Weak Lensing Non Gaussian Statistics in the era of precision Cosmology

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Berkeley 6th September 2022

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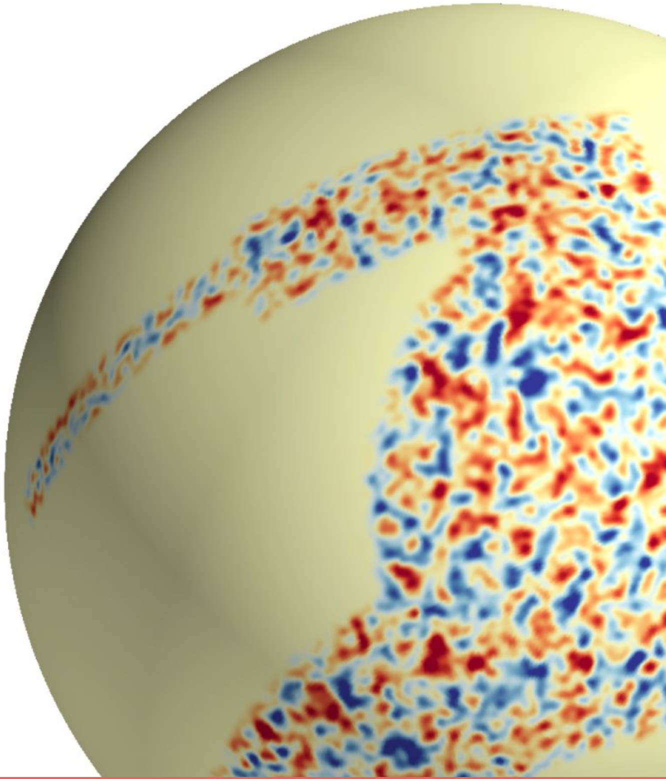
Outline

Main goal: stress-test the standard cosmological model with new methods!

1) Toolbox essential: weak lensing mass maps
2) Why non Gaussian statistics?
3) Results from non Gaussian statistics with DES Y3
4) New promising probes: wavelet based estimators
5) Future obstacles & analyses robustness



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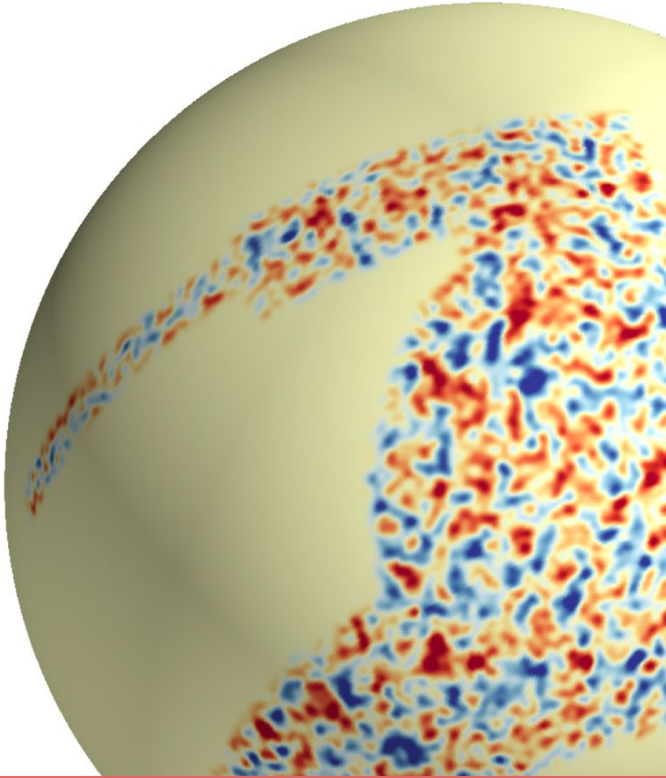
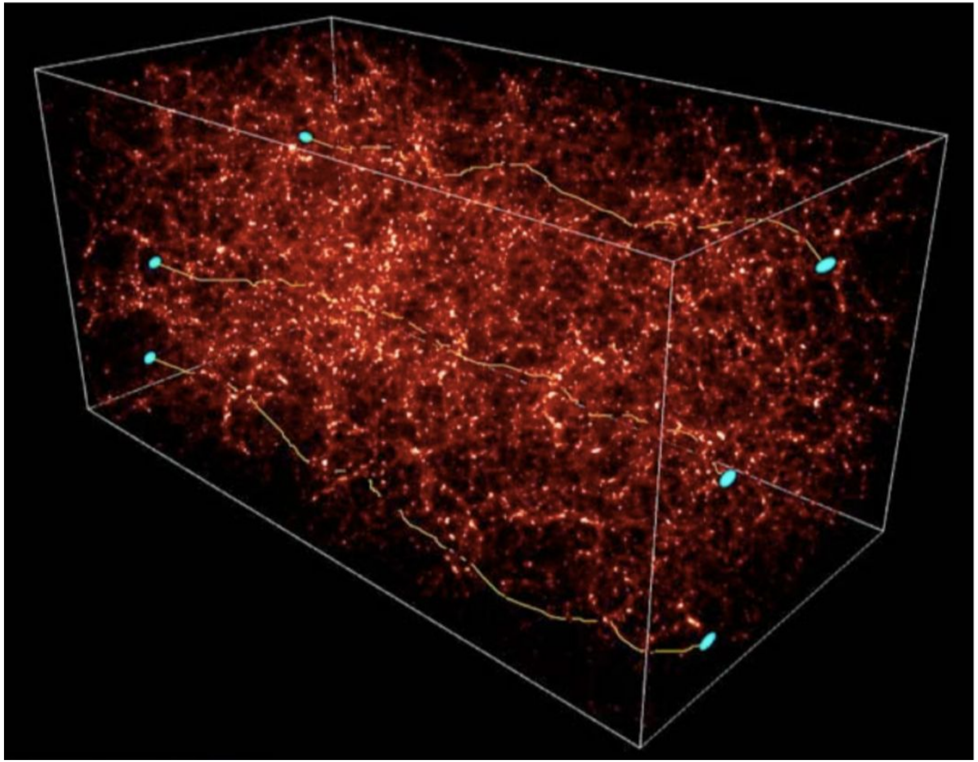


(Weak) Gravitational Lensing



Due to the **Large Scale Structure** of the Universe,
the path followed by the light emitted by distant galaxies will appear distorted

Gravitational lensing allows to probe the matter distribution (mostly dark)

In weak lensing we have to deal with multiple deflections/distortions due to lenses along the line of sight

Lensing potential

$$\phi(\theta, r) = \frac{1}{c^2} \int_0^r dr' \frac{S_k(r - r')}{S_k(r) S_k(r')} \left[\Phi(\theta, r') + \Psi(\theta, r')\right]$$

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Geometrical factor
(redshift-distance relation
depends on cosmology)

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Gravitational potential (clumpiness along the line of sight -> evolution of structures)

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Geometrical factor
(redshift-distance relation depends on cosmology)

Gravitational potential
(clumpiness along the line of sight -> evolution of structures)

laws of gravity

In weak lensing we have to deal with multiple deflections/distortions due to lenses along the line of sight

Lensing potential

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Deflection

$$\alpha = \nabla \phi$$

In weak lensing we have to deal with multiple deflections/distortions due to lenses along the line of sight

Lensing potential

$$\phi(\theta, r) = \frac{1}{c^2}\int_0^r dr' \frac{S_k(r-r')}{S_k(r)S_k(r')}\left[\Phi(\theta, r') + \Psi(\theta, r')\right]$$

Deflection

$$\alpha = \nabla\phi$$

Convergence

$$\kappa = \frac{1}{2}\nabla^2\phi = \frac{1}{2}\left(\phi_{,11} + \phi_{,22}\right)$$

Mass

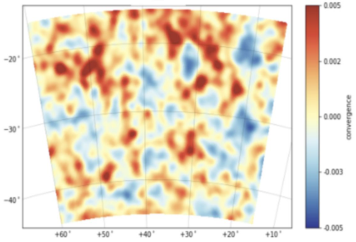
Shear

$$\gamma = \gamma_1 + i\gamma_2 = \frac{1}{2}\left(\phi_{,11} - \phi_{,22}\right) + i\phi_{,12}$$

Observable

(projected) WL mass map (or convergence)

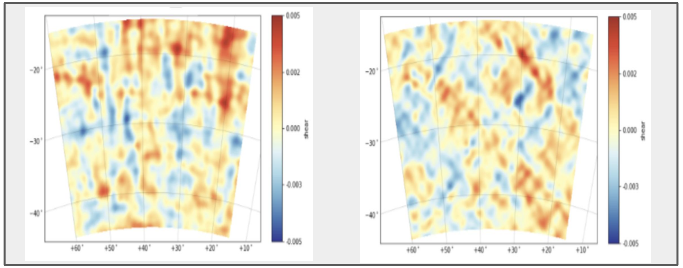
Not observable directly



Mass Map reconstruction (e.g., Kaiser-Squires)

observable!

Using measured galaxies ellipticity, we can estimate the shear field (2 components)



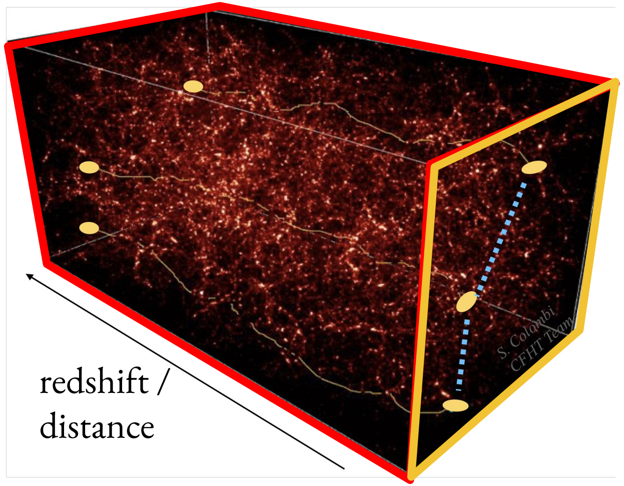
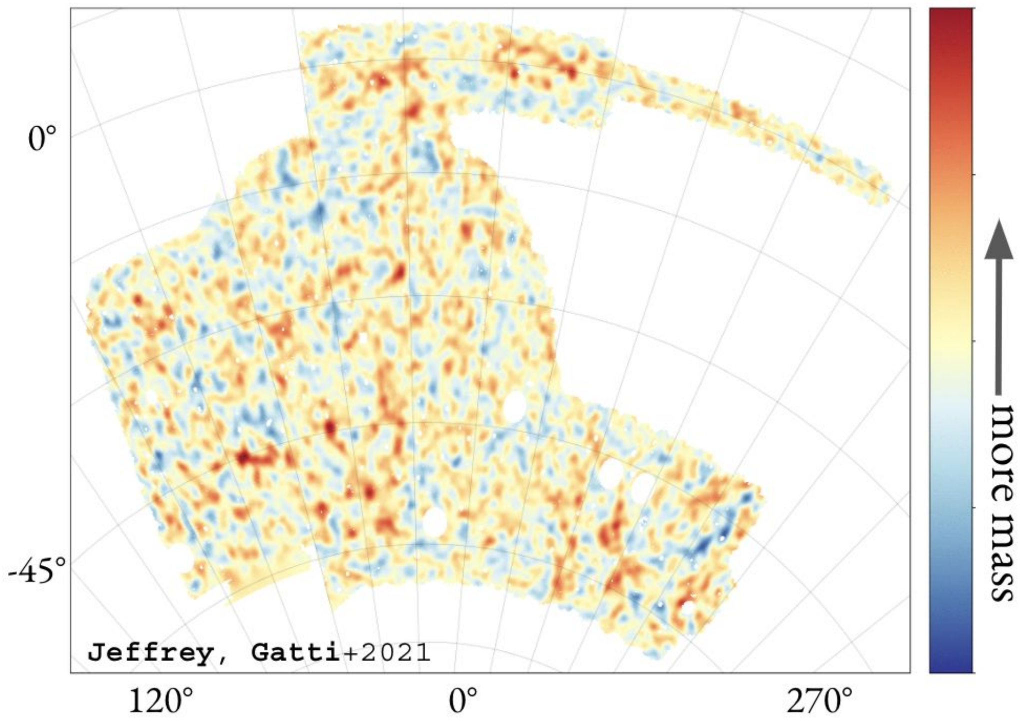
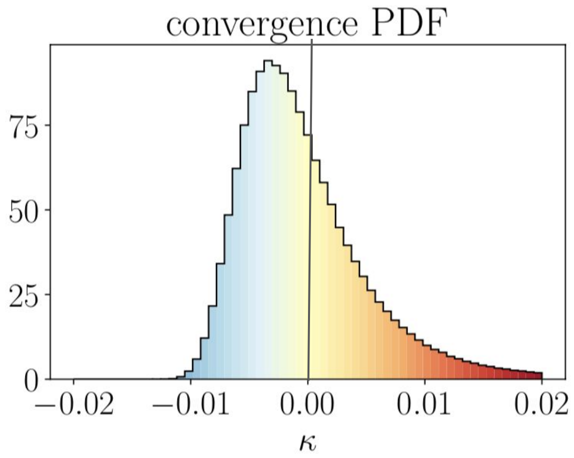


Image plane

Dark Energy Survey Y3 Mass Map



5000 sq. degrees, 100 milion galaxy shapes



The convergence field **is not Gaussian**; high order stats can probe additional cosmological information

A map makes it **easier** to use a wide range of non Gaussian statistics

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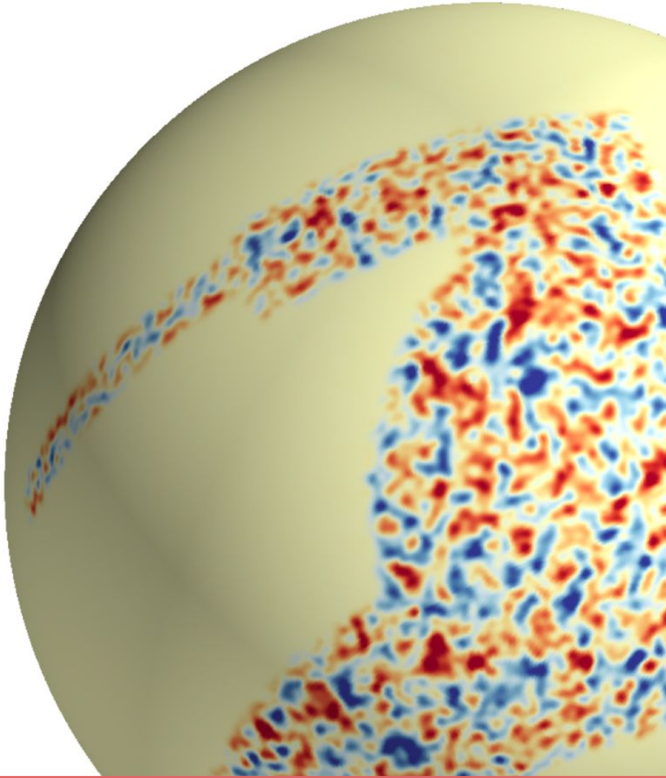
Takeaways:
- WL mass maps = projected matter density maps
- Preserve non Gaussian features of the field
- Easy to study with non Gaussian statistics



