The Handwaver's Guide to Dark Matter Halos

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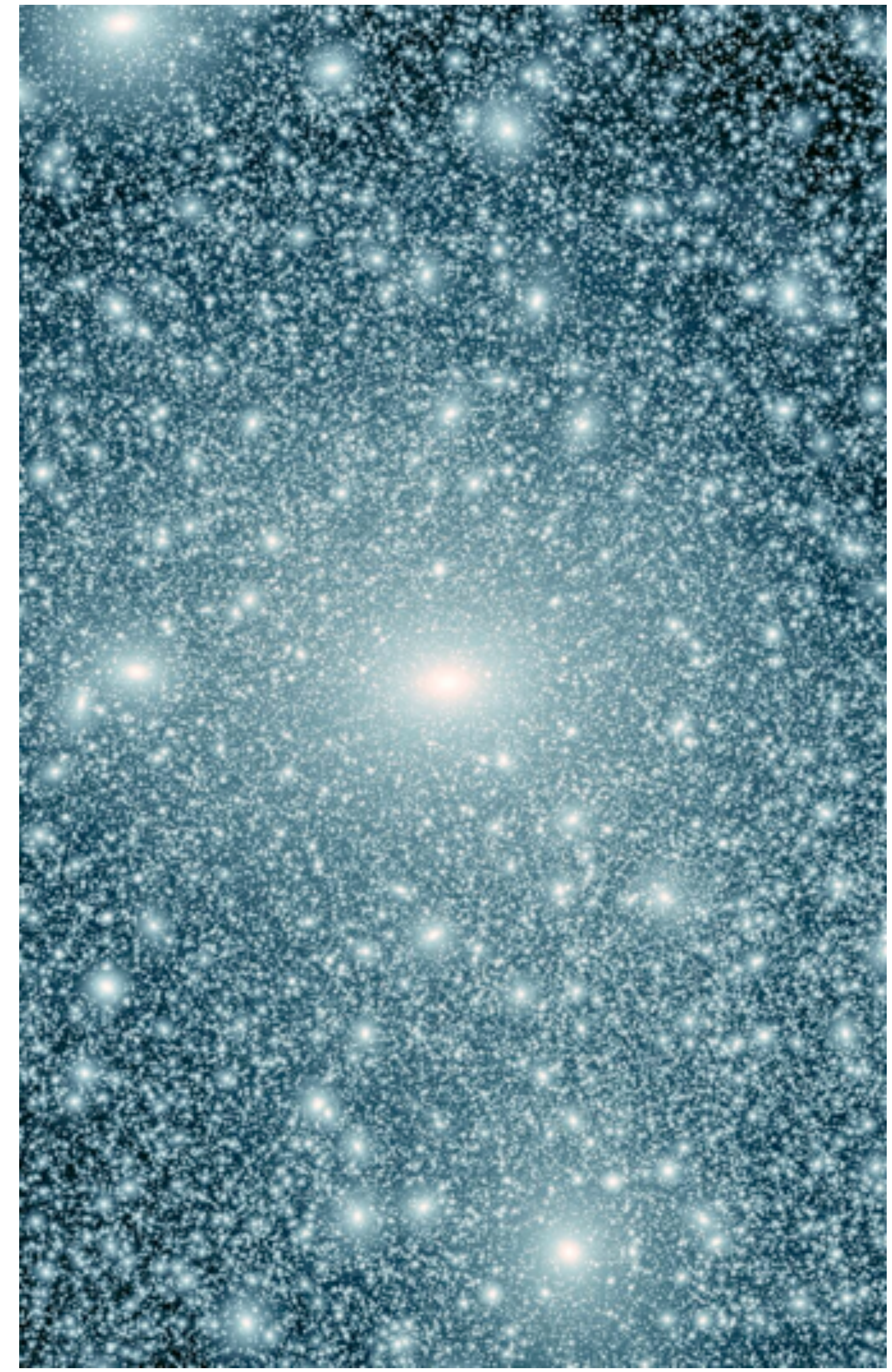
image from Millennium-II simulations
(Boylan-Kolchin et al. 2009)

Outline

- N-body simulations show regularity in halo properties:
  1. density profile
  2. abundance
  3. clustering

- I'll try to give a simple way to understand where these come from

- Then I'll discuss variations, e.g. what changes for cosmologies different than ΛCDM



GHalo (Stadel et al. 2009)

HALOS

halos are:

1. collapsed
2. self-bound
3. virialized

The basic building-blocks of large-scale structure:

- home to all galaxies, quasars, stars, etc.

Millennium-II Simulation
(Boylan-Kolchin et al. 2009)

Do we need a theory of halos?

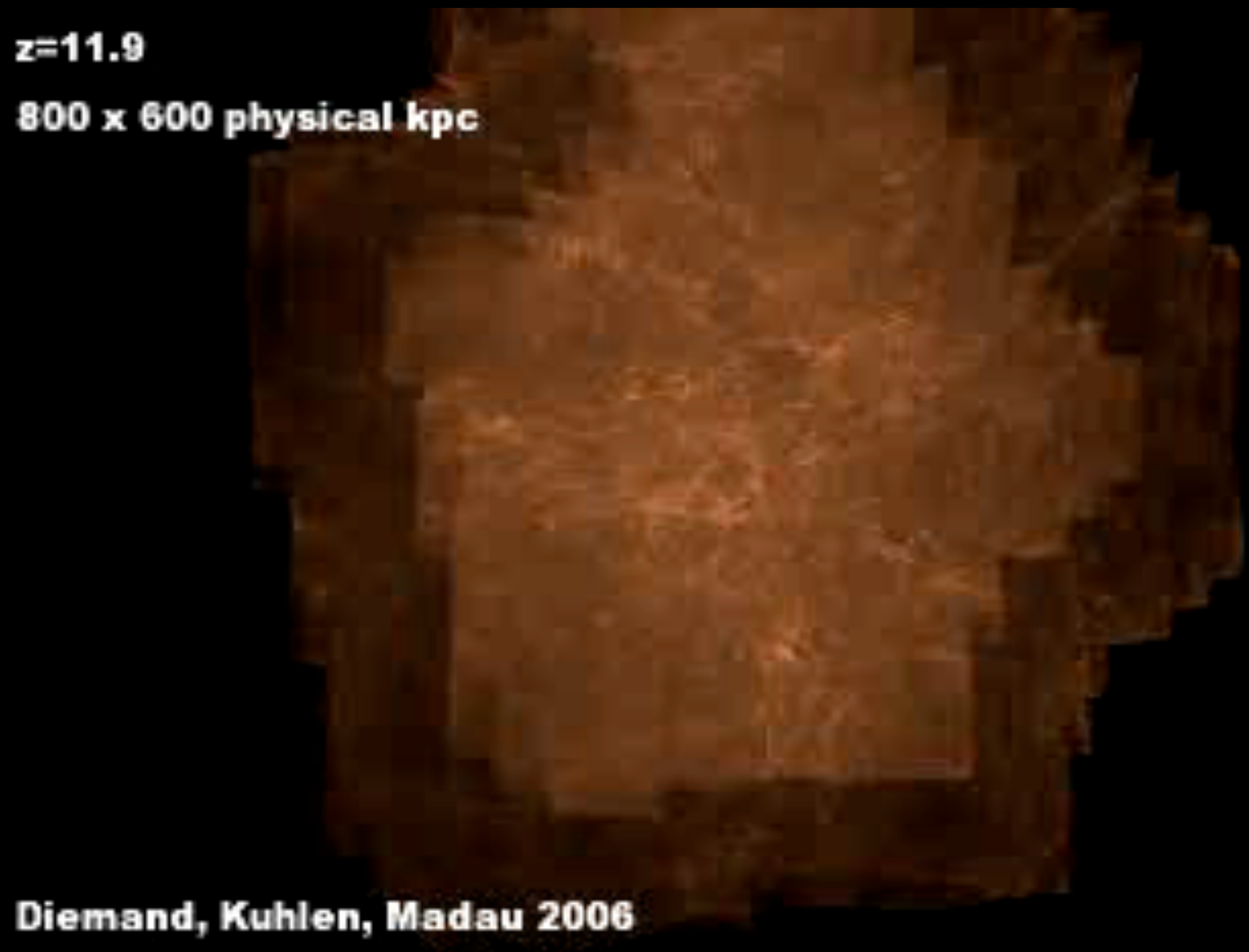
Halo properties are important for a huge range of topics in astrophysics & cosmology, e.g.

- sites of galaxy & star formation
- determines galaxy properties
- DM annihilation signal
- cluster abundance
- large-scale structure
- etc...

So we'd like to understand where halo properties come from, in some simple robust way.

see Lithwick & Dalal (2010), Dalal, Lithwick & Kuhlen (2010)

hierarchical structure formation is a mess (literally!)

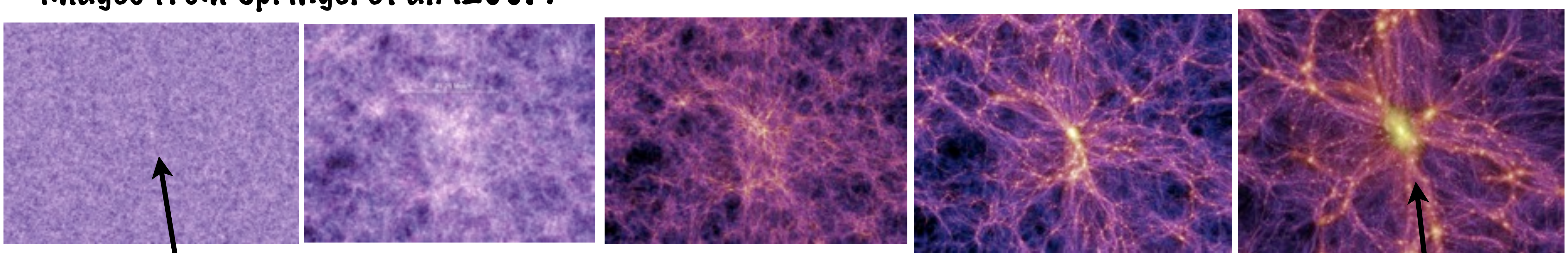


"Via Lactea"
Diemand et al. 2006

despite the mess, we can still understand important halo properties

building a theory of halos

images from Springel et al. (2007)



start with this

end with this

- Halos come from peaks of the initial (Gaussian random) density field, so: **properties of initial peaks ⇒ final halo properties**

- so we need to know:

   1. properties of initial peaks
   2. mapping from peaks → halos (i.e. collapse model)

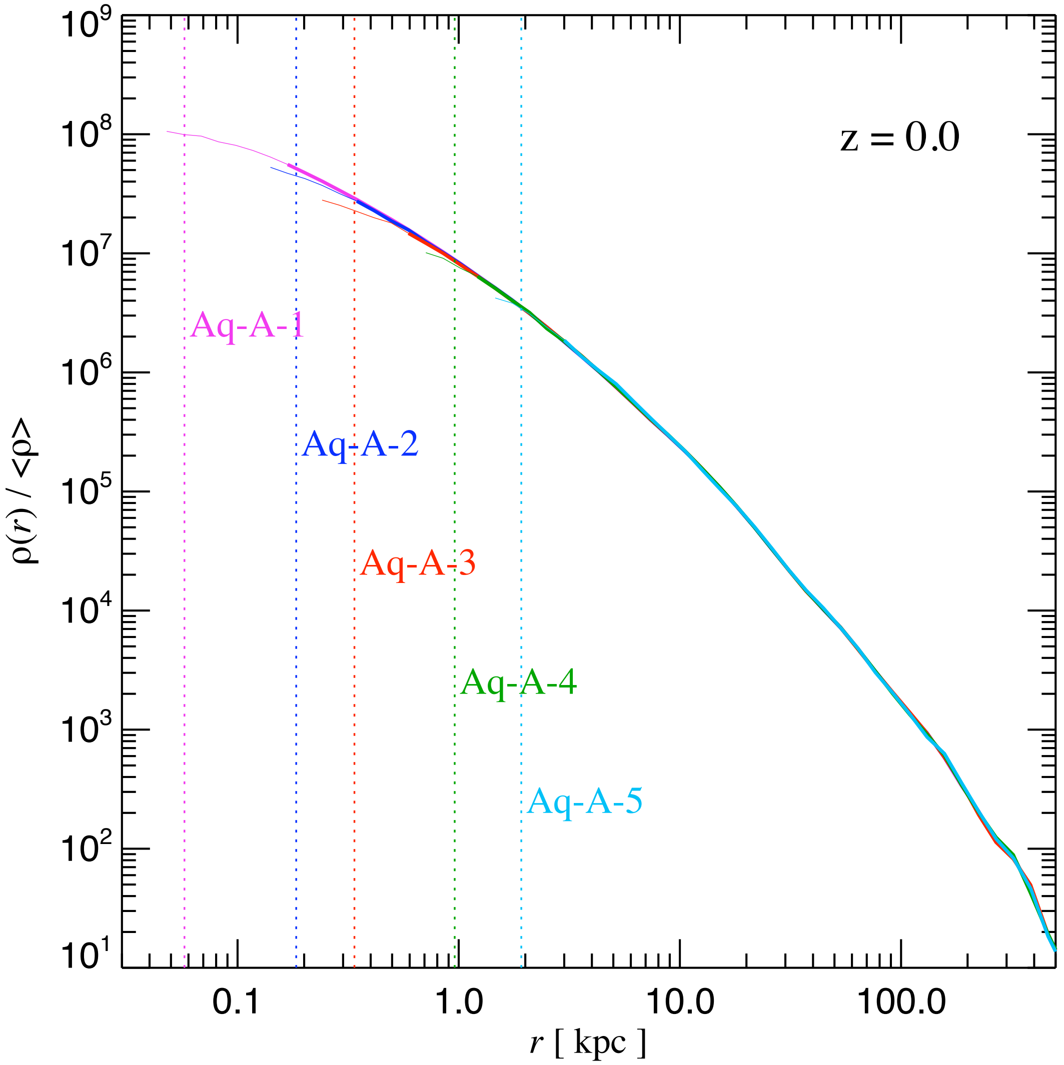
- with this framework, we can understand MANY aspects of halos...

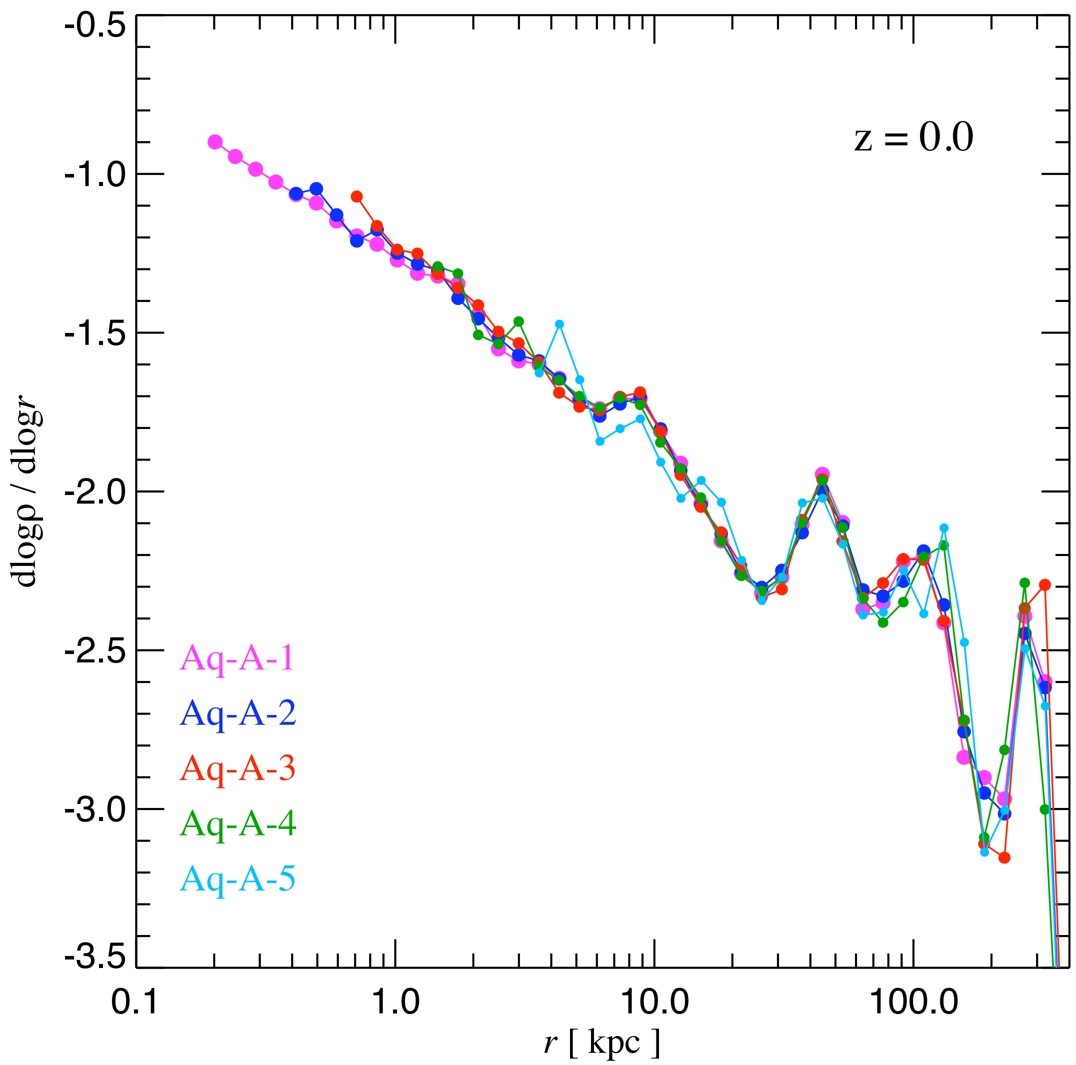
Halo properties

- density profiles
- statistics (abundance, clustering, etc.)

