Large scale structure formation in theories with scale dependent linear growth

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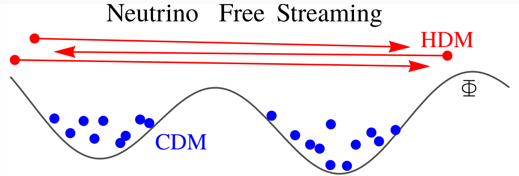
Outline

1. Motivation

Motivation: Neutrino Dark Matter

Massive neutrinos become non-relativistic at intermediate times, but their velocity dispersion is still large nowadays:

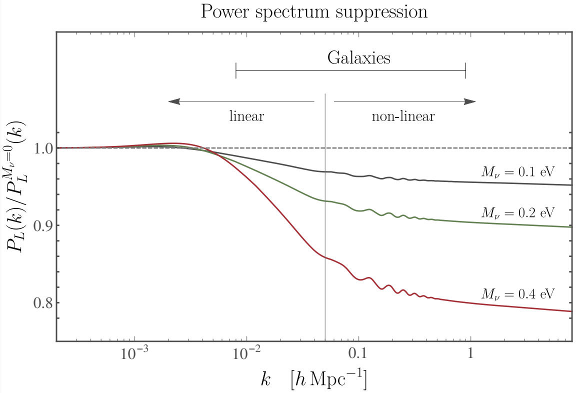
$$\sigma_\nu(z) \simeq 3.6\frac{T_\nu}{m_\nu} \simeq 1805(1+z)\left(\frac{0.1\,\mathrm{eV}}{m_\nu}\right)\ \mathrm{km/s}$$



Schematically: $\ddot{\delta} + 2H\dot{\delta} + (c_s^2 k^2 - 4\pi G\bar{\rho}a^2)\delta = 0$

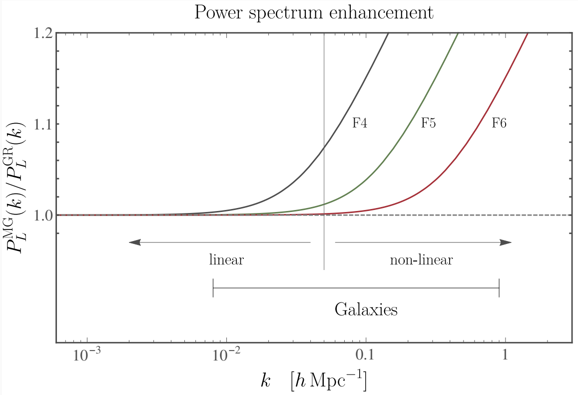
$$k_{\mathrm{FS}} = \left[\frac{4\pi G\bar{\rho}a^2}{\frac{5}{9}\sigma_\nu^2}\right]^{1/2} \simeq 0.0908\frac{H(z)/H_0}{(1+z)^2}\left(\frac{m_\nu}{0.1\,\mathrm{eV}}\right)\ h\,\mathrm{Mpc}^{-1}$$

Matter power spectrum with massive neutrinos



Motivation: Modified gravity

Matter power spectrum in Modified Gravity (MG)



2. Lagrangian framework

with Arka Banerjee [2007.06508],

Lagrangian Perturbation Theory

We follow the trajectories of CDM particles

$$\boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},t).$$

Geodesic equation:

$$\hat{\mathcal{T}}\,\boldsymbol{\Psi}(\boldsymbol{q},t) \equiv \left(\frac{d^2}{dt^2} + 2H\frac{d}{dt}\right)\boldsymbol{\Psi}(\boldsymbol{q},t) = \boldsymbol{\nabla}_{\mathbf{x}}\Phi(\boldsymbol{x},t)\Big|_{\boldsymbol{x}=\boldsymbol{q}+\boldsymbol{\Psi}}$$

Modified Poisson equation:

$$\frac{1}{a^2}\nabla^2_{\mathbf{x}}\Phi(\boldsymbol{x},t)\Big|_{\boldsymbol{x}=\boldsymbol{q}+\Psi} = 4\pi G\bar{\rho}_m\delta(\boldsymbol{x},t) + S(\boldsymbol{x},t)$$

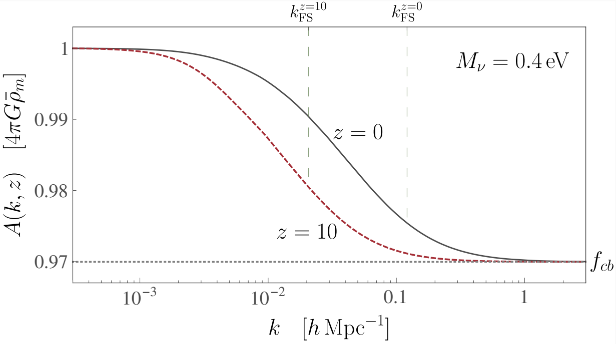
Neutrinos : $\delta = f_{cb}\delta_{cb}$, $S = 4\pi G\bar{\rho}_m f_\nu \delta_\nu$ $f_\nu = \frac{\Omega_\nu}{\Omega_m}$, $f_{cb} = \frac{\Omega_{cb}}{\Omega_m}$

Modified Gravity : $\delta = \delta_m$, $S = -\frac{1}{2a^2}\nabla^2_{\mathbf{x}}\phi_{\mathrm{MG}}.$ e.g. DGP, f(R),..., Horndeski

