum multipoles.
(*Measurements from Philcox & Ivanov, 2021*)

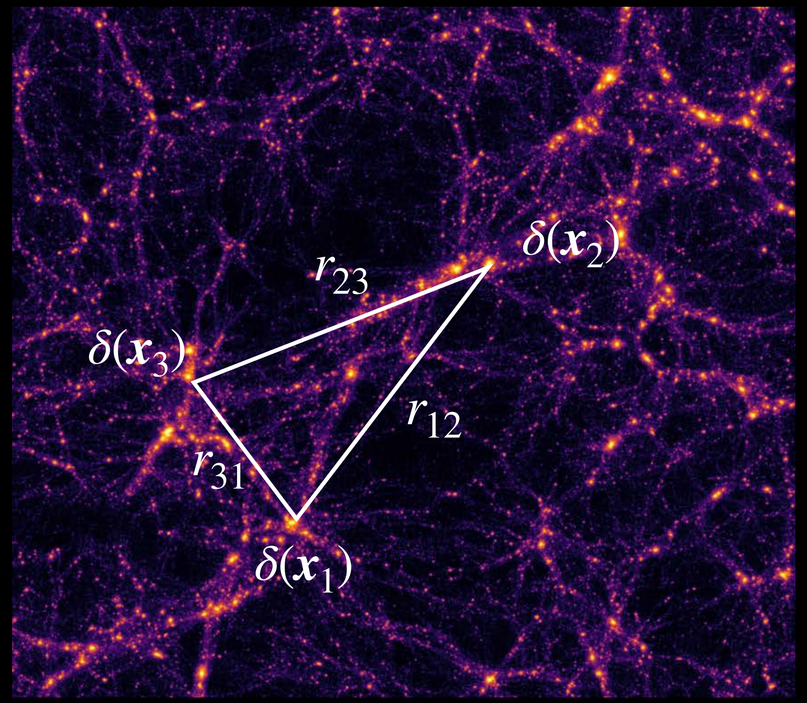
Non-linearities in LSS



Goal: Constrain cosmology from $\delta(\boldsymbol{x}) = \dfrac{\rho(\boldsymbol{x}) - \bar{\rho}}{\bar{\rho}}$

Non-linearities in LSS

- Compress field into its correlation functions



$$\xi_n(\boldsymbol{x}_1, \ldots \boldsymbol{x}_n) \equiv \langle \delta(\boldsymbol{x}_1) \ldots \delta(\boldsymbol{x}_n) \rangle_c$$

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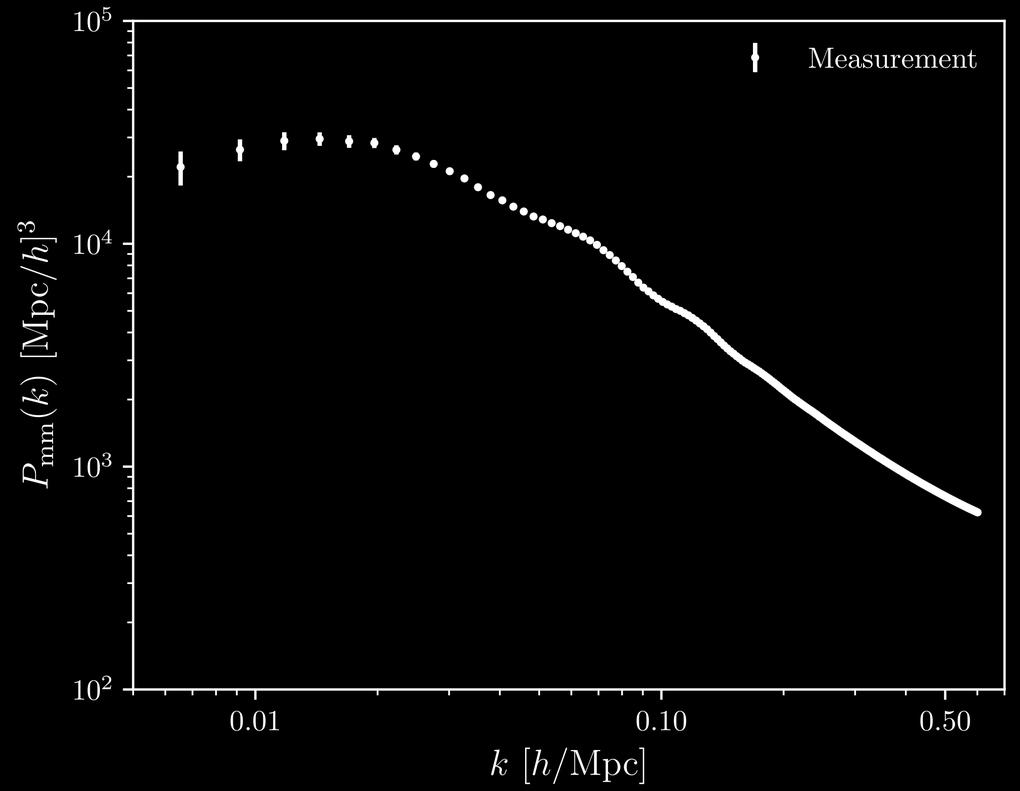

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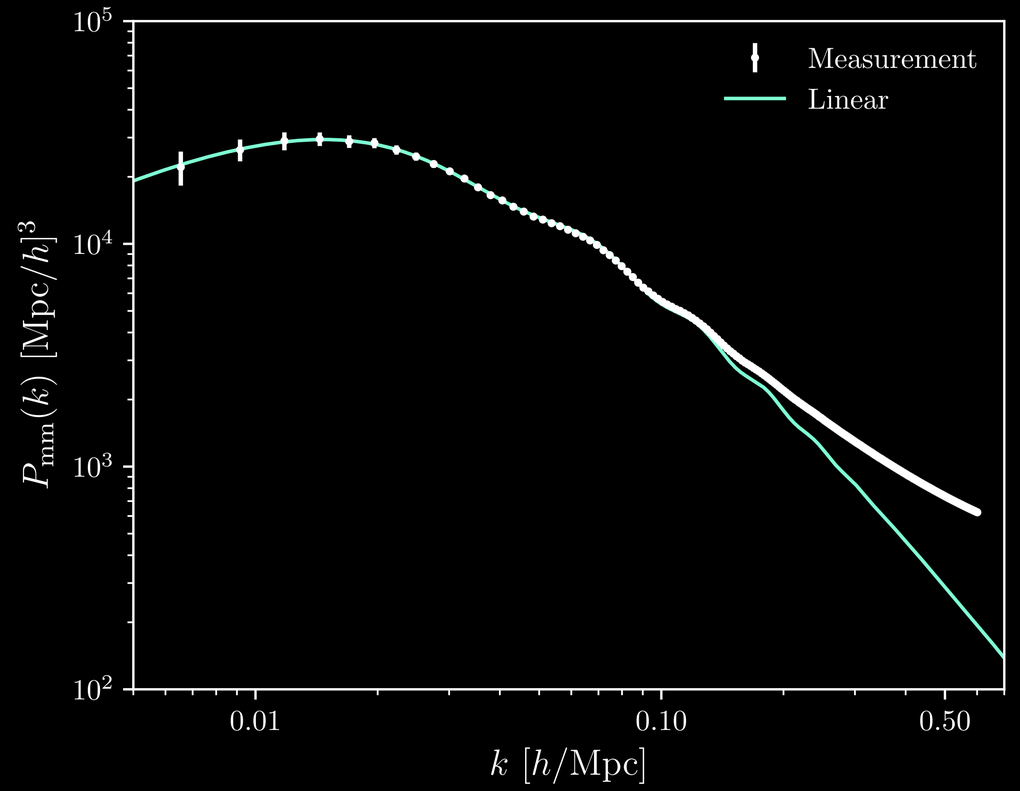

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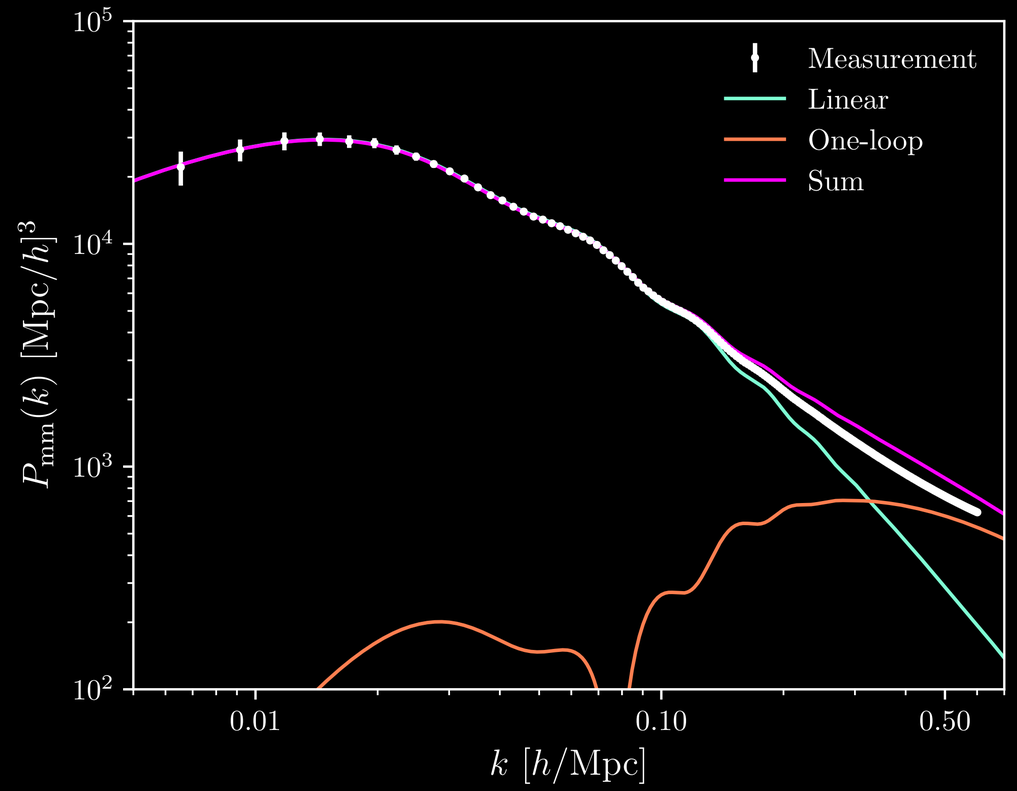

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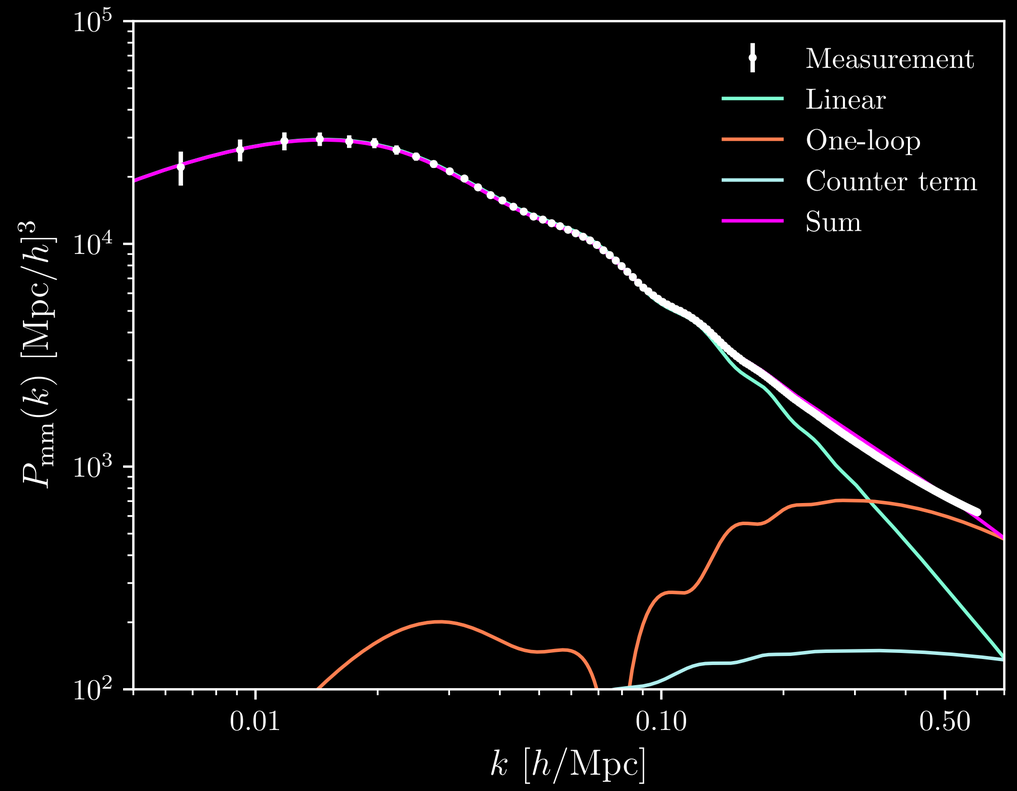

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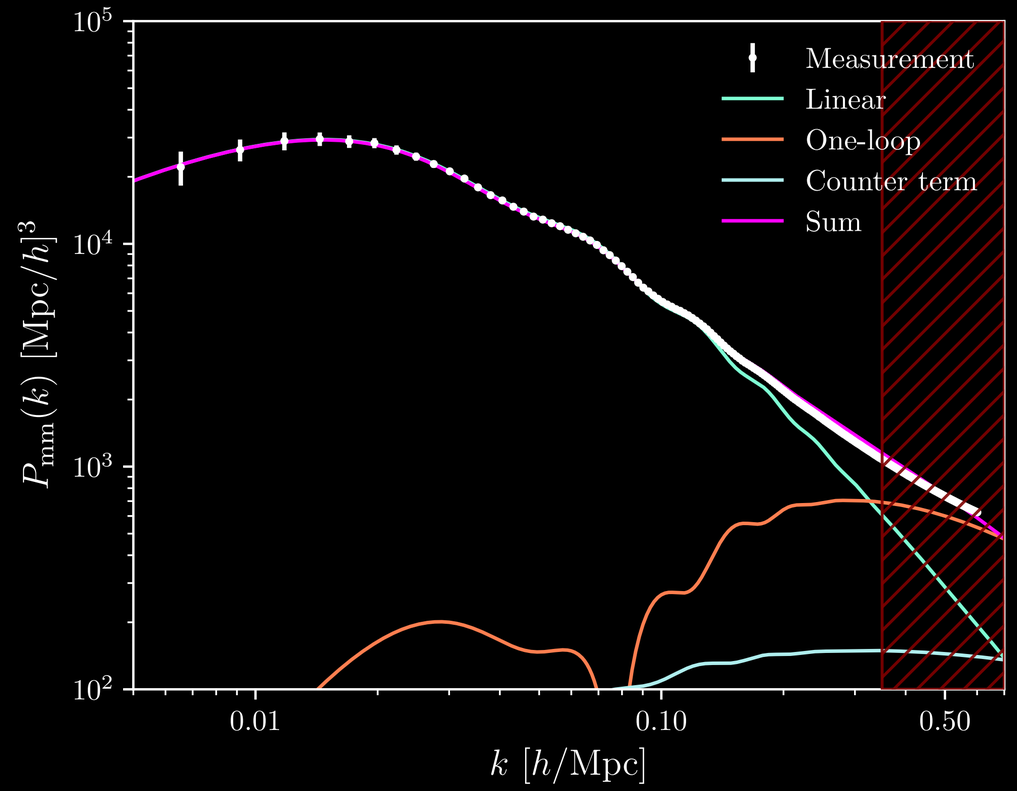

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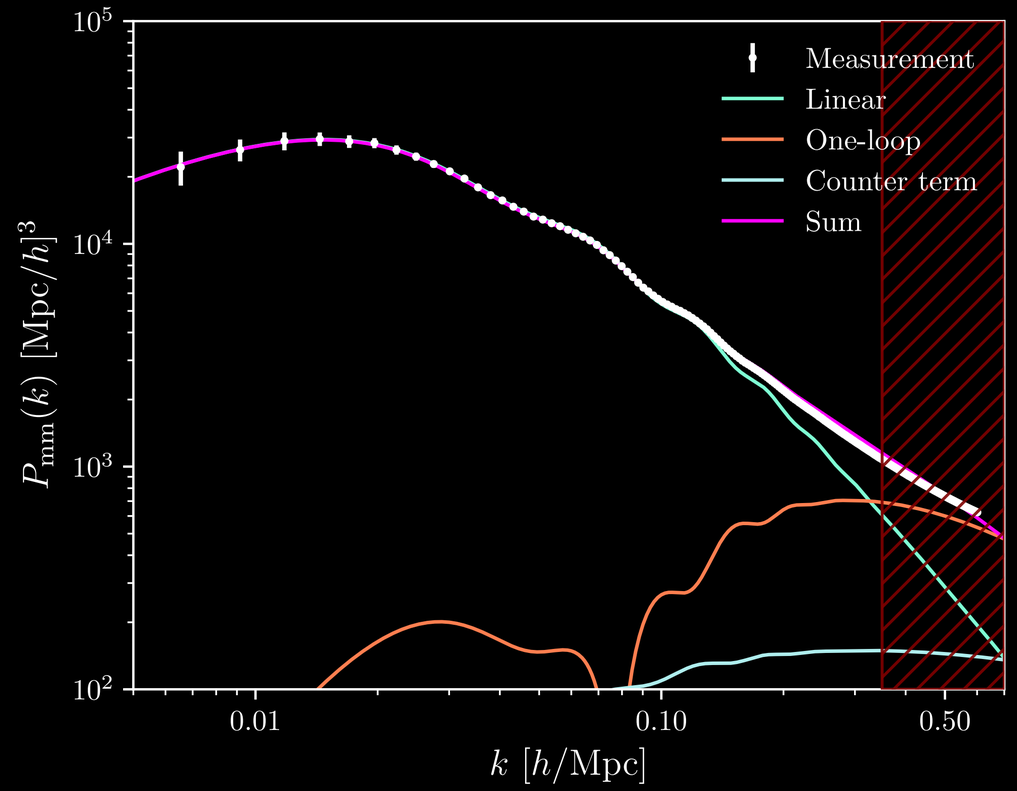

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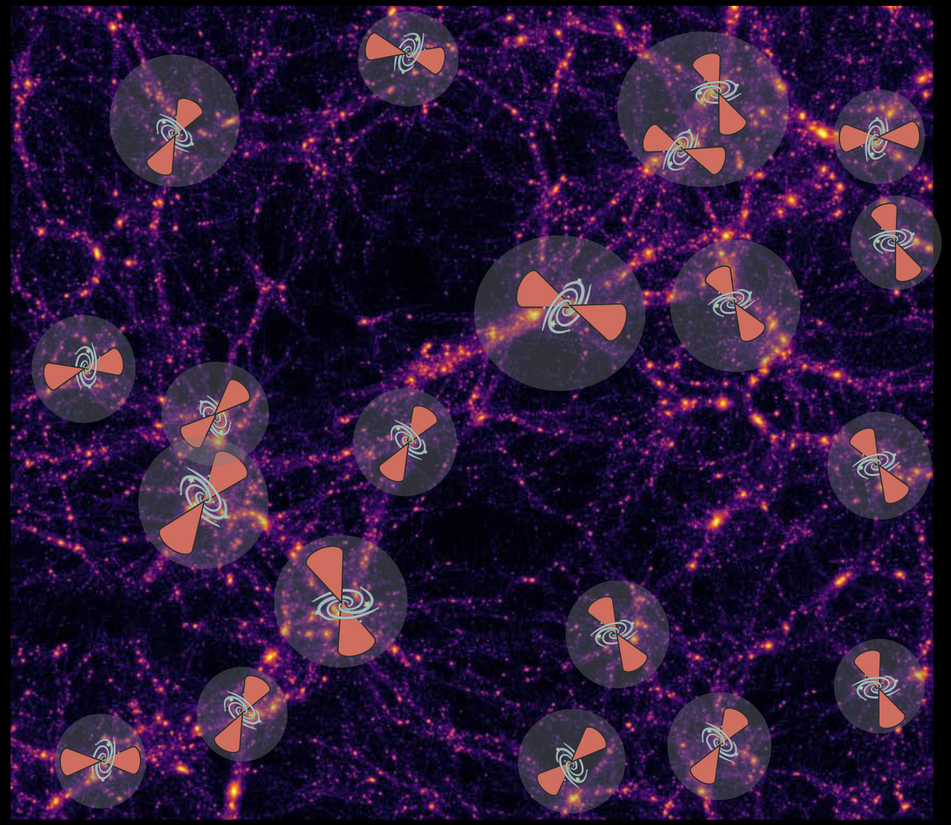

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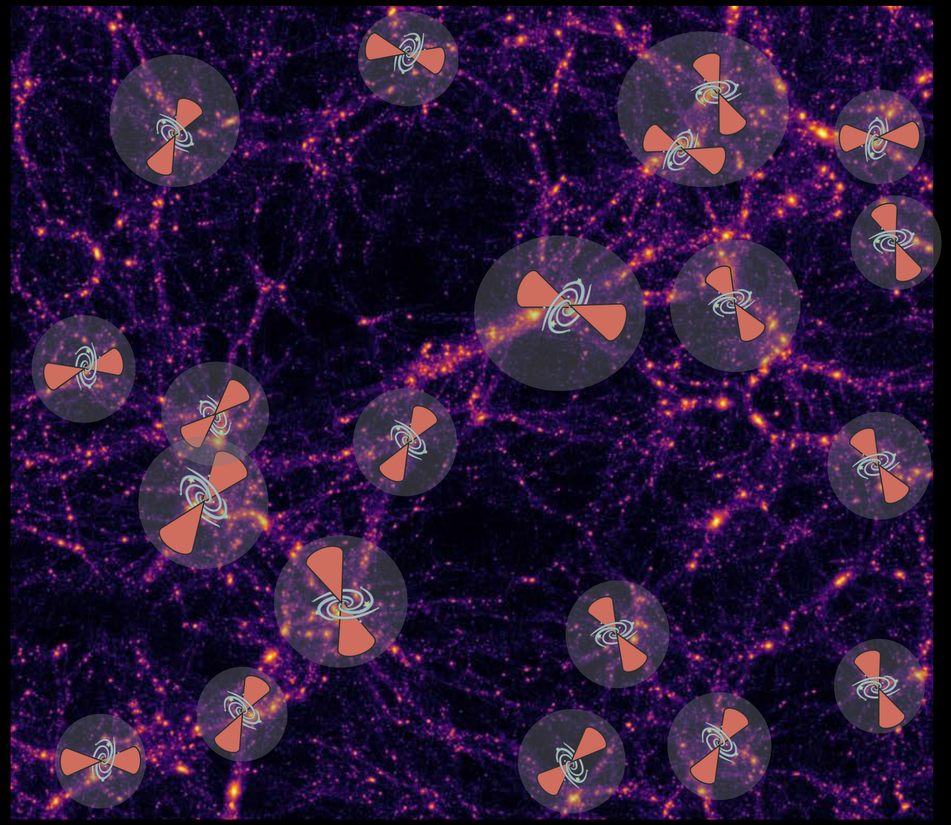

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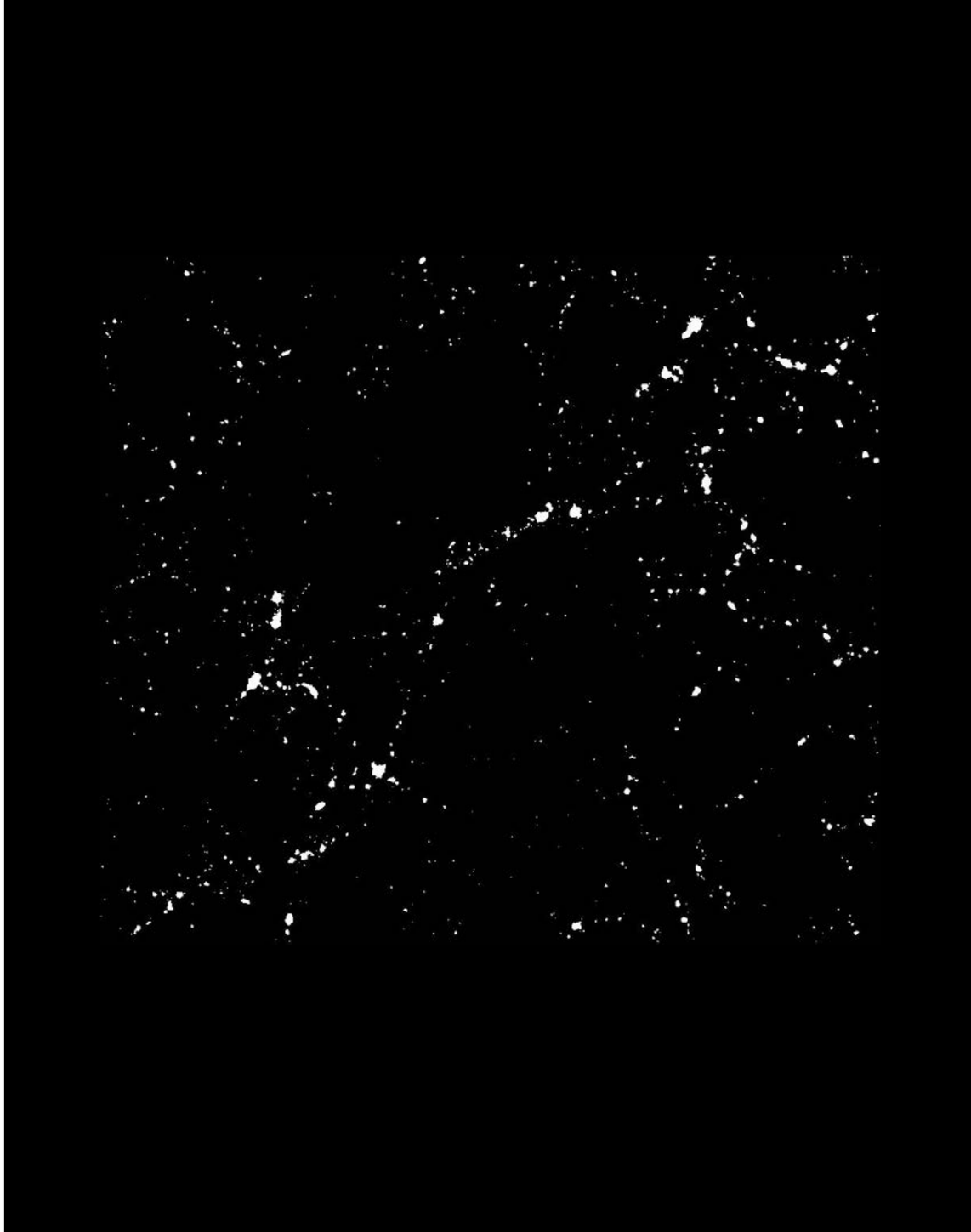

