A bias to CMB lensing measurements from the bispectrum of large-scale structure

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Lensing of the cosmic microwave background

\[ \tilde{T}(x) = T [x + \alpha(x)] = T [x + \nabla \phi(x)] \]
Lensing of the cosmic microwave background

CMB lensing potential

\[ \phi(x) = -2 \int_0^{\chi_*} d\chi \, W(\chi) \psi(x, \chi) \]

\[ W(\chi) = \frac{f_k(\chi_* - \chi)}{f_k(\chi_*) f_k(\chi)} \]

Power spectrum

\[ C_{L\phi\phi} \leftarrow P_\delta(L/\chi; \chi) \]
Lensing of the cosmic microwave background

CMB lensing potential

\[
\phi(x) = -2 \int_0^{x_*} d\chi \ W(\chi) \ \psi(x, \chi)
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W(\chi) = \frac{f_k(x_* - \chi)}{f_k(x_*) f_k(\chi)}
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Power spectrum

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C_L^{\phi\phi} \leftarrow P_\delta(L/\chi; \chi)
\]
Lensing of the cosmic microwave background

Unlensed CMB
Lensing of the cosmic microwave background

Lensed CMB
Lensing of the cosmic microwave background

\[ \langle |\alpha(x)| \rangle \approx 2\text{arcmin} \]
Lensing of the cosmic microwave background

Lensing Potential
Lensing of the cosmic microwave background

Lensing Potential

Coherence Scale: 2°
Lensing of the cosmic microwave background

- **Fixed lens**
  Two-point correlator in Fourier space $\rightarrow$ non-diagonal

- **Average over lens realizations**
  - CMB power spectrum $\rightarrow$
  - Temperature trispectrum $\rightarrow$ non-zero

  T $\rightarrow$ non-Gaussian
CMB lensing reconstruction

Quadratic estimator (Hu 2001)

$$\hat{\phi}(L) = A_L \int g(l, L) T_{\text{expt}}(l) T^*_{\text{expt}}(1 - L)$$

Power spectrum estimate ← mean square of the estimator

$$C^{\hat{\phi}\hat{\phi}}(L) \leftrightarrow \langle \hat{\phi}(L) \hat{\phi}^*(L') \rangle_{(\phi, T)}$$

$$\langle \hat{\phi}(L) \hat{\phi}^*(L') \rangle_{(\phi, T)} \propto \langle T_{\text{expt}}(l_1) T_{\text{expt}}(L - l_1) T_{\text{expt}}(-l_2) T_{\text{expt}}(l_2 - L') \rangle$$
Status of CMB lensing

- **Planck**
  \[ \sum m_\nu < 145 \text{ meV (68\%)} \]
  Planck TT+low P+lensing+BAO

- **Ground-based (Stage-II)**
  - ACTPol
  - SPTPol
  - Polarbear

\[
\frac{[L(L + 1)]^2 C_\ell^{\phi\phi}}{2\pi} \times 10^7
\]

**Figure:** Planck XV 2015
Rapid improvement in precision

- Ground-based experiments
  - **Stage-III**
    - Advanced ACT
    - SPT3G
    - Polarbear2
  - **Stage-IV**
    - CMB Stage 4
    - \( \sigma (\sum m_\nu) \sim 15 \text{ meV} \)

- Increasing precision requires increasingly accurate theoretical modeling
CMB lensing reconstruction

Quadratic estimator (Hu 2001)

\[ \hat{\phi}(L) = A_L \int g(l, L) \tilde{T}_{expt}(l) \tilde{T}_{expt}^*(l - L) \]

Power spectrum estimate \( \leftarrow \) mean square of the estimator

\[ C_{\hat{\phi} \hat{\phi}}(L) \leftarrow \langle \hat{\phi}(L) \hat{\phi}^*(L') \rangle_{(\phi,T)} \]

\[ \langle \hat{\phi}(L) \hat{\phi}^*(L') \rangle_{(\phi,T)} \propto \langle \tilde{T}_{expt}(l_1) \tilde{T}_{expt}(L - l_1) \tilde{T}_{expt}(-l_2) \tilde{T}_{expt}(l_2 - L') \rangle \]
Known biases