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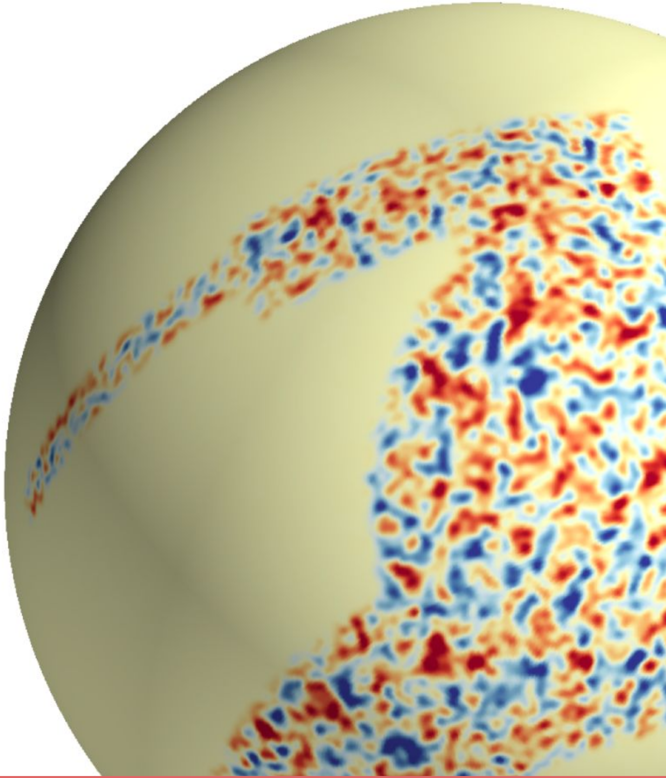
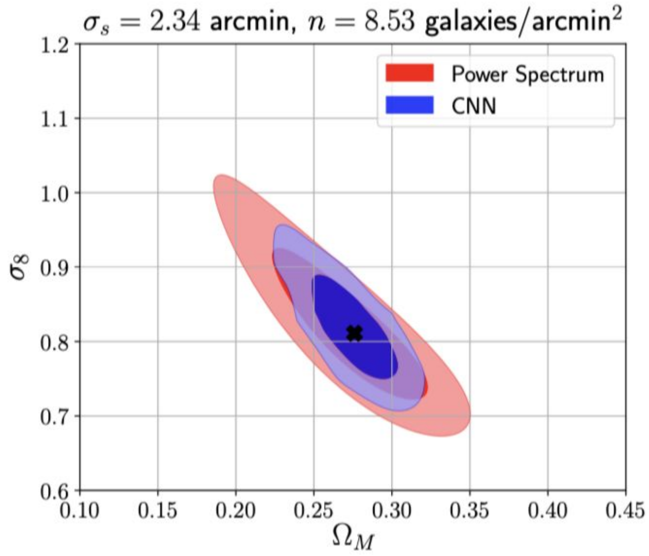
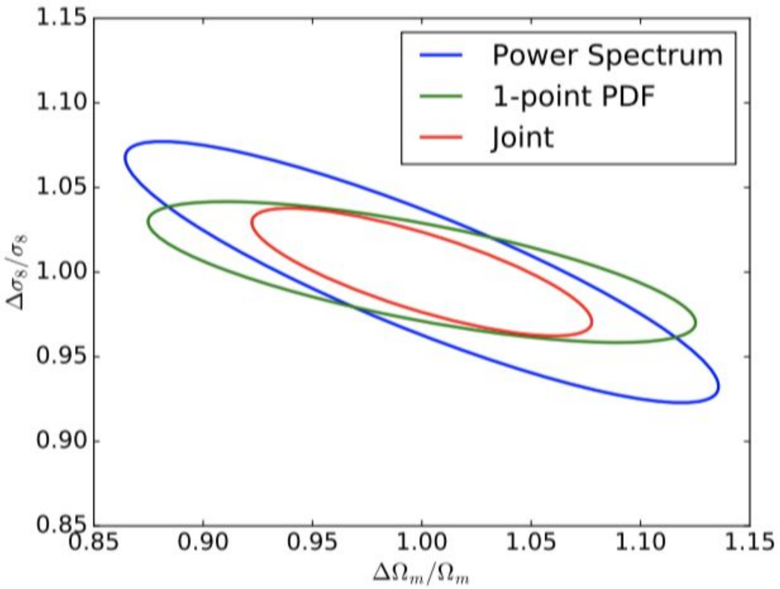


non Gaussian statistics improve cosmological constraints over standard Gaussian statistics



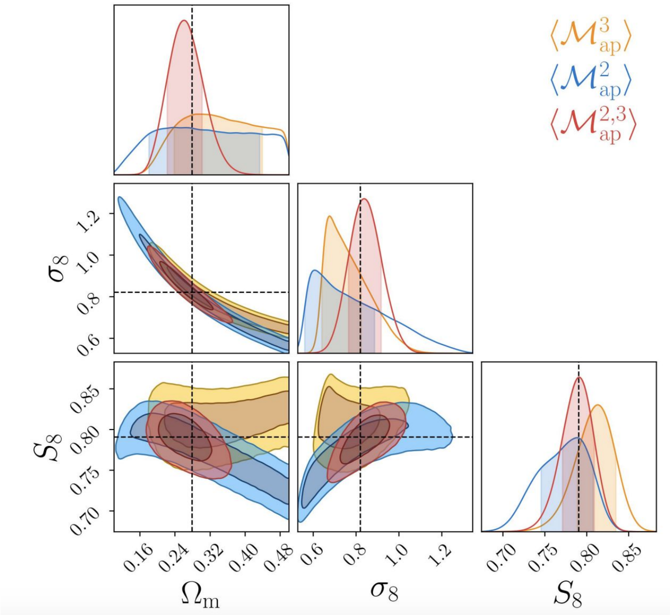
Machine Learning (Fluri+19)

1-point PDF (Patton+18)

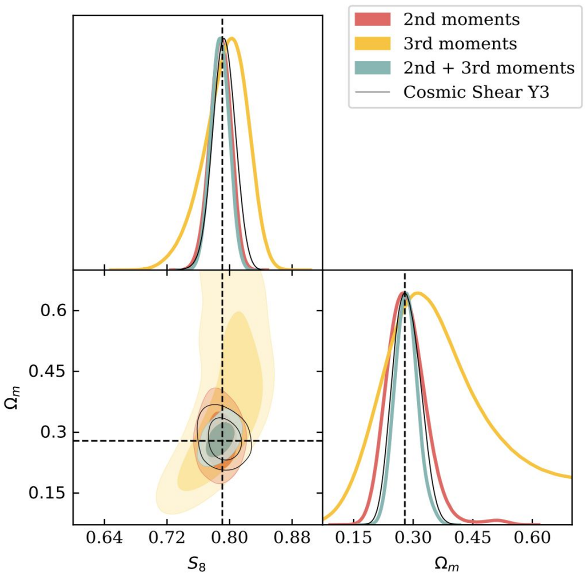


* results shown here are either forecasts or tests on simulations

non Gaussian statistics improve cosmological constraints over standard Gaussian statistics



Mass aperture stat, Heydenreich+22



Moments, Gatti+ 2019

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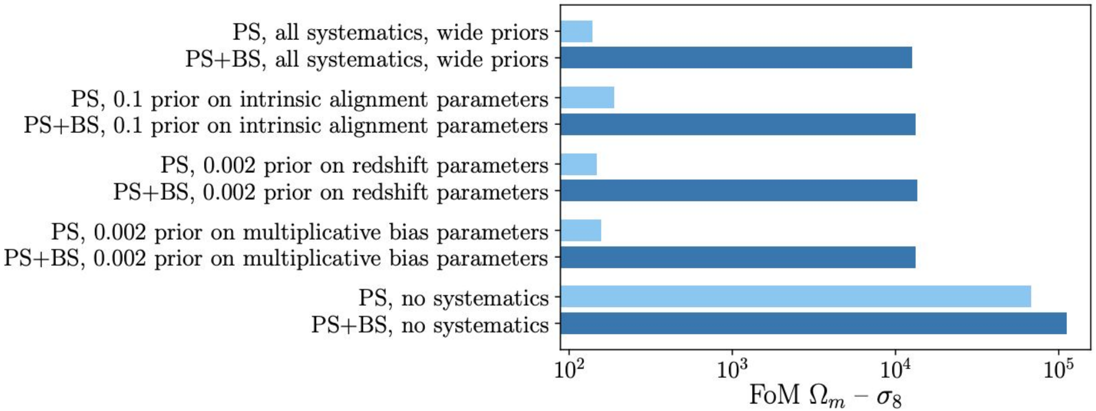
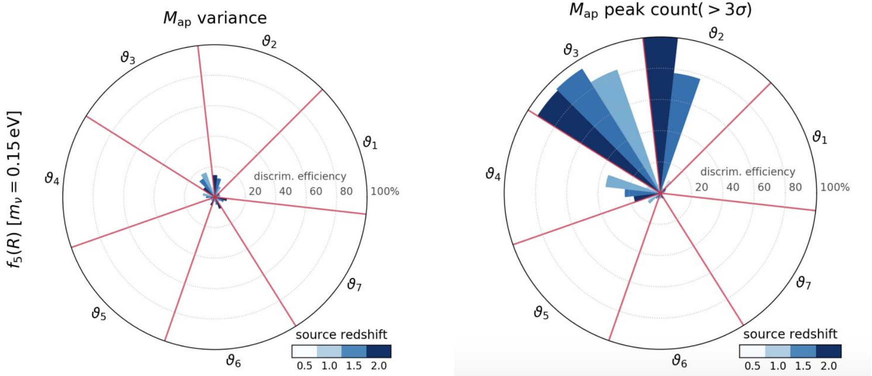
Non Gaussian statistics self-calibrate nuisance parameters



Pyne & Joachimi +22

And can help discriminate between general relativity and modified gravity theories

Peel+18



Growing interest in weak lensing Non Gaussian stats.

- **Peaks statistics** (e.g. Kacprzak et al. 2016; Martinet et al. 2018; Peel et al. 2018; Shan et al. 2018; Ajani et al. 2020; Zürcher et al. 2021a,2021b..)
- **High order Moments** (Chang et al. 2018; Vicinanza et al. 2018; Peel et al. 2018; Gatti et al. 2020,2021...)
- **3pt correlation functions** (Takada & Jain 2003, 2004; Semboloni et al. 2011; Fu et al. 2014,Secco et al 2022...)
- **Minkowski functionals** (Kratochvil et al. 2012; Petri et al. 2015; Vicinanza et al. 2019; Parroni et al. 2020...)
- **Machine Learning** (Ribli et al. 2019; Fluri et al. 2018, 2019; Jeffrey et al. 2021a...)
- **Wavelet-based methods** (Allys 2021, Cheng 2021, Gatti et al in prep....)
- **Others (PDF,minima counts, L1-norm, k-Nearest Neighbor distributions, Minimum Spanning Tree,....)**

  [DISCLAIMER: non exhaustive!]
  Note: only '3pt correlation functions' do not require a map - all the others are map based statistics.

How do I choose? Is one better than the others?

- Human-designed statistics vs. machine designed statistics
- Theory modelling vs. simulation-based modelling
- Impact of systematics / data vector ease of use

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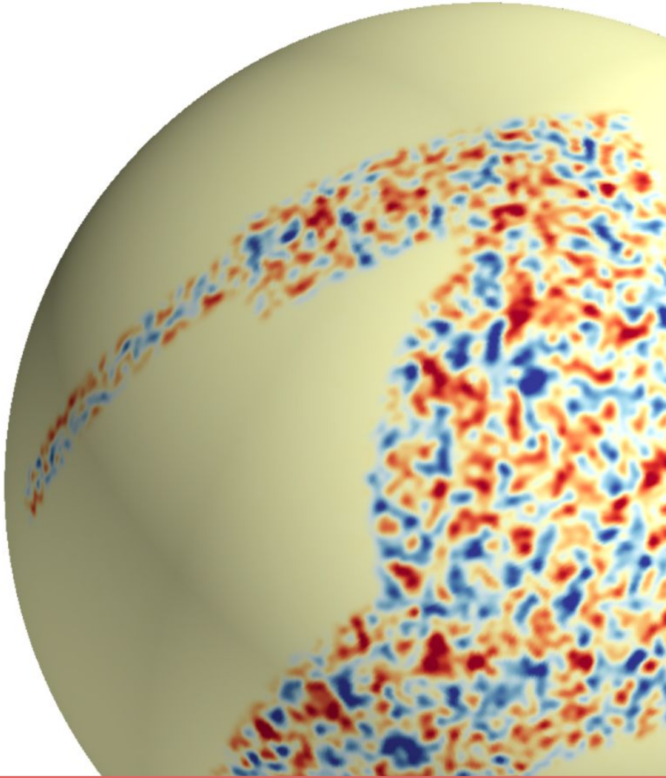
Takeaways:
- Improve cosmological constraints over Gaussian stats.
- Difference dependence on systematics.
- Self-calibrate nuisance parameters.
- Help discriminate between modified gravity theories and GR