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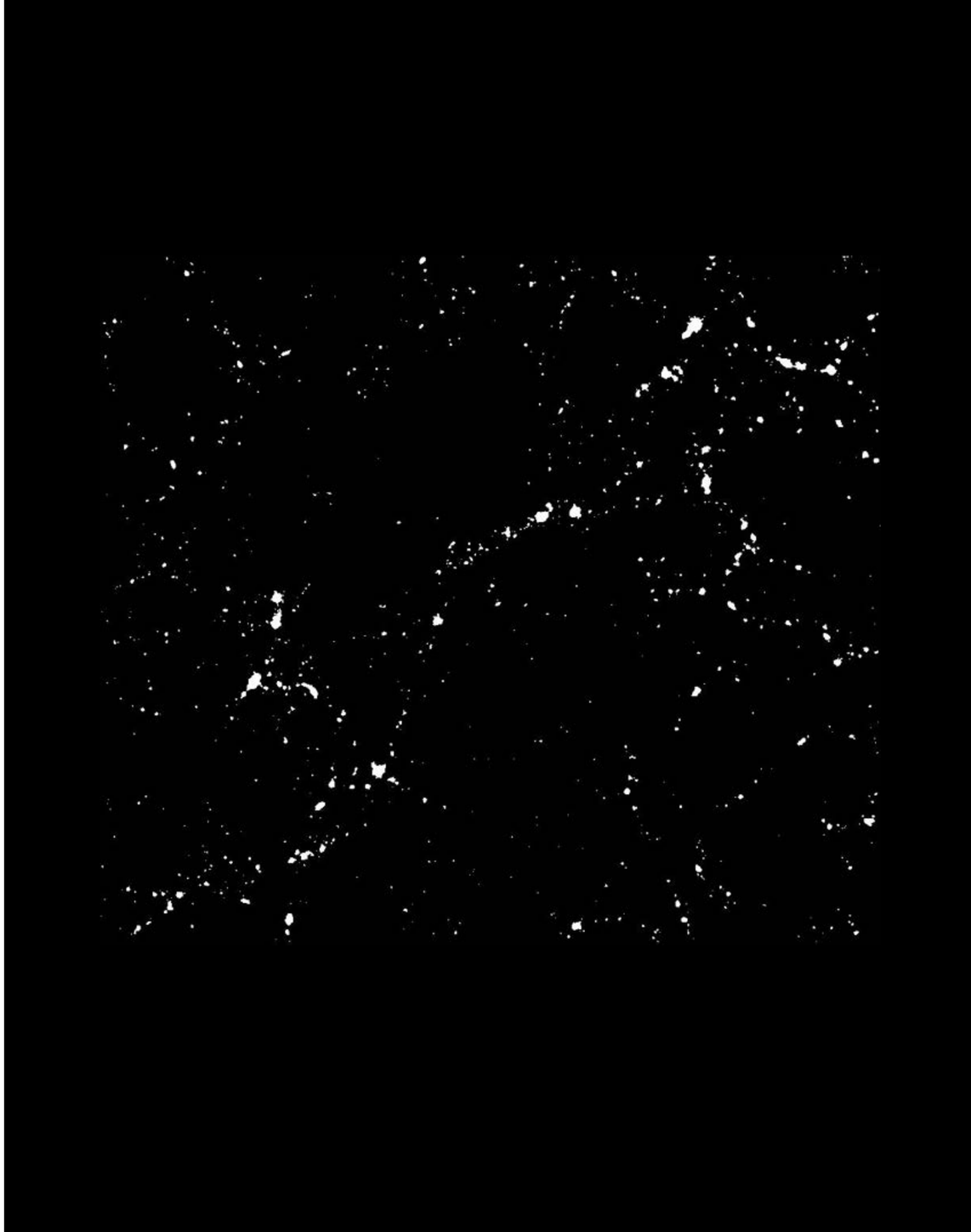

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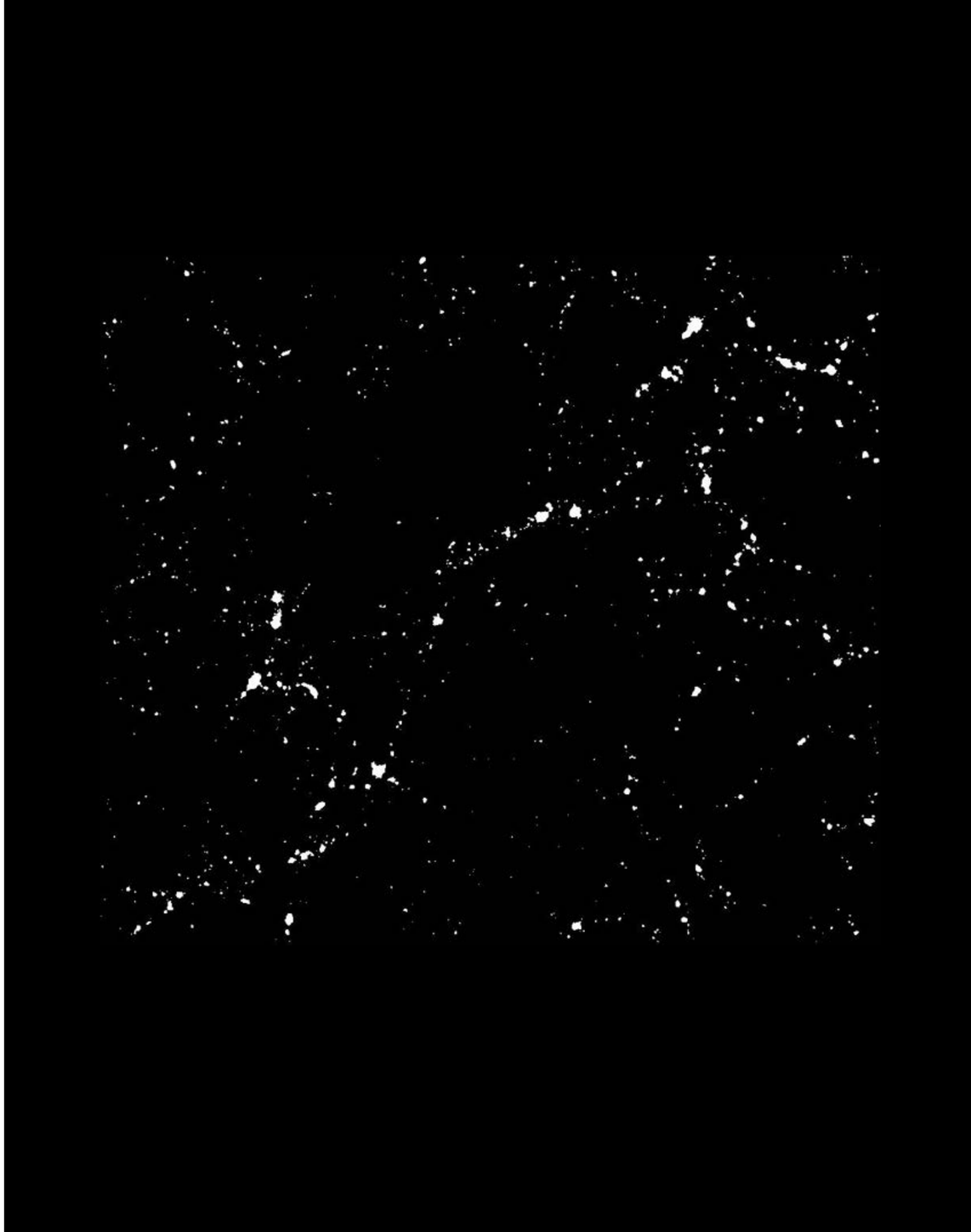

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What to do when we cannot use perturbation theory?

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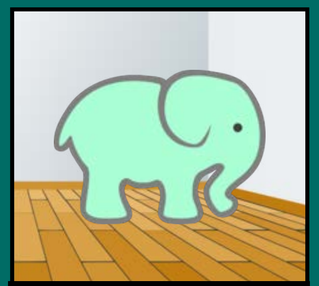

Simulations

Let there be ***freedom from perturbations*** with respect to the things which come from the external cause; and let there be justice in the things ***done by virtue of the internal cause.***

Marcus Aurelius ~170 AD

Let there be ***freedom from perturbations*** with respect to the things which come from the external cause; and let there be justice in the things ***done by symmetries.***

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Consistency relations/soft theorems

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How do symmetries constrain cosmological correlators?

- ***Translational invariance:***

$$\implies \langle \mathcal{O}(\boldsymbol{k_1}) \ldots \mathcal{O}(\boldsymbol{k_n}) \rangle = (2\pi)^3 \delta_D \left(\sum_a \boldsymbol{k_a} \right) \langle \mathcal{O}(\boldsymbol{k_1}) \ldots \mathcal{O}(\boldsymbol{k_n}) \rangle'$$

- ***Rotational symmetry:***

$$\implies \langle \mathcal{O}(\boldsymbol{k_1}) \ldots \mathcal{O}(\boldsymbol{k_n}) \rangle' = F(\boldsymbol{k_i} \cdot \boldsymbol{k_j})$$

- *Examples*:
  - Power spectrum: $\langle \delta(\boldsymbol{k_1})\, \delta(\boldsymbol{k_2}) \rangle = (2\pi)^3 \delta_D(\boldsymbol{k_1} + \boldsymbol{k_2})\, P(k_1)$
  - Bispectrum: $\langle \delta(\boldsymbol{k_1})\, \delta(\boldsymbol{k_2}) \delta(\boldsymbol{k_3}) \rangle = (2\pi)^3 \delta_D(\boldsymbol{k_1} + \boldsymbol{k_2} + \boldsymbol{k_3})\, B(k_1, k_2, k_3)$
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What about more general symmetries?

Consistency relations for LSS

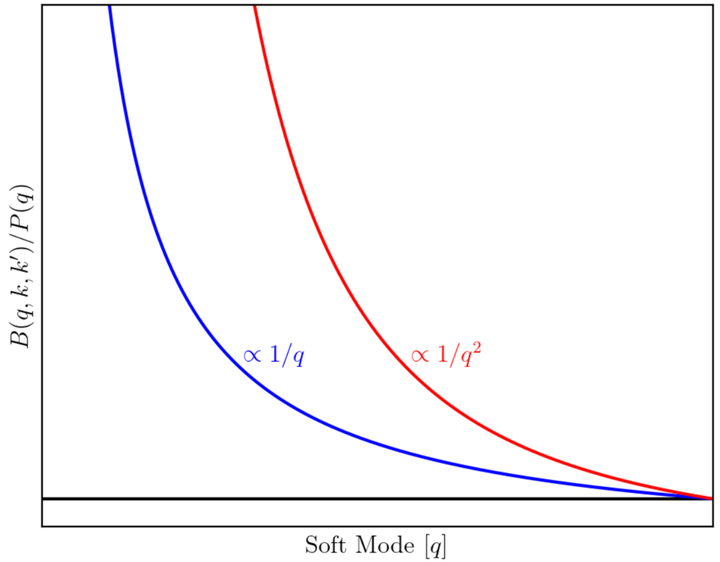
- Equations of motion for δ_m, v_m, Φ

$$\frac{\partial \delta_g}{\partial \tau} + \nabla \cdot [(1+\delta_g) v_g] = 0 \qquad \text{(conservation of mass)}$$

$$\frac{\partial v_g}{\partial \tau} + \mathcal{H} v_g + [v_g \cdot \nabla] v_g = \nabla \Phi \qquad \text{(conservation of momentum)}$$

$$\nabla^2 \Phi = \frac{3}{2} \Omega_m \mathcal{H}^2 \delta \qquad \text{(Poisson equation)}$$

Example of what CR violation looks like



- Possess the following symmetry:

1. *Shift in gravitational potential*: $\Phi \mapsto \Phi + \kappa(\eta)$
2. *Time-dependent translation*: $\boldsymbol{x} \mapsto \boldsymbol{x} + \boldsymbol{n}(\eta)$

$$\Phi \to \Phi - (\mathcal{H}\boldsymbol{n}' + \boldsymbol{n}'') \cdot \boldsymbol{x}, \quad \boldsymbol{v} \to \boldsymbol{v} + \boldsymbol{n}'$$

$\Longrightarrow$

Equal-Time LSS Bispectrum Consistency Relation

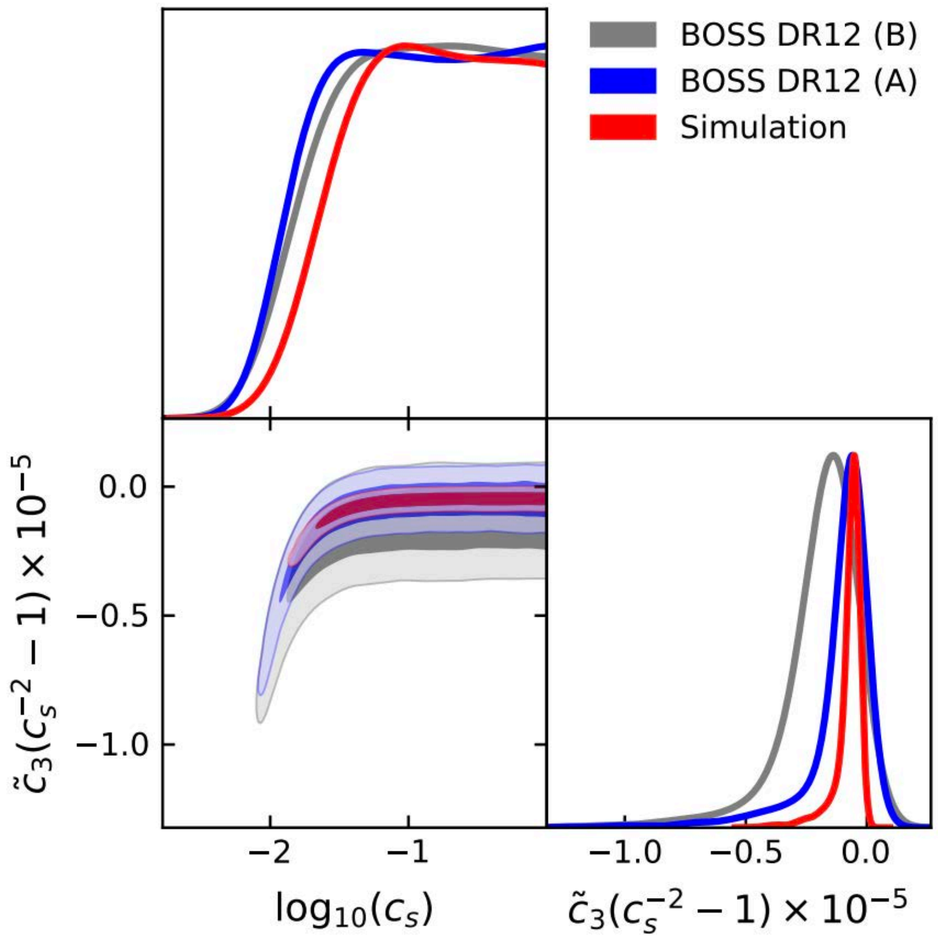
$$\lim_{q \to 0} \left[\frac{B(q,k,k')}{P(q)} \right] \text{ has no } (1/q)^\alpha \text{ poles}$$

** Assumes Gaussian IC's/Equivalence Principle*

Kehagias & Riotto 2012;
Peloso & Pietroni, 2013

Primordial non-Gaussianity

- Simplest single-field models predict ~Gaussian initial conditions
- Classify PNG according to **shapes/amplitudes** ($f_{\rm NL}$) of **bispectra**
  - *Inflaton self-interactions:* $f_{\rm NL}^{\rm equil.}$ and $f_{\rm NL}^{\rm orth.}$
  - *Multiple light fields:* $f_{\rm NL}^{\rm loc.}$
- PNG $\neq f_{\rm NL} \neq f_{\rm NL}^{\rm loc.}$
  - Trispectrum: $g_{\rm NL}$ and $\tau_{\rm NL}$
  - Cosmological collider (Arkani-Hamed & Maldacena, 1503.08043)



Constraints on $f_{\rm NL}$ can be mapped to constraints on physical parameters, e.g. the inflaton sound speed. (from Cabass, Ivanov, Philcox, Simonović, Zaldariagga, 2201.07238)

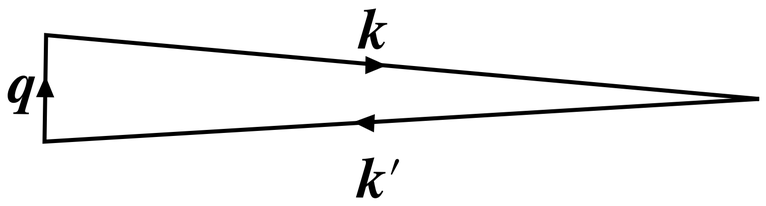
Local primordial non-Gaussianity

- *Local* in configuration space

$$\Phi(\boldsymbol{x}) = \Phi_G(\boldsymbol{x}) + f_{\rm NL}^{\rm loc.}\left(\Phi_G^2(\boldsymbol{x}) - \langle \Phi_G^2(\boldsymbol{x})\rangle\right)$$

Komatsu & Spergel, 2001

- **Correlates long and short modes**
  - Bispectrum peaks in squeezed limit



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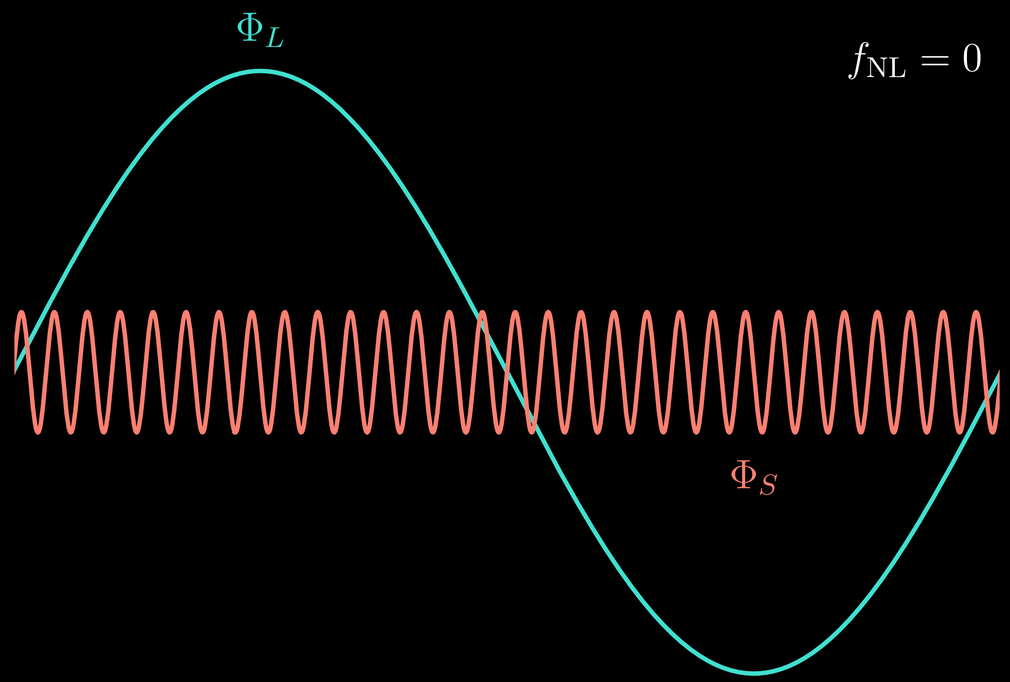
- **Notation:** $f_{\rm NL} \equiv f_{\rm NL}^{\rm loc.}$



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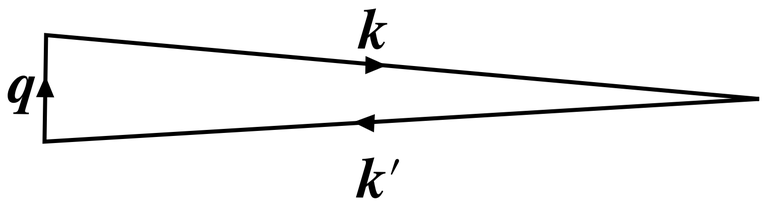

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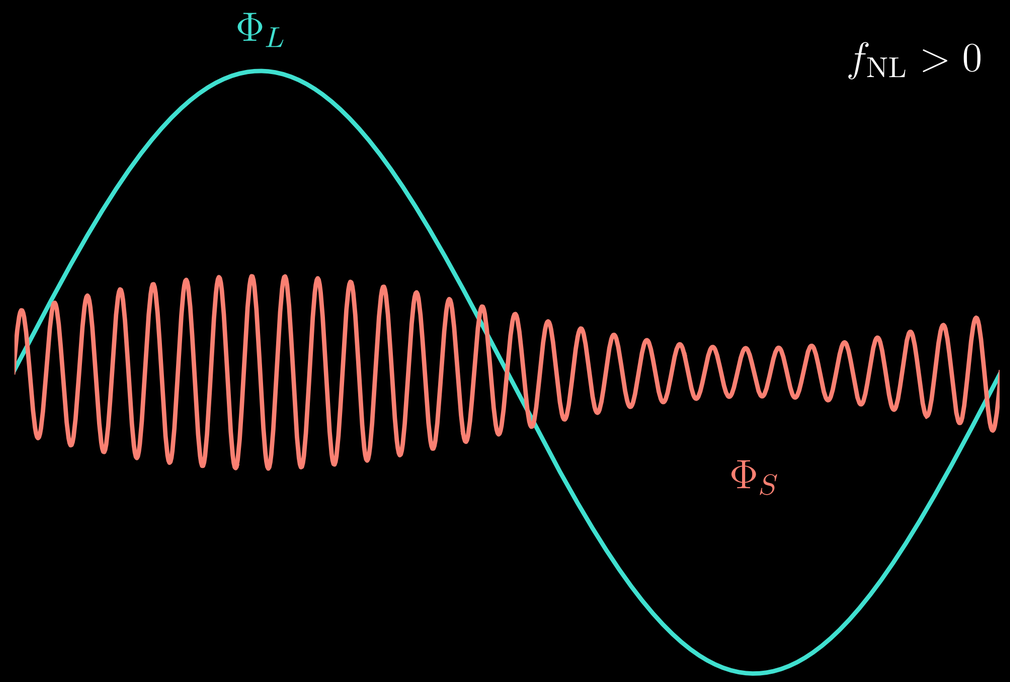

$$\Phi = \Phi_L + f_{\rm NL}(\Phi_L^2 - \langle \Phi_L\rangle^2)$$
$$+(1 + 2f_{\rm NL}\Phi_L)\Phi_S + f_{\rm NL}(\Phi_S^2 - \langle \Phi_S^2\rangle)$$

Mode Coupling

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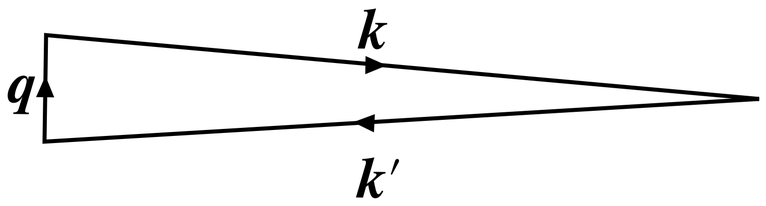

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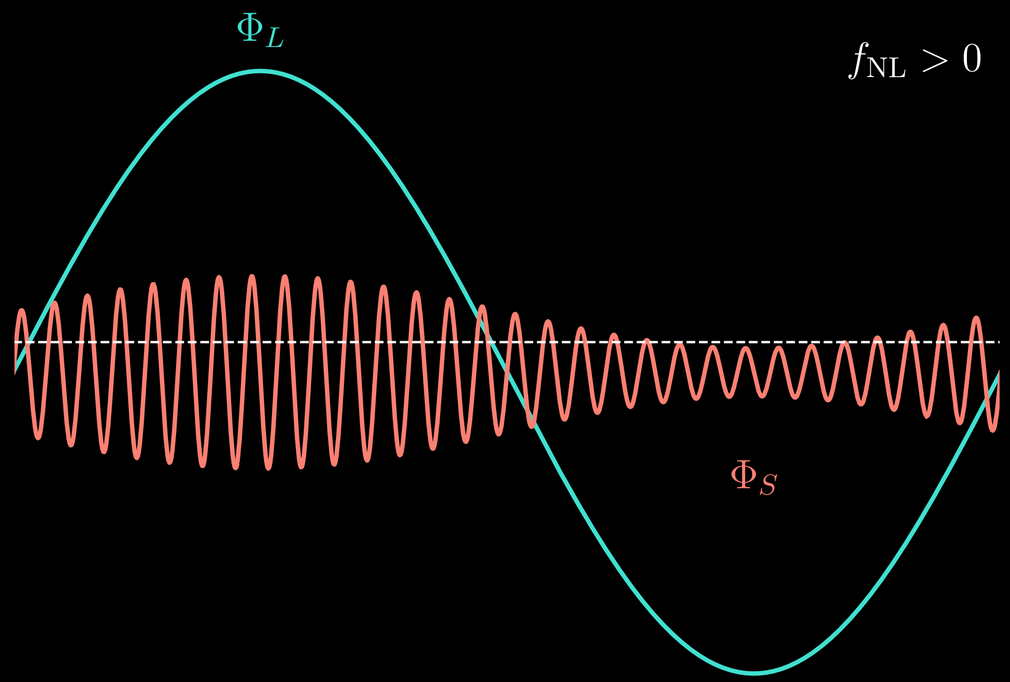

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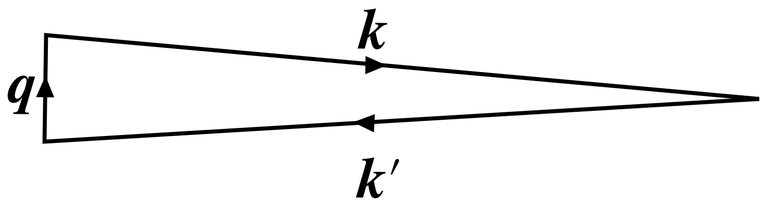
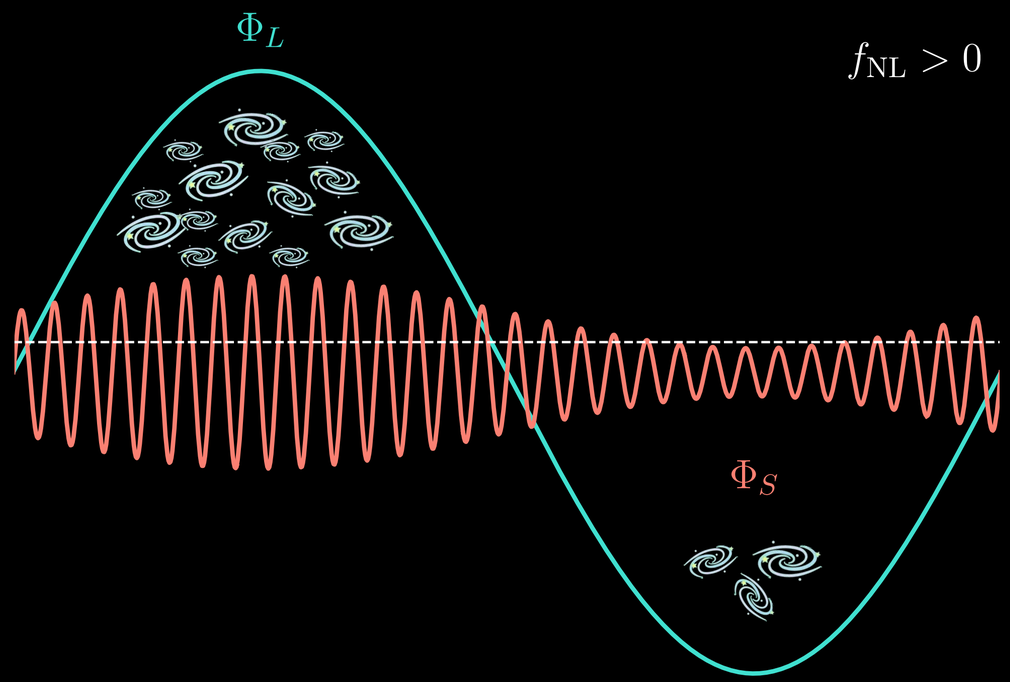
- **Correlates long and short modes**
  - Bispectrum peaks in squeezed limit



"Squeezed" limit

- **Notation:** $f_{\rm NL} \equiv f_{\rm NL}^{\rm loc.}$



"Scale-dependent" bias

Dalal et al./Matarrese & Verde/
Slosar et al./Desjacques, Seljak, & Iliev

Why should we care about $f_{\rm NL}^{\rm loc.}$?

