Probing the size of extra dimension with gravitational wave astronomy

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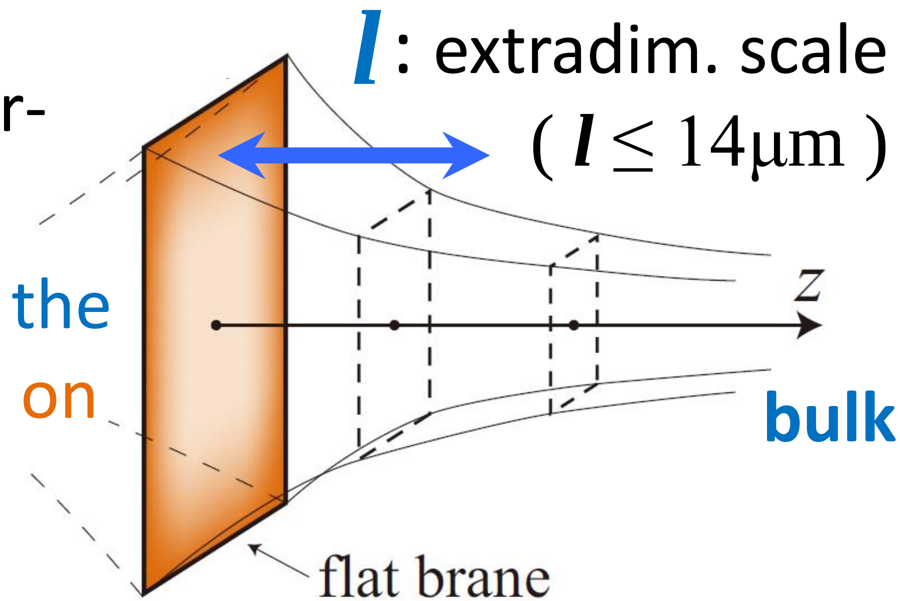
Outline

- In a certain higher-dimensional cosmological model, we may observe the extradimension scale via observations of astrophysical black holes.

1. Model and basic idea
2. Observational constraints on extradim. scale

Braneworld model



- ✓ 4-dimensional **brane** in higher-dimensional spacetime (**bulk**)
- ✓ Only gravity can propagate in the bulk, other fields are confined on the brane

- Randall-Sundrum II (RS-II) model
  - **5D Anti-de Sitter (AdS) bulk / flat 4D spacetime on the brane**
  - Weak gravitation on the brane mimics ordinary 4D gravity

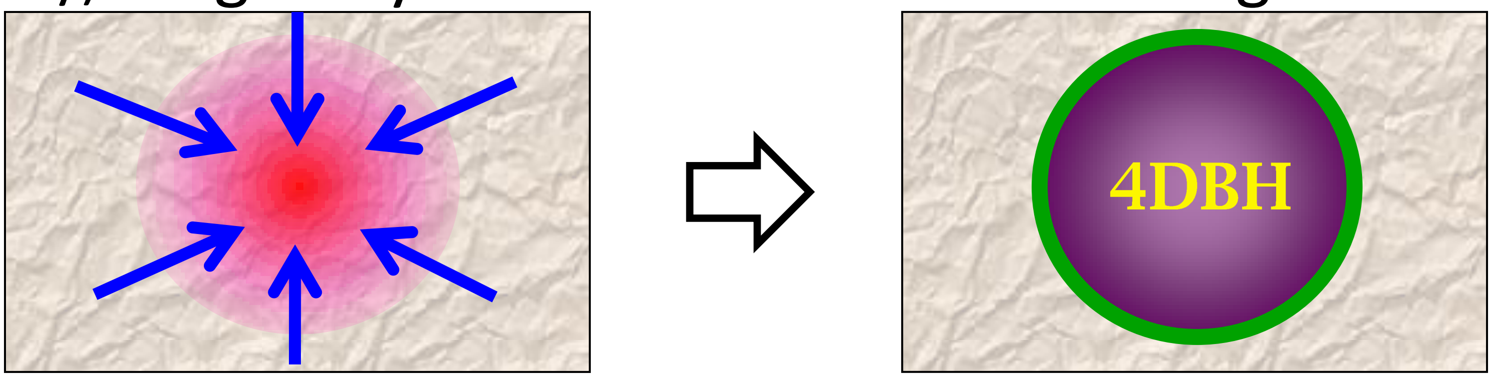
$$V(r) = \frac{Gm_1m_2}{r}\left(1 + \frac{2l^2}{3r^2}\right)$$

5D correction

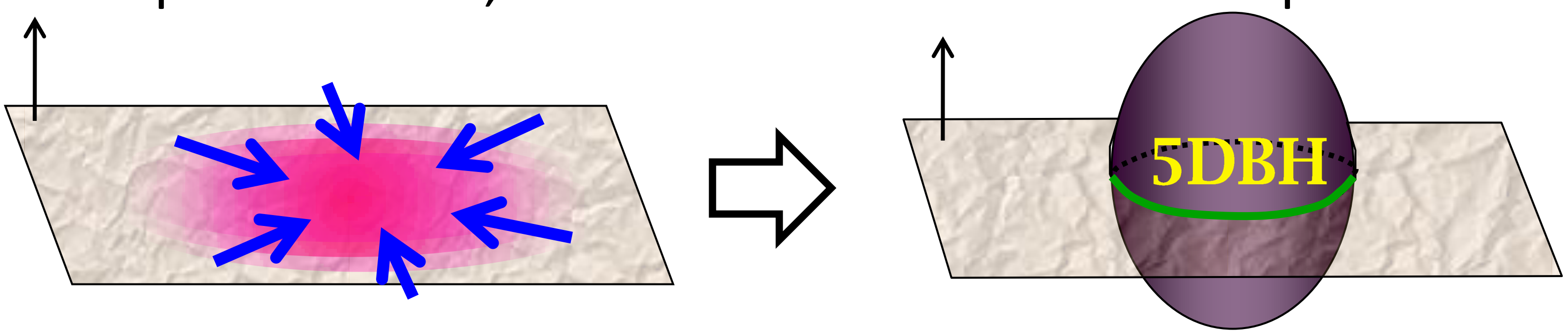
- Randall-Sundrum II (RS-II) model
  - **5D Anti-de Sitter (AdS) bulk** / **flat 4D spacetime on the brane**

Weak gravitation on the brane mimics ordinary 4D gravity

- (ordinary) 4D gravity ➔ 4D BH will form after a grav. Collapse.



- In **5D bulk** point of view, 5D BH localized on brane is expected to form.