Halo Profile

Slope is steep at large radii, and becomes more shallow at small *r*. The rollover is very gradual, occurring over many decades in *r*.





"Aquarius" (Springel et al. 2008)

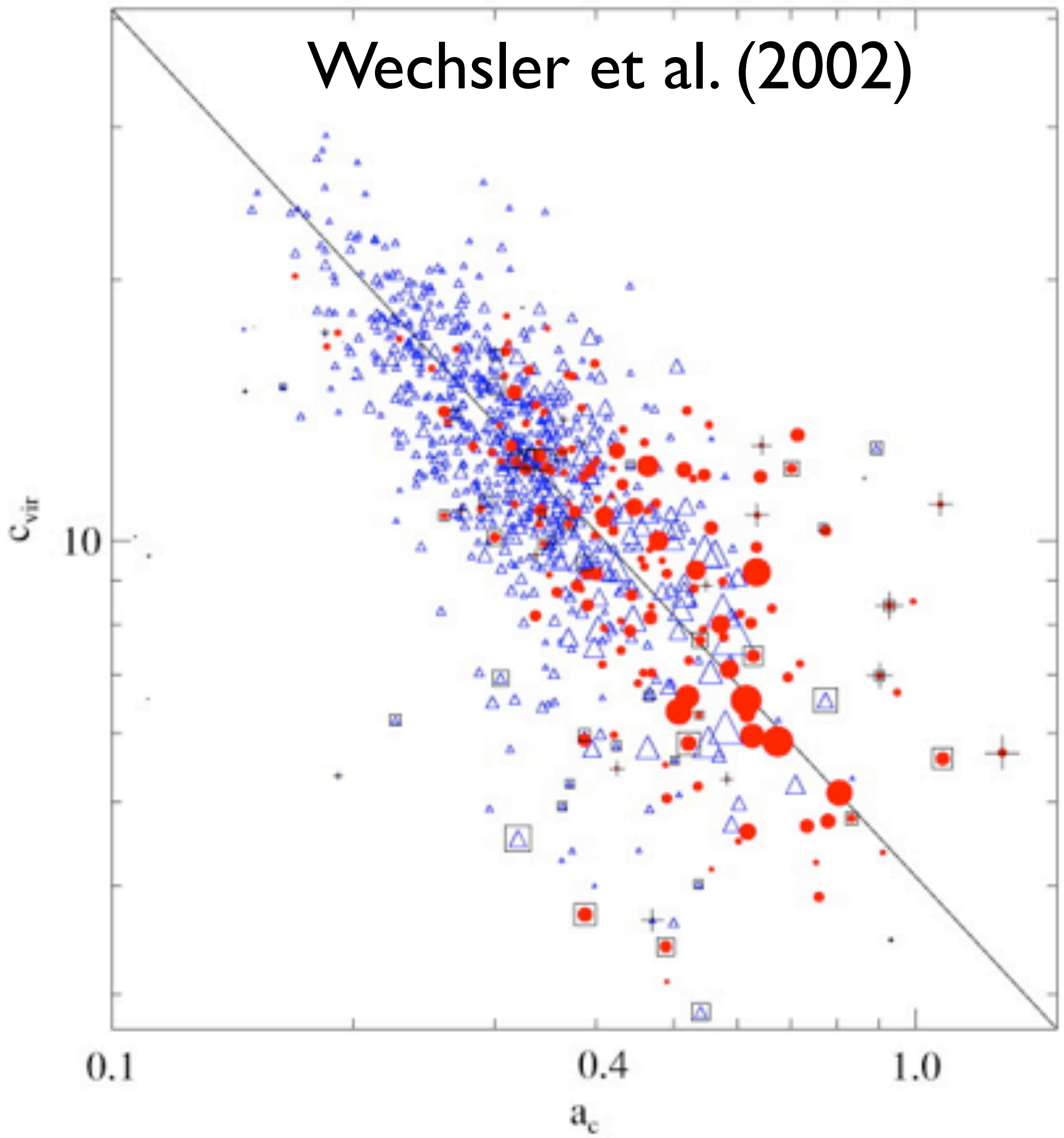
"Universal" NFW profile: the vast majority of simulated halos behave this way; exceptions tend to be recent mergers or bridged halos.

concentrations

$c_{\rm vir}=r_{\rm vir}/r_{-2}$ measures the extent of the outer, steep portion of the profile.

correlates with other parameters, in the sense that

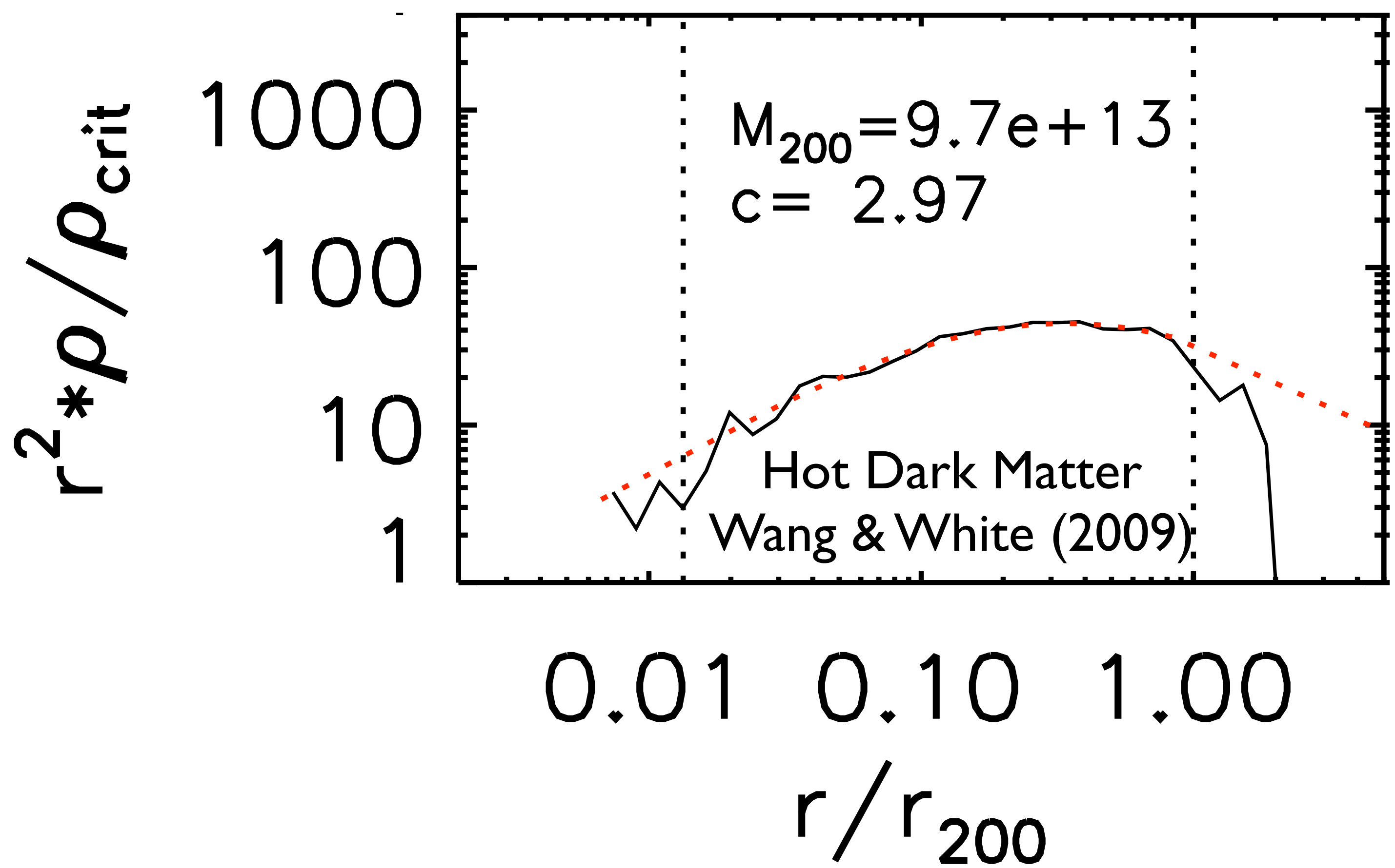
- old, low mass ⇒ high $c_{\rm vir}$
- young, high mass ⇒ low $c_{\rm vir}$



←old young→

Why?

- origin of this profile is a **longstanding** problem.
- lots of suggestions:
  - shape of power spectrum?
    (e.g. Nusser & Sheth 1999)
  - substructure?
    (e.g. Dekel et al. 2003)
  - isotropization of velocities
    (Lu et al. 2007)
  - statistical mechanics
    e.g. maximum entropy??



- **but you always get NFW!**

our approach

- NFW-like profiles occur in many different contexts
- the same underlying physics (likely) occurs in these different cases
- instead of studying this physics in the **messy** cosmological context...
- ... we'll focus on a **simple** case that we can easily understand.

Collapse model

we'll examine one particular example in great detail:

collapse of a scale-free, nonspherical profile $\delta\rho \propto r^{-\gamma} f(\theta,\phi)$

Lithwick & Dalal (2010)

Collapse model

we'll examine one particular example in great detail:

collapse of a scale-free, nonspherical profile $\delta\rho \propto r^{-\gamma} f(\theta,\phi)$

scale-free initial profile + scale-free gravity = self-similar solution

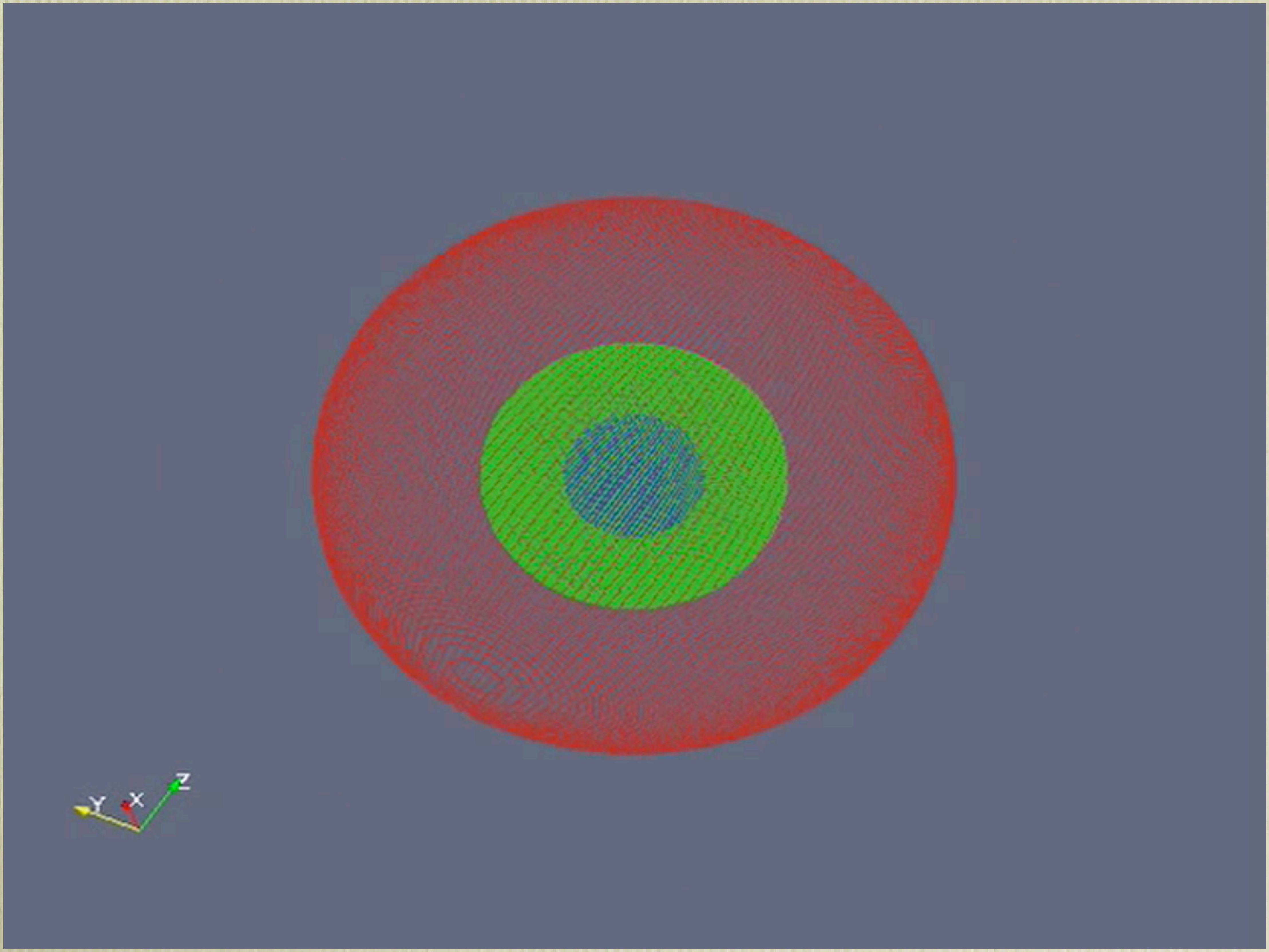
Compared to conventional N-body sim:

- much larger spatial dynamic range (typically $\gtrsim 10^{10}$)
- much MUCH faster run-times

Lithwick & Dalal (2010)

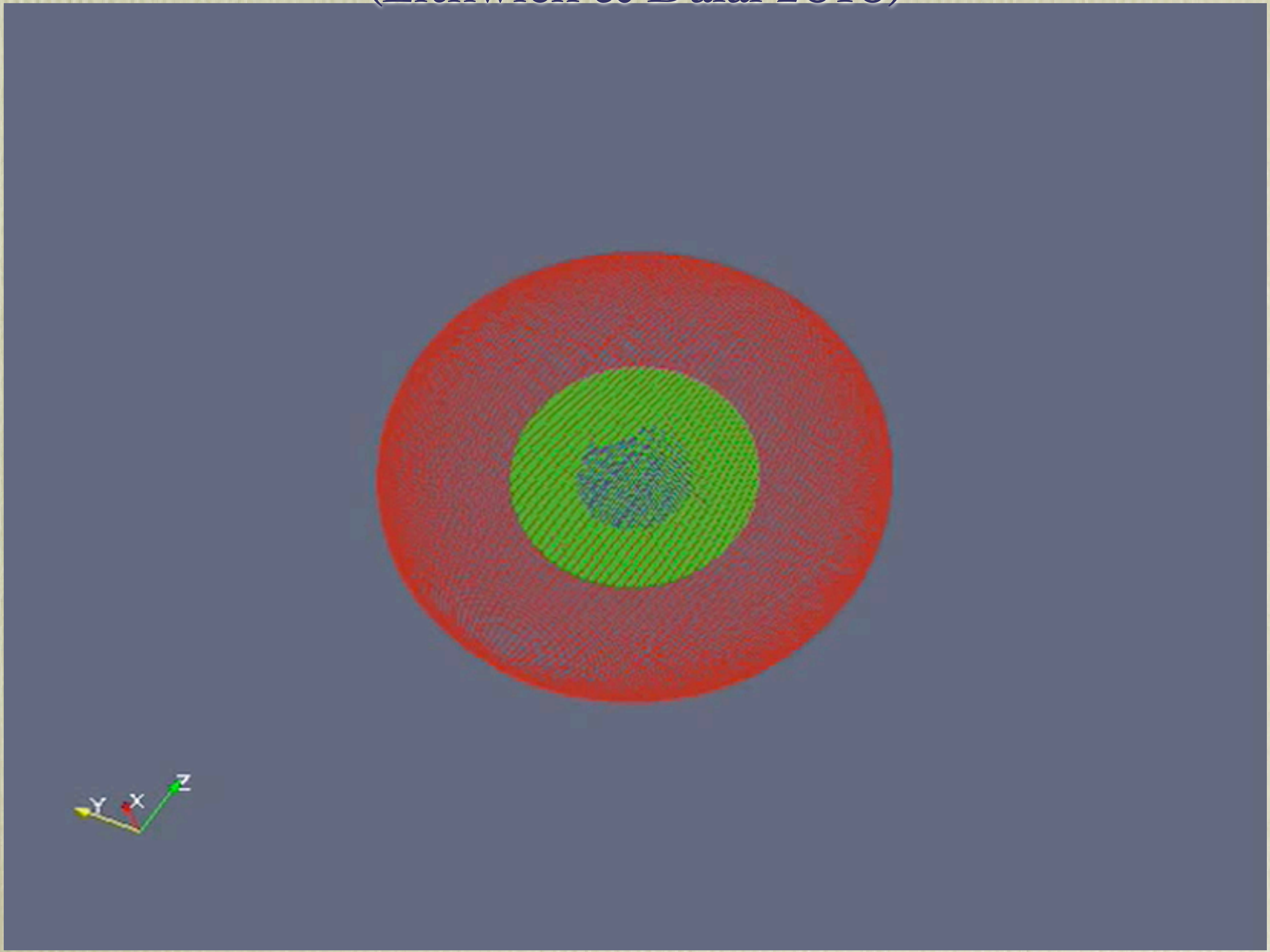
Spherical Self-Similar Solution

(Fillmore & Goldreich 1984, Bertschinger 1985)