- Detection of $f_{\rm NL}$ local **violates single-field consistency condition**

$$f_{\rm NL} \approx \frac{5}{12}(1 - n_s), \quad \text{(assuming single field inflation)}$$

Maldacena, 2002; Creminelli & Zaldarriaga, 2004

- Current best constraints come from CMB
  - $f_{\rm NL} = -0.9 \pm 5.1$ (*Planck* 2018)
- $\sigma_{f_{\rm NL}} < 1$ is an **important theoretical target**
  - Near-term CMB may reach $\sigma_{f_{\rm NL}} \approx 2$
  - LSS(×CMB) can do better!

Local PNG violates LSS consistency relations

- Let's derive the leading order $f_{\rm NL}$ contribution to the **squeezed** matter bispectrum
- Squeezed bispectrum ($q \ll k$) is described by **modulation** of small scale power spectrum to long wavelength gravitational potential Φ_L

$$\lim_{q \ll k_{\rm NL}, k} B_m(q,k,k') = \lim_{q \ll k_{\rm NL}, k} \langle \delta_m(\boldsymbol{q}) \delta_m(\boldsymbol{k}) \delta_m(\boldsymbol{k}') \rangle',$$

(Non-perturbative see, e.g., Lewis, 1107.5431)

$$= \langle \delta_m(\boldsymbol{q}) P_m(k \,|\, \Phi_L) \rangle' = P_{\Phi_L m}(q) \frac{\partial P_m(k)}{\partial \Phi_L(q)},$$

$$= \underbrace{\frac{3\Omega_{m0} H_0^2}{2q^2 T(q) D_{\rm md}(z_q)}}_{\text{From Poisson's equation}} P_m(q) \frac{\partial P_m(k)}{\partial \Phi_L(q)}.$$

Local PNG violates LSS consistency relations

- Evaluate potential derivative using **separate universe**

$$\Phi(\boldsymbol{x}) = \Phi_L(\boldsymbol{x}) + f_{\rm NL}(\Phi_L^2(\boldsymbol{x}) - \langle \Phi_L \rangle^2) + (1 + 2 f_{\rm NL} \Phi_L(\boldsymbol{x}))\Phi_S(\boldsymbol{x}) + f_{\rm NL}(\Phi_S^2(\boldsymbol{x}) - \langle \Phi_S^2 \rangle)$$

(e.g., Giri, Münchmeyer, Smith 2305.03070)

- Equivalent to local rescaling of σ_8

$$\sigma_8^{\rm loc.}(\boldsymbol{x}) = (1 + 2 f_{\rm NL} \Phi_L(\boldsymbol{x}))\sigma_8$$

- Squeezed bispectrum is then

$$\lim_{q \ll k_{\rm NL}, k} B_m(q, k, k') = \frac{6\,\Omega_{m0} H_0^2 f_{\rm NL}}{q^2 T(q) D_{\rm md}(z_q)} \frac{\partial \log P_m(k)}{\partial \log \sigma_8^2} P_m(q) P_m(k) + \mathcal{O}(f_{\rm NL}^2)$$

Can we use this to constrain $f_{\rm NL}$?

“Squeezing f_{NL} out of the matter bispectrum with consistency relations”

2209.06228

SG, *Esposito, Philcox, Hui, Hill, Scoccimarro, Abitbol*

Simulations

- Use nbody simulations to validate squeezed bispectrum model beyond non-linear scale

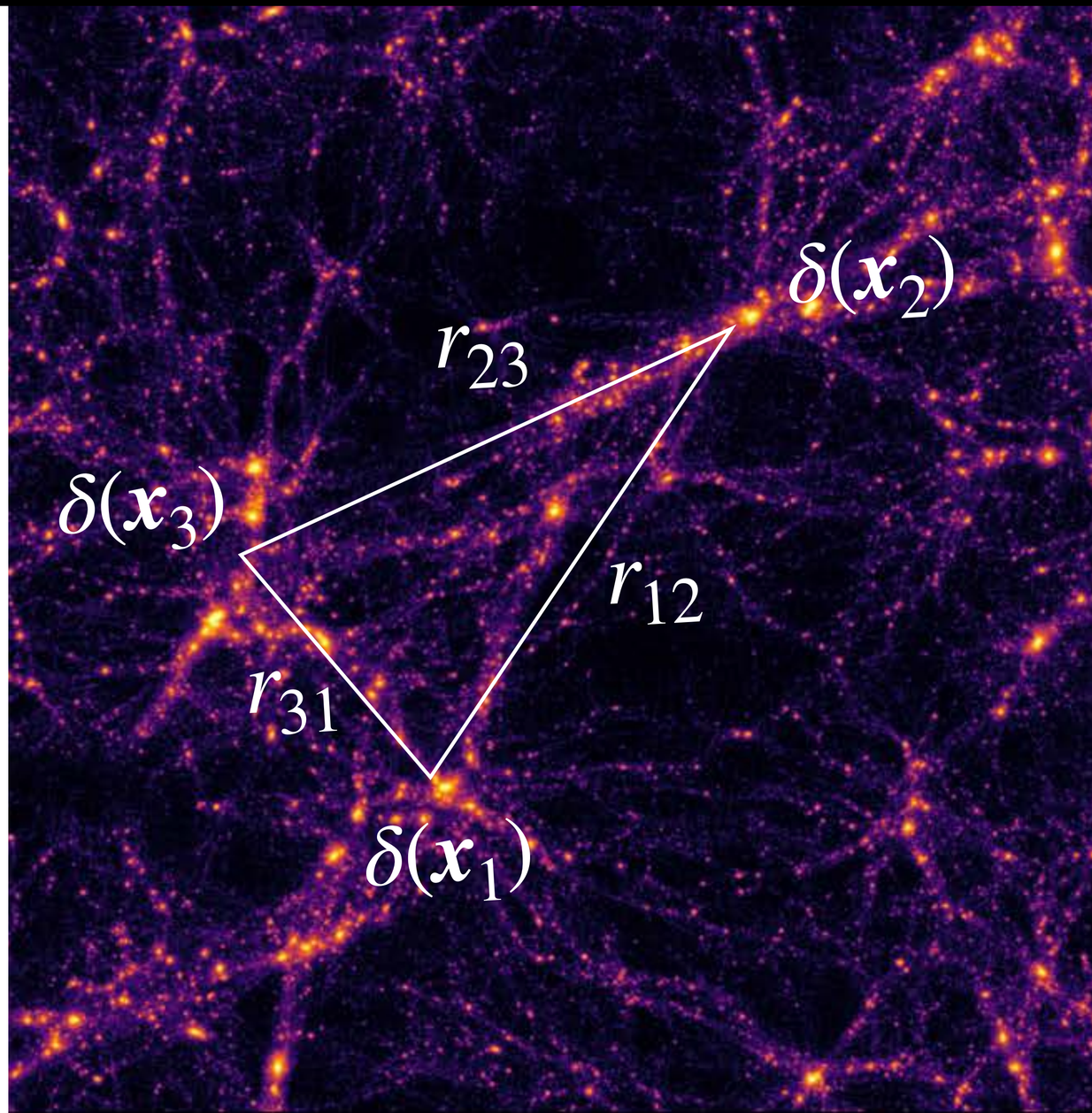
- 40 nbody simulations with Gaussian initial conditions and 12 with $f_{\rm NL} = 100$

  - $L_{\rm box} = 2400\ {\rm Mpc}/h$

  - $N_{\rm particle} = 1280^3$

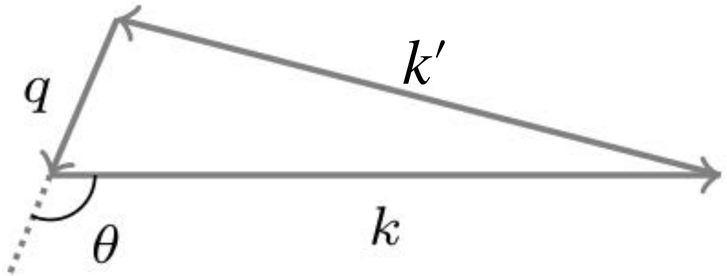
$$\Omega_m = 0.25,\ \Omega_b = 0.04,\ \Omega_\Lambda = 0.75,$$
$$h = 0.7,\ n_s = 1,\ \sigma_8 = 0.8$$

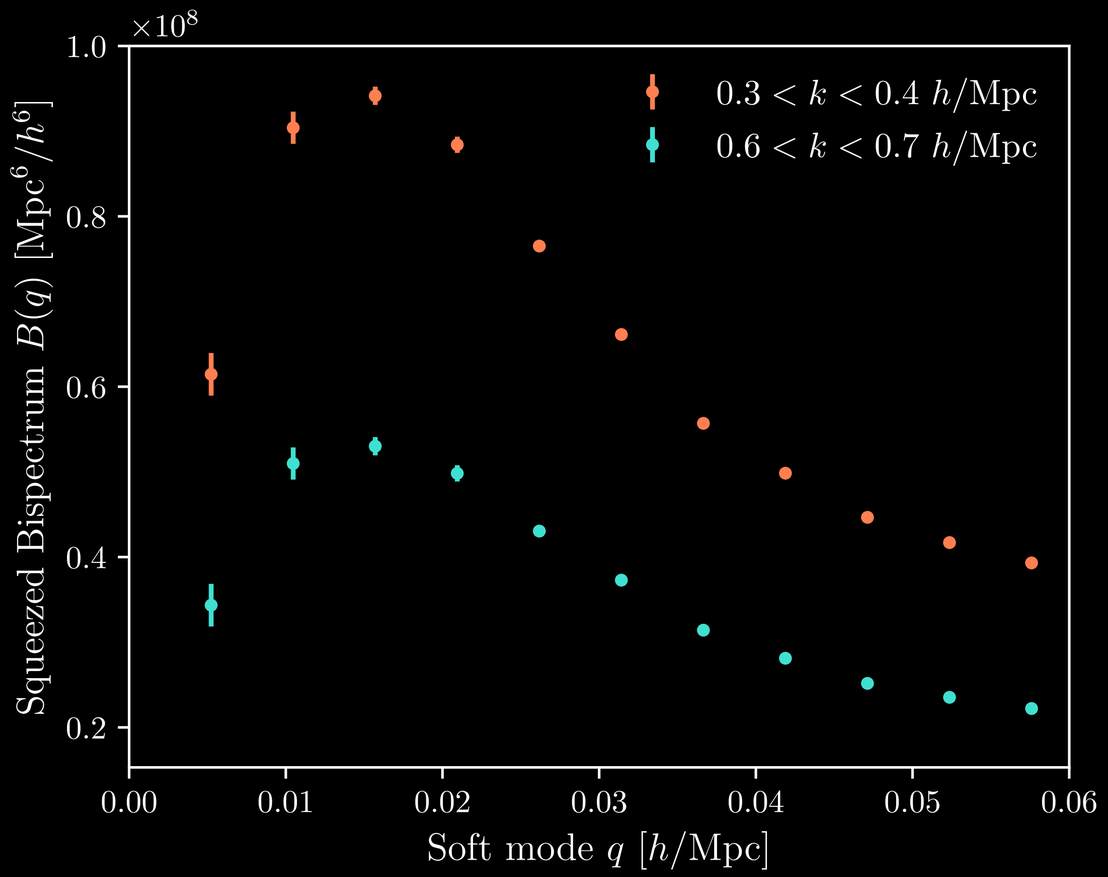
- Measure squeezed bispectrum and power spectrum at **z=0** and **z=0.97**



Measurements

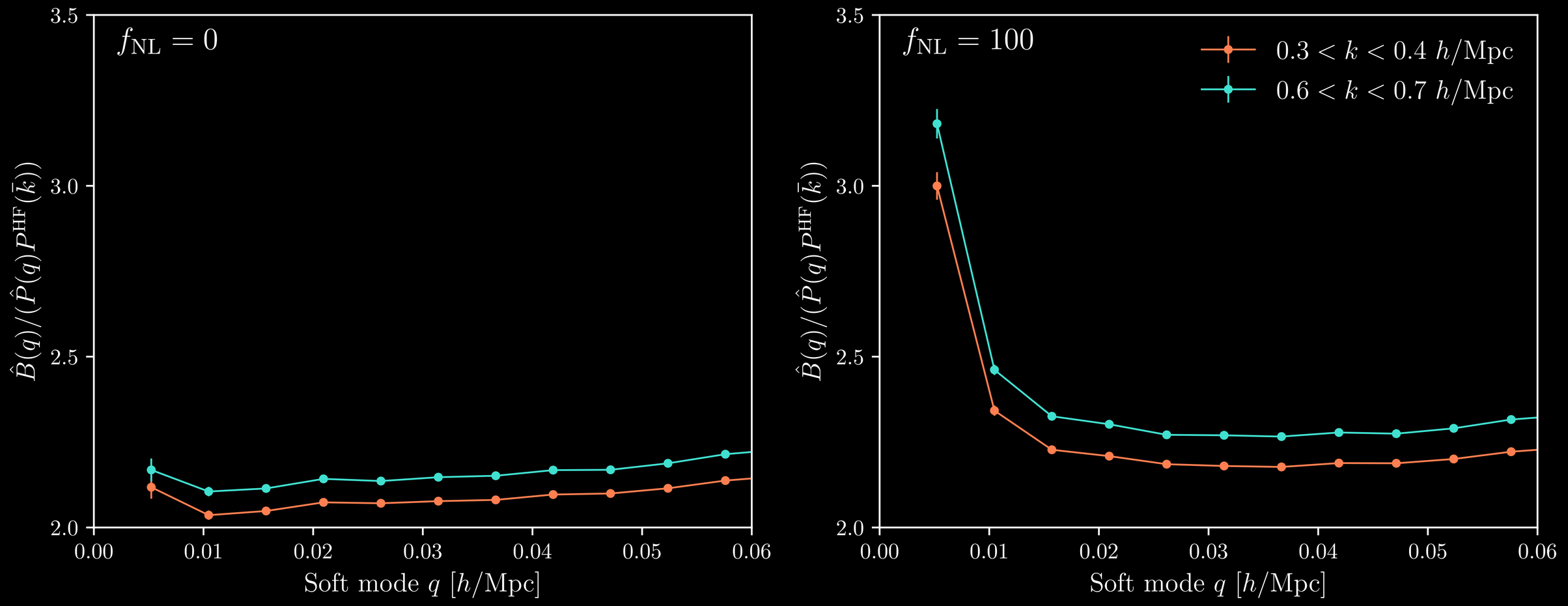
- Measure soft power spectrum ($\hat{P}_m(q)$)
- **Angle averaged** squeezed bispectrum
  - $\hat{B}(q, k_{\rm min}, k_{\rm max}) = \int d\Omega_{\boldsymbol{k}}\, W(q,k)\, B(q,k,k')$
  - Choose weights $W(q,k,\theta) = 1$ (**sub-optimal!**)
  - Average over wide k-bin for hard momenta





Measurements of angle averaged squeezed bispectrum as a function of the soft mode for two different hard momenta bins.

Measurements (continued)



$f_{\rm NL}$ leads to poles in squeezed bispectrum. Amplitude of pole can be used to constrain $f_{\rm NL}$

Theory model

- Squeezed bispectrum has primordial and gravitational contribution

$$B(q,k,k') = B_{\text{prim.}}(q,k,k') + B_{\text{grav.}}(q,k,k')$$

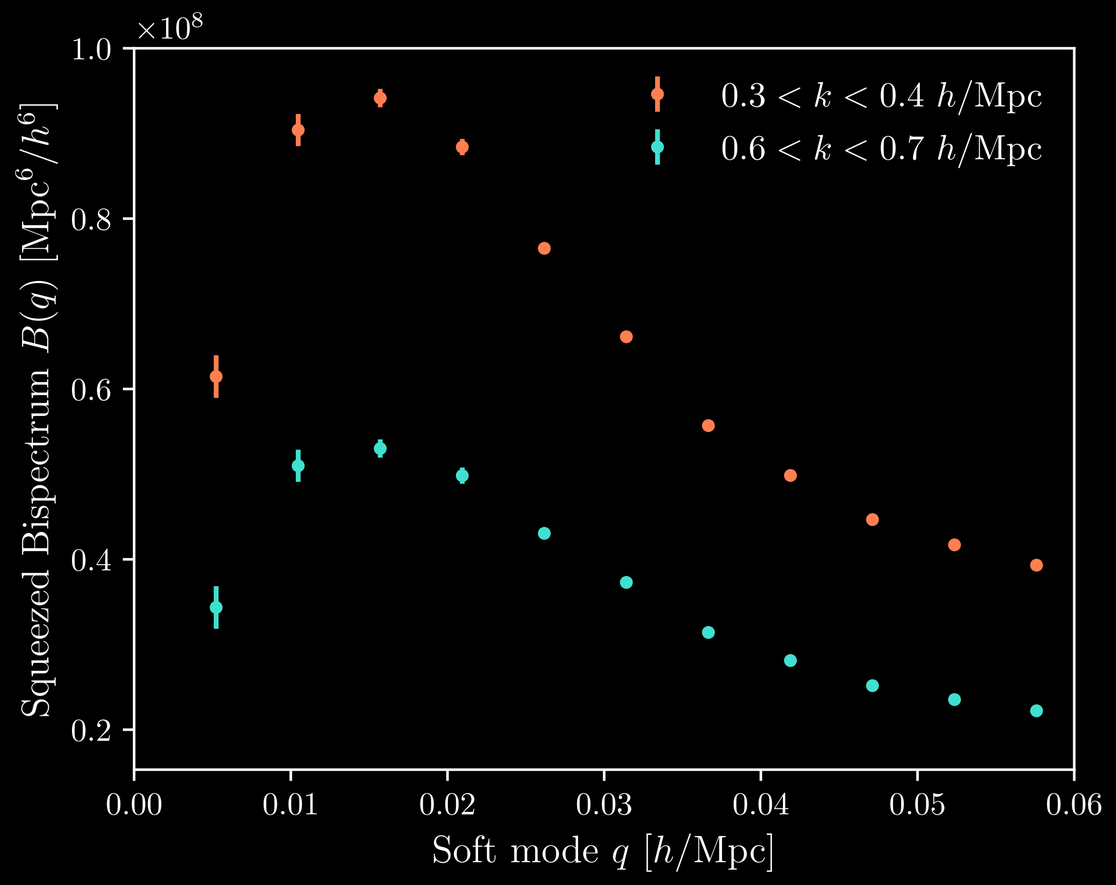
- Already derived primordial contribution

$$B_{\text{prim.}}(q,k,k') = \frac{6\,\Omega_{m0}H_0^2 f_{\text{NL}}}{q^2 T(q) D_{\text{md}}(z_q)} \frac{\partial \log P_m(k)}{\partial \log \sigma_8^2} P_m(q) P_m(k)$$

- Model gravitational contribution based on consistency relations

$$B_{\text{grav.}}(q,k,k') = \sum_{n=0}^{\infty} a_n(k) q^n P(q) P(k)$$

(See also Valageas, 2013; Chiang et al. 2017; Esposito, Hui, Scoccimarro, 2019; Biagetti et al 2022)



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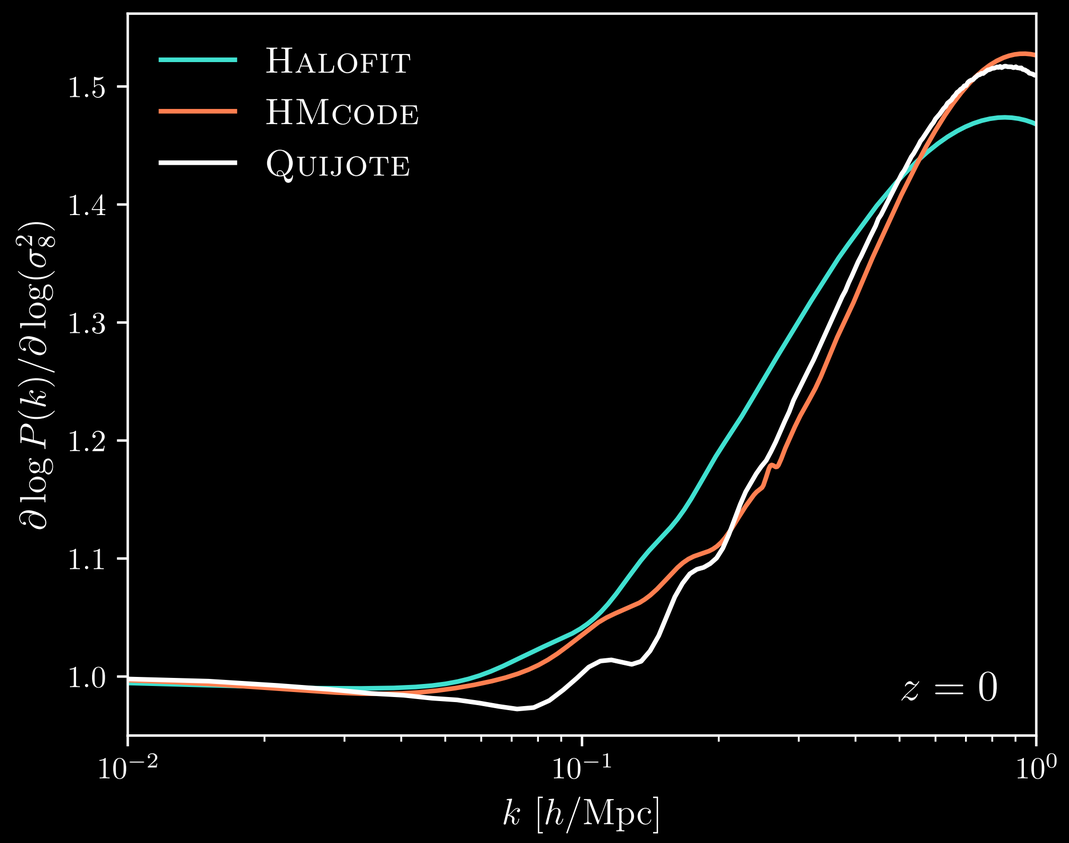

Challenge: f_{NL} is degenerate with logarithmic derivative. Can be modeled with sims for matter. Will be difficult for galaxies (b_ϕ)

Theory model

- Squeezed bispectrum has primordial and gravitational contribution

$$B(q,k,k') = B_{\text{prim.}}(q,k,k') + B_{\text{grav.}}(q,k,k')$$

- Already derived primordial contribution

$$B_{\text{prim.}}(q,k,k') = \frac{6\,\Omega_{m0}H_0^2 f_{\text{NL}}}{q^2 T(q) D_{\text{md}}(z_q)} \frac{\partial \log P_m(k)}{\partial \log \sigma_8^2} P_m(q) P_m(k)$$