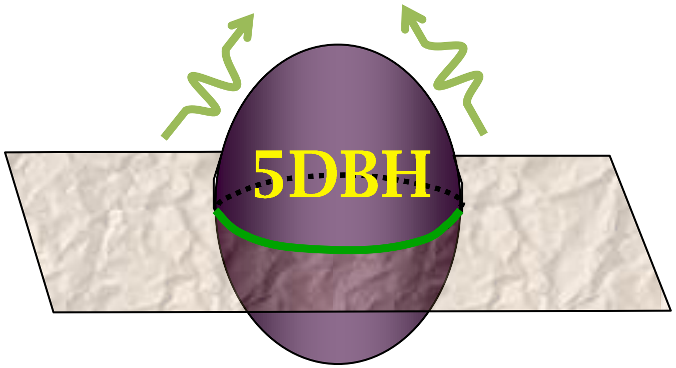
- ***Rapid BH evaporation*** in RS-II model

[Tanaka, 2002; Emparan et al., 2002]

5D picture :

5D BH deformation

by some instability



4D picture :

4D BH

losing its mass via

Hawking radiation

where radiation is amplified by

$$l^2/G_4 = \mathbf{10^{60}} \times (l /10\mu m)^2$$

compared to ordinary Hawking radiation.

(l: extradim. scale)

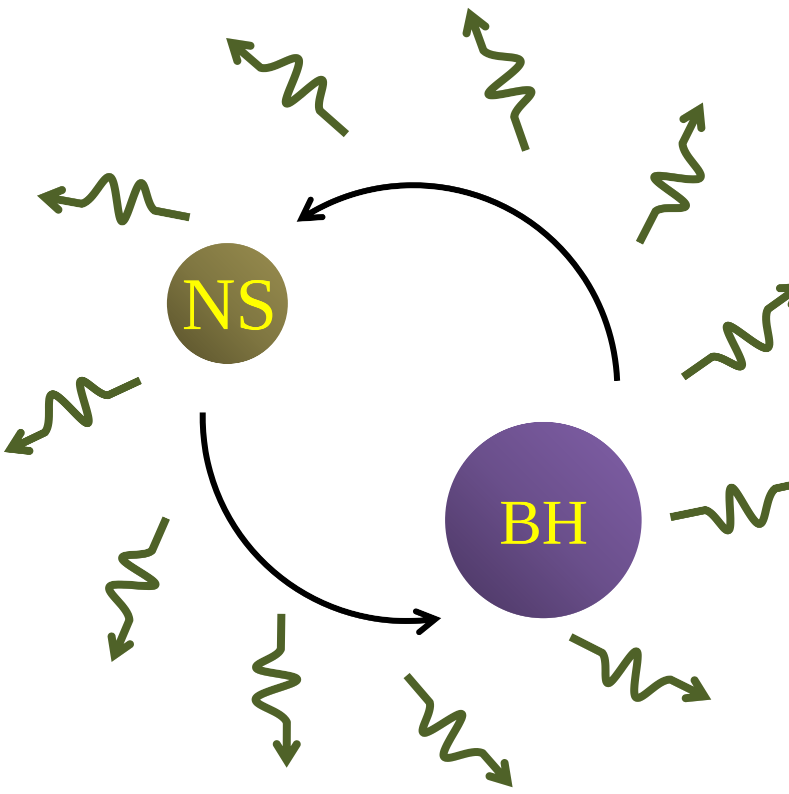
Constrain l by Gravitational wave observation

[Yagi, NT & Tanaka, in prep.]

- Binary of BH and Neutron star

--- Gravitational wave emission

→ Binary separation decreases.



Constrain l by Gravitational wave observation

[Yagi, NT & Tanaka, in prep.]

- Binary of BH and Neutron star

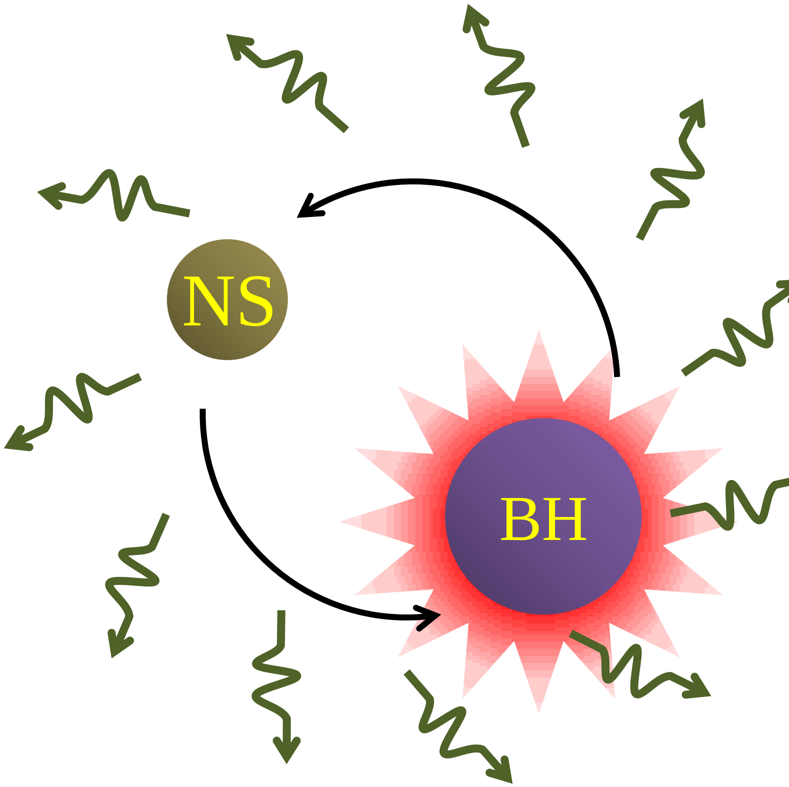
--- Gravitational wave emission

→ Binary separation decreases.

--- BH evaporation

→ Mass & Momentum loss

→ Binary separation ***increases***.

