New Developments in Analyzing Gravitational Wave Data For Compact Binary Mergers

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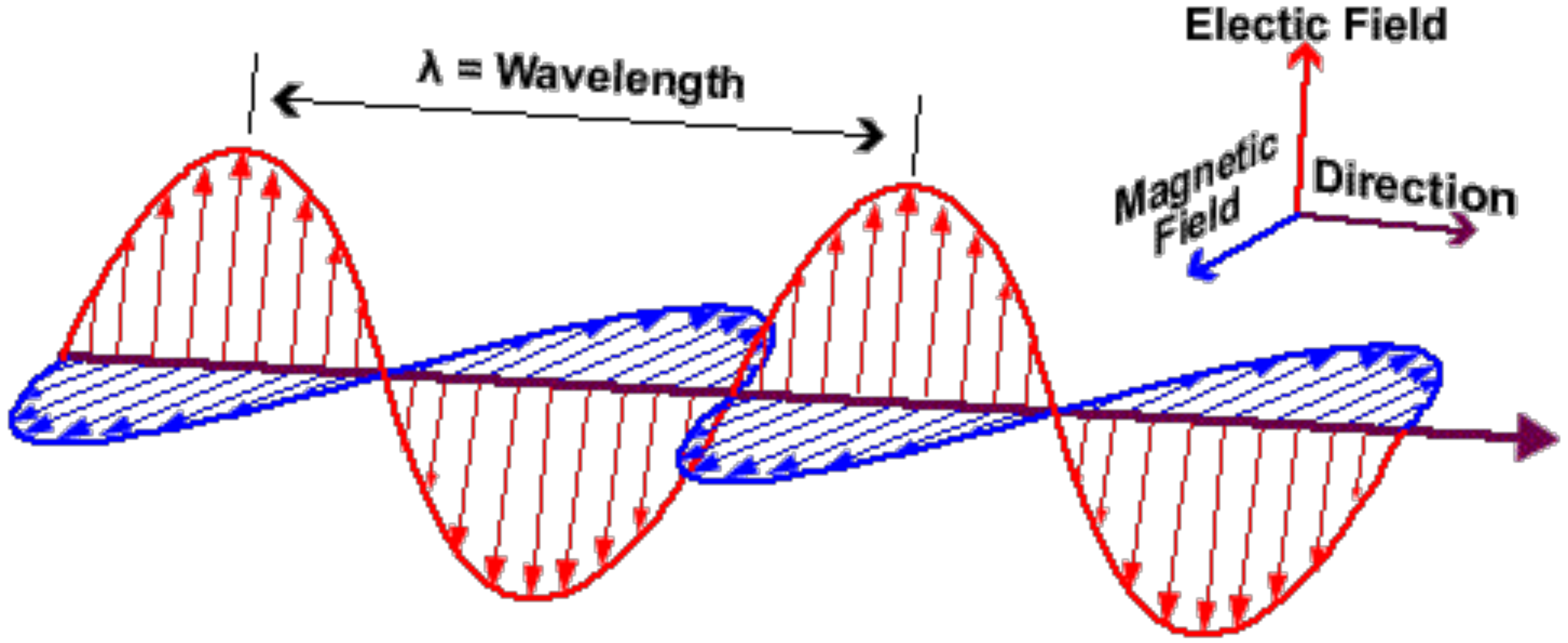
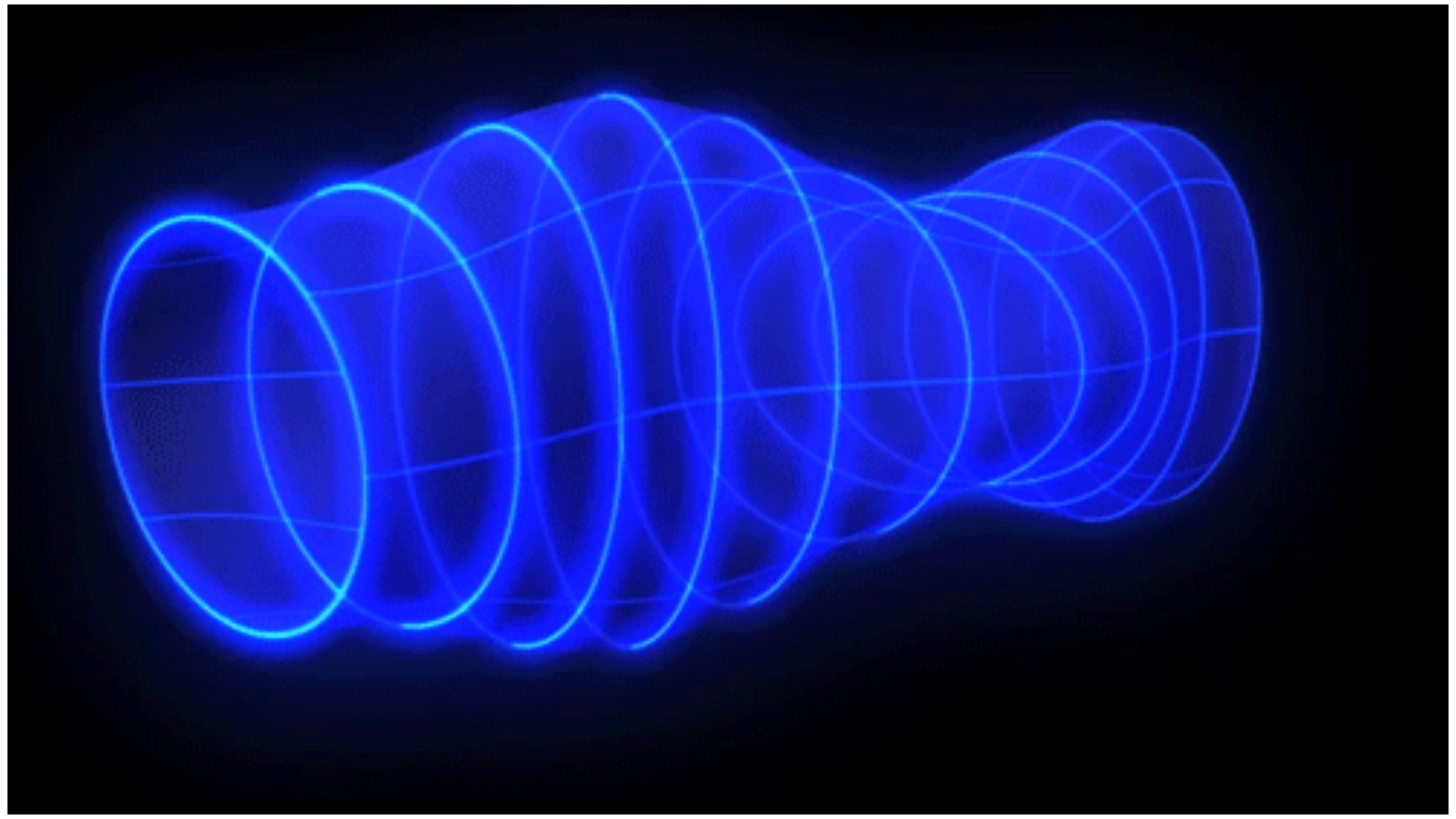
A New Window Into the Universe

Electromagnetic waves

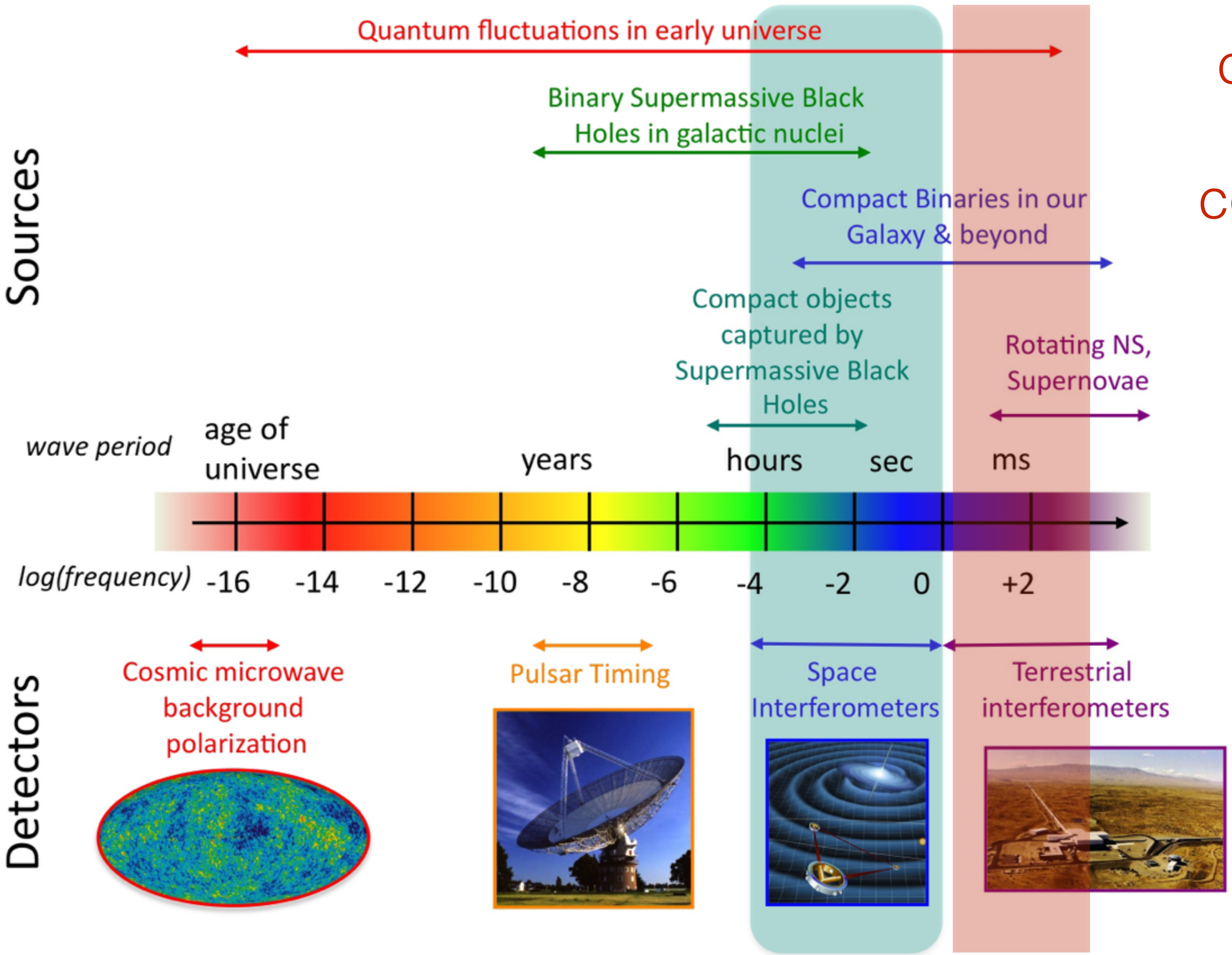


Gravitational waves:

Transverse sinusoidal distortion in the **space-time metric**



Gravitational Wave Spectrum



Sources

Quantum fluctuations in early universe

Binary Supermassive Black Holes in galactic nuclei

Compact Binaries in our Galaxy & beyond

Compact objects captured by Supermassive Black Holes

Rotating NS, Supernovae

wave period age of universe years hours sec ms

log(frequency) -16 -14 -12 -10 -8 -6 -4 -2 0 +2

Detectors

Cosmic microwave background polarization

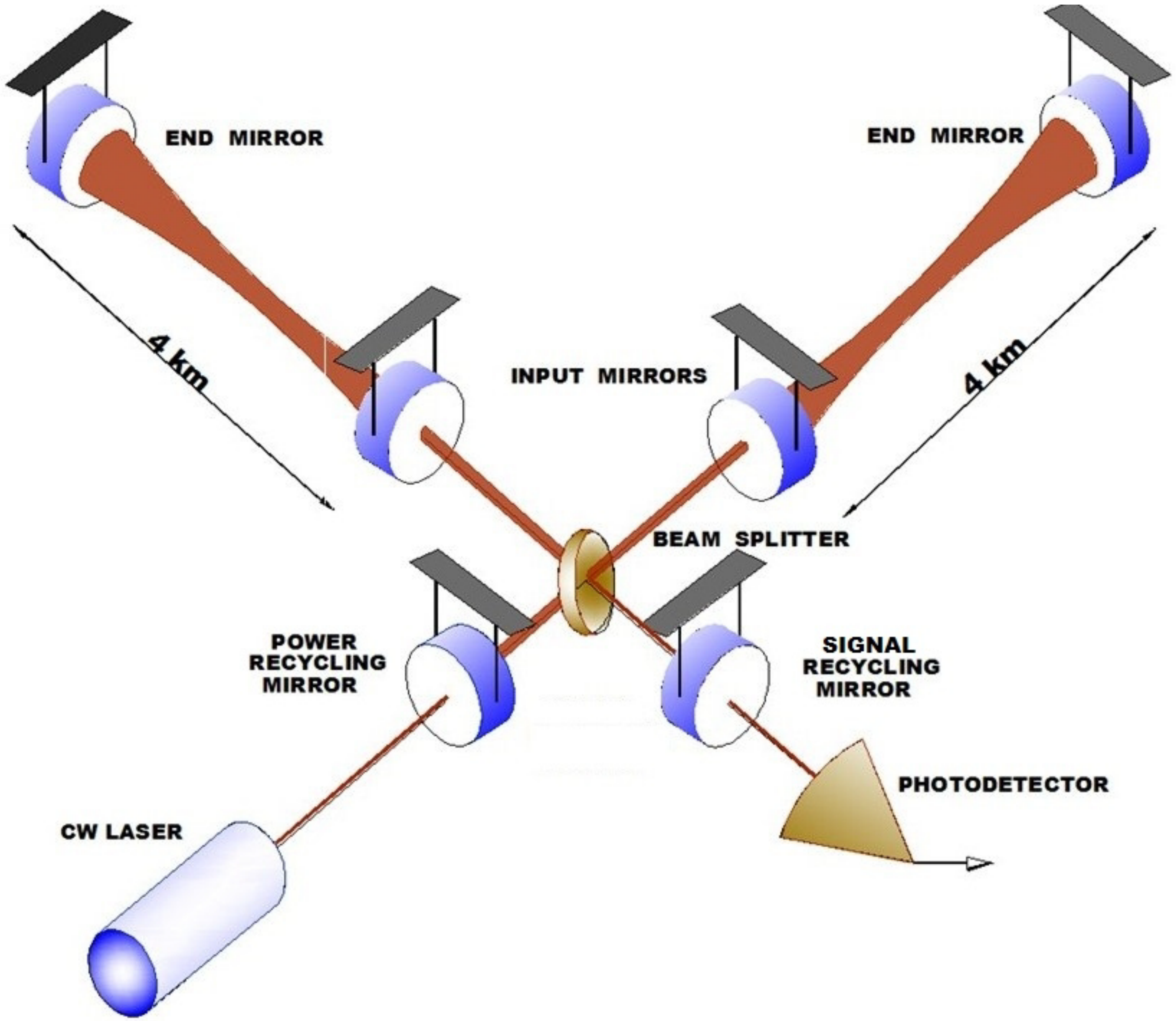
Pulsar Timing

Space Interferometers

Terrestrial interferometers

coalescence of stellar mass compact objects

Laser Interferometry



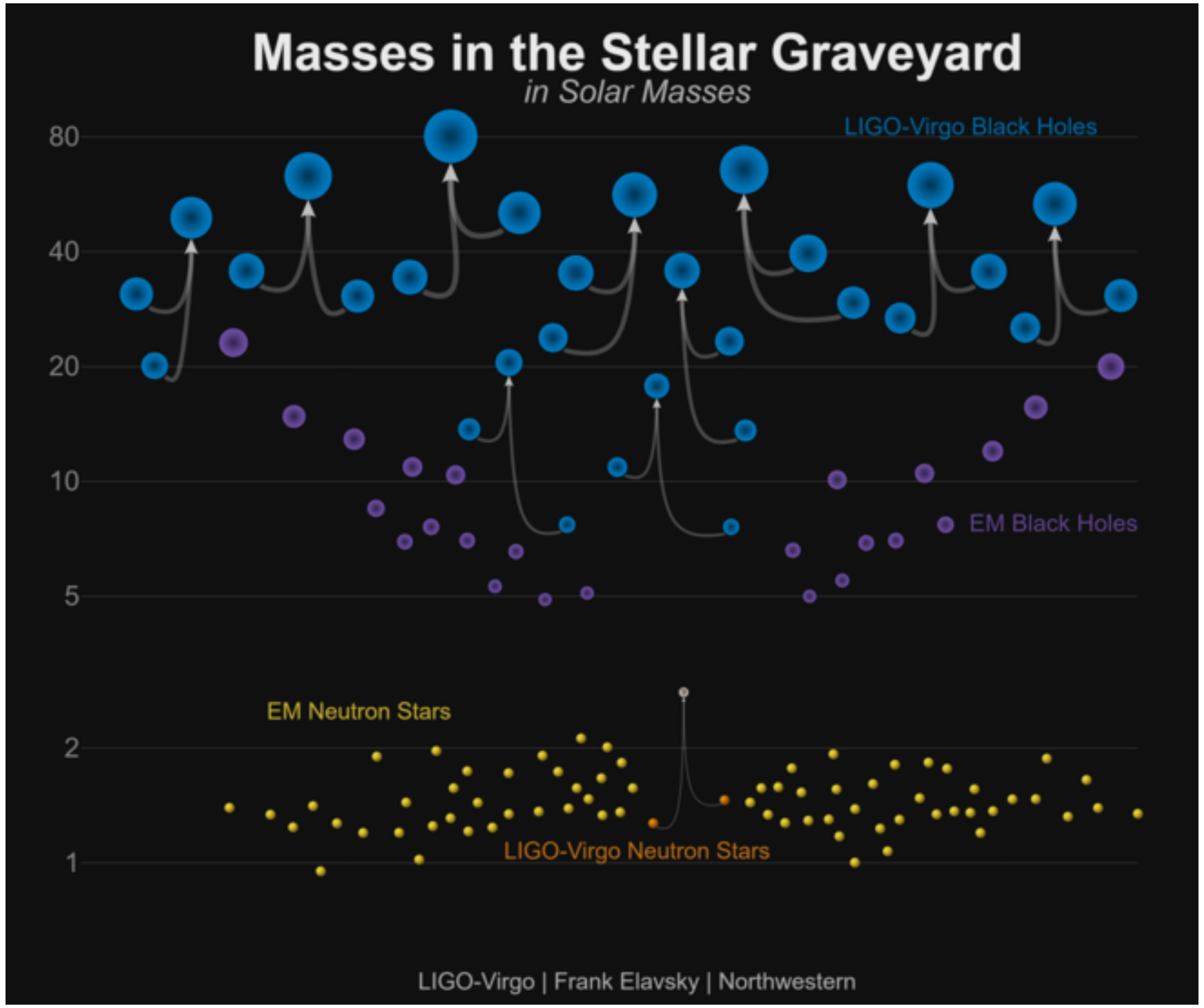
Measure **strain** as a function of time

$$h(t) = \frac{L(t)}{L_0} - 1$$

GW observation differs from (most) light observation:

Full information about the **amplitude** and the **phase** of the wave

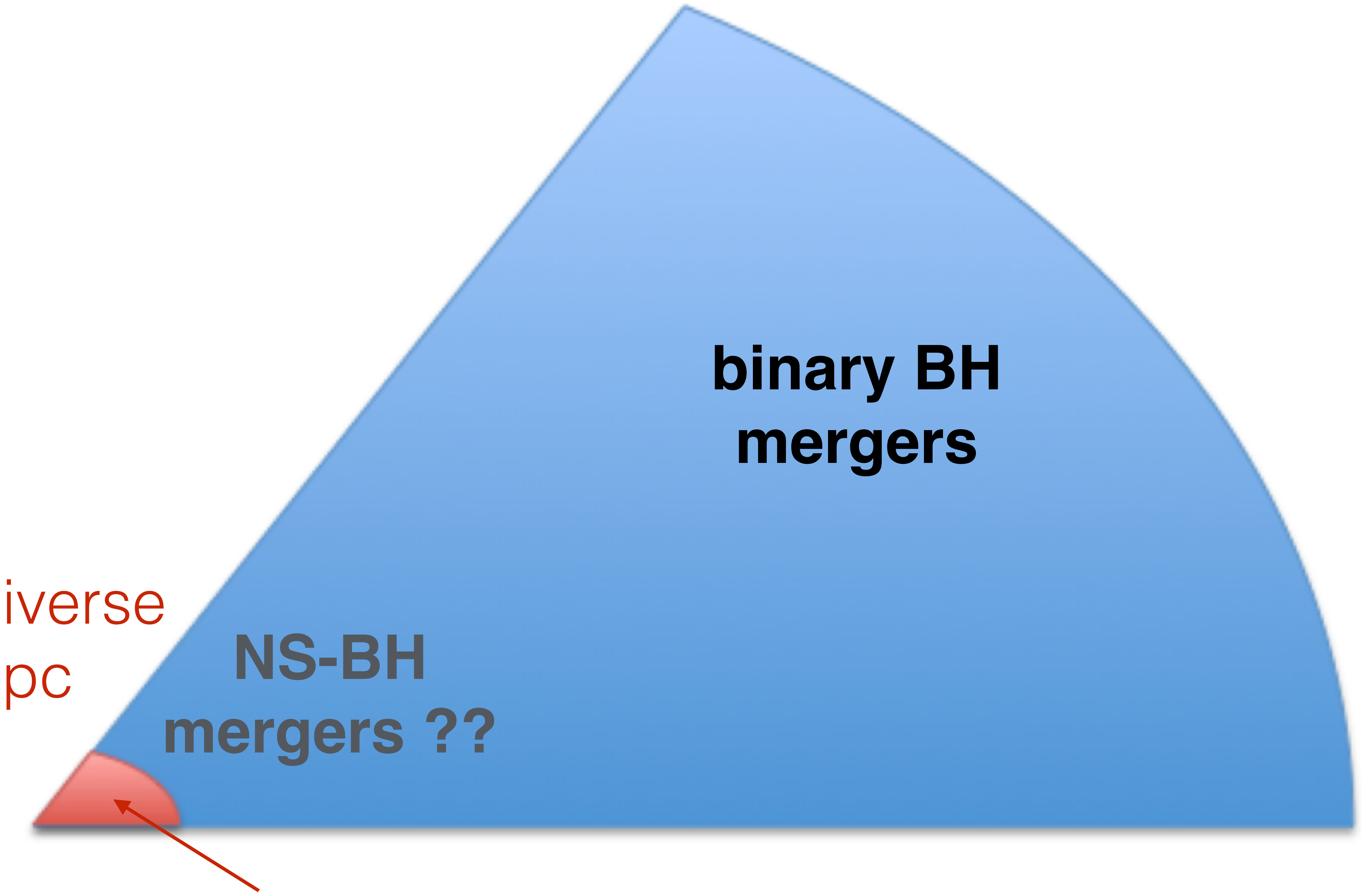
Compact Binary Mergers



Masses in the Stellar Graveyard

in Solar Masses

LIGO-Virgo | Frank Elavsky | Northwestern



cosmological distances
~ 1-3 Gpc

binary BH
mergers

low-z universe
~ 100 Mpc

NS-BH
mergers ??