Lensed temperature perturbation series

$$\tilde{T}(l) = T(l) + \delta T(l) + \delta^2 T(l) + O(\phi^3)$$

$$\left\langle \tilde{T}\tilde{T}'\tilde{T}'\right\rangle = \left\langle TTT'T'\right\rangle + \left\langle \delta T\delta TT'T'\right\rangle + \left\langle \delta^2 TTT'T'\right\rangle + \ldots + O(\phi^4)$$

Different couplings give rise to different bias terms

$$\left\langle \hat{C}^\phi\phi \right\rangle = N_L^{(0)} + C_L^{\phi\phi} + N_L^{(1)} \quad \text{(Gaussian } \phi)$$

Kesden et al. 2003, Hanson et al. 2011, Lewis et al. 2011
Known biases

- **PS**
  foreground-sourced biases

- **MC**
  simulation-based corrections
  e.g. leakage from masking

Figure: Planck XV 2015

Van Engelen et al. (2014)
Osborne et al. (2014)
Namikawa et al. (2013)
Lensing potential
validity of Gaussian approximation

- 1-point PDF, peak count distribution → adds skewness
  Liu, Hill, Sherwin, Petri, VB, Haiman 2016

- Non-negligible bispectrum
Lensing potential bispectrum

• Lensing potential is sourced by large-scale matter distribution

• Non-linear structure formation $\rightarrow$ matter distribution becomes non-Gaussian and acquires bispectrum

\[ B_\phi(l_1, l_2, l_3) \leftarrow B_\delta(l_1/\chi, l_2/\chi, l_3/\chi) \]
Lensing potential bispectrum

- Lensing potential is sourced by large-scale matter distribution

- Non-linear structure formation $\rightarrow$ matter distribution becomes non-Gaussian and acquires bispectrum

$$B_\phi(l_1, l_2, l_3) \leftarrow B_\delta(l_1/\chi, l_2/\chi, l_3/\chi)$$

- Does the lensing potential become non-Gaussian, too?  
  Many lenses along LOS should 'gaussianize' the lensing potential...

- Namikawa 2016: bispectrum as signal $\rightarrow$ significant detection with a Stage-IV experiment
New 4-point bias

\[ \langle \tilde{T} \tilde{T} \tilde{\tilde{T}}' \tilde{T}' \rangle = O(\phi^0) + O(\phi^2) + O(\phi^3) + O(\phi^4) + \ldots \]

Contributions involving the lensing potential bispectrum

\[ \langle \delta T \delta T \delta T' T' \rangle, \langle \delta^2 T \delta T T' T' \rangle, \langle \delta^2 T T \delta T' T' \rangle, \langle \delta^3 T T T' T' \rangle \]

Type A, Type B, Type C, Type D

VB, Schmitfull, Sherwin 2016
New 4-point bias

\[ \langle \delta T \delta T \delta T' T' \rangle, \langle \delta^2 T \delta T T' T' \rangle, \langle \delta^2 T T \delta T' T' \rangle, \langle \delta^3 T T T' T' \rangle \]

Type A \hspace{2cm} Type B \hspace{2cm} Type C \hspace{2cm} Type D

More or less coupled 6D integrals over lensing bispectrum and temperature power spectra
New 4-point bias

\[ \langle \delta T \delta T \delta T' T' \rangle, \quad \langle \delta^2 T \delta T T' T' \rangle, \quad \langle \delta^2 T T \delta T' T' \rangle, \quad \langle \delta^3 T T T' T' \rangle \]

**Type A**

\[ \langle T, i \phi, i T, j \phi, j T', k \phi', k T' \rangle \]

**Type B**

\[ \langle T, i \phi, i T, j \phi, j T', k \phi', k T' \rangle \]

**Type C**

\[ \langle T, i \phi, i T, j \phi, j T', k \phi', k T' \rangle \]

**Type D**
New 4-point bias

\[
\langle \delta T \delta T \delta T' T' \rangle, \quad \langle \delta^2 T \delta T T' T' \rangle, \quad \langle \delta^2 T T \delta T' T' \rangle, \quad \langle \delta^3 T T T' T' \rangle
\]

Type A

\[
\langle T, i \phi, i T, j \phi, j T', k \phi', k T' \rangle
\]

Type B

\[
\langle T, i j \phi, i \phi, j T T', k \phi', k T' \rangle
\]