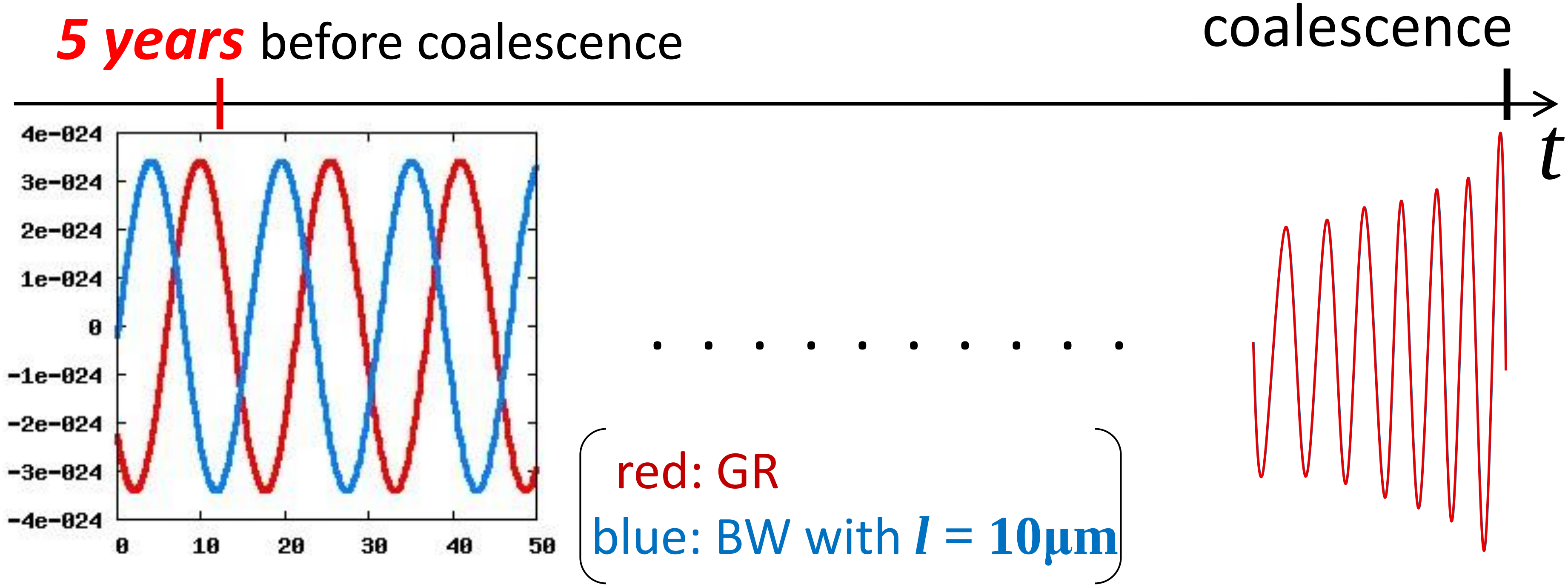


Constrain l by Gravitational wave observation

[Yagi, NT & Tanaka, in prep.]

- Read out l from ***GW waveform***

ex.) GW from (1.4+10) M_{solar} at D_L=3Gpc

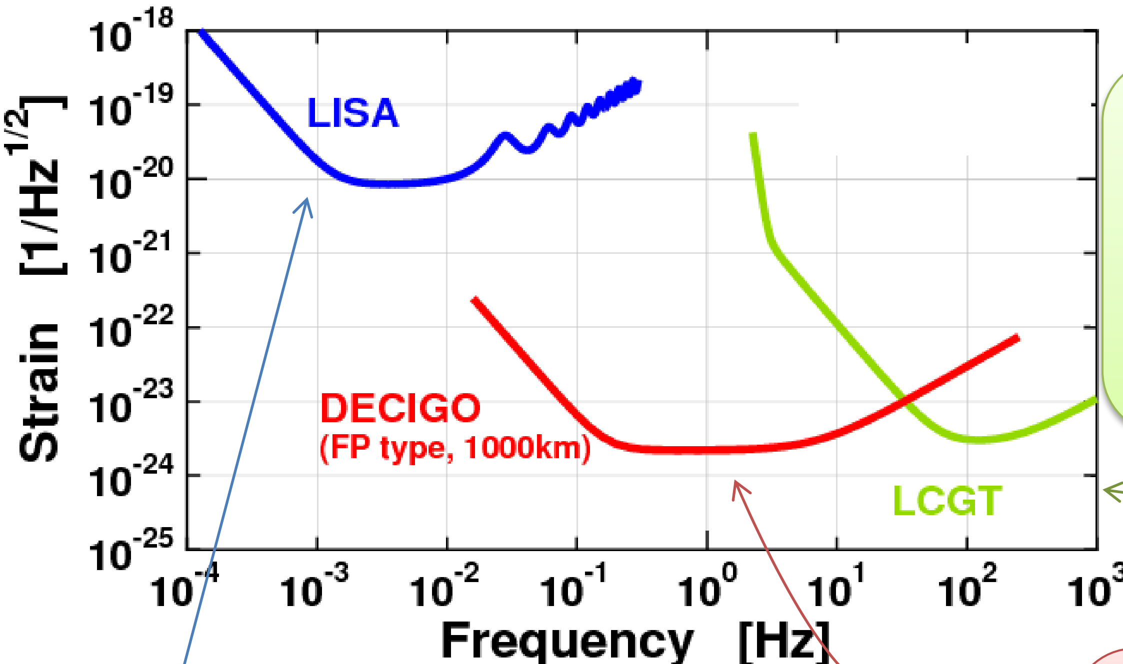


Phase difference accumulates to be $\mathcal{O}$(a cycle) ← Detectable!

Constrain l by DECIGO/BBO

[Yagi, NT & Tanaka, in prep.]

- GW observations



<u>100Hz band : LIGO, LCGT, ...</u>
- NS/BH merger events
- GRB central engine
- Primordial black hole

<u>1Hz band : DECIGO, BBO</u>
- GW from inflation
- IMBH binary, EMRI
- Cosmological binary

<u>mHz band : LISA</u>
- SMBH binary
- Extreme mass-ratio inspiral (EMRI)
- Galactic binary

Constrain *l* by DECIGO/BBO

[Yagi, NT & Tanaka, in prep.]

- Results:
  - Suppose that signal that obeys pure GR (l=0) is observed.
    → Upper bound on l up to statistical error
  - DECIGO/BBO will observe 10^4 NS/BH in a year
    → statistical analysis using Monte Carlo simulations

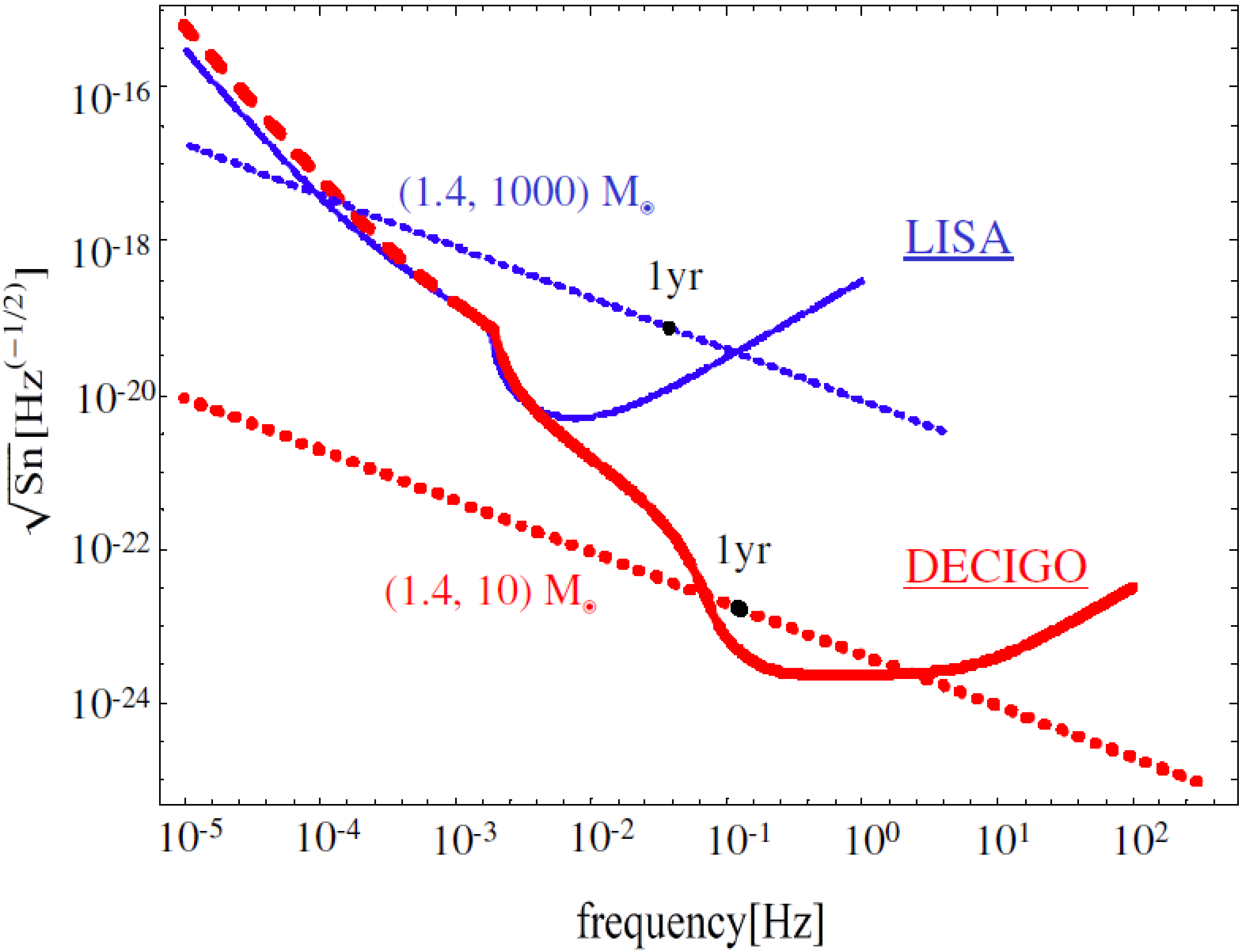
BBO (4 clusters), $(1.4+10)M_\odot$,
statistical anlysis of 10^4 binaries