- Model gravitational contribution based on consistency relations

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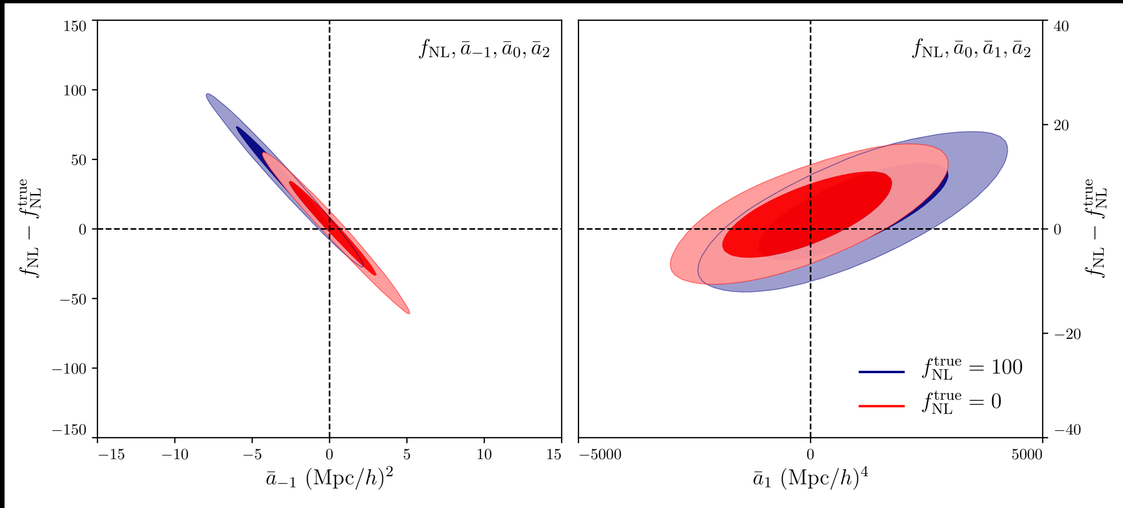

Theory model

- Squeezed bispectrum has primordial and gravitational contribution

$$B(q,k,k') = B_{\text{prim.}}(q,k,k') + B_{\text{grav.}}(q,k,k')$$

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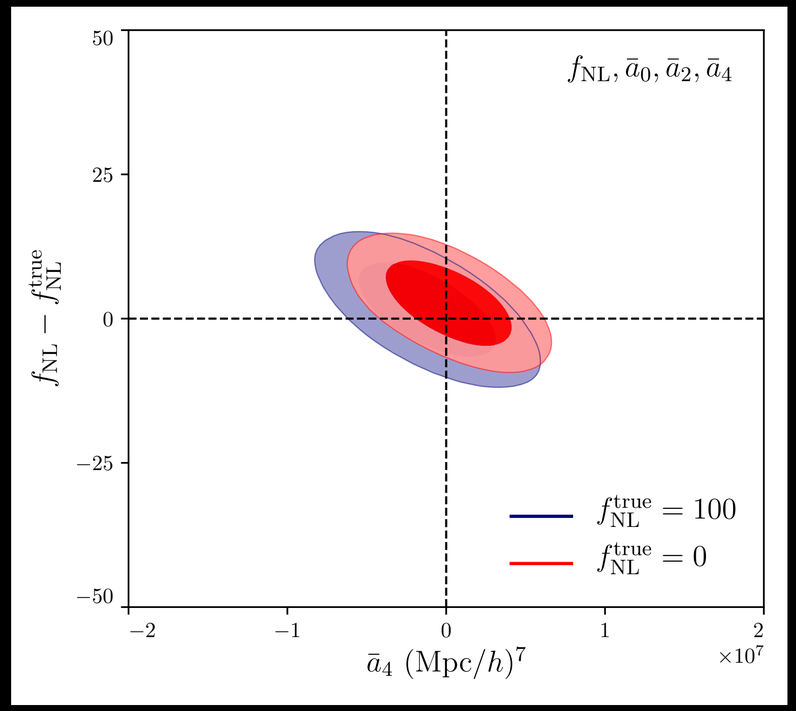
- Model gravitational contribution based on consistency relations

$$B_{\text{grav.}}(q,k,k') = a_0(k)P(q)P(k) + a_2(k)q^2 P(q)P(k)$$

(See also Valageas, 2013; Chiang et al. 2017; Esposito, Hui, Scoccimarro, 2019; Biagetti et al 2022)



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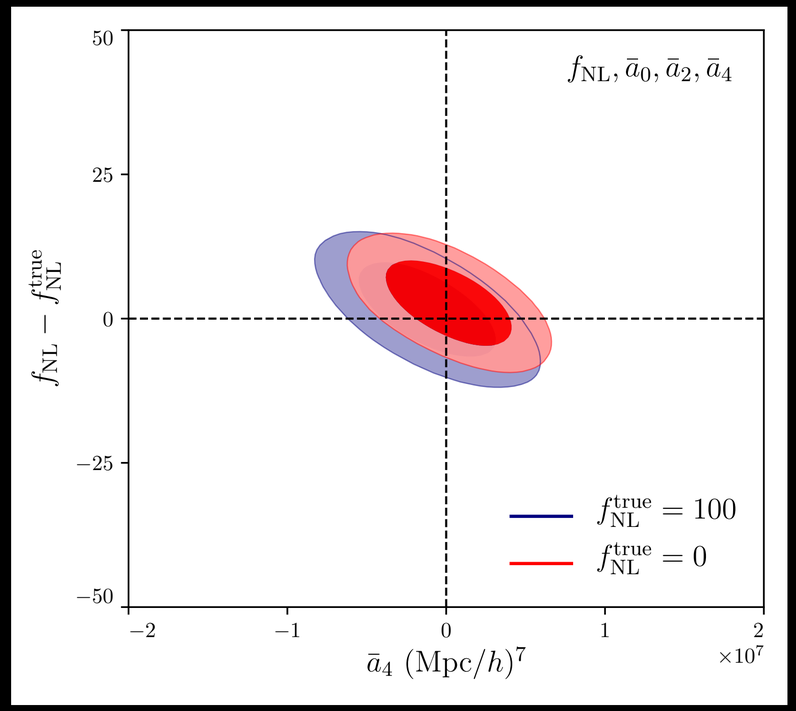
- Model gravitational contribution based on consistency relations

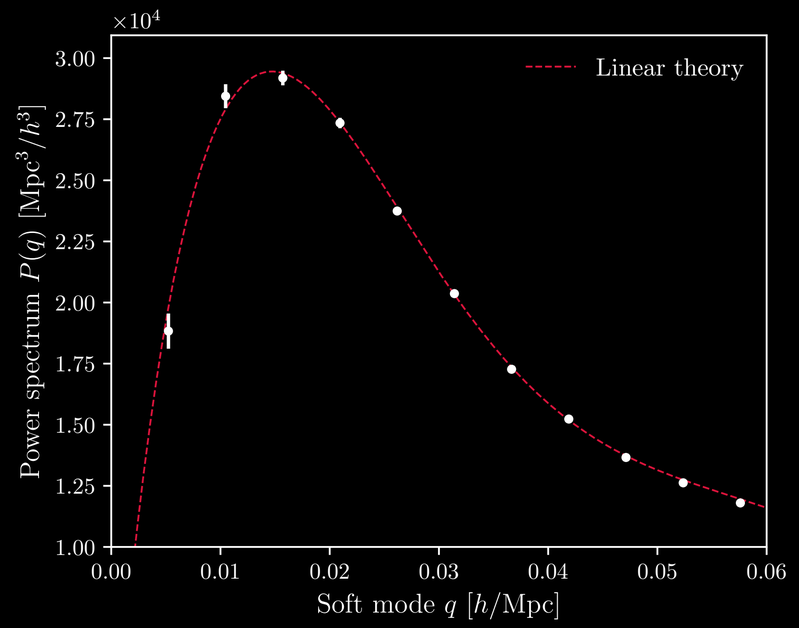
$$B_{\text{grav.}}(q,k,k') = \bar{a}_0 P(q) + \bar{a}_2 q^2 P(q)$$

(See also Valageas, 2013; Chiang et al. 2017; Esposito, Hui, Scoccimarro, 2019; Biagetti et al 2022)

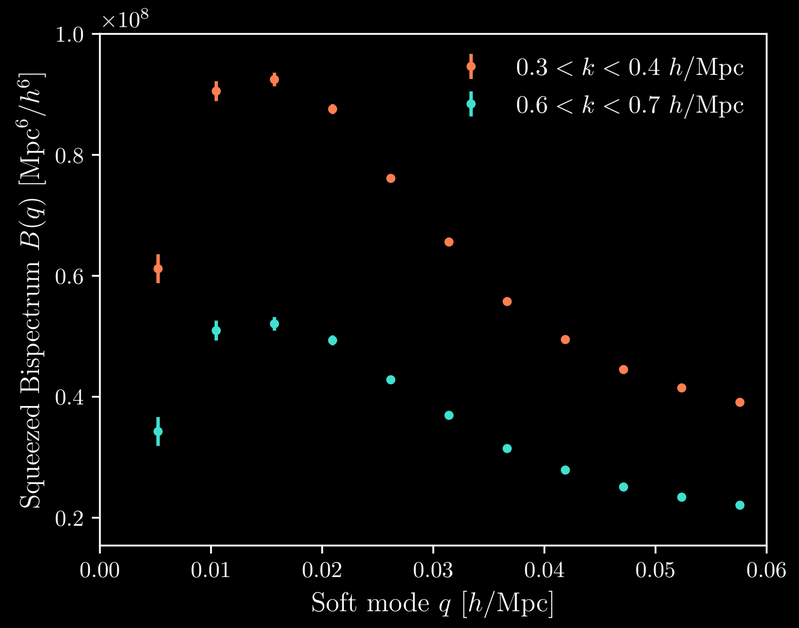


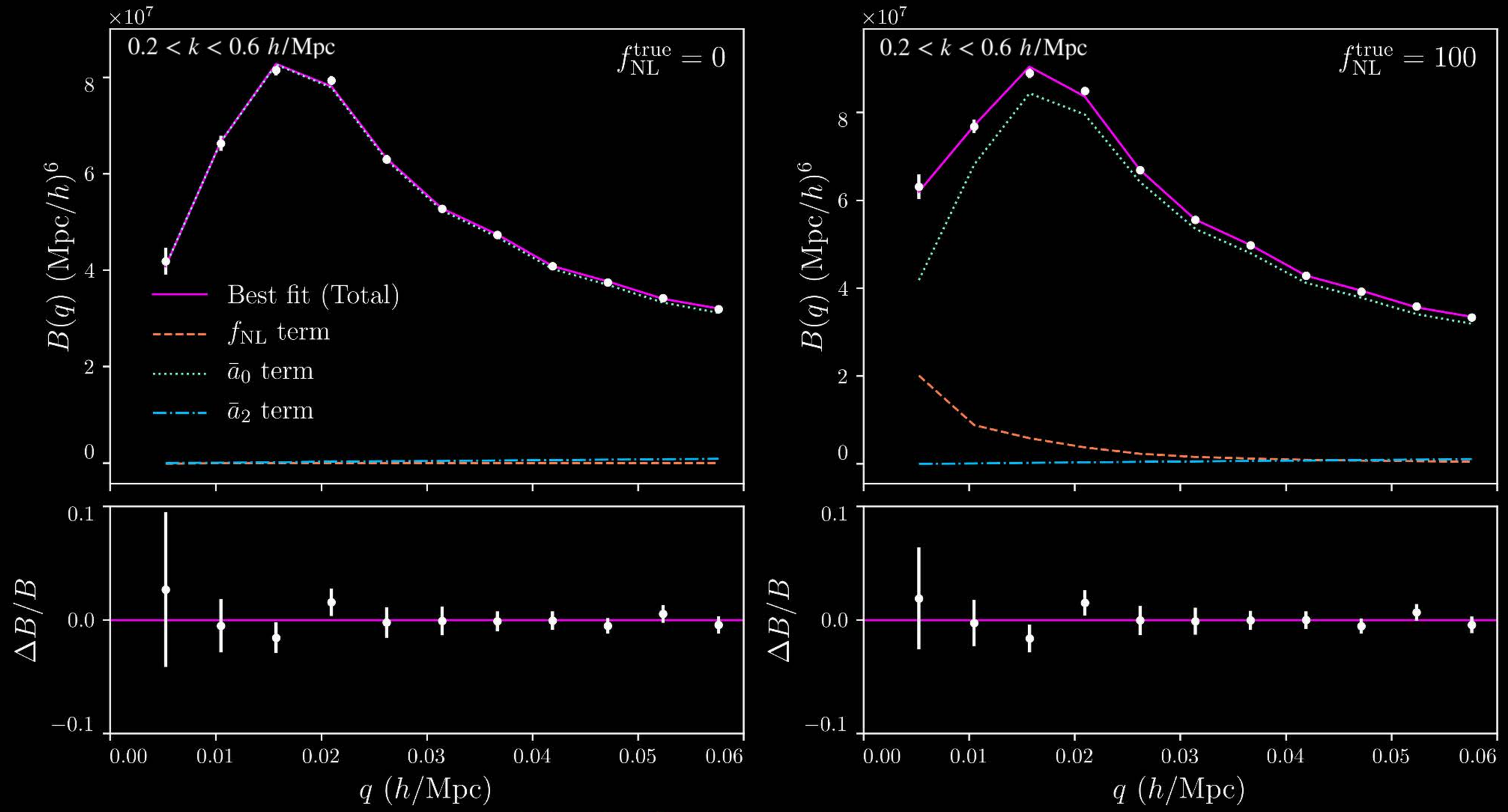
Measurements/Likelihood

- Measure power spectrum and **angle averaged** squeezed bispectrum
- Joint likelihood for $\hat{P}(q)$ and $\hat{B}(q)$
  - **Sample variance cancellation**
- Covariance estimated from simulations **without** PNG
  - Multivariate-t likelihood (**Sellentin & Heavens, 2015**)
- Fit for $f_{\rm NL}$, a_0, and a_2

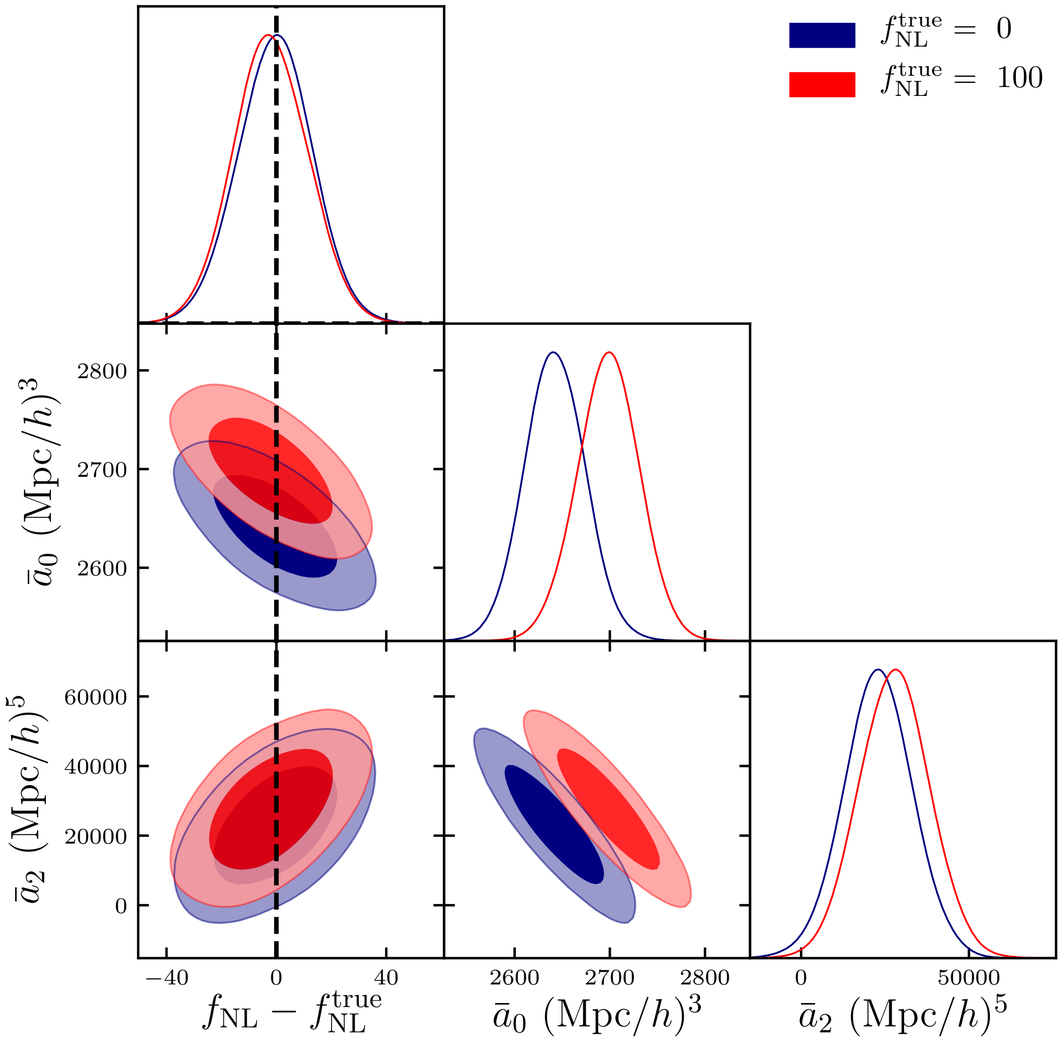


Highly correlated

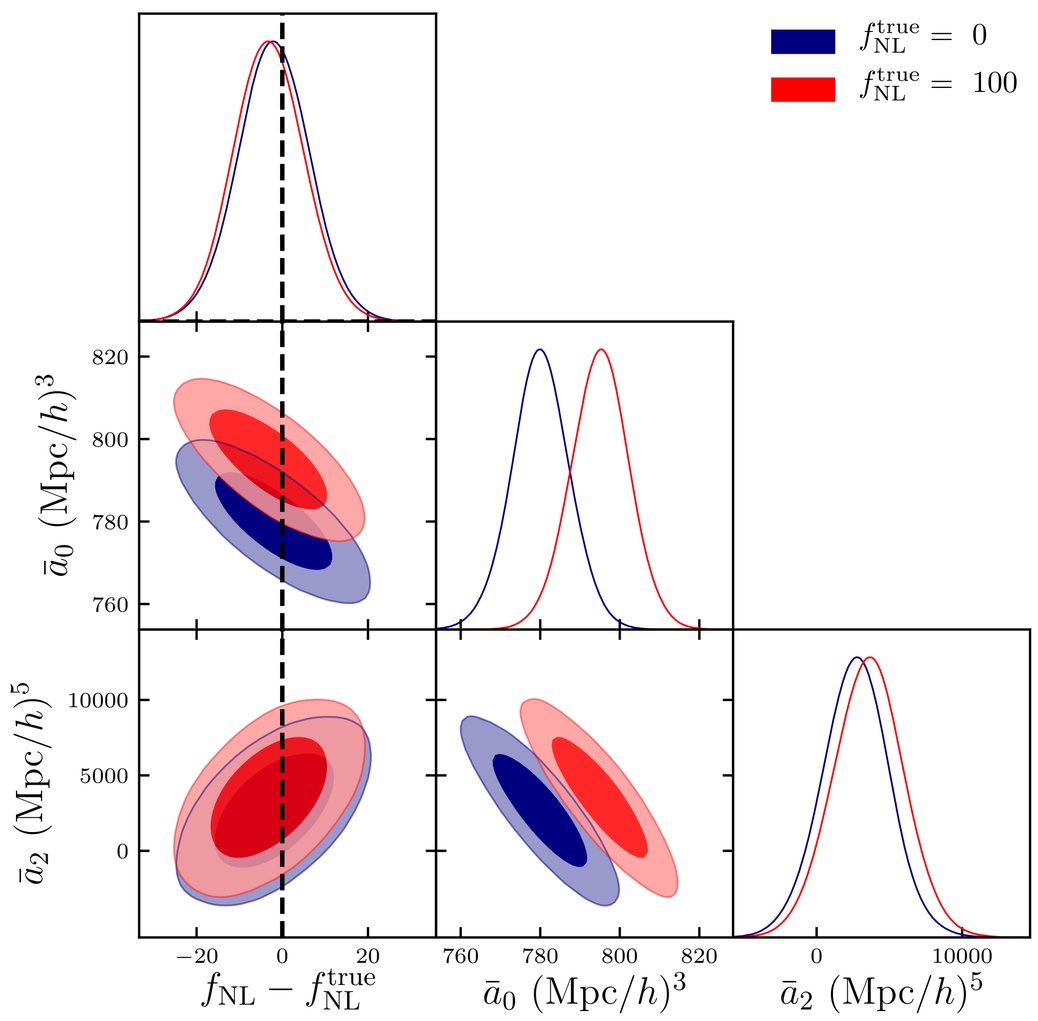




$$\hat{B}(q) = \frac{6 f_{\mathrm{NL}} \Omega_{m,0} H_0^2}{D_{\mathrm{md}}(z)} \bar{P}(k) \frac{\partial \log \bar{P}(k)}{\partial \log \sigma_8^2} \frac{\hat{P}(q)}{q^2} + \bar{a}_0 \hat{P}(q) + \bar{a}_2 q^2 \hat{P}(q)$$

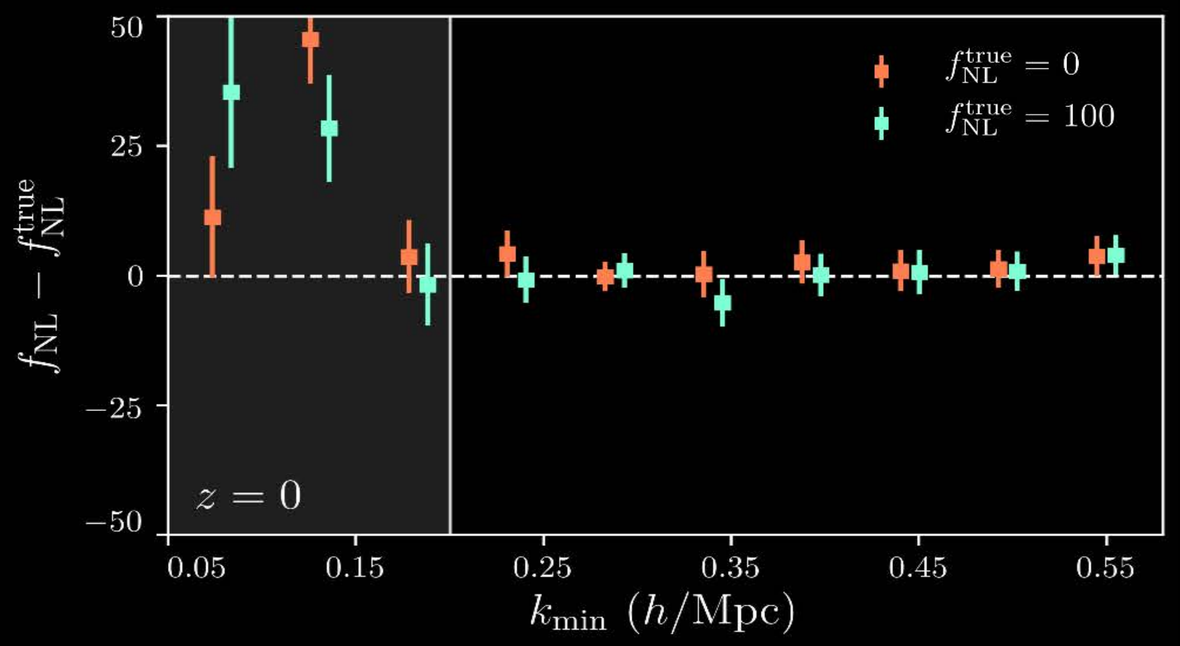


Marginalized Posterior at z=0



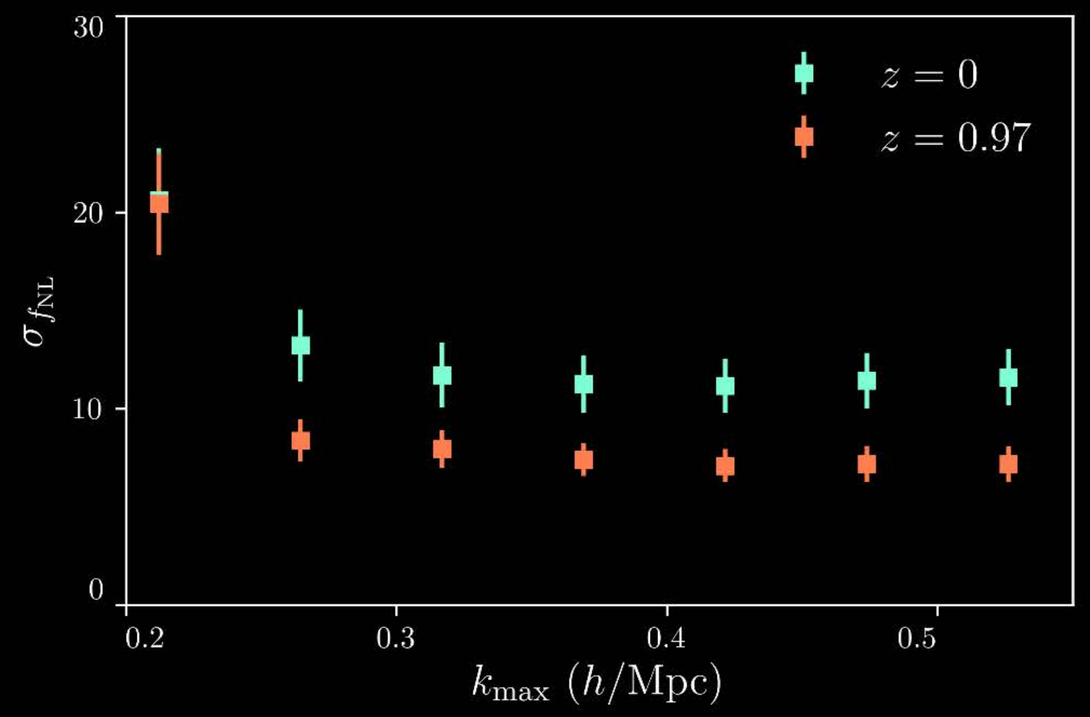
Marginalized Posterior at z=0.97

(i) How squeezed do the triangles need to be?