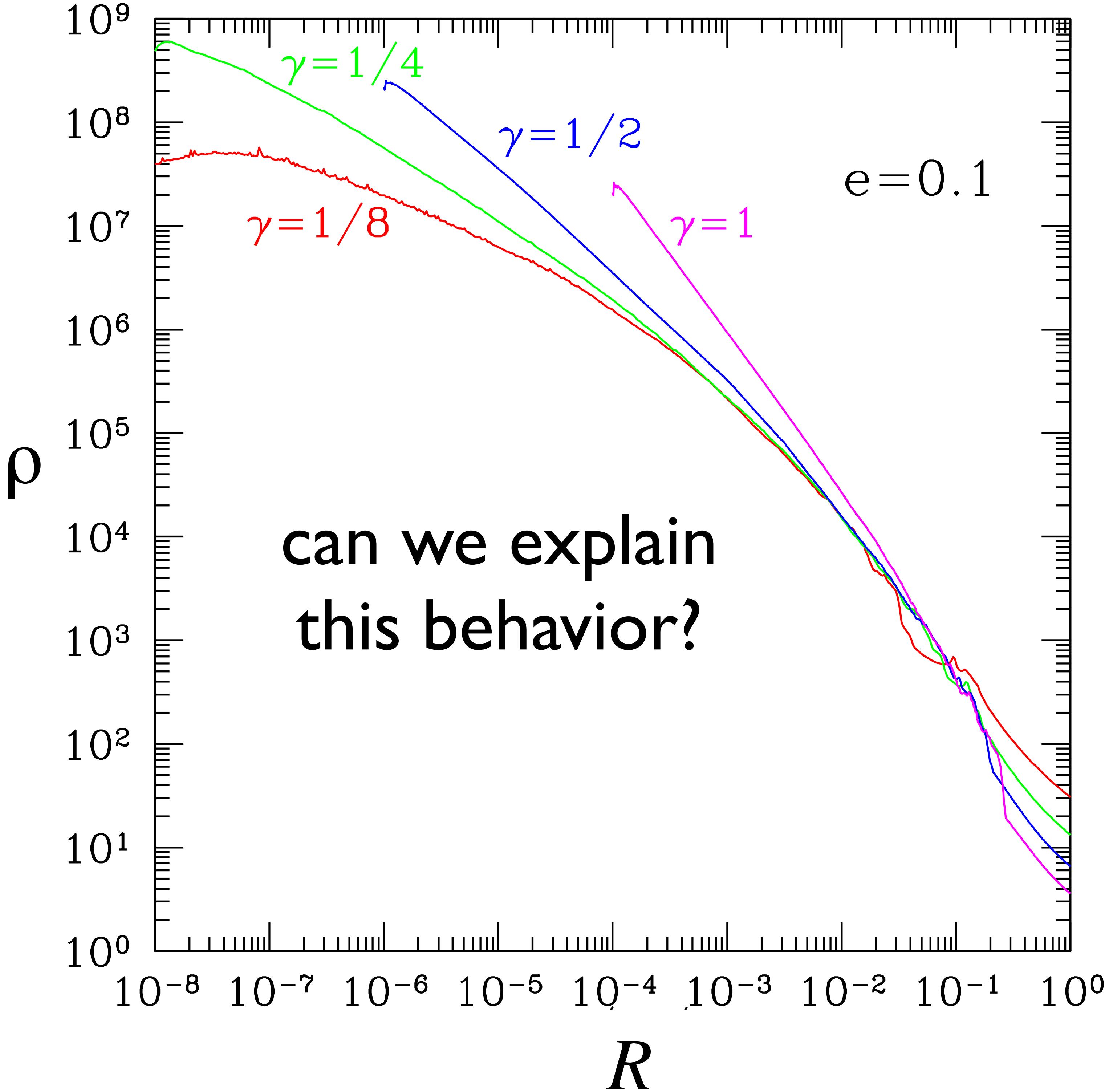


Nonspherical Self-Similar Solution

(Lithwick & Dalal 2010)

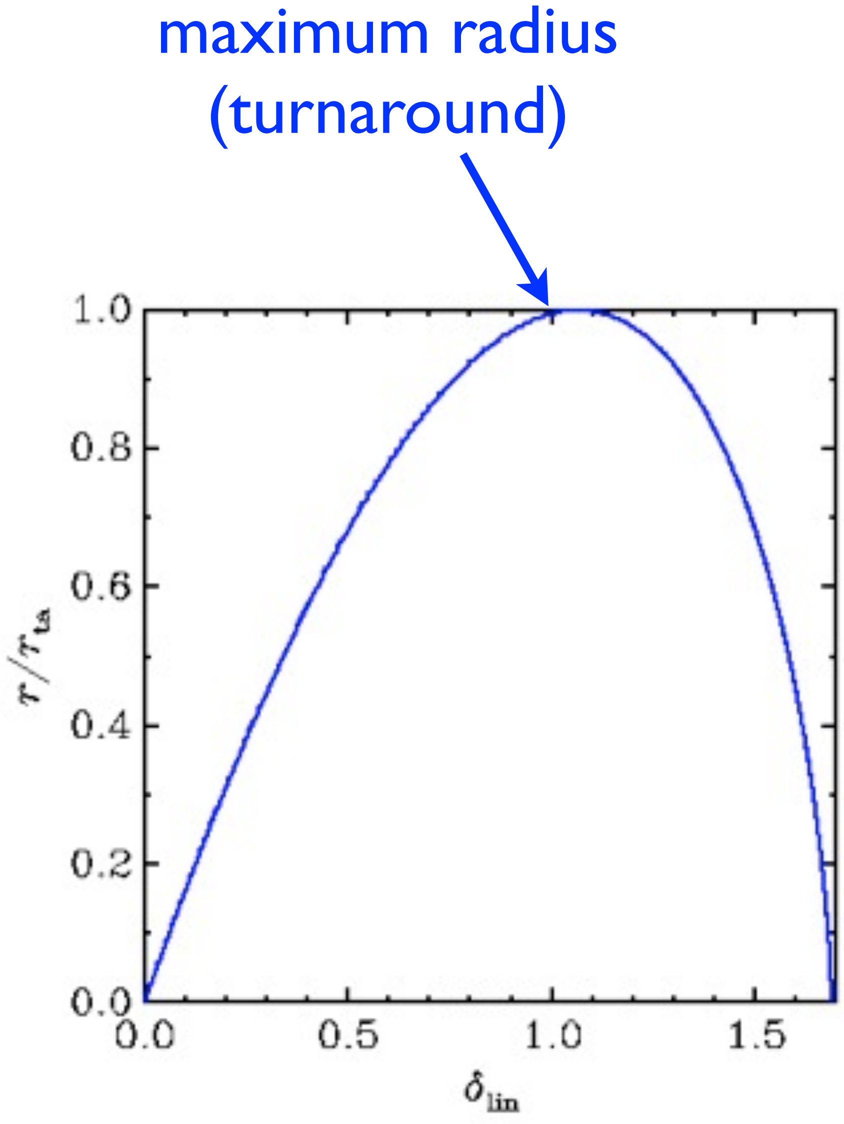


example: density profile



spherical collapse model

- Gunn & Gott (1972)
- entire model: solve $\ddot{r} = -\frac{GM}{r^2}$
- results that I'll use:
  1. max radius ($r_{\text{turnaround}}$)
  2. time of turnaround (when $\delta \approx 1$)



(outer) Density profile

see Fillmore & Goldreich (1984)

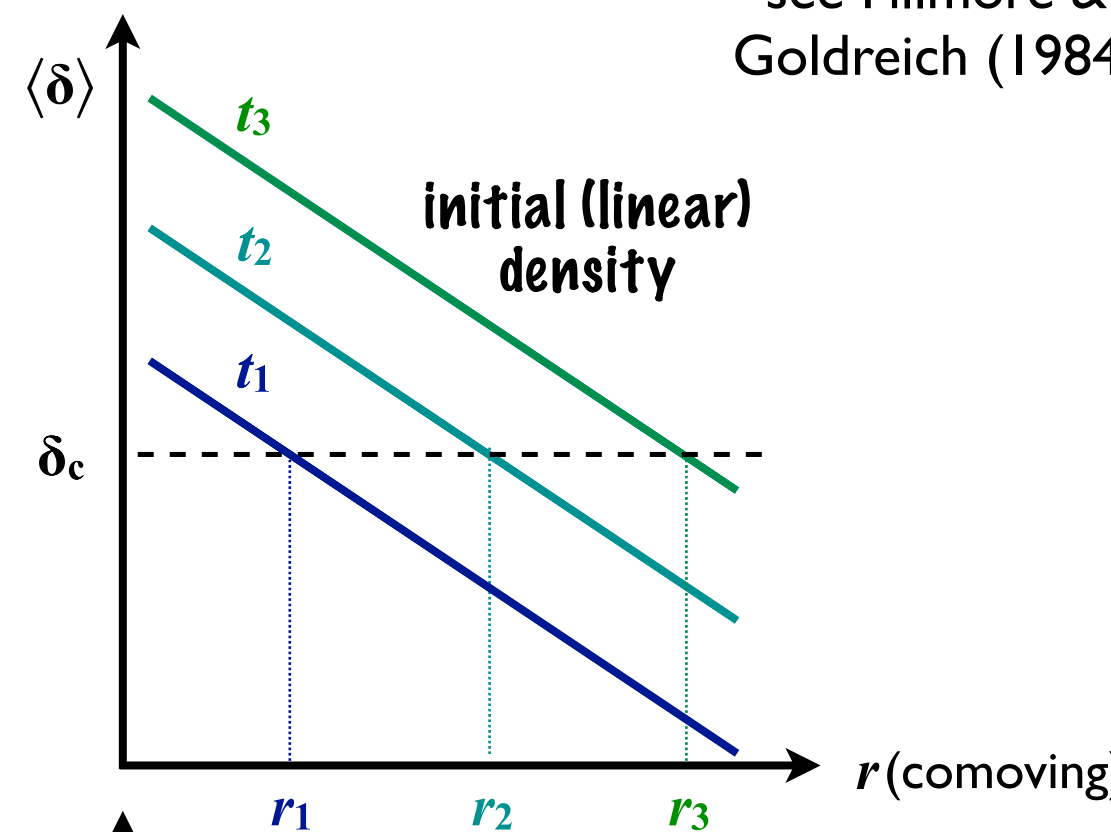
Suppose linear density profile has local slope γ, so that

$$\delta(r,a) \propto a\, r^{-\gamma}$$

Turnaround occurs when $\delta \sim 1$, so

$r_{ta} \propto a^{1/\gamma}$ (comoving)

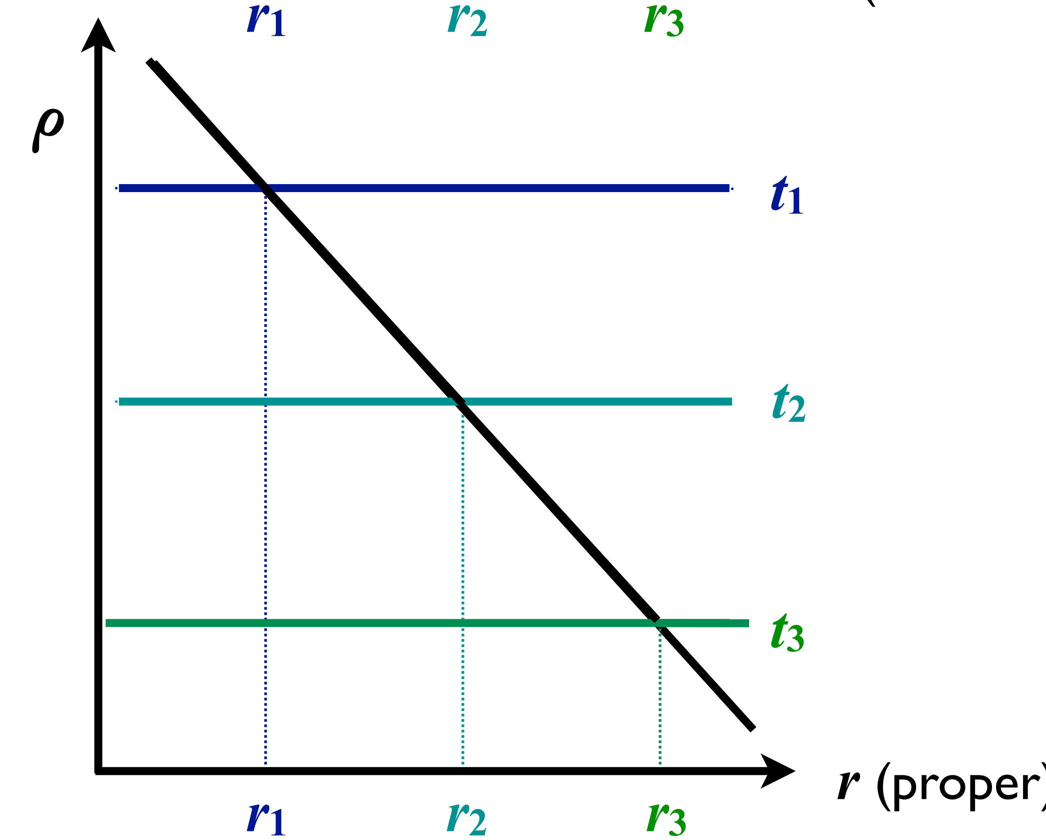
$r_{ta} \propto a^{(1+\gamma)/\gamma}$ (proper)



Suppose (for now) that all particles execute circular orbits, so there is *no shell crossing*.

Background $\rho \propto a^{-3}$, and $a_{ta} \propto r_{ta}^{\gamma/(1+\gamma)}$, so the slope of the density is

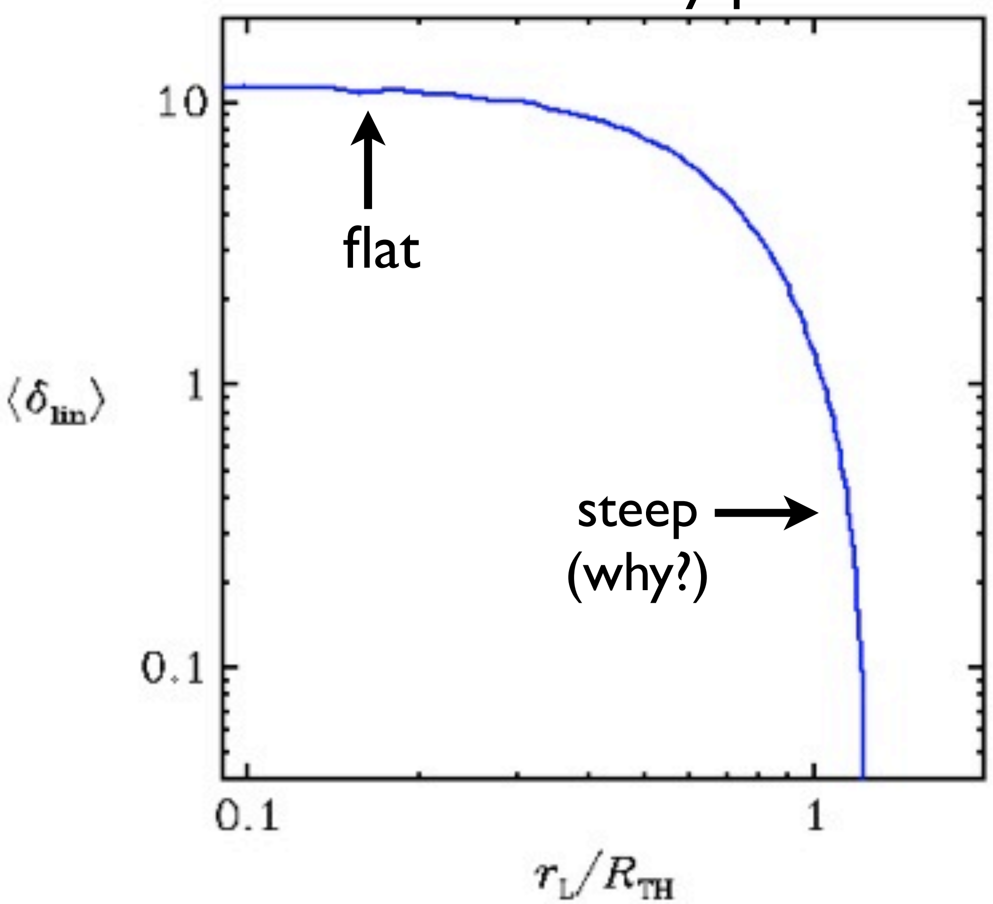
$$\rho \propto r^{-\alpha}, \quad \alpha = \frac{3\gamma}{1+\gamma}$$



(outer) Density profile

The preceding argument ($\rho \propto d^3 r_L / d^3 r$) can be used to predict the halo profile given the initial peak profile:

- initial radial density profile



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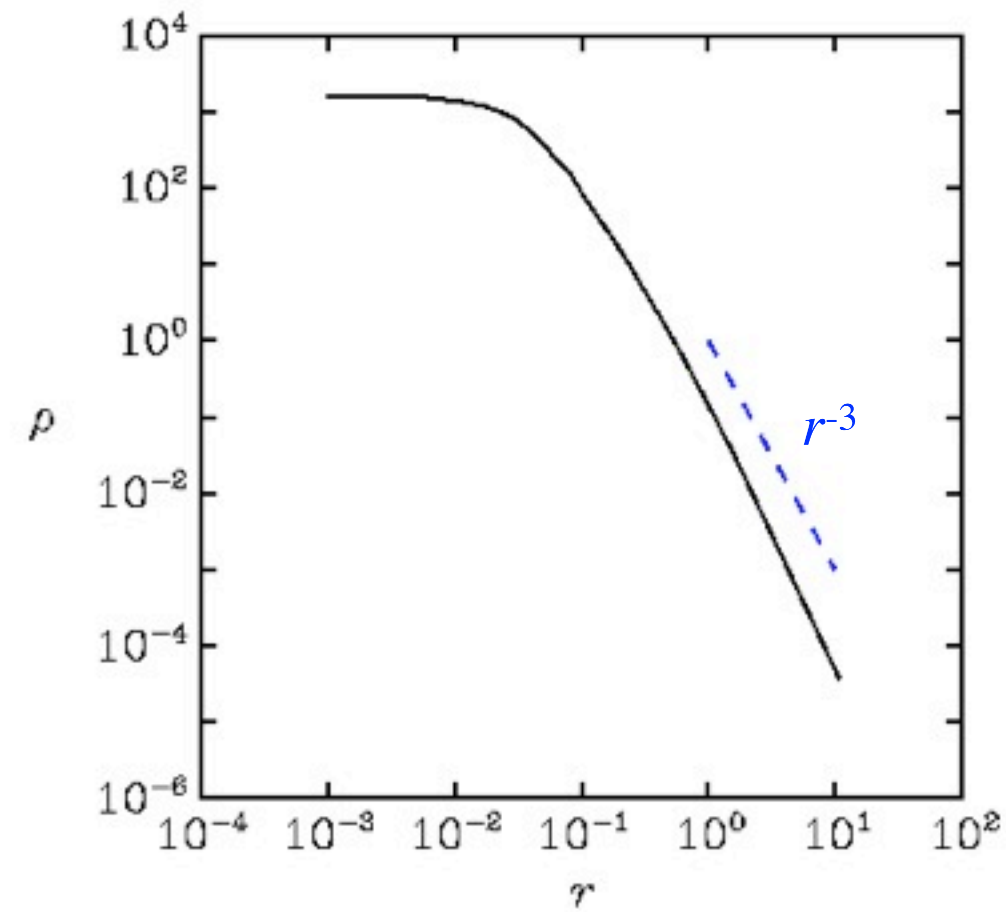


recall slope $\alpha \approx 3\gamma/(1+\gamma)$

see also Gunn & Ryden (1988), Ascasibar et al. (2004), Lu et al. (2006)