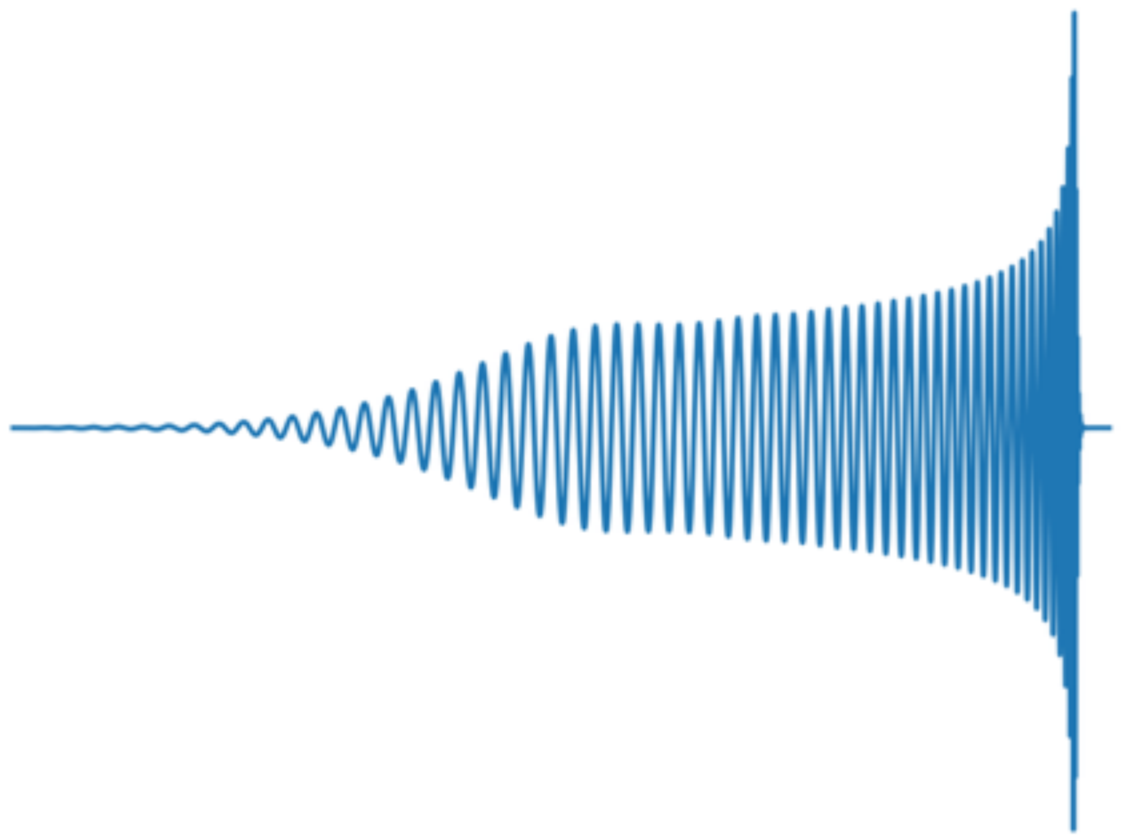
Milky Way

binary NS
mergers

GW Signal Detection

strain signal
expected from GR
waveform
“template”

$$h(t)$$



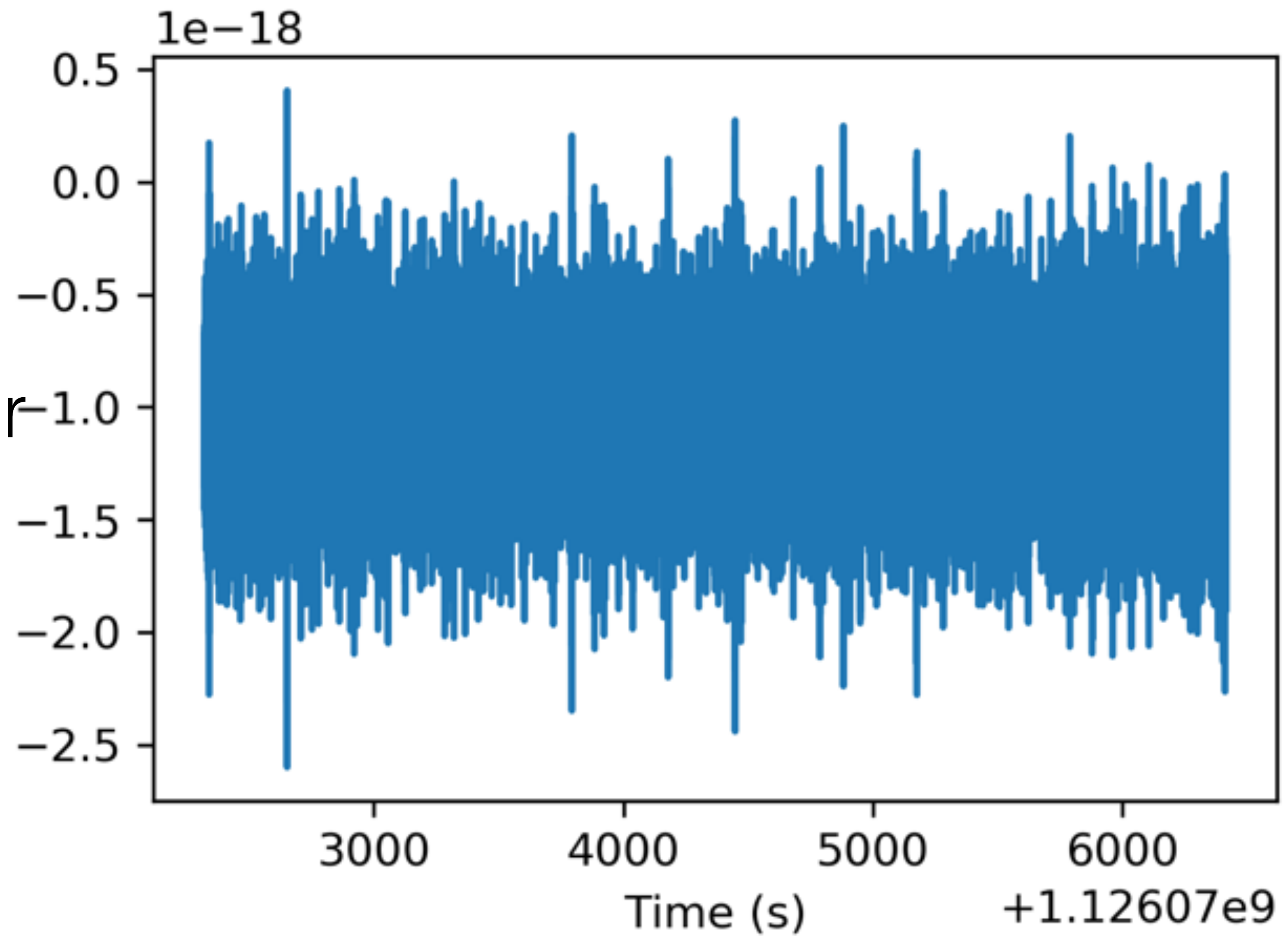
To find the needle in the haystack, use the technique of **matched filter**

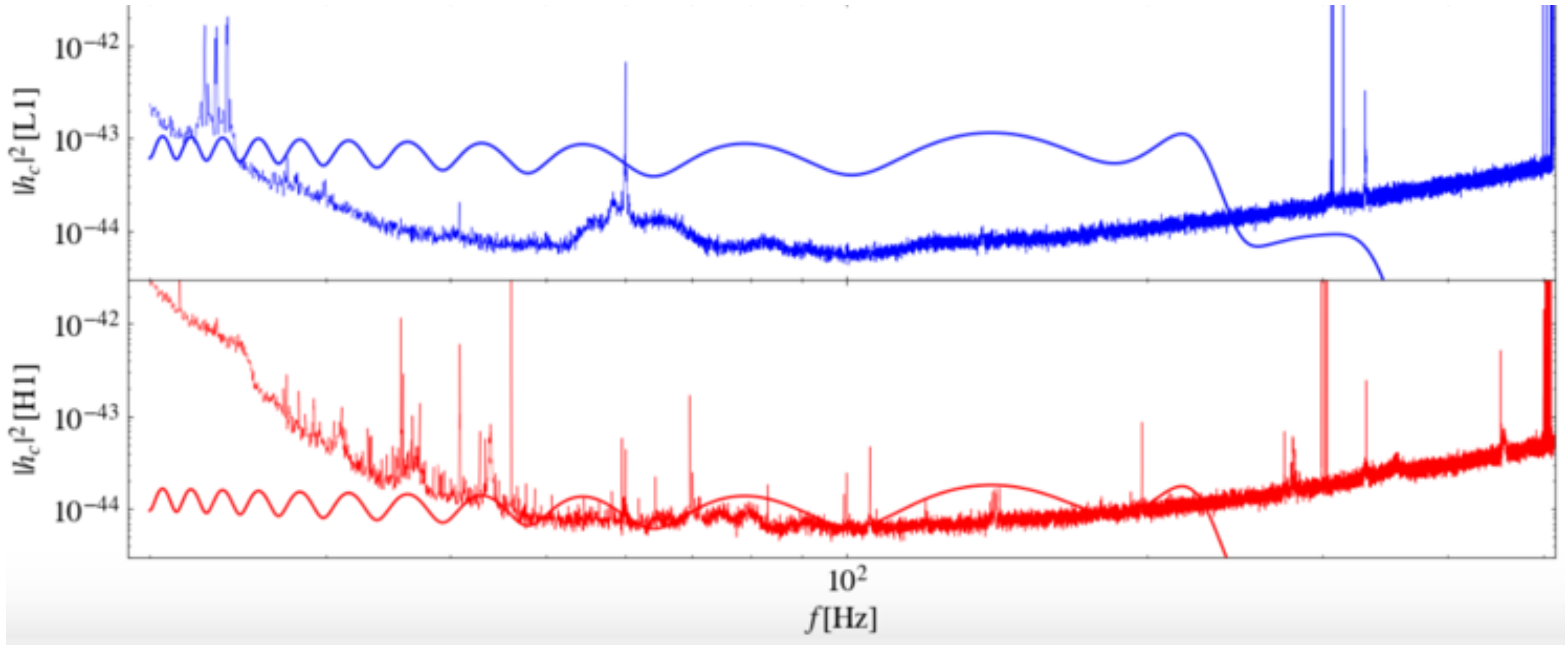
$$(d(t)|h(t+\tau)) := \sum_f \frac{d(f)\, h^*(f)\, e^{i\, 2\pi\, f\, \tau}}{S_N(f)/4}$$

noise **power spectral density** (PSD)

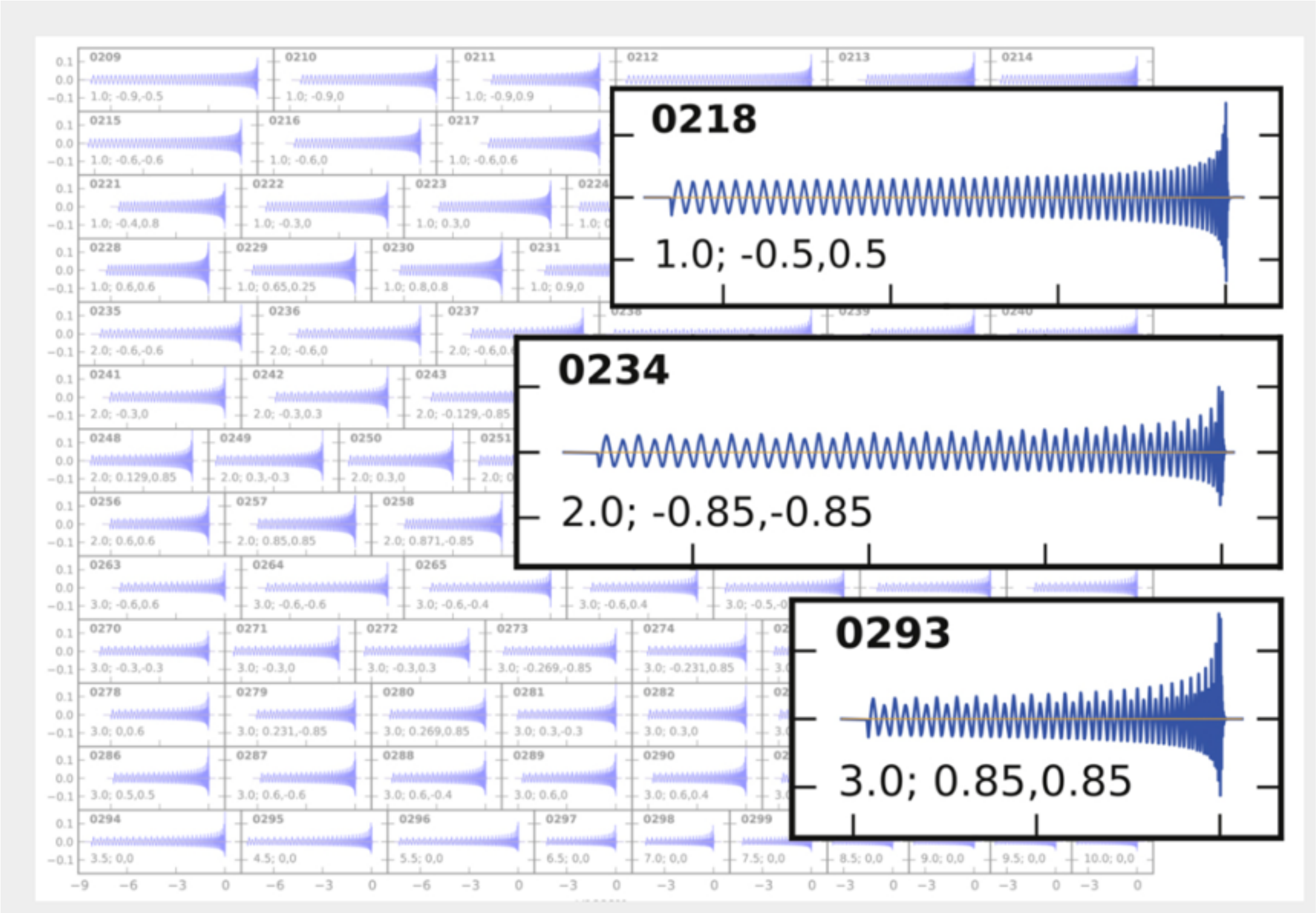
strain data as
recorded in detector

$$d(t)$$





Template Bank



- Necessary to try out a large number of waveform templates

- Should not try out templates that cannot be realized in any physical binary sources

- Should not repeat trying out templates that are indistinguishable from each other at given noise level.

- Should not repeat trying out templates that correspond to different source parameters but are actually (nearly) identical

Need an economic but effectual template bank !!

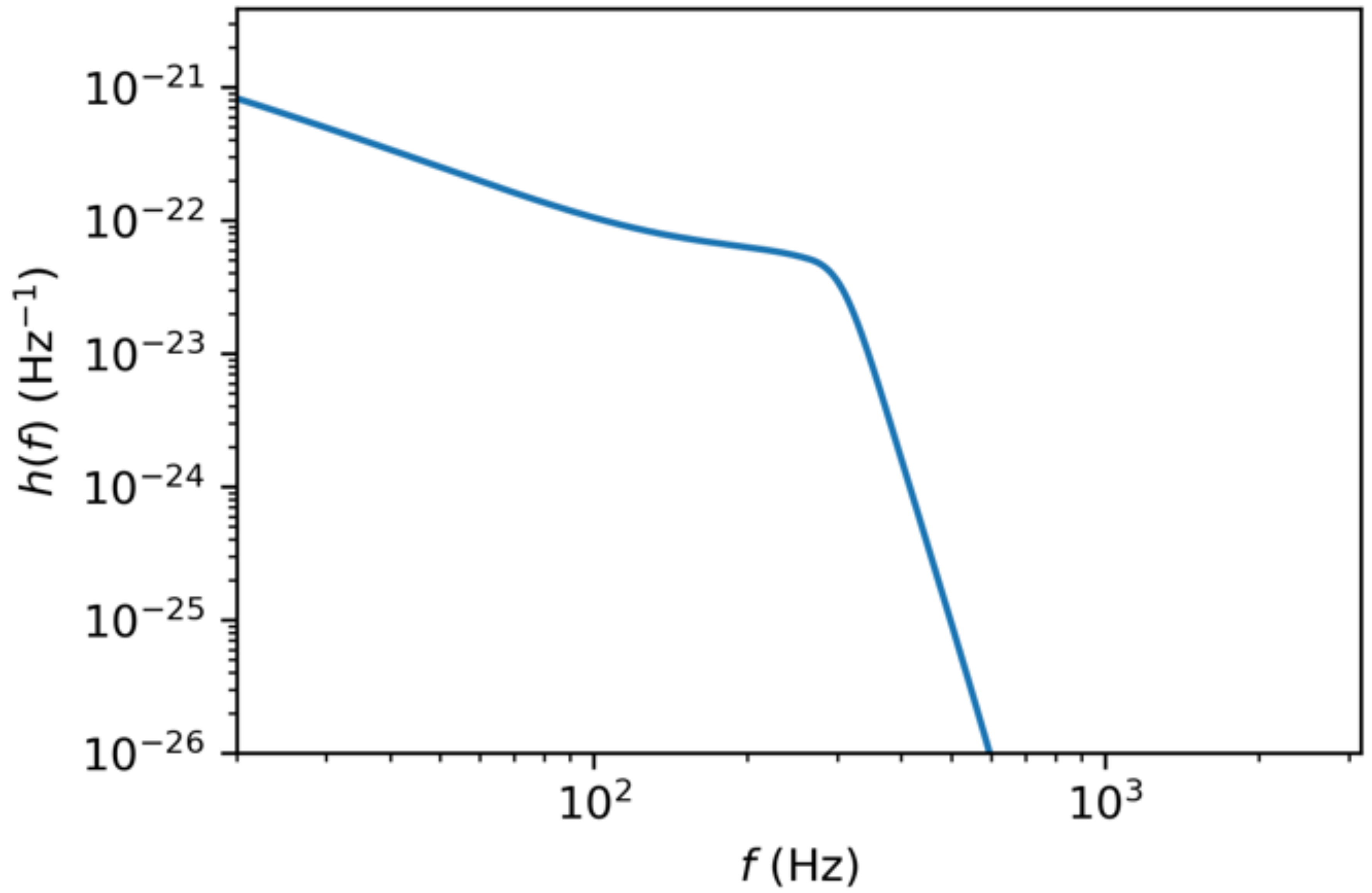
Template Bank: Amplitude and Phase

In frequency domain

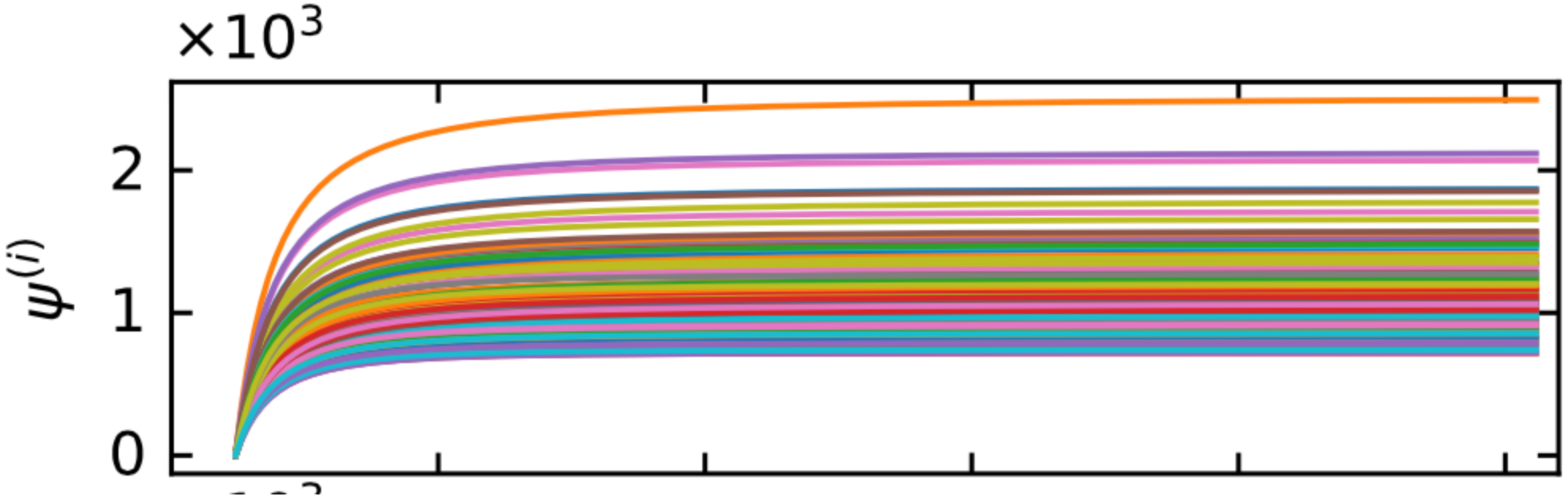
(unwrapped) phase profile

$$h(f;\boldsymbol{p}) = \boxed{A(f;\boldsymbol{p})}\, e^{i\boxed{\phi(f;\boldsymbol{p})}}$$

amplitude profile



Amplitude is smooth and non-oscillatory



The (unwrapped) phase evolves over many many radian.