Obs. time	Upper bound on l
1yr	9.62 (μ m)
3yr	3.73 (μ m)
5yr	2.62 (μ m)

10 time better than table-top experiments!

Summary

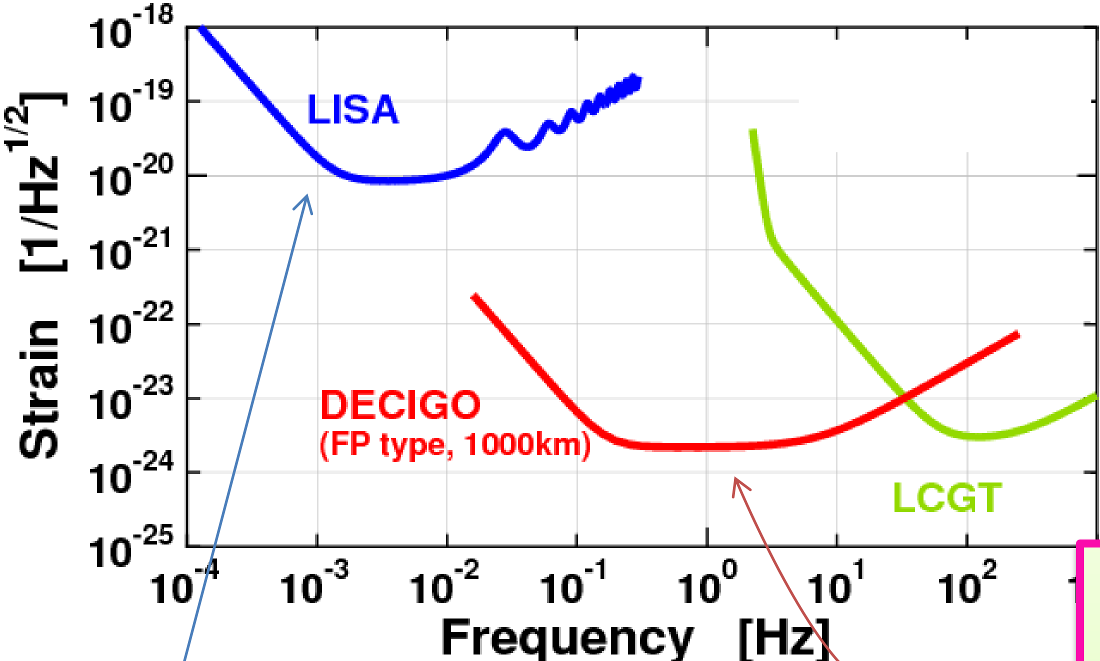
- In the RS-II braneworld model, we may observe the extradimension scale l via observations of astrophysical black holes.
- We can constrain l by
  - observing BH mass loss , or
  - finding BHs with small mass & long lifetime.
- GW observation by DECIGO/BBO
  → can make the upperbound ***10 times stronger***.
- Issues:
  - Validity of the scenario
  - Other effects: eccentricity, BH spin, precession, . . .
  - Other observations?



10-16
10-18
10-20
10-22
10-24
√Sn [Hz^(−1/2)]
(1.4, 1000) M⊙
LISA
1yr
1yr
DECIGO
(1.4, 10) M⊙
10-5
10-4
10-3
10-2
10-1
10^0
10^1
10^2
frequency[Hz]

Constrain l by DECIGO/BBO

- GW observations



<u>100Hz band : LIGO, LCGT, …</u>

NS/BH merger events

GRB central engine

Primordial black hole

<u>mHz band : LISA</u>

SMBH binary

Extreme mass-ratio inspiral (EMRI)

Galactic binary

- BHs of primordial origin
- may be $M < M_{\rm solar}$
- Part of DM?
- Age ∼ Universe age

