ARIEL UNIVERSITY

Bouncing Models and Sourced Fluctuations

Ido Ben-Dayan
Ariel University

1604.07899, 1812.06970, 2011.00626 and WiP with Udaykrishna Thattarampilly.

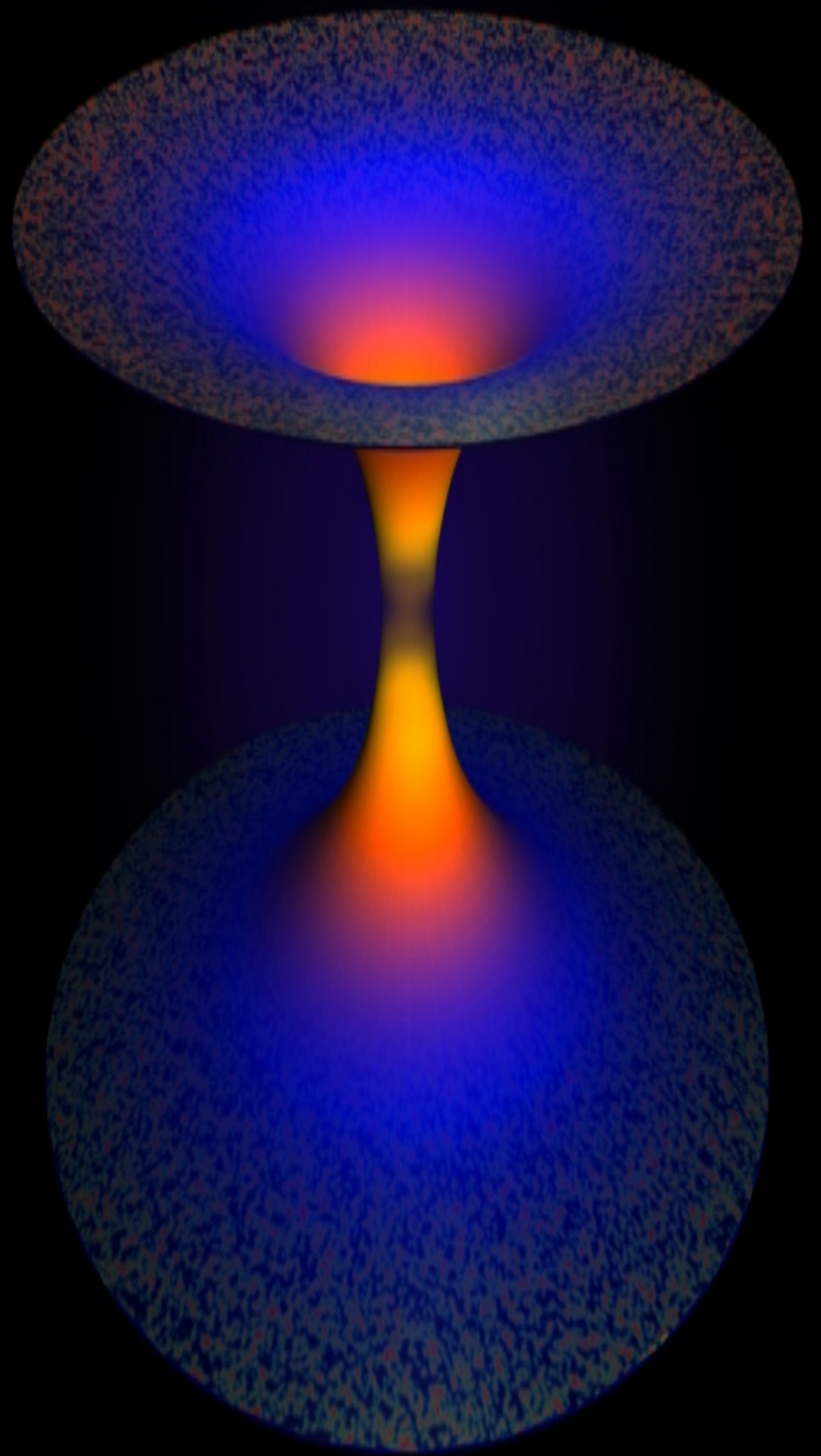
Is measuring "r" a proof of Inflation?

No!

Bouncing model with large r and predictions for LISA/LIGO

Outline

- Preliminaries- "Philosophy", disclaimer, notations and status of observations.
- Questions about Inflation.
- Bouncing Model - basic idea, solution of the "classical" Early Universe problems, generic results
- Sourced Fluctuations in Bouncing Models- model and results.



History and Philosophy

A. The Universe - Something from Nothing or Eternal?
B. Pedagogical Review
C. Effective Field Theory
D. Focus on Phenomenology

Disclaimer 1: The Inflationary paradigm actually works pretty well, especially in the context of CMB observations and the flatness and isotropy problem. However, there are virtues in Bouncing Models and there are certain possible future observations that should make bounces preferable. It is important to continue researching both possibilities.

Disclaimer 2: It is clear that most of what I will claim can be modeled around. However, at a certain point we should consider whether the remaining models are “elegant” or “simple” enough.

Disclaimer 3: The would be theory of Quantum Gravity could invalidate the entire discussion regarding both Inflation and Bouncing Models. My working assumption will be that QG will not do that.

Stochastic GW discerns between paradigms!

- Mukhanov-Sasaki eq. for GW

$$\left[\partial_\tau^2 + \left(k^2 - \frac{a''}{a}\right)\right] Q_k = \left[\partial_\tau^2 + \left(k^2 - \frac{b(b-1)}{\tau^2}\right)\right] Q_k = 0$$

$$a = a_1(-\tau)^b \qquad Q_k = \frac{c}{\sqrt{2k}}\sqrt{-k\tau}\,H^{(1)}_{b-1/2}(-k\tau)$$

$$P_{t,s} \sim \langle QQ \rangle \sim k^{2+2b}$$

- Inflation b=-1 => SCALE INV. (also matter bounce b=2) Ekpyrotic, b<<1 => $P_{t,s}$~k^2 BAD!!!

SECOND FIELD IS A MUST!

Vacuum Fluctuations

Inflation $r=?$	Bounce $r\sim 10^{-30}$
Possible Detection on CMB scales. No Detection on LI scales	No Detection on CMB scales. Possible Detection on LI scales

Stochastic GW detection at any scale discerns paradigms!

As stated in the disclaimer, this can be modeled around and Inflation models with possible detection on LI scales exist, but they are certainly a considerable deviation from the standard picture of Inflation or of LCDM.

Stochastic GW discerns between paradigms!

$$\left[\partial_\tau^2 + \left(k^2 - \frac{a''}{a}\right)\right] Q_k = \left[\partial_\tau^2 + \left(k^2 - \frac{b(b-1)}{\tau^2}\right)\right] Q_k = 0$$

MG

Source

- Really probing the geometry.
- Scalar pert. -slightly more freedom.
- All the rest is model building (amplitude, n_s, curvaton, isocurv., NG, sources…)

Notations and Conventions

- Metric and units

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j$$

$$\gamma_{ij} \equiv \frac{\delta_{ij}}{\left(1 + \frac{K}{4}\delta_{mn}x^m x^n\right)^2}$$

$$dt = ad\eta \;\; OR \;\; ad\tau$$

$$\hbar = c = 8\pi G_N = M_{pl}^{-2} \equiv 1$$

- Friedmann Eqs. for perfect fluids

$$H^2 + \frac{K}{a^2} = \frac{1}{3}\rho$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3P)$$

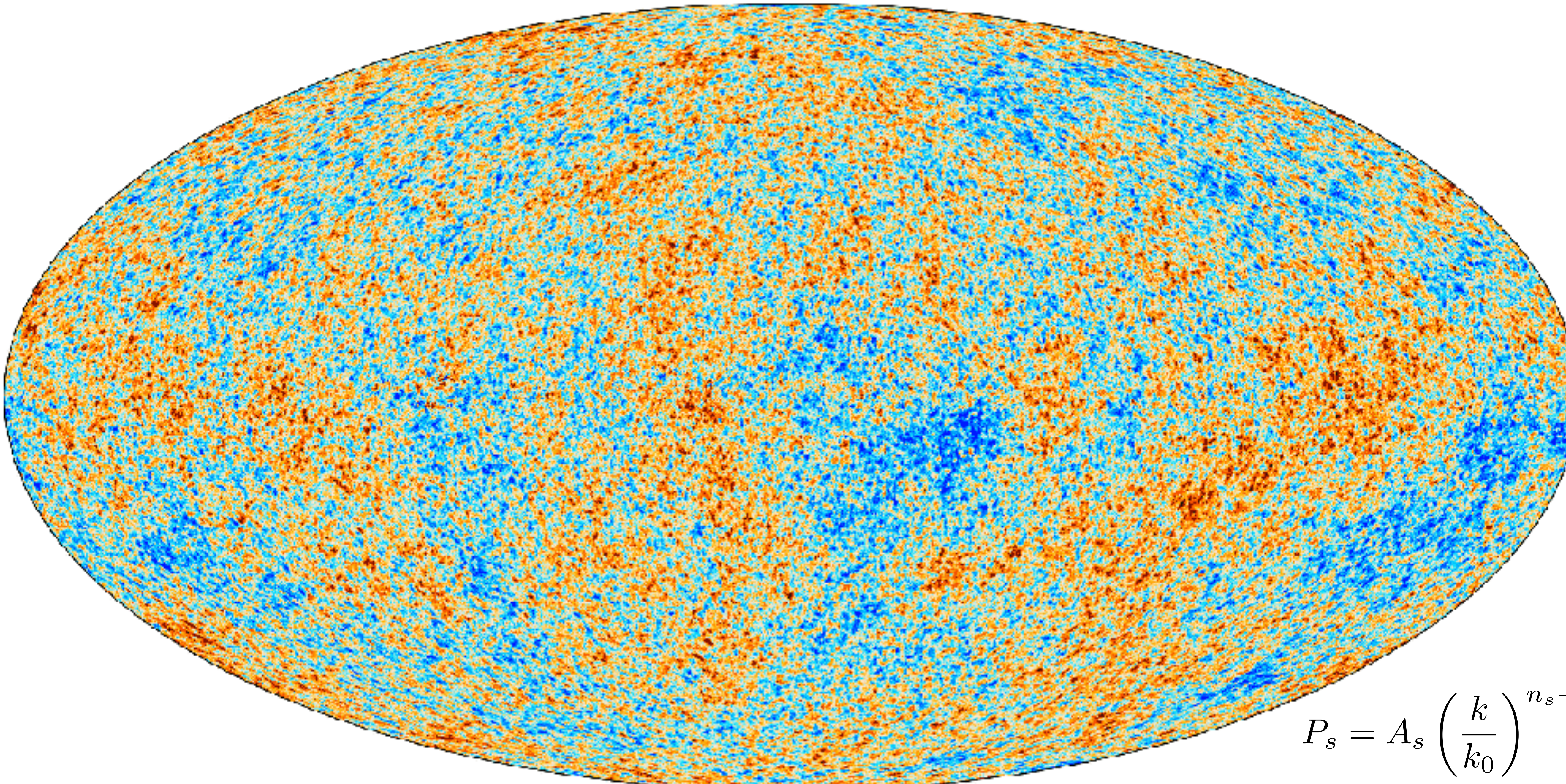
$$w \equiv \frac{P}{\rho}$$

- Canonical scalar field

$$\mathcal{L}_{can}[\phi] = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$