Need to track to within a small fraction of an radian

Otherwise, “match” is lost.

A Geometric Solution

Roulet+ 1904.01683

Construct a linear space of phases

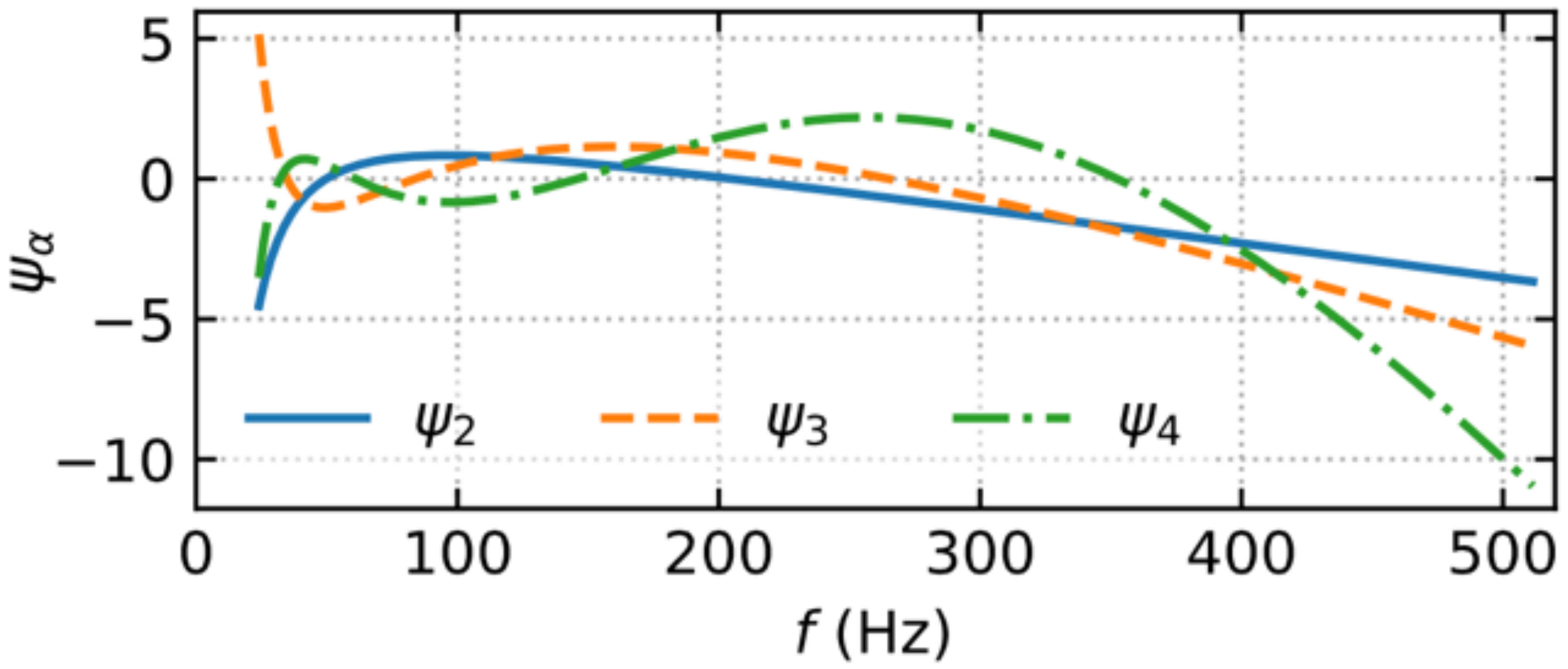
$$\phi(f) = c_0 + c_1 f + \sum_{\alpha=2}^{n} c_\alpha \psi_\alpha(f)$$

Orthonormalize the basis phase profiles

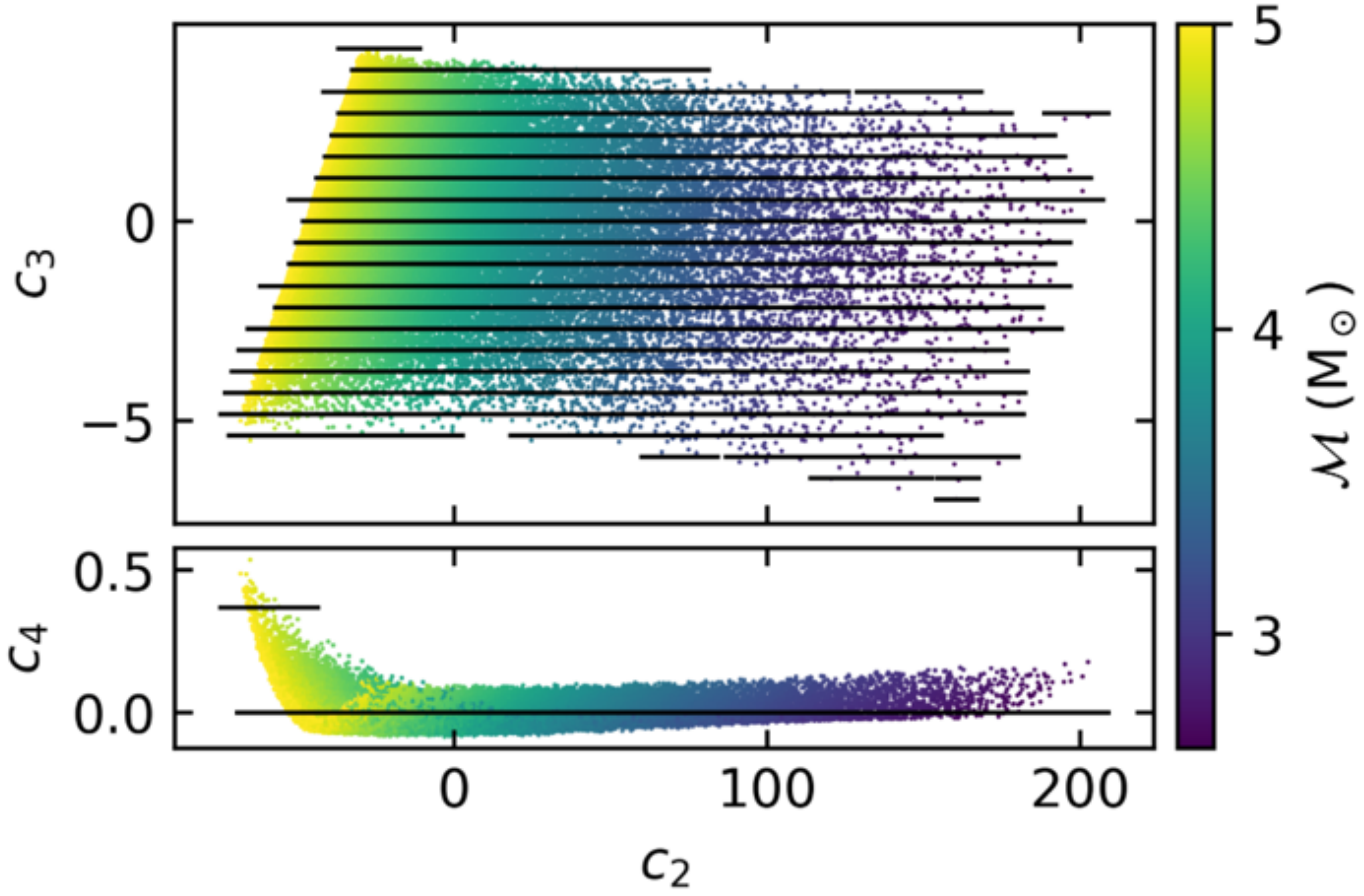
$$\langle \psi_\alpha, \psi_\beta \rangle = \sum_f \frac{\bar{A}^2(f)}{S_N(f)/4} \psi_\alpha(f) \psi_\beta(f) = \delta_{\alpha\beta}$$

Such that the "Euclidean" distance in terms of the c-coefficients measure "mismatch"

Only a few extra bases are needed



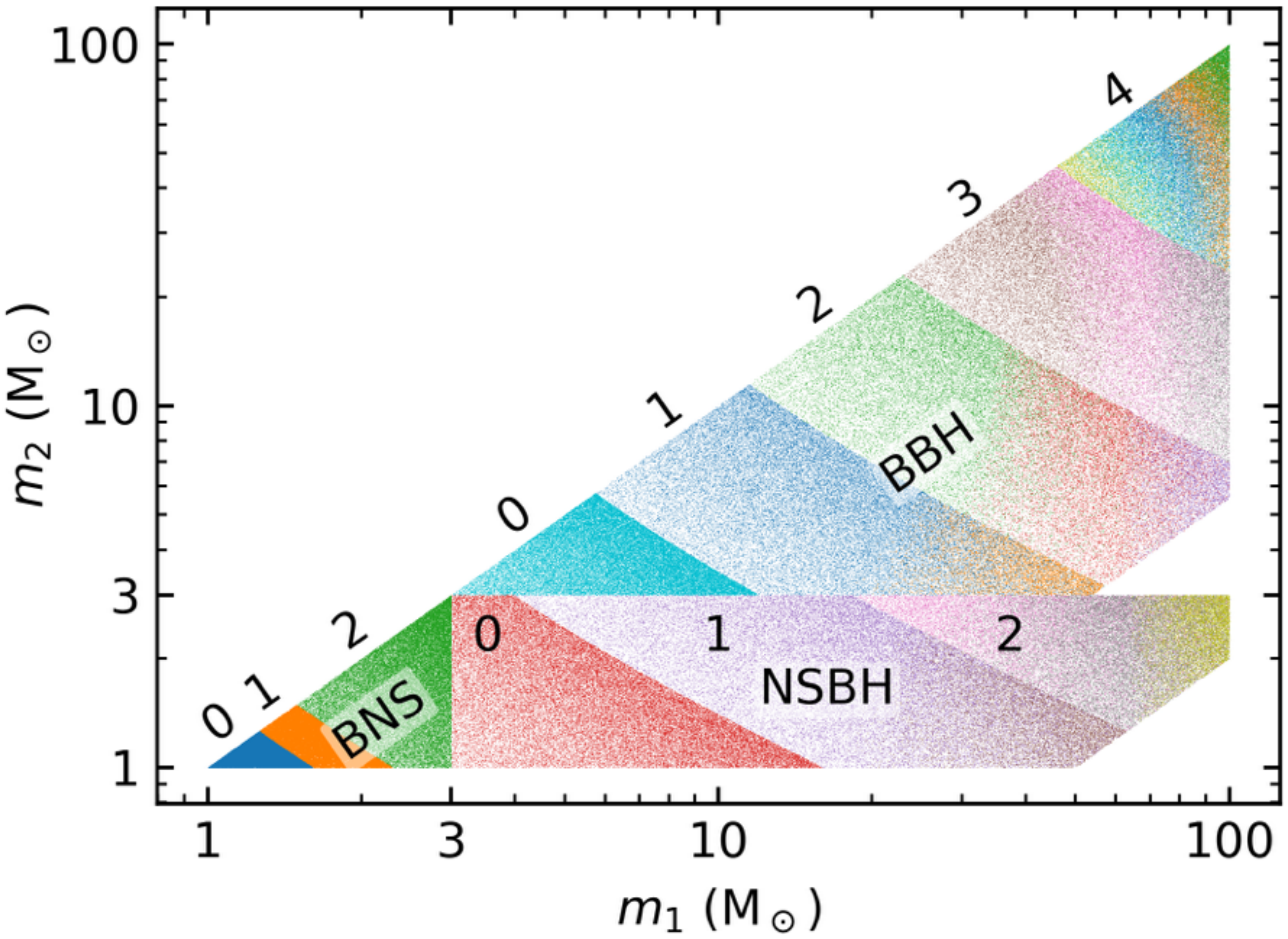
The bank is defined as a lattice



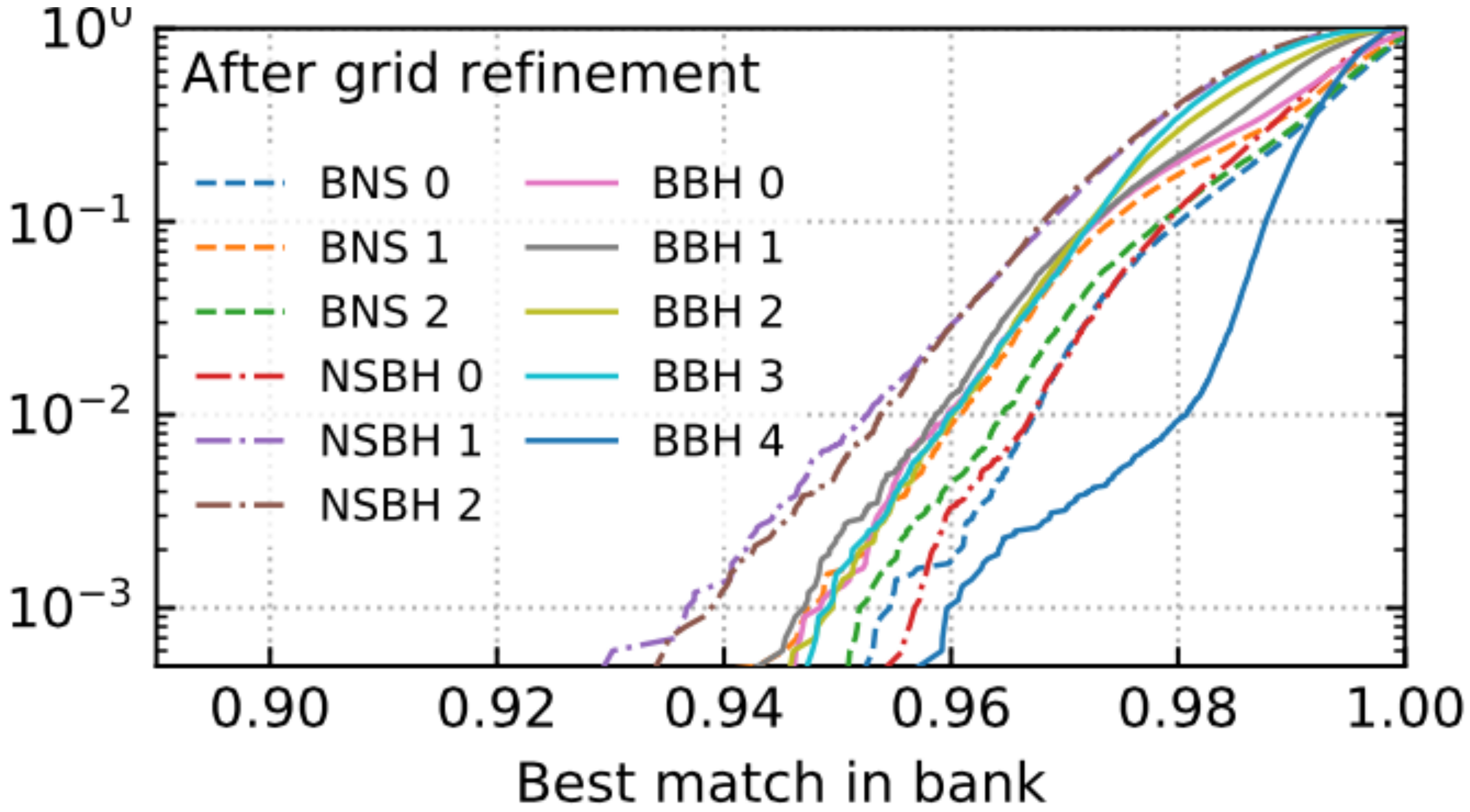
Template Banks According to Amplitude

Construct this metric space of phases for a group of waveforms that share similar amplitude profiles.

If used in search, only lose **a few percent in SNR² (i.e. match)**



Roulet+ 1904.01683



Matched Filter: Trigger Statistics

A template waveform only needs to be defined up to:

Babak+ 2013
Abbott+ 2017

An amplitude normalization;
A phase constant;
The time of arrival.

$$h(f) \longrightarrow A\, e^{i\,\phi_c + i\,2\pi\, f\, t_c}\, h(f)$$

For normalized template

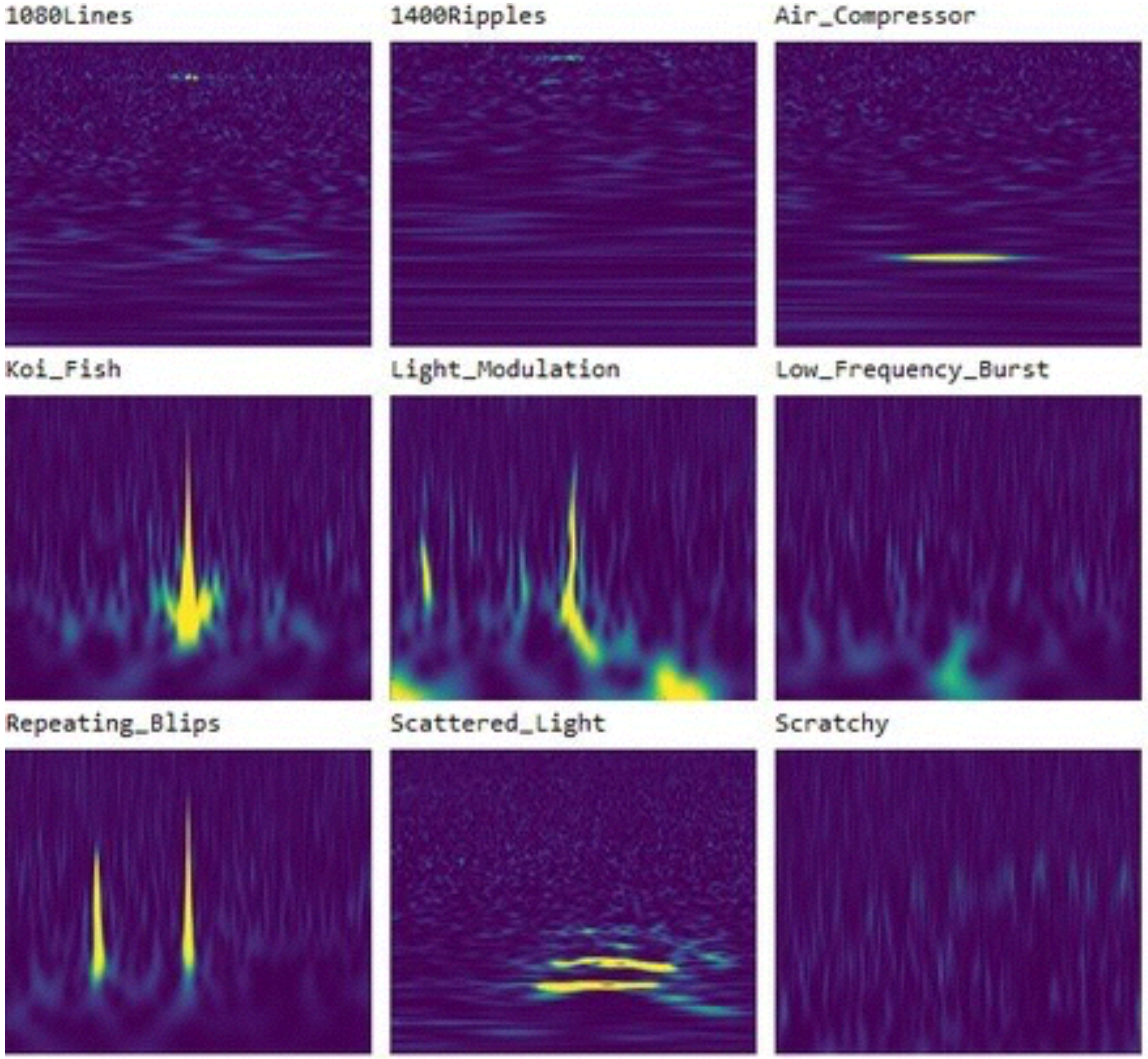
$$(h|h) = \sum_f \frac{|h(f)|^2}{S_N(f)/4} = 1$$