→ With one PBH detection,

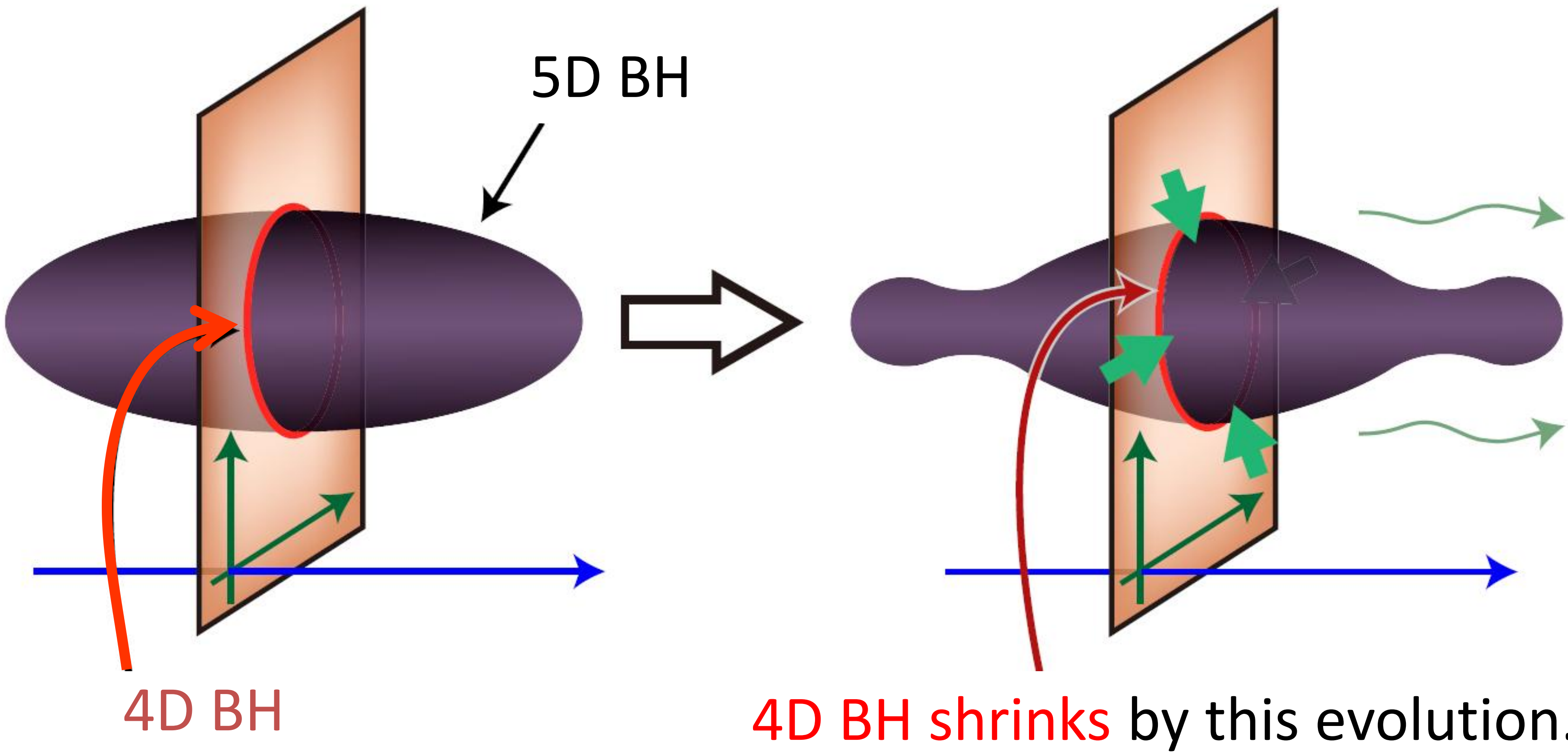
$$l < 0.1(M/M_{\rm solar})^{3/2}\ \mu{\rm m}$$

(Conjecture on) Property of this 5D BH:

5D BH deforms and escapes into bulk

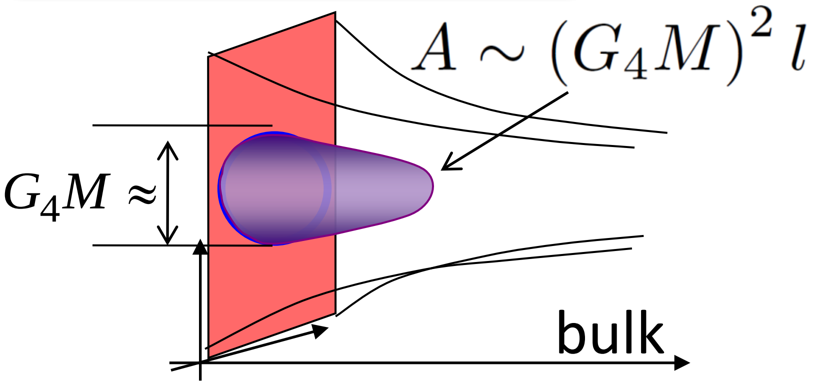
due to Gregory-Laflamme-like instability
and constant acceleration toward bulk.

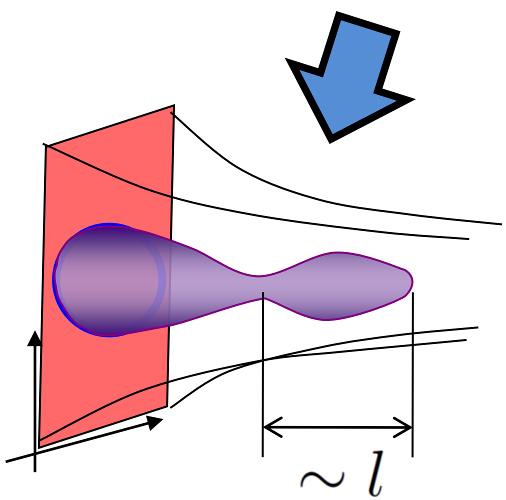
[Tanaka, 2002; Emparan et al., 2002]



- $\color{blue}{\text{5D}}$ and 4D theories give the same 4D BH mass-loss rate!

5D classical gravity





4D QFT + gravity

Hawking temp. : $T_H \sim \dfrac{1}{G_4 M}$

$$\rightarrow \dot{M} \sim N^2 \times T_H^4 r_g^2 \sim \frac{l^2}{G_4^3 M^2}$$

$$\therefore \frac{\dot{M}}{M} \sim \frac{l^2}{G_4^3 M^3}$$

Deforms in dynamical timescale: $t_{\rm dyn.} \sim G_4 M$

BHs of area $A \sim l^3$ will fall into the bulk in $t_{\rm dyn.}$

$$\Rightarrow \frac{dA}{dt} \sim \frac{d}{dt}\left(G_4^2 M^2 l\right) \sim \frac{l^3}{G_4 M}\,, \quad \frac{\dot{M}}{M} \sim \frac{l^2}{G_4^3 M^3}$$

coincide

Constrain l by observing BH evaporation

[Emparan, Garcia-Bellido & Kaloper (2003)]

- Consider in 4D point of view from now on.
- Mass-loss of BH (mass M) via Hawking radiation is quite slow :
  - $F = \sigma T^4, \quad T = \kappa/2\pi \propto 1/M \rightarrow dM/dt \propto 1/M^2$
  - (evaporation time) ~ 10^{75} x (M / M_{solar})^3 years
- In the current set up,
  - Radiation is amplified by $l^2/G_4 = 10^{73}$ x (l / 1 mm)^2
  - (evap. time) ~ 100 x (l / 1mm)$^{-2}$ (M / M_{solar})^3 yrs

    ~ 10^6 x (l /14μm)$^{-2}$ (M / M_{solar})^3 yrs

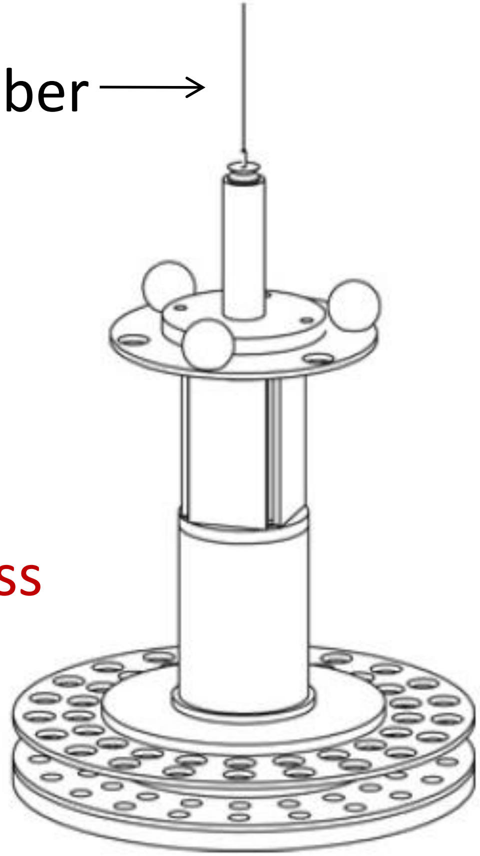
Short enough to be observable!