Type C

\[
\langle T, i j \phi, i \phi, j T T, k \phi', k T' \rangle
\]

Type D

\[
-4 A^2_L S_L \int_{l_1, 1} g(l_1, L) [(l_1 - 1) \cdot 1][(l_1 - 1) \cdot (L - 1)] C_{l_1-1}^{TT} B_\phi (1, L - 1, -L)
\]

\[
S_L = \int_{l_2} g(l_2, L) (l_2 \cdot L) C_{l_2}^{TT}
\]
New 4-point bias

\[ \langle \delta T \delta T \delta T' T' \rangle, \ \langle \delta^2 T \delta T T' T' \rangle, \ \langle \delta^2 TT \delta T' T' \rangle, \ \langle \delta^3 TT T' T' \rangle \]

**Type A**

\[ \langle T, i \phi, i T, j \phi, j T', k \phi', k T' \rangle \]

**Type B**

\[ \langle T, i j \phi, i \phi, j T T', k \phi', k T' \rangle \]

**Type C**

\[ \langle T, i j \phi, i \phi, j T T', k \phi', k T' \rangle \]

**Type D**

\[ \langle C_L^{\phi \phi} \rangle = N_L^{(0)} + C_L^{\phi \phi} + N_L^{(1)} + N_L^{(3/2)} \]
Results
Temperature TT,TT

\[ \Theta = 7 \text{ arcmin} \]
\[ \sigma = 30 \mu \text{K arcmin} \]
\[ \text{fsky} = 0.63 \]

\[ L^4 N_L^{(3/2)} / 2\pi \]

\[ N^{(3/2)} \text{ lensing bias} \]
\[ -N^{(3/2)} \text{ lensing bias} \]
\[ C_L^{\phi \phi} \text{ lensing signal} \]
\[ N_{A1}^{(3/2)} \]
\[ -N_{A1}^{(3/2)} \]
\[ N_{C1}^{(3/2)} \]
\[ -N_{C1}^{(3/2)} \]
Results
Temperature TT,TT

VB, Schmittfull, Sherwin 2016

$\sigma = 1 \mu K_{arcmin}$
$\Theta = 1 \text{ arcmin}$
$f_{sky} = 0.5$
Results

Temperature TT, TT

Stage-III:
\( \sigma = 6 \mu \text{Karcmin} \)
\( \Theta = 1.4 \text{ arcmin} \)
\( \text{fsky} = 0.4 \)

VB, Schmittfull, Sherwin 2016
Results
Temperature TT, TT

VB, Schmittfull, Sherwin 2016
Lensing Bispectrum Upgrades
Semi-analytic Fit

So far
Eulerian perturbation theory at leading-order (tree-level)

\[ B_\delta(k_1, k_2, k_3; \eta) = 2 F_2(k_1, k_2) P_\delta(k_1, \eta) P_\delta(k_2, \eta) + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \]

Upgrade
Fitting formula

\[ F_2(k_i, k_j) \rightarrow F_2^{\text{eff}}(k_i, k_j, n_i, n_j) \]

Scoccimarro&Couchman 2001,
Gil-Marin et al. 2012

Figure: Namikawa 2016
Lensing Bispectrum Upgrades
Post-Born Corrections

Update
Post-Born corrections \(\rightarrow\) non-negligible contributions to Bispectrum
Pratten&Lewis 2016
Results

Updated TT,TT Bias/Signal

Stage-III experiment

Bias/Signal

$N^{3/2}(L)/C^{\phi\phi}(L)$

-3.3% -2.7% -27.4%

$L$

tree-level

fit

fit + post-Born
Results

Updated TT,TT Bias/Noise

Stage-III experiment

\[ \Delta L = 100 \]

![Graph showing bias/noise results with different lines for tree-level, fit, and fit+post-Born.]