Define trigger score

$$Z(d|h) := (d|h) = \sum_f \frac{d(f)\, h^*(f)}{S_N(f)/4}$$

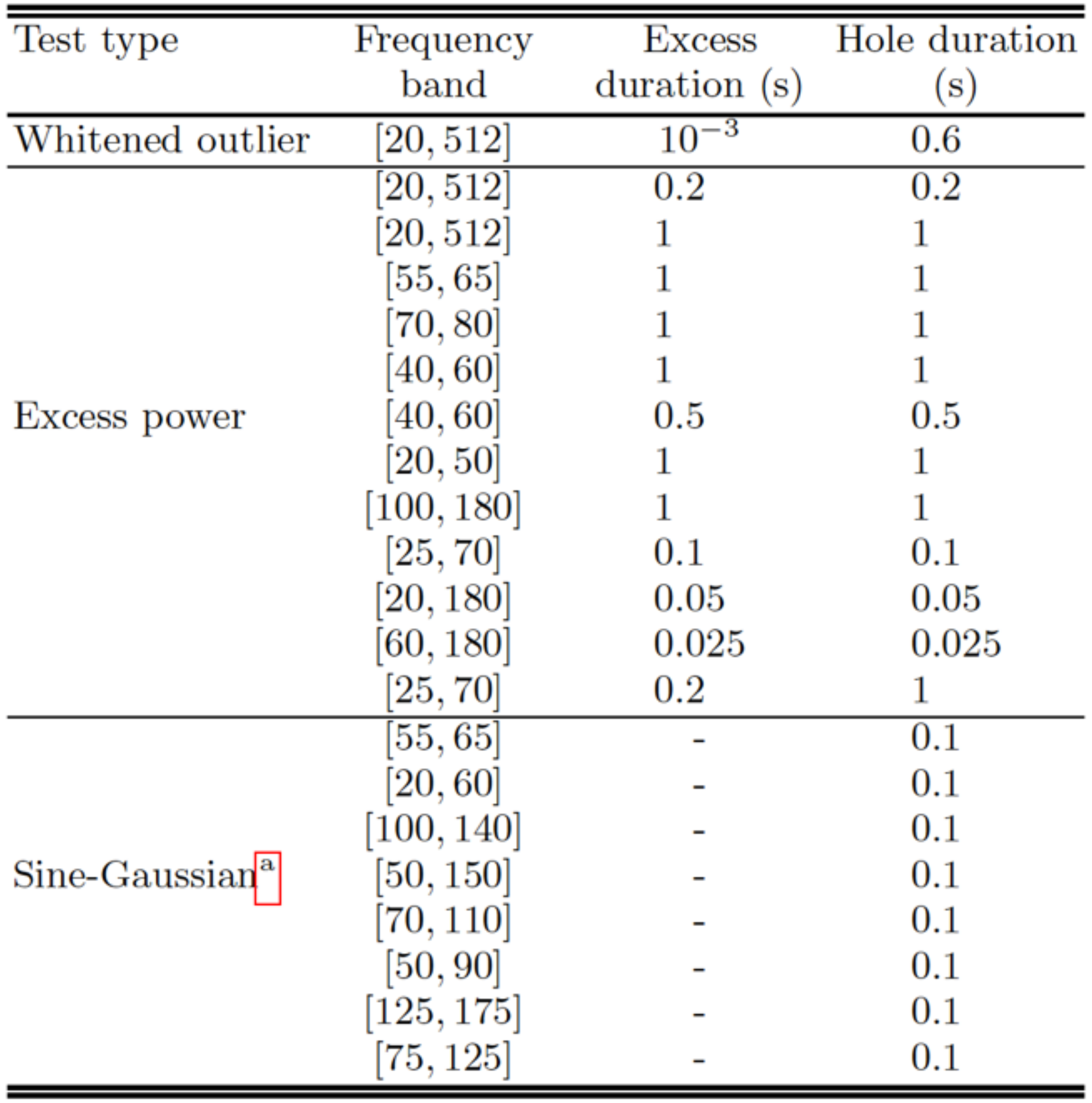
If perfect stationary gaussian noise, Z has **chi-square statistics with 2 DOF**
Astrophysical signals would stand out of the tail (decay exponentially with $|Z|^2$)

Noise Transients (“Glitches”)



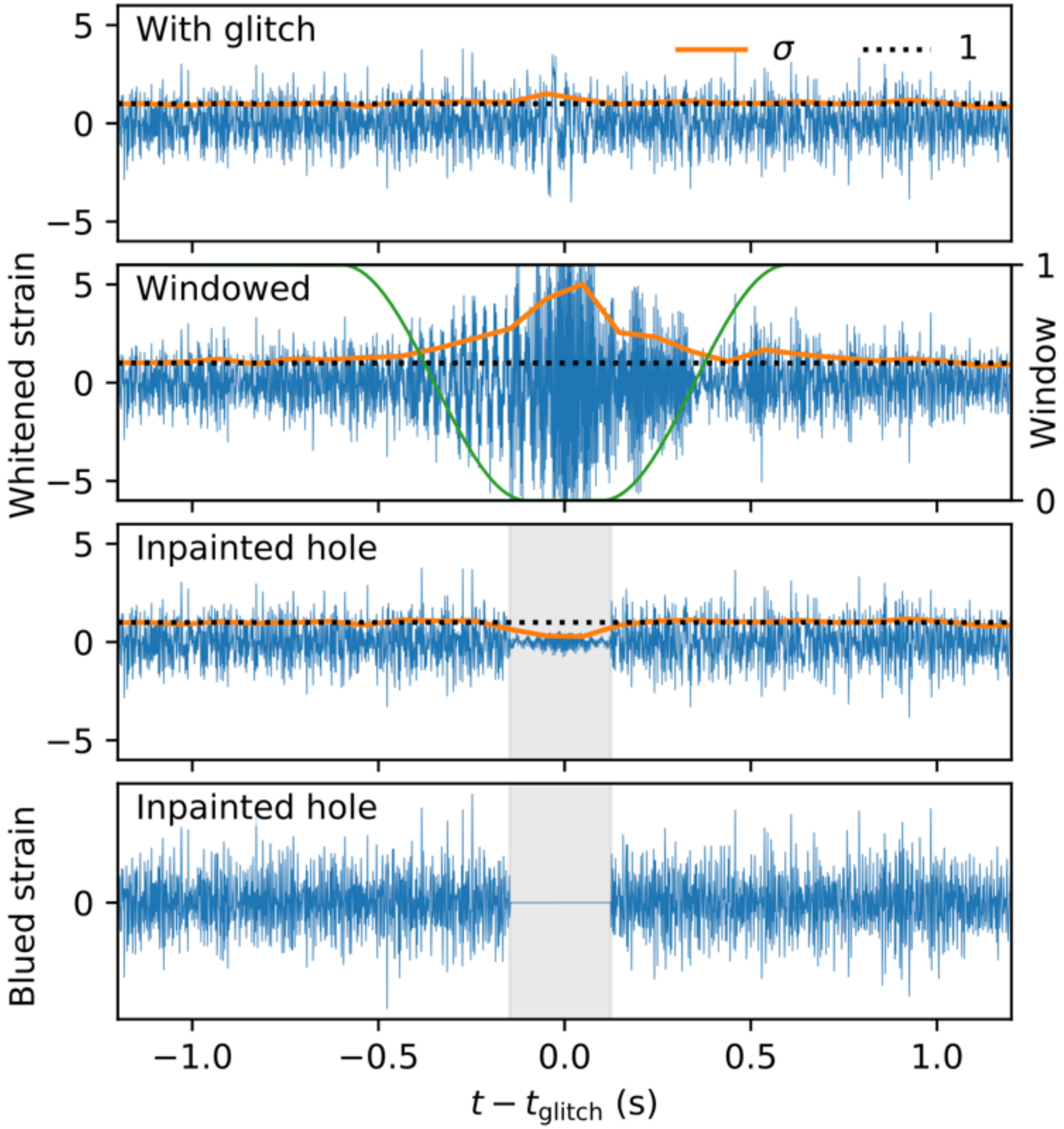
Abbott+ 2017

We identify and mask out bad seconds



Test type	Frequency band	Excess duration (s)	Hole duration (s)
Whitened outlier	$[20, 512]$	10^{-3}	0.6
Excess power	$[20, 512]$	0.2	0.2
	$[20, 512]$	1	1
	$[55, 65]$	1	1
	$[70, 80]$	1	1
	$[40, 60]$	1	1
	$[40, 60]$	0.5	0.5
	$[20, 50]$	1	1
	$[100, 180]$	1	1
	$[25, 70]$	0.1	0.1
	$[20, 180]$	0.05	0.05
	$[60, 180]$	0.025	0.025
	$[25, 70]$	0.2	1
Sine-Gaussian[a]	$[55, 65]$	-	0.1
	$[20, 60]$	-	0.1
	$[100, 140]$	-	0.1
	$[50, 150]$	-	0.1
	$[70, 110]$	-	0.1
	$[50, 90]$	-	0.1
	$[125, 175]$	-	0.1
	$[75, 125]$	-	0.1

Careful Treatment of Masked Data



Matched filter is **not local in time**

Zeroing the bad samples in time causes leakage of ringing artifacts

Conventional to apply smooth windows.

We solve a linear algebra problem to guarantee no leakage (analogous to inpainting the masked CMB sky within the Galactic plane)

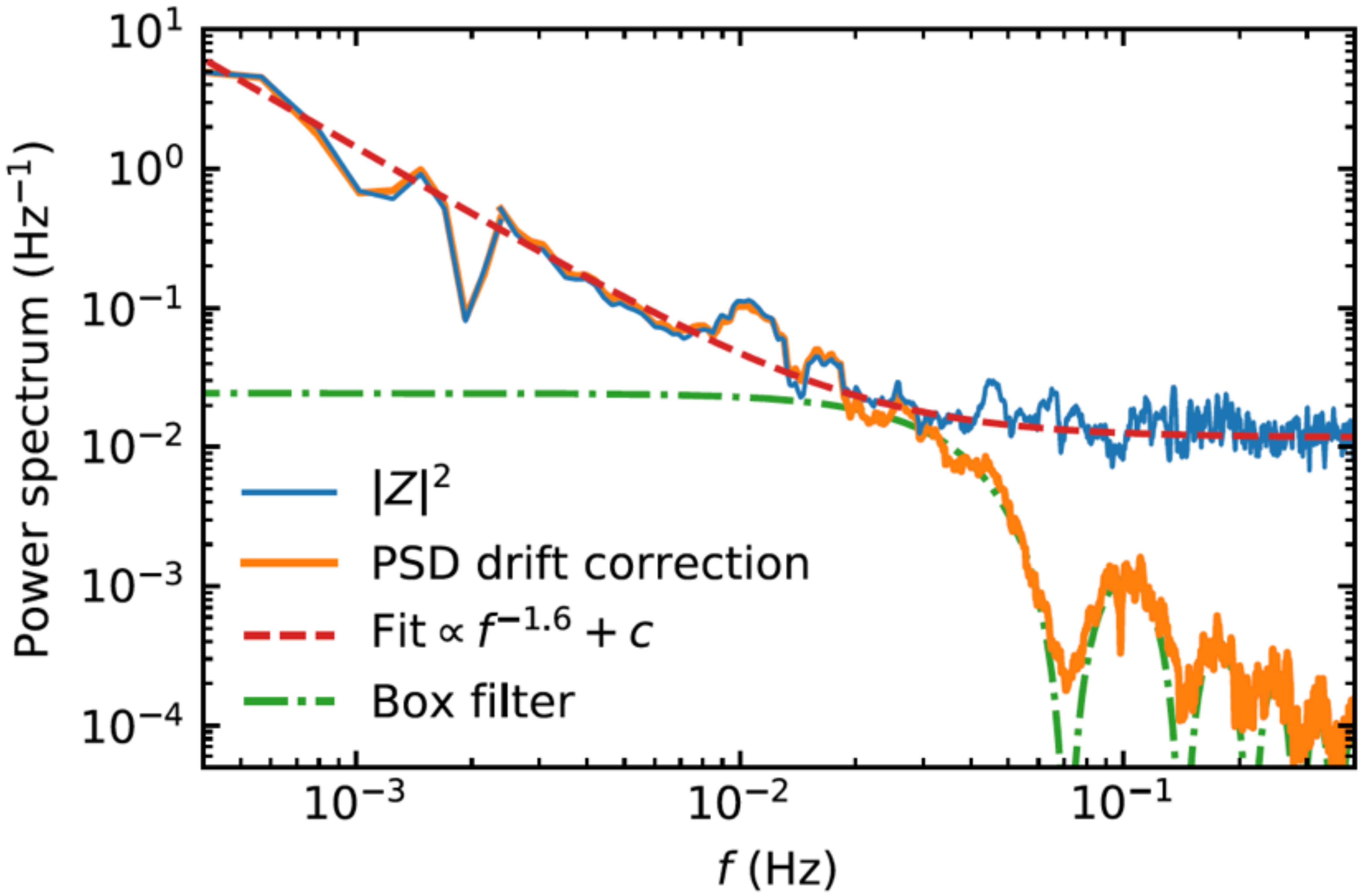
$$(d|h) = \sum_f \boxed{\frac{d(f)}{S_N(f)/4}} h^*(f)$$

Should just zero **the blued strain**

PSD Drift

In order to well resolve the lines in the PSD, need to measure over **O(1000) seconds of data**

Empirically, we found that the PSD is changing over O(10) seconds, by ~ 10%



Tail of the Z distribution is badly over-produced !

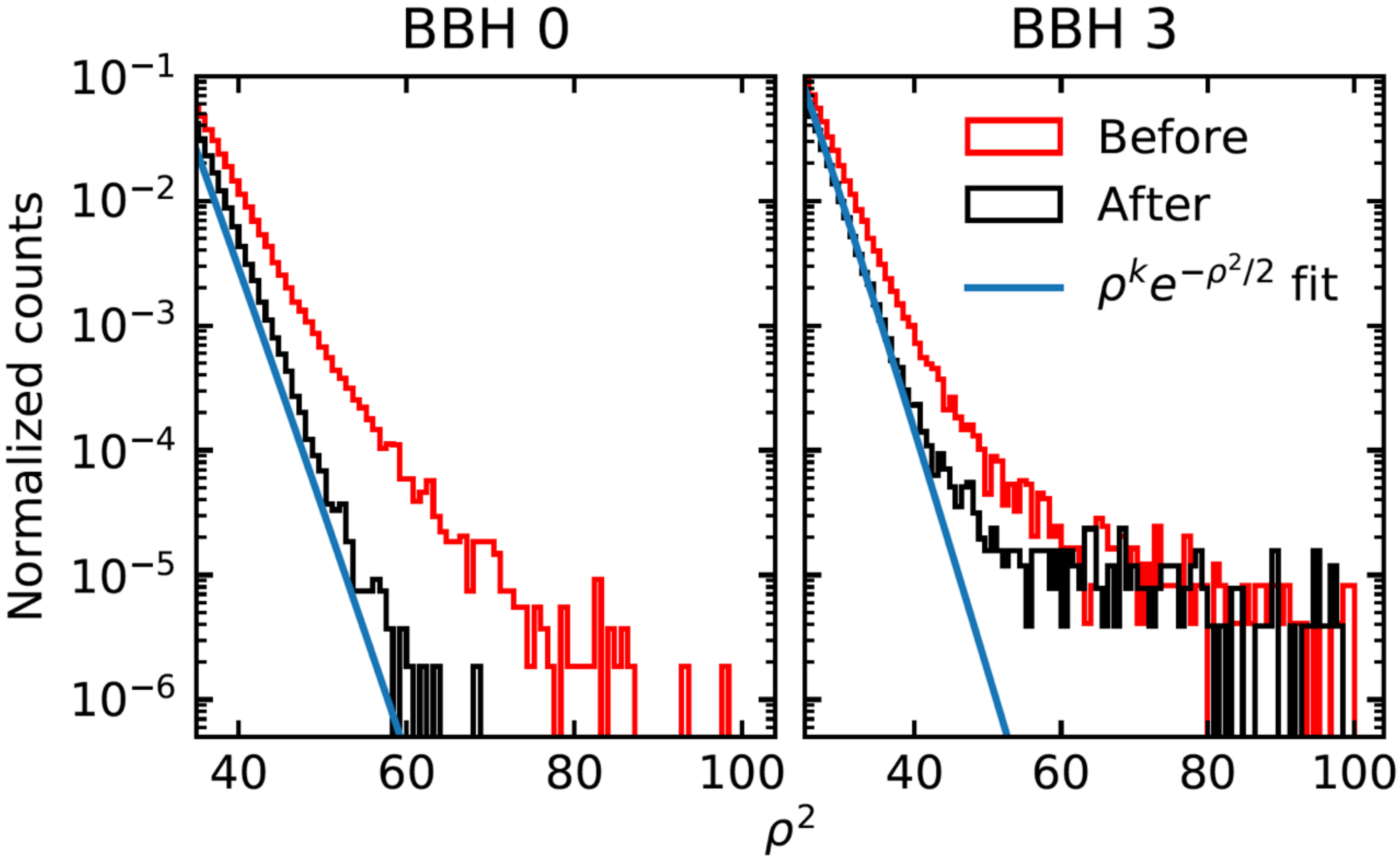
$$Z(d|h) := (d|h) = \sum_f \frac{d(f)\, h^*(f)}{S_N(f)/4}$$

Solution:
We track the std of Z to ~ 1% precision on time scales of O(10) seconds!

This correction only depends on the amplitude profile of h(f), but not the phase!

We therefore track PSD drift bank by bank.

Trigger Distribution After PSD Drift Correction

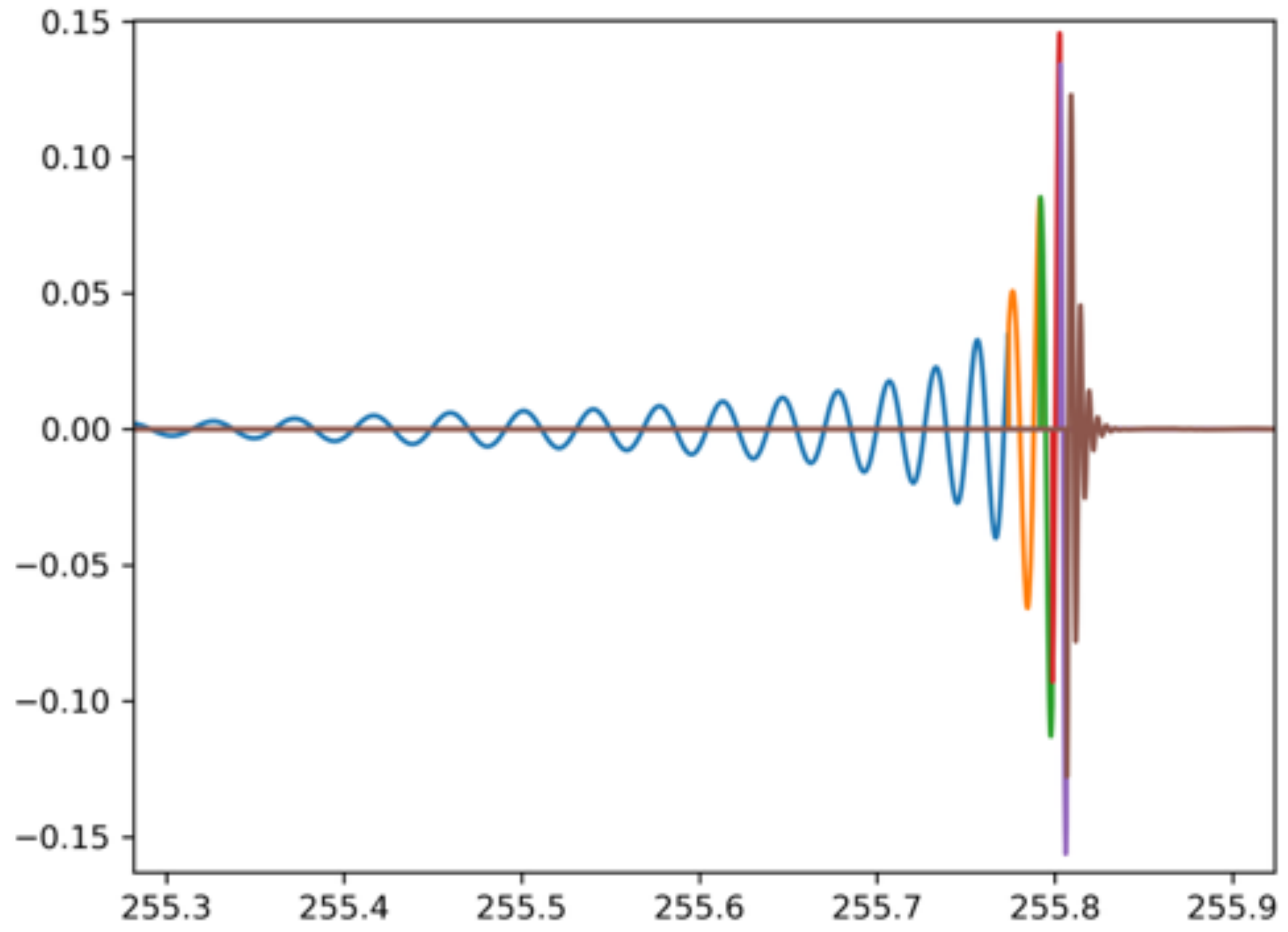


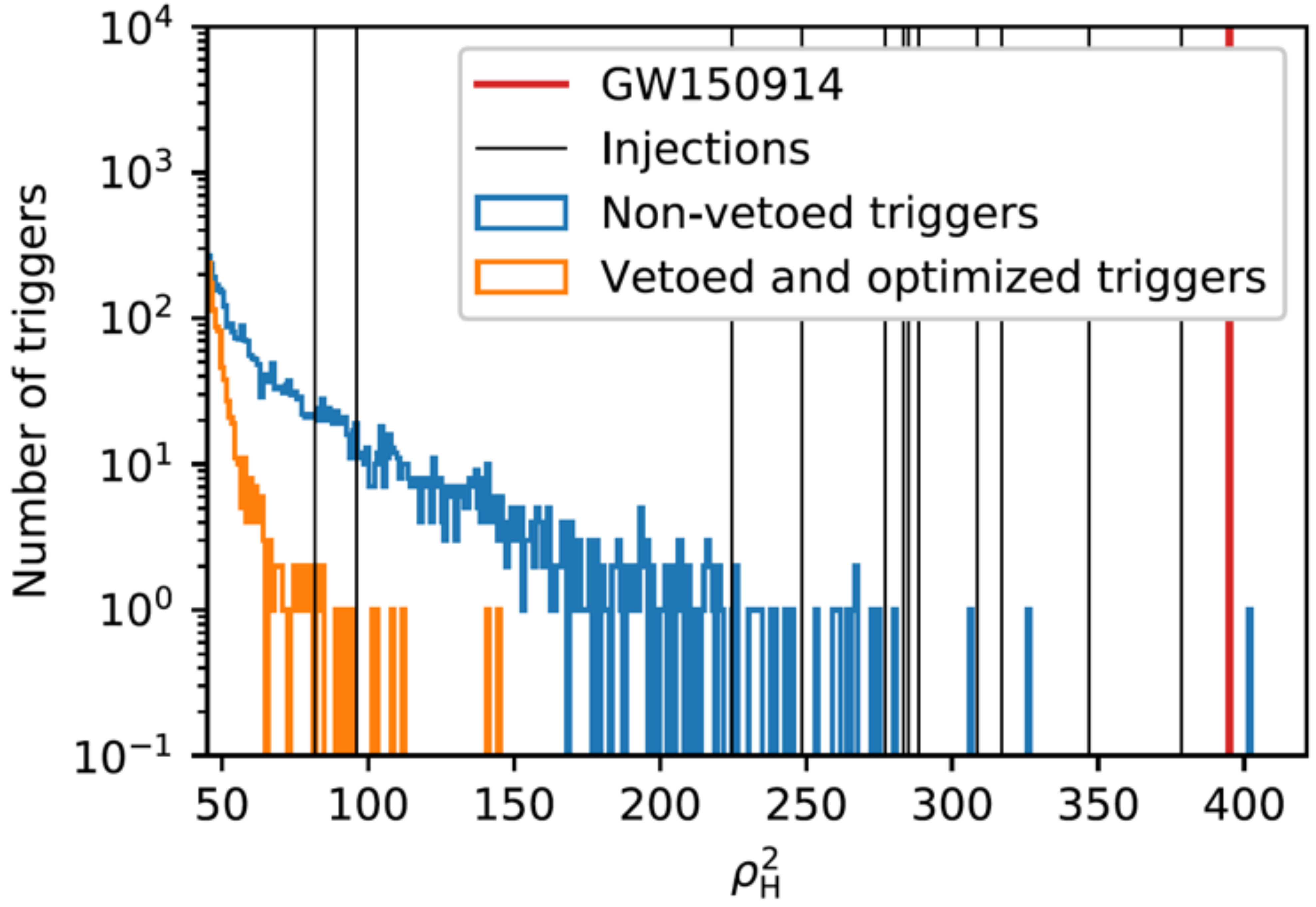
Vetoing Triggers

Check if different frequency ranges contribute to the “match” according to theoretical expectations