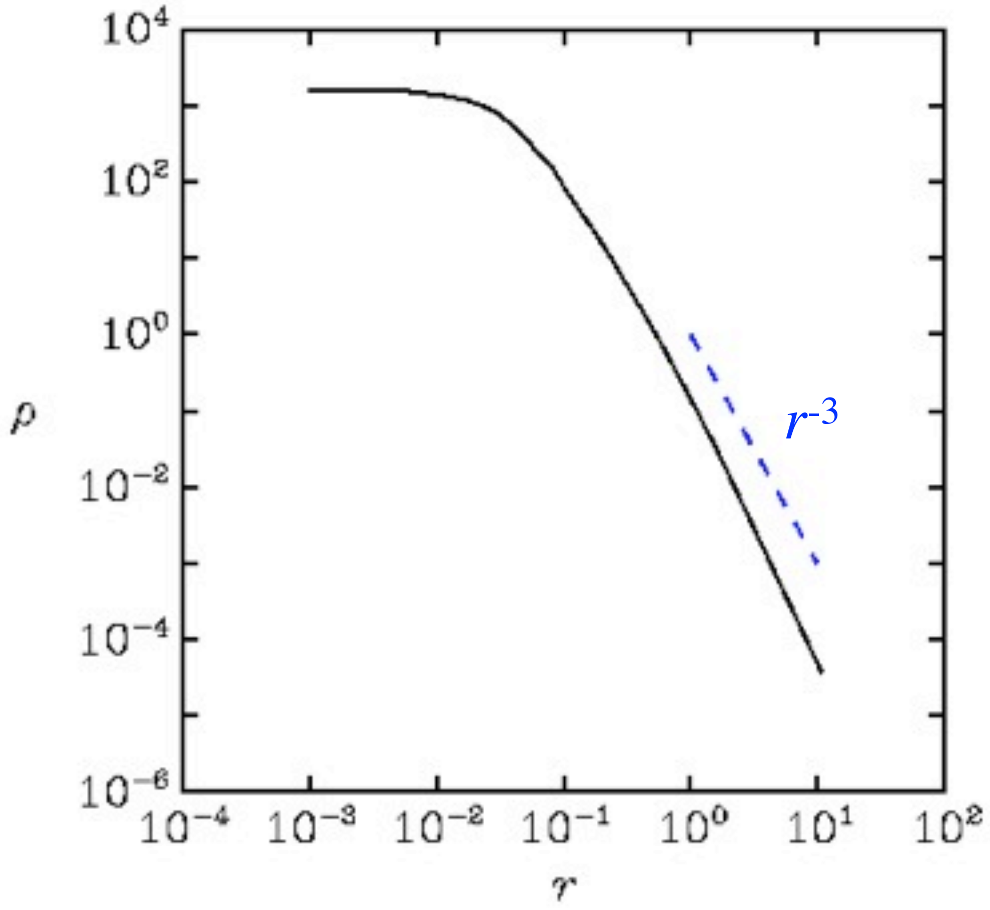
(inner) Density profile

This works for outer profile, but does not explain the inner profiles.

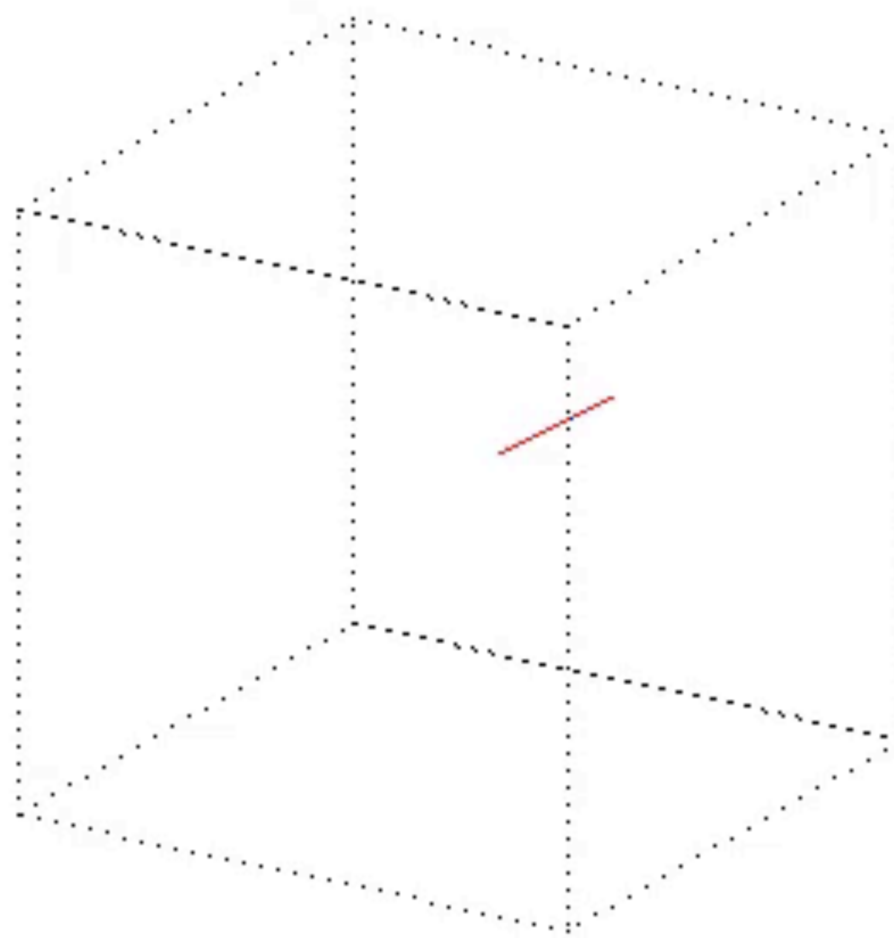
So far, we've assumed circular orbits with no shell crossing, but reality is not so simple!

See examples:

- box orbit
- loop orbit
- banana orbit
- ...



example: banana orbit



more examples:
box, loop

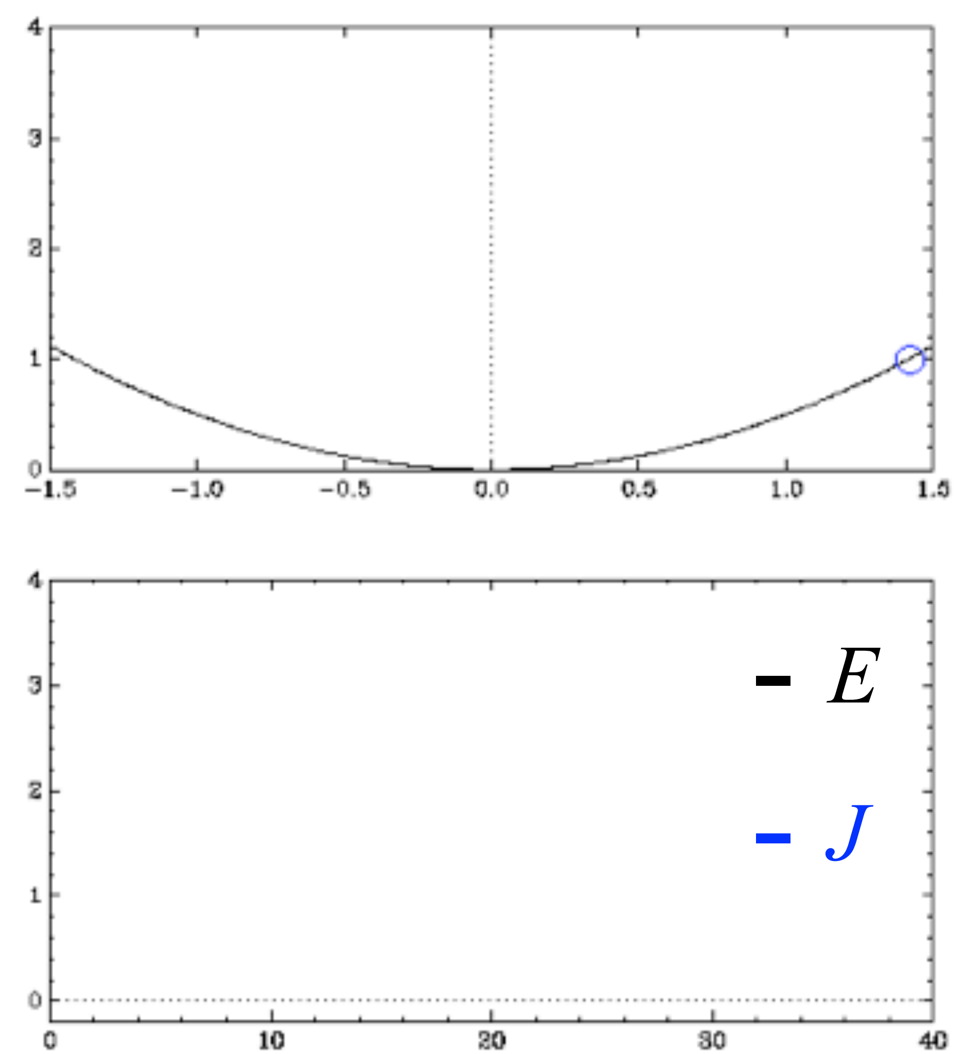
orbits respond to evolving potential

adiabatic contraction

- if potential changes slowly compared to orbital time, then orbits respond adiabatically and conserve *adiabatic invariants*
- for one-dimensional system, action $J = \int v\, dx \sim v\, x \sim \Phi^{1/2}\, x$
- example: harmonic oscillator

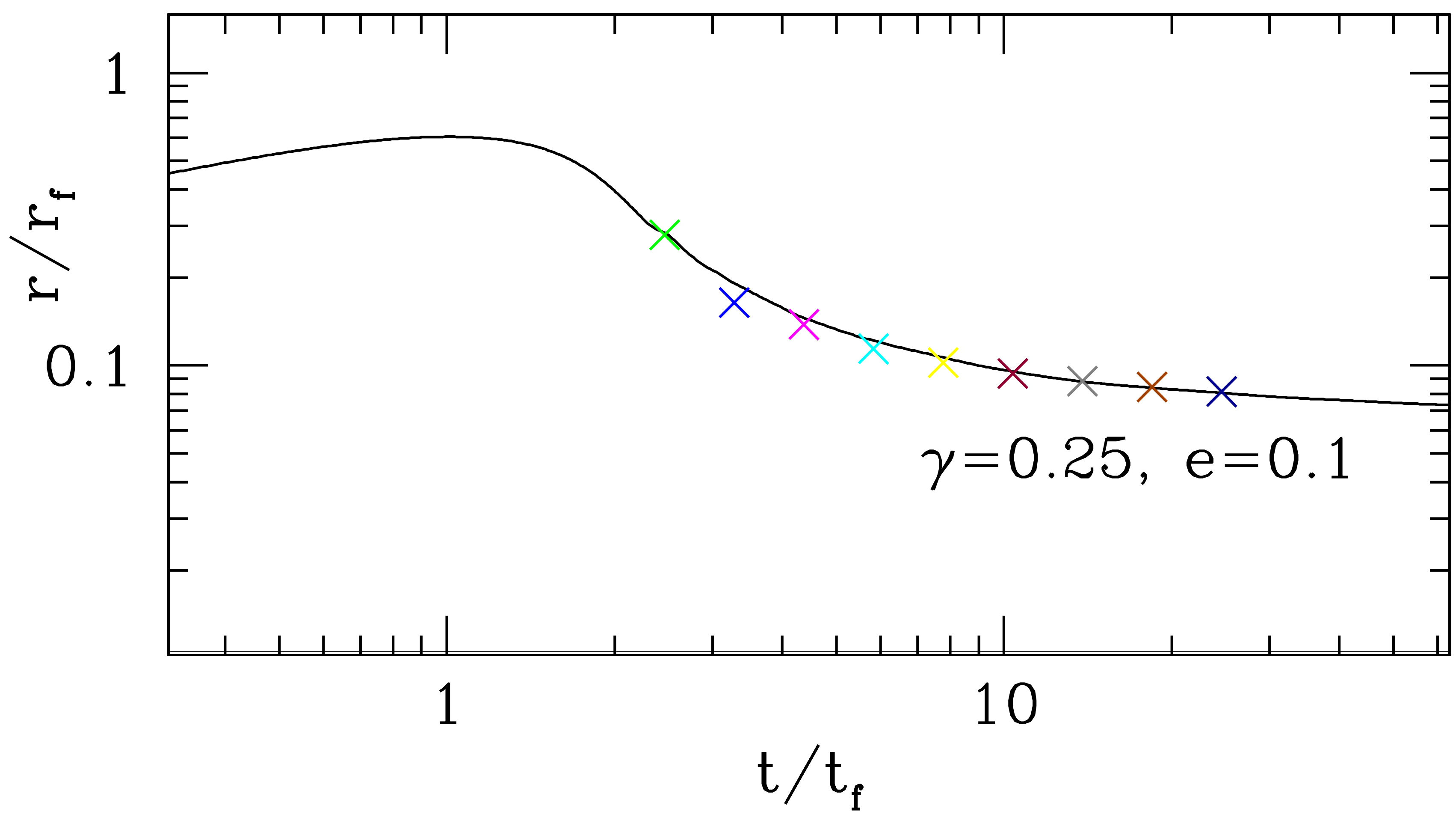
$\Phi(x)=\frac{1}{2}\,\omega^2 x^2$

$\Rightarrow J \sim E/\omega$



adiabatic contraction

- in spherical systems, the radial action $J_r = \int v_r\, dr \propto (M\, r)^{1/2}$ is an adiabatic invariant
- our halos are not spherical, but shells are consistent with conserving J_r



adiabatic invariants

- the conserved adiabatic invariants may be *predicted* from the initial peak profile, using the spherical collapse model:

spherical collapse model

- Gunn & Gott (1972)
- entire model: solve $\ddot{r} = -\frac{GM}{r^2}$
- results that I'll use:

1. max radius ($r_{\text{turnaround}}$)
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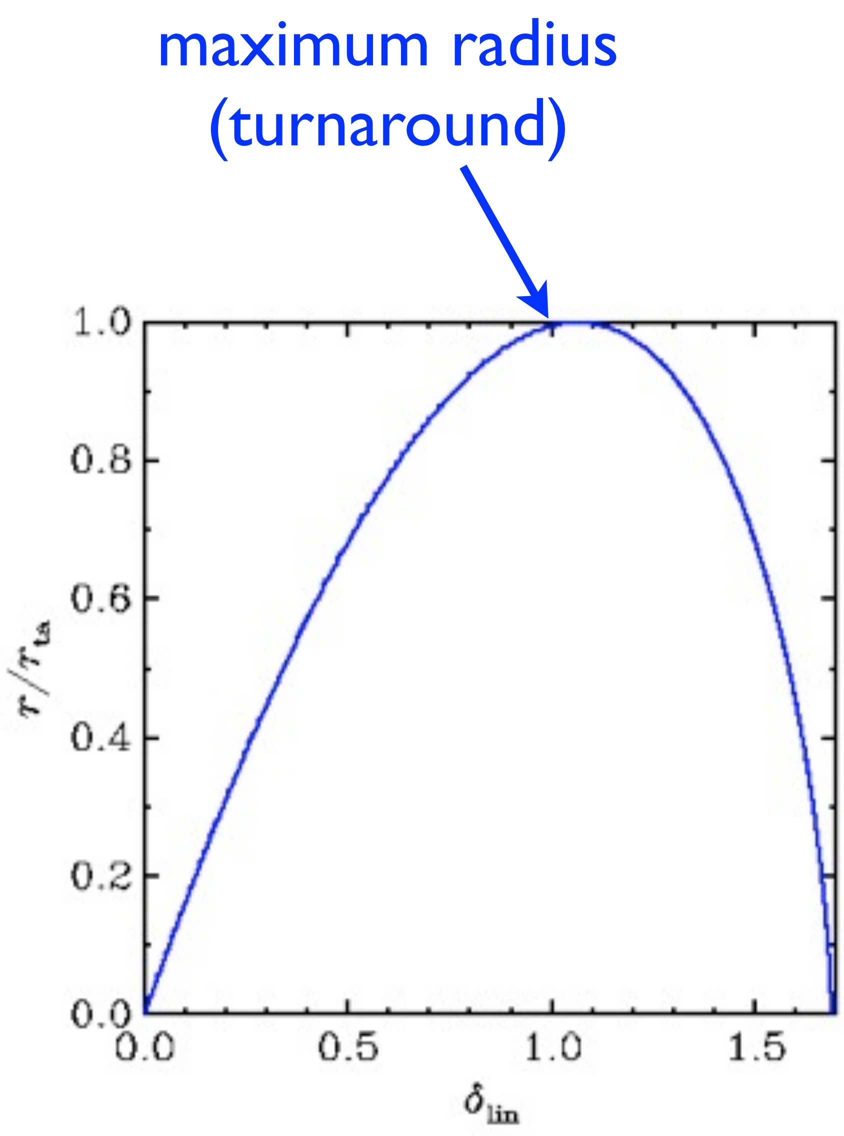
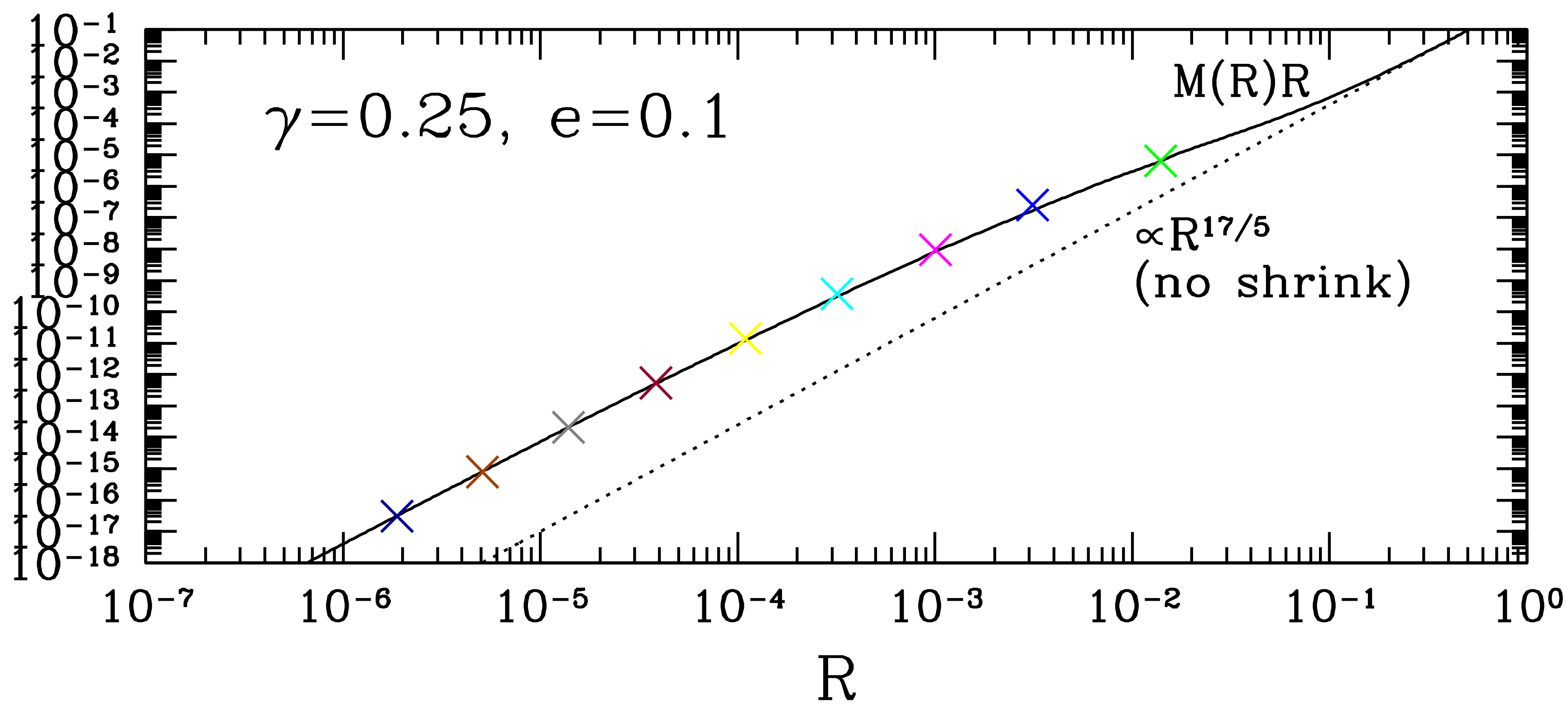


adiabatic invariants

- the conserved adiabatic invariants may be *predicted* from the initial peak profile, using the spherical collapse model:
- assume that $M_{\mathrm{L}} \times r_{\mathrm{ta}}$ is conserved



hooray!

- Ok: (we think) we understand the **simple** case of self-similar collapse:
  - ➡ the important physics is adiabatic contraction applied to the initial peaks
- does the same physics explain the messier, more realistic case of CDM halos?

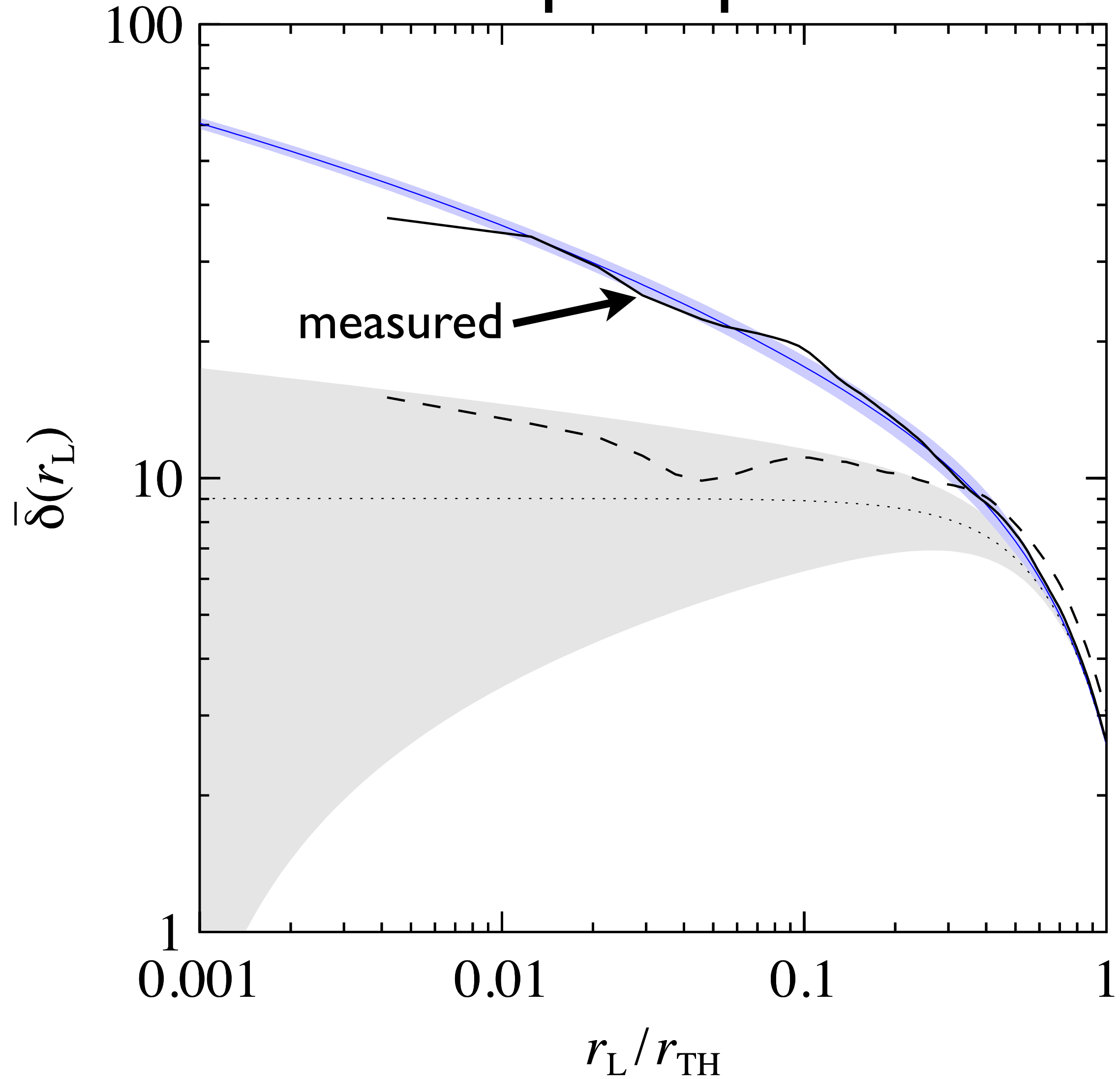
Via Lactea-2

- High resolution simulation of halo similar to MW
- 40 Mpc box, with over 10^9 particles inside virialized region at z=0
- profile resolved down to 10^{-3} of r_{200}
- snapshots extending from z=104.3 to z=0

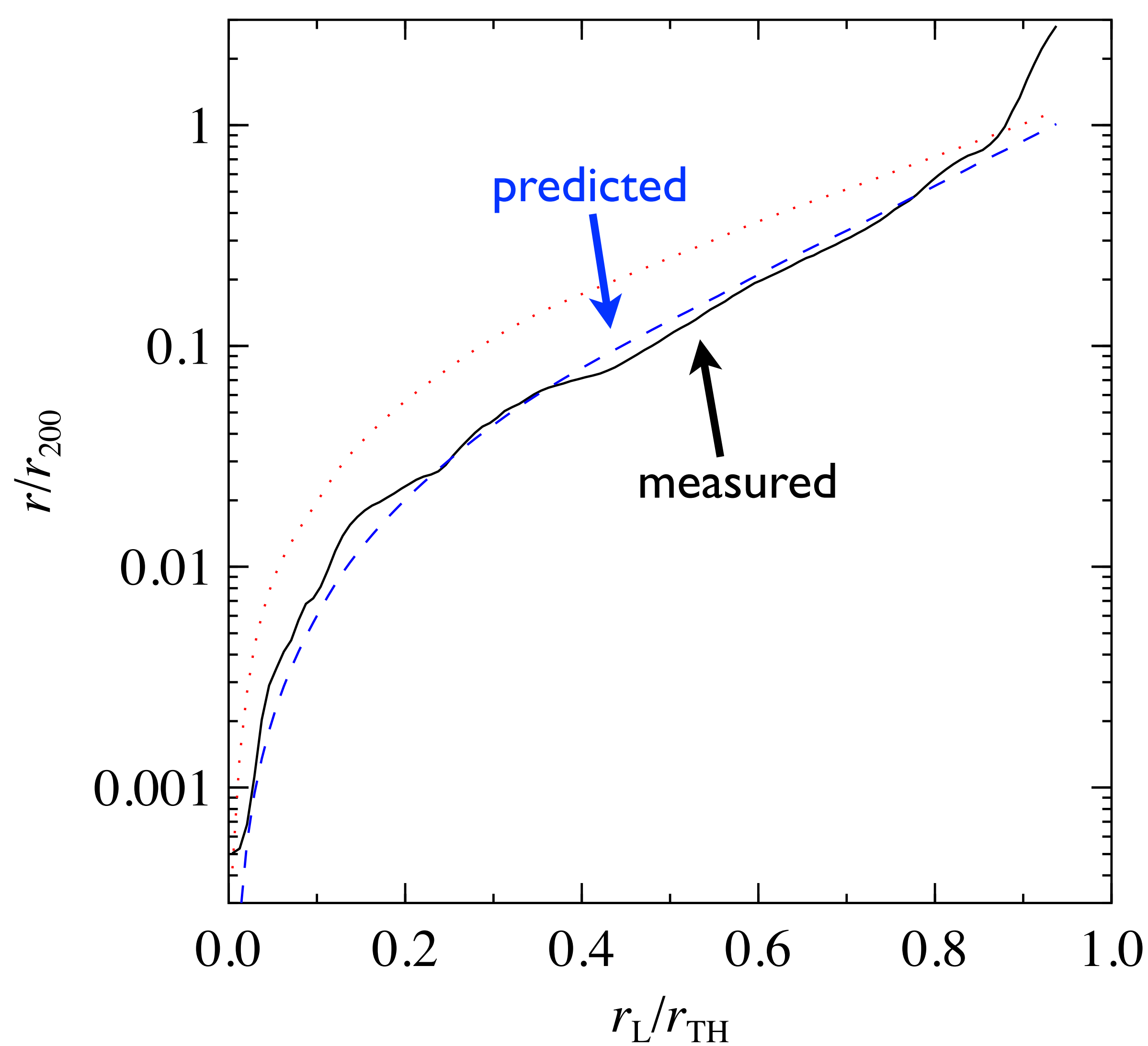
VL-2 comparison

Dalal, Lithwick & Kuhlen (2010)

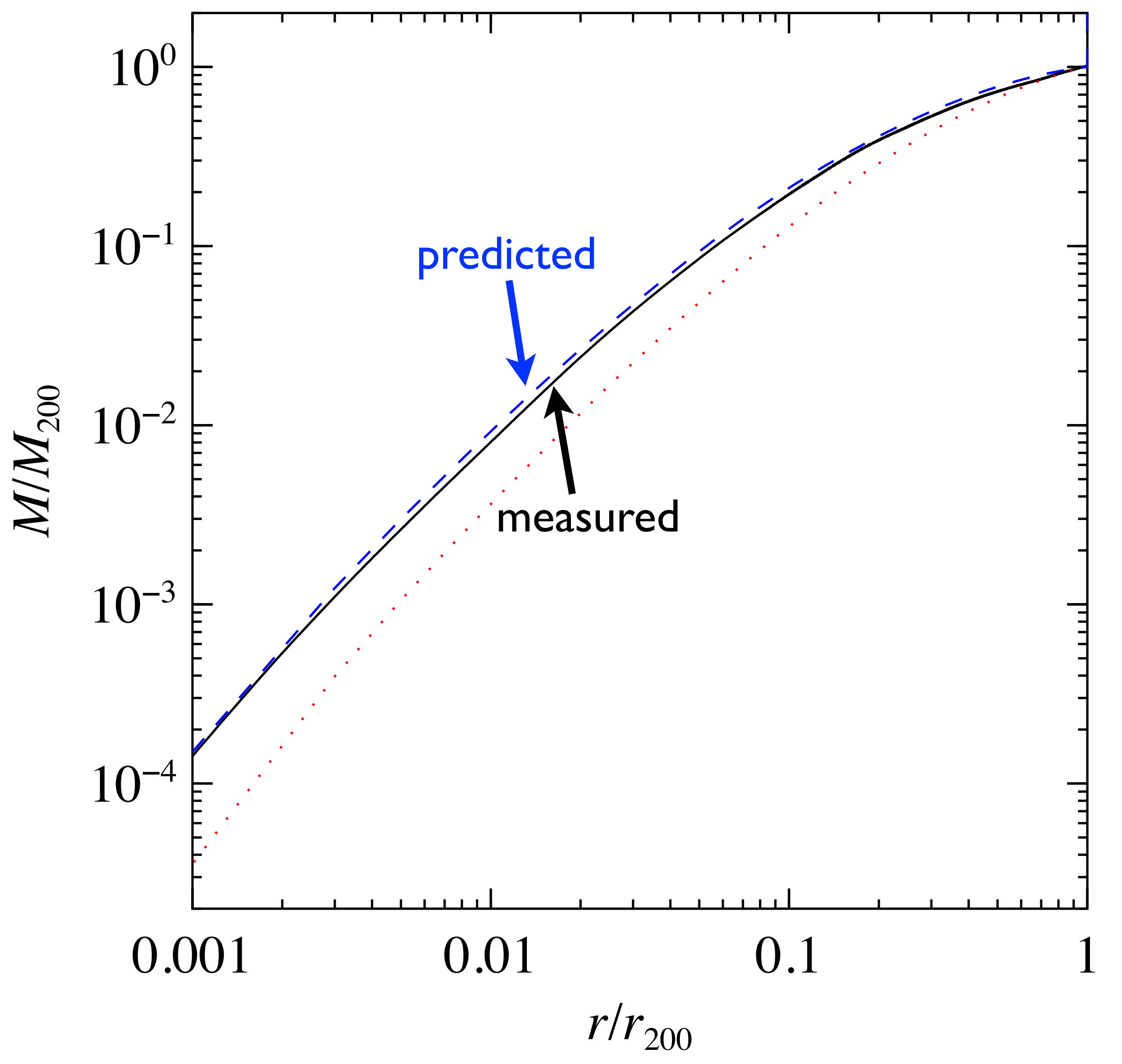
initial peak profile



final shell radii



VL-2: mass profile



success?