- 0.5σ
- 1.5σ
- 3.7σ
Results

test against ray-traced lensing simulations

- Evaluation of bias is numerically involved
- Neglected, tightly coupled, terms might not be negligible
- Possibly non-negligible higher order contributions
- Possible shortcomings of bispectrum model

independent test with simulations

VB/Sherwin

(Jia Liu, Colin Hill, Marcel Schmittfull,
Andrea Petri)
Results
test against ray-traced lensing simulations

• **10 000 fully non-linear** convergence fields
  ← ray-tracing through an N-body simulation
  many lens planes → include post-Born corrections

• **10 000 Gaussian** realizations of the convergence
  with same (measured) power spectrum as N-body result

• Lens same background CMB with Gaussian and Non-
  Gaussian simulations and add same noise maps
  → cancels cosmic variance

• Reconstruct lensing power spectrum from lensed maps
  → Compare the residuals of the reconstructions
Results

test against ray-traced lensing simulations

VB, Sherwin, Liu, Hill in prep.
Results

Test against ray-traced lensing simulations

VB, Sherwin, Liu, Hill in prep.
Results

test against ray-traced lensing simulations

Including post-Born corrections

VB, Sherwin, Liu, Hill in prep.
Results

test against ray-traced lensing simulations

without post-Born corrections

VB, Sherwin, Liu, Hill in prep.
Non-Gaussianity in 1-Point PDF

Liu, Hill, Sherwin, Petri, VB, Haiman 2016

Gaussian random fields (GRF) before vs after reconstruction

Gaussian random fields (GRF) vs N-body

Bias from reconstruction
Non-Gaussianity in 1-Point PDF

Gaussian vs Non-Gaussian PDF after reconstruction

9 σ difference for Wiener filtered maps

additional constraints from PDF and Peaks

Liu, Hill, Sherwin, Petri, VB, Haiman 2016
Conclusions

- The **CMB lensing potential** is **non-Gaussian**
  - non-Gaussianity of the large-scale structure
  - correlated deflections (post-Born corrections)
- The **bispectrum** of the lensing potential induces a **bias** to measurements of the lensing power spectrum
- For temperature-based reconstruction the bias from LSS alone is of order $\sim 3\sigma$ for a **Stage-III** experiment
- **Post-Born** corrections seem to reduce its magnitude to $<1\sigma$ for single bandpower → but cumulative effect matters!
- Its exact magnitude still needs to be **confirmed with simulations**
  but preliminary results look promising
Outlook

- Evaluation of the bias for **polarization-based reconstruction**
  (probably reduced for EB-EB but possibly similar for EE-EE)
- Evaluation of **bias in cross-correlation** measurements with other tracers of large-scale structure
- Higher-order statistics are now becoming detectable
  (PDF, peak counts, bispectrum)
  → need to characterize biases/noise in measurements of these statistics
Outlook

- Evaluation of the bias for polarization-based reconstruction

As measurement precision of CMB lensing increases we need a more and more refined theoretical modeling

→ lots of work still ahead to make full use of the upcoming data

(PDF, peak counts, bispectrum)

→ need to characterize biases/noise in measurements of these statistics
Backups

• CMB Lensing Theory
• CMB Lensing Reconstruction
• Lensing potential bispectrum
\[ C_L^{\phi \phi} = \int_0^{\chi_*} d\chi \frac{W(\chi)^2}{\chi^2} \frac{\gamma(\chi)^2}{(L/\chi)^4} P_\delta(L/\chi; \chi) \quad \gamma(\chi) \equiv \frac{3}{2} \frac{H_0^2 \Omega_{m0}}{c^2 a(\chi)} \]
CMB lensing parameter constraints

Figure: Planck XV 2015
CMB lensing parameter constraints

Figure: Planck XV 2015
Planck constraints on sum of neutrino masses

\[ \sum m_\nu < 0.72 \text{ eV} \quad \text{Planck TT+lowP} ; \quad (54a) \]
\[ \sum m_\nu < 0.21 \text{ eV} \quad \text{Planck TT+lowP+BAO} ; \quad (54b) \]
\[ \sum m_\nu < 0.49 \text{ eV} \quad \text{Planck TT, TE, EE+lowP} ; \quad (54c) \]
\[ \sum m_\nu < 0.17 \text{ eV} \quad \text{Planck TT, TE, EE+lowP+BAO} . \quad (54d) \]
Convergence Power Spectrum scale and redshift dependence

Figure: Zaldarriaga & Seljak 1998
CMB lensing reconstruction

Quadratic estimator (Hu & Okamoto 2002)

\[ \hat{\phi}(L) = A_L \int g(l, L) \tilde{T}_{\text{expt}}(l) \tilde{T}_{\text{expt}}^{*}(l - L) \]