Massive neutrinos

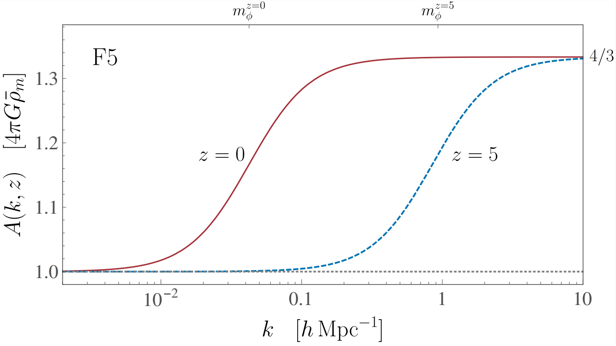
$$-\frac{k^2}{a^2}\Phi(\boldsymbol{k}) = 4\pi G\bar{\rho}_m(f_{cb}\delta_{cb}(\boldsymbol{k}) + f_\nu\delta_\nu(\boldsymbol{k})) = A(k,t)\delta_{cb}(\boldsymbol{k})$$

$$A(k,t) \equiv 4\pi G\bar{\rho}_m(f_{cb} + f_\nu\frac{\delta_\nu}{\delta_{cb}}) \approx \frac{3\Omega_m H^2}{2}(f_{cb} + f_\nu\frac{T_\nu(k,t)}{T_{cb}(k,t)})$$



Modified Gravity

$$-\frac{k^2}{a^2}\Phi(\boldsymbol{k}) = 4\pi G\bar{\rho}_m\delta_m(\boldsymbol{k}) + S(\boldsymbol{k},t)$$
$$= A(k,t)\delta_m(\boldsymbol{k}) + \text{Non-linear}$$



The equation of motion becomes

$$\left[\boldsymbol{\nabla}_{\mathbf{x}} \cdot \hat{\mathcal{T}} \boldsymbol{\Psi}(\boldsymbol{q})\right](\boldsymbol{k}) = -4\pi G \bar{\rho}_m \tilde{\delta}(\boldsymbol{k}) - \tilde{S}(\boldsymbol{k}).$$

$\left[(\cdots)\right](\boldsymbol{k})$ indicates the Fourier transform of $(\cdots)(\boldsymbol{q})$: $\int d^3q\, e^{-i\boldsymbol{k}\cdot\boldsymbol{q}}(\cdots)$

A tilde means the q-Fourier Transform of a function that takes arguments in Eulerian space:

$$\begin{aligned}
\tilde{S}(\boldsymbol{k}) &\equiv \int d^3q\, e^{-i\boldsymbol{k}\cdot\boldsymbol{q}}\, S(\boldsymbol{x}) = \int d^3q\, e^{-i\boldsymbol{k}\cdot\boldsymbol{q}} S(\boldsymbol{q} + \Psi) \\
&= \int d^3q\, e^{-i\boldsymbol{k}\cdot\boldsymbol{q}} \Big(S(\boldsymbol{q}) + \Psi_i(\boldsymbol{q}) S_{,i}(\boldsymbol{q}) + \frac{1}{2} \Psi_i(\boldsymbol{q}) \Psi_j(\boldsymbol{q}) S_{,ij}(\boldsymbol{q}) + \cdots \Big) \\
&= S(\boldsymbol{k}) \;+ \int_{\boldsymbol{k}_{12}=\boldsymbol{k}} i k_1^i S(\boldsymbol{k}_1) \Psi_i(\boldsymbol{k}_2) \;-\; \frac{1}{2} \int_{\boldsymbol{k}_{123}=\boldsymbol{k}} k_1^i k_1^j S(\boldsymbol{k}_1) \Psi_i(\boldsymbol{k}_2) \Psi_j(\boldsymbol{k}_3) + \cdots
\end{aligned}$$

Notation: $$\int_{\boldsymbol{k}_{1\cdots n}=\boldsymbol{k}} = \int \frac{d^3k_1 \cdots d^3k_n}{(2\pi)^{3n}} (2\pi)^3 \delta_{\mathrm{D}}(\boldsymbol{k}_1 + \cdots + \boldsymbol{k}_n - \boldsymbol{k})$$

Standard Lagrangian approach in vanilla ΛCDM: $\left[\boldsymbol{\nabla}_{\mathbf{x}} \cdot \hat{\mathcal{T}}\, \boldsymbol{\Psi}(\boldsymbol{q})\right](\boldsymbol{k}) = -4\pi G \bar{\rho}_m \tilde{\delta}(\boldsymbol{k})$

$$J_{ij}(\boldsymbol{q},t) = \frac{\partial x^i(\boldsymbol{q},t)}{\partial q^j} = \delta_{ij} + \Psi_{i,j}, \qquad J(\boldsymbol{q},t) = \det J_{ij}$$

- $\boldsymbol{\nabla}_{\mathbf{x}} \cdot \hat{\mathcal{T}}\, \boldsymbol{\Psi}(\boldsymbol{q}) = J^{-1}_{ij} \nabla_j \hat{\mathcal{T}} \Psi_i(\boldsymbol{q}) = \hat{\mathcal{T}} \Psi_{i,i} - \Psi_{i,j} \hat{\mathcal{T}} \Psi_{j,i} + \Psi_{i,k} \Psi_{k,j} \hat{\mathcal{T}} \Psi_{i,j}.$