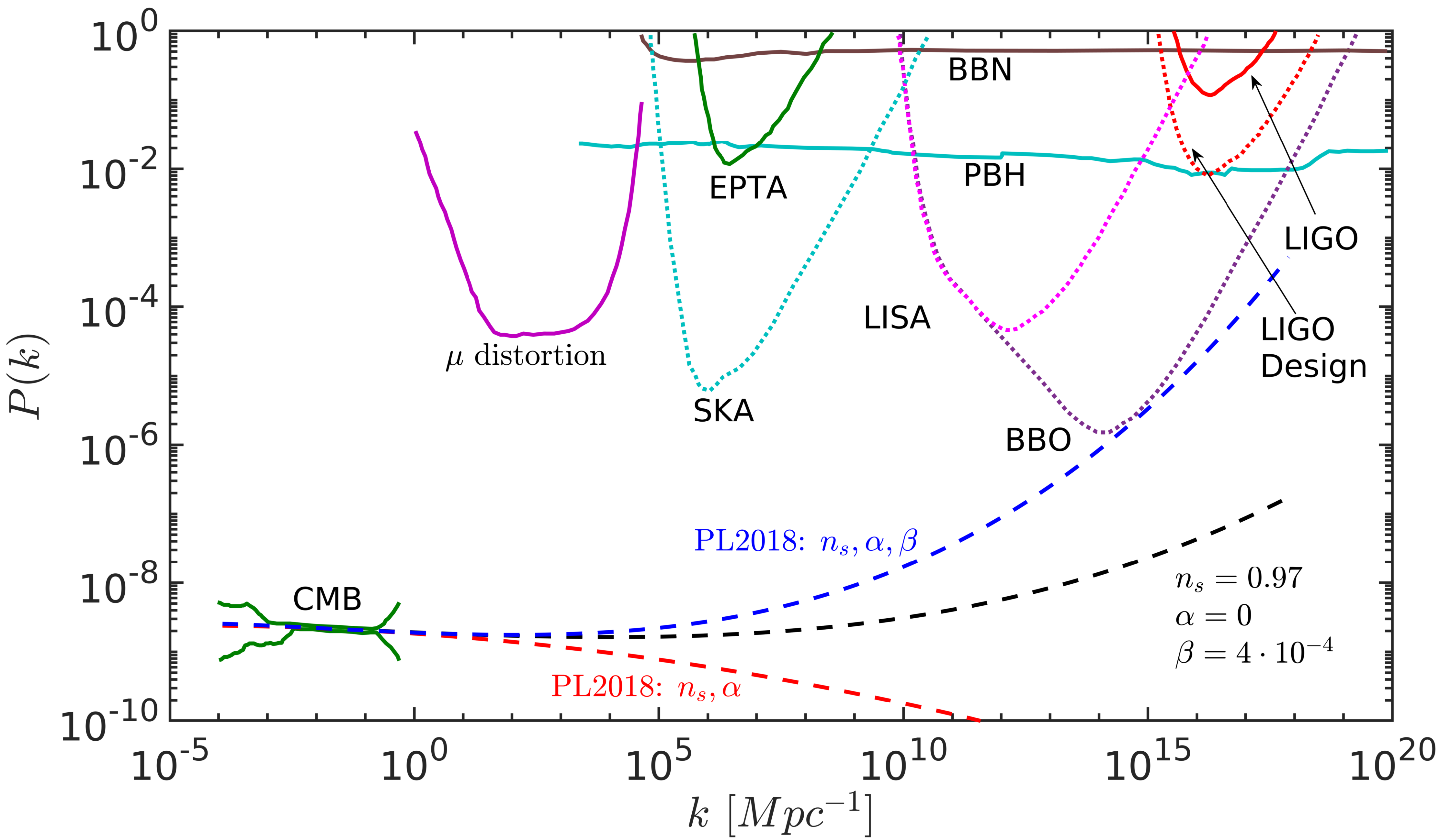
$$P_s = A_s \left(\frac{k}{k_0}\right)^{n_s-1}$$

$$P_t = A_t \left(\frac{k}{k_0}\right)^{n_t}$$

$$r(k_0) = \frac{P_t(k_0)}{P_s(k_0)}$$

Temp. anisotropies->filtered to 5 deg->+pol.

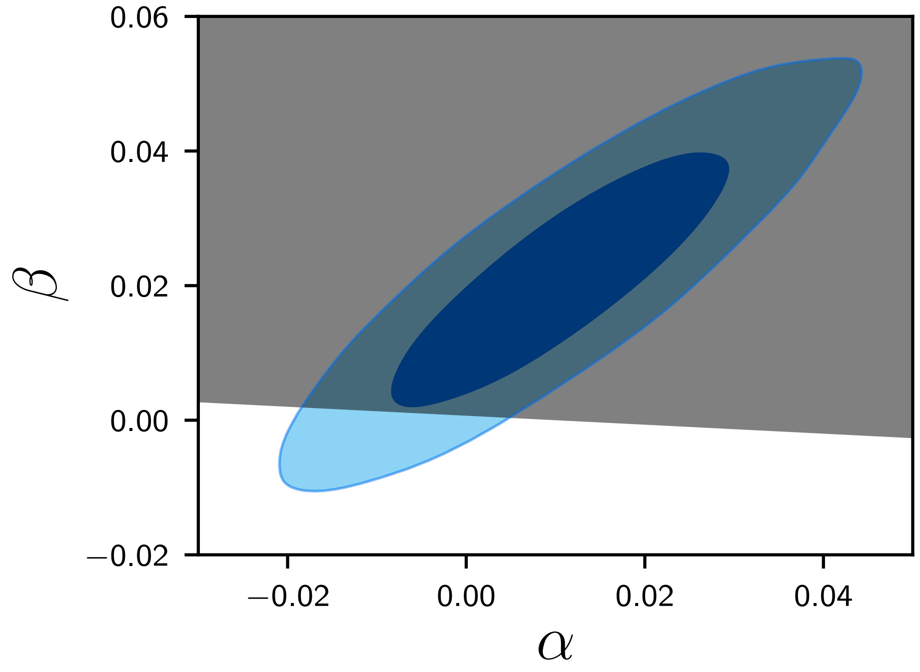
$$A_s \simeq 2.1 \times 10^{-9}, \quad n_s \simeq 0.965, \quad r < 0.036, \quad n_t?$$



2nd order GW, Neff and LI

IBD, B. Keating, D. Leon and I. Wolfson

2019



Inflation

- A period of accelerated expansion. H~const., a~e^{Ht} , for at least 50 e-folds.
- Various realizations. Simplest and most common- single canonical scalar field.
- Slow-roll approximation.
- Predictions for CMB and LI

Observations

$$P_s = A_s \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$P_t = A_t \left(\frac{k}{k_0}\right)^{n_t}$$

$$r(k_0) = \frac{P_t(k_0)}{P_s(k_0)}$$

$$\mathbf{A_s \simeq 2.1 \times 10^{-9}, \quad n_s \simeq 0.965, \quad r < 0.036, \quad n_t?}$$

Theory

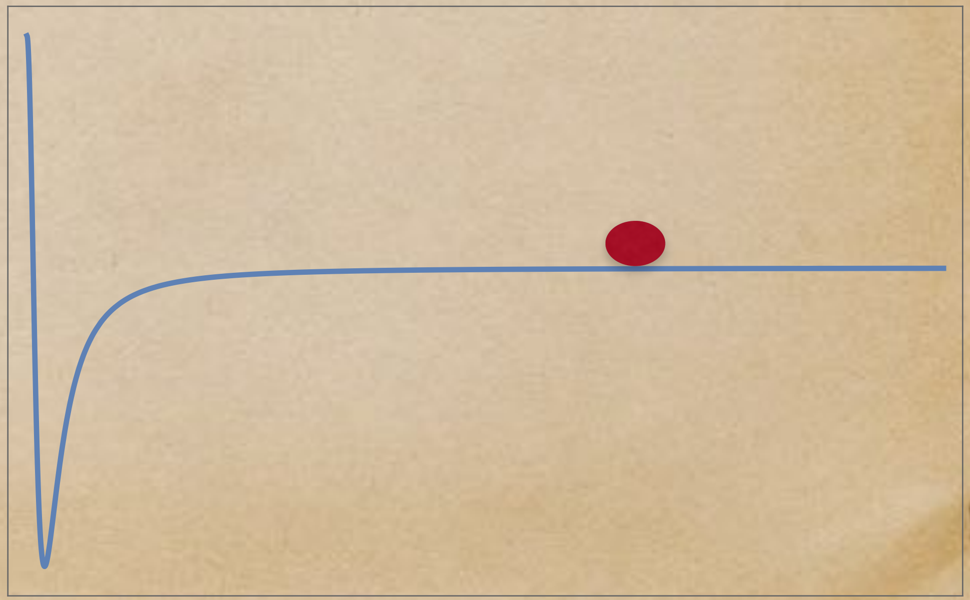
$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$\eta = \frac{V''}{V} \ll 1$$

$$n_s = 1 + 2\eta - 6\epsilon$$

$$r = 16\epsilon$$

$$n_t = -2\epsilon = -r/8 \lesssim 0$$



Questions about Inflation

- Biggest Question: The Big Bang Singularity. Resolving the Big Bang Singularity, can presumably be done without touching inflation. However, such a “solution” will simply be another patch added to the model, without any additional predictions, or ways to confirm or falsify such a patch.

Questions about Inflation

- Other "Issues" with descending levels of seriousness. None of them is a show stopper.