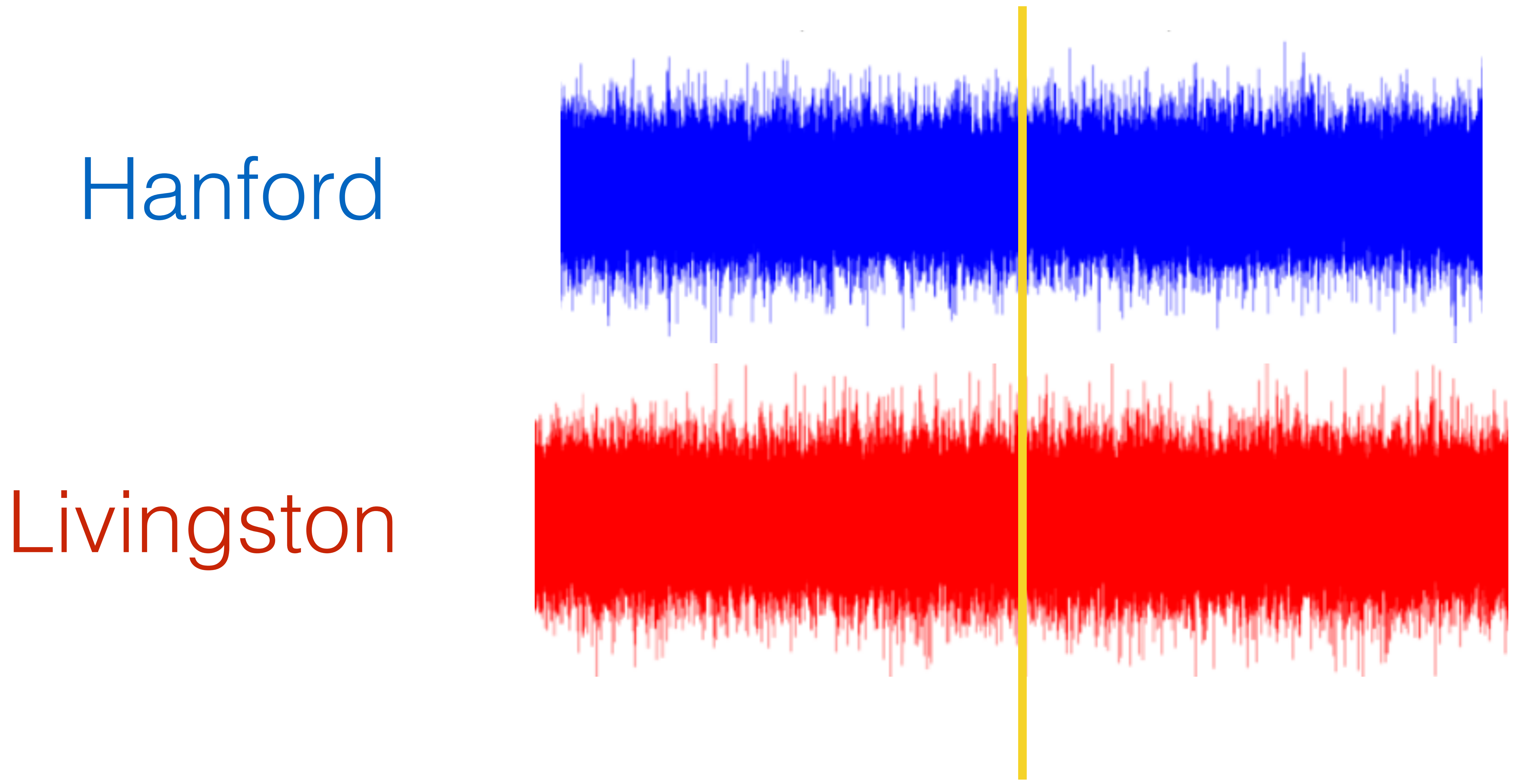
Promise:

- **False positive rate** < 1% for perfect Gaussian noise
- Robust to **PSD drift** and to **template bank inefficiency**





Search For Coincidence



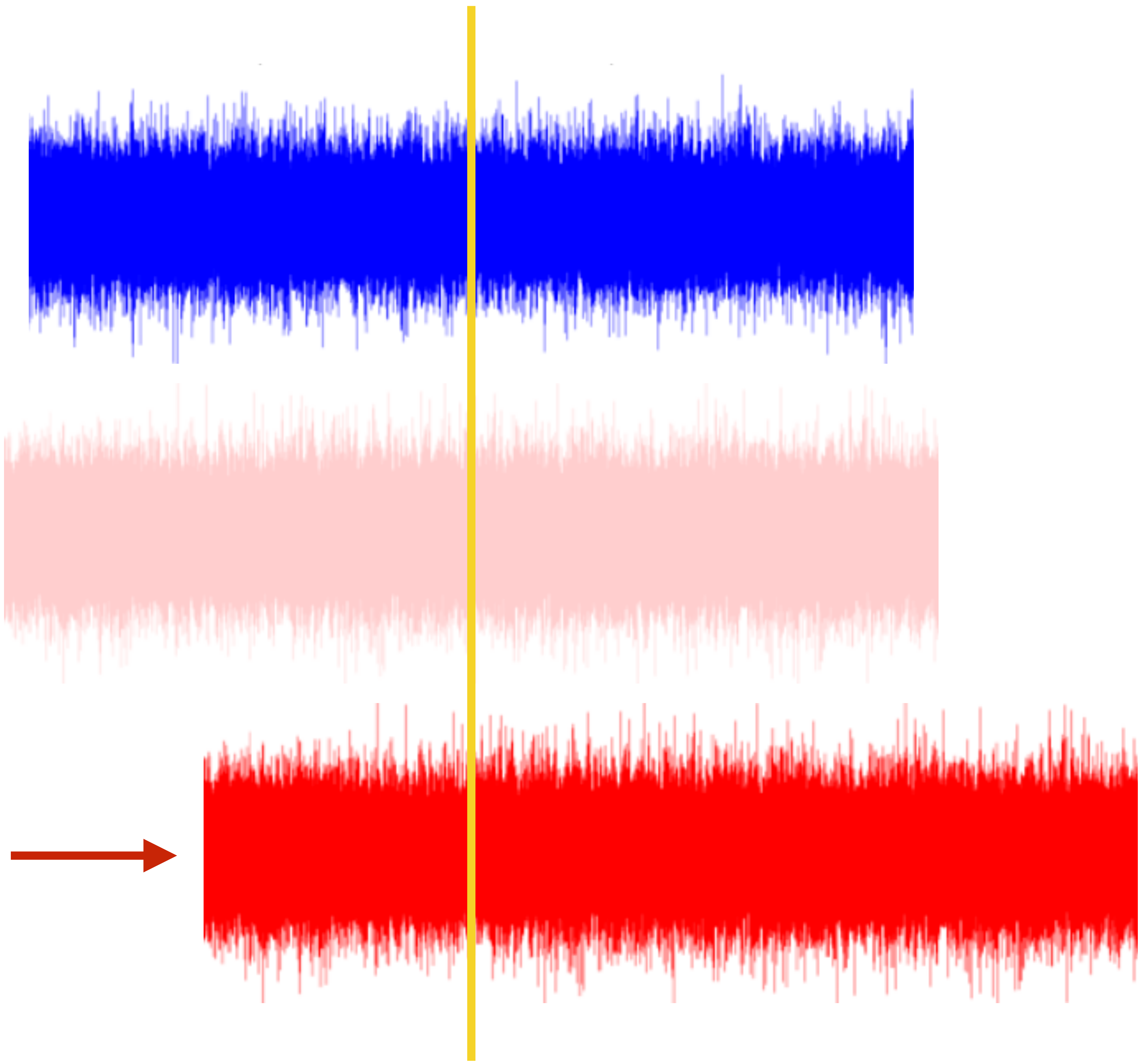
Hanford

Livingston

Zero time lag

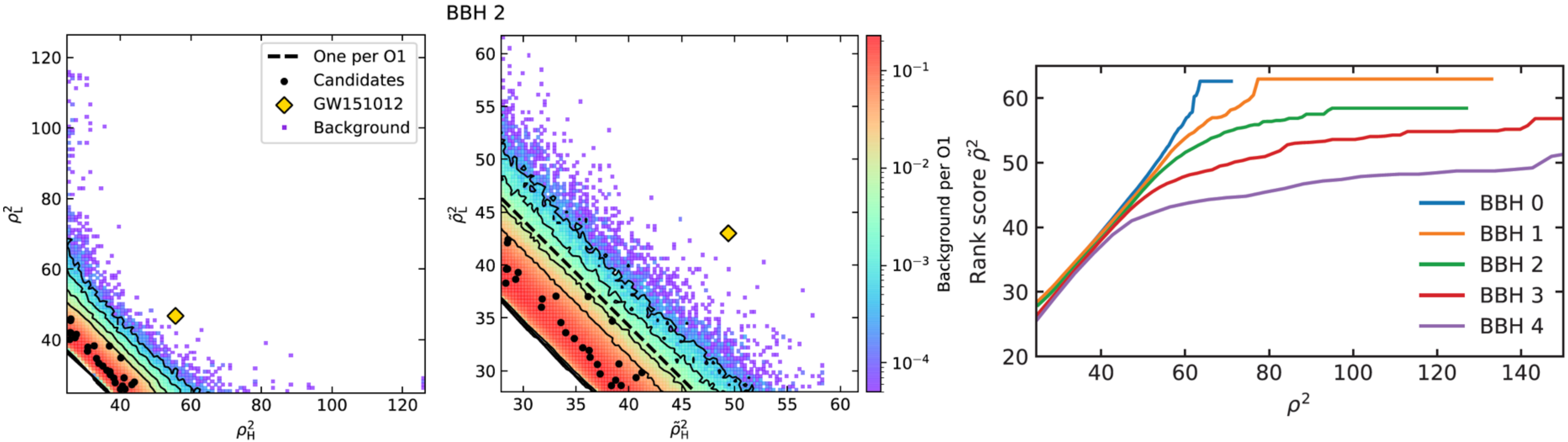
Record if the same template matches in two detectors at (nearly) the same time.

Genuine GW signal is subject to **a maximal time delay** between any two detectors



Unphysical time lags provide an empirical way to generate coincidences of **uncorrelated noise transients**.

(Incoherent) Ranking Score



In this way we calculate the false alarm rate (FAR)

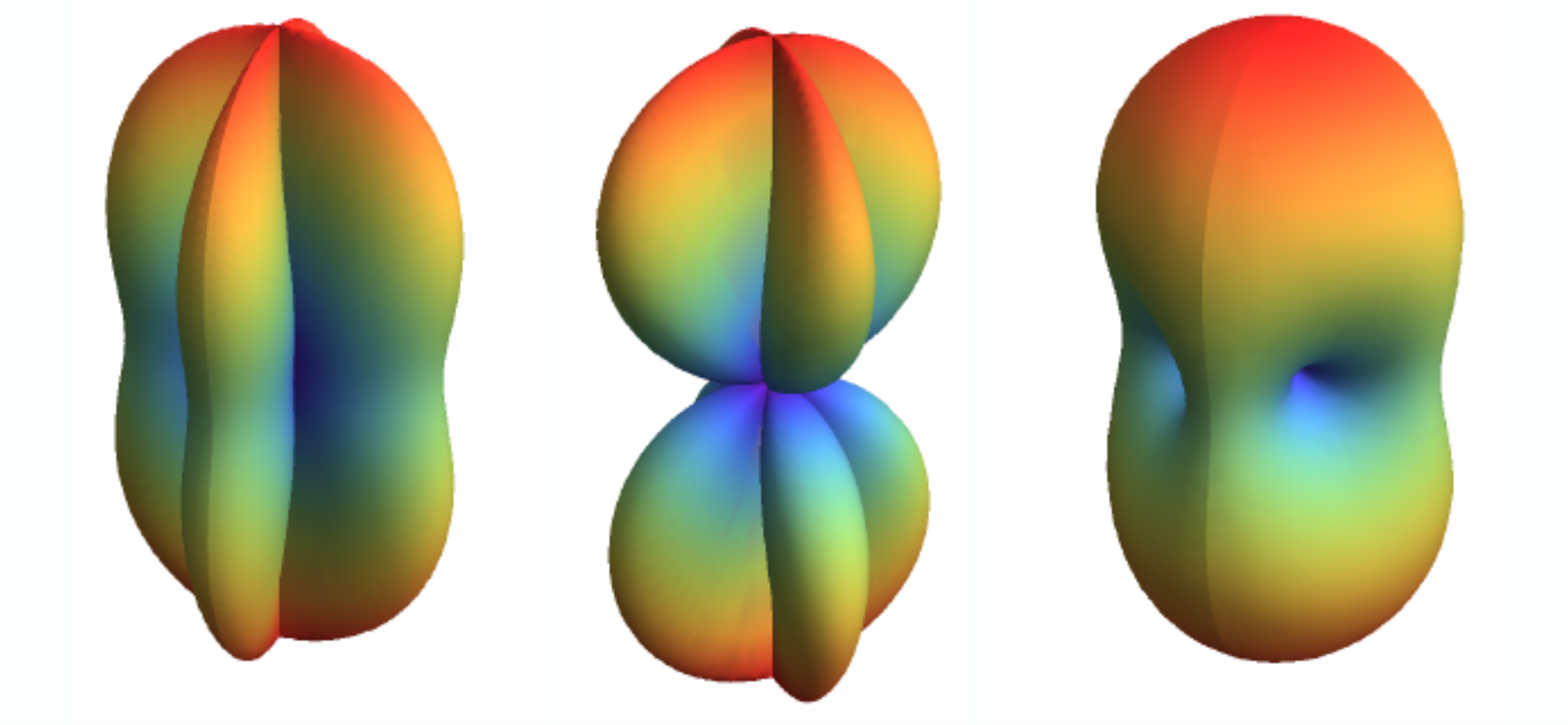
Coherent Score

Amplitude, phase and time correlations between detectors

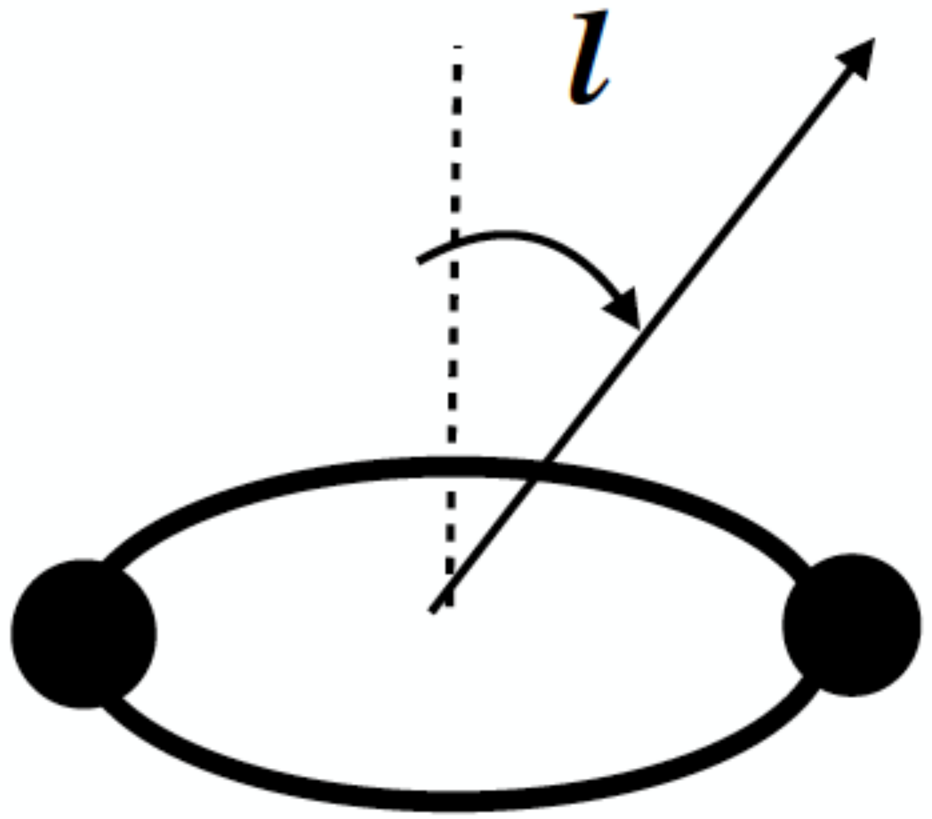
$$h_+ = A(1+\cos^2\iota)\,\cos\phi_{\mathrm{GW}}$$

$$h_\times = -2A\cos\iota\,\sin\phi_{\mathrm{GW}}$$

$$h = h_+F_+ + h_\times F_\times$$



detector angular response

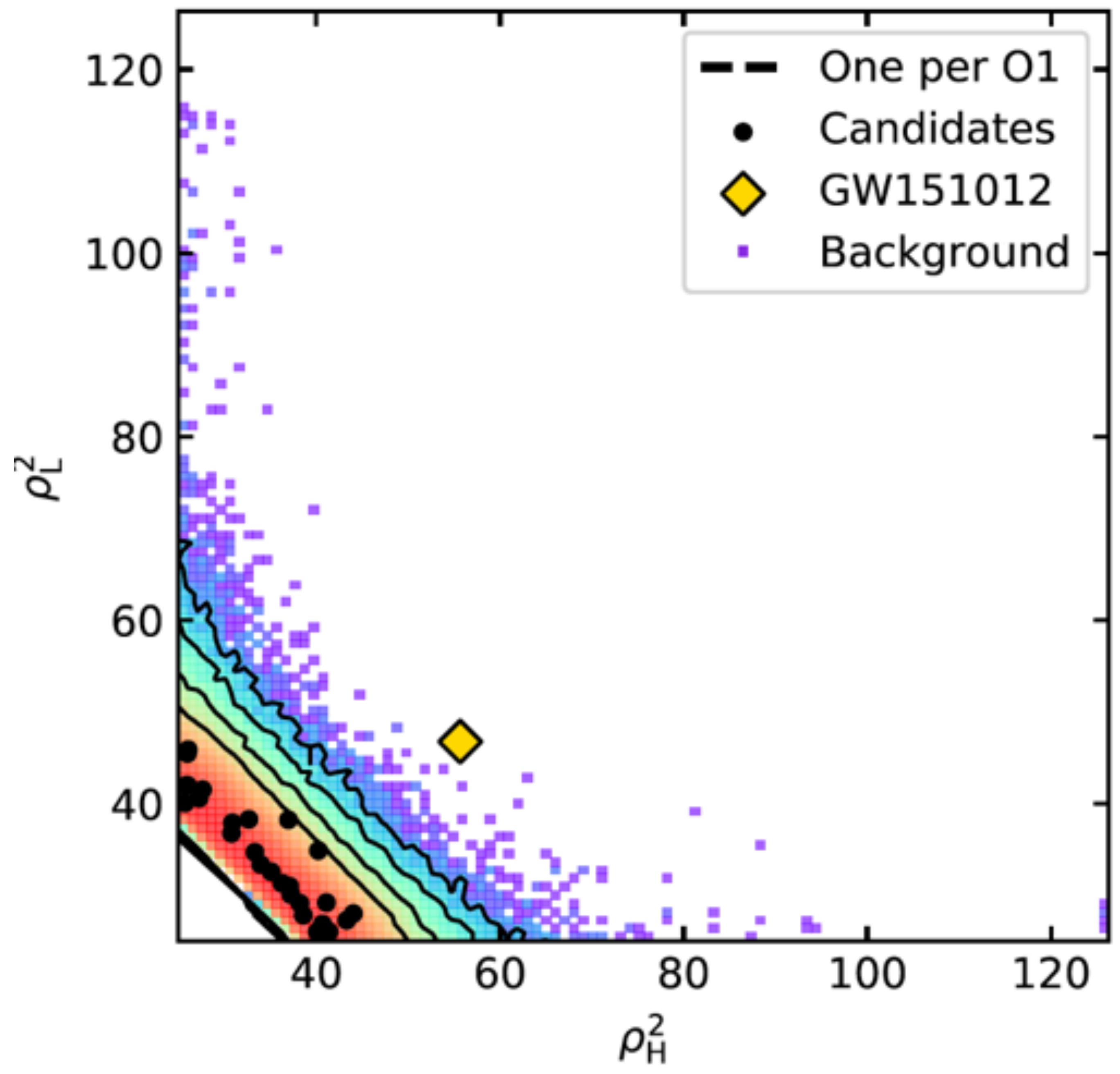


inclination

Adhikari+ 2013

We adjust the score by the log of the PDF ratio

$$\frac{P\left(\rho_1^2, \rho_2^2, \Delta t, \Delta\phi | H1\right)}{P\left(\rho_1^2, \rho_2^2, \Delta t, \Delta\phi | H0\right)}$$