- matter conservation, $(1+\delta(\boldsymbol{x}))d^3x = (1+\delta(\boldsymbol{q}))d^3q$, implies

$$\tilde{\delta}(\boldsymbol{k}) = \left[\frac{1-J(\boldsymbol{q})}{J(\boldsymbol{q})}\right](\boldsymbol{k}) = -\left[\Psi_{i,i} - \tfrac{1}{2}\left((\Psi_{i,i})^2 + \Psi_{i,j}\Psi_{j,i}\right) + \mathcal{O}(\Psi^3)\right](\boldsymbol{k})$$

$$\left[\hat{\mathcal{T}} - 4\pi G \bar{\rho}_m\right] \Psi_{i,i}(\boldsymbol{q},t) + \cdots = 0$$

$$\Psi_i(\boldsymbol{k},t) = i \frac{k_i}{k^2} \delta(\boldsymbol{k},t) + \cdots$$

With additional scales, the equation of motion becomes

$$\left[\boldsymbol{\nabla}_{\mathbf{x}} \cdot \hat{\mathcal{T}}\Psi(\boldsymbol{q})\right](\boldsymbol{k}) = -4\pi G\bar{\rho}_m\tilde{\delta}(\boldsymbol{k}) - \tilde{S}(\boldsymbol{k}) = -A(k)\tilde{\delta}(\boldsymbol{k}) + \mathrm{FL}.$$

FL (Frame lagging) appears because one changes from $q-$FT to x-FT

Up to third order:

$$\begin{aligned}\left(\hat{\mathcal{T}} - A(k)\right)[\Psi_{i,i}](\boldsymbol{k}) &= [\Psi_{i,j}\hat{\mathcal{T}}\Psi_{j,i}](\boldsymbol{k}) - \frac{A(k)}{2}[\Psi_{i,j}\Psi_{j,i}](\boldsymbol{k}) - \frac{A(k)}{2}[(\Psi_{l,l})^2](\boldsymbol{k}) \\ &\quad -[\Psi_{i,k}\Psi_{k,j}\hat{\mathcal{T}}\Psi_{j,i}](\boldsymbol{k}) + \frac{A(k)}{6}[(\Psi_{l,l})^3](\boldsymbol{k}) + \frac{A(k)}{2}[\Psi_{l,l}\Psi_{i,j}\Psi_{j,i}](\boldsymbol{k}) + \frac{A(k)}{3}[\Psi_{i,k}\Psi_{k,j}\Psi_{j,i}](\boldsymbol{k}) \\ &\quad - \int_{\boldsymbol{k}_{12}=\boldsymbol{k}} \mathcal{K}^{\mathrm{FL}}_{ki}(\boldsymbol{k}_1,\boldsymbol{k}_2)\Psi_k(\boldsymbol{k}_1)\Psi_i(\boldsymbol{k}_2) - \int_{\boldsymbol{k}_{123}=\boldsymbol{k}} \mathcal{K}^{\mathrm{FL}}_{kij}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\Psi_k(\boldsymbol{k}_1)\Psi_i(\boldsymbol{k}_2)\Psi_j(\boldsymbol{k}_3)\end{aligned}$$

Lagrangian Perturbation Theory

Expand $\Psi = \Psi^{(1)} + \Psi^{(2)} + \Psi^{(3)} + \cdots$. To n-th order

$$\Psi_i^{(n)}(\boldsymbol{k}, t) = \frac{i}{n!} \int_{\boldsymbol{k}_{1\cdots n} = \boldsymbol{k}} L_i^{(n)}(\boldsymbol{k}_1, \cdots, \boldsymbol{k}_n; t) \delta^{(1)}(\boldsymbol{k}_1, t) \cdots \delta^{(1)}(\boldsymbol{k}_n, t)$$

To linear order

$$\Psi_i^{(1)}(\boldsymbol{k}, t) = i \frac{k_i}{k^2} \delta^{(1)}(\boldsymbol{k}, t), \qquad \text{with} \qquad \delta^{(1)}(\boldsymbol{k}, t) = D_+(k, t) \delta^{(1)}(\boldsymbol{k}, t_0)$$

and the scale-dependent linear growth function D_+ is the growing solution to

$$\left[\frac{d^2}{dt^2} + 2H \frac{d}{dt} - \frac{3}{2} \Omega_m H^2 \left(1 + \alpha(k)\right) \right] D_+(k, t) = \left[\hat{\mathcal{T}} - A(k) \right] D_+(k, t) = 0$$

The second order kernel becomes

$$L_i^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2,t) = \frac{3}{7}\frac{k_i}{k^2}\left(\mathcal{A}(\boldsymbol{k}_1,\boldsymbol{k}_2,t) - \mathcal{B}(\boldsymbol{k}_1,\boldsymbol{k}_2,t)\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2k_2^2}\right),$$

with

$$\mathcal{A}(\boldsymbol{k}_1,\boldsymbol{k}_2,t) = \frac{7}{3}\frac{D_{\mathcal{A}}(\boldsymbol{k}_1,\boldsymbol{k}_2,t)}{D_+(k_1)D_+(k_2)}, \qquad \mathcal{B}(\boldsymbol{k}_1,\boldsymbol{k}_2,t) = \frac{7}{3}\frac{D_{\mathcal{B}}(\boldsymbol{k}_1,\boldsymbol{k}_2,t)}{D_+(k_1)D_+(k_2)}$$

with $D_{\mathcal{A},\mathcal{B}}$ given by the diferential equations

$$\left[\hat{\mathcal{T}} - A(k)\right]D_{\mathcal{A}} = \left(A(k) + (A(k)-A(k_2))\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1^2} + (A(k)-A(k_1))\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_2^2}\right)D_+(k_1)D_+(k_2),$$

$$\left[\hat{\mathcal{T}} - A(k)\right]D_{\mathcal{B}} = \Big(A(k_1) + A(k_2) - A(k)\Big)D_+(k_1)D_+(k_2).$$

For ΛCDM, these reduce to

$$D^{(2)\Lambda\mathrm{CDM}}_{\mathcal{A},\mathcal{B}}(t) = \frac{3}{7}D_+^2(t) + \frac{4}{7}\left(\hat{\mathcal{T}} - \frac{3}{2}\Omega_m H^2\right)^{-1}\left[\frac{3}{2}\Omega_m H^2\left(1-\frac{f^2}{\Omega_m}\right)\right],$$

such that $\mathcal{A}^{\Lambda\mathrm{CDM}}(t) = \mathcal{B}^{\Lambda\mathrm{CDM}}(t) \approx 1.01$ are only time dependent and close to unity.