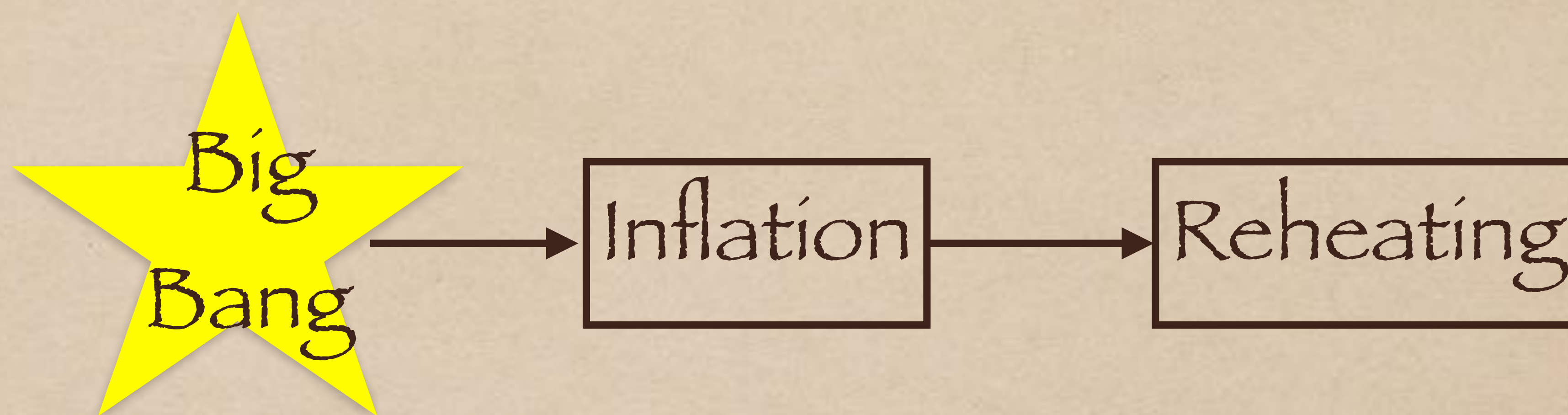
1. Initial Conditions Problem:

- Phase space measure? The number of trajectories that lead to inflation does not grow with time.
- Did we scan enough the space of metrics that converge to Inflation? (Senatore and Kleban 2016)

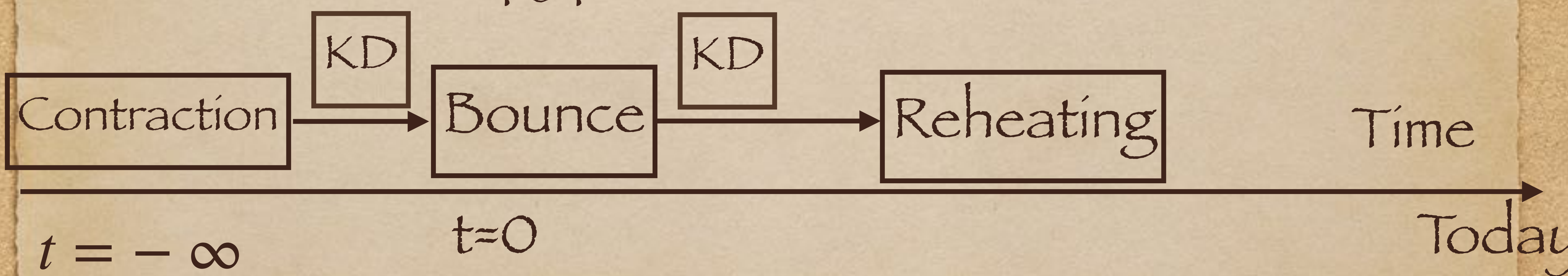
2. The Multiverse of Eternal Inflation. If there is a region where the scalar spectrum is greater than unity, $P_s > 1$, then we have a "multiverse" with the majority of the volume still inflating. Do Inflationary predictions make sense in such case? (IBD, Hadad and Michaelis 2021)
3. Simplest Models of Inflation are ruled out.
4. Quantum Gravity- Various conjectures about the would be QG theory, and 20+ years of experience in String Theory suggest that Inflation or metastable dS are very difficult to realize from basic ingredients. However, see IBD "Draining the Swampland" 2019

$$\Delta\phi \lesssim 1 \qquad \frac{V'}{V} \gtrsim c \sim 1 \qquad \frac{V''}{V} \gtrsim c' \sim 1$$

Bouncing Cosmology



- Requirement: predict a valid CMB spectrum and solve the flatness and isotropy problems.



- "Non-singular Bounce". EFT is valid

Immediate Obstacles for a Bounce

Ekpyrosis w>>1

- Shear and BKL Instability

$$H^2 = \frac{1}{3}\left[\frac{-3K}{a^2} + \frac{\rho_{m0}}{a^3} + \frac{\rho_{r0}}{a^4} + \frac{\rho_{\theta 0}}{a^6} + \cdots + \frac{\rho_{\phi 0}}{a^{3(1+w_\phi)}}\right]$$

- NEC violation (unless K=+1, where you need inflation to dilute it)

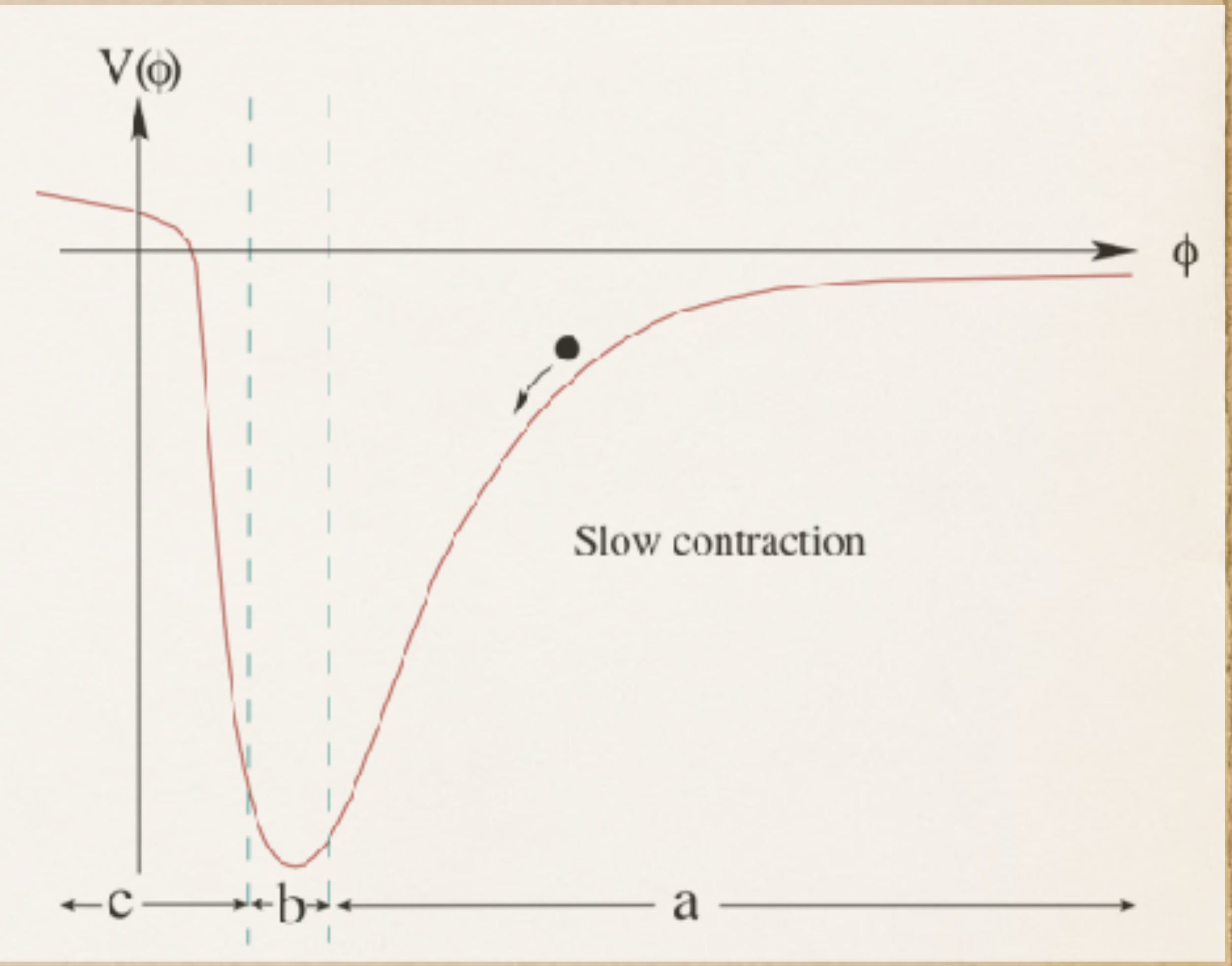
$$\dot{H} = -\frac{1}{2}(\rho + P) = -\frac{1}{2}\dot{\phi}^2$$

- => Modified Gravity, non-canonical kinetic term, beyond scalar field (Artymowski, IBD, Kumar 2019,2020,2021)

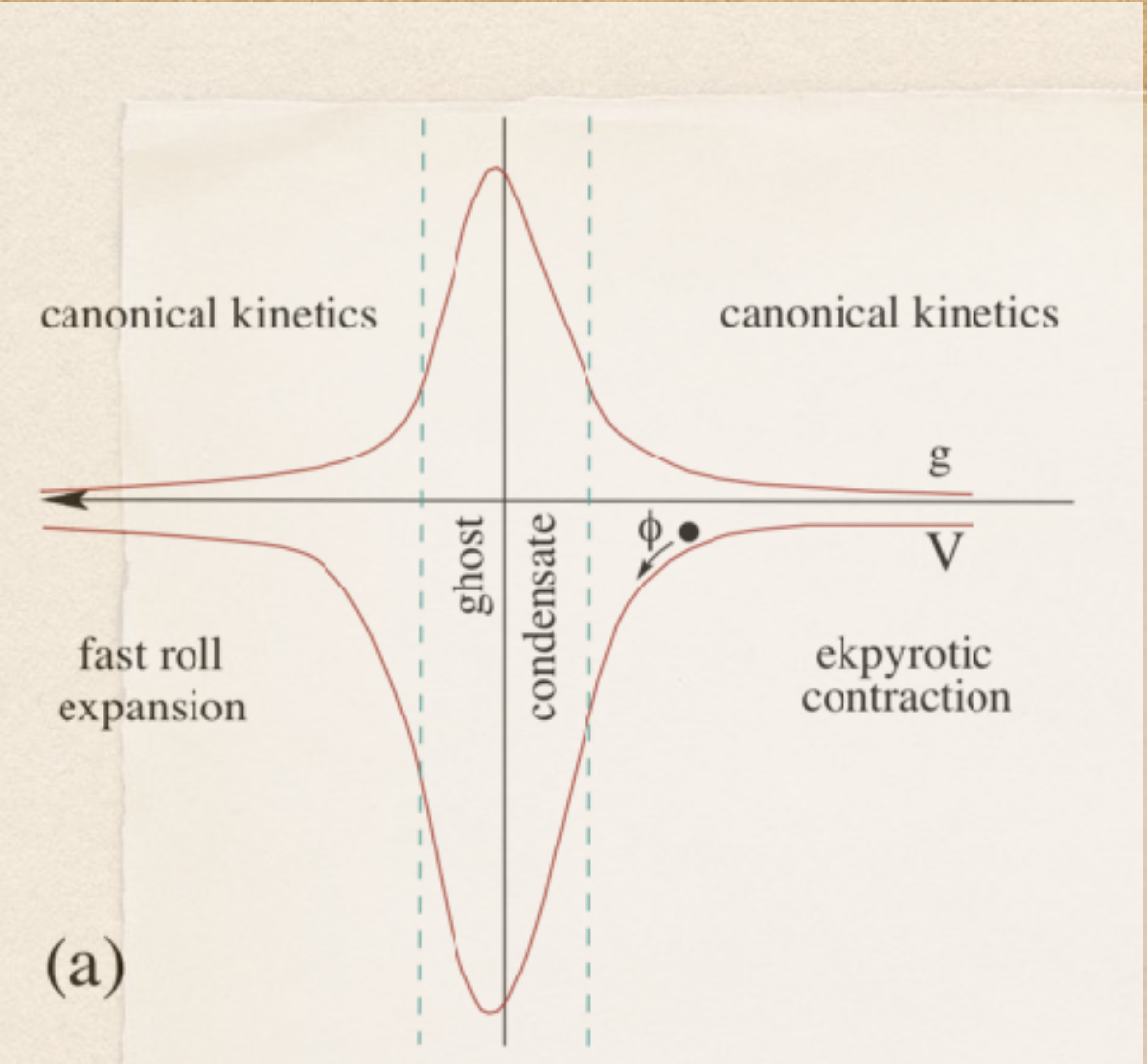
- Bouncing models solve the horizon and flatness problems.