Power spectrum estimate → mean square estimator

\[ \langle \hat{\phi}(L) \hat{\phi}^{*}(L') \rangle_{(\phi,T)} = A_L^2 \int_{l_1} \int_{l_2} g(l_1, L) g(l_2, L') \]

\[ \langle \tilde{T}_{\text{expt}}(l_1) \tilde{T}_{\text{expt}}(L - l_1) \tilde{T}_{\text{expt}}(-l_2) \tilde{T}_{\text{expt}}(l_2 - L') \rangle \]
CMB lensing reconstruction

**Quadratic estimator**

Weight

\[ g(1, L) = \frac{(L - 1) \cdot L C_{\tilde{T}\tilde{T}}^{L-1} + 1 \cdot L C_{\tilde{T}\tilde{T}}^L}{2 C_{\tilde{T}, \text{expt}}^{L-1} \cdot C_{\tilde{T}\tilde{T}}^L} \]

Normalization

\[ A_L^{-1} = 2 \int_1 g(1, L) L C_{\tilde{T}\tilde{T}}^L \]

**Gaussian variance**

\[ \sigma^2(L) = \frac{1}{f_{\text{sky}}} \frac{2}{(2L + 1)} \left( N_L^{(0)} + C_{\phi\phi}^L + N_L^{(1)} \right)^2 \]
# CMB lensing reconstruction

<table>
<thead>
<tr>
<th>Representative experiment</th>
<th>Stage-IV (CMB-S4)</th>
<th>Stage-III (AdvancedACT-like)</th>
<th>Planck</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{\text{FWHM}}$ [arcmin]</td>
<td>1.0</td>
<td>1.4</td>
<td>7.0</td>
</tr>
<tr>
<td>$\sigma_N^{TT}$ [$\mu$K arcmin]</td>
<td>1.0</td>
<td>6.0</td>
<td>30.0</td>
</tr>
<tr>
<td>$f_{\text{sky}}$</td>
<td>0.5</td>
<td>0.4</td>
<td>0.63</td>
</tr>
</tbody>
</table>
Lensing potential bispectrum

Weighted projection of LSS bispectrum

\[ B_\phi(l_1, l_2, l_3) = - \int_0^{X^*} d\chi \chi^2 W(\chi)^3 \frac{\gamma(\chi)^3}{(l_1 l_2 l_3)^2} B_\delta(l_1/\chi, l_2/\chi, l_3/\chi; \chi) \]

LSS bispectrum in Eulerian perturbation theory at leading order

\[ B_\delta(k_1, k_2, k_3; \eta) = 2 F_2(k_1, k_2) P_\delta(k_1, \eta) P_\delta(k_2, \eta) + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \]

\[ F_2(k_i, k_j) = F_2(k_i, k_j) = \frac{5}{7} + \frac{1}{2} \left( \frac{k_i}{k_j} + \frac{k_j}{k_i} \right) \hat{k}_i \cdot \hat{k}_j + \frac{2}{7} (\hat{k}_i \cdot \hat{k}_j)^2 \]
Lensing potential bispectrum

- Contributions to equilateral configuration
  - From different LSS modes
  - From different redshifts
Bispectrum of large-scale structure

- No exact analytical model for LSS bispectrum
  - Standard perturbation theory at leading order (tree-level)

Figure: Lazanu et al. (2016)
1-Point PDF

Liu, Hill, Sherwin, Petri, VB, Haiman 2016

Bias from reconstruction

Before reconstruction

Gaussian random fields (GRF) before vs after reconstruction
Bispectrum of large-scale structure

- No exact analytical model for LSS bispectrum
- Eulerian perturbation theory at leading-order (tree-level)

\[ B_\delta(k_1, k_2, k_3; \eta) = 2 F_2(k_1, k_2) P_\delta(k_1, \eta) P_\delta(k_2, \eta) + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \]
Lensing potential bispectrum

- Contributions to equilaterial configuration
  - From different LSS modes
  - From different redshifts

VB, Schmittfull, Sherwin 2016
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\[ W(\chi) = \frac{f_k(\chi_* - \chi)}{f_k(\chi_*) f_k(\chi)} \]

Power spectrum

\[ C_L^{\phi \phi} \leftarrow P_\delta(L/\chi; \chi) \]