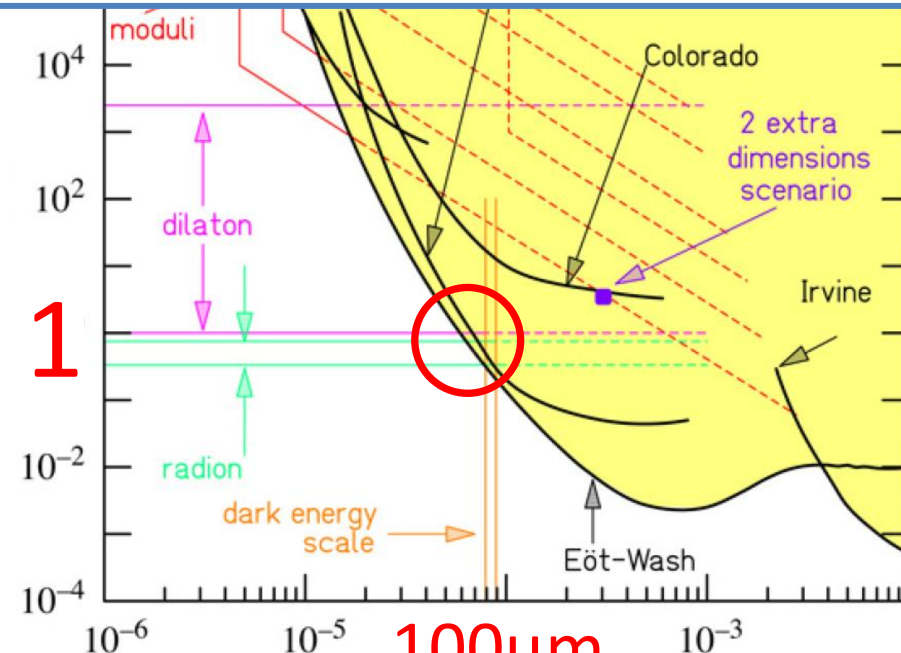
Constrain *l* by table-top experiments

Torsion balance experiment by **Eöt-Wash Group**

[Adelberger et al. (2009)]

$$V(r) = -G_N \frac{m_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$





Attraction between plates ➔ $|\alpha| < 1,\ \lambda = 56\mu m$ ➔ $\boldsymbol{l = 14\mu m}$

Constrain l by observing BH evaporation

[Emparan, Garcia-Bellido & Kaloper (2003)]

- In the current set up,
  - Radiation is amplified by $l^2/G_4 = 10^{73}$ x $(l / 1 \text{ mm})^2$
  - (evap. time) ~ 100 x $(l / 1\text{mm})^{-2} (M / M_{solar})^3$ yrs
    $\sim 10^6$ x $(l / 14\mu\text{m})^{-2} (M / M_{solar})^3$ yrs

- Some constraints on l from observations
  - Orbit evolution of X-ray binary [Johansen et al. (2009)]

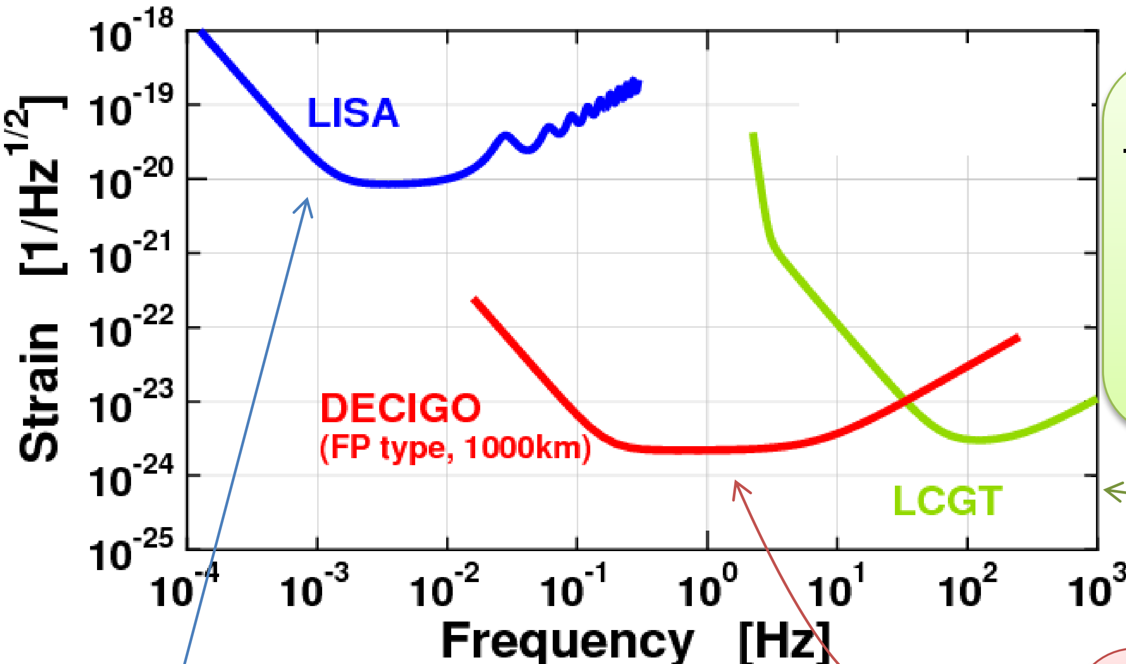
    A0620-00: $10 M_{solar}$ BH + K star → $l < 0.132$ mm
  - Age of BH in globular cluster RZ2109

    [Gnedin et al. (2009)]

    $10\ M_{solar}$ BH, Age = 10^9yrs (?) → $l < 0.003$ mm (?)