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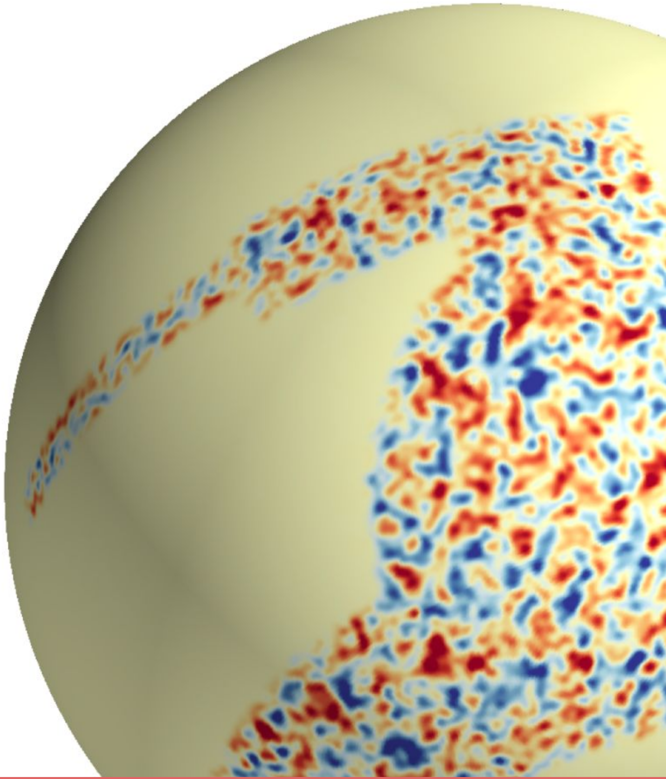
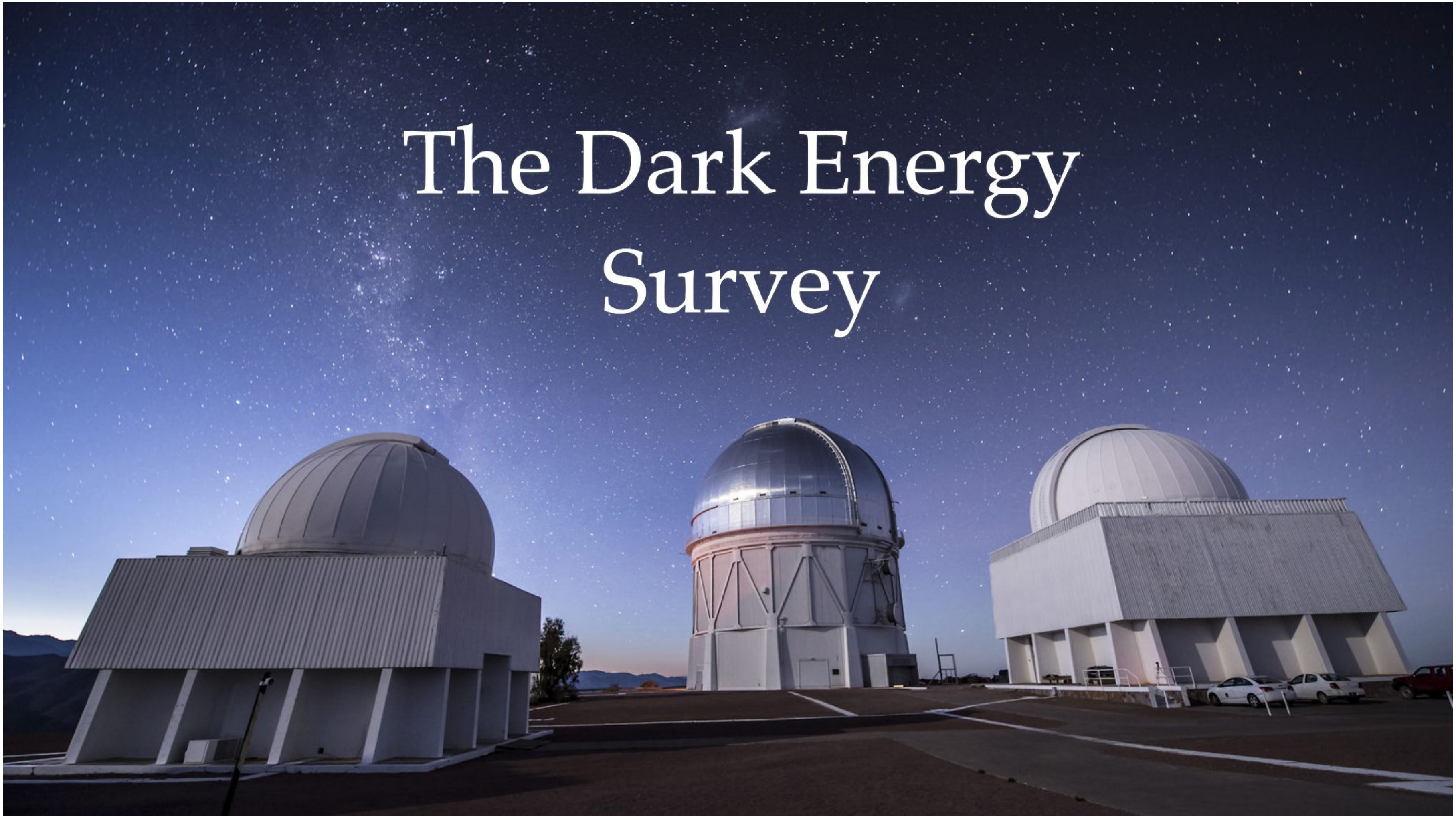
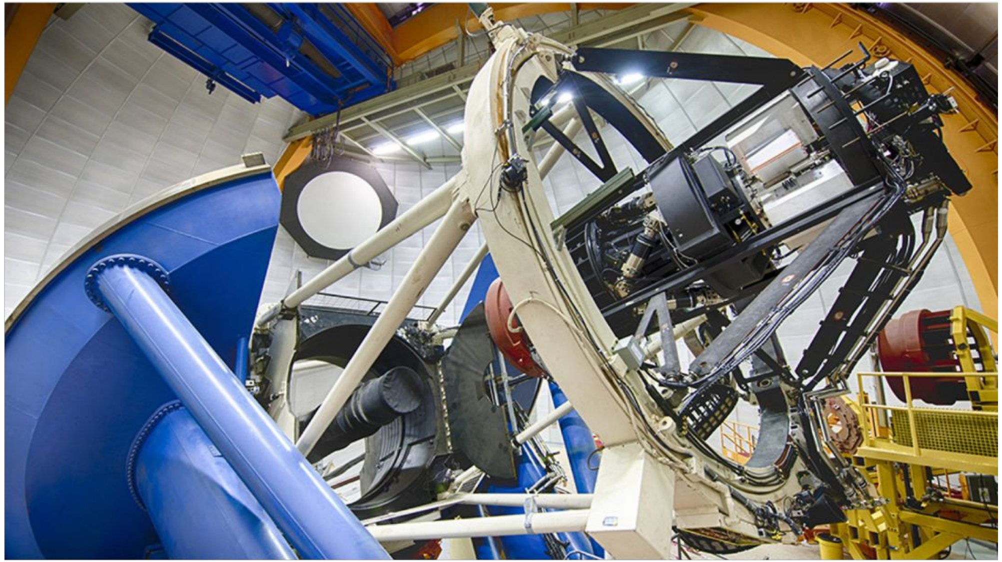
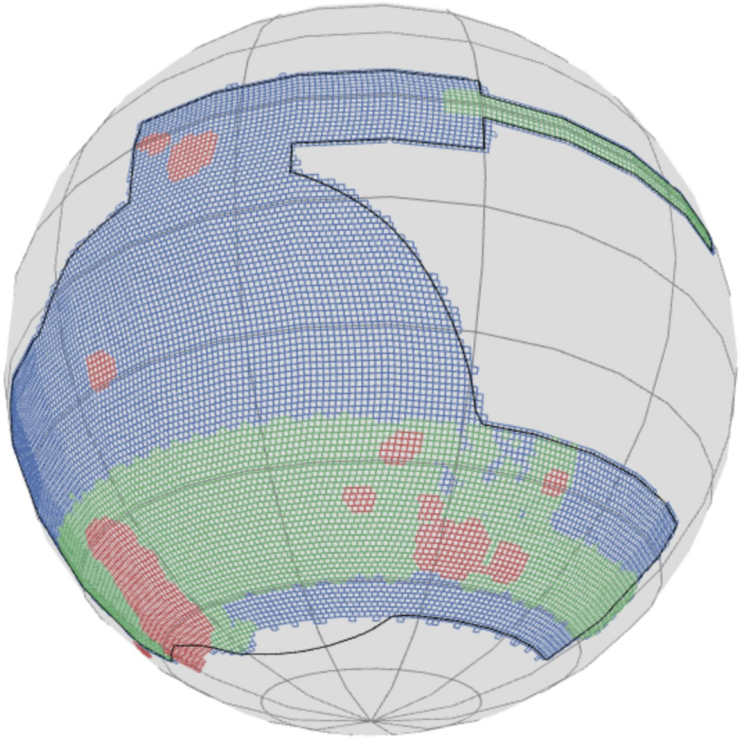



The Dark Energy
Survey

The Dark Energy Survey

- Imaging galaxy survey.
- ~5000 sq. deg. after 6 years (2013-2019)
- Shapes, photometric redshifts and positions for 300 million galaxies.





Red : Science verification data
Green: DES Y1
Blue: DES Y3

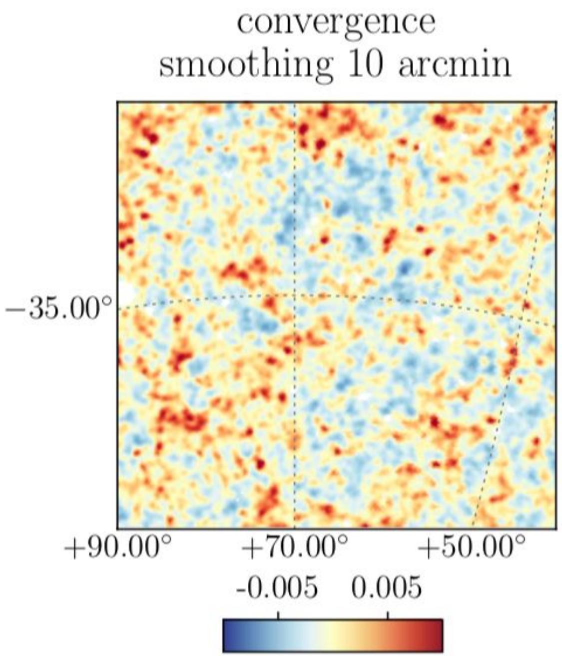
- The DES Y3 data spans the full footprint (4134 sq deg). 100 million galaxy shapes, 10 million galaxy positions

- In 2021 we released the so called '**3x2pt**' DES Y3 cosmological analysis which featured the analysis of 3 different 2pt correlation functions (shear-shear, galaxy-shear, galaxy-galaxy). In January 2022, we released our DES Y3 catalogs.

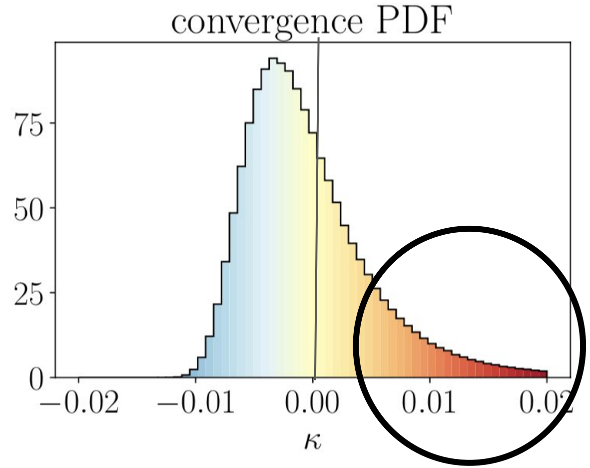
-

Non Gaussian statistics in DES

(WL mass map)



Map of the mass distribution of the Universe (integrated along the line-of-sight).

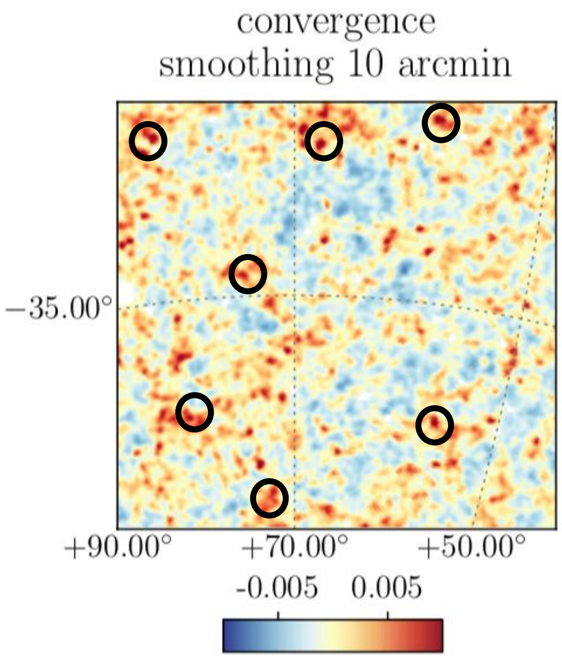


The convergence field **is not Gaussian**; high order stats can probe additional cosmological information

DES Y3 moments analysis, Gatti+21, [2110.10141]

Non Gaussian statistics in DES

(WL mass map)



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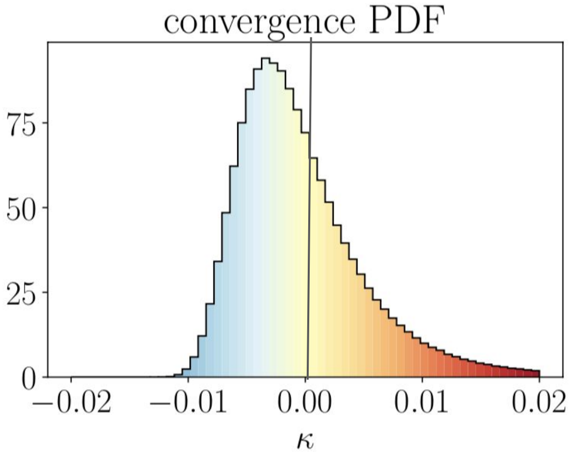

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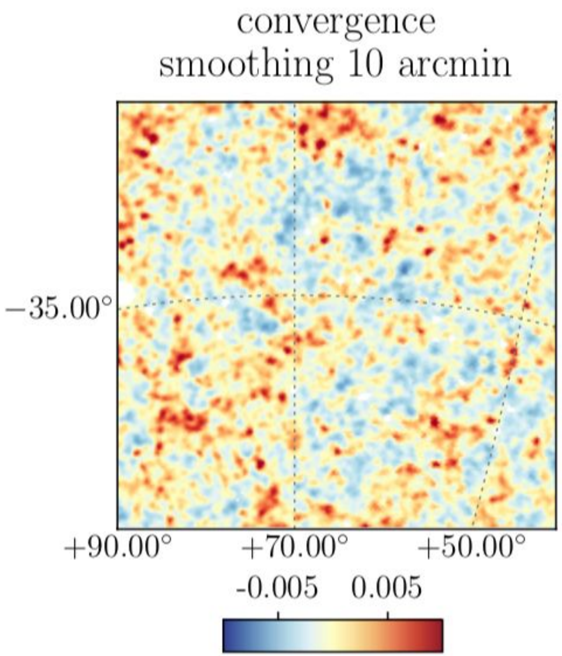
DES Y3 moments analysis, Gatti+21, [2110.10141]
DES Y3 peaks analysis, Zuercher+22, [2110.10135]
DES Y3 LFI peaks analysis & CNN (Jeffrey+ in prep.)