Probability of Astrophysical Origin

Need to compare the **GW hypothesis** H$_1$ to the **noise hypothesis** H$_0$

$$p_{\rm astro}({\rm event}|\mathcal{R}_{\rm GW}) = \frac{\mathcal{R}({\rm event}|H_1)}{\mathcal{R}({\rm event}|H_0) + \mathcal{R}({\rm event}|H_1)}$$

Noise hypothesis

measure empirically

$$\begin{aligned}\mathcal{R}({\rm event} \mid H_0) &= \mathcal{R}_{\rm bg} P(\Delta t, \Delta\phi, \rho_{\rm H}^2, \rho_{\rm L}^2 \mid H_0) \\ &= \mathcal{R}_{\rm bg} \frac{P(\rho_{\rm H}^2 \mid H_0) P(\rho_{\rm L}^2 \mid H_0)}{2\pi T};\end{aligned}$$

GW hypothesis

$$\mathcal{R}({\rm event}|H_1) = \boxed{\mathcal{R}_{\rm GW}} P\left(\Delta t, \Delta\phi, \rho_{\rm H}^2, \rho_{\rm L}^2 | H_1\right)$$

GW rate normalization is a free parameter subbank by subbing

Bayesian inference for rate normalization

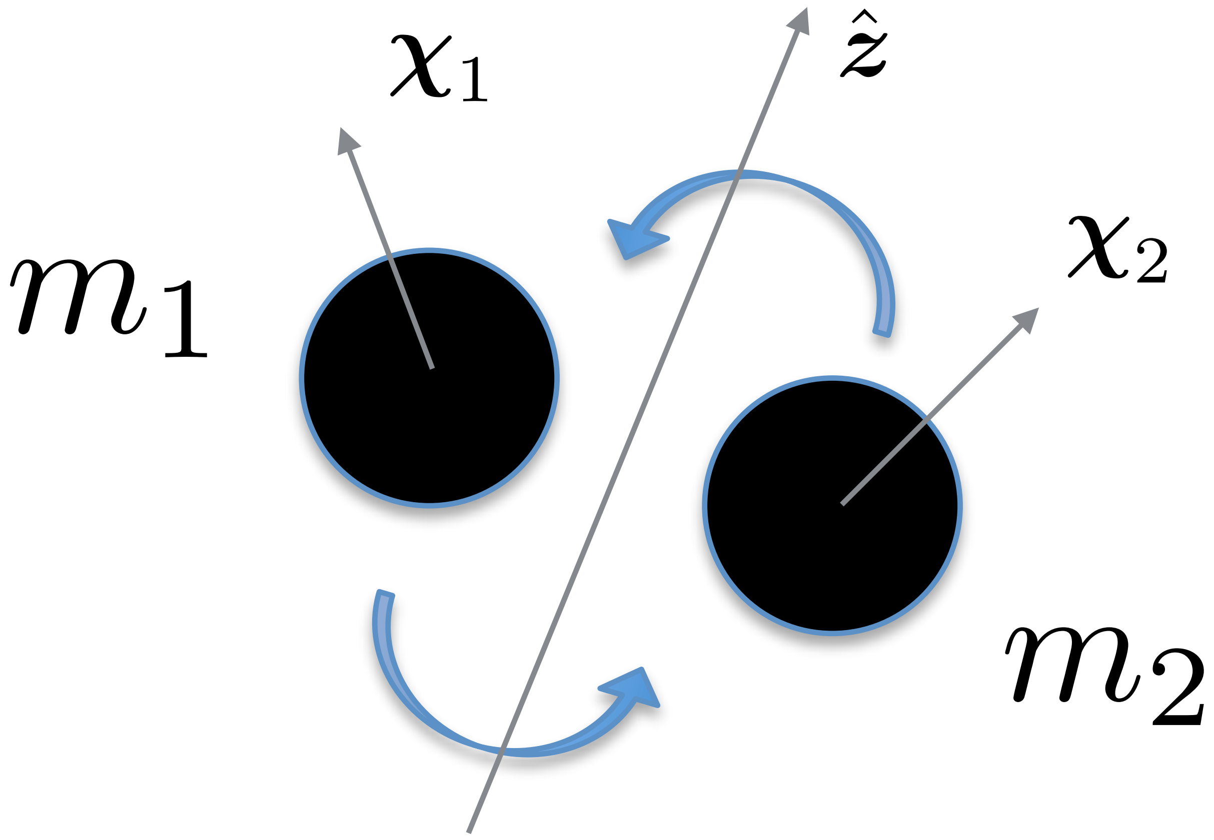
$$\mathcal{L}(\mathcal{R}_{\rm GW}) \sim e^{-\mathcal{R}_{\rm GW}} \prod_{\rm events} [\mathcal{R}({\rm event}|H_0) + \mathcal{R}({\rm event}|H_1)]$$

Marginalized probability

$$p_{\rm astro}({\rm event}) = \int d\mathcal{R}_{\rm GW}\, P(\mathcal{R}_{\rm GW})\, p_{\rm astro}({\rm event}|\mathcal{R}_{\rm GW})$$

Parameters For Binary Mergers

Intrinsic parameters



component masses
and spins

"chirp" mass $\mathcal{M} = \frac{(m_1\, m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$ $\dot{f} \sim \mathcal{M}^{5/3}\, f^{11/3}$

In detector frame $\mathcal{M}^{\mathrm{det}} = \mathcal{M}\,(1+z)$

effective spin parameter $\chi_{\mathrm{eff}} = \frac{m_1\, \chi_{1z} + m_2\, \chi_{2z}}{m_1 + m_2}$

Extrinsic parameters

(Geocentric) arrival time
Luminosity distance
Orbital phase
Inclination
Sky position: RA, Dec
Roll angle of the orbit on the sky

Parameter Estimation

Parameter estimation is important to address many astrophysics questions

Intrinsic parameters

Component mass distribution?
Mass cutoff (e.g. due to pair-instability SNe)?
Mass ratio?

Fast spinning or slowly/non-spinning?
(Binary stellar evolution? Dynamic formation?)
Spins aligned, anti-aligned, or random?

Identify NS-BH mergers ?!

Extrinsic parameters

(RA, Dec) for EM follow-ups

Luminosity distance for understanding **redshift evolution** of the mergers;
And for the **standard siren** test.

Inclination has important implications for the luminosity distance, mass ratio measurement, and spin-precession effect.

Likelihood Evaluation

Need to calculate the "overlap"

$$(d(t)|h(t+\tau)) := \sum_f \frac{d(f)\, h^*(f)\, e^{i\, 2\pi\, f\, \tau}}{S_N(f)/4}$$

To analyze a chunk of **T** seconds at a sampling rate **Fs** Hz, perform ing FFT on a regular frequency grid requires **O(N log N)** flops, where **N = T*Fs**

Parameter estimation may require us to evaluate the above FFT for **O(10^6–10^8)** parameter combinations.

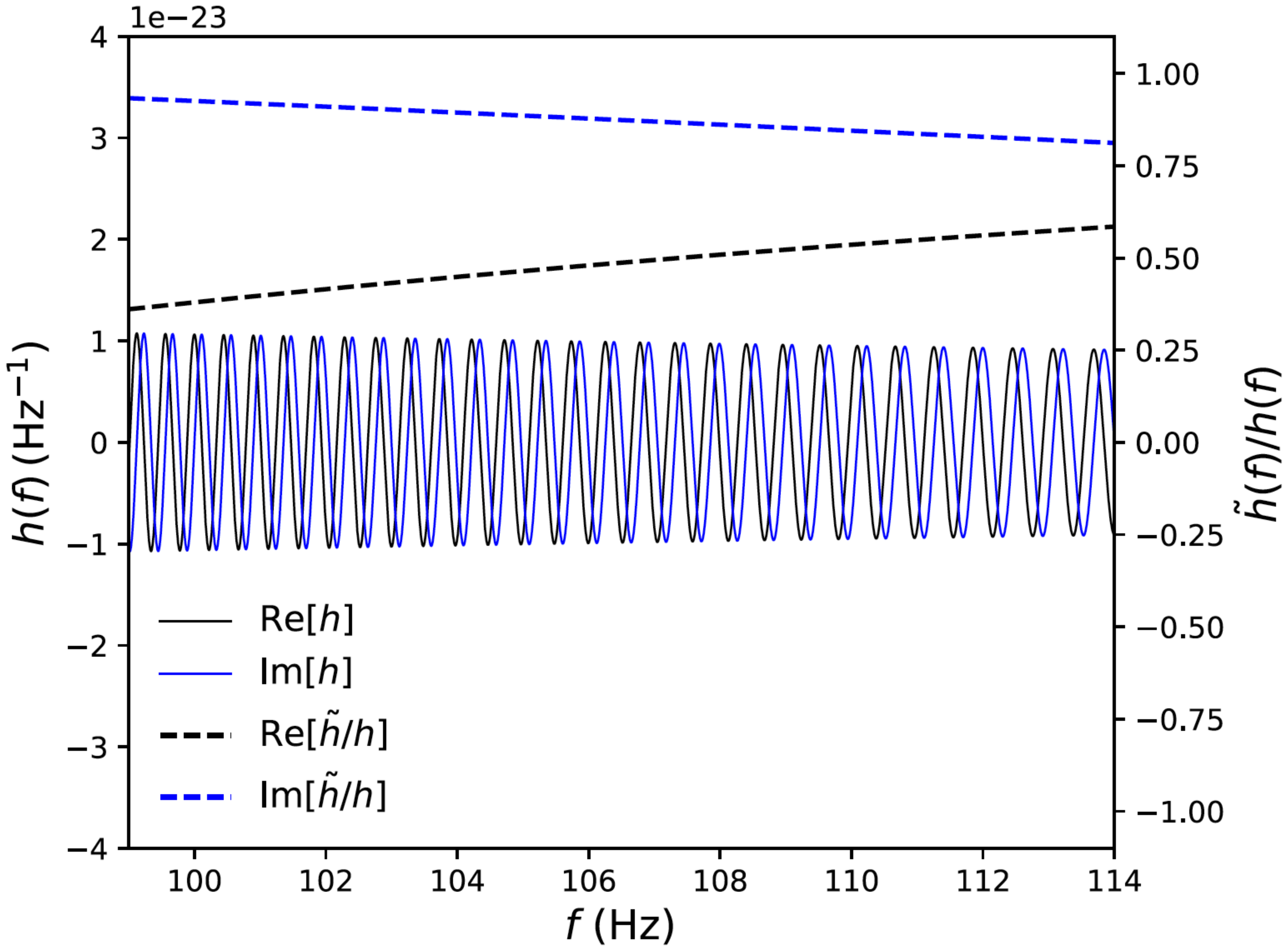
Become increasingly challenging when events are visible in band for a very long period of time.

Fast Likelihood Evaluation: Relative Binning

Binary neutron star merger GW170817

$$\mathcal{M}^{\rm det} = 1.1975 \pm 0.0001\, M_\odot$$

Let us compare $\mathcal{M}^{\rm det} = 1.1975\, M_\odot$, $\mathcal{M}^{\rm det} = 1.1985\, M_\odot$



Compute (frequency-domain) **waveform ratio** only on a sparse frequency grid

$$r(f) = \frac{h(f)}{h_0(f)} = r_0(h, \mathrm{b}) + \mathrm{r_1(h, b)\,(f - f_m(b))} + \cdots$$

Match can be approximated as

$$Z(d, h) = 4 \sum_f \frac{d(f) h^*(f)}{S_n(f)/T}$$

$$\approx \sum_{\mathrm{b}} \left[A_0(\mathrm{b})\, r_0^*(h, \mathrm{b}) + A_1(\mathrm{b})\, r_1^*(h, \mathrm{b})\right]$$

Pre-compute moments (on FFT grid, but for once)

$$A_0(\mathrm{b}) = 4 \sum_{f \in \mathrm{b}} \frac{d(f)\, h_0^*(f)}{S_n(f)/T}$$

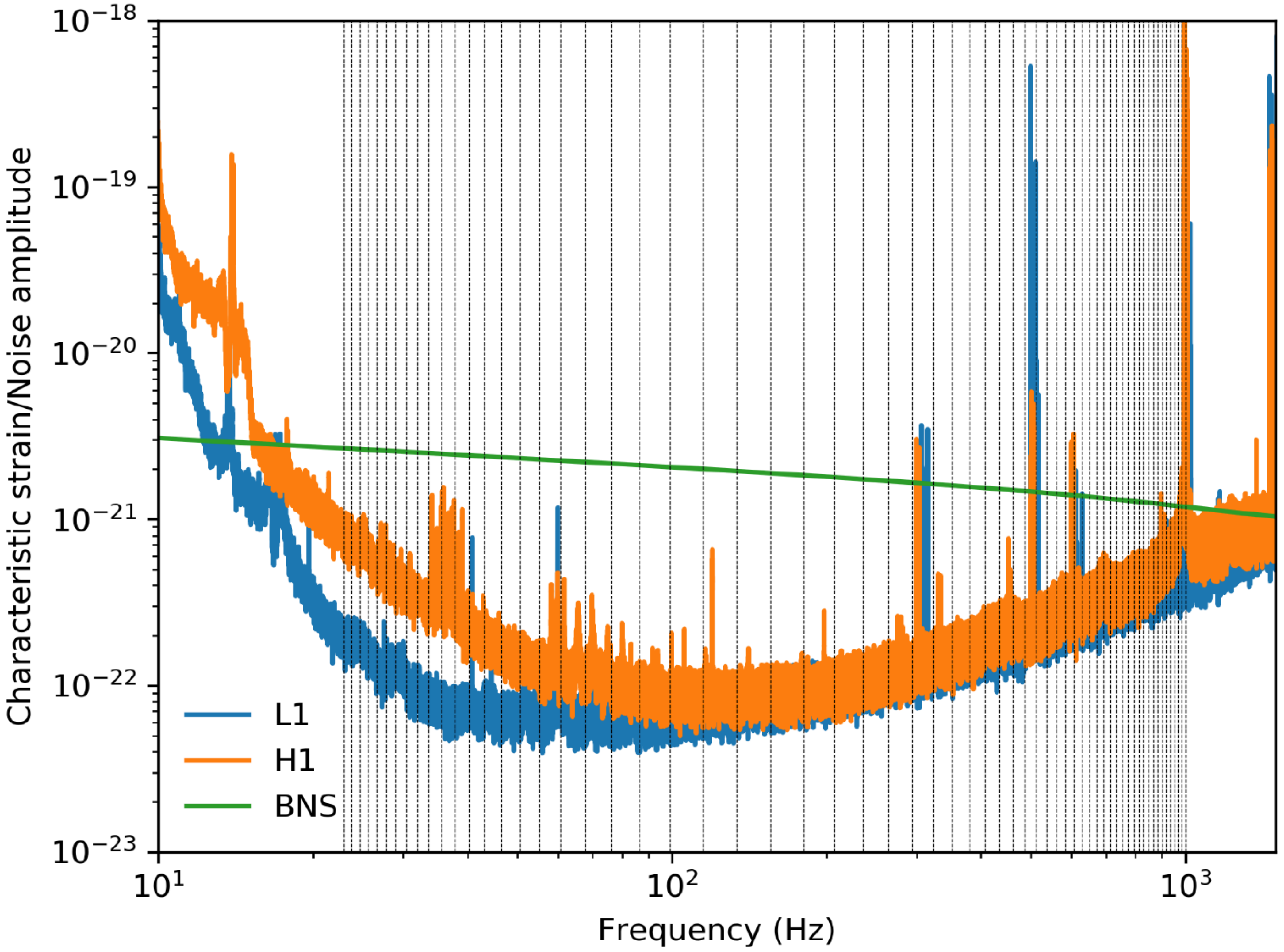
$$A_1(\mathrm{b}) = 4 \sum_{f \in \mathrm{b}} \frac{d(f)\, h_0^*(f)}{S_n(f)/T} (f - f_{\rm m}(\mathrm{b}))$$

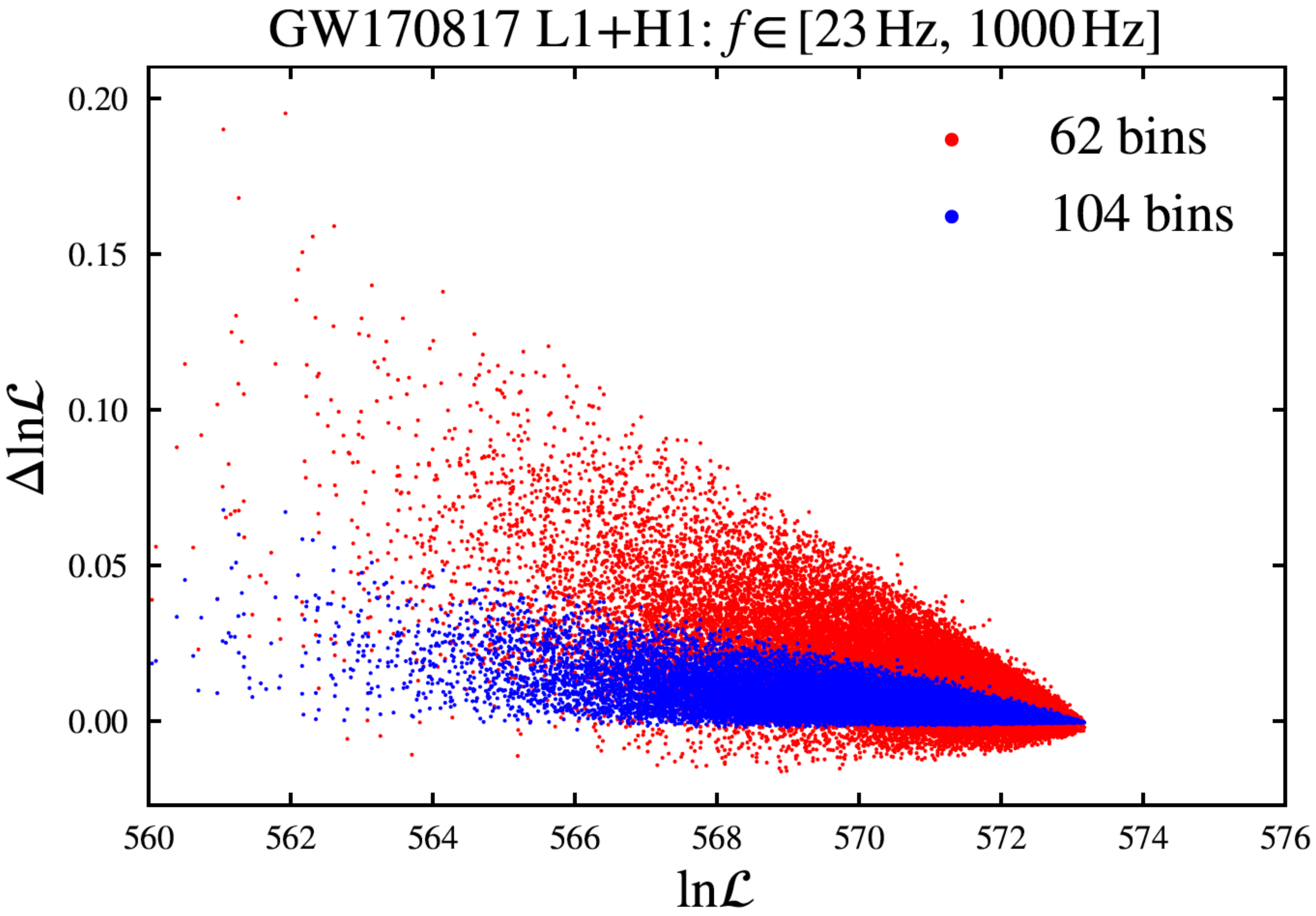
Relative Binning: GW170817

Zackay, Dai & Venumadhav 1806.08792
Also see earlier exploration:
Tanaka & Tagoshi (2000)
N. Cornish 1007.4820

Absolute error on the log likelihood under control
Use O(100) frequency bins

Non-uniform frequency bins





Relative Binning: Comments

- Reference waveform $h_0(f)$ found by iteration
- For $h_0(f)$, necessary to pinpoint only **intrinsic parameters**
- Except for the **arrival time**, which we treat in the same way as the intrinsic parameters. We give up on computing for all times at once using FFT.
- Routinely used in our analysis.
- A similar formalism applicable to **time domain waveform** $h(t)$. However some requirement on how the waveform model works in order to save computation.
- Further directions: Spin-precession; Eccentricity; etc.