Constrain l by observing BH evaporation

- GW observations



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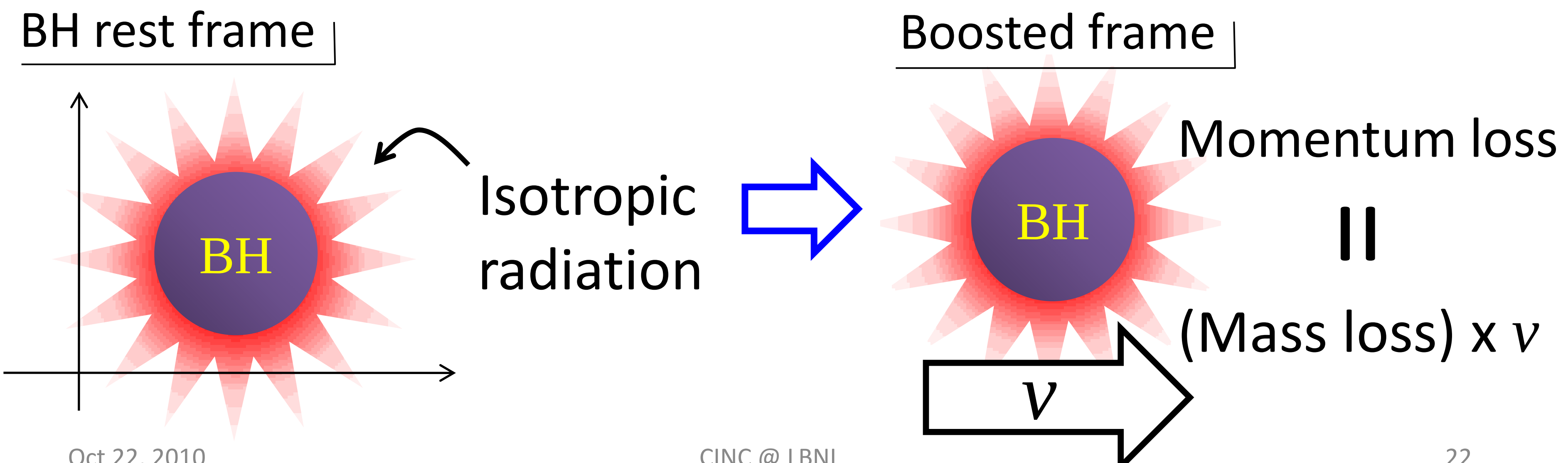
Constraining the braneworld with gravitational wave observations

[McWilliams (2010)]

- Galactic NS/(Evaporating)BH binary
  - Mass loss by radiation
  - Momentum loss = (mass loss) x (velocity)

→ Specific angular momentum J/M will be conserved.



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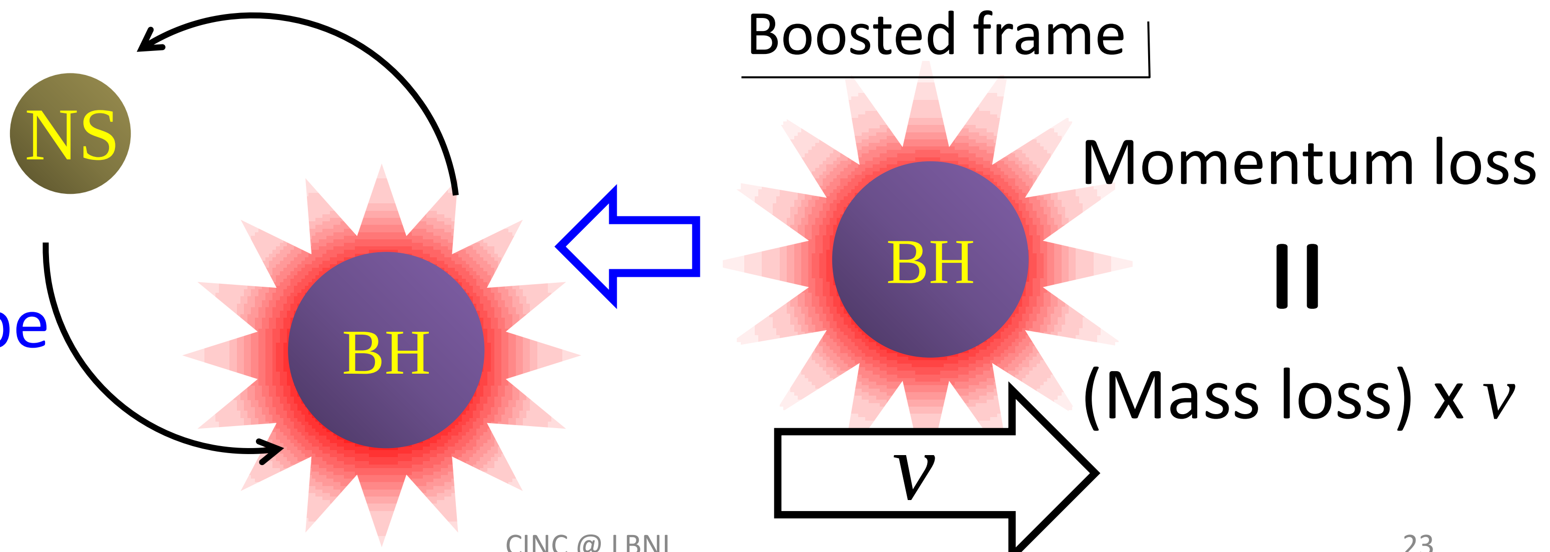
J ↘, while J/M will be conserved.



Constraining the braneworld with gravitational wave observations

[McWilliams (2010)]

- Galactic NS/(Evaporating)BH binary

→ Specific angular momentum $\boldsymbol{j = J/M}$ will be conserved.

$$\begin{cases} j = \sqrt{r\left(m_{\rm NS} + m_{\rm BH}\right)} \\ \dot{j} = 0 \end{cases}$$

$$\dot{r} = -\frac{r\,\frac{d}{dt}\left(m_{\rm NS} + m_{\rm BH}\right)}{m_{\rm NS} + m_{\rm BH}} = -\frac{r\,\dot{m}_{\rm BH}}{m_{\rm NS} + m_{\rm BH}}$$

$$= 3.2 \times 10^{-8} \left(\frac{r}{1\rm AU}\right) \left(\frac{7M_\odot}{m_{\rm NS} + m_{\rm BH}}\right) \left(\frac{5M_\odot}{m_{\rm BH}}\right)^2 \left(\frac{\ell}{44\mu{\rm m}}\right)^2 {\rm AU\ yr^{-1}}$$

Binary separation increases with time!

Constraining the braneworld with gravitational wave observations

[McWilliams (2010)]

- GW emission → Inspiral, effective for small *r*

$$\dot{r}_{\rm GW} = -5.2 \times 10^{-17} \left(\frac{1{\rm AU}}{r}\right)^3 \left(\frac{m_{\rm BH}}{5M_\odot}\right) \left(\frac{M}{2M_\odot}\right) \left(\frac{m_{\rm NS}+m_{\rm BH}}{7M_\odot}\right) {\rm AU\ yr}^{-1}$$

- BH evaporation → Outspiral, effective for large *r*

$$\dot{r}_{\rm rad} = 3.2 \times 10^{-8} \left(\frac{r}{1{\rm AU}}\right) \left(\frac{7M_\odot}{m_{\rm NS}+m_{\rm BH}}\right) \left(\frac{5M_\odot}{m_{\rm BH}}\right)^2 \left(\frac{\ell}{44\mu{\rm m}}\right)^2 {\rm AU\ yr}^{-1}$$

➔ When $r > r_{\rm crit} = 6.3 \times 10^{-3} \left(\frac{m_{\rm BH}}{5M_\odot}\right)^{\frac{4}{3}} \left(\frac{m_{\rm NS}}{2M_\odot}\right)^{\frac{1}{4}} \sqrt{\left(\frac{m_{\rm BH}+m_{\rm NS}}{7M_\odot}\right)\left(\frac{44\mu{\rm m}}{\ell}\right)}\,{\rm AU}$,

the binary will keep outspiraling.