Non Gaussian statistics in DES



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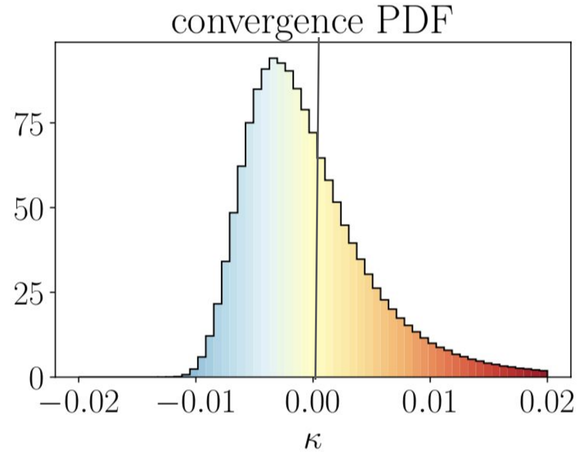

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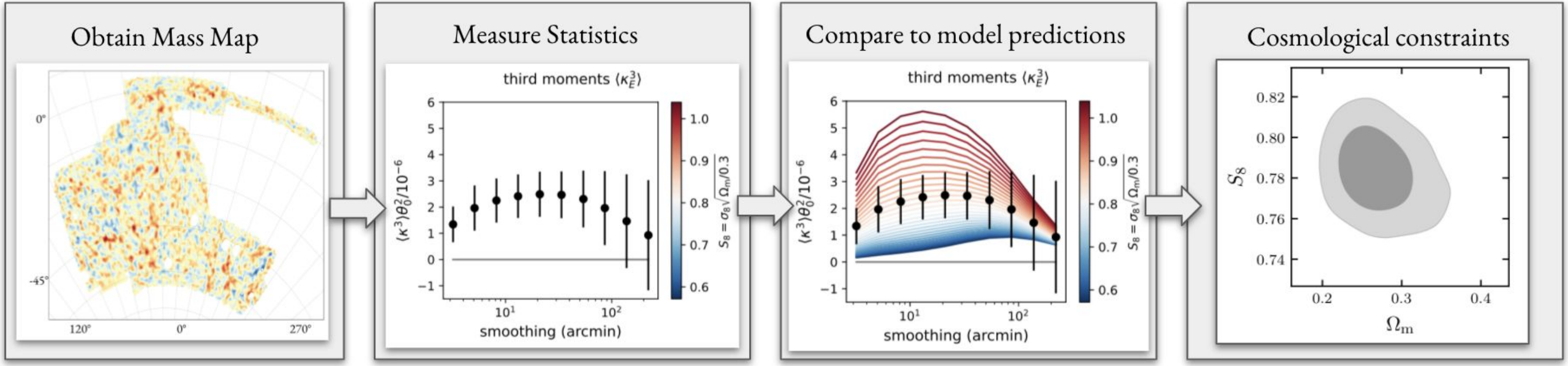
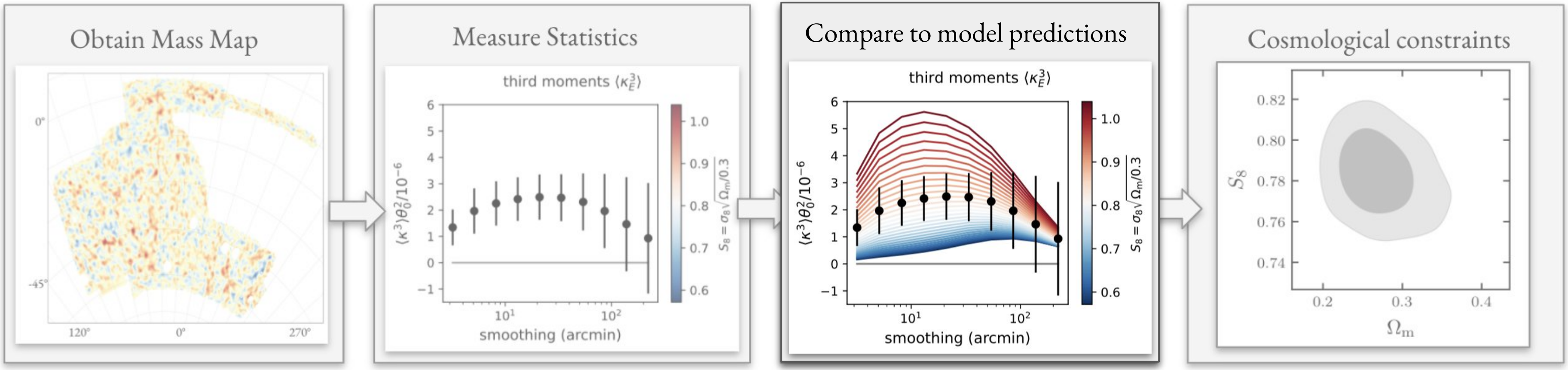
DES Y3 Wavelet Phase Harmonics (Gatti+ in prep.)

From maps to cosmology



From maps to cosmology



Two different strategies to model high order statistics

Analytical modelling

☹ complex to develop; not always feasible

☺ not computationally expensive

adopted in the moments analysis [Gatti+21]

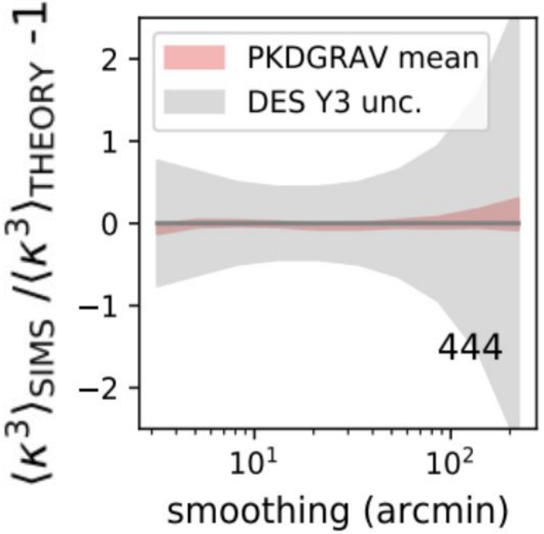
Simulation-based forward modelling

☺ possible for any statistic

☹ computationally expensive

adopted in the peaks analysis [Zuercher+21]

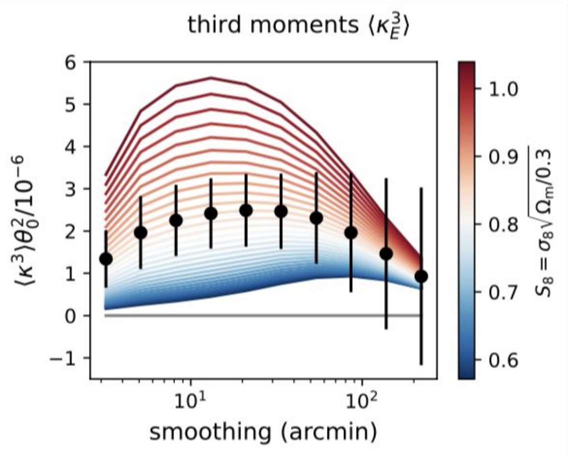
Analytical predictions: complex to develop, but computationally cheap to evaluate.



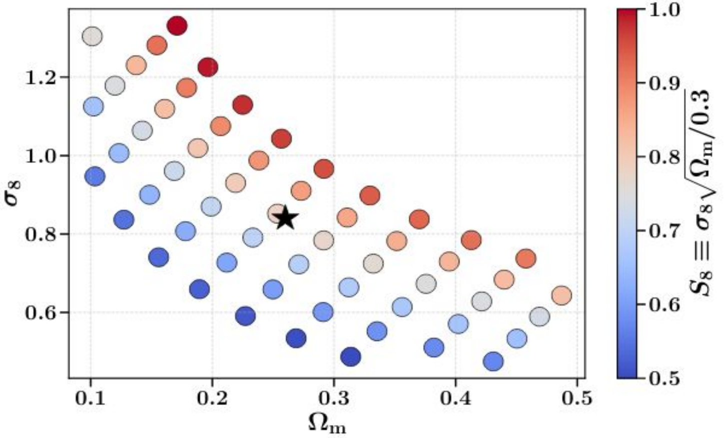
$$\langle \delta^3_{\theta_0,\mathrm{lin}}\rangle(\tau) = \frac{6}{(2\pi)^3}\int \mathrm{d}^2k_1\mathrm{d}^2k_2 W(\mathbf{k}_1,\theta_0)W(\mathbf{k}_2,\theta_0)W(\mathbf{k}_1+\mathbf{k}_2,\theta_0)$$
$$\times P_{\mathrm{lin}}(\mathbf{k}_1,\tau), P_{\mathrm{lin}}(\mathbf{k}_2,\tau)F_2(\mathbf{k}_1,\mathbf{k}_2,\tau), \qquad \text{(A11)}$$

$$F_2(\mathbf{k}_1,\mathbf{k}_2,\tau) = \frac{1}{2}[(1+\frac{k_1}{k_2}\cos\phi)+(1+\frac{k_2}{k_1}\cos\phi)]+[1-\mu(\tau)](\cos^2\phi-1),$$

$$F_2(\mathbf{k}_1,\mathbf{k}_2,\tau) = \frac{1}{2}b_1b_2[(1+\frac{k_1}{k_2}\cos\phi)+(1+\frac{k_2}{k_1}\cos\phi)]$$
$$+[1-\mu(\tau)]c_1c_2(\cos^2\phi-1)+[a_1a_2\mu(\tau)-b_1b_2+[1-\mu(\tau)]c_1c_2].$$



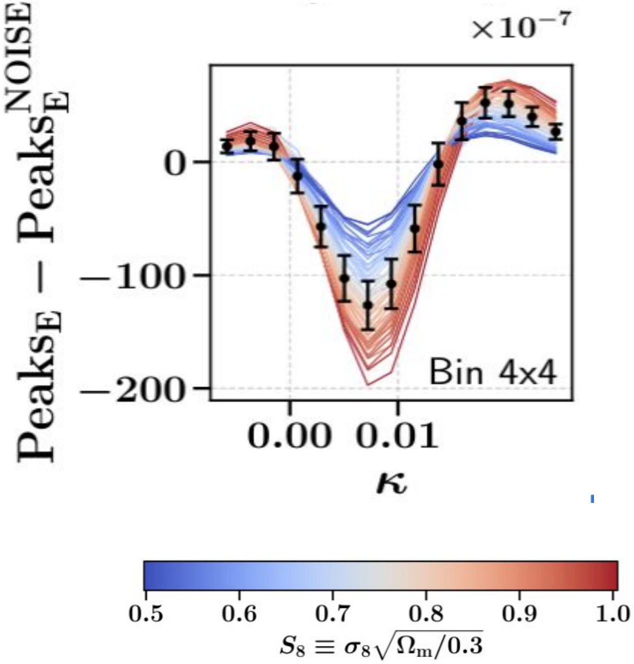
Simulation-based forward modelling:
lots of simulations required!



Predictions interpolated using an emulator

Forward model DES Y3-like mass maps

Predictions for peak functions



Credit: D. Zuercher

Validation & systematics control

These analyses rely on the **data validation** from the DES 3x2 efforts (source sample validation, redshift & shear calibration).

Similar modelling complexity of the DES 3x2:

- ΛCDM, **5 cosmological parameters**
- **Intrinsic Alignment** (NLA)
- **Calibration systematics** (redshift & shear)

Parameter	Prior
Cosmological Parameters	
Ω_{m}	U[0.1, 0.9]
σ_8	U[0.5, 1.4]
Ω_{b}	U[0.03, 0.07]
n_{s}	U[0.87, 1.07]
h	U[0.55, 0.91]
Calibration Parameters	
m_1	$\mathcal{N}(-0.0063, 0.0091)$
m_2	$\mathcal{N}(-0.0198, 0.0078)$
m_3	$\mathcal{N}(-0.0241, 0.0076)$
m_4	$\mathcal{N}(-0.0369, 0.0076)$
Δz_1	$\mathcal{N}(0.0, 0.018)$
Δz_2	$\mathcal{N}(0.0, 0.015)$
Δz_3	$\mathcal{N}(0.0, 0.011)$
Δz_4	$\mathcal{N}(0.0, 0.017)$
Intrinsic Alignment Parameters	
$A_{\mathrm{IA},0}$	U[−5, 5]
α_{IA}	U[−5, 5]

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Similar modelling complexity of the DES 3x2:
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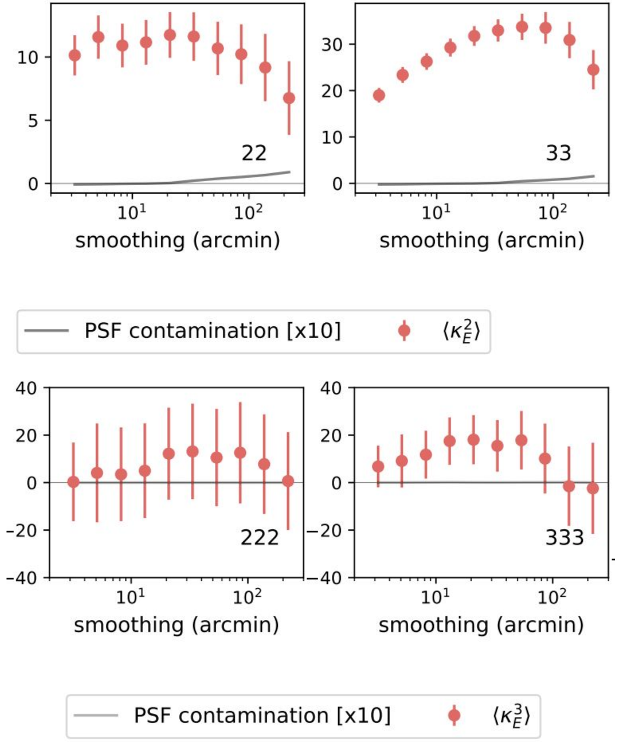
- Calibration systematics (redshift & shear)

+ Extra specific tests for high order statistics
(validation pipeline & systematics)



Second moments

Third moments

Cosmology from DES Y3 2nd+3rd moments

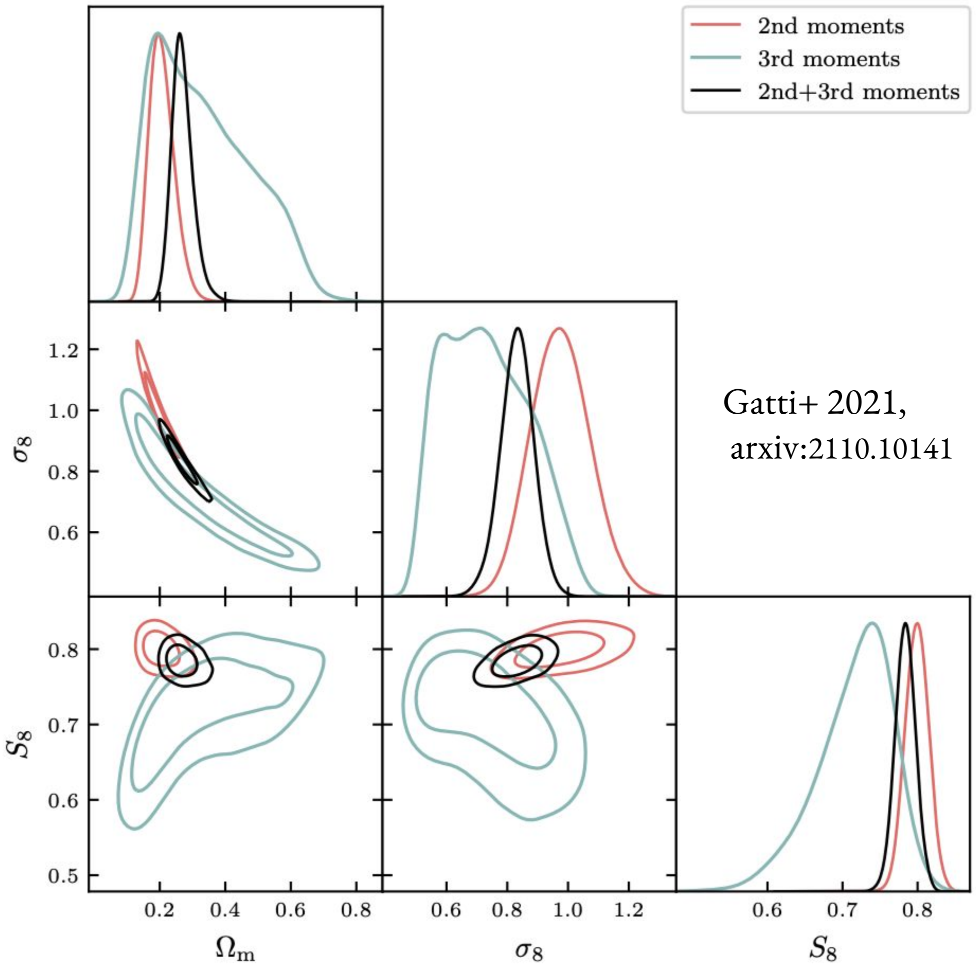
3rd moments probe additional non Gaussian information & break σ_8 -Ω_m degeneracy

3rd moments is partially independent of second -> different impact of systematics.