The third order kernel becomes

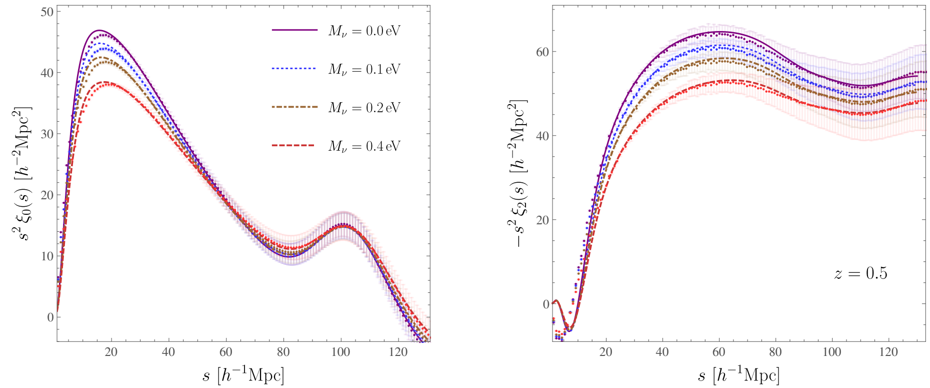
$$L_i^{(3)}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{k^i}{k^2}\left\{\frac{5}{7}\left(\mathcal{A}^{(3)} - \mathcal{B}^{(3)}\frac{(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)^2}{k_2^2 k_2^3}\right)\left(1-\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_{23})^2}{k_1^2 k_{23}^2}\right)\right.$$
$$\left. -\frac{1}{3}\left(\mathcal{C}^{(3)} - 3\mathcal{D}^{(3)}\frac{(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)^2}{k_2^2 k_3^2} + 2\mathcal{E}^{(3)}\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)(\boldsymbol{k}_3\cdot\boldsymbol{k}_1)}{k_1^2 k_2^2 k_3^2}\right)\right\},$$

with $\mathcal{A}^{(3)}$, $\mathcal{B}^{(3)}$, $\mathcal{C}^{(3)}$, $\mathcal{D}^{(3)}$ and $\mathcal{E}^{(3)}$ are time and scale-($\boldsymbol{k}_1$, $\boldsymbol{k}_2$, $\boldsymbol{k}_3$)dependent, and solutions to second order differential equations. These functions become unity in EdS.

Applications: MG-COLA, Winther ++ 1703.00879

I.C. for N-Body simulations with massive neutrinos, Elbers++ 2202.00670

Correlation function multipoles



A. A & A. Banerjee 2007.06508

3. From Lagrangian to Eulerian perturbation theory

with Arka Banerjee et al [2012.05077], [2106.13771]

LPT: Lagrangian displacements

$$\Psi_i^{(n)}(\boldsymbol{k}) = \frac{i}{n!}\int\left(\prod_{i=1}^{n}\frac{d^3k_i}{(2\pi)^3}\right)\delta_{\rm D}(\boldsymbol{k}-\sum_{a=1}^{n}\boldsymbol{k}_a)L_i^{(n)}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)\delta_{cb}^{(1)}(\boldsymbol{k}_1)\cdots\delta^{(n)}(\boldsymbol{k}_n)$$

SPT: density and velocity fluctuations

$$\delta^{(n)}(\boldsymbol{k}) = \int\left(\prod_{i=1}^{n}\frac{d^3k_i}{(2\pi)^3}\right)\delta_{\rm D}(\boldsymbol{k}-\sum_{a=1}^{n}\boldsymbol{k}_a)\,F_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)\delta^{(1)}(\boldsymbol{k}_1)\cdots\delta^{(n)}(\boldsymbol{k}_n)$$

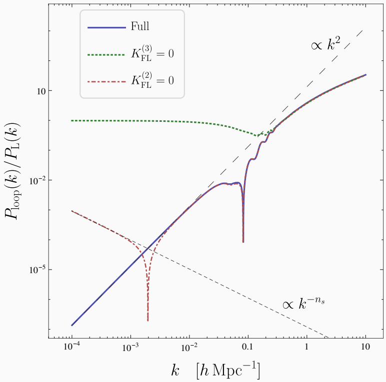
$$\theta^{(n)}(\boldsymbol{k}) = \int\left(\prod_{i=1}^{n}\frac{d^3k_i}{(2\pi)^3}\right)\delta_{\rm D}(\boldsymbol{k}-\sum_{a=1}^{n}\boldsymbol{k}_a)\,G_n(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n)\delta^{(1)}(\boldsymbol{k}_1)\cdots\delta^{(n)}(\boldsymbol{k}_n)$$