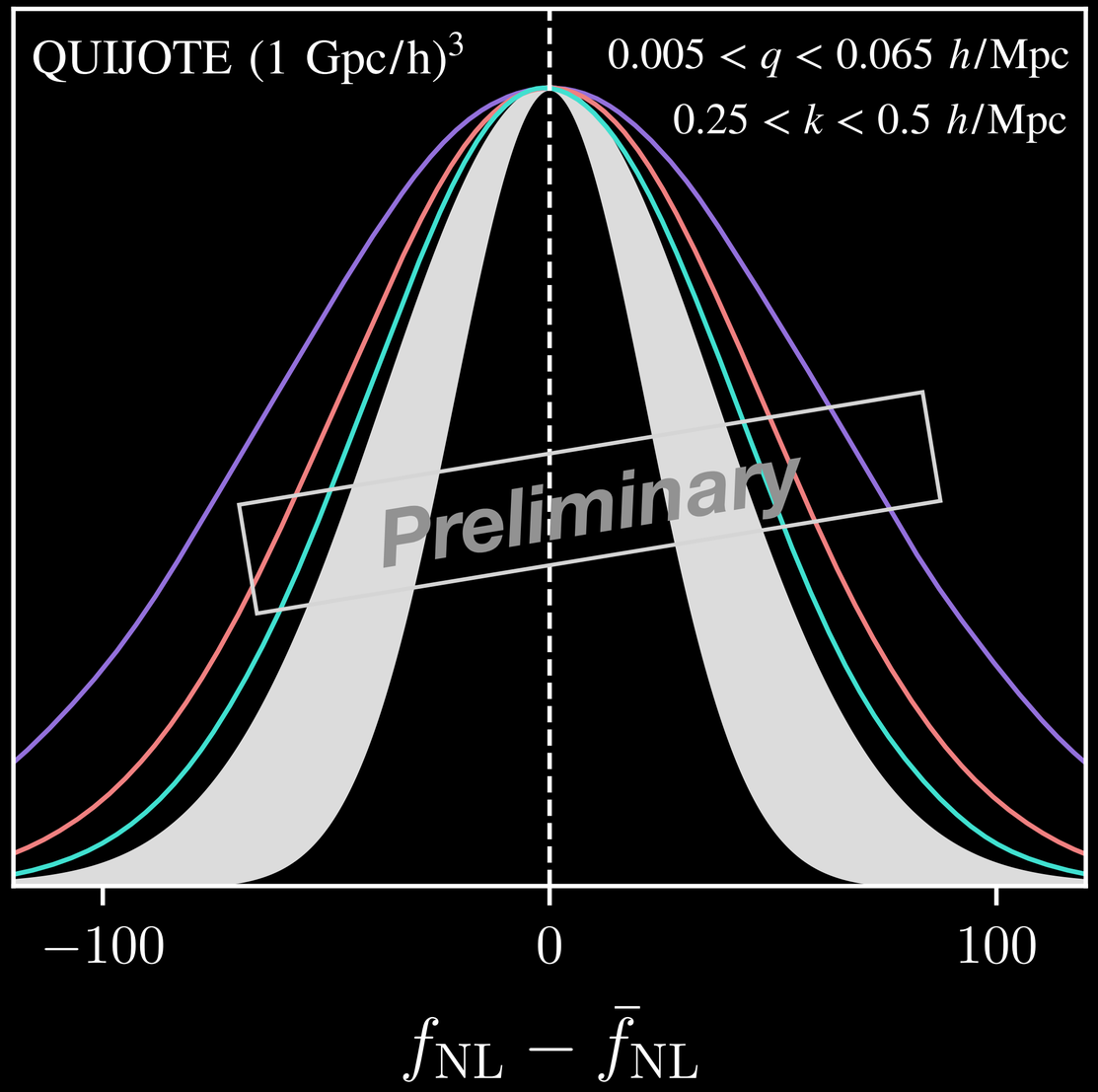
We recover the true value of fNL for kmin>0.2h/Mpc

(ii) How much information is in non-linear regime?



Constraints saturate for k>0.3h/Mpc due to ***non-Gaussian covariance*** *and* ***sub-optimal weighting***

Optimal weighting can help!



Instead of summing all k-bins, split into sub bins

— Single bin ($\sigma_{f_{\mathrm{NL}}} = 62$)

— Two bins ($\sigma_{f_{\mathrm{NL}}} = 47$)

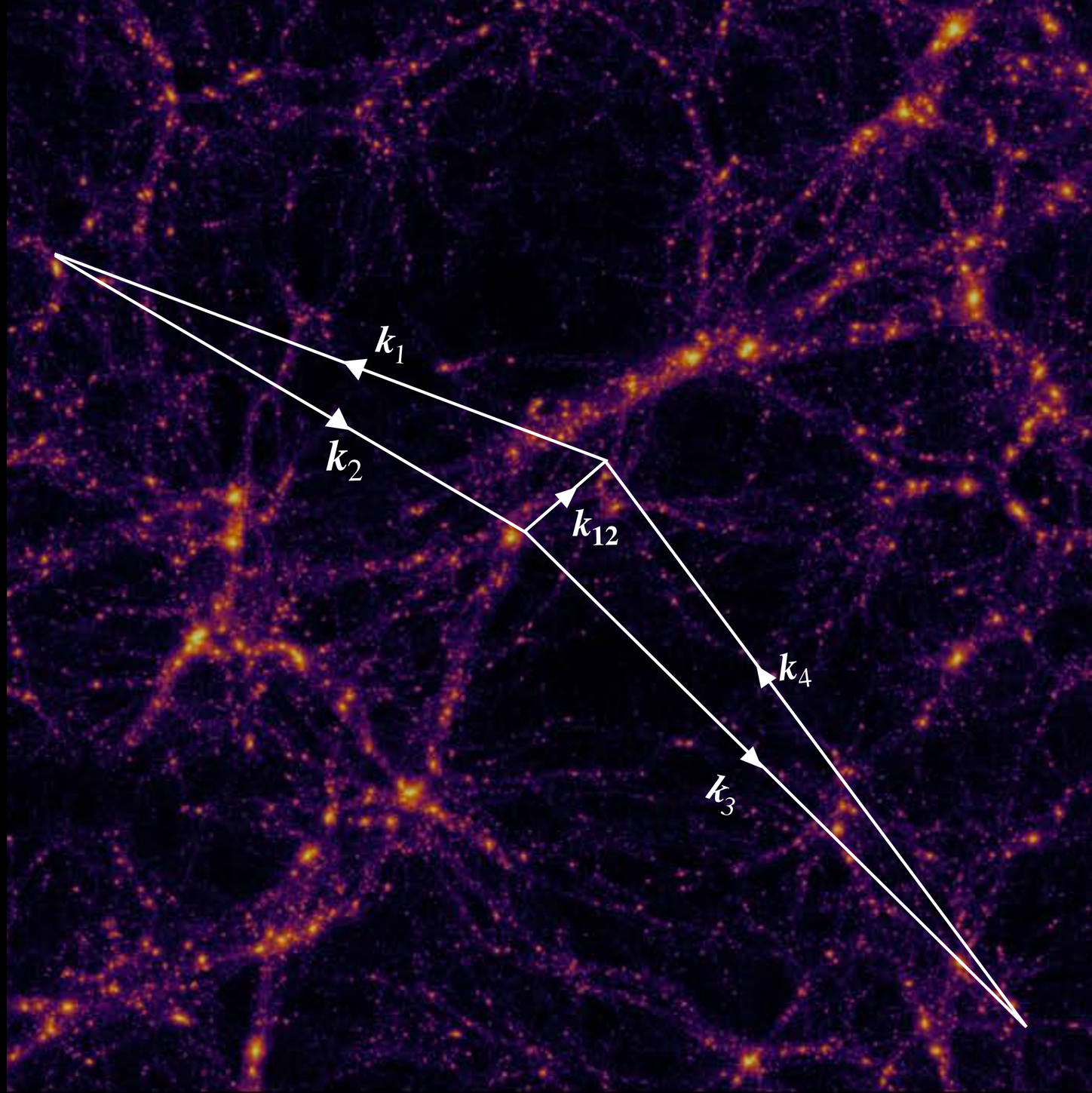
— Four bins ($\sigma_{f_{\mathrm{NL}}} = 41$)

Shaded region: Fisher error for B+P up to $k_{\mathrm{max}} = 0.5\ h/\mathrm{Mpc}$ from Quijote PNG analysis (Coulton, 2022) for different marginalizations

Error from fitting squeezed bispectrum is close to Fisher error from full bispectrum!

Can we do better with the collapsed trispectrum?

(see also Giri, Münchmeyer, Smith 2305.03070)



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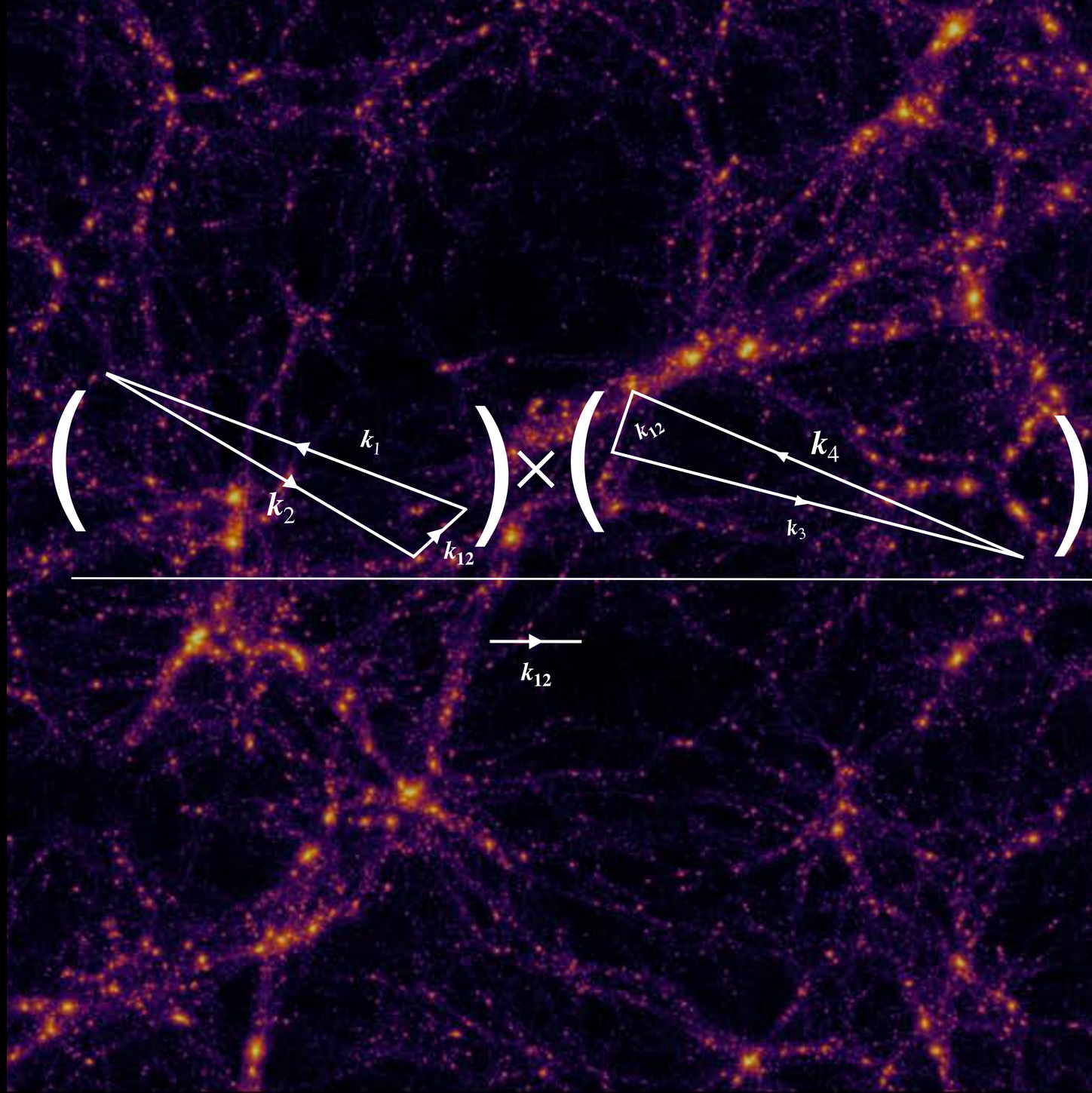
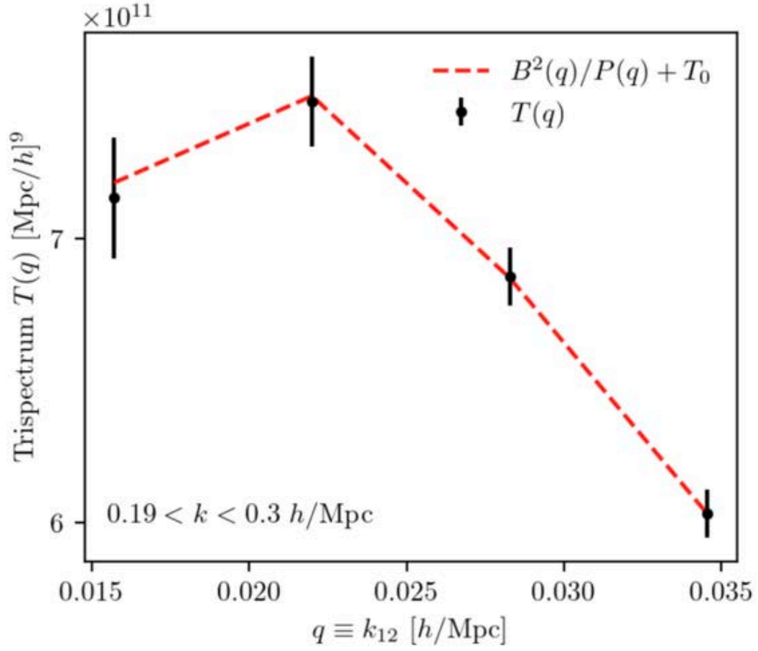


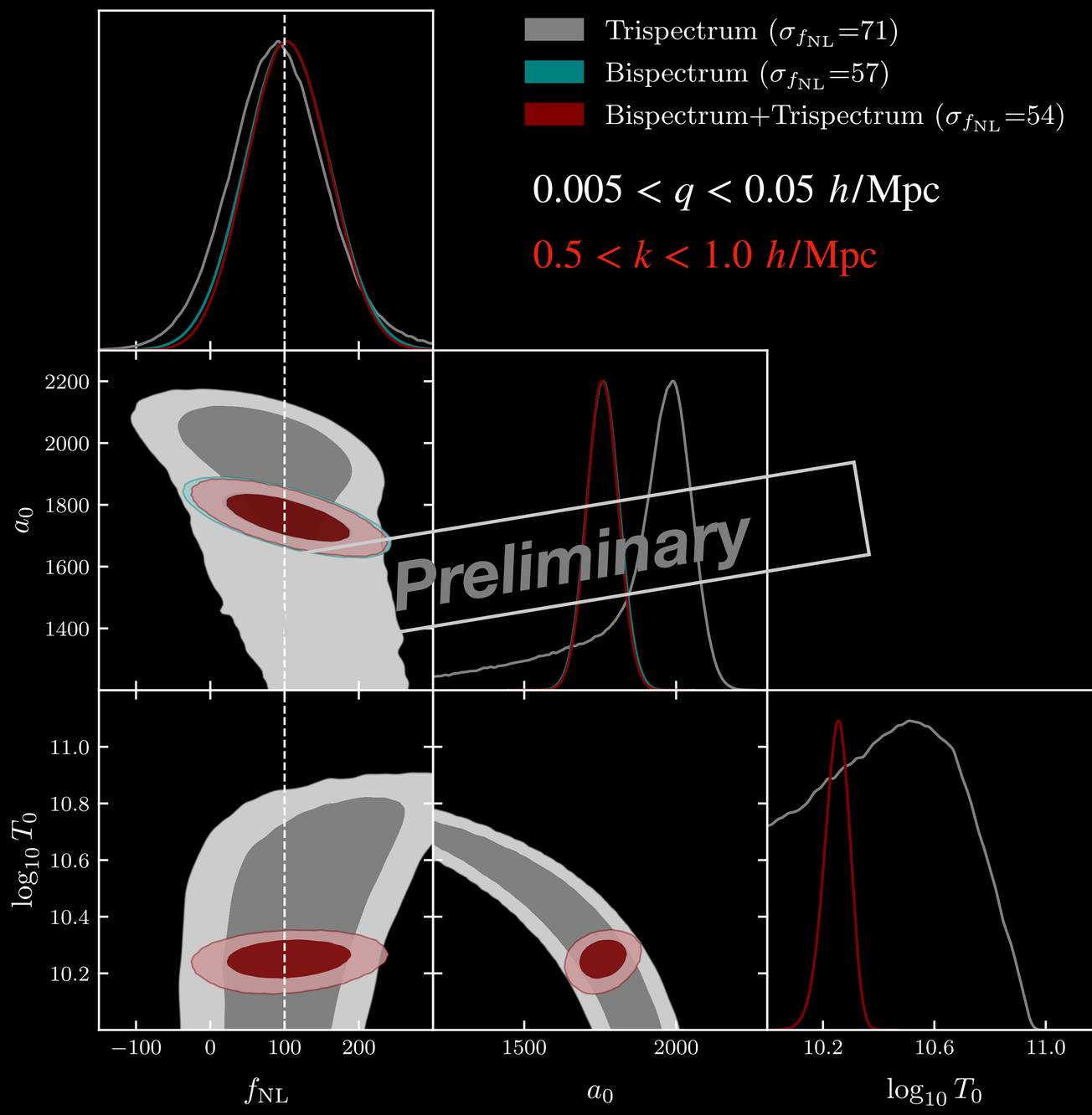
Multi-bin analysis

- Collapsed trispectrum is product of **squeezed bispectra**

$$\lim_{k_{12} \ll k_{\rm NL} \ldots} T(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) = \frac{B(\boldsymbol{k}_{12}, \boldsymbol{k}_1) B(\boldsymbol{k}_{12}, \boldsymbol{k}_3)}{P_{\rm L}(k_{12})} + T_0 \,.$$

- We get this for free! (after coding a trispectrum estimator..)
  - Joint analysis B+T



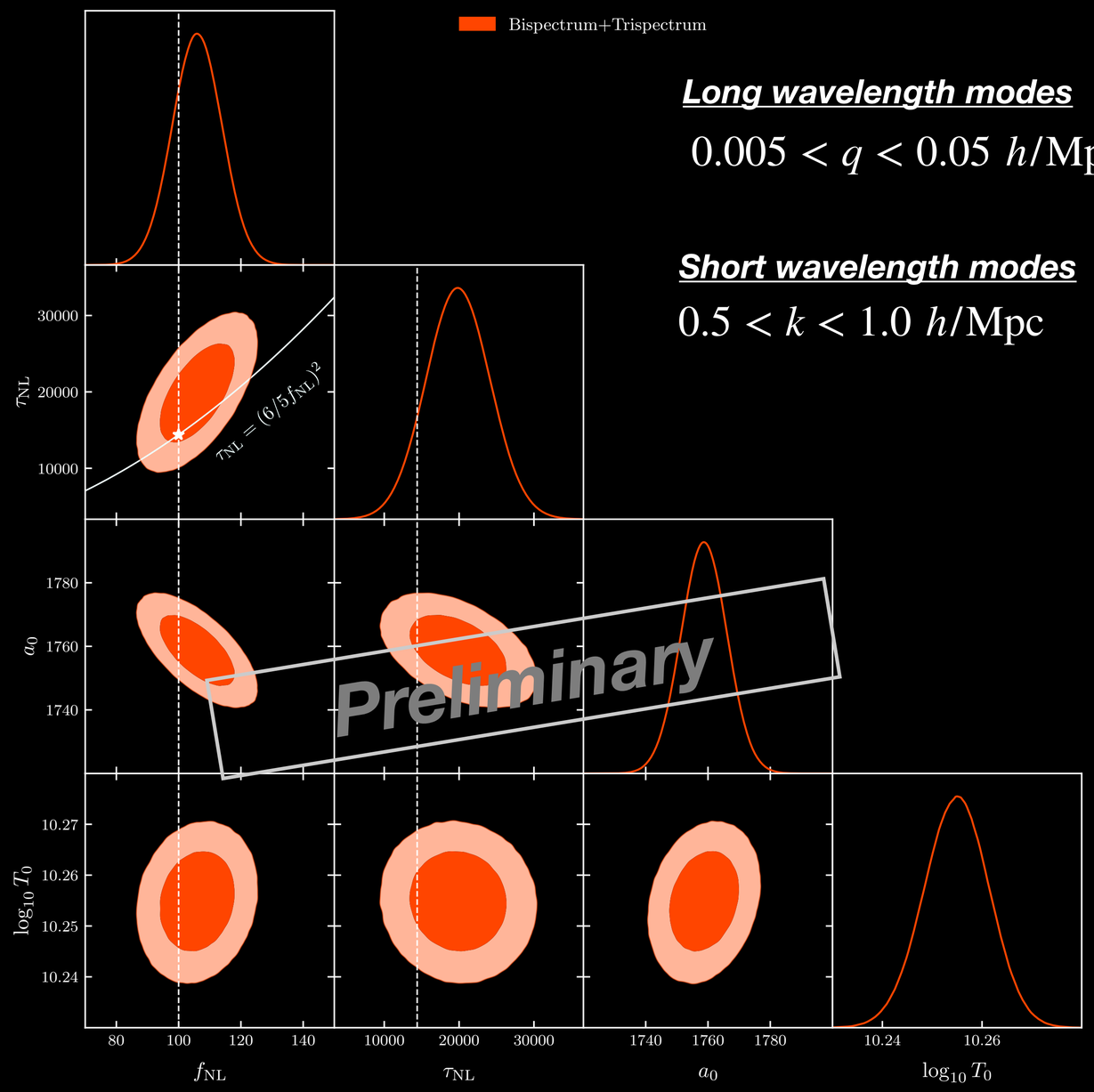


New possibilities with trispectrum

- With trispectrum we can go beyond $f_{\rm NL}$
- **Example**: $\tau_{\rm NL}$
  - Collapsed T not associated with squeezed B (excess d.o.f. during inflation)

$$\tau_{\rm NL} \geq \left(\frac{6}{5} f_{\rm NL})\right)^2$$ **(Suyama-Yamaguchi)**

  - Can fit for $f_{\rm NL}$ and $\tau_{\rm NL}$ using joint likelihood in bispectrum and trispectrum measured from Quijote
- These results are a **proof of concept**
  - Need to implement optimal weighting and more configurations
  - Should learn a lot from bispectrum



Constraints on ($f_{\rm NL}$, $\tau_{\rm NL}$) from squeezed bispectrum and collapsed trispectrum using Quijote simulations with $f_{\rm NL} = 100$!

“Consistently constraining f_{NL} with the squeezing lensing bispectrum using consistency relations”

2310.12959

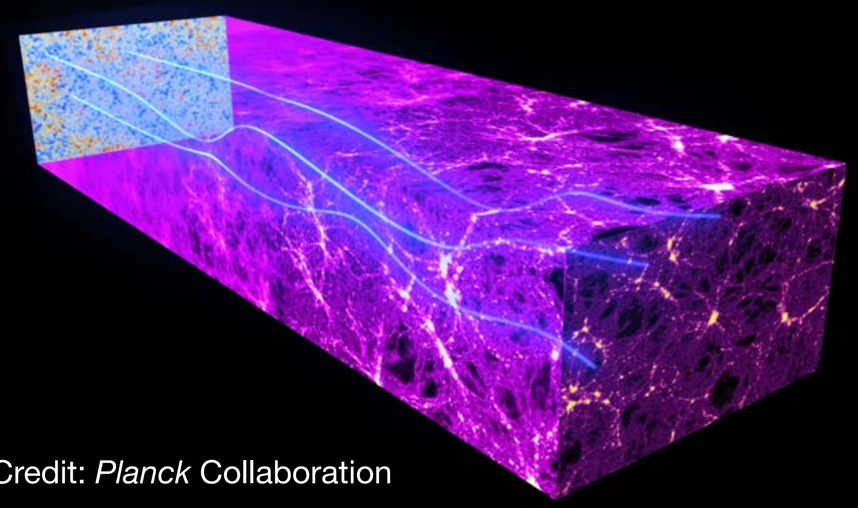
SG, *Philcox, Hill, Esposito, Hui*

Constraining fNL with the squeezed lensing bispectra

- Can't observe the **matter bispectrum** directly, but can observe integrals of it with lensing!