3rd+2nd moments improve constraints by 30% over 2nd moments only

$$\Omega_m = 0.27 \pm 0.03$$
$$\sigma_8 = 0.83 \pm 0.05$$
$$S_8 = 0.784 \pm 0.013$$

Most stringent constraints on S8 from a WL analysis to date!



Gatti+ 2021, arxiv:2110.10141

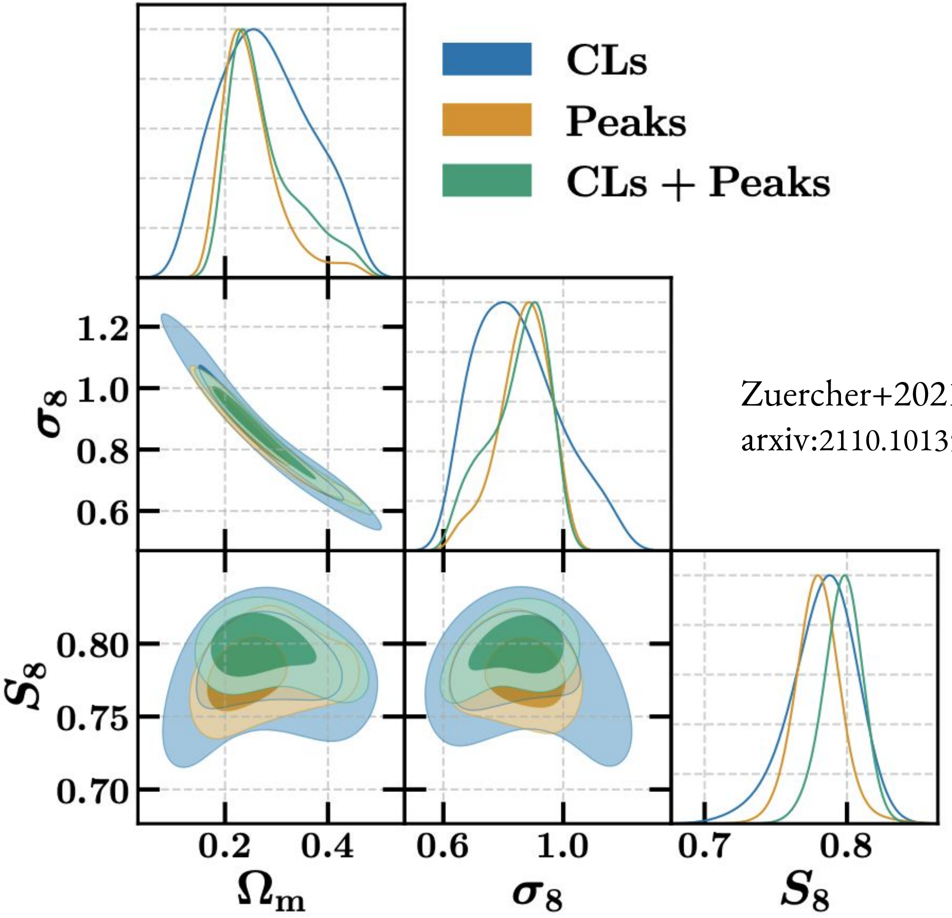
Cosmology from DES Y3 Power Spectra+ Peaks

Peaks probe additional non Gaussian information & break σ_8 -Ω_m degeneracy

Peaks+Power Spectra(CL) improve constraints by 40% over Power Spectra only

$$\Omega_m = 0.276^{+0.034}_{-0.086}$$
$$\sigma_8 = 0.850^{+0.13}_{-0.068}$$
$$S_8 = 0.797^{+0.015}_{-0.013}$$

Similar constraining power on S8 of the moments analysis

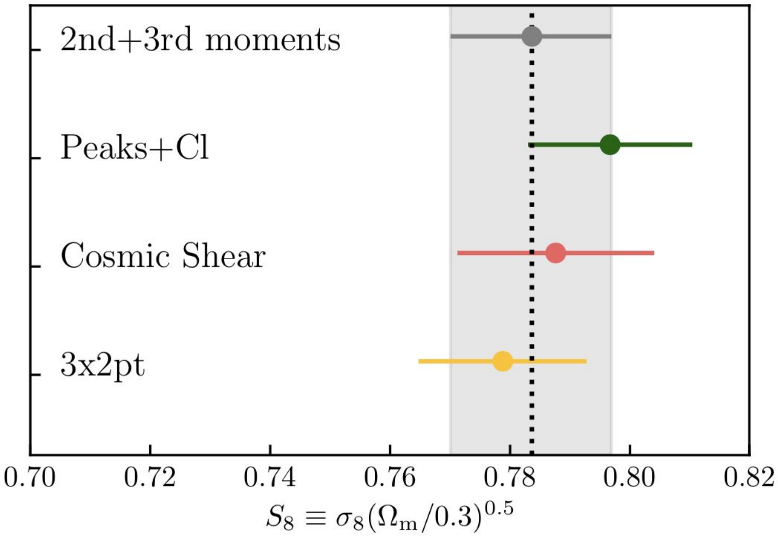


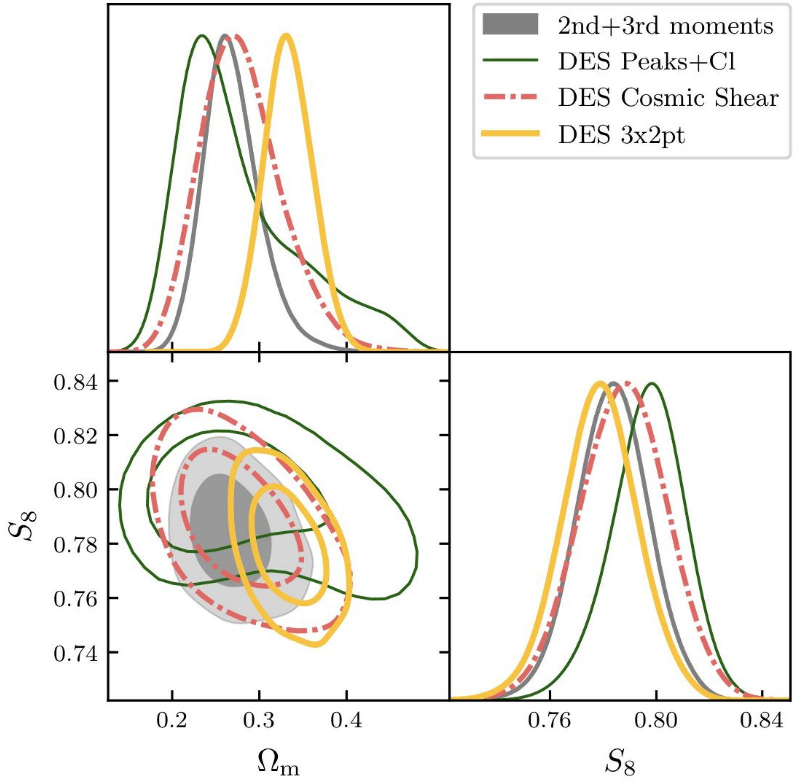
Zuercher+2021, arxiv:2110.10135

Are moments & peaks consistent with DES 3x2 results?

The moments, peaks and DES 3x2 analyses
use 3 different pipelines

results are consistent!

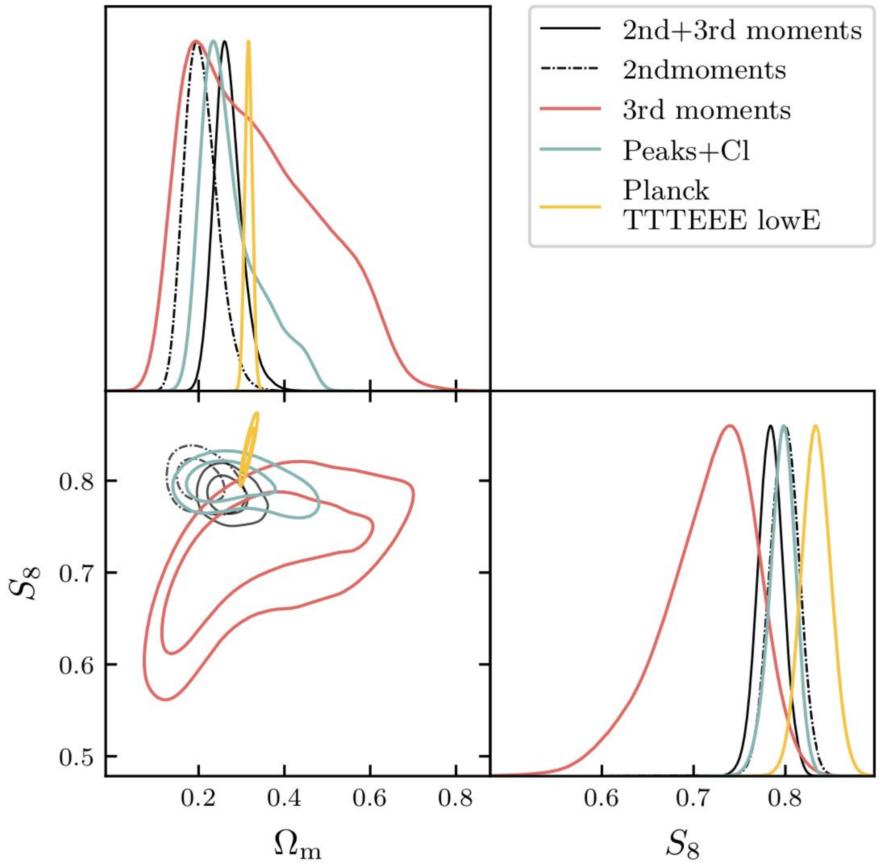




note: modelling & analysis choices are very
similar among analyses but *not identical*

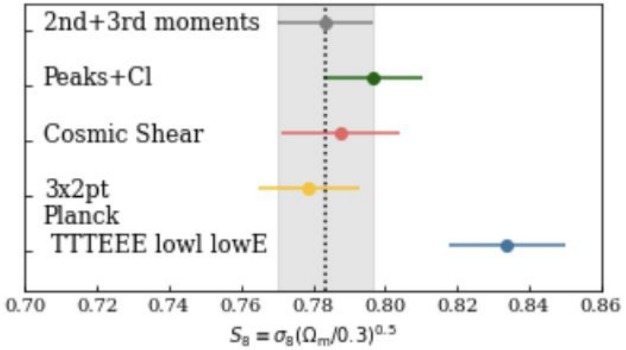
DES Y3 Cosmic Shear, Amon+21,Secco&Samuroff+21
DES Y3 3x2, DES collaboration (2021)

Are moments & peaks consistent with Planck?



They are consistent (<3σ), although note that 3rd moments alone shows a 2.8 tension

	Planck **TTTEEE lowl lowE**
2nd moments	2.7σ
3rd moments	2.8σ
2nd+3rd moments	2.2σ
Peaks+Cl	1.5σ



Outline

Main goal: stress-test the standard cosmological model with new methods!

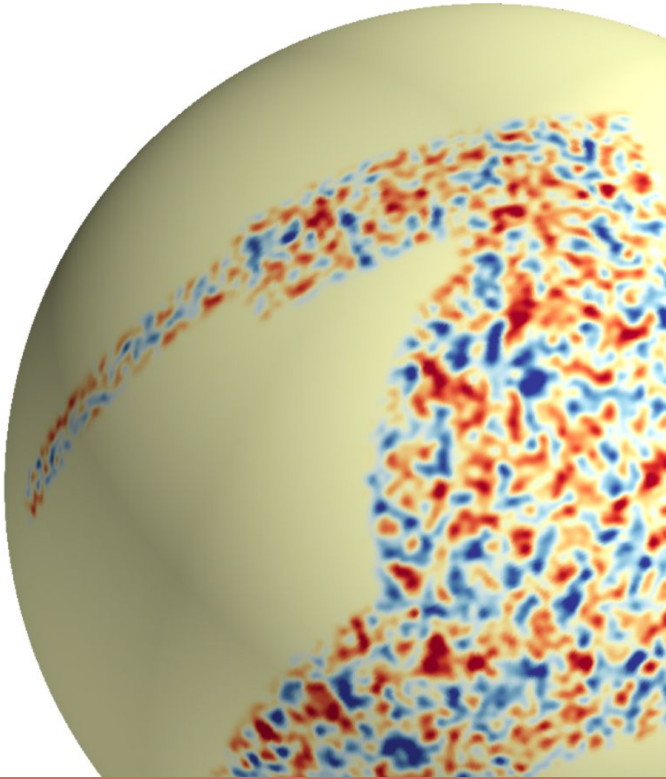
1) Toolbox essential: weak lensing mass maps

2) Why non Gaussian statistics?

3) **Results from non Gaussian statistics with DES Y3**

4) New promising probes: wavelet based estimators

5) Future obstacles & analyses robustness



Takeaways:

- Best constraints on S8 from a WL analysis to date
- Results compatible with other DES constraints
- ~2 sigma tension with Planck

Outline

Main goal: stress-test the standard cosmological model with new methods!