Canonical Example of Contraction Phase-Ekpyrosis

- Exact solution of EOM
- p>>1, V>0 Power law Inflation. Viable models are small deviations from it.
- p<<1, V<0 Ekpyrotic, w>>1
- p=2/3, V<0 Matter dominated contraction



$$a = a_0(-t)^p, \quad t < 0, \quad V(\phi) = -V_0 e^{-\sqrt{2/p}\phi}$$



The Model!

Scalar field+potential+
U(1) gauge field

Gauge field source

$$\mathscr{L} = [1 - g(\phi)]X + \beta X^2 + \Upsilon X \Box \phi - V(\phi) - \frac{I^2(\phi)}{4}\left(F^2 - \gamma F\tilde{F}\right)$$

Kinetic

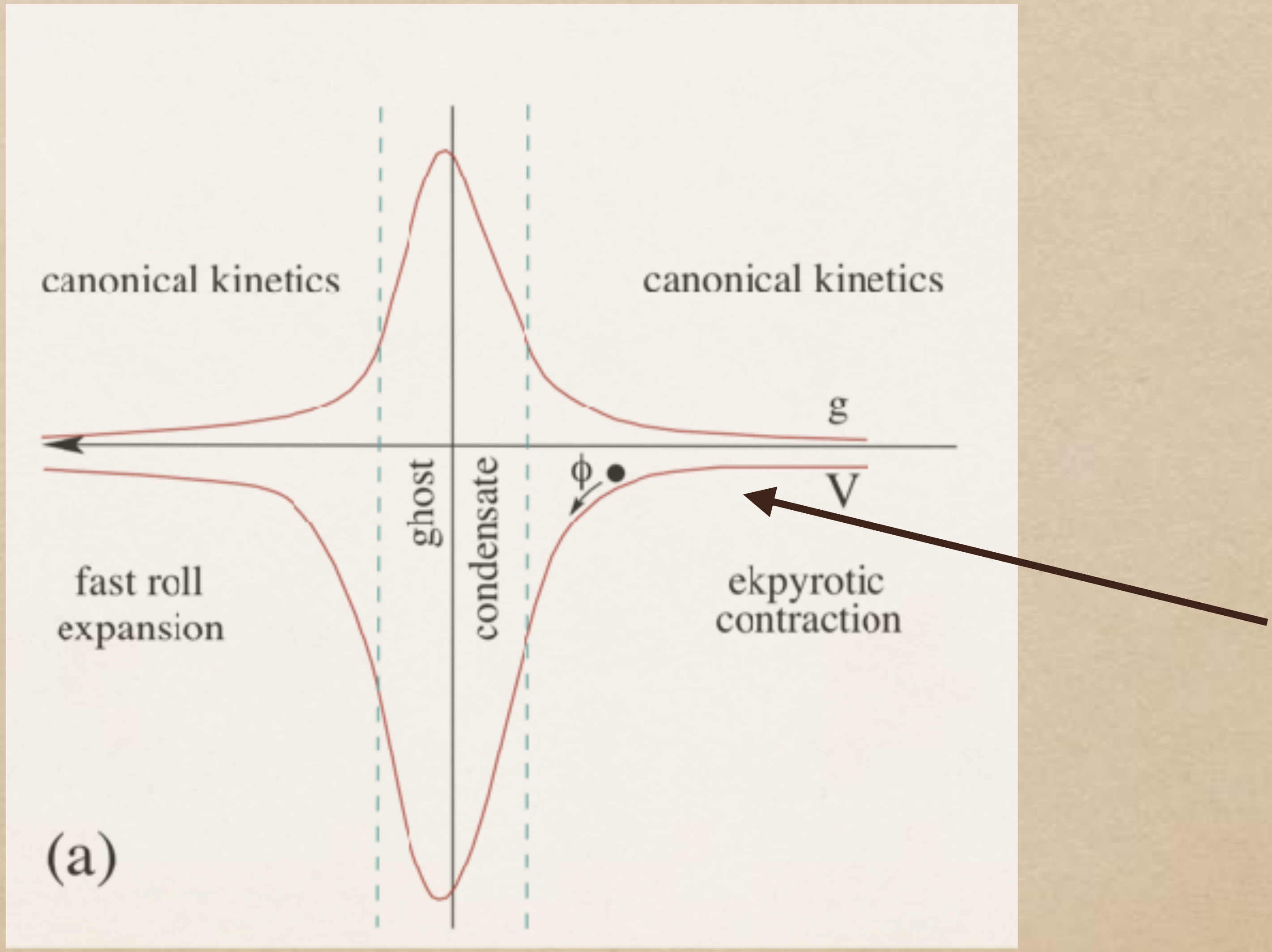
Potential

$$X = -\frac{1}{2}(\partial\phi)^2$$

$$V(\phi) = -\frac{2V_0}{e^{-\sqrt{2/q}\phi} + e^{b\sqrt{2/q}\phi}}$$

$$g(\phi) = -\frac{2g_0}{e^{-\sqrt{2/p}\phi} + e^{b_g\sqrt{2/p}\phi}}$$

Contraction Phase



Sourced Fluctuations

$$\left[\partial_\tau^2 + \left(k^2 - \frac{a''}{a}\right)\right] Q_k = J$$

- Vacuum spectra and sourced spectra. No cross terms.

$$P_{t,s}^{tot.} = P_{t,s}^{v} + P_{t,s}^{s}$$

$$P_{t,s}^{v} \sim k^2$$

- Coupling between the scalar field(s) and gauge or fermion fields. Avoids BBN constraints.

- Backreaction Bound: $\frac{H}{M_{pl}} << \sqrt{3/D_{1,2}(n) p^2 \xi^{3/2} e^{-\pi\xi}}$

$$I(\varphi_1) = (-\tau)^{-n}$$

Parity Breaking

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial\varphi_1)^2 - V(\varphi_1) - I^2(\varphi_1)\left\{\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\gamma}{4}\tilde{F}^{\mu\nu}F_{\mu\nu}\right\}\right]$$

Gauge Field Mode Equation

- Invariant under: $n \to -1-n, \quad \gamma \to -\frac{n}{1+n}\gamma$
- Controlled Backreaction. $\tilde{A}''_\lambda + \left(k^2 + 2\,\lambda\,\xi\,\frac{k}{\tau} - \frac{n\,(n+1)}{\tau^2}\right)\tilde{A}_\lambda = 0$

$$\xi \equiv -n\gamma, \quad \tilde{A} \equiv I(\tau)A$$

- Exponential enhancement of gauge quanta of only one polarization.

$$\tilde{A}(k,\,\tau) \simeq \sqrt{-\frac{\tau}{2\,\pi}}\,e^{\pi\xi}\,\Gamma\left(|2n+1|\right)\,|2\,\xi\,k\,\tau|^{-|n+1/2|}$$

$$-k\tau \ll 1/\xi$$

- Sourced spectra, with the same invariance, uncorrelated with the adiabatic one.

$$\mathcal{P}_T(n,\xi;\, n \geq -1/2) = \mathcal{P}_T(-1-n,\xi;\, n \leq -1/2).$$

Sourced GW Spectrum

$$J_\lambda \sim \tilde{A}^2 \sim e^{2\pi\gamma|n|}$$

$$\mathcal{P}_t^s \sim J_\lambda^2 \sim e^{4\pi\gamma|n|} k^{4(2+n)}$$

- Exponential enhancement, scale invariant (n=-2,1)
- Chiral tensor spectrum (only + pol.) due to the parity breaking term

Sourced Scalar Spectrum

- To avoid gauge artifacts: Full solution of EFE to second order.
- Same leading source term =>Same tilt, almost same amplitude. Only important difference in time dependent term $\mathcal{I}^2_{s,t}$
- Exact momentum integration using dim. reg.
- For scale inv. n->-2,1