- **Lensing convergence:**

$$\kappa^{(i)}(\hat{\boldsymbol{n}}) = \int_0^{\chi_s} d\chi\, W^{(i)}(\chi)\, \delta_m(\chi\hat{\boldsymbol{n}}, \chi)\,, \text{ where } \begin{cases} \overbrace{W^{\kappa_{\rm CMB}}(\chi)}^{\text{CMB}} = \dfrac{3\,H_0^2\,\Omega_m \chi}{2\,a(\chi)} \left(\dfrac{\chi_* - \chi}{\chi_*}\right), \\ \underbrace{W^{\kappa_{\rm g},(i)}(\chi)}_{\text{Cosmic shear/galaxy surveys}} = \dfrac{3\,H_0^2\,\Omega_m \chi}{2\,a(\chi)} \displaystyle\int_\chi^\infty d\chi'\, p_s^{(i)}(\chi') \left(\dfrac{\chi' - \chi}{\chi'}\right). \end{cases}$$



Credit: *Planck* Collaboration

$$\kappa_\ell \approx \int W(\chi)\, \delta_m(\ell/\chi, \chi)\, d\chi \implies \begin{cases} C_\ell \propto \displaystyle\int d\chi \frac{W(\chi)}{\chi^2}\, P_m(\ell/\chi, \chi) \\ b_{\ell_1\ell_2\ell_3} \propto \displaystyle\int d\chi \frac{W^3(\chi)}{\chi^4}\, B_m(\ell_1/\chi, \ell_2/\chi, \ell_3/\chi, \chi) \end{cases}$$

$$\text{where } B_m(q, k, k', \chi) = \frac{6 f_{\rm NL} \Omega_{m,0} H_0^2}{D_{\rm md}(\chi)} \frac{\partial P(k,\chi)}{\partial \log \sigma_8^2} \frac{P(q,\chi)}{q^2 T(q)} + \bar{a}_0(k,\chi) P(q,\chi)$$

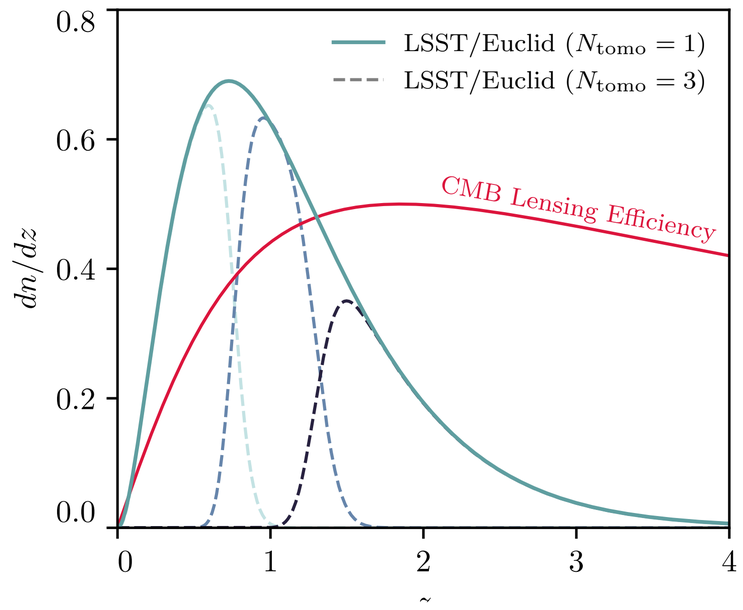
Forecast Setup

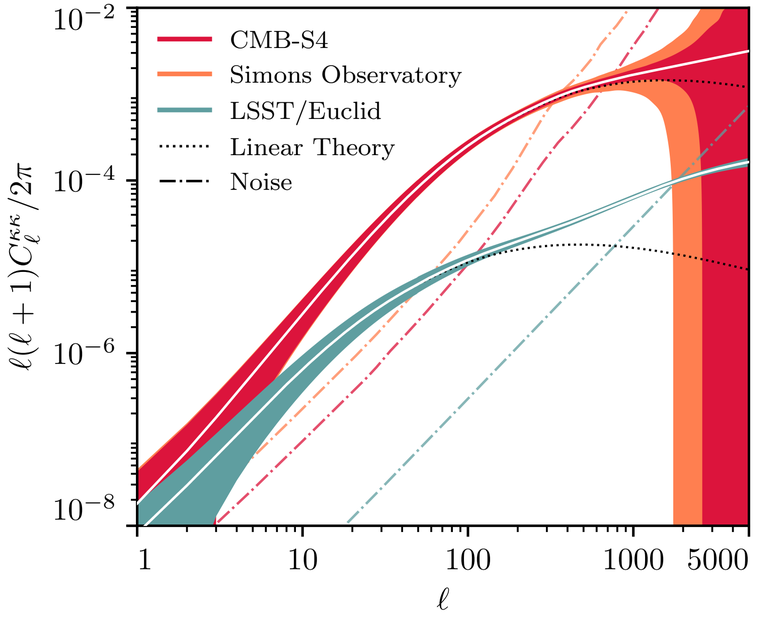
- Lensing surveys
  - **CMB Lensing**: Simons Observatory and CMB-S4
  - **Cosmic Shear:** LSST/Euclid-like survey
    - *Specifications:* $\bar{n}_g = 31\ \mathrm{arcmin}^{-2}$, $\sigma_\epsilon = 0.26$, $\sigma_z = 0.05(1+z)$, vary N_{tomo}
- **Fisher matrix**: f_{NL} is only free parameter (optimistic)

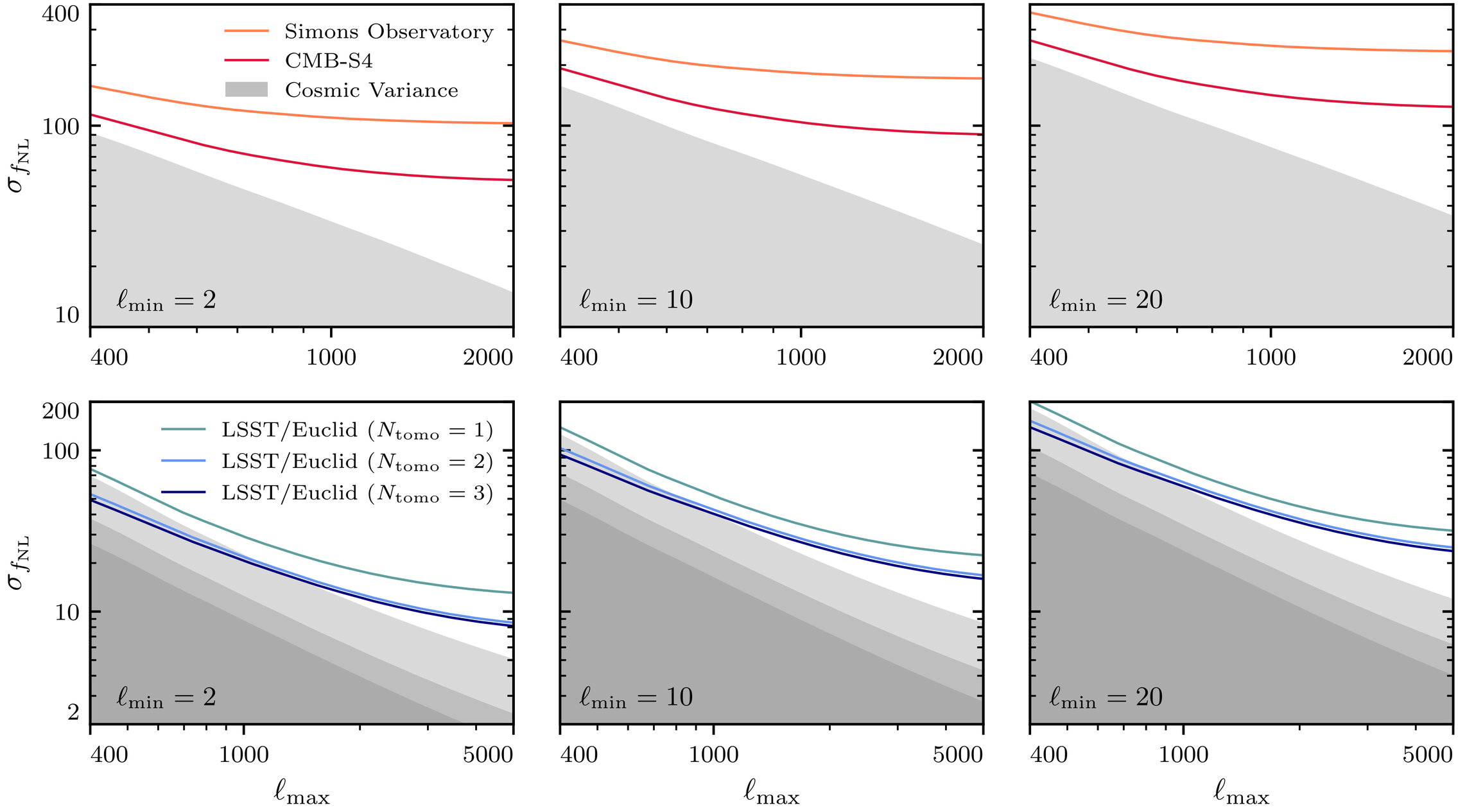
$$\mathcal{F} = \underbrace{\sum_{ijk} \sum_{\ell_1\ell_2\ell_3}}_{abc} \frac{\partial b^{(ijk)}_{\ell_1\ell_2\ell_3}}{\partial f_{\mathrm{NL}}} \underbrace{\mathrm{Cov}^{-1}\left[b^{(ijk)}_{\ell_1\ell_2\ell_3}, b^{(abc)}_{\ell_1\ell_2\ell_3}\right]} \frac{\partial b^{(abc)}_{\ell_1\ell_2\ell_3}}{\partial f_{\mathrm{NL}}},$$

Sum over tomographic bins and multipoles

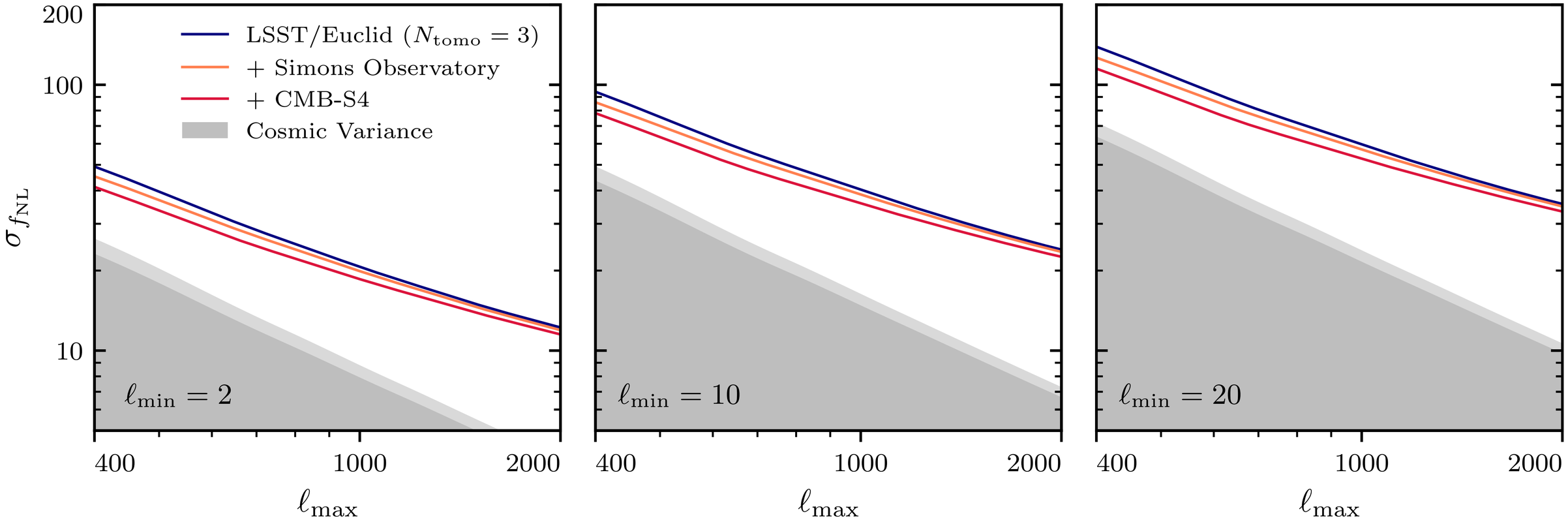
Assume Gaussian covariance with reconstruction/shape noise



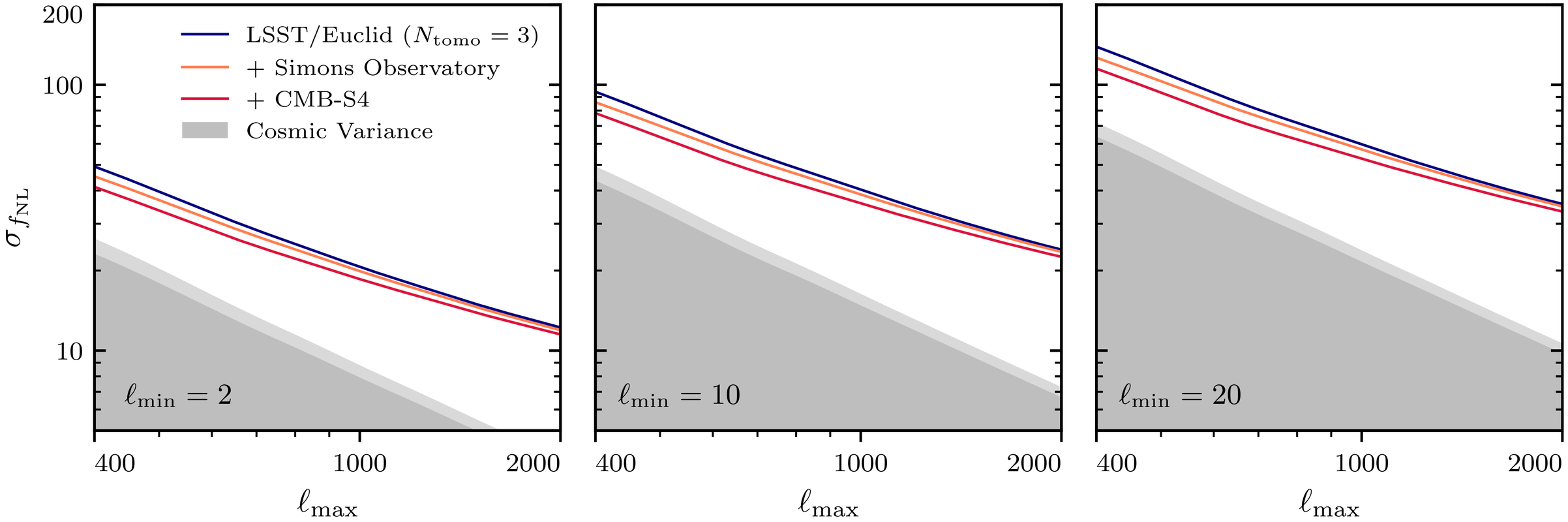



Simons Observatory
CMB-S4
Cosmic Variance
LSST/Euclid ($N_{\rm tomo} = 1$)
LSST/Euclid ($N_{\rm tomo} = 2$)
LSST/Euclid ($N_{\rm tomo} = 3$)
$\sigma_{f_{\rm NL}}$
$\ell_{\rm max}$
$\ell_{\rm min} = 2$
$\ell_{\rm min} = 10$
$\ell_{\rm min} = 20$
400
100
10
200
2
1000
2000
5000

Does shear+CMB lensing help?



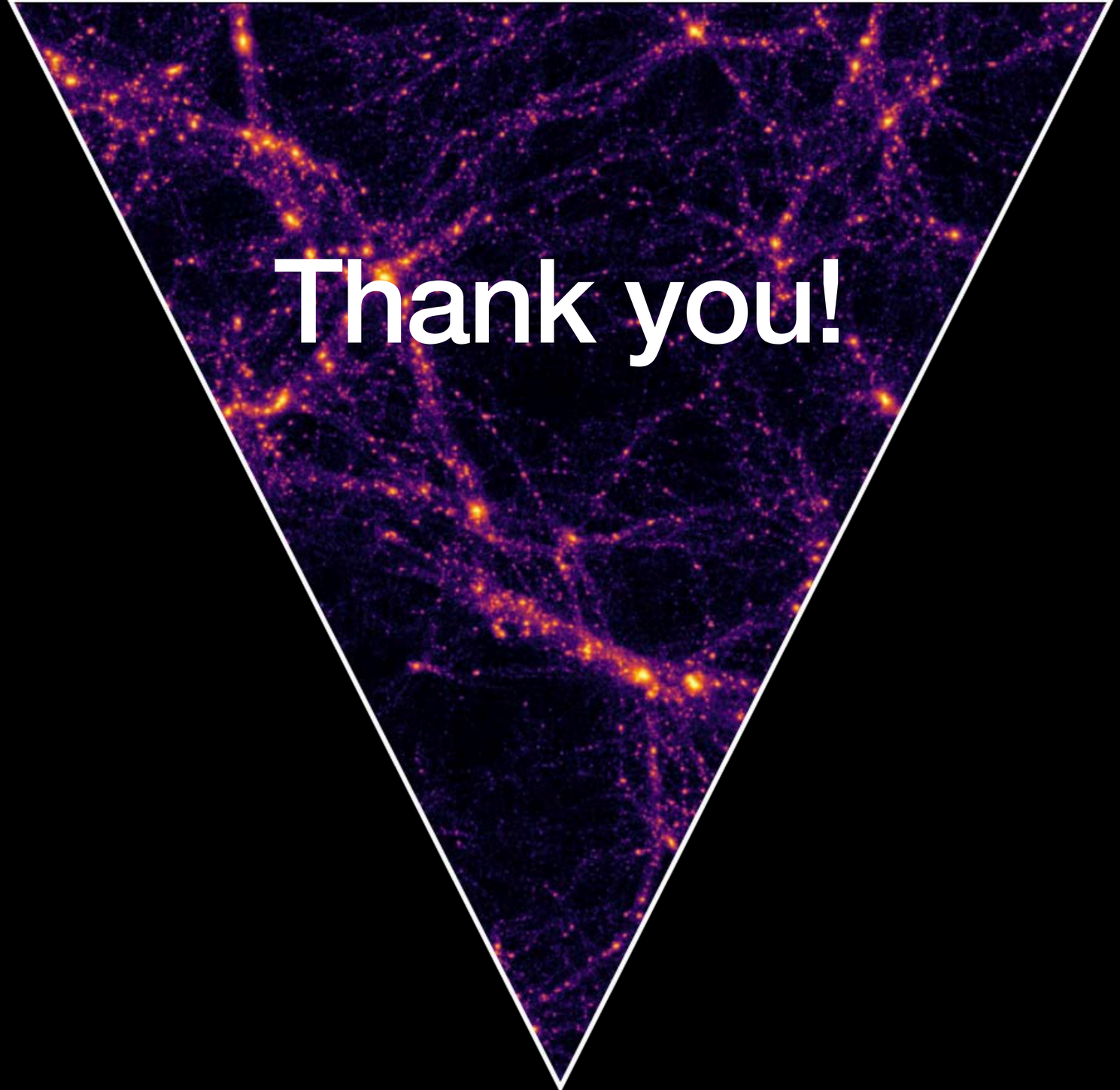
Does shear+CMB lensing help?



Under optimistic assumptions may reach $\sigma_{f_{\rm NL}} \approx 10$ from lensing bispectra.
Completely independent of galaxy field!

Conclusions

- Consistency relations can be used to constrain PNG with LSS using info from non-linear regime
- Many possibilities (and challenges) in the future
  - Optimize weighting for hard modes in bispectrum and trispectrum
    - Understand information content
  - Constrain more general models that constrain consistency relations
    - **PNG:** Quasi-single field inflation/cosmological collider: fractional poles/oscillatory features
    - Test equivalence principle/modified gravity
  - **Extend framework to galaxies/halos**
    - Bias, RSD, shot noise
    - Potential use
      - Multi-tracer, systematics


Thank you!

Backup

Measurements

- ***Power spectrum:*** need to be careful with binning effects

- ***Bispectrum:***

$$\hat{M}_{in} \equiv \begin{cases} \left\langle q^n \delta_{\boldsymbol{q}} \delta_{-\boldsymbol{q}} / T(q) \right\rangle_i, \text{if } n < 0 \\ \left\langle q^n \delta_{\boldsymbol{q}} \delta_{-\boldsymbol{q}} \right\rangle_i, \qquad \text{otherwise} \end{cases},$$

$$\hat{B}_i \equiv \left\langle \mathrm{Re}\, \delta_{\boldsymbol{q}} \delta_{\boldsymbol{k}} \delta_{-\boldsymbol{q}-\boldsymbol{k}} \right\rangle_i, \quad \text{with } k \in [k_{\min}, k_{\max}]$$

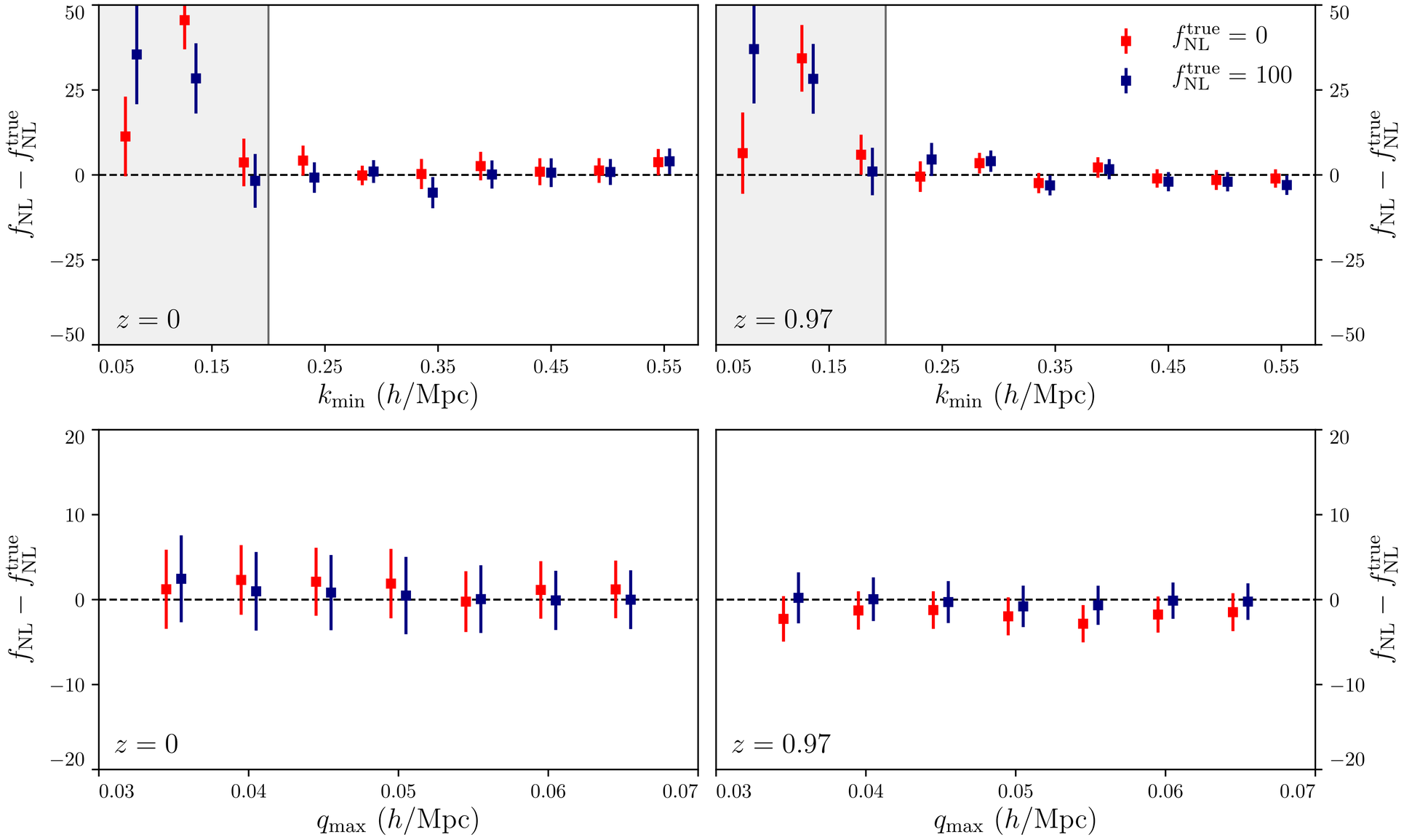
$$\hat{\boldsymbol{B}} = \sum_{n=-2,0,2} \bar{a}_n \hat{\boldsymbol{M}}_\alpha$$

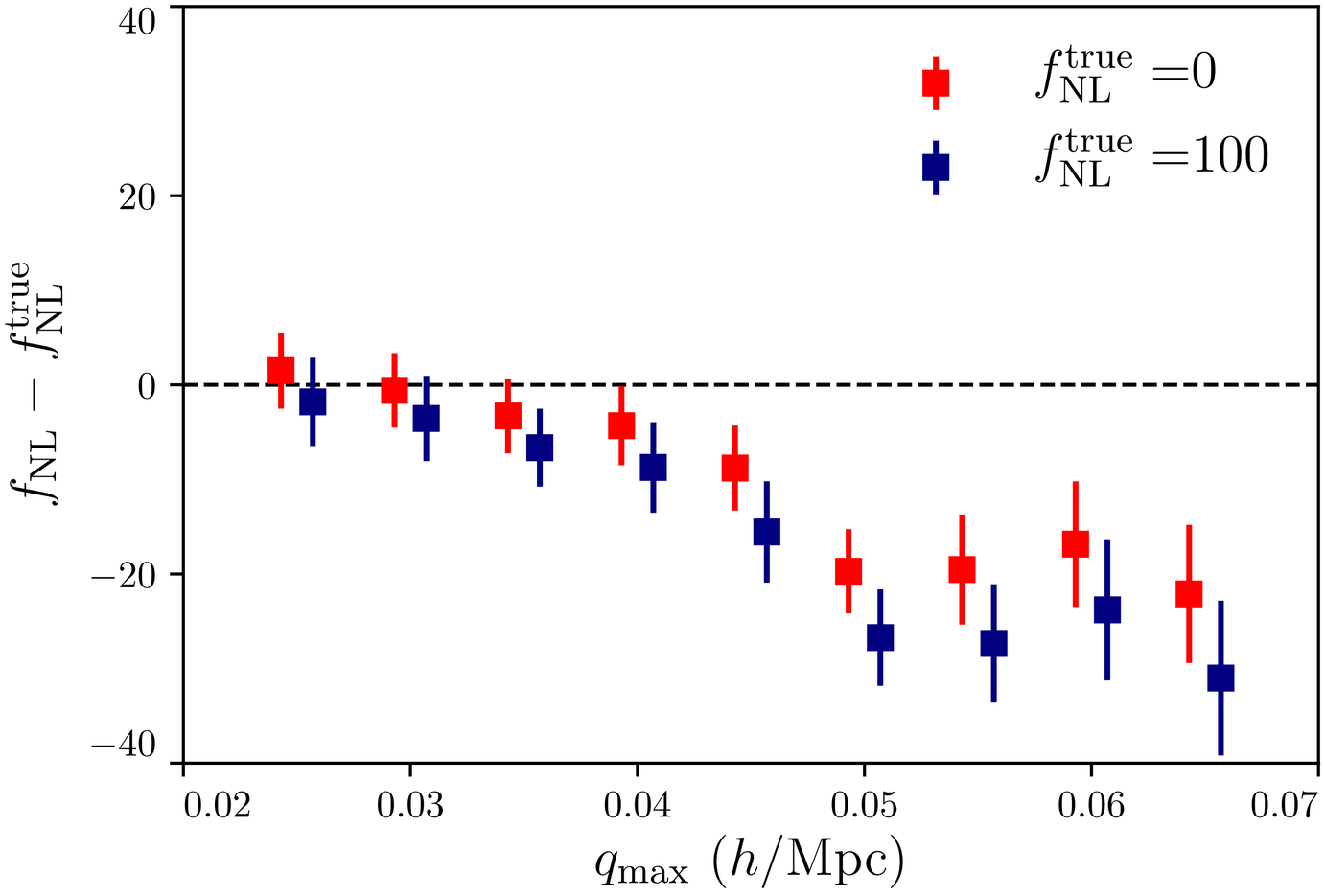
  - Average over all angles and hard momenta