From LPT to SPT kernels [1809.07713, 2012.05077]

$$L^{(n)} \longrightarrow F_n, G_n$$

Density field

$$L^{(1)}, L^{(2)} \quad \longrightarrow \quad F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{1}{2}\left(k_i L_i^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2) + k_i k_j L_i^{(1)}(\boldsymbol{k}_1) L_j^{(1)}(\boldsymbol{k}_2)\right)$$



$$P_{\rm loop}(k \to 0)\Big|_{K_{\rm FL}^{(2)}=0} \sim \frac{9}{98} \int_{p \gg k} \frac{dp}{4\pi^2} p^2 P_L^2(p) \left[\mathcal{A}(-\boldsymbol{p}, \boldsymbol{p})\Big|_{\text{No FL}} - \mathcal{B}(-\boldsymbol{p}, \boldsymbol{p})\right]^2,$$

$$P_{\rm loop}(k \to 0)\Big|_{K_{\rm FL}^{(3)}=0} \propto P_L(k) \int_{p \gg k} \frac{dp}{4\pi^2} p^2 P_L(p),$$

Velocity field

$$\theta(\boldsymbol{x}, t) \equiv -\frac{1}{aHf_0}\nabla \cdot \boldsymbol{v}(\boldsymbol{x}, t),$$

with f_0 the growth rate at a very large scales ($k \ll k_{\rm FS}$).

That is,

$$f(k,t) = \frac{d\ln D_+(k,t)}{d\ln a(t)}, \qquad f_0(t) = f(k \to 0, t).$$

At linear order we obtain

$$\theta^{(1)}(\boldsymbol{k}) = \frac{f(k)}{f_0}\delta^{(1)}(\boldsymbol{k}).$$

The dipole in the second order density kernel F_2 arises from expanding $\delta(\boldsymbol{x}+\boldsymbol{\Psi})$ to lowest order:

$$\delta^{(2)}(\boldsymbol{x}) \ni \delta^{(1)}(\boldsymbol{x}+\Psi) - \delta^{(1)}(\boldsymbol{x}) = \Psi_i^{(1)}\partial_i\delta^{(1)}(\boldsymbol{x}) = \partial_i\left[\nabla^{-2}\delta^{(1)}(\boldsymbol{x})\right]\partial_i\delta^{(1)}(\boldsymbol{x})$$

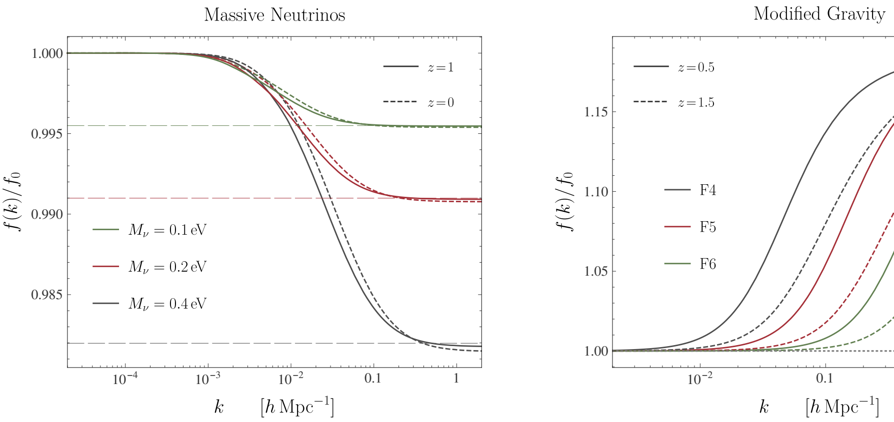
Moving to Fourier space: $$\frac{\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2}{2}\left(\frac{k_2}{k_1}+\frac{k_1}{k_2}\right) \in F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)$$

Similarly, for the velocity field,

$$\theta^{(2)}(\boldsymbol{x}) \ni \theta^{(1)}(\boldsymbol{x}+\Psi) - \theta^{(1)}(\boldsymbol{x}) = \partial_i\left[\nabla^{-2}\delta^{(1)}(\boldsymbol{x})\right]\partial_i\theta^{(1)}(\boldsymbol{x})$$

Since $\theta^{(1)}(\boldsymbol{k}) = \dfrac{f(k)}{f_0}\delta^{(1)}(\boldsymbol{k})$,

$$\frac{\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2}{2}\left(\frac{f(k_2)}{f_0}\frac{k_2}{k_1}+\frac{f(k_1)}{f_0}\frac{k_1}{k_2}\right) \in G_2(\boldsymbol{k}_1,\boldsymbol{k}_2).$$

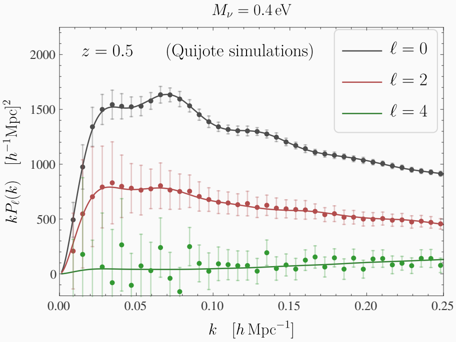


Massive Neutrinos
$f(k)/f_0$
1.000
0.995
0.990
0.985
$z=1$
$z=0$
$M_\nu = 0.1\,\mathrm{eV}$
$M_\nu = 0.2\,\mathrm{eV}$
$M_\nu = 0.4\,\mathrm{eV}$
10^{-4}
10^{-3}
10^{-2}
0.1
1
$k \quad [h\,\mathrm{Mpc}^{-1}]$
Modified Gravity
$f(k)/f_0$
1.15
1.10
1.05
1.00
$z=0.5$
$z=1.5$
F4
F5
F6
10^{-2}
0.1
1
$k \quad [h\,\mathrm{Mpc}^{-1}]$