- this shows we can predict the final halo profile, **given** the initial peak profile
- then, if we can predict the initial profiles as well, then we have a complete model!
- these are peaks of Gaussian random fields: $\Rightarrow$ try Gaussian statistics to predict profiles

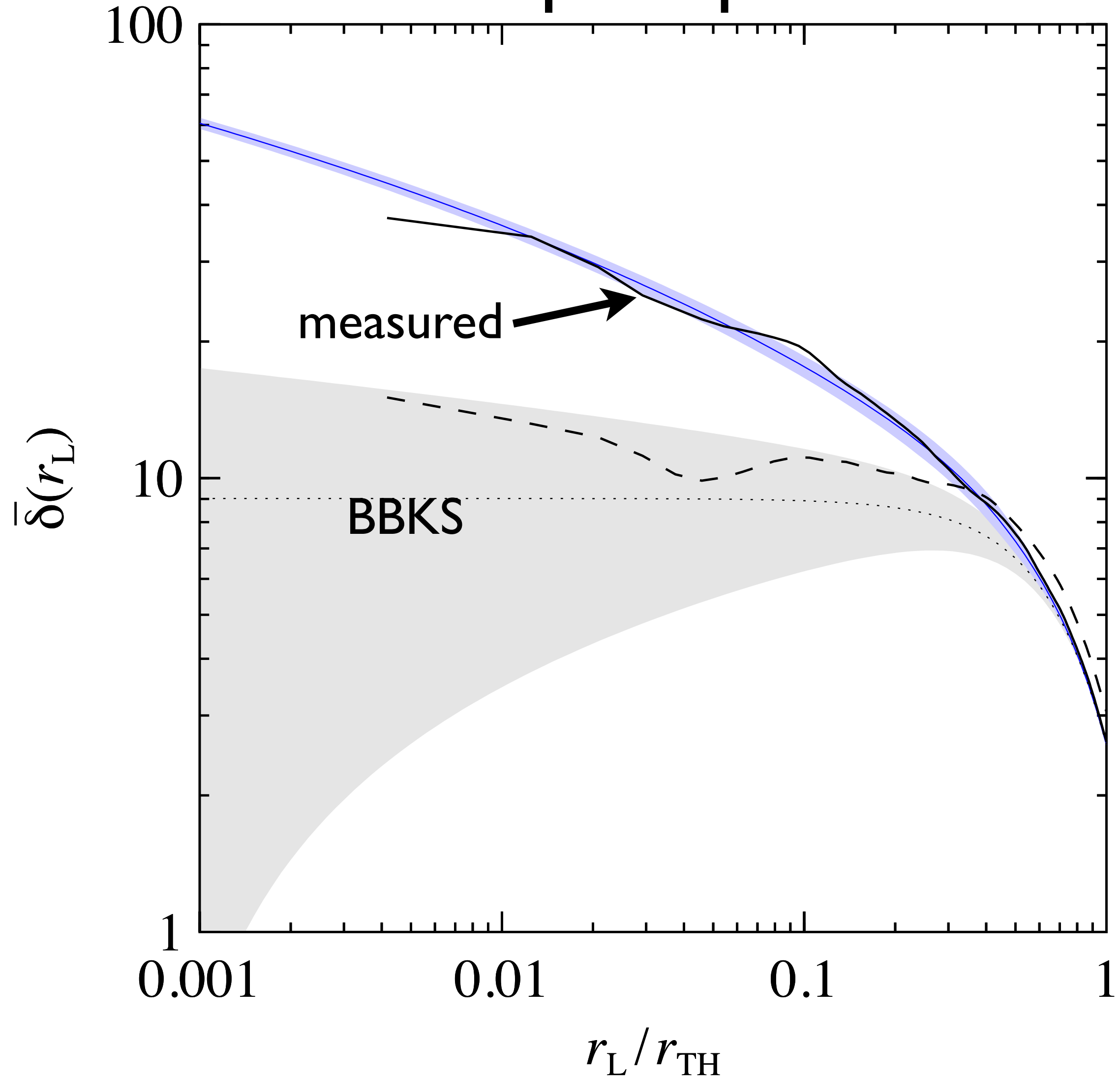
simplest Gaussian prediction

- nicely explained in BBKS (1986)
- compute (average) peak profile using the probability distribution of density δ
- this is a **conditional** probability, since we know that $\delta = \delta_{\rm crit}$ at radius $r_{\rm halo}$
- so the average profile is $\langle \delta(r) | \, \delta(r_{\rm halo}) \rangle$, which only depends on the matter power spectrum

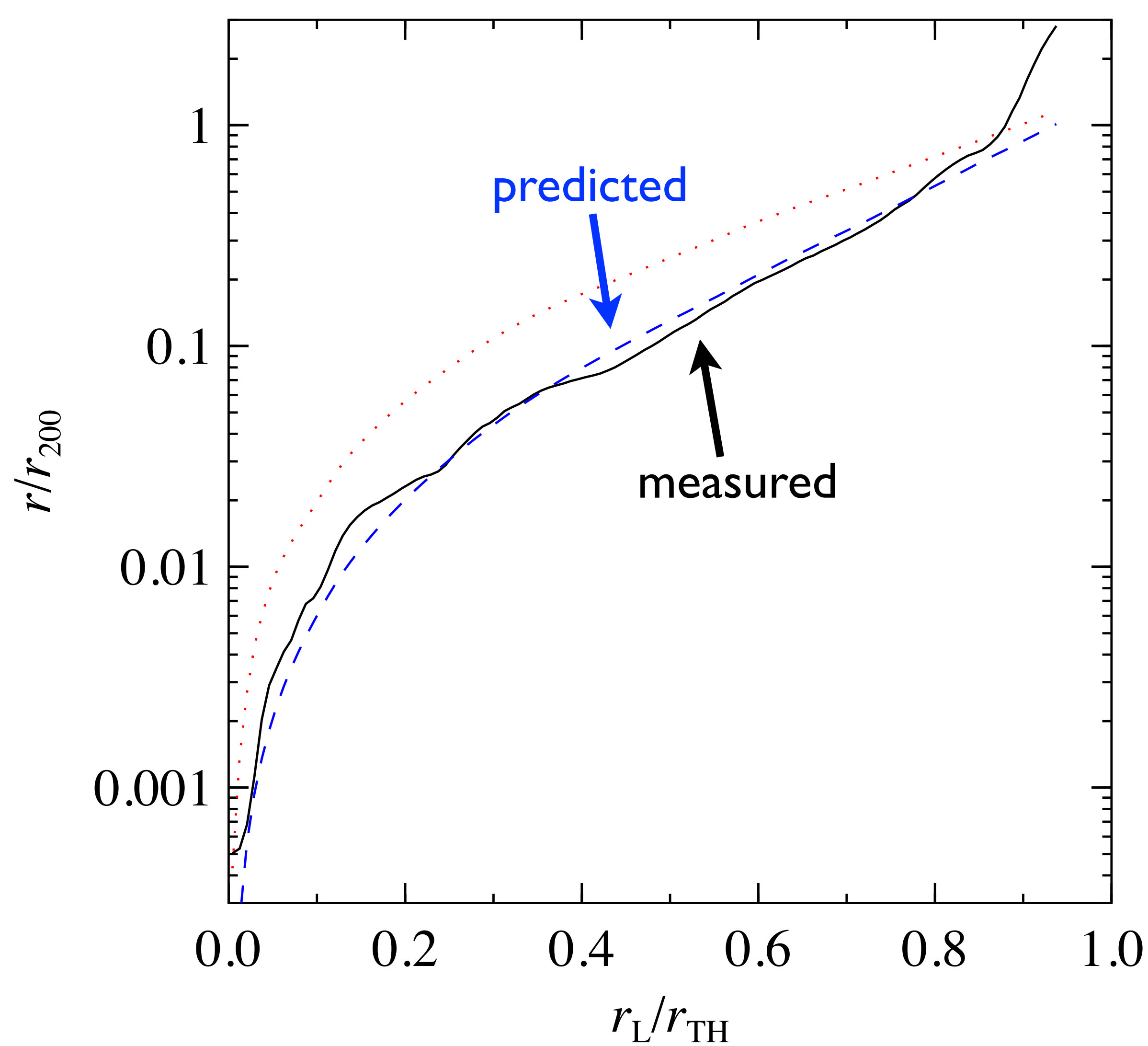
VL-2 comparison

Dalal, Lithwick & Kuhlen (2010)

initial peak profile



final shell radii



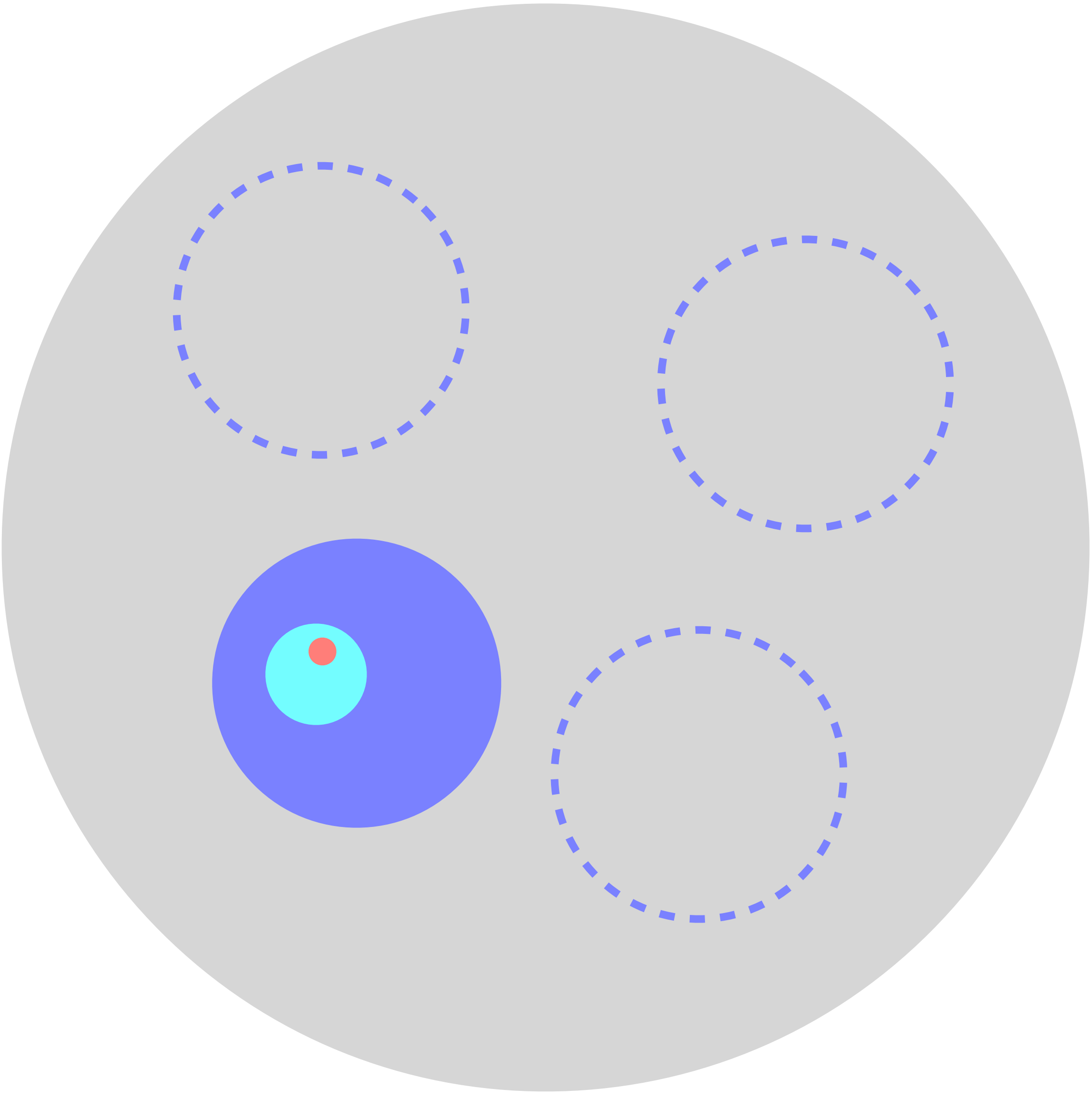
peak profiles

- naive Gaussian statistics (BBKS) does not match the actual peak profiles – why?
- the naive calculation ignores the **hierarchy** of peaks within peaks for cold dark matter
- we proposed a simple way to account for this hierarchy, still using simple Gaussian statistics...

peak profiles

- naive Gaussian statistics (BBKS) does not match the actual peak profiles -- why?
- processes during collapse (e.g. **dynamical friction**) can rearrange matter, dragging high density material to the center
- simple model for this: assume that the **densest** material comes from the **highest** subpeaks that are the first to collapse

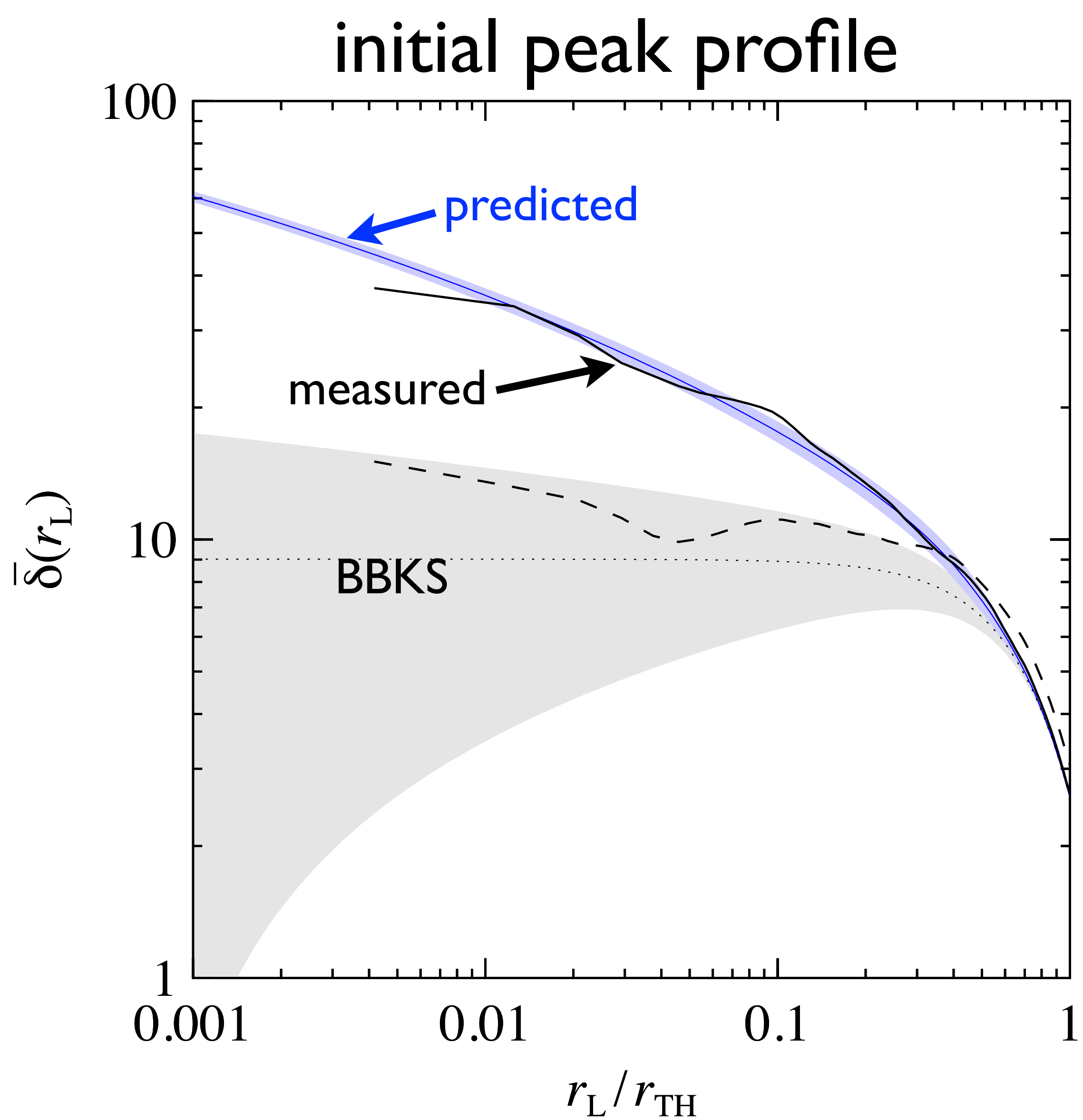
schematically:

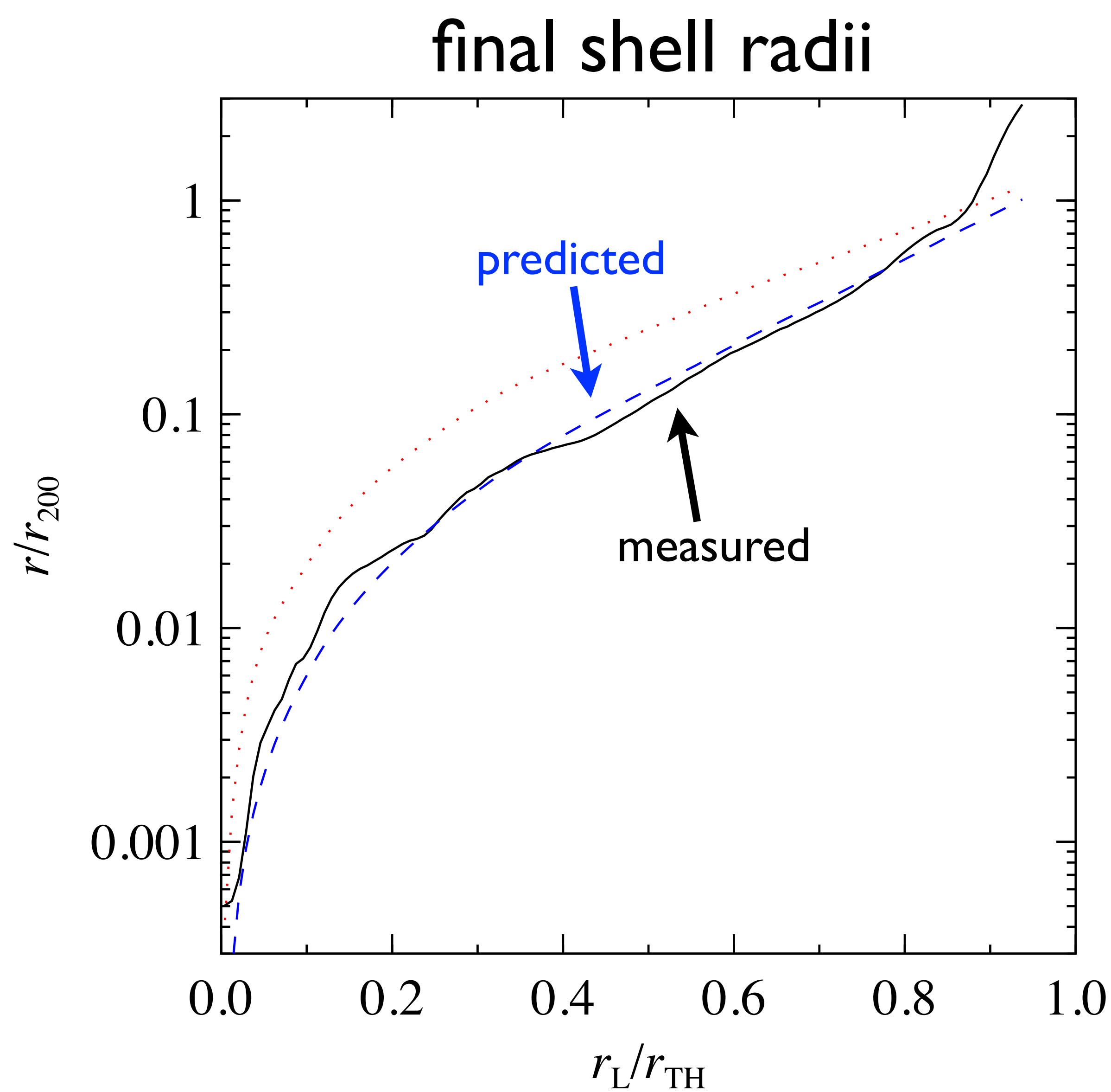


initial volume

VL-2 comparison

Dalal, Lithwick & Kuhlen (2010)





what does it mean?