- ***Matter response:***

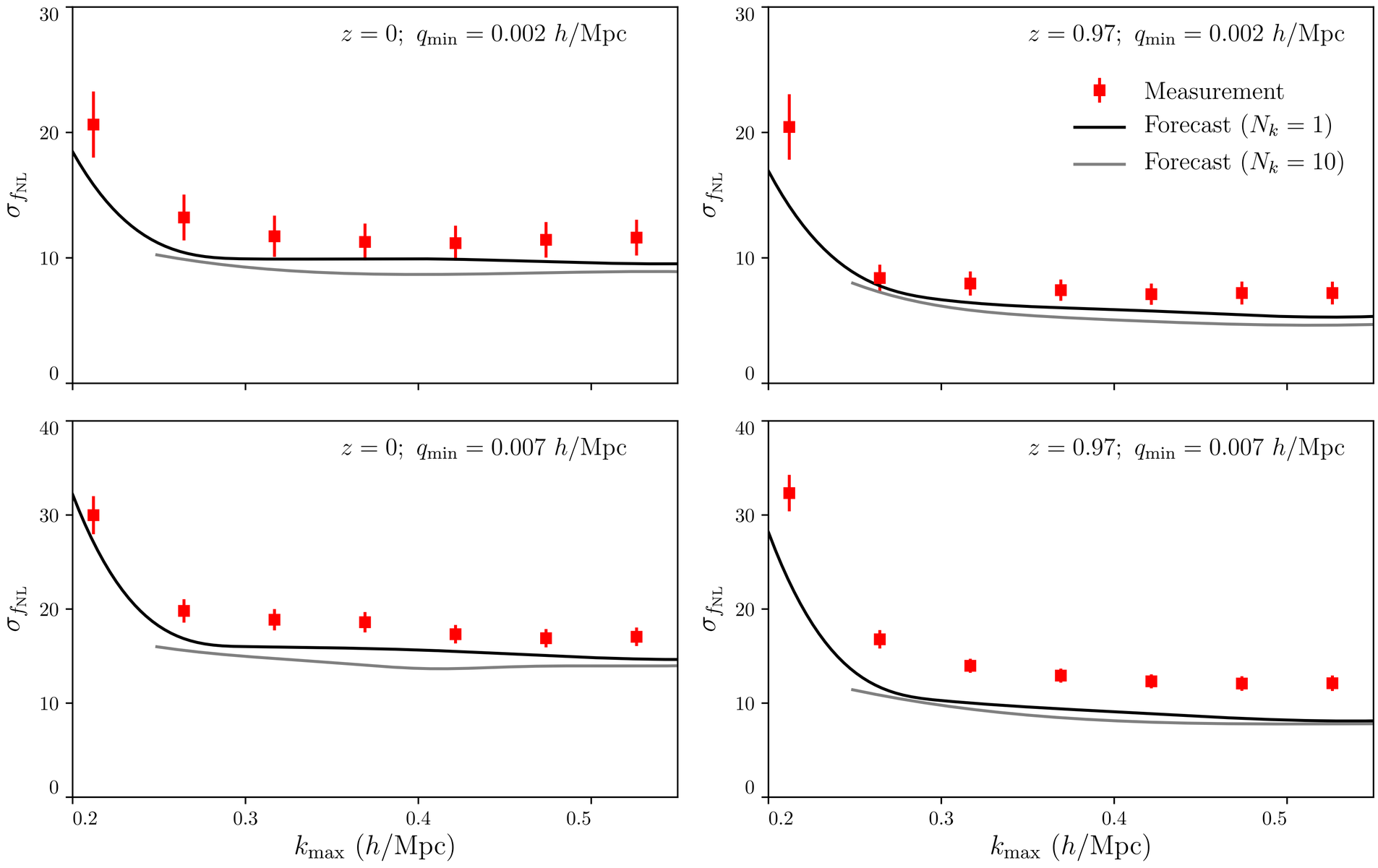
$$\bar{F} \equiv \left\langle \delta_{\boldsymbol{k}} \delta_{-\boldsymbol{k}} \underbrace{\partial \log P(k) / \partial \log \sigma_8^2}_{\text{from Halofit}} \right\rangle \text{ with } k \in [k_{\min}, k_{\max}]$$

How squeezed is squeezed?




$f_{\rm NL}^{\rm true}$ =0
$f_{\rm NL}^{\rm true}$ =100
40
20
0
−20
−40
$f_{\rm NL} - f_{\rm NL}^{\rm true}$
0.02
0.03
0.04
0.05
0.06
0.07
$q_{\rm max}$ $(h/{\rm Mpc})$

Where is the information?



Bispectrum Estimator

Suppose we want to estimate $B(k_1, k_2, k_3)$ in bins k_i

Define $W_{k_i}(p_i) \equiv \begin{cases} 1, & p_i \in k_i \\ 0, & \text{otherwise} \end{cases}$

$$B(k_1, k_2, k_3) \propto \int_{\boldsymbol{p}_1} \int_{\boldsymbol{p}_2} \int_{\boldsymbol{p}_3} \delta(\boldsymbol{p}_1) W_{k_1}(p_1)\, \delta(\boldsymbol{p}_2) W_{k_2}(p_2)\, \delta(\boldsymbol{p}_3) W_{k_3}(p_3) \underbrace{(2\pi)^3 \delta_D(\boldsymbol{p}_{123})}_{= \int d^3x e^{-i\boldsymbol{p}_{123}\cdot\boldsymbol{x}}}$$

$$\implies B(k_1, k_2, k_3) \propto \int d^3x \prod_{j=1}^{3} \left(\int_{\boldsymbol{p}_i} \delta(\boldsymbol{p}_i) W_{k_i}(p_i) e^{-i\boldsymbol{p}_i\cdot\boldsymbol{x}} \right)$$

Use exponential representation of δ_D to write in separable form

$$\therefore \hat{B}(k_1, k_2, k_3) = \frac{\sum_{i=1}^{N_{\text{grid}}^3} \delta_{k_1}(x_i)\delta_{k_2}(x_i)\delta_{k_3}(x_i)}{\sum_{i=1}^{N_{\text{grid}}^3} I_{k_1}(x_i) I_{k_2}(x_i) I_{k_3}(x_i)}$$

Bispectrum Estimator Validation

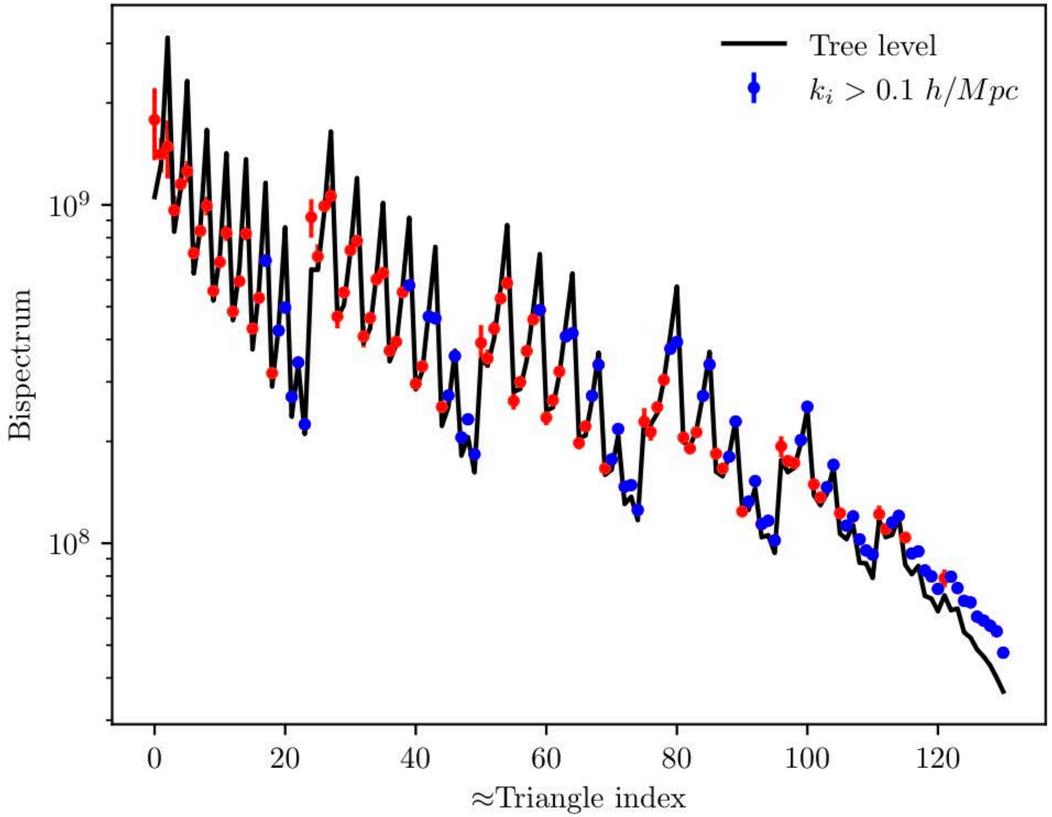


Figure 1: Validation on measurements from Quijote

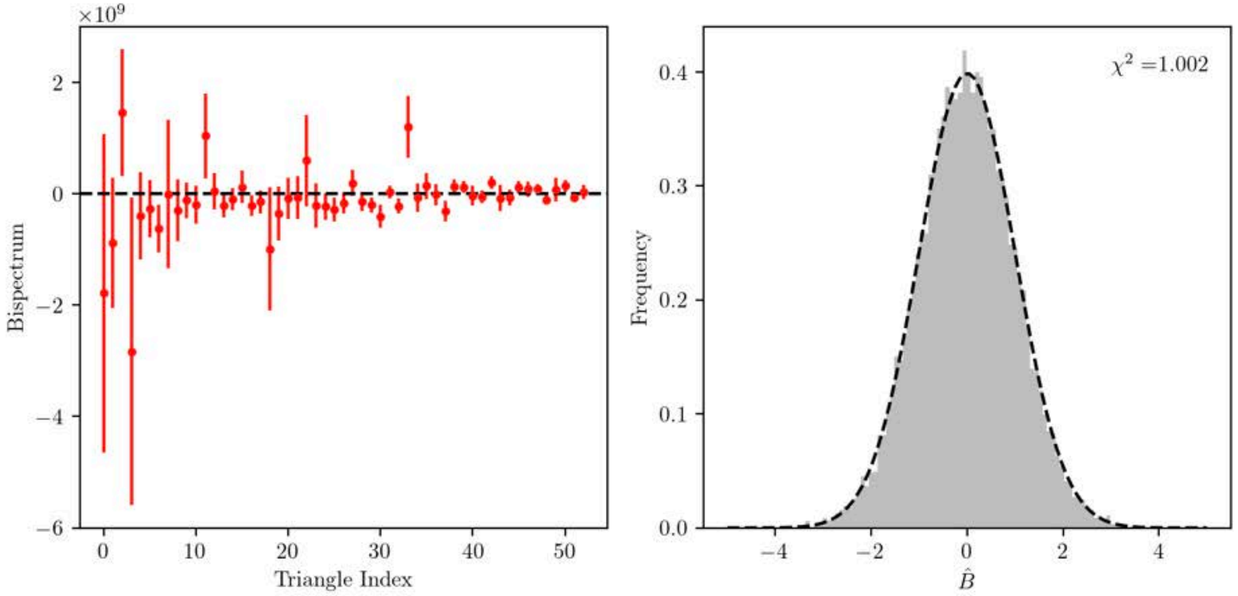


Figure 2: Validation on Gaussian mocks

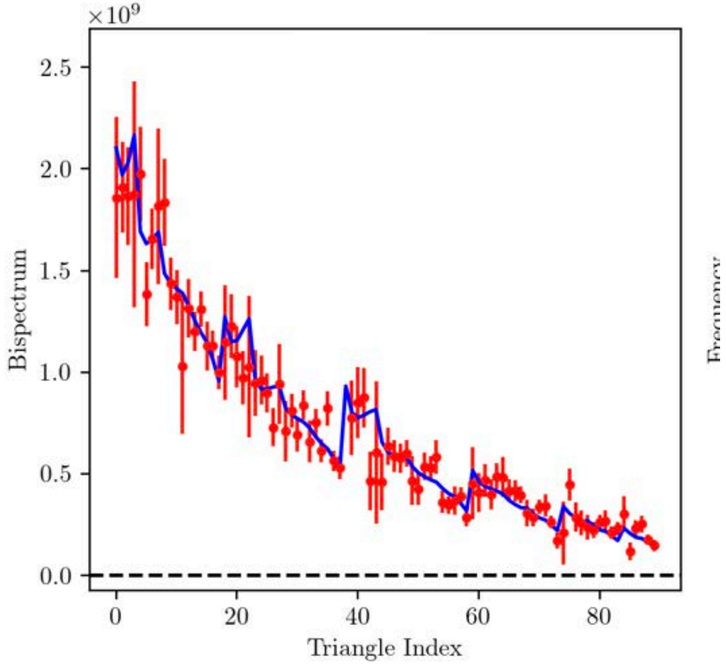


Figure 3: Validation on mocks with $f_{\rm NL}^{\rm loc.}$

Trispectrum Estimator

- Trispectrum estimator in terms of external legs $k_a, \ldots, k_d$ and diagonals k_{ab}, k_{bc} is **not separable**
  - Use estimator integrated over q_{23} (see Appendix A of 2306.11782)

$$\left\langle \hat{T}_{\rm tot}(k_a, k_b, k_c, k_d, k_E) \right\rangle = \frac{1}{N_{a,b,c,d,E}} \int_{\boldsymbol{Q}} W_E(Q) \int_{\boldsymbol{k}_1,\ldots,\boldsymbol{k}_4} \Big\{ W_a(k_1) W_b(k_2) W_c(k_3) W_d(k_4) \left\langle \delta_{\boldsymbol{k}_1} \delta_{\boldsymbol{k}_2} \delta_{\boldsymbol{k}_3} \delta_{\boldsymbol{k}_4} \right\rangle (2\pi)^3 \delta_D^{(3)}(\boldsymbol{k}_{12} - \boldsymbol{Q}) (2\pi)^3 \delta_D^{(3)}(\boldsymbol{k}_{34} + \boldsymbol{Q}) \Big\},$$

- Can be efficiently evaluated by computing FFT's of product fields

$$D_{ij}(\boldsymbol{Q}) \equiv \int d^3x \, e^{-i\boldsymbol{Q}\cdot\boldsymbol{x}} \delta_{W_i}(\boldsymbol{x}) \delta_{W_j}(\boldsymbol{x}).$$

$$\boxed{\hat{T}_{\rm tot}(k_a, k_b, k_c, k_d, k_E) = \frac{1}{N_{a,b,c,d,E}} \int_{\boldsymbol{Q}} W_E(Q) D_{ab}(\boldsymbol{Q}) D^*_{cd}(\boldsymbol{Q}),}$$

Biased estimator because it includes disconnected terms!!!

Trispectrum Estimator (continued)

- Just need an estimator for disconnected terms and can subtract them
  - Simple estimator: $\delta^4 - 3\langle\delta^2\rangle^2$
  - More optimal estimator: $\delta^4 - 6\,\delta^2\langle\delta^2\rangle^2 + 3\langle\delta^2\rangle^2$ (See Appendix F. of **Shen, Schaan, Ferraro, 24**)
- Can be efficiently implemented using FFT's

Depends on fiducial P(k)

$$F^P_{ij}(\boldsymbol{x}) \equiv \int_{\boldsymbol{k}} W_i(k) W_j(k) P(k) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

Depends on realization of δ

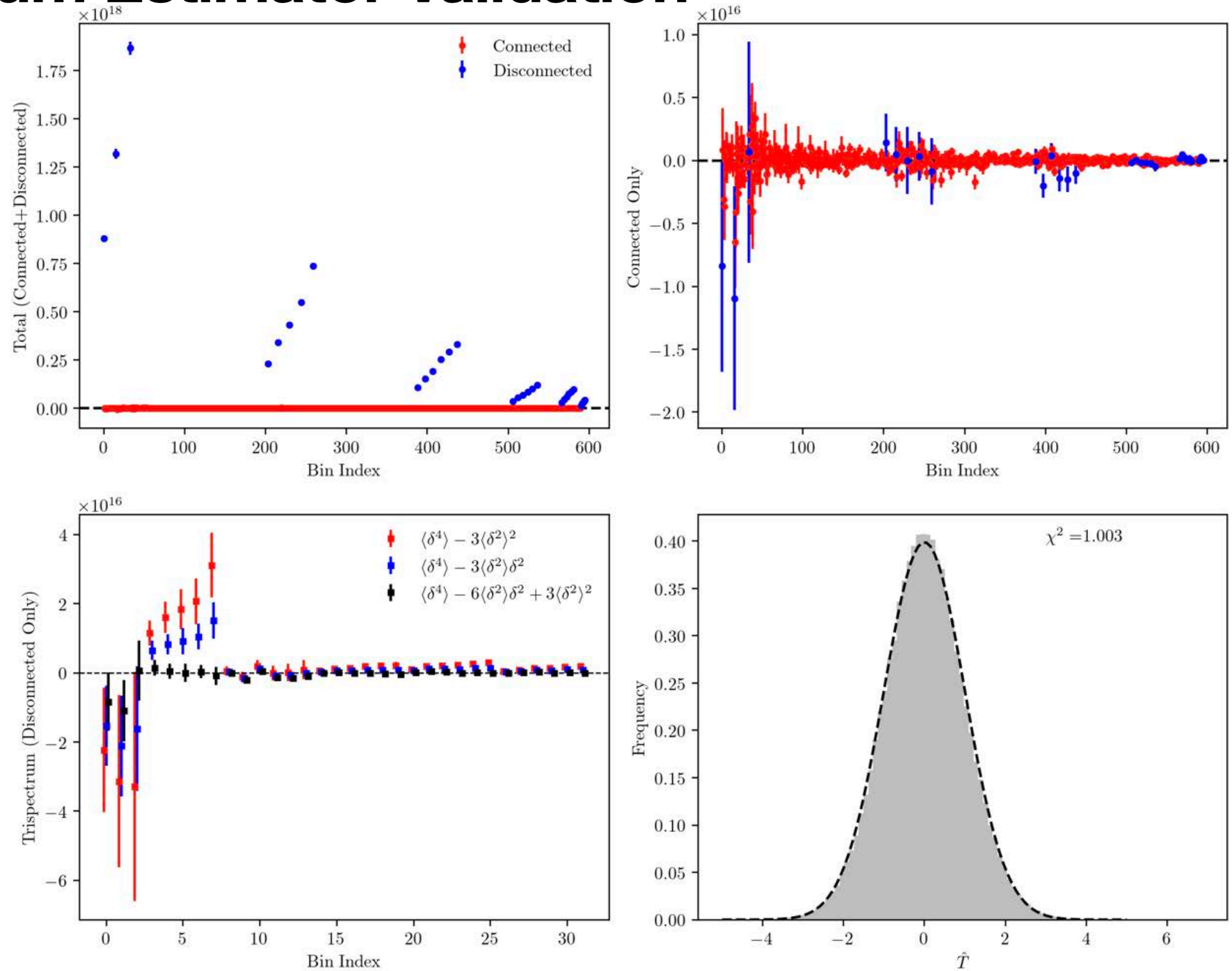
$$F^\delta_{ij}(\boldsymbol{x}) \equiv \int_{\boldsymbol{k}} W_i(k) W_j(k) |\delta_{\boldsymbol{k}}|^2 e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\hat{T}^{\langle\delta^2\rangle^2}_{\rm disc} = \frac{V}{N_{a,b,c,d,E}} \int_{\boldsymbol{Q}} W_E(Q) \int d^3x\, e^{-i\boldsymbol{Q}\cdot\boldsymbol{x}} \left[F^P_{ac}(\boldsymbol{x}) F^P_{bd}(\boldsymbol{x}) + F^P_{ad}(\boldsymbol{x}) F^P_{bc}(\boldsymbol{x})\right]$$

$$\hat{T}^{\delta^2\langle\delta^2\rangle}_{\rm disc} = \frac{V}{N_{a,b,c,d,E}} \int_{\boldsymbol{Q}} W_E(Q) \int d^3x\, e^{-i\boldsymbol{Q}\cdot\boldsymbol{x}} [F^P_{ac}(\boldsymbol{x}) F^\delta_{bd}(\boldsymbol{x}) + F^\delta_{ac}(\boldsymbol{x}) F^P_{bd}(\boldsymbol{x}) + F^P_{ad}(\boldsymbol{x}) F^\delta_{bc}(\boldsymbol{x}) + F^\delta_{ad}(\boldsymbol{x}) F^P_{bc}(\boldsymbol{x})].$$

Can compute both types of disconnected terms and subtract accordingly!

Trispectrum Estimator Validation



Super sample covariance (SSC)

- SSC for observable can be computed from derivative w.r.t DC mode

$$\mathrm{Cov}^{\mathrm{SSC}}(\mathcal{O},\mathcal{O})' = \sigma^2_{R_{\mathrm{box}}} \frac{\partial \mathcal{O}}{\partial \delta_{\mathrm{b}}} \frac{\partial \mathcal{O}'}{\partial \delta_{\mathrm{b}}} \qquad \left.\frac{\partial P(k)}{\partial \delta_{\mathrm{b}}}\right|_{\delta_{\mathrm{b}}=0} = P(k) + \frac{\partial P_{\mathrm{SU}}(k)}{\partial \delta_{\mathrm{b}}} - \frac{1}{3}\frac{\partial P(k)}{\partial \log(k)}$$

- Can estimate from separate universe (SU). For bispectrum looks like

$$\frac{\partial B(k_1,k_2,k_3)}{\partial \delta_b} = B(k_1,k_2,k_3) + \frac{\partial B_{\mathrm{SU}}(k_1,k_2,k_3)}{\partial \delta_b} - \frac{1}{3}\sum_{i=1}^{3}\frac{\partial B(k_1,k_2,k_3)}{\partial \log k_i}.$$

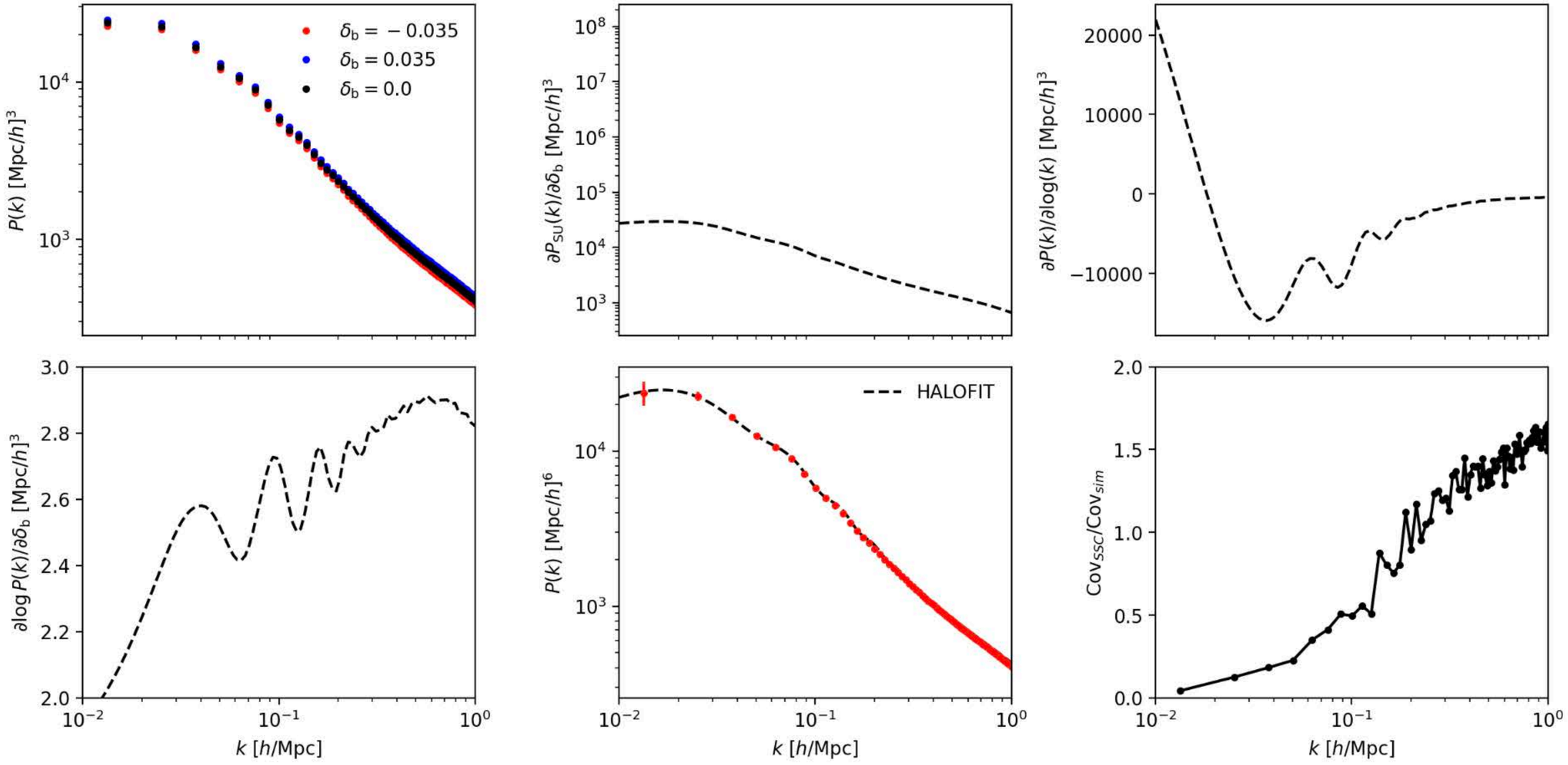
- Painful to compute for angle-averaged bispectrum in wide bins. Instead rewrite in terms of power spectrum SSC

$$B^{\mathrm{thr}}(k_1,k_2,k_3) = a_0(k)P(k_1)P(k_2),$$ ***(Assuming squeezed triangles)***