Full Model [Scoccimarro 2004; Vlah & White 2018; & 2012.05077]

$$P_s(\boldsymbol{k}) = \int d^3x\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \Big\langle \big(1+\delta(\boldsymbol{x}_1)\big)\big(1+\delta(\boldsymbol{x}_2)\big) e^{-i\boldsymbol{k}\cdot\Delta\boldsymbol{v}_{\rm LoS}} \Big\rangle = \sum_{m=0}^{\infty} \frac{(-i)^m}{m!} (k\mu)^m \tilde{\Xi}^{(m)}_{\hat{\boldsymbol{n}}}(\boldsymbol{k})$$

- + Biasing scheme
- + EFT counterterms
- + Stochastic Noise
- + IR-Resummations



Power spectrum

The power spectrum can be recast as

$$P(k,\mu) = \left[1 + (\alpha_0 + \alpha_2\mu^2)k^2\right] P_{\text{Kaiser}}(k,\mu) + (\alpha_0^{\text{sn}} + \alpha_2^{\text{sn}} k^2\mu^2 + \cdots)$$
$$+ \sum_{m=0}^{\infty}\sum_{n=0}^{m} \mu^{2n} f_0^m I_{nm}(k)$$

with the functions $I_{nm}(k)$ of the form

$$I_{mn}(k) = \int d\boldsymbol{p}\, \mathcal{I}_{mn}(\boldsymbol{k},\boldsymbol{p})$$

Performing these loop calculations is computationally slow, since at each volume element of the integration we need to solve a system of differential equations for second and third order growth functions to find the values of $\mathcal{A}, \mathcal{B}(\boldsymbol{k},\boldsymbol{p})$ and their third order counterparts for constructing $\mathcal{I}_{mn}(\boldsymbol{k},\boldsymbol{p})$. The slowness of this procedure precludes the use of efficient parameter sampling algorithms for estimation of cosmological parameters.

4. `fk` kernels and FFTLog method

with A. Banerjee ++ [2106.13771]

and H. Noriega ++ [2208.02791]

Power spectrum

However, the dominant contributions to the loop corrections come from the growth rates instead of the computationally costly $\mathcal{A}$, $\mathcal{B}$... functions, which mainly play a normalization role, and their effect is similar to how they affect the ΛCDM kernels, as oppose to the use of EdS kernels. Therefore, one may keep only the growth function contributions.

We define the `fk` kernels as

$$F_2^{\mathtt{fk}}(\boldsymbol{k}_1, \boldsymbol{k}_2) = F_2(\boldsymbol{k}_1, \boldsymbol{k}_2)\Big|_{\mathcal{A}=\mathcal{B}=\mathcal{A}^{\Lambda\text{CDM}}},$$

$$G_2^{\mathtt{fk}}(\boldsymbol{k}_1, \boldsymbol{k}_2) = G_2(\boldsymbol{k}_1, \boldsymbol{k}_2)\Big|_{\mathcal{A}=\mathcal{B}=\mathcal{A}^{\Lambda\text{CDM}}},$$

and similar for $n > 2$.

For example, one of the $I_{mn}(k)$ functions is

$$P_{\theta\theta}^{22}(k) = 2\int_{\boldsymbol{p}} [G_2(\boldsymbol{p}, \boldsymbol{k}-\boldsymbol{p})]^2 P_L(p) P_L(\boldsymbol{k}-\boldsymbol{p}),$$

with $$G_2(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{3\left[f(k_1)+f(k_2)\right]\mathcal{A} + 3\dot{\mathcal{A}}/H}{14 f_0} + \frac{\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2}{2}\left(\frac{f(k_2)}{f_0}\frac{k_2}{k_1} + \frac{f(k_1)}{f_0}\frac{k_1}{k_2}\right) + \left(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2\right)^2\left(\frac{f(k_1)+f(k_2)}{2f_0} - \frac{3\left[f(k_1)+f(k_2)\right]\mathcal{B} + 3\dot{\mathcal{B}}/H}{14 f_0}\right)$$

- `fk`-kernels:

  Replace $\mathcal{A}(\boldsymbol{k}_1, \boldsymbol{k}_2, t) \to \mathcal{A}^{\text{LS}}(t) = \mathcal{A}(\boldsymbol{k}_1 \to 0, \boldsymbol{k}_2 \to 0, t) \simeq 1.01$ (and the same for $\mathcal{B}$).

$$
\begin{aligned}
P^{22}_{\theta\theta}(k) &= 2\int_{\boldsymbol{p}} \left[G_2(\boldsymbol{p}, \boldsymbol{k}-\boldsymbol{p})\right]^2 P_L(p) P_L(|\boldsymbol{k}-\boldsymbol{p}|) \\
&= 2\int_{\boldsymbol{p}} K^I_{22} \frac{f(p)}{f_0} P_L(p) \frac{f(|\boldsymbol{k}-\boldsymbol{p}|)}{f_0} P_L(|\boldsymbol{k}-\boldsymbol{p}|) + 4\int_{\boldsymbol{p}} K^{II}_{22} \frac{f^2(p)}{f_0^2} P_L(p) P_L(|\boldsymbol{k}-\boldsymbol{p}|)
\end{aligned}
$$