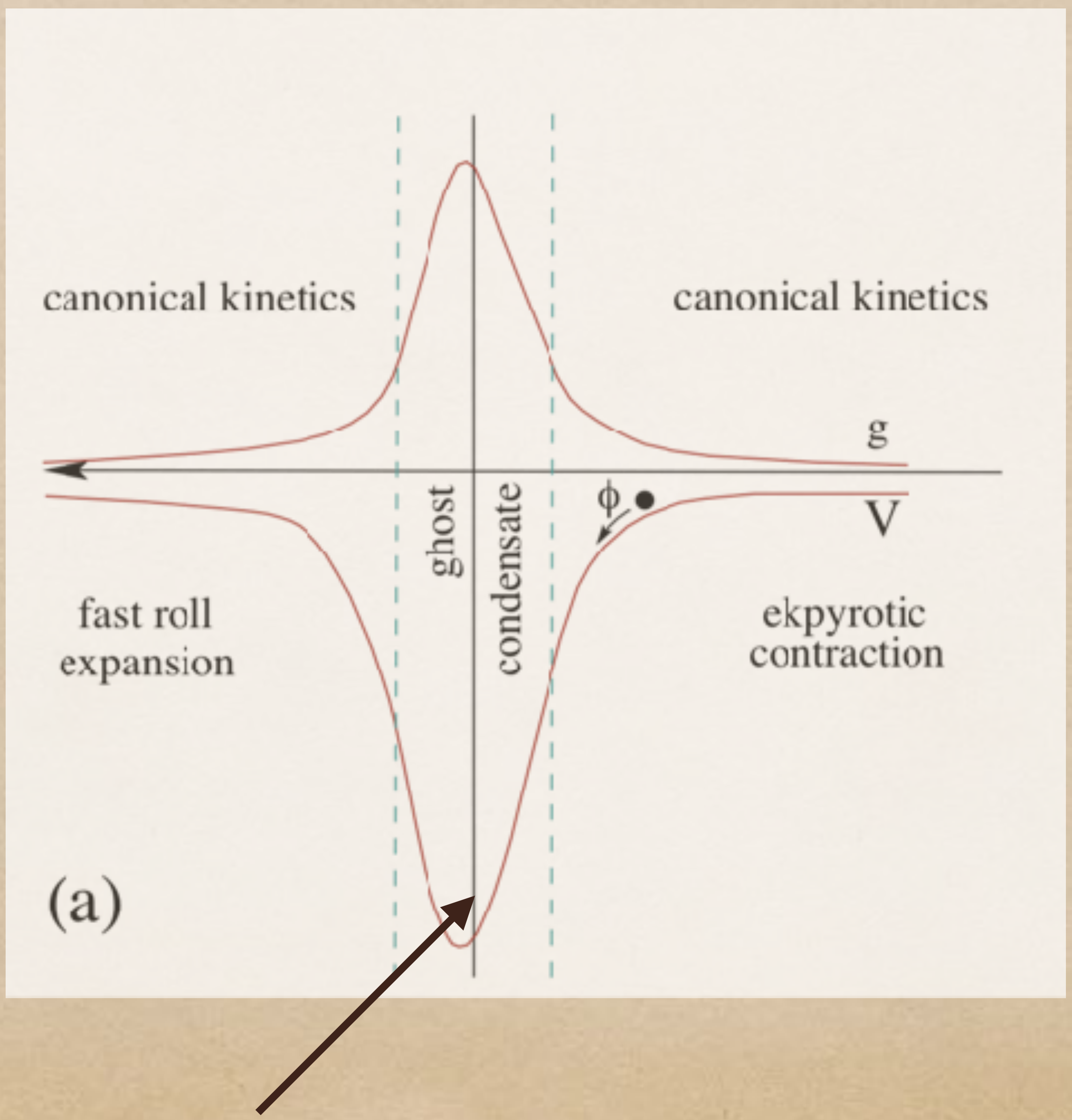
$$< X_k X_{k'} >= \frac{2\pi^2}{k^3}\delta(\vec{k}+\vec{k}')(\mathcal{P}^v_X(k)+\mathcal{P}^s_X(k)).$$

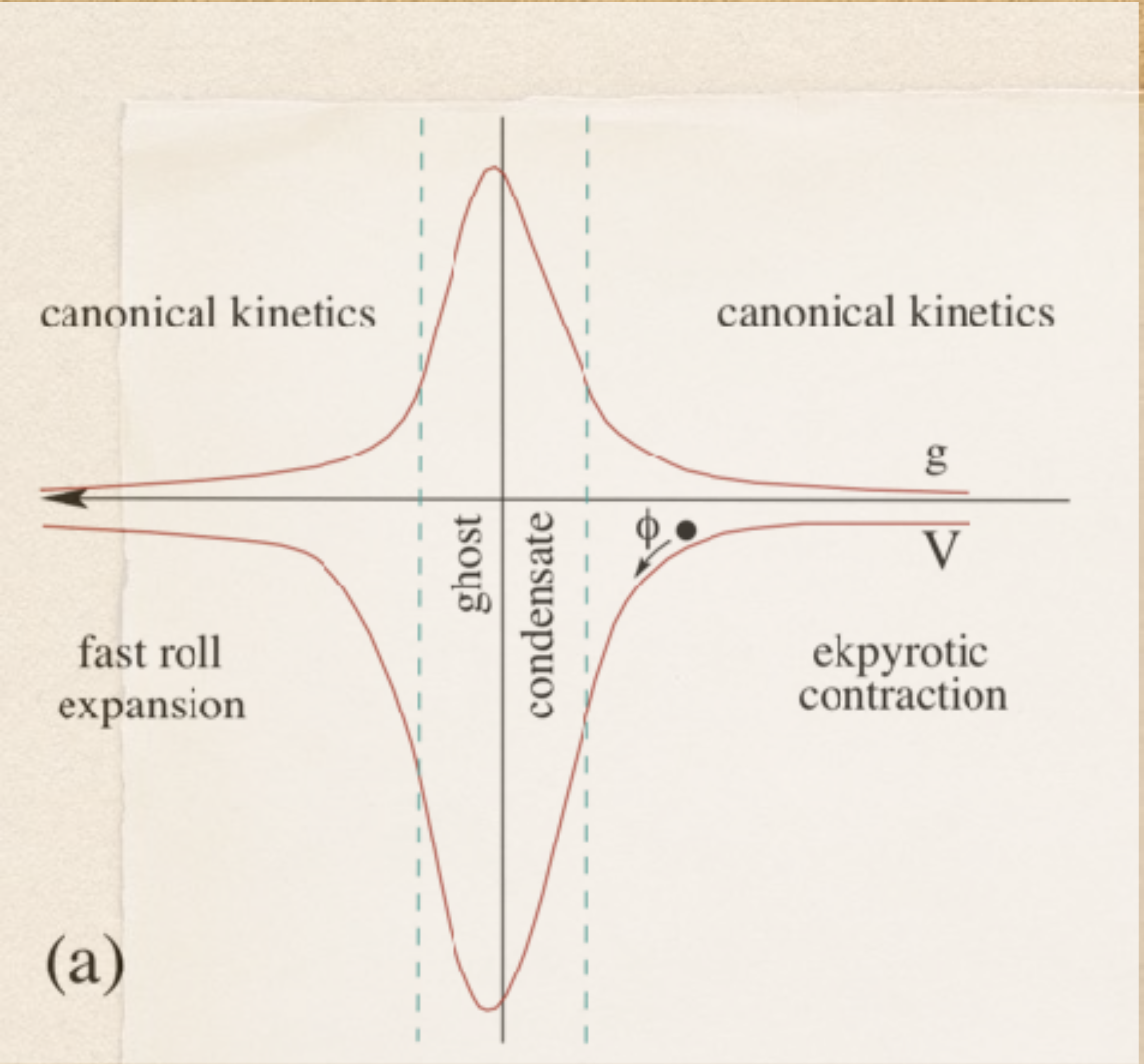
$$\mathcal{P}^s_{T,S} = \frac{2\mathcal{N}^{T,S\,2}_m \mathcal{I}^2_m}{2\pi^2}\frac{e^{4\pi\xi}\xi^{2\alpha}}{M^4_{pl}}k^{6+2\alpha}\times f^{T,S}(q)$$

$$r \simeq \frac{1}{(1-n)^2} \to \frac{1}{9}$$

r too large and not too small??

Crossing the Bounce





Crossing the bounce

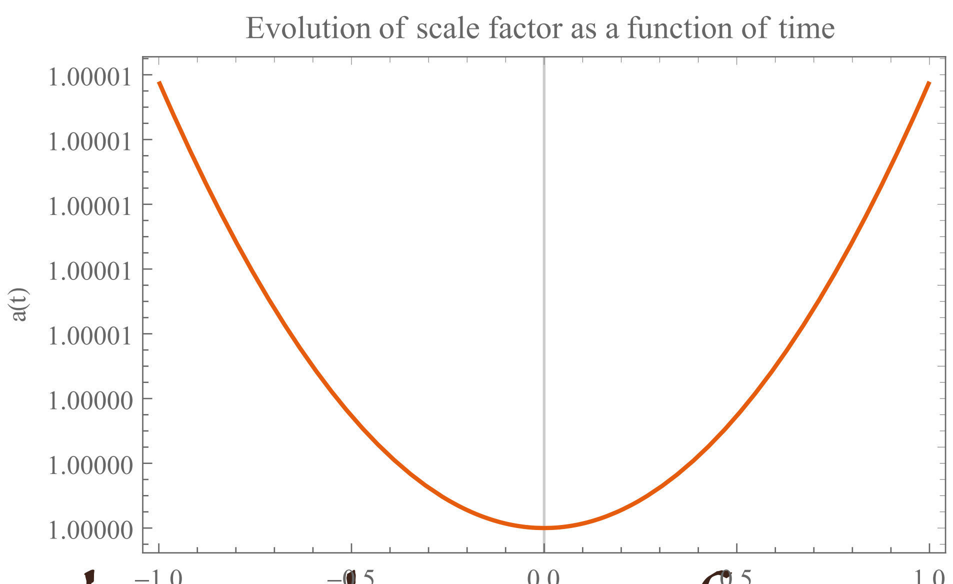
$$\mathcal{L} = [1 - g(\phi)]X + \beta X^2 + \Upsilon X \Box \phi - V(\phi) - \frac{I^2(\phi)}{4}\left(f^2 - \gamma F\tilde{F}\right)$$