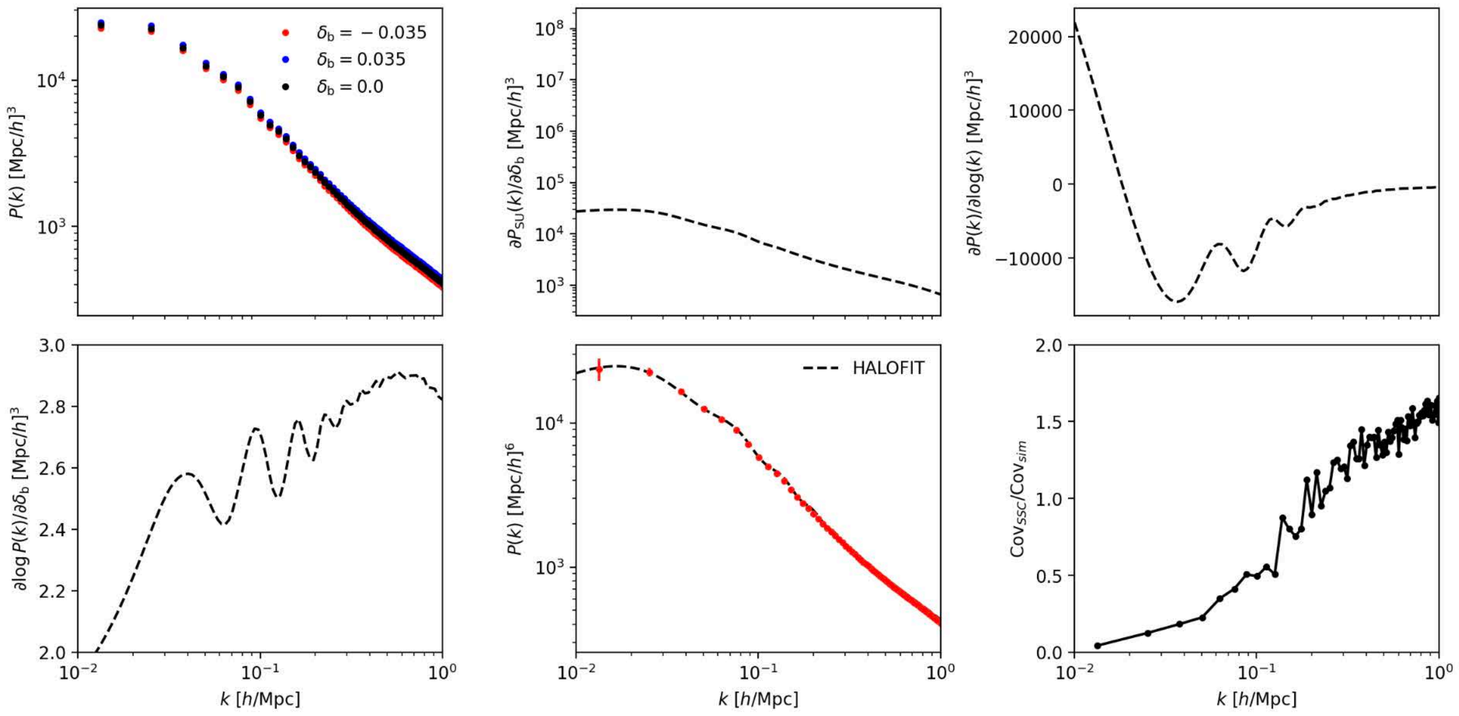
$$\mathrm{Cov}^{\mathrm{SSC}}(B(q,k),B(q',k')) \approx a_0(k)a_0(k')\Big[P(q)P(k')\,\mathrm{Cov}^{\mathrm{SSC}}(P(k),P(k')) + P(q)P(k')\,\mathrm{Cov}^{\mathrm{SSC}}(P(k),P(q')) + P(k)P(q')\,\mathrm{Cov}^{\mathrm{SSC}}(P(q),P(k')) + P(k)P(k')\,\mathrm{Cov}^{\mathrm{SSC}}(P(q),P(q'))\Big].$$

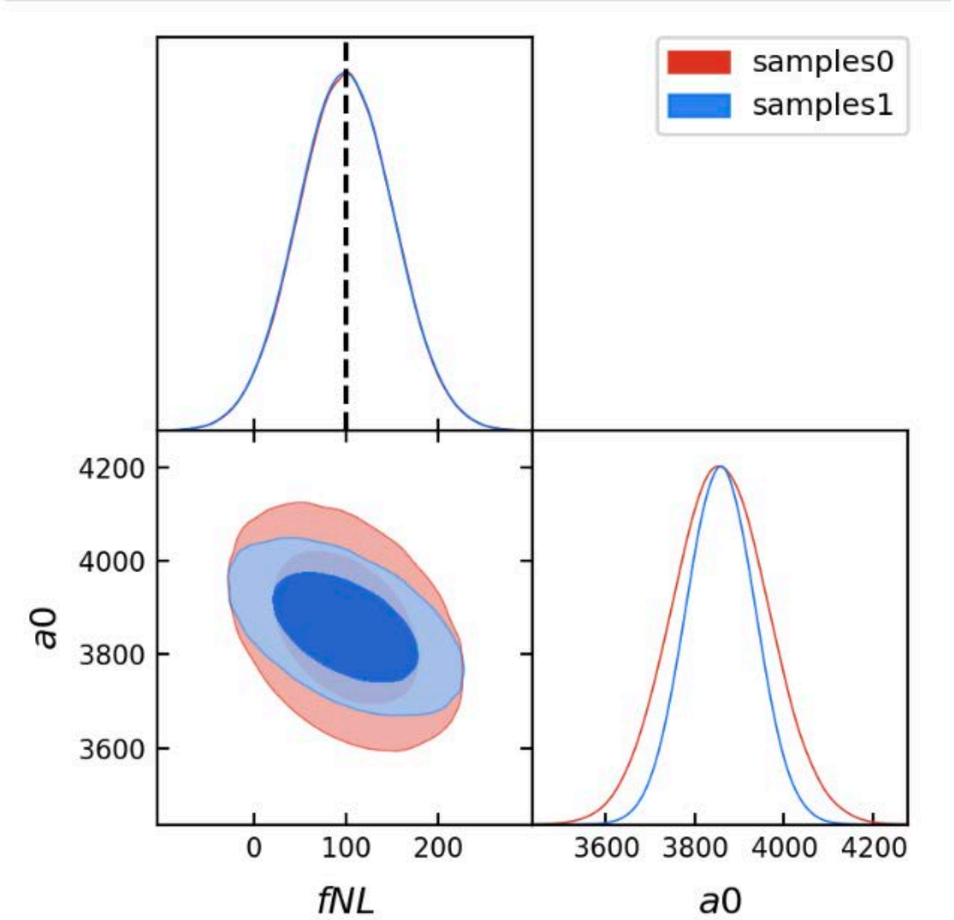
- Checked that this equation agrees with SU within ~20% for squeezed bispectrum from Quijote in thin bin

SSC (continued)



SSC (continued)

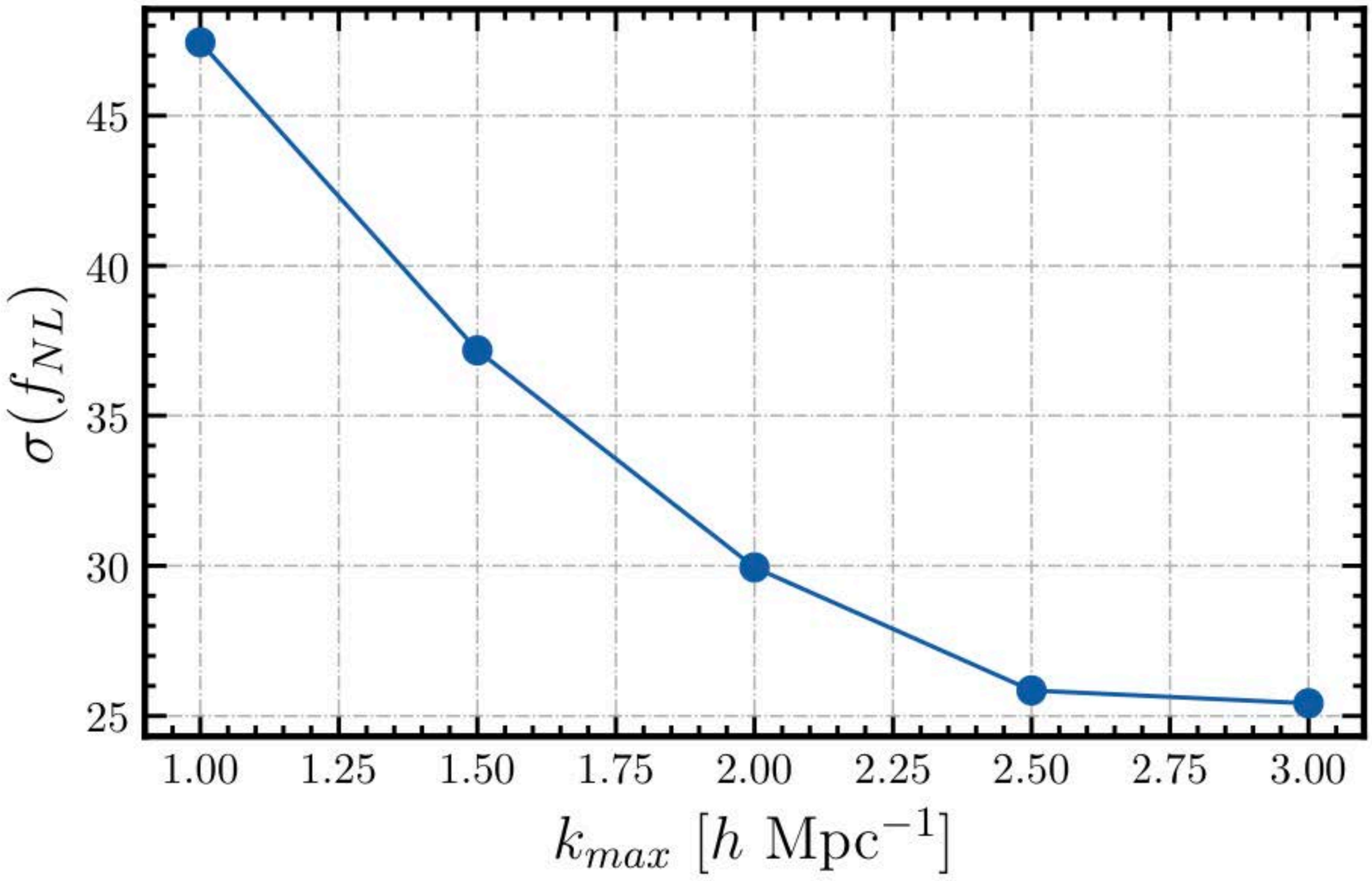


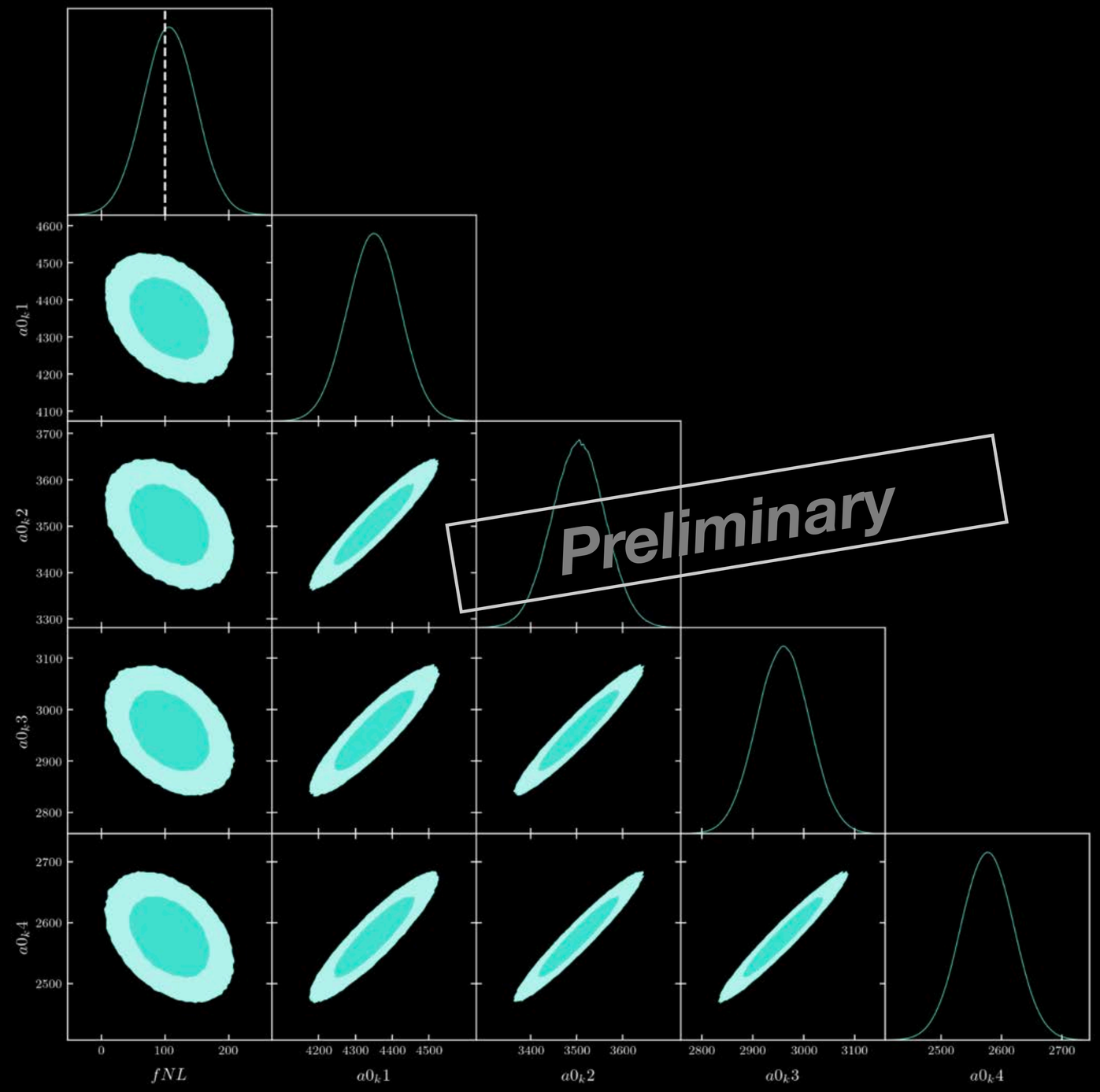


Left: Validating P(k) SSC using Quijote.

Right: including SSC in Quijote degrades constraints on a0, but has negligible impac
fNL

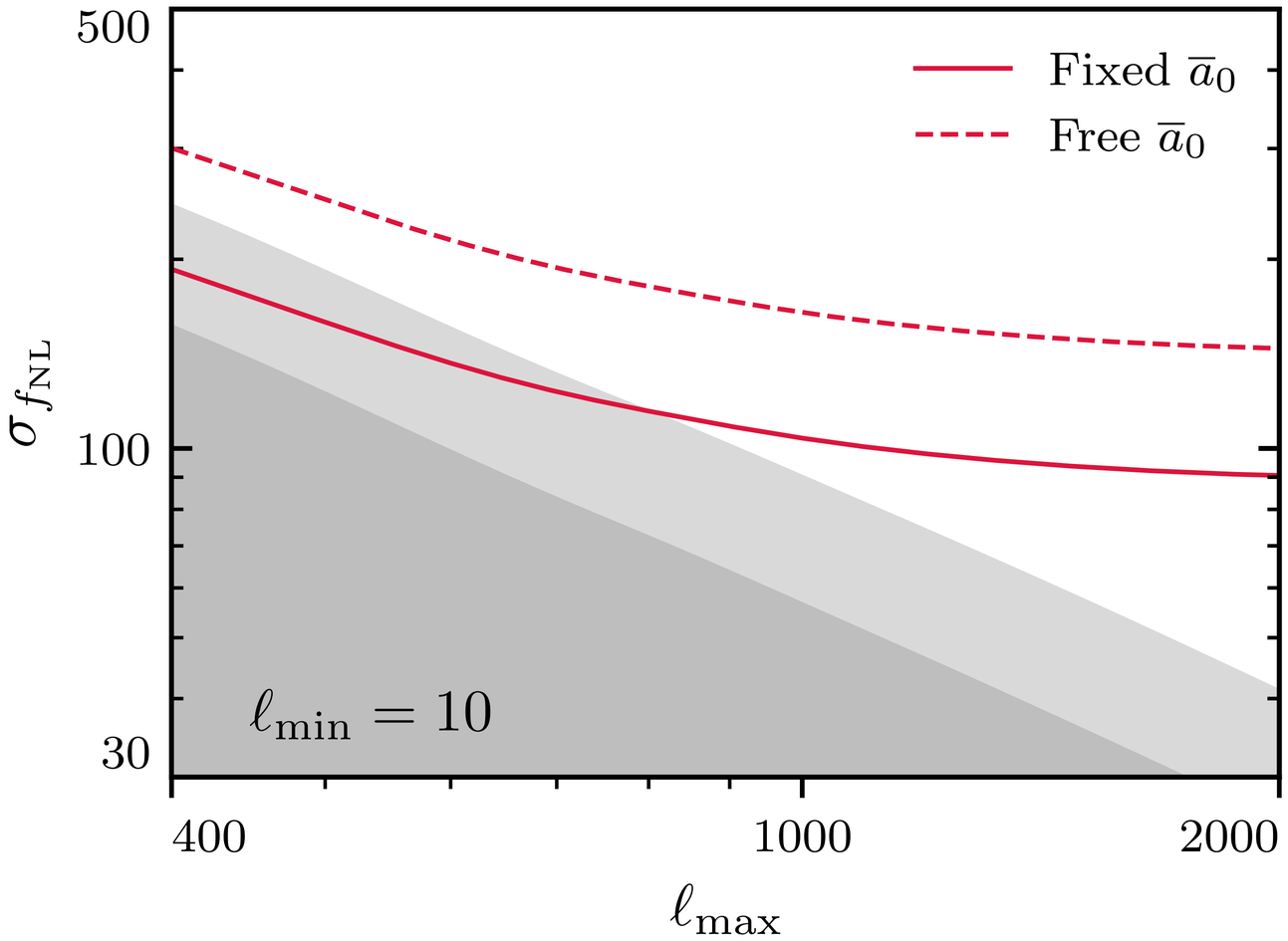
arxiv.org/pdf/2305.03070

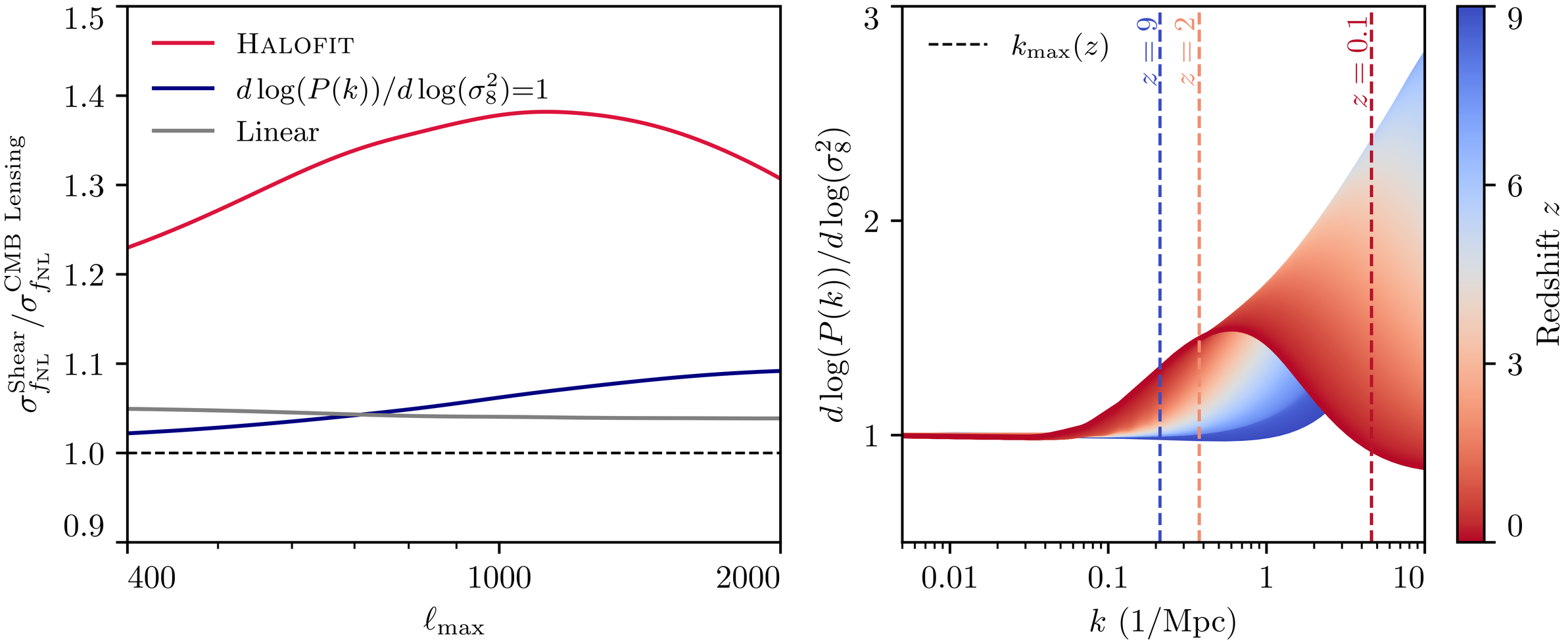



Preliminary
fNL
$a0_k1$
$a0_k2$
$a0_k3$
$a0_k4$
0
100
200
4200
4300
4400
4500
3400
3500
3600
2800
2900
3000
3100
2500
2600
2700
4100
4200
4300
4400
4500
4600
3300
3400
3500
3600
3700
2500
2600
2700

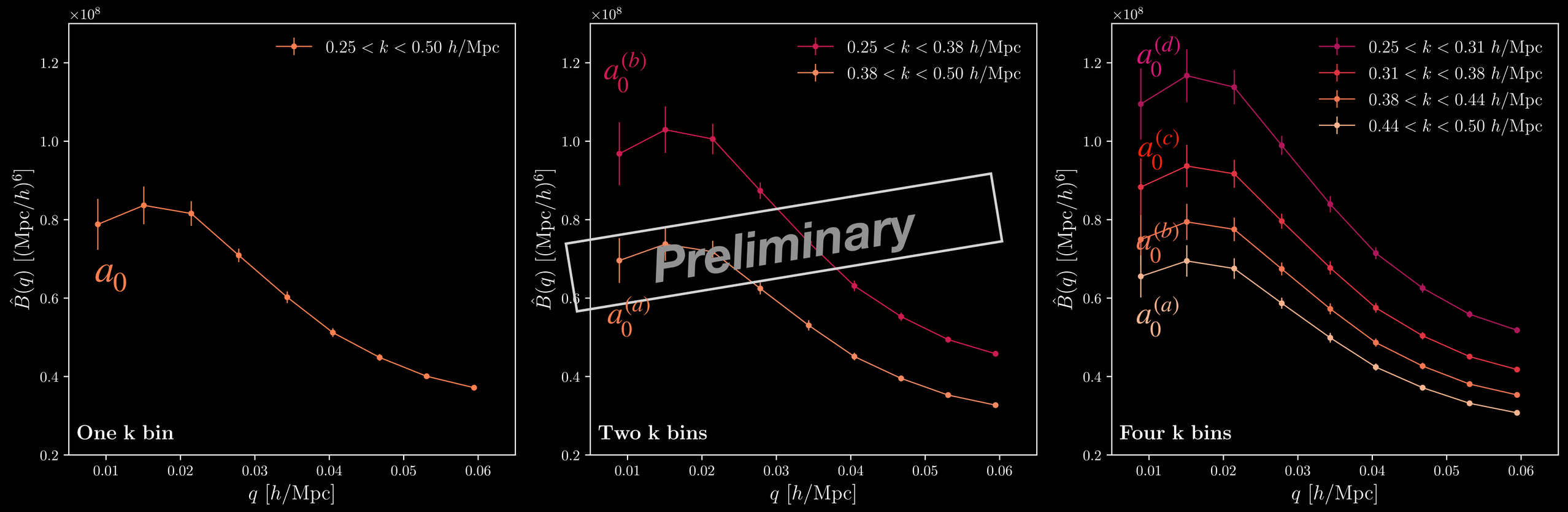
Marginalization of gravitational non-Gaussianity

- Marginalization over gravitational NG increases forecasted error on fNL by ~50%




HALOFIT
$d\log(P(k))/d\log(\sigma_8^2)$=1
Linear
$\sigma_{f_{\rm NL}}^{\rm Shear}/\sigma_{f_{\rm NL}}^{\rm CMB\ Lensing}$
$\ell_{\rm max}$
$k_{\rm max}(z)$
$z=9$
$z=2$
$z=0.1$
$d\log(P(k))/d\log(\sigma_8^2)$
k (1/Mpc)
Redshift z

Optimizing the estimator



Instead of summing over all non-linear modes, we can split into k-bins.

- Example: squeezed bispectrum measurements from 100 realizations of Quijote with $f_{\mathrm{NL}} = 100$ with $0.25 < k < 0.5\ h/\mathrm{Mpc}$ (see also Giri, Münchmeyer, Smith 2305.03070)