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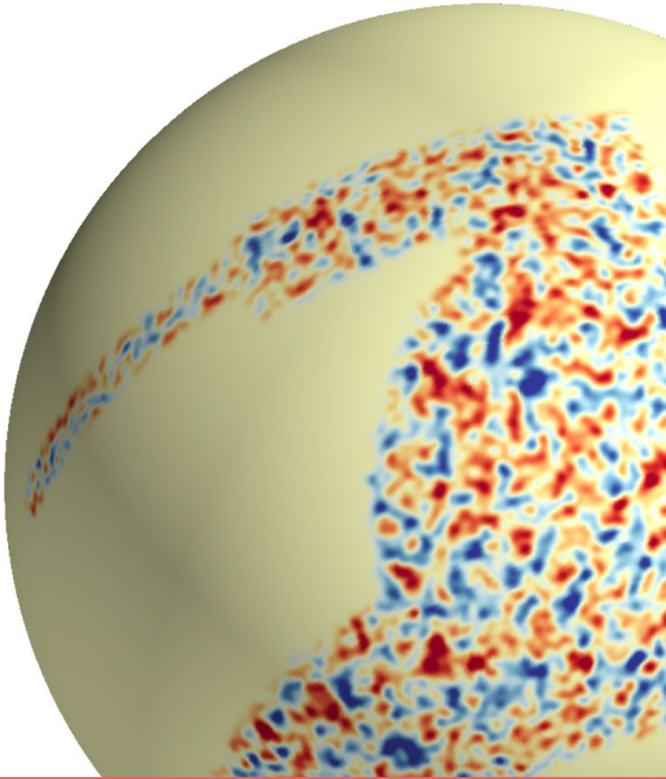
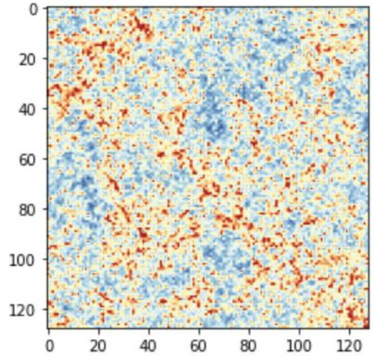


Wavelet-based Non Gaussian estimators

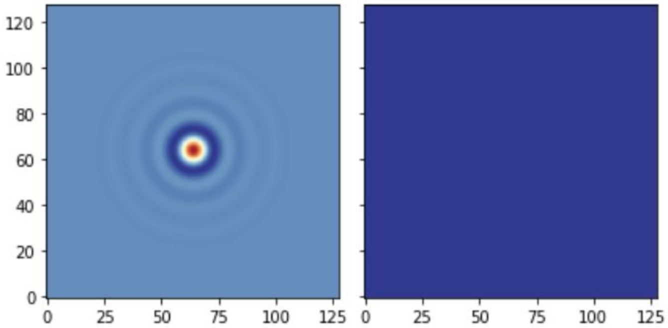
E.g.: Isotropic Wavelets (Jeffrey in prep.), Wavelet Phase Harmonics (Allys 2021), Scattering Transform (Cheng 2021)

Wavelet-based Non Gaussian estimators

2D convergence map



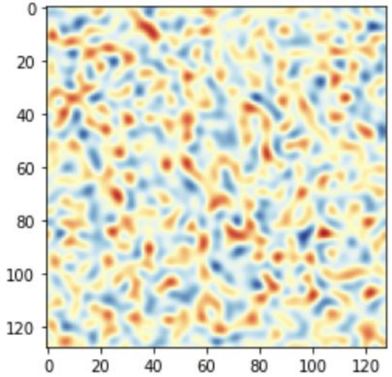
Isotropic wavelet filter



Real part

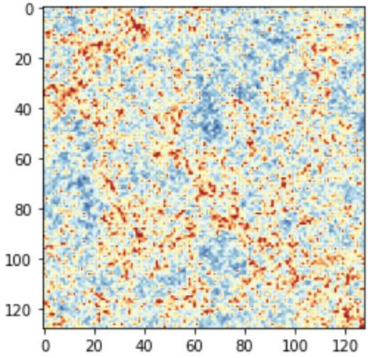
Imaginary part

smoothed convergence map

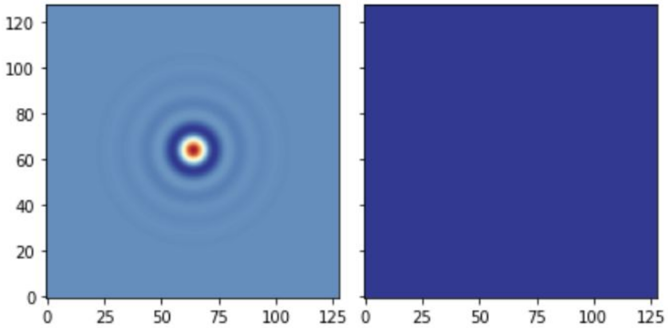


Wavelet-based Non Gaussian estimators

2D convergence map



Isotropic wavelet filter



Real part

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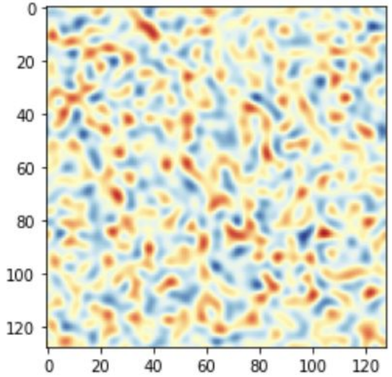
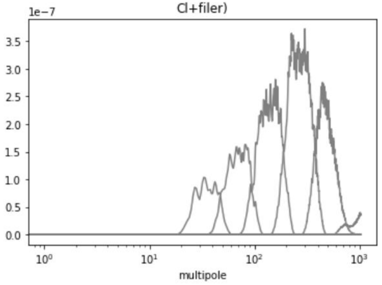
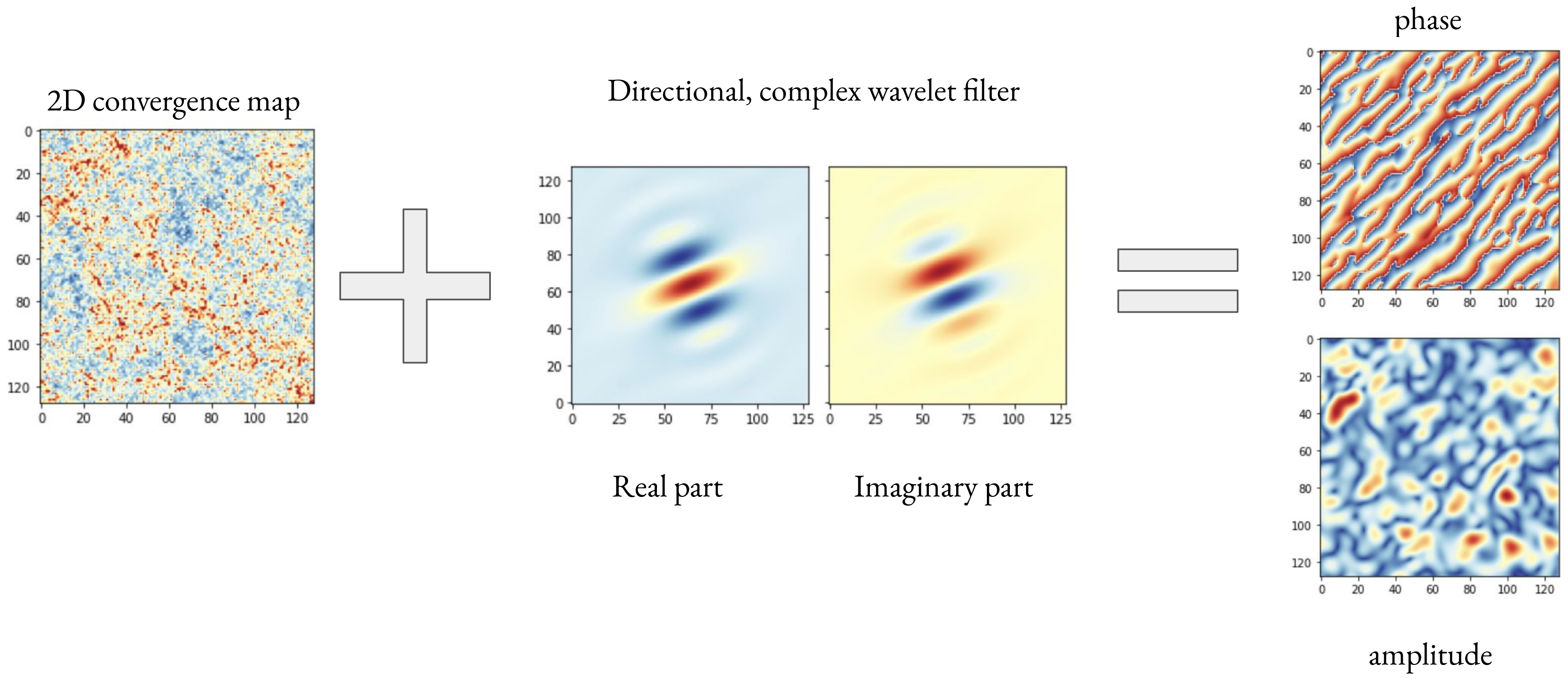




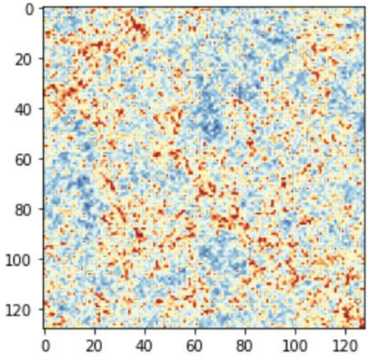
Smoothed maps well localised in real space and Fourier space

Wavelet-based Non Gaussian estimators

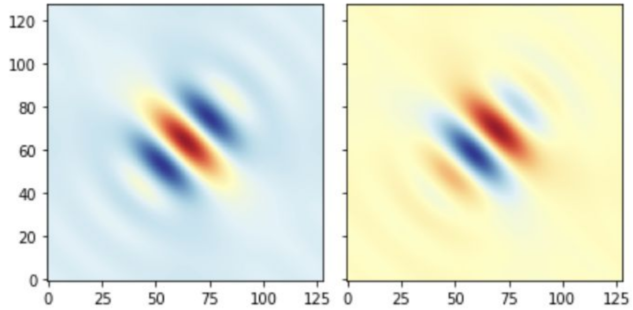


Wavelet-based Non Gaussian estimators

2D convergence map



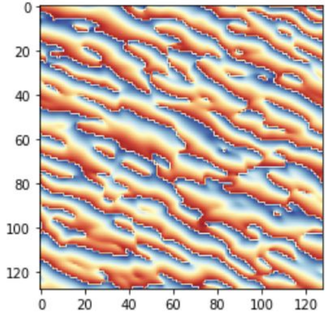
Directional, complex wavelet filter

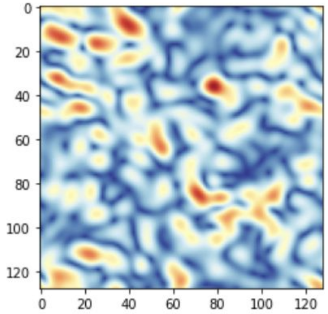


Real part

Imaginary part

phase

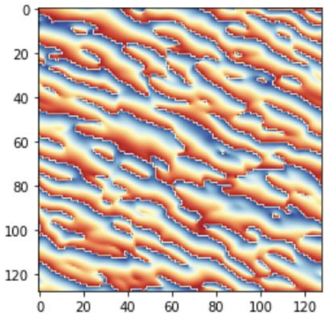


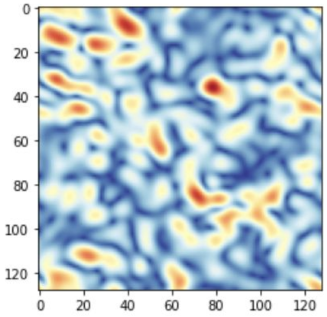


amplitude

Wavelet Phase Harmonics

phase



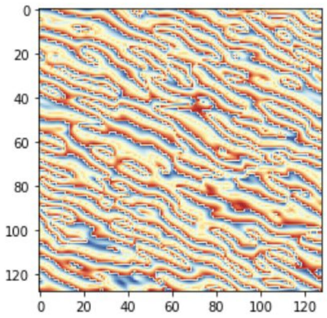


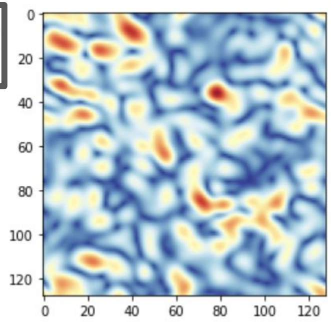
amplitude

$$\mathrm{PH}(\kappa, p) \equiv |\kappa| \exp^{i\, p \arg(\kappa)}$$

Accelerates phases, but leaves amplitude unaltered

phase





amplitude

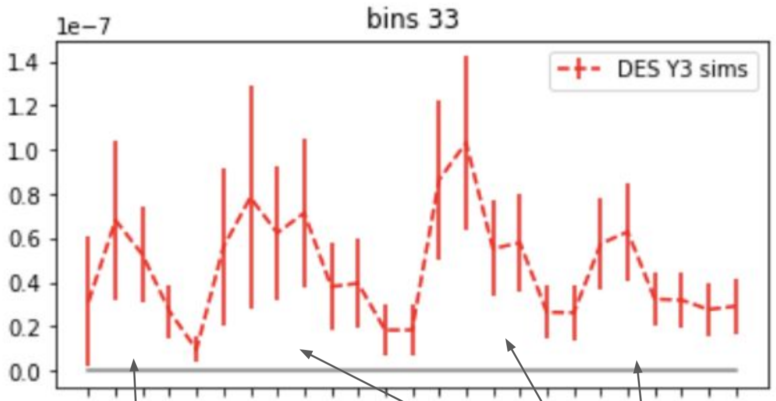
Second moments of the smoothed & accelerated maps:

$$\langle \mathrm{PH}(\kappa_{j_1,\ell_1}, p = p_1), \mathrm{PH}(\kappa_{j_2,\ell_2}, p = p_2) \rangle,$$

- + Probe couplings betweens scales & non Gaussian features of the fields
- + More robust against noise outliers

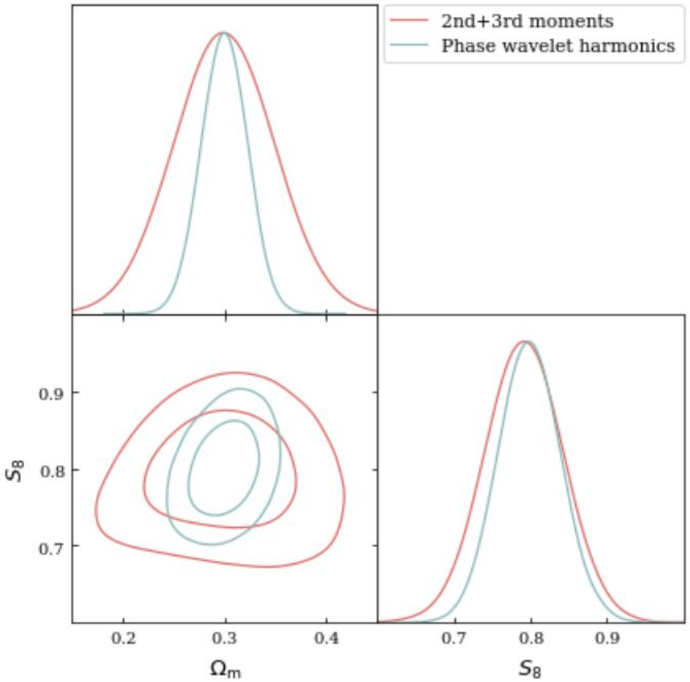
Wavelet Phase Harmonics

$$\langle \mathrm{PH}(\kappa_{j_1,\ell_1}, p = p_1), \mathrm{PH}(\kappa_{j_2,\ell_2}, p = p_2) \rangle,$$



p1= 0, p2=1,
same filter scale (j1=j2)

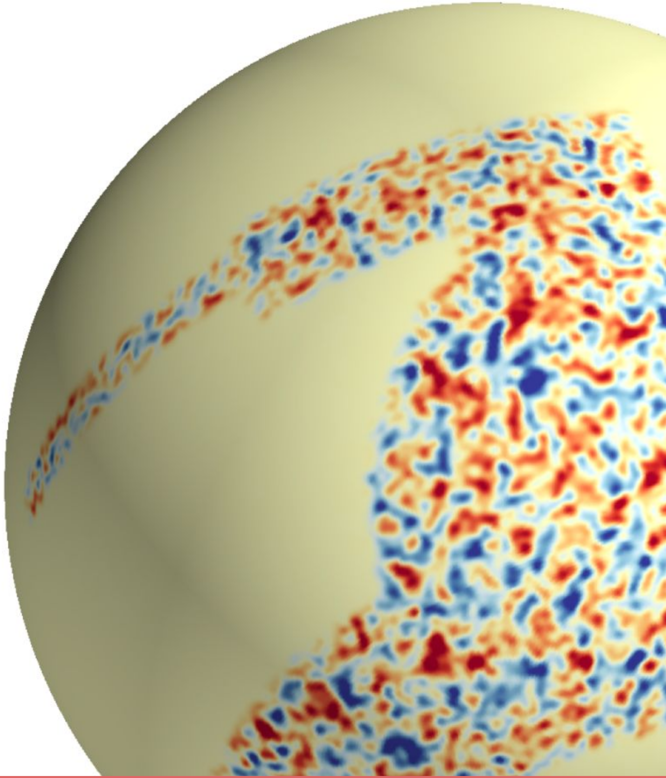
p1= 0, p2=1, different filter scales (j1 !=j2)



PRELIMINARY (Gatti et al in prep.)