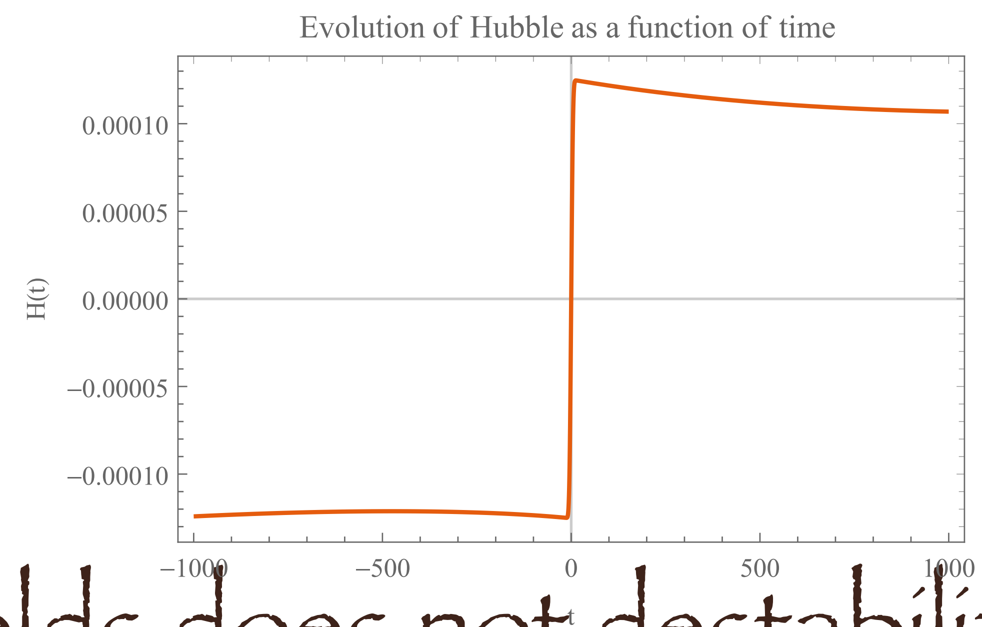
$$X = -\frac{1}{2}(\partial\phi)^2$$

$$V(\phi) = -\frac{2V_0}{e^{-\sqrt{2/q}\phi} + e^{b\sqrt{2/q}\phi}}$$

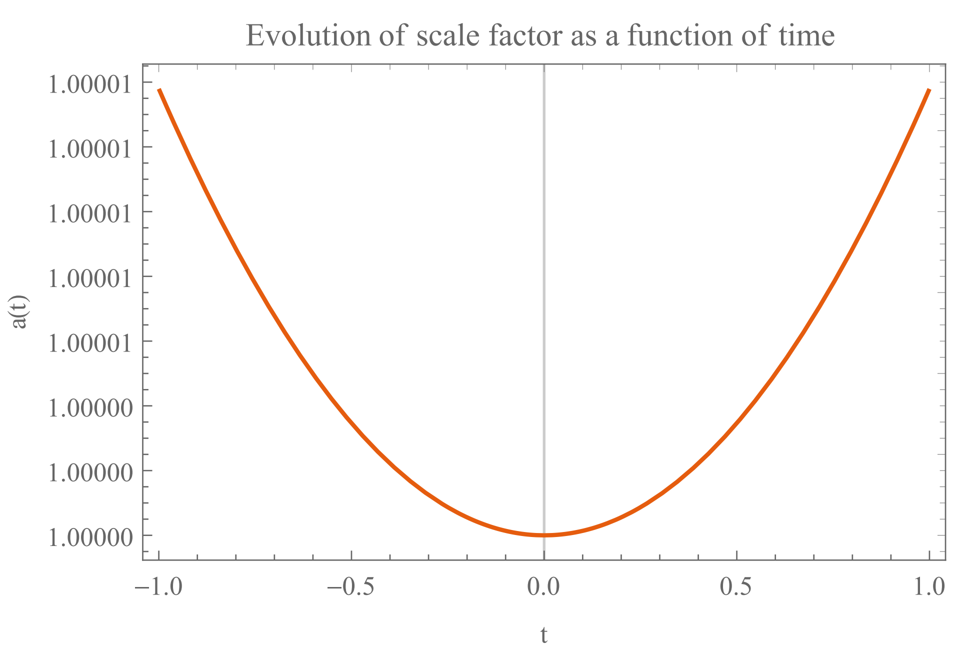
$$g(\phi) = -\frac{2g_0}{e^{-\sqrt{2/p}\phi} + e^{b_g\sqrt{2/p}\phi}}$$

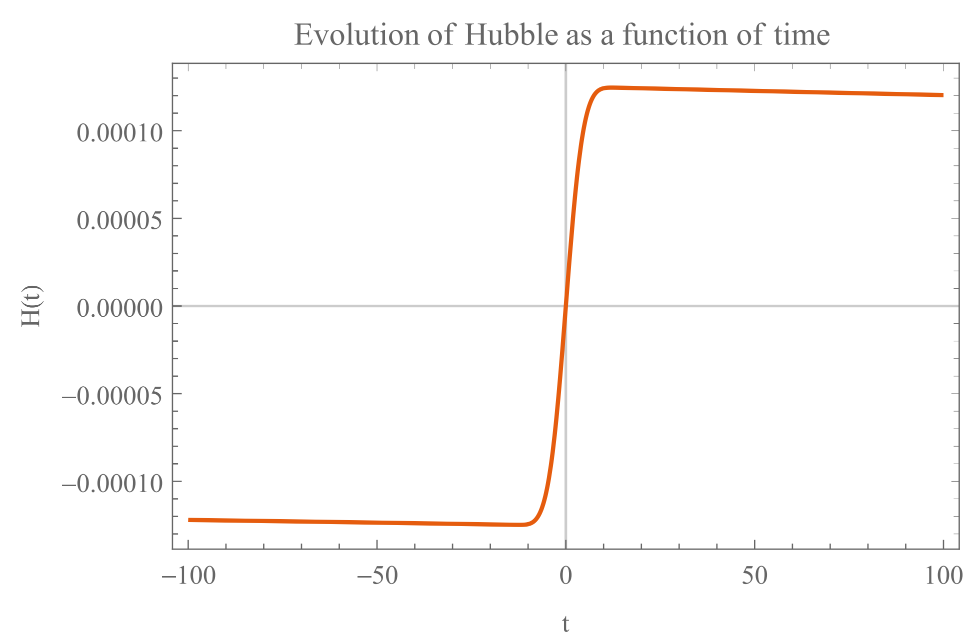
Xing the Bounce- background evolution





- The inclusion of gauge fields does not destabilize the bounce.

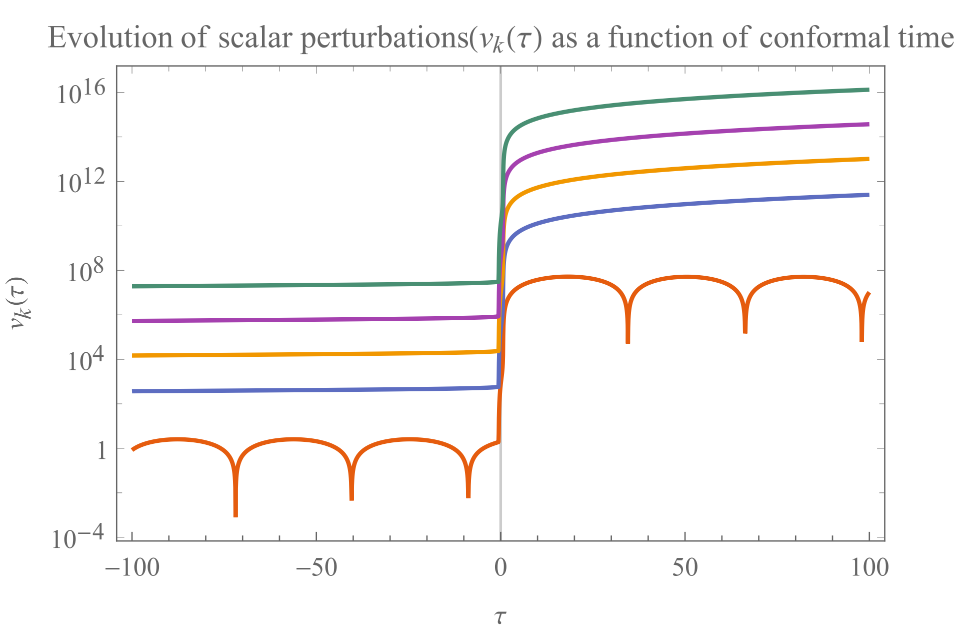


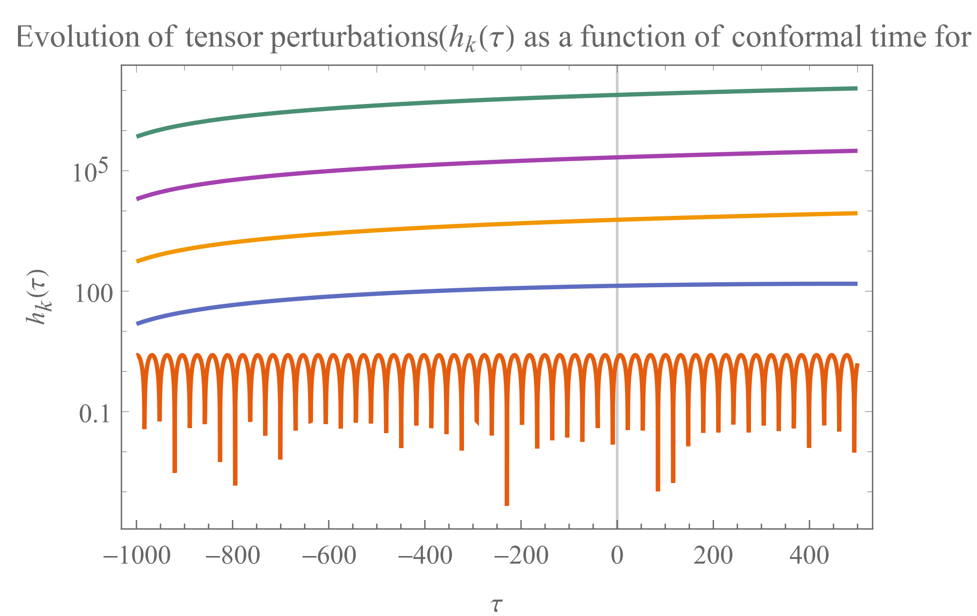


Xing the Bounce- vacuum spectrum

$$\left[\partial_\tau^2 + \left(c_s^2 k^2 - \frac{z''}{z}\right)\right] Q_k = 0$$

$$\left[\partial_\tau^2 + \left(k^2 - \frac{a''}{a}\right)\right] Q_k = 0$$





Xing the Bounce- vacuum spectrum

- The tilt of both spectra is unchanged.
- The amplitude of the scalar spectrum is enhanced when crossing the bounce.

$$\mathcal{F} = e^{\int_{B_-}^{B_+} \omega d\tau} \simeq e^{\left(\sqrt{\Upsilon + \frac{2}{T^2}}\,t + \frac{2 + 3\Upsilon T^2 + \Upsilon^2 T^4}{3T^4\sqrt{\Upsilon + \frac{2}{T^2}}}\,t^3\right)\Big|_{B_-}^{B_+}}$$

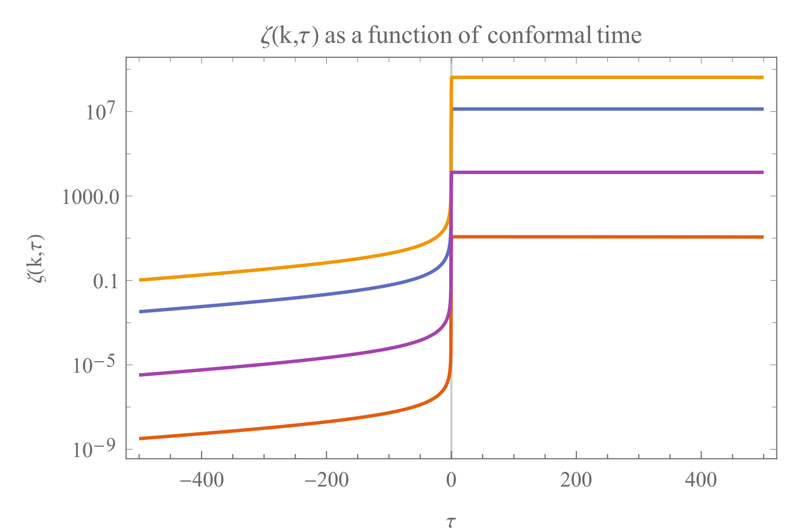
- Matching hom. solutions across hyper-surfaces.