Analyzing O1 and O2

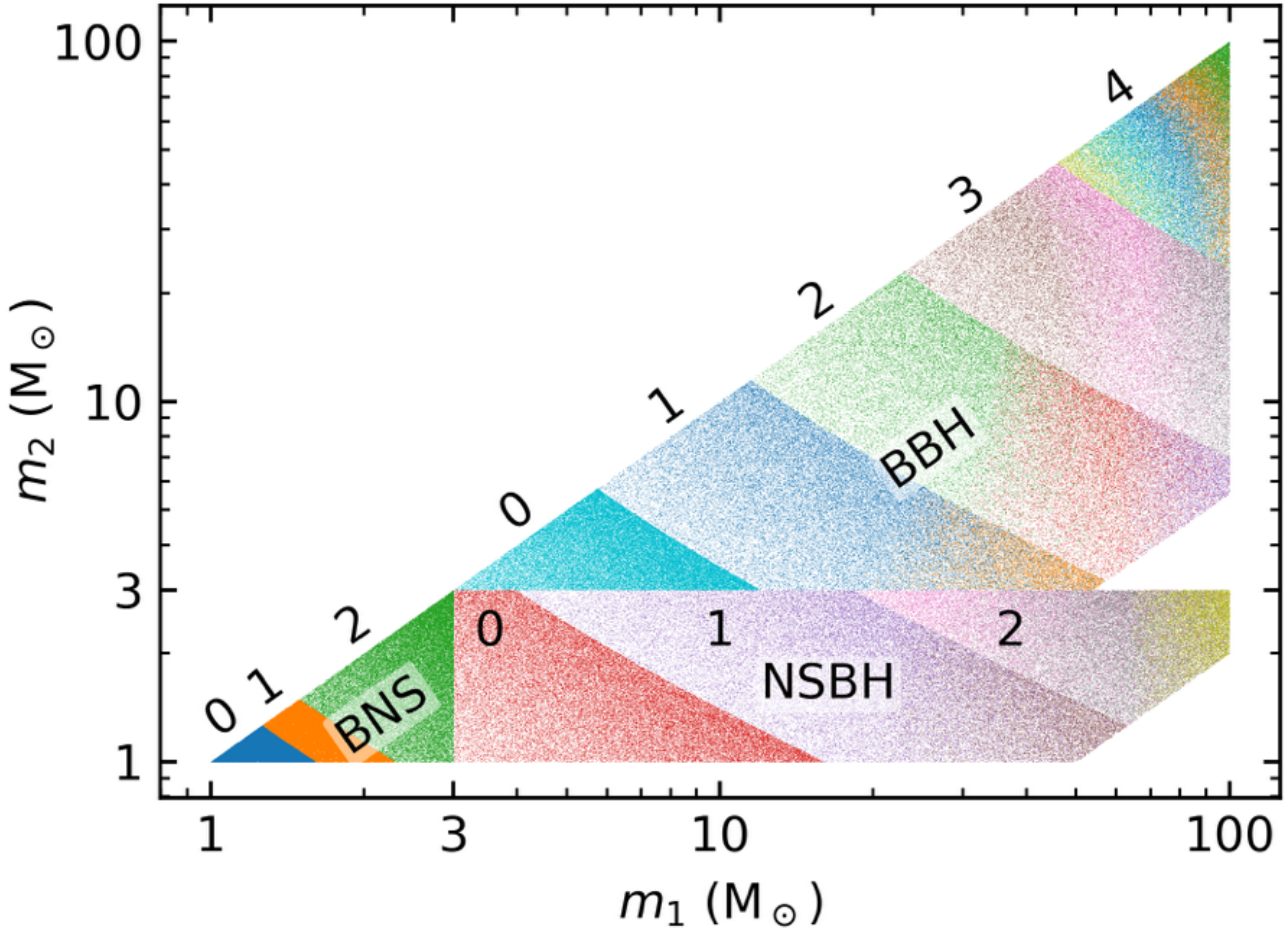
Astrophysical probability estimation done **sub-bank by sub-bank**

Why?

- Signal processing/vetoing differs from bank to bank
- Different banks/sub-banks have wildly different number of templates.
- Glitch property differs substantially between (sub)-banks.

Venumadhav+ 1902.10341

Venumadhav+ in prep



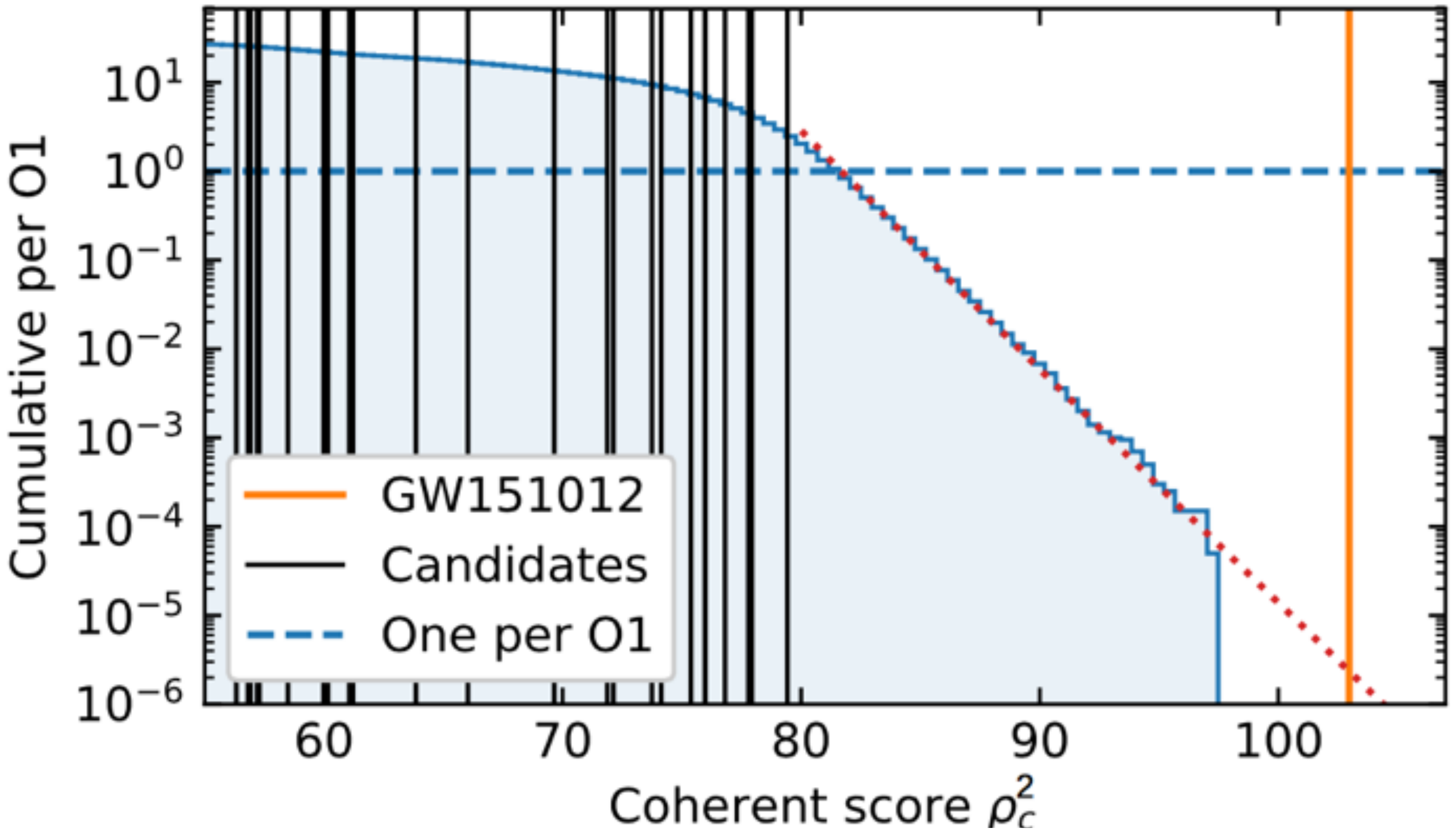
Heavy BBHs in O1 and O2

(Note: so far only Hanford-Living joint analysis)

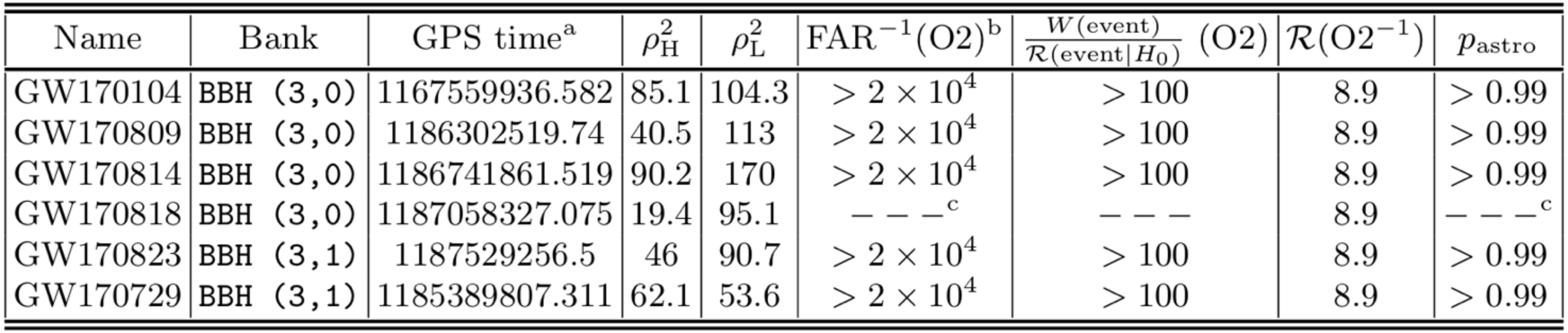
First of all we confirm LIGO/Virgo detections

Abbott+ 2018

GW151012 was originally LVT151012



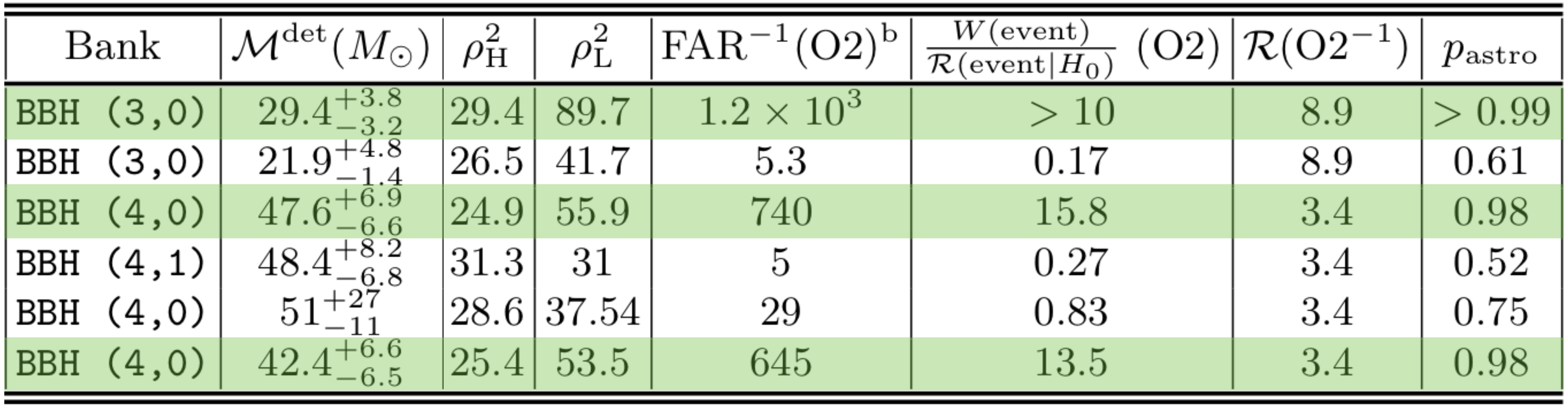
(preliminary) Venumadhav+ in prep



Name	Bank	GPS time[a]	ρ_{H}^2	ρ_{L}^2	FAR^{-1}(O2)[b]	$\frac{W(\mathrm{event})}{\mathcal{R}(\mathrm{event}\mid H_0)}$ (O2)	$\mathcal{R}(\mathrm{O2}^{-1})$	p_{astro}
GW170104	`BBH (3,0)`	1167559936.582	85.1	104.3	$> 2 \times 10^4$	> 100	8.9	> 0.99
GW170809	`BBH (3,0)`	1186302519.74	40.5	113	$> 2 \times 10^4$	> 100	8.9	> 0.99
GW170814	`BBH (3,0)`	1186741861.519	90.2	170	$> 2 \times 10^4$	> 100	8.9	> 0.99
GW170818	`BBH (3,0)`	1187058327.075	19.4	95.1	— — —[c]	— — —	8.9	— — —[c]
GW170823	`BBH (3,1)`	1187529256.5	46	90.7	$> 2 \times 10^4$	> 100	8.9	> 0.99
GW170729	`BBH (3,1)`	1185389807.311	62.1	53.6	$> 2 \times 10^4$	> 100	8.9	> 0.99

New BBH Events in O1/O2

(preliminary) Venumadhav+ in prep

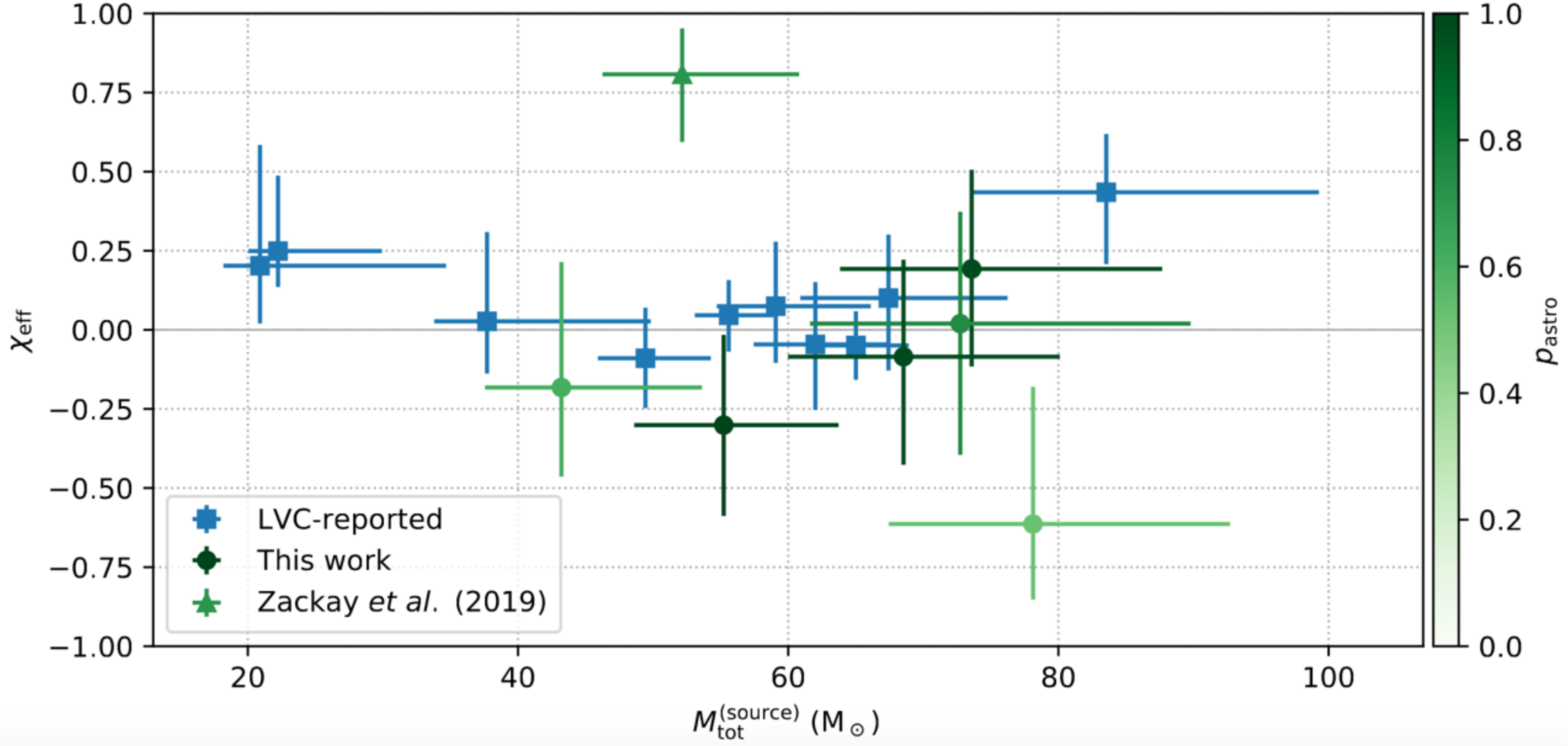


Bank	$\mathcal{M}^{\mathrm{det}}(M_\odot)$	ρ_{H}^2	ρ_{L}^2	$\mathrm{FAR}^{-1}(\mathrm{O2})^{\mathrm{b}}$	$\frac{W(\mathrm{event})}{\mathcal{R}(\mathrm{event}\|H_0)}$ (O2)	$\mathcal{R}(\mathrm{O2}^{-1})$	p_{astro}
`BBH (3,0)`	$29.4^{+3.8}_{-3.2}$	29.4	89.7	1.2×10^3	> 10	8.9	> 0.99
`BBH (3,0)`	$21.9^{+4.8}_{-1.4}$	26.5	41.7	5.3	0.17	8.9	0.61
`BBH (4,0)`	$47.6^{+6.9}_{-6.6}$	24.9	55.9	740	15.8	3.4	0.98
`BBH (4,1)`	$48.4^{+8.2}_{-6.8}$	31.3	31	5	0.27	3.4	0.52
`BBH (4,0)`	51^{+27}_{-11}	28.6	37.54	29	0.83	3.4	0.75
`BBH (4,0)`	$42.4^{+6.6}_{-6.5}$	25.4	53.5	645	13.5	3.4	0.98

In this Table, we list all candidates with $p_{\mathrm{astro}} > 0.5$

BBH population properties

(preliminary)



Zackay+ 1902.10331

Venumadhav+ in prep

Confident BBH events:

- Heavier than X-ray Binaries
- Consistent with a mass cutoff
- Consistent with non-/slowly-spinning

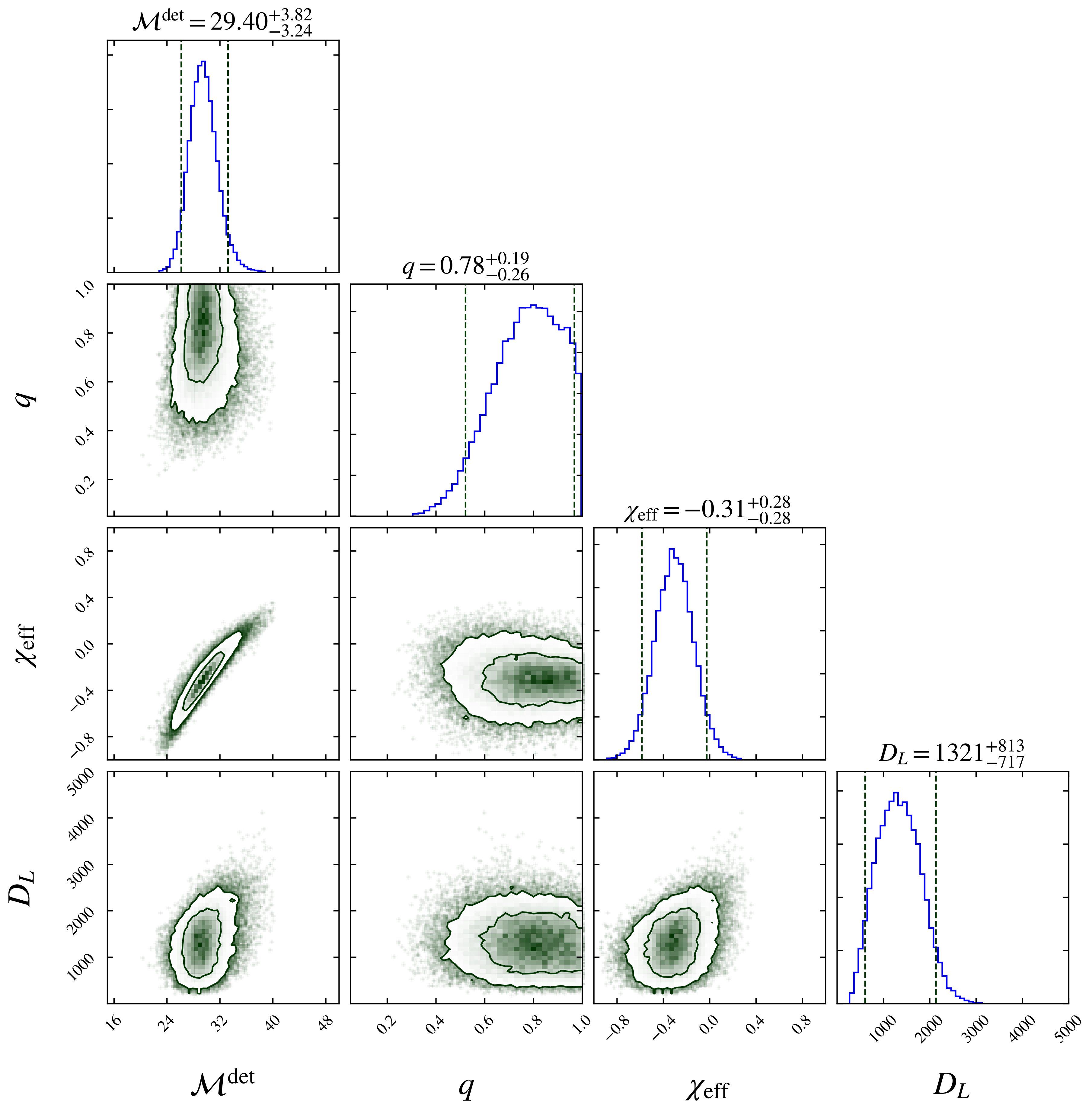
Conclusion

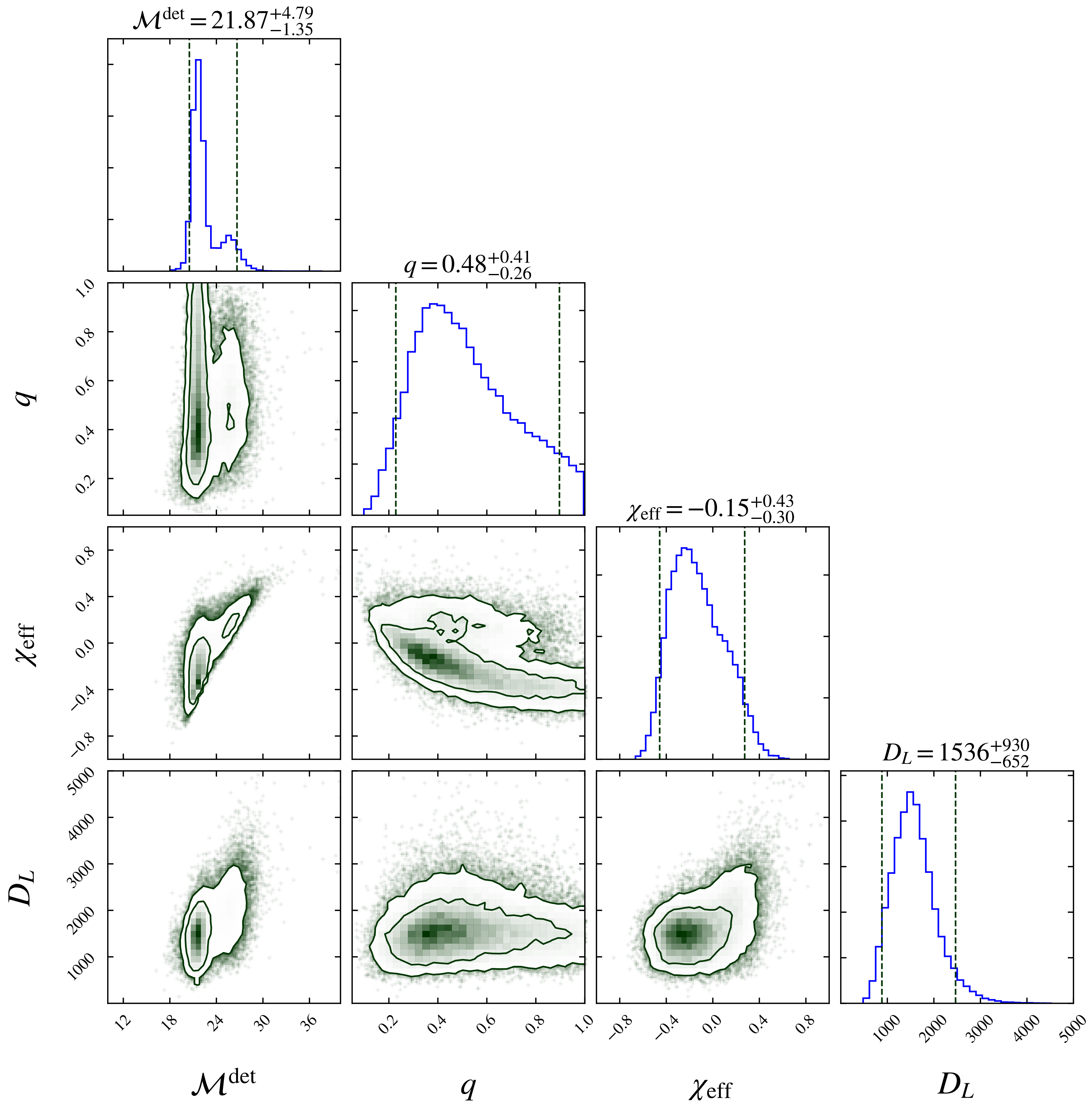
- LIGO/Virgo public release of bulk data is benefiting and will continue to tremendously benefit the community.
  We are very grateful.