Outline

Main goal: stress-test the standard cosmological model with new methods!



Takeaways:

- WBEs are CNN without training
- WBEs isolate better scales (= easier handle on systematics)
- More robust against noise outliers
- Very constraining!

Outline

Main goal: stress-test the standard cosmological model with new methods!

1) Toolbox essential: weak lensing mass maps
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3) Results from non Gaussian statistics with DES Y3
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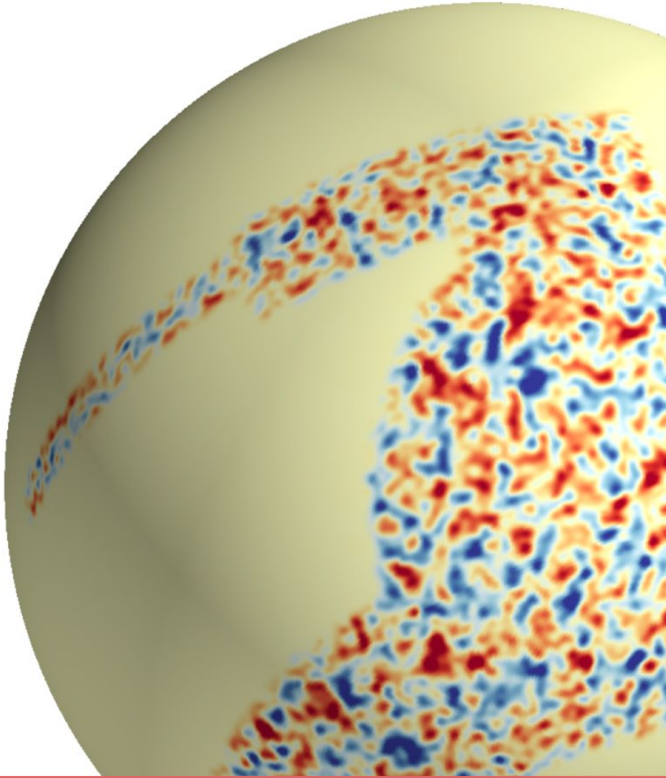
5) **Future obstacles & analyses robustness**



Exciting future perspectives:

- Larger datasets with stage IV surveys = more constraining power!
- Larger parameter space: neutrinos, wCDM, modified gravity, baryonic feedback
- Non Gaussian statistics of LSS combined probes (galaxies, CMB secondary anisotropies, etc).

Obstacles:

- Computational needs.
  - In the forward modelling approach, the number of simulations needed to explore a larger parameter space increases exponentially. We need faster simulations & approximate methods to sample the posterior, and more efficient ways to include baryonic physics.

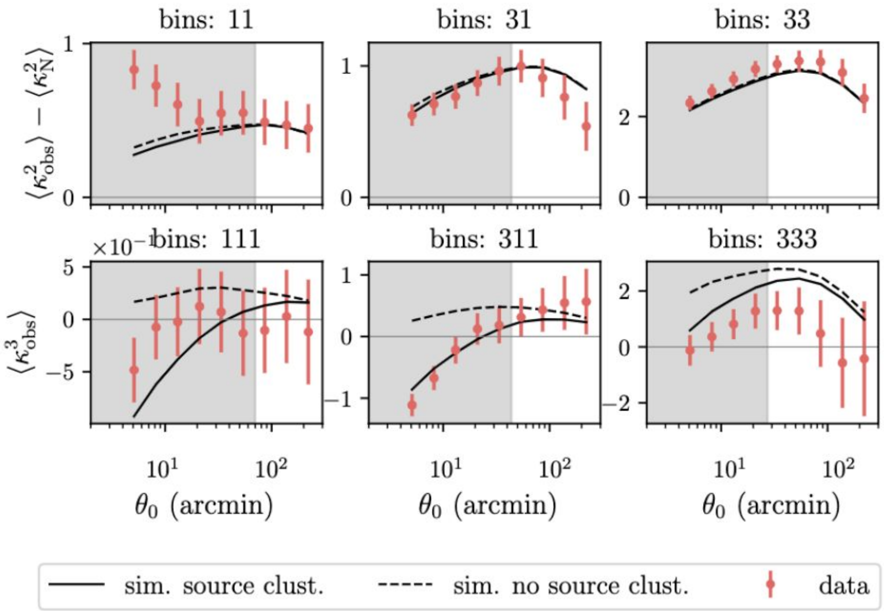
- Better control over systematics.
  - Blind simulated challenges!

More about systematics

Many of the effects/systematics we thought are negligible for Gaussian statistics might not be negligible for non Gaussian statistics. We cannot rely on our ‘Gaussian experience’

More about systematics

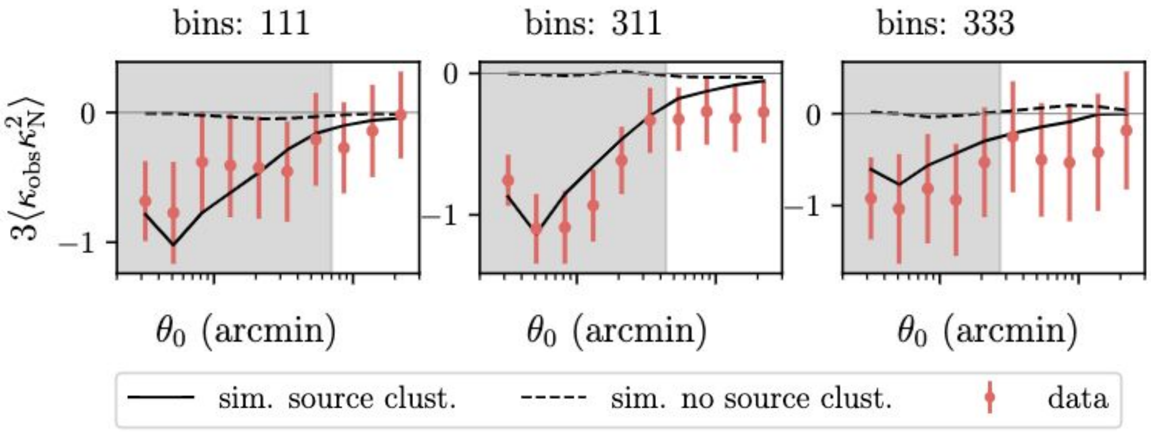
Source clustering: we preferentially sample the shear field in overdense location.
It has a much larger effect on map-based non Gaussian statistics compared to Gaussian statistics



Gatti et al., in prep.

More about systematics

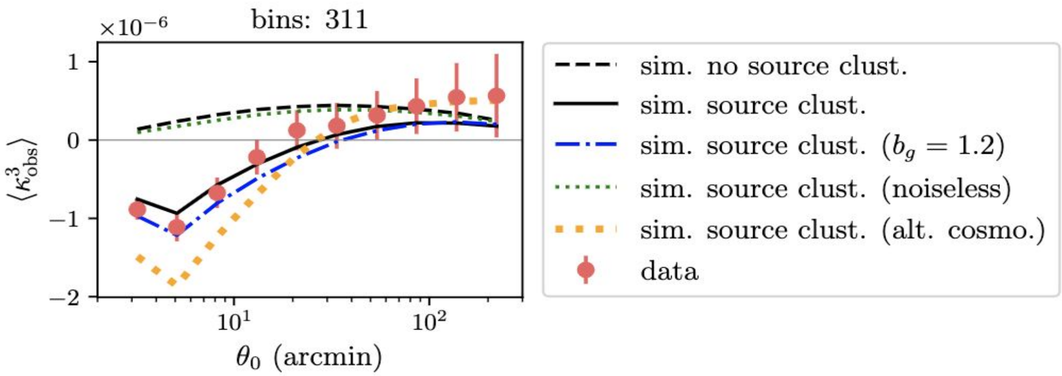
Source clustering introduces a *spurious correlation between pixel noise and shear signal;*



Gatti et al., in prep.

More about systematics

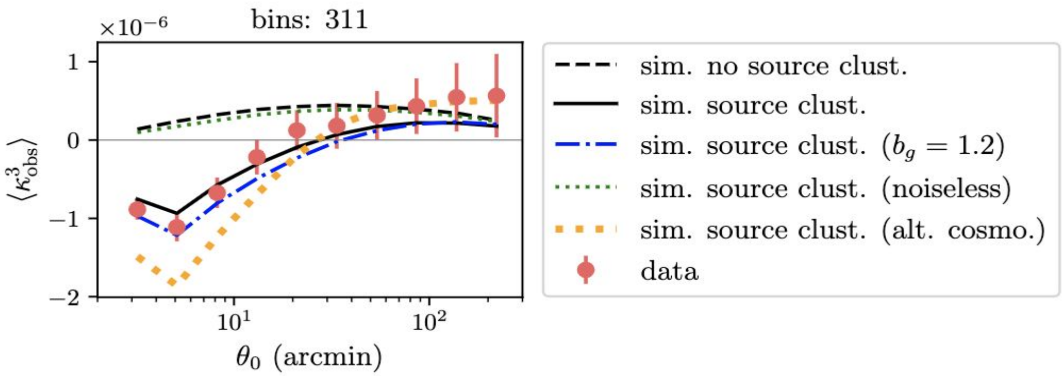
Source clustering depends on cosmology & on the galaxy-matter bias of the source sample



Gatti et al., in prep.

More about systematics

Source clustering: we know how to incorporate it into simulations!

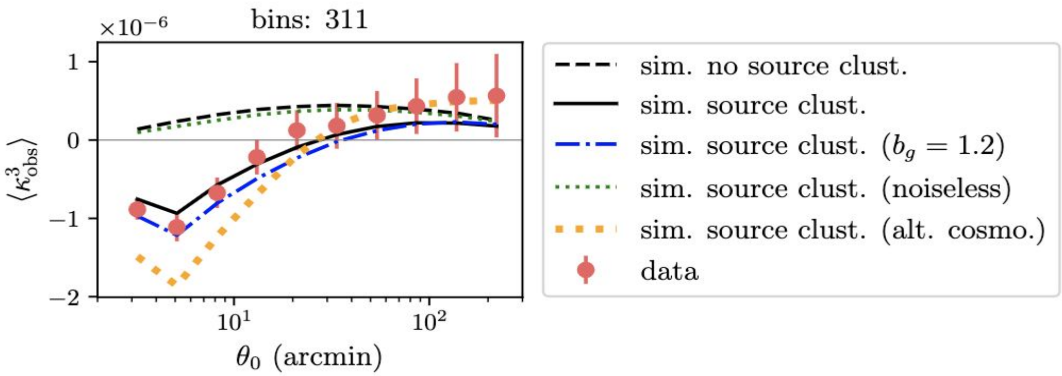


Gatti et al., in prep.

More about systematics

Source clustering - results.

- We found that the impact is larger for non Gaussian statistics compared to Gaussian statistics.
- It has been overlooked so far.
- Cutting scale is (for now!) a sufficient mitigation strategy for the analyses I presented.
- Every non Gaussian analysis has to test the impact of this effect.



Gatti et al., in prep.

Intro to a public challenge for WL non Gaussian statistics

Marco Gatti, Bhuv Jain with Elisabeth Krause, Francois Lanusse, and others – all welcome!

How to establish community trust in say deep learning applied to lensing data?

- By running the pipeline on mocks developed by a 'third party'
- By including systematics that are unknown, in both the model and the details (e.g. whether IA is NLA or TATT with unknown parameterization)

Intro to a public challenge for WL non Gaussian statistics

Goal: groups aim at recovering input cosmology from maps provided by a third party

Learning phase 'A'

No blinding, no systematics

Learning phase 'B'

No blinding, with known systematics

Challenge phase

With blinding, simulated data with unknown systematics

Timeline

The challenge will be divided into phases:

- Learning phase

  A) convergence maps/catalogs are provided, along with input cosmology. No systematics included.

  B) convergence maps/catalogs are provided, along with input cosmology. Systematics are included one at a time and fully described.

- Challenge phase: convergence maps/catalogs are provided, with multiple unknown systematics and blinded cosmology.

https://github.com/mgatti29/ML_challenge_cosmology

Outline

Main goal: stress-test the standard cosmological model with new methods!

1) Toolbox essential: weak lensing mass maps

2) Why non Gaussian statistics?