  $H = \Upsilon t$, $\tau_{B\pm}$ onset and end of bounce , $T = (\tau_{B+} - \tau_{B-})/4$

Xing the Bounce- Sourced Spectrum

$$< X_k X_{k'} >= \frac{2\pi^2}{k^3}\delta(\vec{k}+\vec{k}')(\mathcal{P}^v_X(k)+\mathcal{P}^s_X(k)).$$

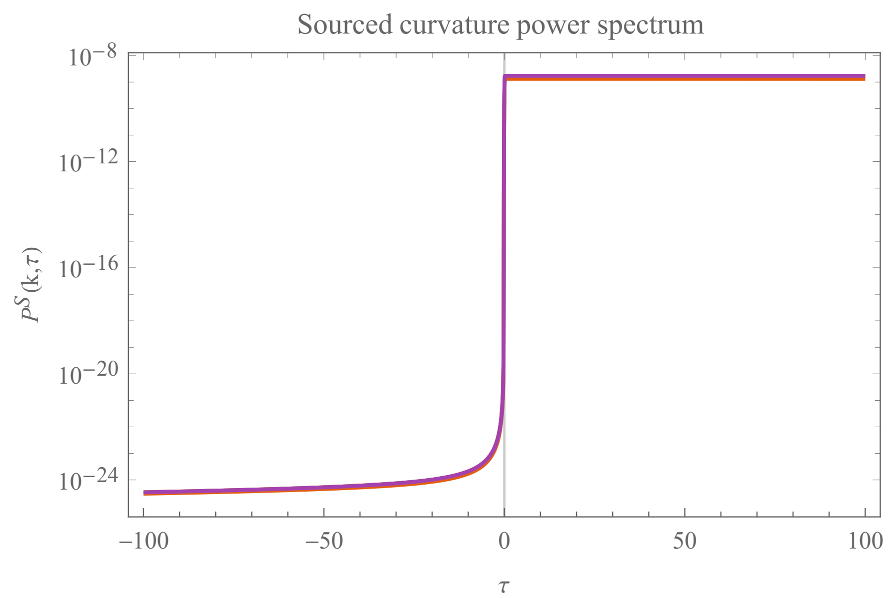
$$\mathcal{P}^s_{T,S} = \frac{2\mathcal{N}_m^{T,S\,2}\mathcal{I}_m^2}{2\pi^2}\frac{e^{4\pi\xi}\xi^{2\alpha}}{M_{pl}^4}k^{6+2\alpha}\times f^{T,S}(q)$$



- Matching inhom. Solutions
- GW-unchanged. Scalar enhanced.

$$\mathcal{I}_S^{after} \simeq \mathcal{I}_S^{before}\mathcal{F}_S > \mathcal{I}_S^{before}e^{4\sqrt{2}}$$

- Shorter Bounce- larger enhancement



CMB results

$$P_S(k) = \frac{11.1}{256 r \pi^6 (1 - n_s)} \frac{e^{4\pi\xi}}{b^4 \xi^6} \left(\frac{H_{end}}{M_{pl}} \right)^4 \left(\frac{k}{H_{end}} \right)^{n_s - 1} F^s(T)^2$$

$$A_s \simeq 2.1 \times 10^{-9}, \quad n_s \simeq 0.965$$

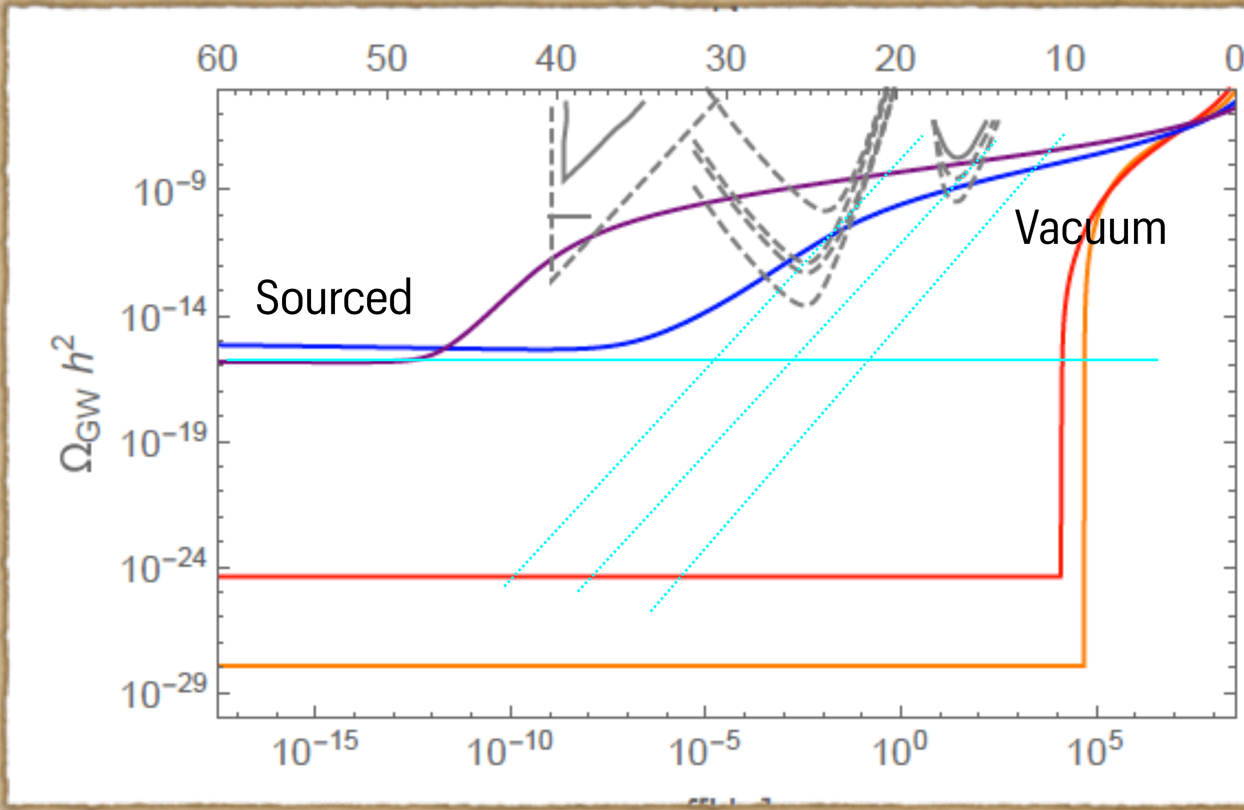
- Valid scalar spectrum.
- Tensor to scalar ratio: $$r_{after} = \frac{r_{before}}{\mathcal{F}_S^2} = \frac{1/9}{e^{8\sqrt{2}}} \lesssim 10^{-5}$$
- What about LIGO and LISA?

LIGO and LISA

- The sourced spectrum is slightly red tilted to match CMB - no observation by LI.
  $n_s^s - 1 = n_t^s \simeq -0.04, \quad n_s^v - 1 = n_t^v \simeq 2$

- The vacuum spectrum is unconstrained except for BBN/ Neff. Contrary to bounces without gauge fields.

- Contraction can happen at larger H => Observable by LISA/LIGO



	CMB	LI	n_T	Chirality
Slow-roll Inflation	✔	✖	~<0	✖
Sourced Inflation	✔	✔	~<0, +blue	✔ (LI)
Bounce	✖	?	~2-3	✖
Sourced Bounce	✔	✔	~0< and n_T~2	✔ (CMB)

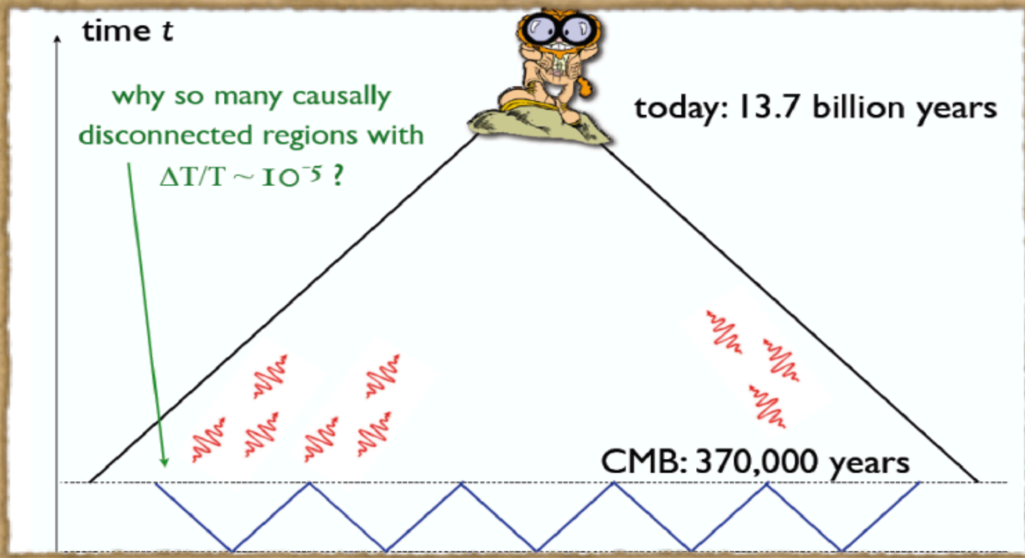
Summary of Results

Summary of GW detection

- Null detection - status quo maintained.
- Detection on CMB scales only - favors Inflation!
- Detection on LI scales only - favors Bounce!
- Detection on CMB and LI scales is allowed by only a small subset of models! Non-generic! No paradigms!
- Detection of chirality of GW - favors Sourced fluctuations!

Conclusions

- We have constructed a full fledged Bouncing Model with Sourced Fluctuations.
- Predictions: In accord with CMB, sizeable r, chiral GW, and observable by LIGO or LISA.
- Future work - Non gaussianity ? Other sources? Other models?
- Bouncing models are still underdeveloped compared to Inflation.
- Bouncing models are very delicate - BKL instability, NEC violation, 2nd field. Singular vs. Delicate? And what about QG?
- Sourced fluctuations allows for new phenomena in both bouncing and inflationary background.
- Any type of GW detection (CMB, LI, chirality) will revolutionize the field. Specifically detection on LI scales will make bouncing models more favorable.
- Are we still in the regime of "simple" or "elegant" models?