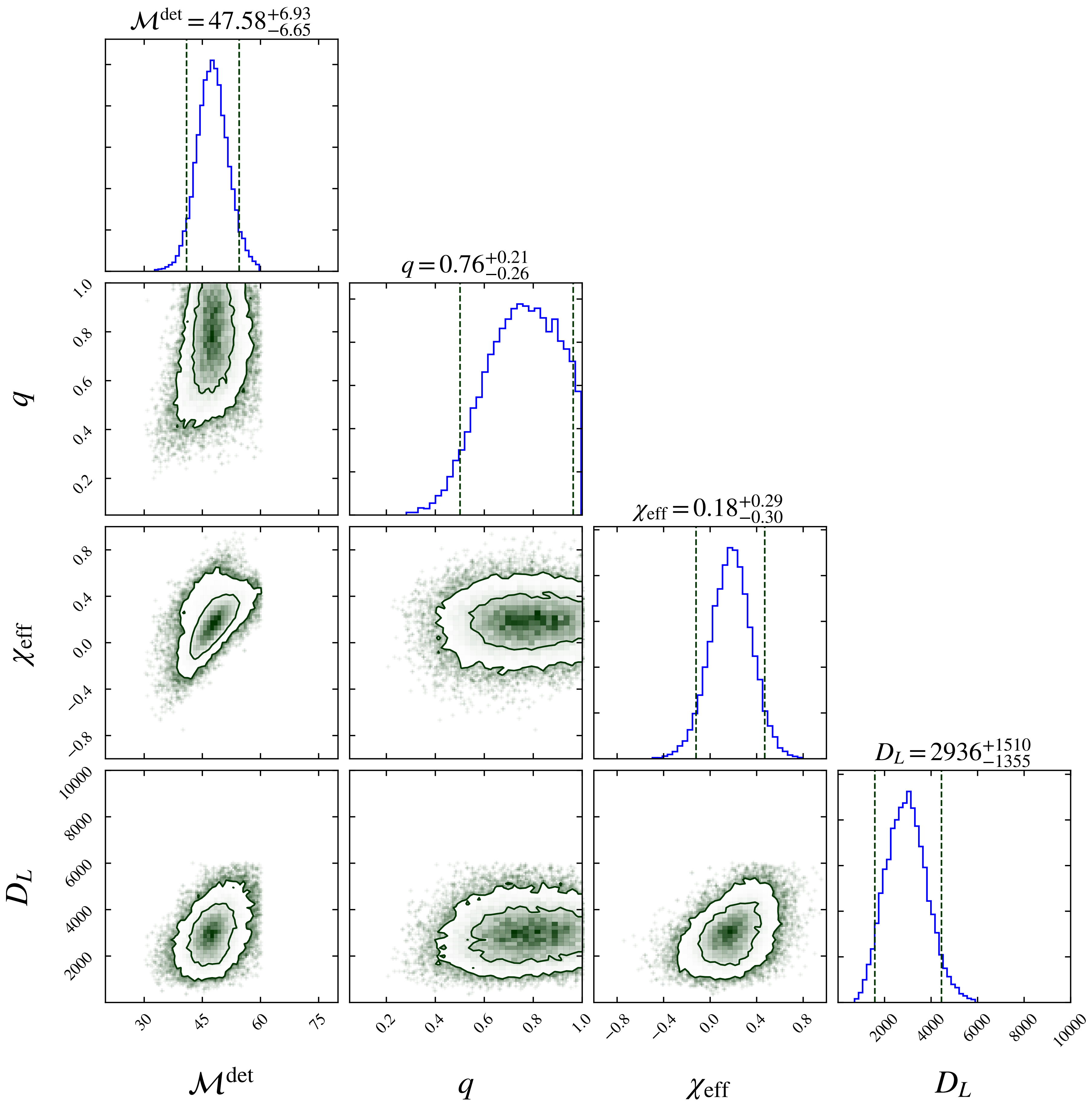
e.g. Nitz, Nielsen & Capano 1902.09496
Nitz+ 1811.01921

- We have developed a new search pipeline, for which we independently developed methods for template construction, signal precessing, score ranking, estimation of FAR and astro probability, as well as efficient parameter inference.

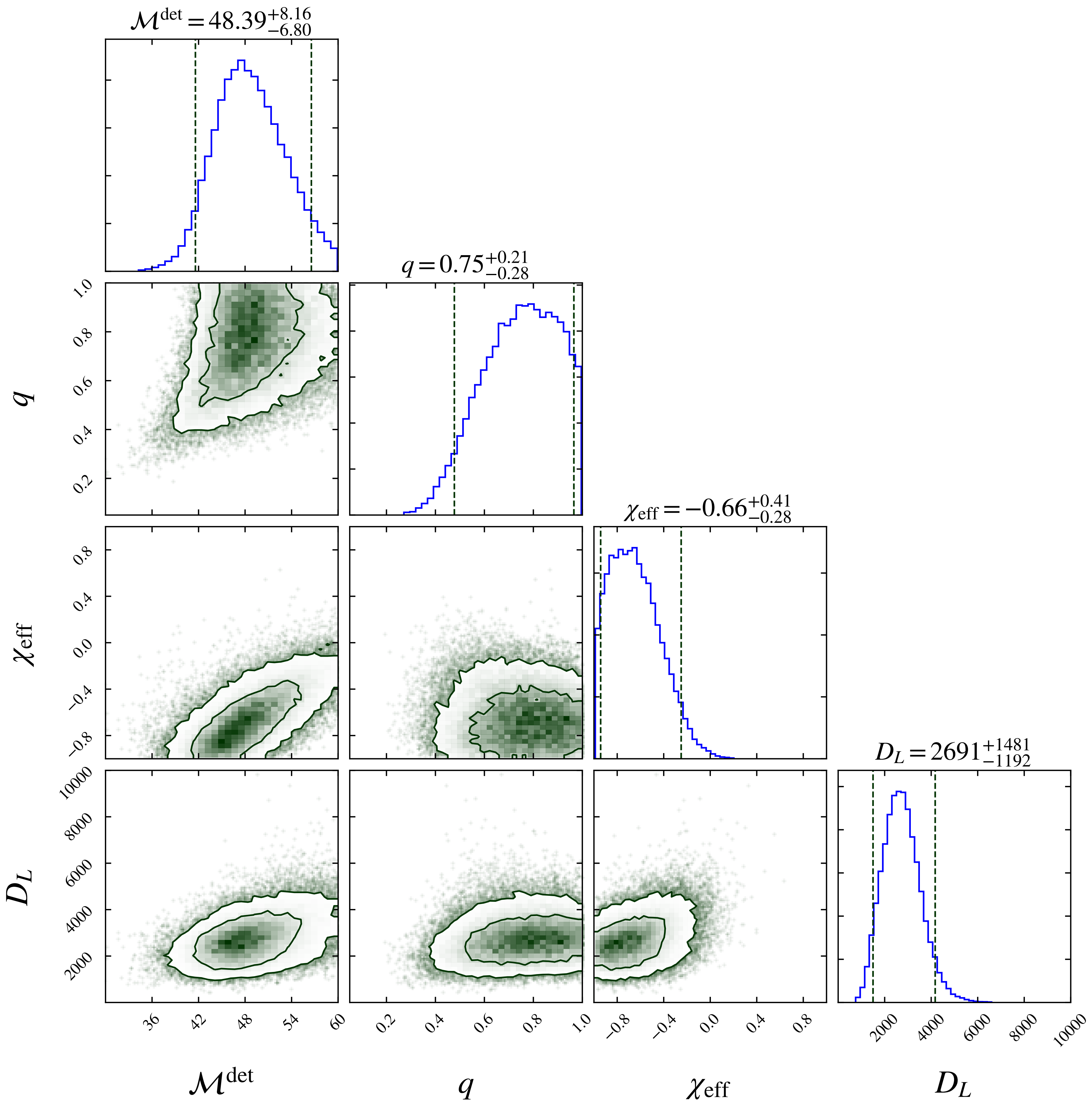
- We have applied our pipeline to the bulk O1 and O2 data. We found a few new BBH events that are reliable.


$\mathcal{M}^{\rm det} = 29.40^{+3.82}_{-3.24}$
$q = 0.78^{+0.19}_{-0.26}$
$\chi_{\rm eff} = -0.31^{+0.28}_{-0.28}$
$D_L = 1321^{+813}_{-717}$
q
$\chi_{\rm eff}$
D_L
$\mathcal{M}^{\rm det}$
q
$\chi_{\rm eff}$
D_L
16
24
32
40
48
0.2
0.4
0.6
0.8
1.0
−0.8
−0.4
0.0
0.4
0.8
1000
2000
3000
4000
5000

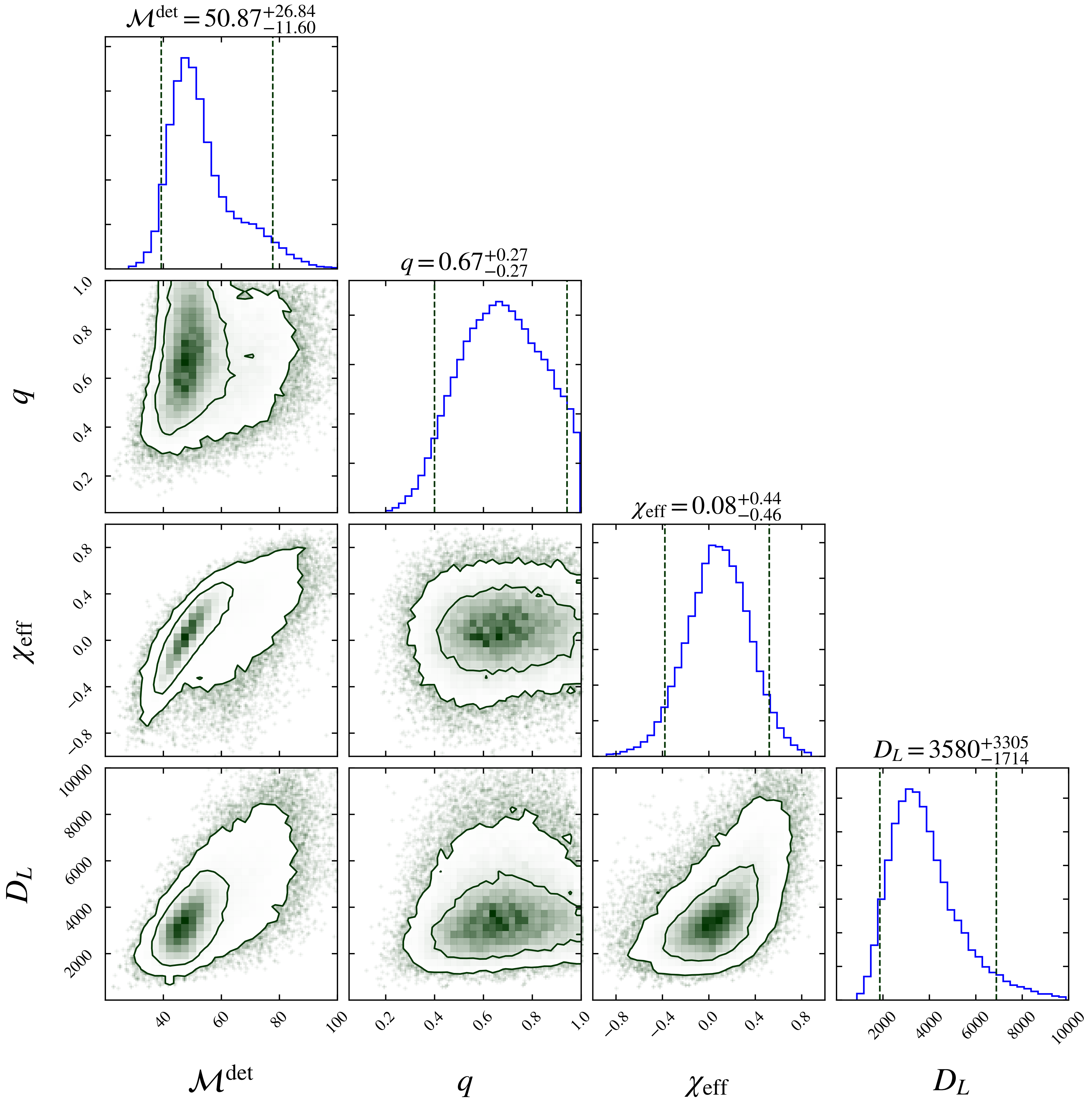

$\mathcal{M}^{\rm det} = 21.87^{+4.79}_{-1.35}$
$q = 0.48^{+0.41}_{-0.26}$
$\chi_{\rm eff} = -0.15^{+0.43}_{-0.30}$
$D_L = 1536^{+930}_{-652}$
q
$\chi_{\rm eff}$
D_L
$\mathcal{M}^{\rm det}$
q
$\chi_{\rm eff}$
D_L
12
18
24
30
36
0.2
0.4
0.6
0.8
1.0
−0.8
−0.4
0.0
0.4
0.8
1000
2000
3000
4000
5000



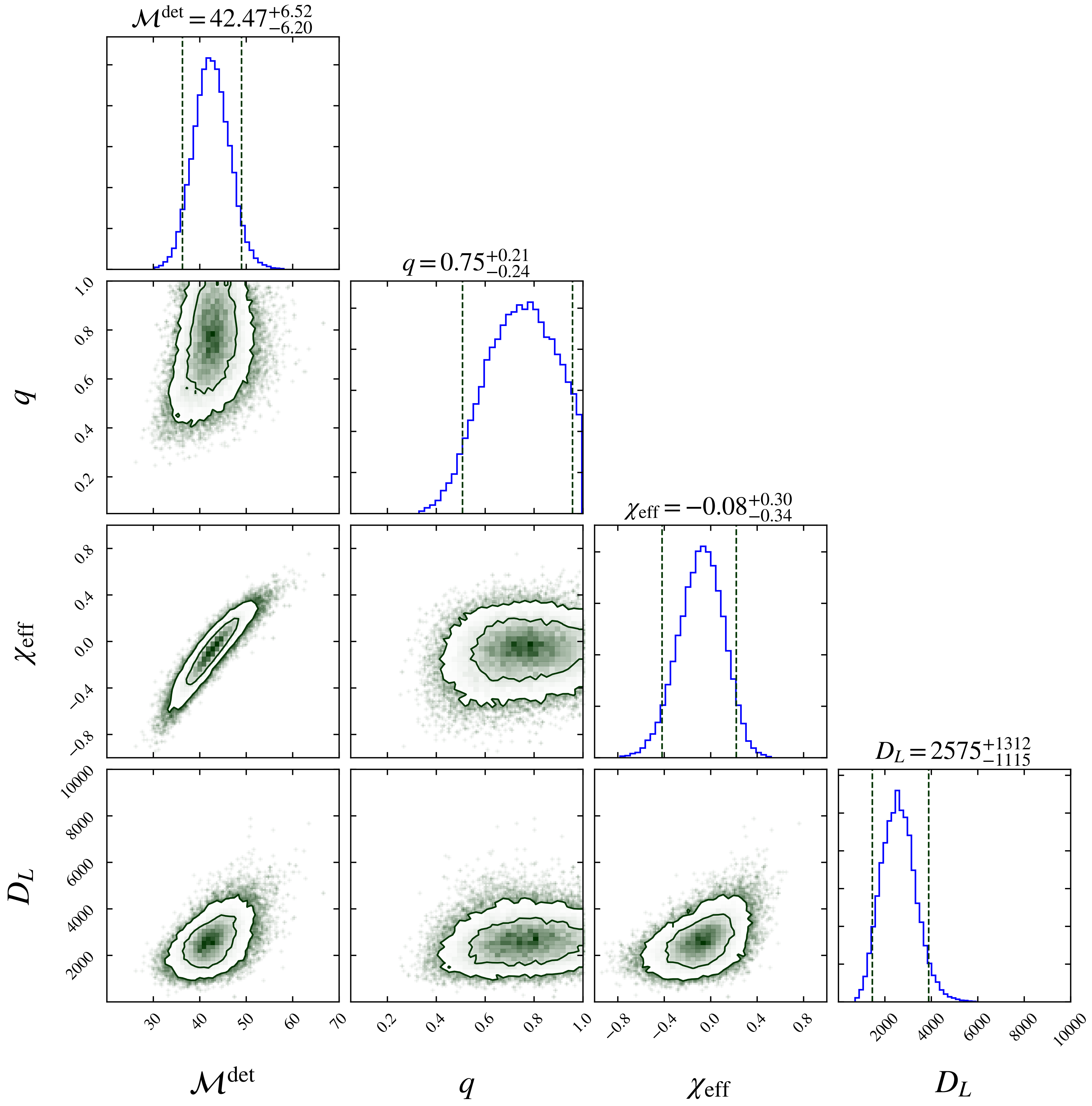
$\mathcal{M}^{\rm det} = 47.58^{+6.93}_{-6.65}$
$q = 0.76^{+0.21}_{-0.26}$
$\chi_{\rm eff} = 0.18^{+0.29}_{-0.30}$
$D_L = 2936^{+1510}_{-1355}$
q
$\chi_{\rm eff}$
D_L
$\mathcal{M}^{\rm det}$
30
45
60
75
0.2
0.4
0.6
0.8
1.0
−0.8
−0.4
0.0
0.4
0.8
2000
4000
6000
8000
10000



$\mathcal{M}^{\rm det} = 48.39^{+8.16}_{-6.80}$
$q = 0.75^{+0.21}_{-0.28}$
$\chi_{\rm eff} = -0.66^{+0.41}_{-0.28}$
$D_L = 2691^{+1481}_{-1192}$
q
$\chi_{\rm eff}$
D_L
$\mathcal{M}^{\rm det}$
36
42
48
54
60
0.2
0.4
0.6
0.8
1.0
−0.8
−0.4
0.0
0.4
0.8
2000
4000
6000
8000
10000



$\mathcal{M}^{\rm det} = 50.87^{+26.84}_{-11.60}$
$q = 0.67^{+0.27}_{-0.27}$
$\chi_{\rm eff} = 0.08^{+0.44}_{-0.46}$
$D_L = 3580^{+3305}_{-1714}$
q
$\chi_{\rm eff}$
D_L
$\mathcal{M}^{\rm det}$
0.2 0.4 0.6 0.8 1.0
−0.8 −0.4 0.0 0.4 0.8
2000 4000 6000 8000 10000
40 60 80 100



$\mathcal{M}^{\rm det} = 42.47^{+6.52}_{-6.20}$
$q = 0.75^{+0.21}_{-0.24}$
$\chi_{\rm eff} = -0.08^{+0.30}_{-0.34}$
$D_L = 2575^{+1312}_{-1115}$
q
$\chi_{\rm eff}$
D_L
$\mathcal{M}^{\rm det}$
0.2 0.4 0.6 0.8 1.0
−0.8 −0.4 0.0 0.4 0.8
2000 4000 6000 8000 10000
30 40 50 60 70