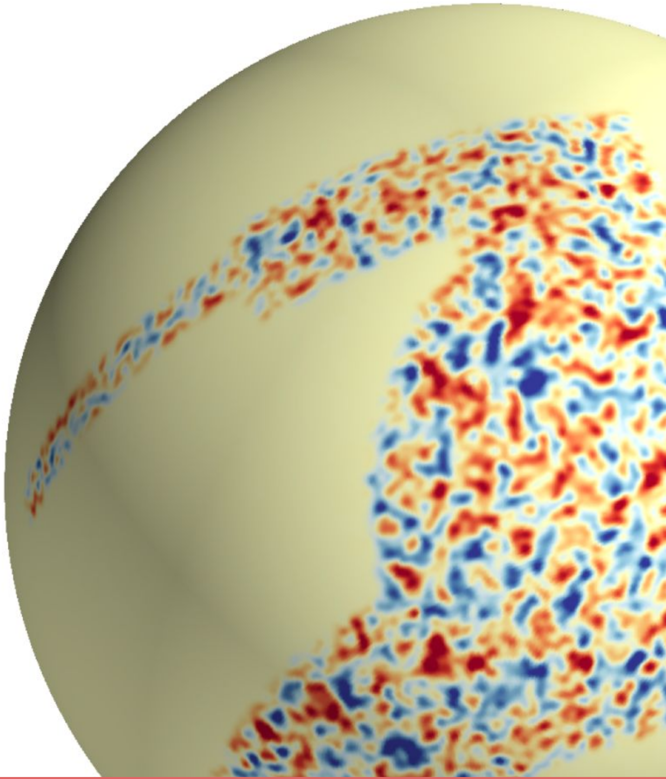
3) Results from non Gaussian statistics with DES Y3

4) New promising probes: wavelet based estimators

5) **Future obstacles & analyses robustness**

Takeaways:

- Problem: scalability / computing resources
- Systematics affect non Gaussian stats. differently
- Blind challenges can establish community trust



Summary

1) Toolbox essential: weak lensing mass maps

2) Why non Gaussian statistics?

3) Results from non Gaussian statistics with DES Y3

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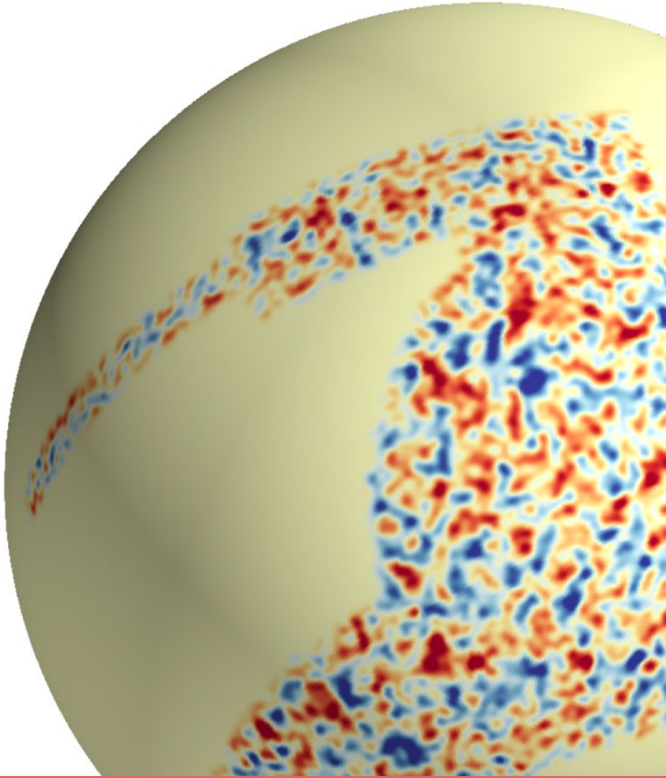


Main goal: stress-test the standard cosmological model with new methods. Non Gaussian statistics are a great tool to achieve this!

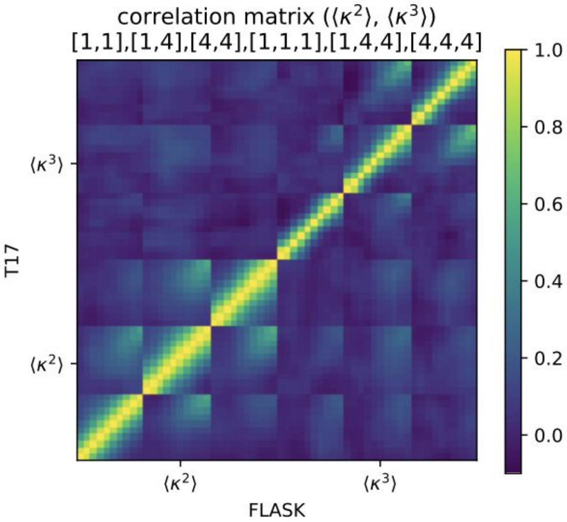
Summary

- There's a growing interest in non Gaussian analyses of WL data - great benefits: improved constraints & robustness checks against systematics!

- Results from 2 independent analyses using high order statistics and DES Y3 data (peaks & moments). Consistent results with other DES analyses, <3 sigma tension with Planck.

- More non Gaussian analyses very soon with DES! LFI peaks, wavelet-based moments, deep learning, etc.

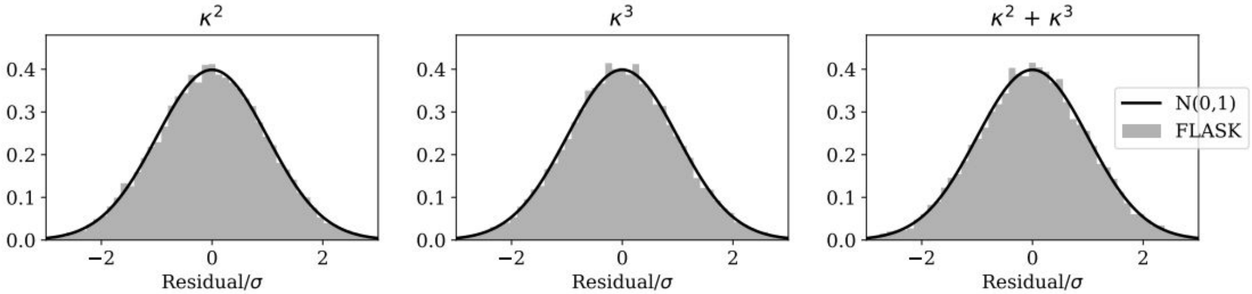
Covariance, likelihood & data compression



Covariance matrix - it's usually estimated from mocks. To avoid biases, # mocks >> length data vector!

Data vector compression - it reduces the dimensionality of the DV, and 'Gaussianizes' the likelihood.

$$d_i^{\mathrm{compr}} = \langle d \rangle_{,i}^{T} \hat{C}^{-1} d \equiv b_i d,$$



$$S01^{i,k}_{j} \equiv \frac{1}{N_{\rm tot}} \sum_{\rm pix}^{N_{\rm tot}} \sum_{\ell}^{L} \mathrm{PH}(\kappa^{i}_{j,\ell}, 0)\mathrm{PH}(\kappa^{k}_{j,\ell}, 1) =$$

coupling between spatial frequencies
within a single wavelet band

$$\frac{1}{N_{\rm tot}} \sum_{\rm pix}^{N_{\rm tot}} \sum_{\ell}^{L} |\kappa^{i}_{j,\ell}| \kappa^{k}_{j,\ell} \qquad (11)$$

$$C01\delta\ell0^{i,k}_{j1,j2\neq j1} \equiv \frac{1}{N_{\rm tot}} \sum_{\rm pix}^{N_{\rm tot}} \sum_{\ell}^{L} \mathrm{PH}(\kappa^{i}_{j1,\ell}, 0)\mathrm{PH}(\kappa^{k}_{j2,\ell}, 1) =$$

$$\frac{1}{N_{\rm tot}} \sum_{\rm pix}^{N_{\rm tot}} \sum_{\ell}^{L} |\kappa^{i}_{j1,\ell}| \kappa^{k}_{j2,\ell} \qquad (12)$$

$$C01\delta\ell1^{i,k}_{j1,j2\neq j1} \equiv \frac{1}{N_{\rm tot}} \sum_{\rm pix}^{N_{\rm tot}} \sum_{\ell}^{L} \mathrm{PH}(\kappa^{i}_{j1,\ell}, 0)\mathrm{PH}(\kappa^{k}_{j2,\ell+1}, 1) =$$

$$\frac{1}{N_{\rm tot}} \sum_{\rm pix}^{N_{\rm tot}} \sum_{\ell}^{L} |\kappa^{i}_{j1,\ell+1}| \kappa^{k}_{j2,\ell} \qquad (13)$$

$$C\mathrm{phase}^{i,k}_{j1,j2\geq j1} \equiv \frac{1}{N_{\rm tot}} \sum_{\rm pix}^{N_{\rm tot}} \sum_{\ell}^{L} \mathrm{PH}(\kappa^{i}_{j1,\ell+1}, 1)\mathrm{PH}(\kappa^{k}_{j2,\ell}, 2^{j2-j1}) =$$

coupling between spatial
frequencies *and*
different wavelet bands

$$\frac{1}{N_{\rm tot}} \sum_{\rm pix}^{N_{\rm tot}} \sum_{\ell}^{L} \kappa^{i}_{j1,\ell+1} \mathrm{PH}(\kappa^{k}_{j2,\ell}, 2^{j2-j1}) \qquad (14)$$

$$S00^{i,k}_{j} \equiv \frac{1}{N_{\rm tot}} \sum_{\rm pix}^{N_{\rm tot}} \sum_{\ell}^{L} \mathrm{PH}(\kappa^{i}_{j,\ell}, 0)\mathrm{PH}(\kappa^{k}_{j,\ell}, 0) =$$

L1 - sparsity of the field

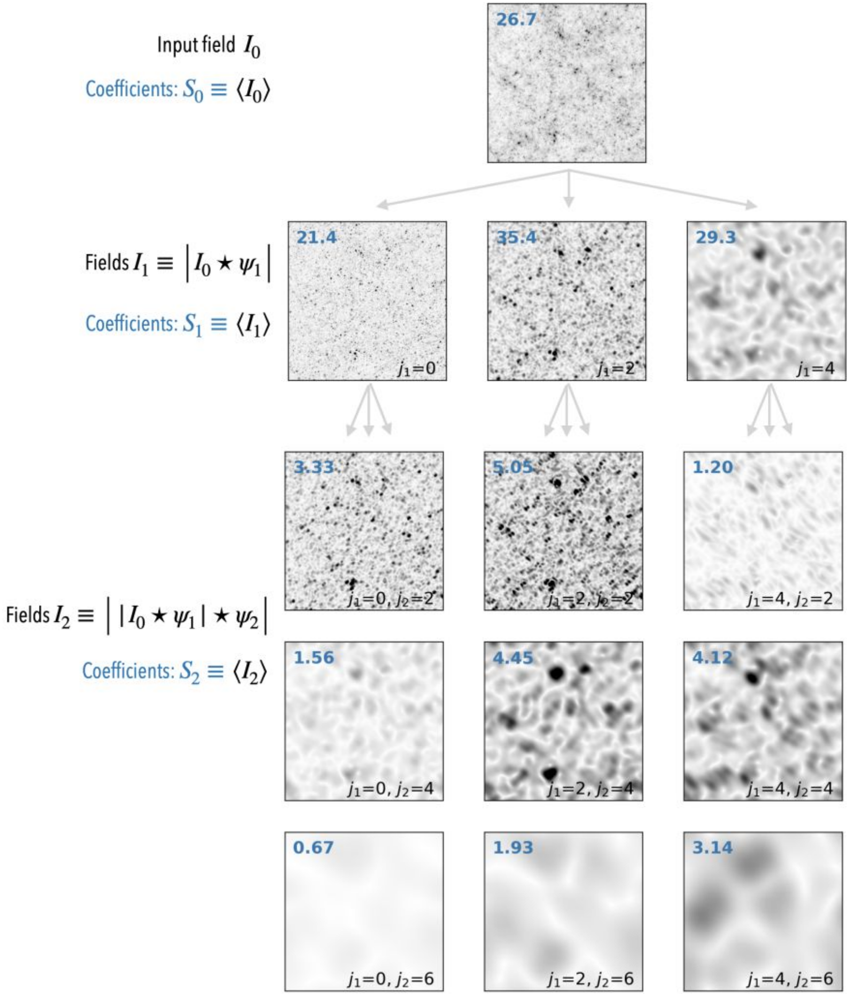
$$\frac{1}{N_{\rm tot}} \sum_{\rm pix}^{N_{\rm tot}} \sum_{\ell}^{L} |\kappa^{i}_{j,\ell}||\kappa^{k}_{j,\ell}|$$

$$S11^{i,k}_{j} \equiv \frac{1}{N_{\rm tot}} \sum_{\rm pix}^{N_{\rm tot}} \sum_{\ell}^{L} \mathrm{PH}(\kappa^{i}_{j,\ell}, 1)\mathrm{PH}(\kappa^{k}_{j,\ell}, 1) =$$

power spectrum

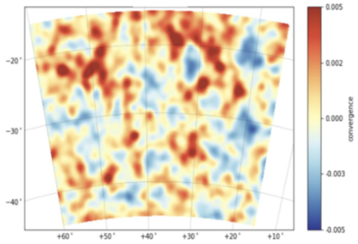
$$\frac{1}{N_{\rm tot}} \sum_{\rm pix}^{N_{\rm tot}} \sum_{\ell}^{L} \kappa^{i}_{j,\ell} \kappa^{k}_{j,\ell}$$

Scattering transform



(projected) WL mass map (or convergence)

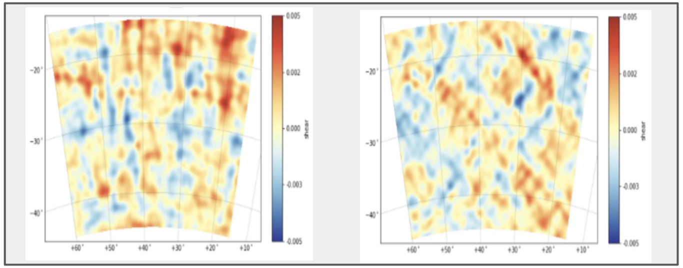
Not observable directly

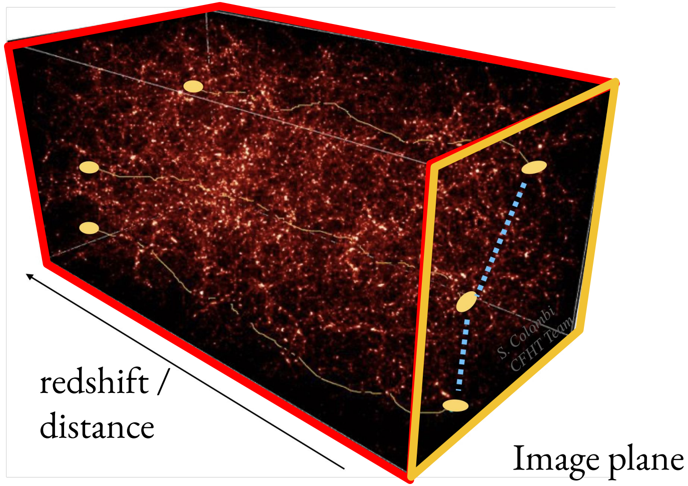


Mass Map reconstruction (e.g., Kaiser-Squires)

observable!

Using measured galaxies ellipticity, we can estimate the shear field (2 components)





Convergence	$\kappa = \frac{1}{2}\nabla^2\phi = \frac{1}{2}\left(\phi_{,11} + \phi_{,22}\right)$	**Mass**
Shear	$\gamma = \gamma_1 + i\gamma_2 = \frac{1}{2}\left(\phi_{,11} - \phi_{,22}\right) + i\phi_{,12}$	**Observable**

Analysis robust against different analysis choices