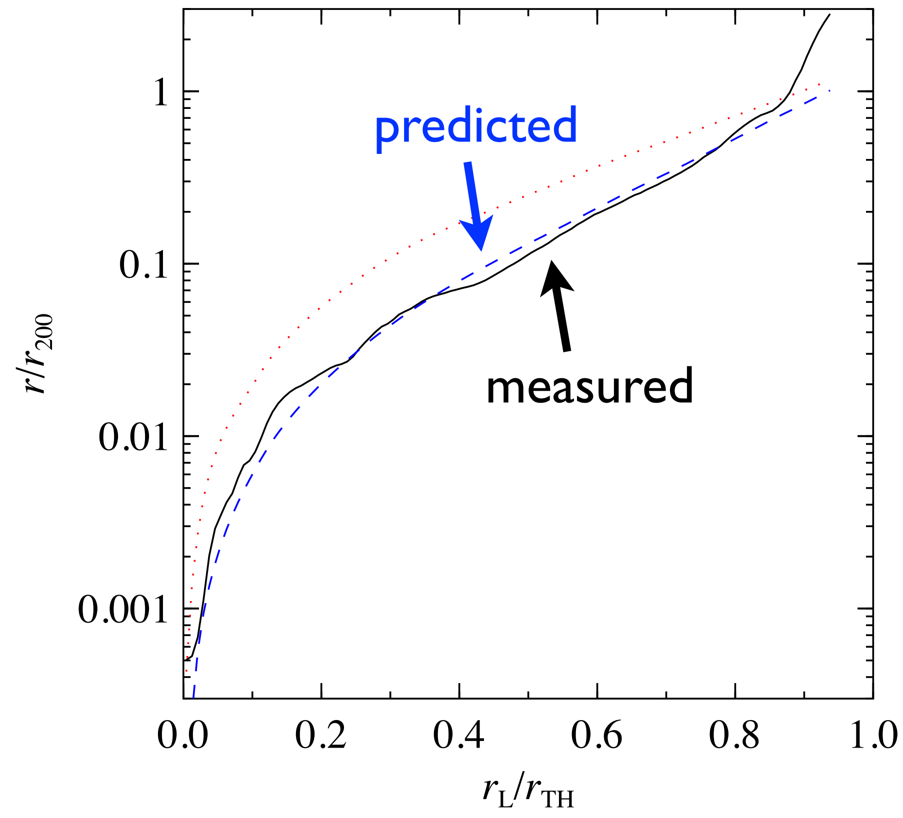
Upshot:

1. we know how to translate from initial peak profile to final halo profile
2. we know how to calculate the statistics of initial peak profiles for Gaussian random fields.

➡ We are done! (problem solved?!)

some implications

- most halos do not **violently relax**



some implications

- most halos do not **violently relax**
- we find no reason for a $\rho \propto r^{-1}$ cusp as $r \rightarrow 0$
- instead, the gradual roll-over in slope continues down all the way to $r=0$

broader appplications

why is this important?

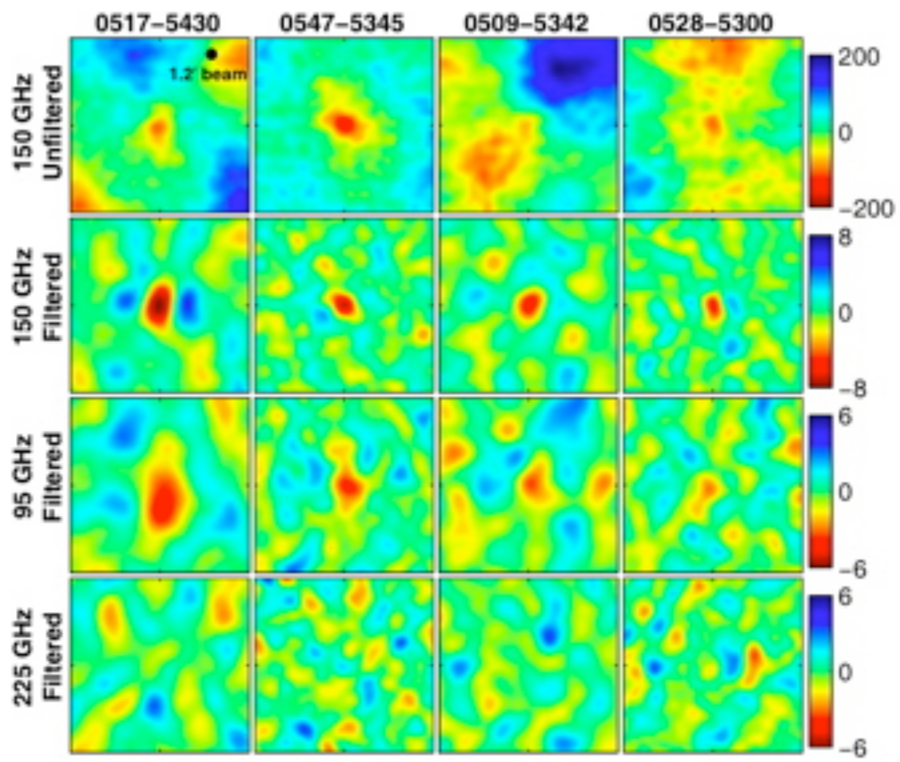
1. now we know how to compute statistics
2. now we know what changes as we vary things
3. now we know what aspects of the model are tested by various observations...

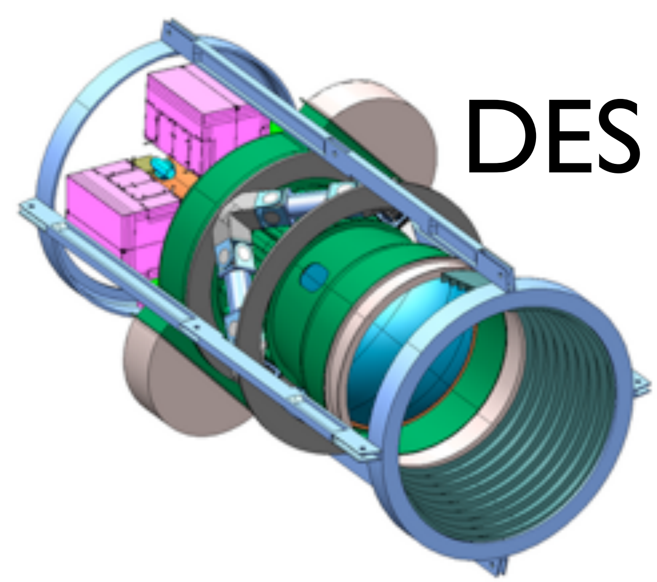
Halo statistics

Halo mass function

- The number of halos as a function of mass
- One of the most fundamental statistics in cosmology
- This (largely) controls the number of galaxies, clusters, etc.
- Time dependence tells us how **fast** objects grow, how often they merge, etc.

Halo mass function





... and many more

halo mass function

our approach is to compute halo statistics using

1. peak statistics, and
2. our self-similar collapse model

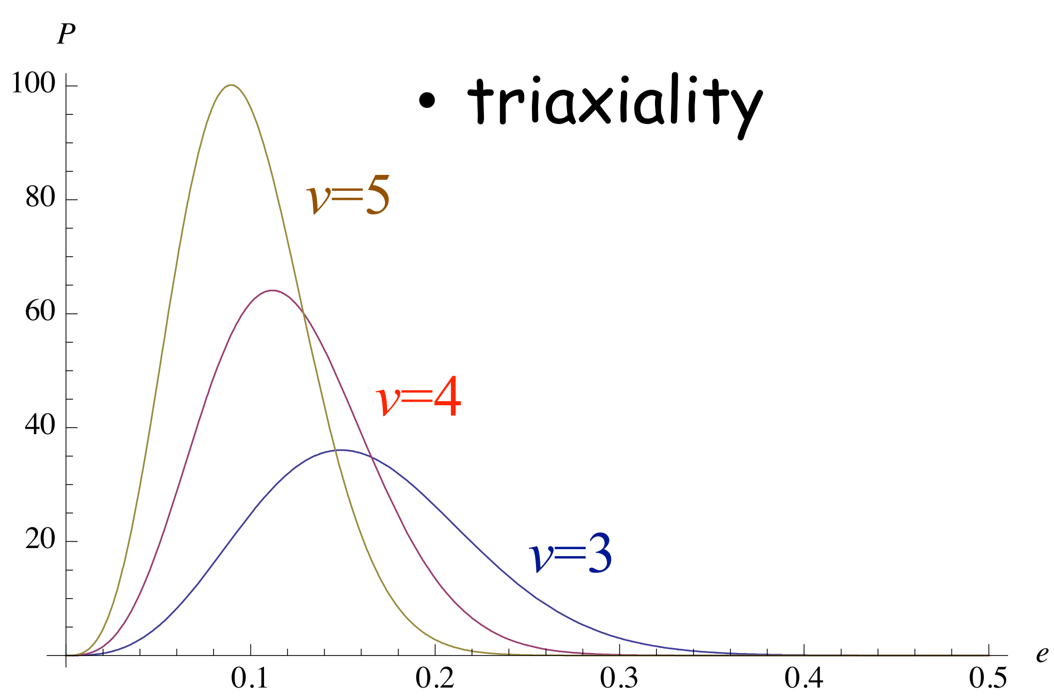
peak statistics (Gaussian)

- peak statistics ..... already worked out by BBKS (1986), e.g.:

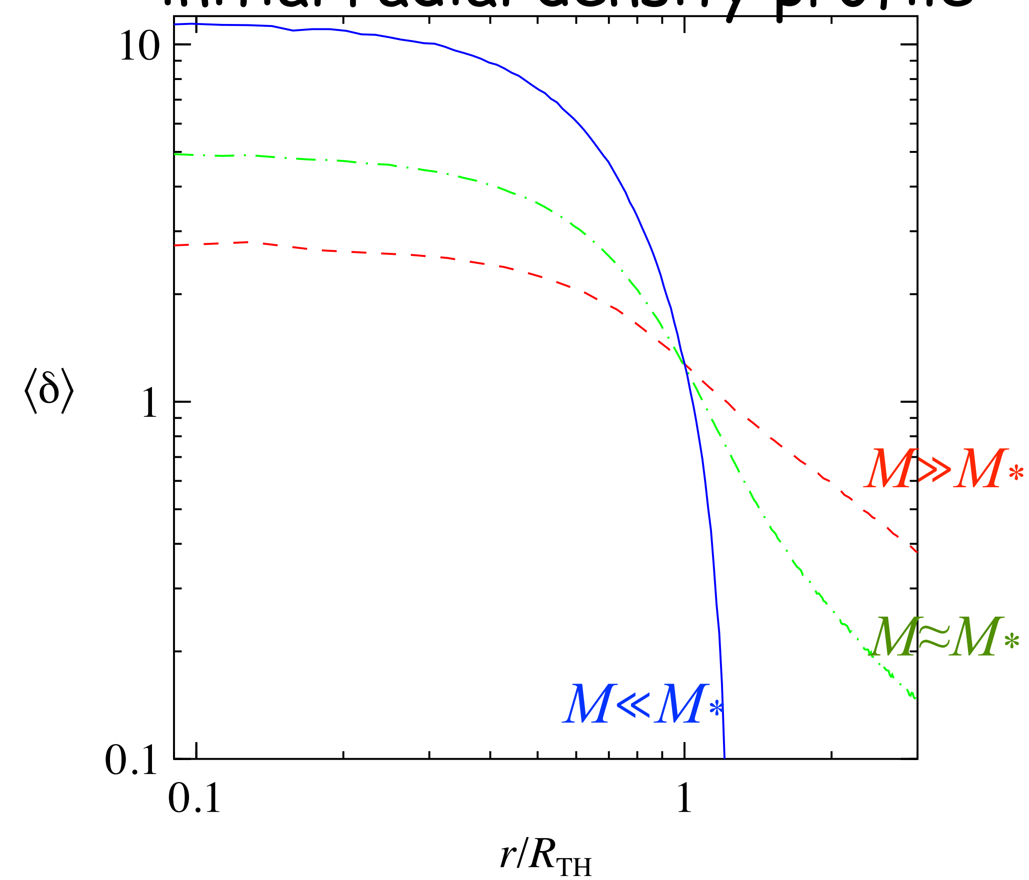
$$\mathcal{N}_{\rm pk}(\nu) \approx \frac{(\sigma^2_{\delta\nabla^2\delta}/3\sigma^2_\delta)^{3/2}}{2\pi^2}(\nu^3 - 3\nu)e^{-\nu^2/2}, \qquad \nu \to \infty$$

- Also: properties of the peaks: e.g. statistics of...

- triaxiality



- initial radial density profile



halo statistics

- Now, **combine** peak stats with our collapse model
- for example:

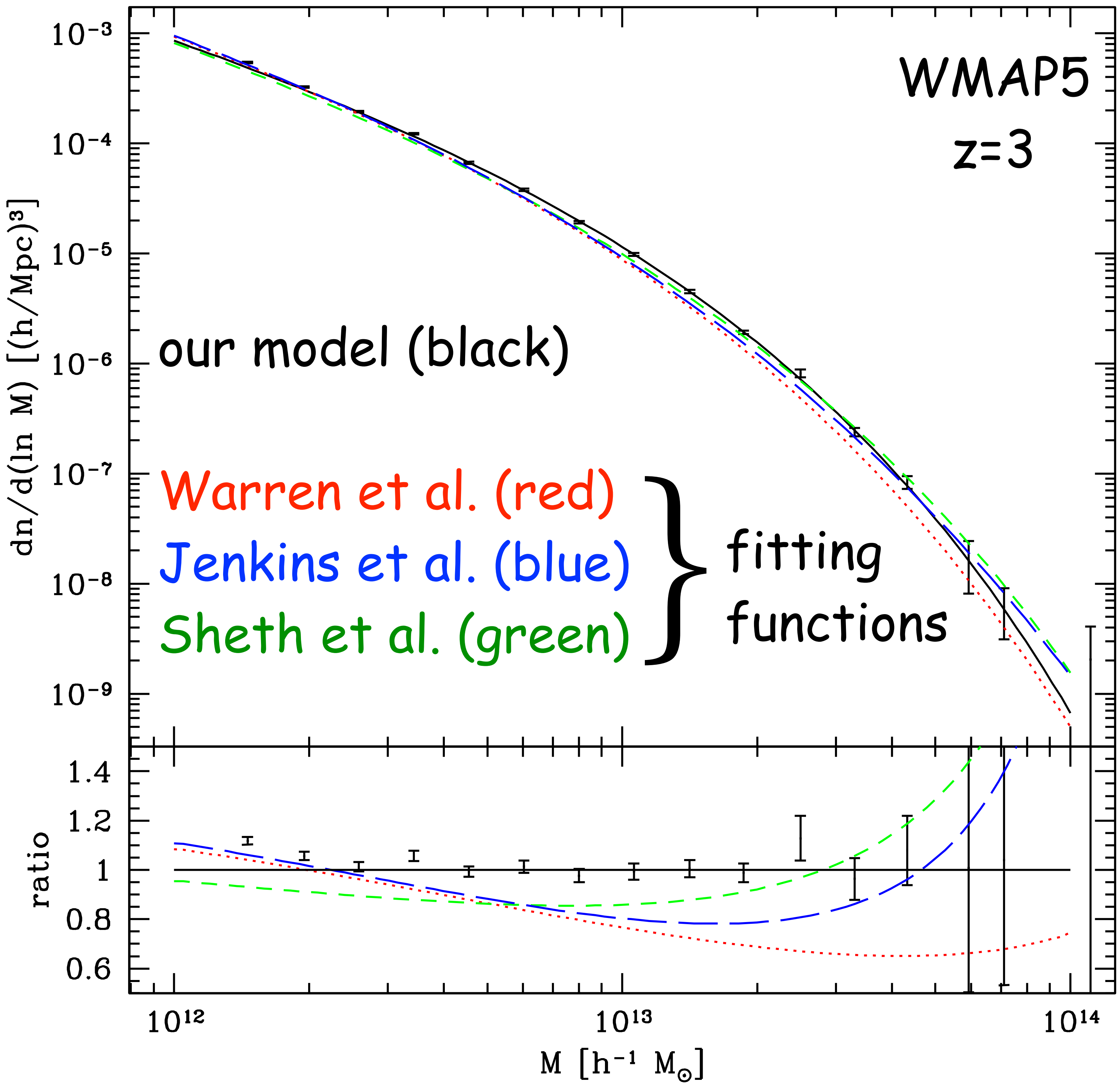
$$n = \int de\, dp \ldots \int_{\nu_c}^{\infty} \mathcal{N}(\nu, e, p, \ldots) d\nu$$

  see also BBKS, Bond & Myers (1996)

- Peaks are complicated, but we assume that just a few peak properties are important:
  - radial slope γ
  - triaxiality e,p
- Our self-similar collapse model allows us to compute post-collapse properties as a function of γ, e, p

Dalal et al. (2011)