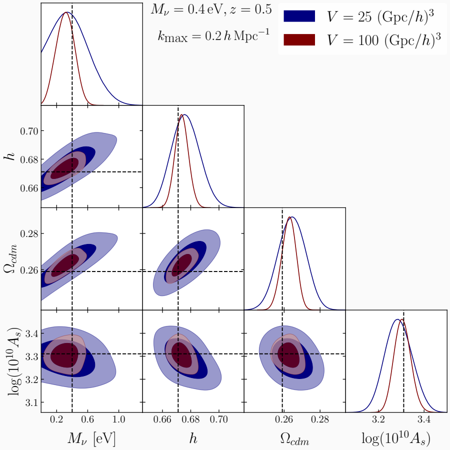
$$
\text{with} \qquad K^{I,II}_{22} \equiv \sum_{n_1,n_2=-2}^{2} A^{I,II}_{n_1 n_2}\, k^{-2(n_1+n_2)} p^{2n_1} |\boldsymbol{k}-\boldsymbol{p}|^{2n_2}
$$

- FFTLog: We can decompose the linear spectra in series of power laws: e.g. $\dfrac{f(k)}{f_0} P_L(k) = \sum_m c^f_m k^{i\nu_m}$, and convert the loop integral to a matrix multiplication (as in Simonovic++ 2017)

$$
P^{22}_{\theta\theta}(k) = \sum_{m,n} k^{-i\nu_m} c^f_m M^I_{mn} c^f_n k^{-i\nu_n} + \sum_{m,n} k^{-i\nu_m} c^{ff}_m M^{II}_{mn} c_n k^{-i\nu_n}\,,
$$

where the coefficients c_m are computed via FFT.

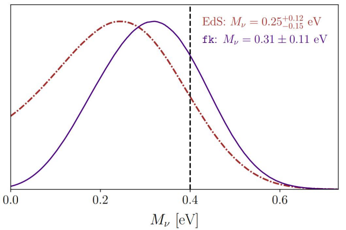
FOLPS



H. Noriega ++ 2208.02791

github.com/henoriega/FOLPS-nu

time $\approx 0.2\,\mathrm{s}$ ($N_{\mathrm{FFTLog}} = 128$)



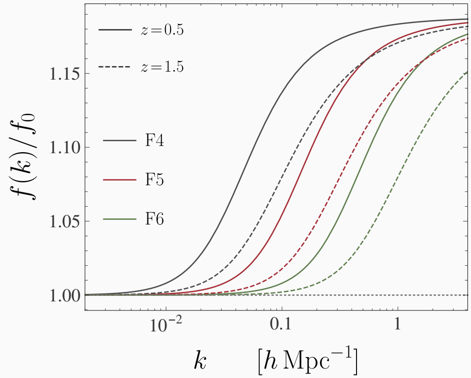
5. Constraints from BOSS and eBOSS data

The case of modified gravity

$$\nabla^2\Phi = 4\pi G a^2 \mu(k,a) \sum_i \rho_i \Delta_i, \qquad \nabla^2(\Phi+\Psi) = 4\pi G a^2 \Sigma(k,a) \sum_i \rho_i \Delta_i$$

e.g. Hu-Sawicky f(R) gravity

$$\mu(k,a) = 1 + \frac{1}{3}\frac{k^2}{k^2 + a^2 m^2(a)}, \qquad m^2(a) = \frac{H_0^2(\Omega_m a^{-3} + 4\Omega_\Lambda)^3}{2|\mathrm{f_{R0}}|(\Omega_m + 4\Omega_\Lambda)^2}$$



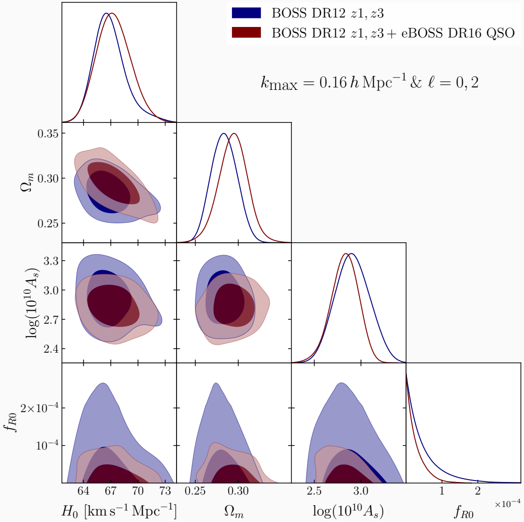
DR12 BOSS LRGs and eBOSS QSO

github.com/alejandroaviles/fkpt

BOSS DR12 z1,z3 $|f_{R0}| < 3.29 \times 10^{-5}$

+ eBOSS DR16 QSOs $|f_{R0}| < 3.23 \times 10^{-5}$

[In preparation]