Solution of the Horizon Problem

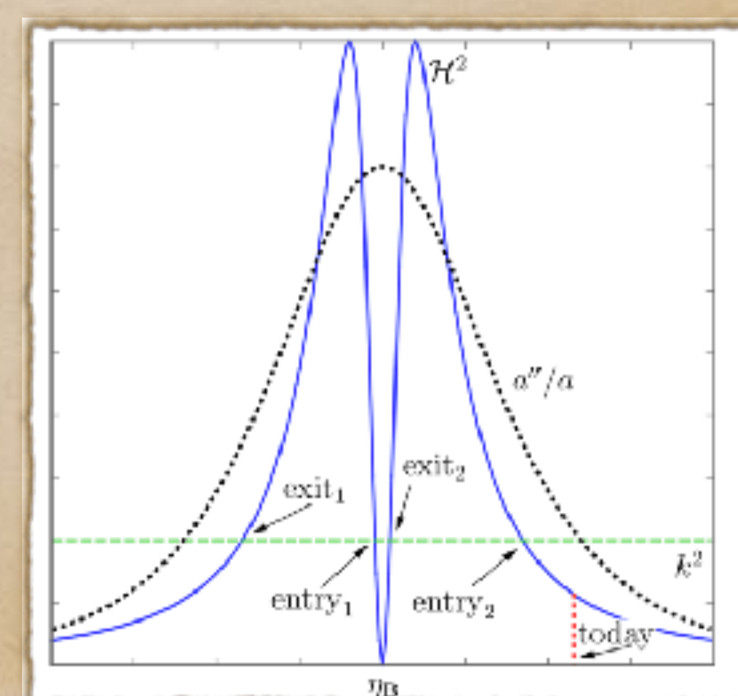
$$d_H \simeq \frac{1}{H(1+z_{LSS})^{3/2}}$$

$$d_A \simeq \frac{1}{H(1+z_{LSS})}$$

$$\frac{d_H}{d_A}|_{z_{LSS}=1100} \simeq \frac{1}{H(1+z_{LSS})^{1/2}}|_{z_{LSS}=1100} \simeq 1.6^\circ$$

- In the Hot Big Bang:
- If we have a period of contraction - plenty of time to come into causal contact. d_H can be arbitrarily large for $w>-1/3$. t_{ini} beginning of contraction, t_{end} end of contraction.

$$d_H^{cont} = \frac{3(1+w)}{1+3w} t_{end} \left\{ 1 - \left(\frac{t_{ini}}{t_{end}}\right)^{(1+3w)/[3(1+w)]} \right\}$$



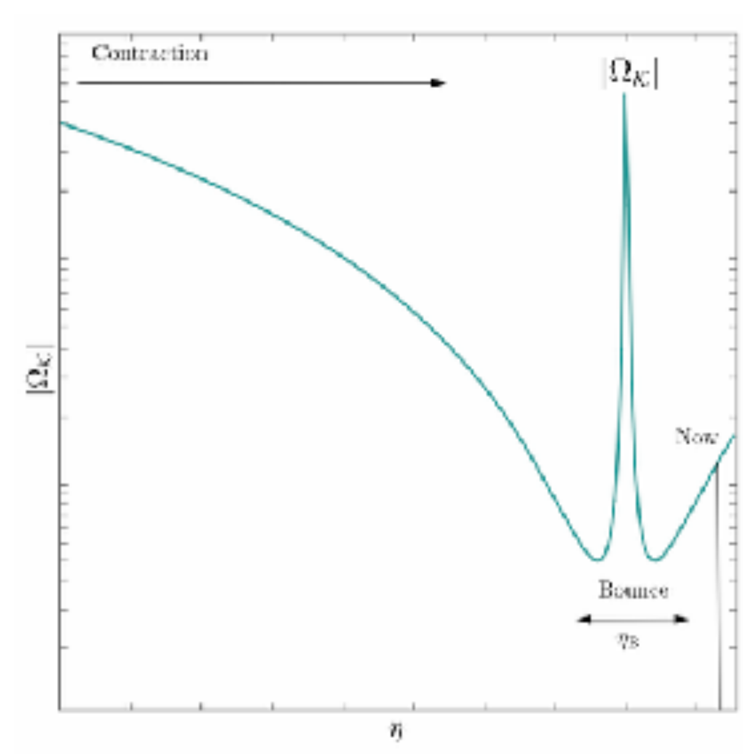
Solution of the Flatness Problem

$$\Omega_{total} = \Omega_\Lambda + \Omega_m + \Omega_r + \Omega_K \simeq 1$$

$$|\Omega_K| \equiv \frac{|K|}{a^2 H^2} \ll 1$$

$$\frac{d|\Omega_K|}{dt} = -2|K|\frac{\ddot{a}}{\dot{a}^3}$$

- In the Hot Big Bang curvature should always increase. $\frac{d|\Omega_K|}{dt} > 0$ We observe a flat $\ddot{a} < 0, \quad \dot{a} > 0$ Universe=>Fine-tuned.
- Decelerating and contracting $\ddot{a} < 0, \quad \dot{a} < 0$ => arbitrarily small curvature. $\frac{d|\Omega_K|}{dt} < 0$
- Remains negligible for a symmetric bounce.



Bouncing Cosmology - Tensor Spectrum (GW)

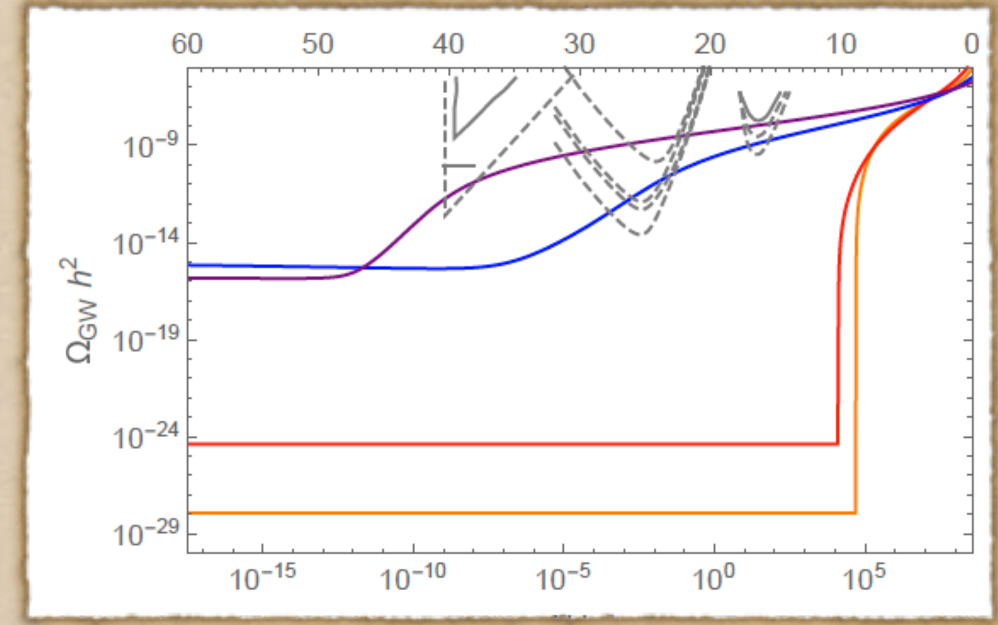
- "Fiddling" with the potential generated a viable scalar spectrum, as is usual in inflation model building.
- GW - a metric perturbation, no place to "fiddle". The scale factor a defines your evolution.

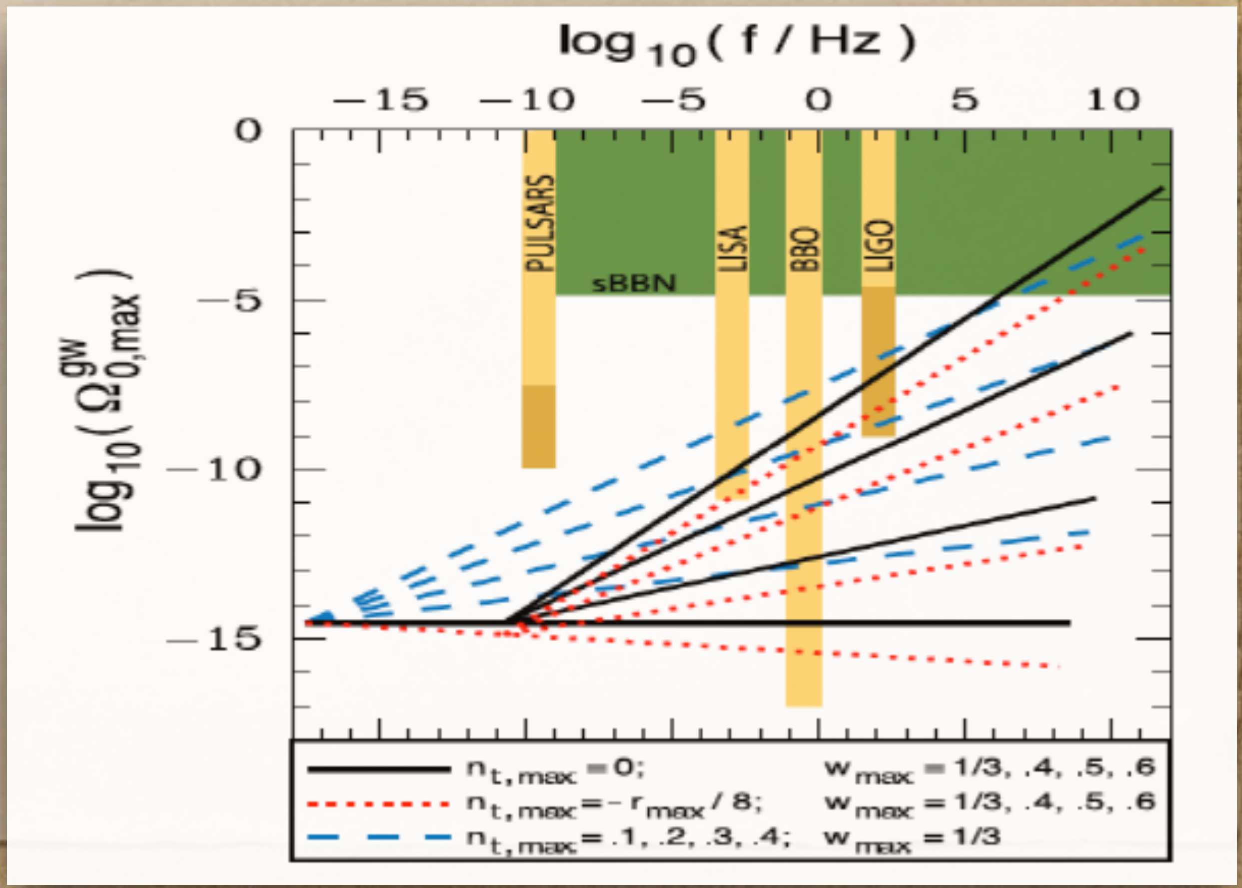
$$\left[\partial_\tau^2 + \left(k^2 - \frac{a''}{a}\right)\right] Q_k = 0, \Rightarrow P_t \sim k^{2+2b} \sim k^2$$

- Since ekpyrosis happens before the bounce, there is some intermediate phase where the spectrum could break. (Gasperini 2016, Brustein et al 1995 etc.....)

Other Possibilities

- Sourced fluc. in Inflation.
- Additional epochs in the Early Universe, "stiff matter domination"





Sourced Fluctuations

- Thoroughly investigated in Inflationary background. Rich phenomena - additional scalar and tensor spectra, non-gaussianity, magnetogenesis…
- Coupling of the scalar field to gauge fields will generate these phenomena in bouncing models as well. (IBD 2016, Chowdhury et al. 2018…)

$$I(\varphi_1) = (-\tau)^{-n}$$

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial\varphi_1)^2 - V(\varphi_1) - I^2(\varphi_1)\left\{\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\gamma}{4}\tilde{F}^{\mu\nu}F_{\mu\nu}\right\}\right]$$