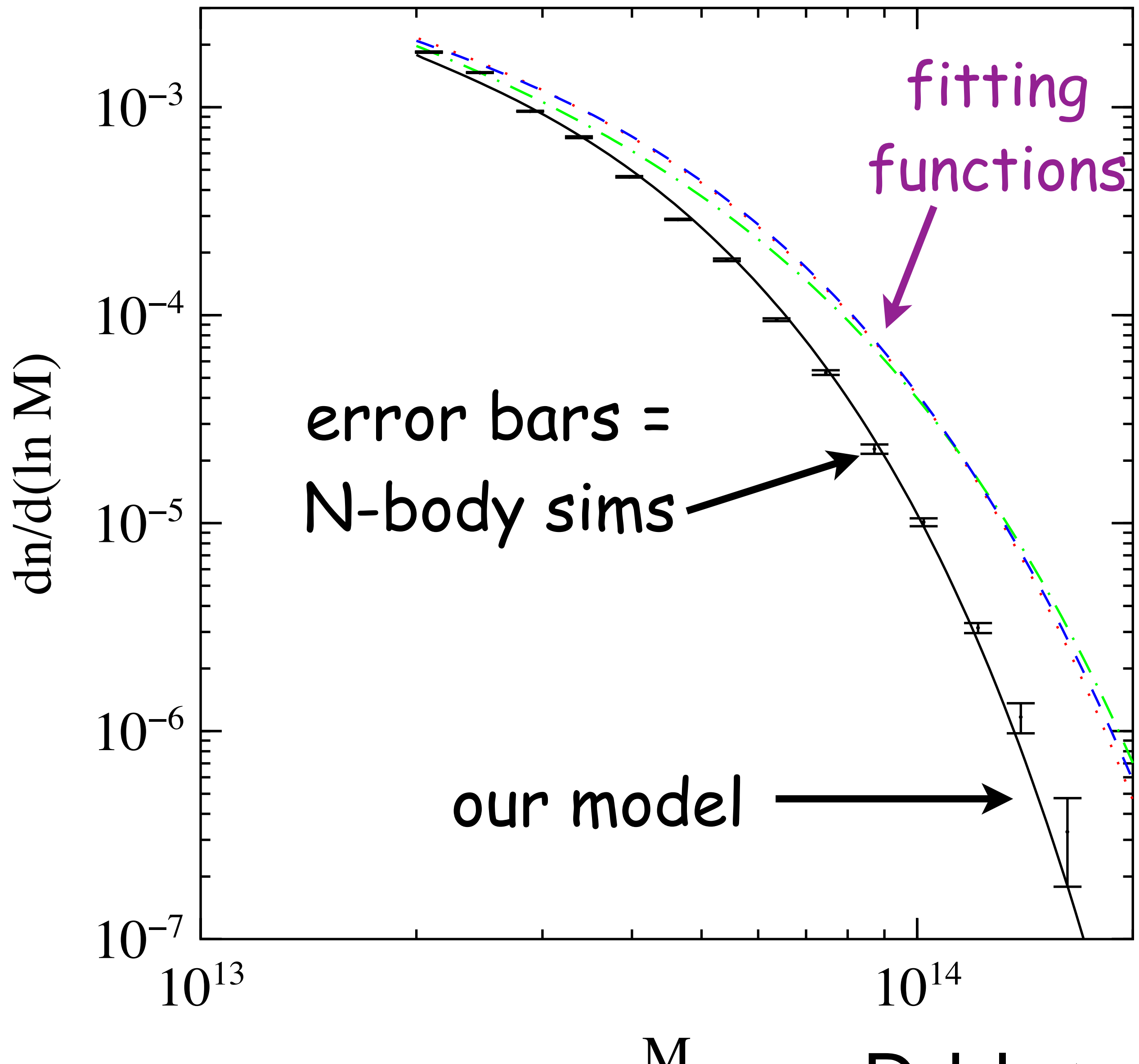
ΛCDM mass function



Dalal et al. (2011)

halo mass function

Example: FoF dn/dM for Ω_m=1, $P(k) \propto$ const.



our model (black)
Warren et al. (red)
Jenkins et al. (blue)
Sheth et al. (green)

Dalal et al. (2011)

other statistics

the same model trivially predicts other important halo statistics, like

- <u>clustering</u> (e.g. 2-pt function)
- halo <u>concentrations</u>
- halo <u>growth rates</u>
- <u>assembly bias</u>

other applications

- using this approach, we can predict how halo properties change for alternative cosmologies:

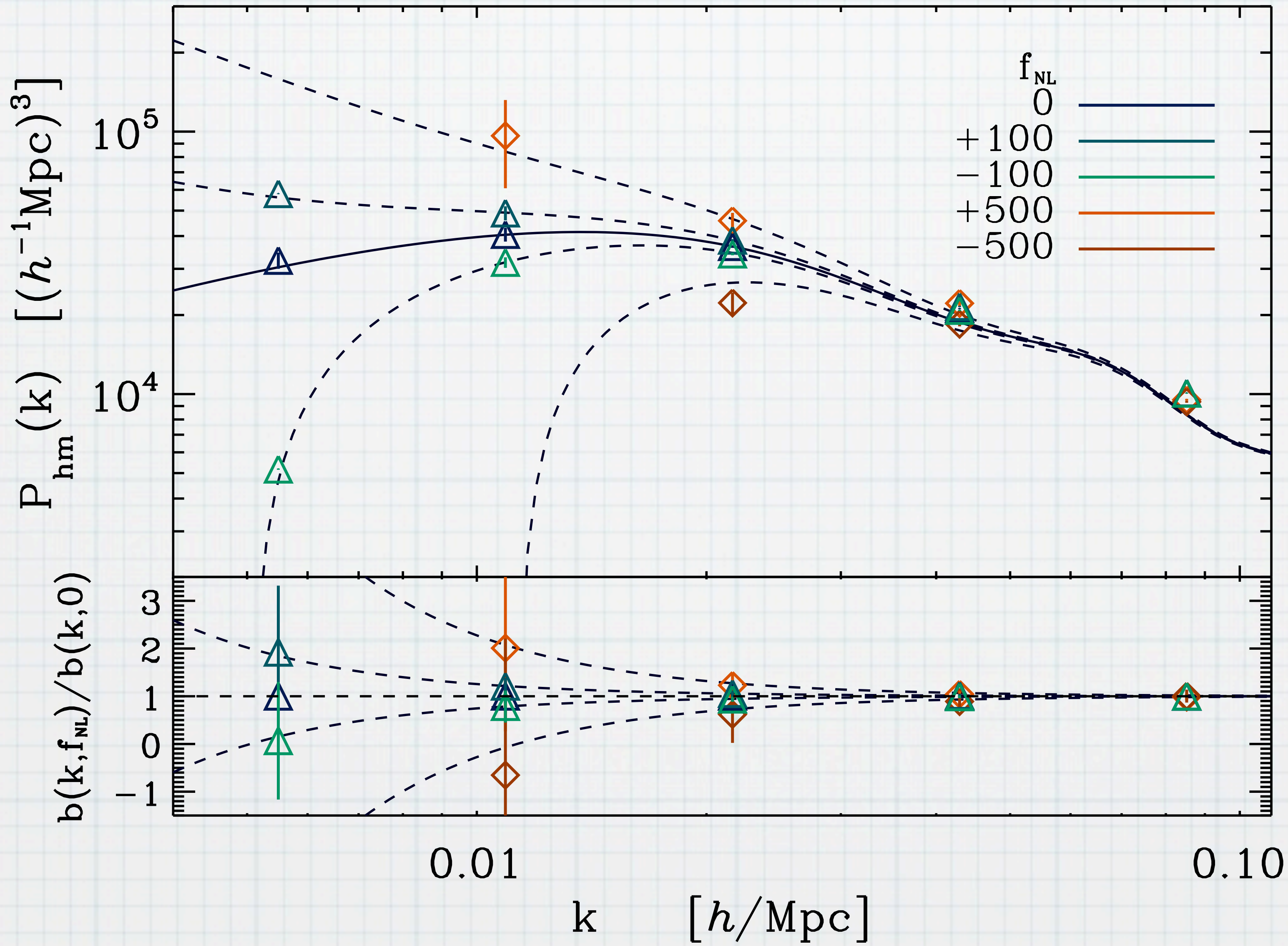
  1. non-gaussianity (Dalal et al. 2008a)
  2. warm dark matter (Villaescusa-Navarro & Dalal 2010)
  3. modified gravity (in prep.)

1. Non-Gaussianity

- Primordial NG is a powerful probe of early universe physics
- Essentially every early universe model (e.g. inflation, cyclic, etc...) all predict **some** NG
- the detailed form of the NG contains lots of info on the physics of the early universe
- so this is a HUGE industry in cosmology

Non-gaussianity

$$\Delta b(k) = 2 b_L f_{\rm NL} \delta_{\rm crit} \frac{3\Omega_m}{2 a g(a) T(k) r_H^2 k^2}$$

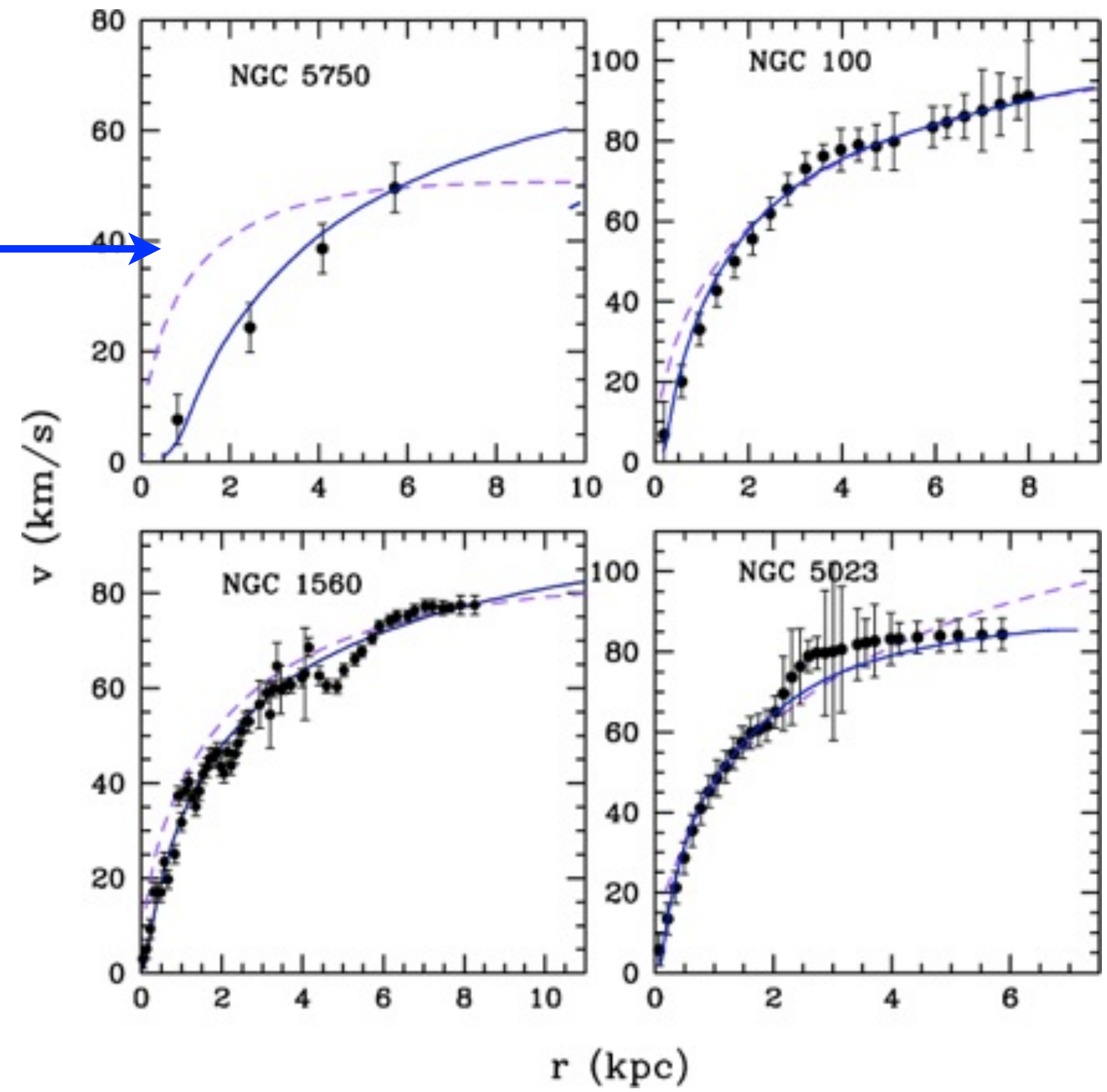


allows constraints $|f_{NL}| \sim 1$, approaching guaranteed detection regime!

2. Warm dark matter

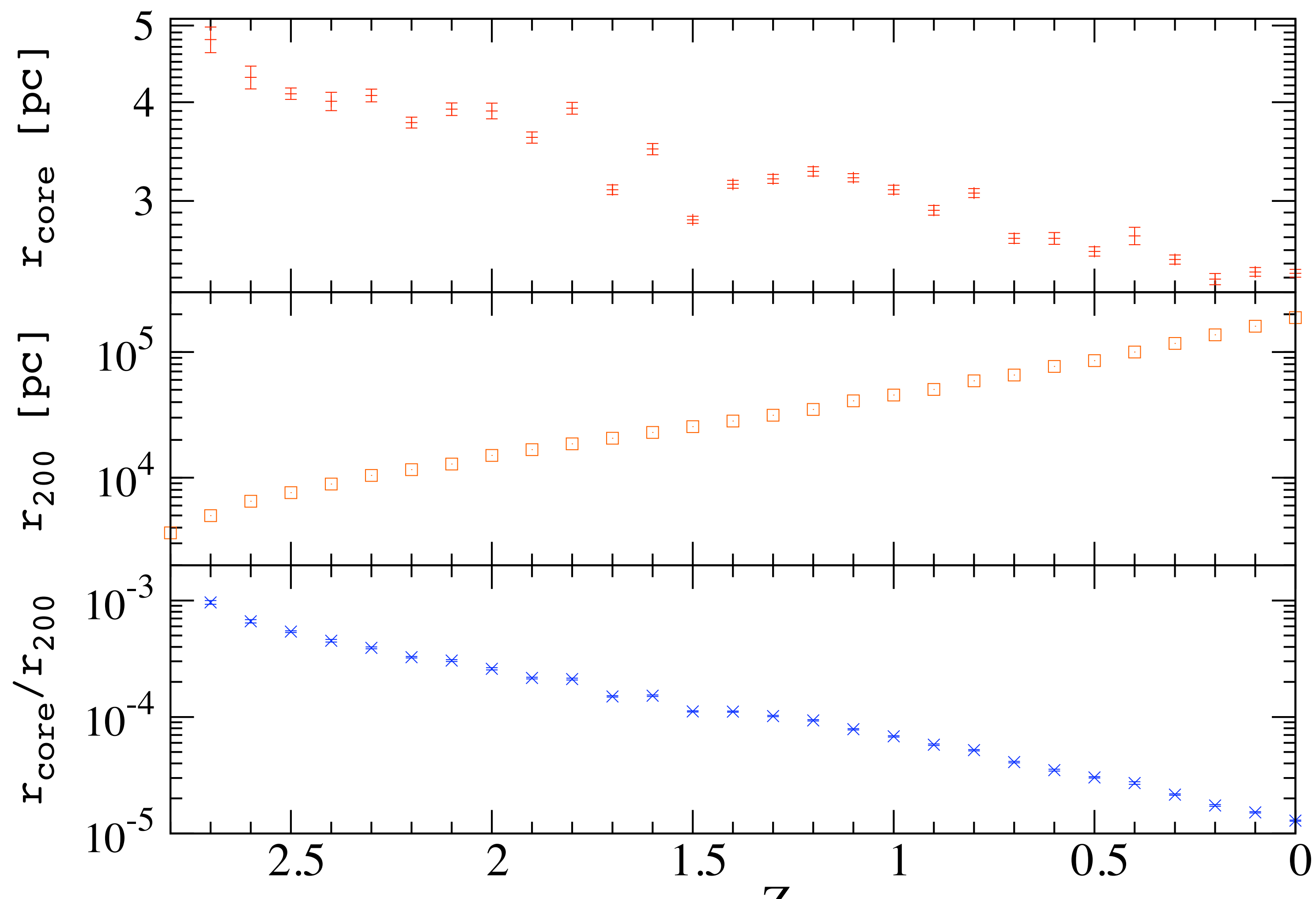
- for Cold DM, we expect high central densities of DM in halos
- observationally, some dwarfs may have cores instead of cusps

cusp/core problem?



WDM: tiny cores

core size is typically $< 10^{-3}$ of halo size



Villaescusa-Navarro & Dalal (2010)

Summary (1)

- dark matter halos are fundamental to modern cosmology
- we have presented a new, **simple** way to understand the properties of DM halos
- internal structure of halos may be understood by applying **adiabatic contraction** to the profiles of initial peaks

Summary (2)

- halo statistics (abundance, clustering, etc.) may be understood from the statistics of the progenitor peaks
- our framework allows us to understand what happens to halos in different cosmologies, e.g.
    - primordial non-gaussianity
    - warm dark matter, modified gravity, etc.

future

- Dynamics of triaxial halos
  - orbital families, resonance/chaos
- Properties of substructure from sub-peaks
- Generalization to include hydro / dissipation
  - building towards understanding how baryons affect dark matter
- Beyond the SM
  - modified gravity theories (f(R), DGP, ...)

Summary

- we can understand many properties of halos by considering peak properties
- in this talk I focused on basic properties like profile, mass function, etc., but the peaks viewpoint also helps illuminate more detailed properties (e.g. assembly bias)
- the same basic formalism can be used to see what changes for different cosmologies, e.g. with nongaussianity or modified gravity...