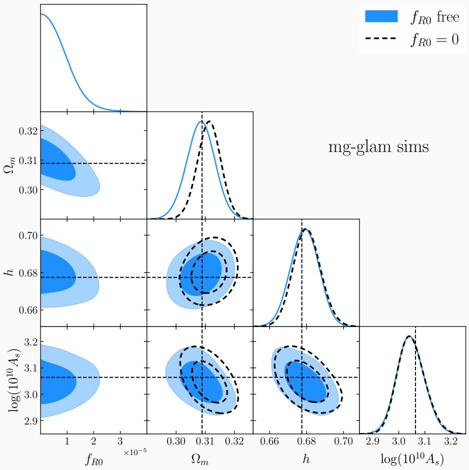
6. Wiggles vs broadband

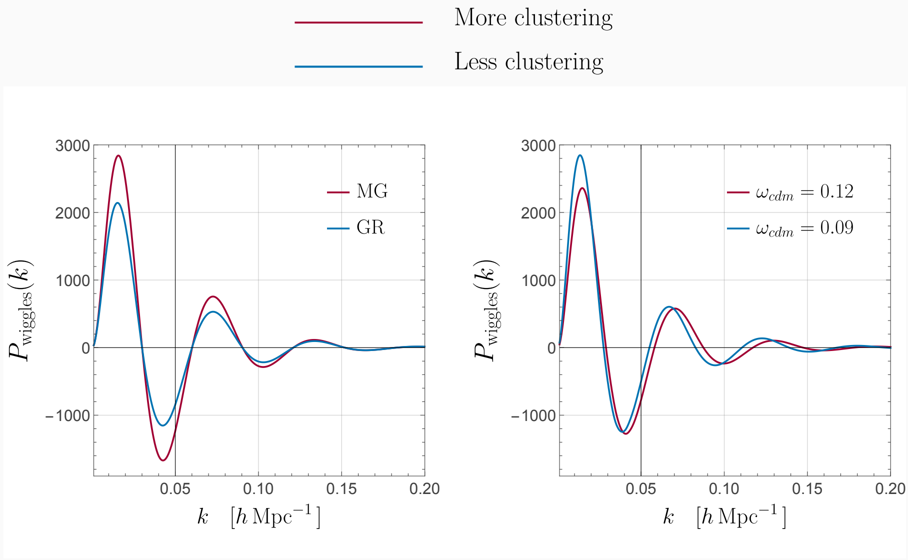
- In theories with additional scales, e.g. MG or M_ν, one expects to have a change on the broadband power spectrum at some k.
    - - For example, in MG this is an enhancement, while in M_ν this is a suppression.

- The claim is that one can attempt to measure the suppression (or enhancement) in the broadband, and in that way constrain the mass of the neutrinos or MG models.

- That is, if I split the information of the power spectrum in wiggles + Broadband + RSD, one expect to obtain the information of massive neutrinos from the BB. I no longer believe this is the case.

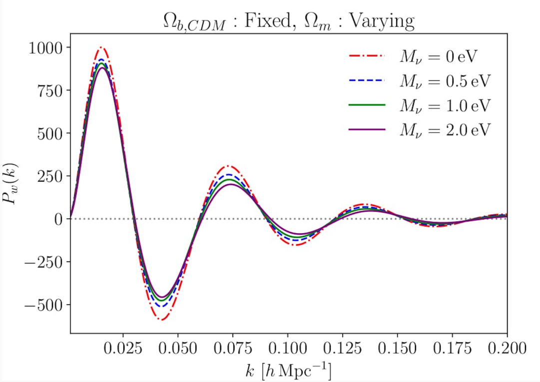
Take a look at this figure on which there is a MG parameter f_{R0} whose effect is to enhance the power spectrum above a scale $k \propto |f_{R0}^{-1}|$:

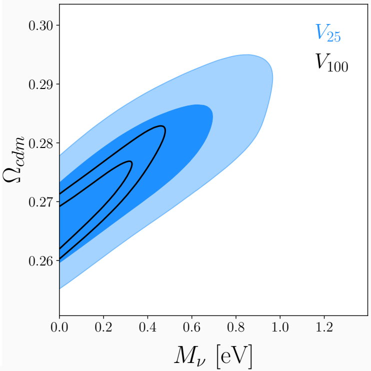




More clustering
Less clustering
3000
2000
1000
0
−1000
0.05
0.10
0.15
0.20
MG
GR
$P_{\rm wiggles}(k)$
$k \quad [h\,{\rm Mpc}^{-1}]$
$\omega_{cdm} = 0.12$
$\omega_{cdm} = 0.09$

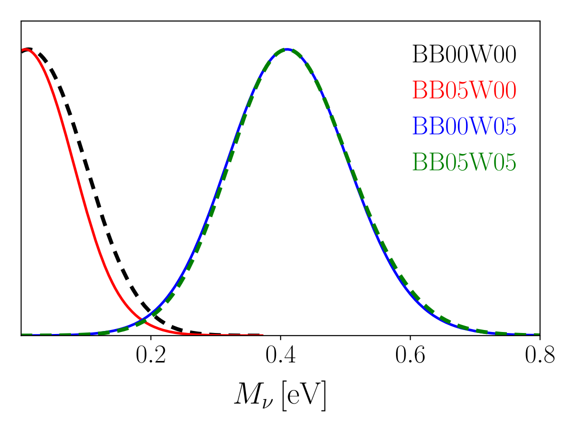
The effects of massive neutrinos in the wiggles is mainly a modulation in the relative amplitude of the wiggles, with only a very small shift in their frequency (if $\Omega_{b,cdm}$ fixed). More damping for bigger neutrino masses.





We decomposed the two linear power spectra into broadband + wiggles and mixed the pieces to get four models. Then, we evolve them non linearly and generate model generated synthetic data with Patchy mock covariance, and construct four data sets:

- BB00W00 - Broadband of the model with mass $M_\nu = 0.0$ eV, wiggles of the model with mass $M_\nu = 0.0$ eV
- BB00W05 - Broadband of the model with mass $M_\nu = 0.0$ eV, wiggles of the model with mass $M_\nu = 0.5$ eV
- BB05W00 - Broadband of the model with mass $M_\nu = 0.5$ eV, wiggles of the model with mass $M_\nu = 0.0$ eV
- BB05W05 - Broadband of the model with mass $M_\nu = 0.5$ eV, wiggles of the model with mass $M_\nu = 0.0$ eV



It seems that a PT/EFT analysis extracts the information of the neutrino mass from the wiggles, while it is blind to the broadband suppression.

Thank you!