Constraining the braneworld with gravitational wave observations

[McWilliams (2010)]

➔ When $r > r_{\rm crit} = 6.3 \times 10^{-3} \left(\frac{m_{\rm BH}}{5M_\odot}\right)^{\frac{4}{3}} \left(\frac{m_{\rm NS}}{2M_\odot}\right)^{\frac{1}{4}} \sqrt{\left(\frac{m_{\rm BH}+m_{\rm NS}}{7M_\odot}\right)\left(\frac{44\mu{\rm m}}{\ell}\right)}\,{\rm AU}$,

the binary will keep outspiraling.

Suppose that LISA found GW from inspiral with $f_{\rm GW} \sim 0.1$ mHz.

➔ $r_{\rm crit}$ must be larger than r for $f_{\rm GW}$.

➔ Extradimension scale is constrained as

$$\ell \leq 22 \left(\frac{m_{\rm BH}}{5M_\odot}\right)\left(\frac{m_{\rm BH}+m_{\rm NS}}{7M_\odot}\right)\sqrt{\frac{m_{\rm NS}}{2M_\odot}}\left(\frac{f_{\rm gw}}{0.1{\rm mHz}}\right)^{3/4} \mu{\rm m}\,.$$

Constraining the braneworld with gravitational wave observations

[McWilliams (2010)]

Suppose that LISA found GW from inspiral with $f_{\rm GW} \sim 0.1$ mHz.

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Problems:

- Such binaries are almost monochromatic in LISA lifetime

  → Difficult to distinguish outspiral from inspiral .

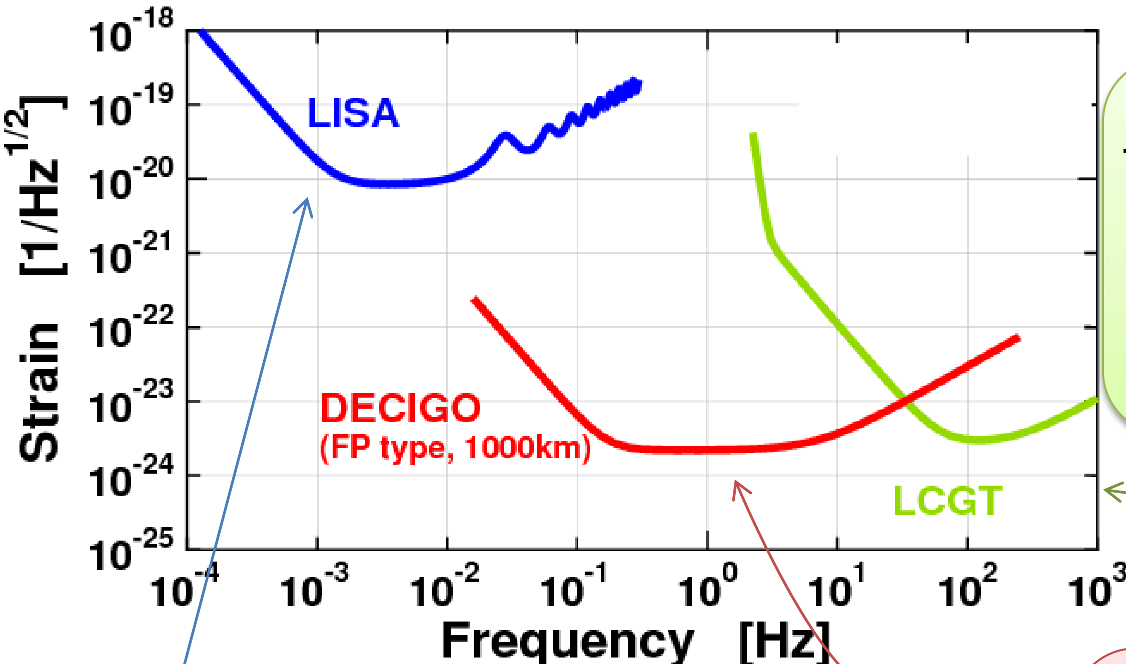
  → We should take

  more direct approach using GW wave form.

Constrain l by DECIGO/BBO

[Yagi, NT & Tanaka, in prep.]

- GW observations



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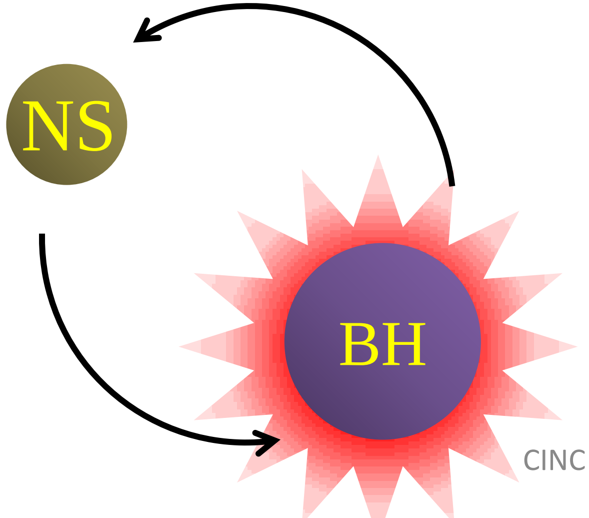
Constrain l by Gravitational wave observation

[Yagi, NT & Tanaka, in prep.]

- Analysis outline
  1. Solve the binary motion, and write down the GW phase in terms of physical parameters.
  2. Apply the matched filtering analysis to estimate the parameters.

  → ***Upper bound on l up to statistical error***



For detail, see e.g.
Yagi & Tanaka,
PRD **81** 064008 (2010),
arXiv:0908.3283 [gr-qc].

Constrain l by DECIGO/BBO

[Yagi, NT & Tanaka, in prep.]

1. Solve the binary motion, and write down the GW phase in terms of physical parameters.

- BH binary of mass M & m
- Binary orbit evolution is given by

$$\dot{r}_{\rm GW} = -\frac{64}{5}\frac{\mu M_t^2}{r^3},$$

$$\dot{r}_{\rm rad} = -\frac{\dot{M}_t}{M_t} r$$

$$= \quad A\frac{(m^2+M^2)l^2}{\mu^2 M_t^3} r$$

$$M_t = m + M$$
$$\mu = mM/M_t$$
$$\eta = \mu/M_t$$
$$\mathcal{M} = M_t^{2/5}\mu^{3/5}$$
$$\Omega = M_t^{1/2} r^{-3/2}$$

$$\dot{m} = -A\frac{l^2}{m^2}$$

Constrain *l* by DECIGO/BBO

[Yagi, NT & Tanaka, in prep.]

1. Solve the binary motion, and write down the GW phase in terms of physical parameters.

- $\dot{r} = \dot{r}_{\rm GW} + \dot{r}_{\rm rad}$ & $\Omega = M_t^{1/2} r^{-3/2}$

$\downarrow$

$$\dot{f} = \frac{\dot{\Omega}}{\pi}$$

$$= \frac{96}{5}\pi^{8/3}\mathcal{M}^{5/3} f^{11/3}\left(1 - \frac{5}{48}A\frac{(m^2+M^2)l^2}{\mu^3 M_t}(\pi M_t f)^{-8/3}\right)$$

$$\begin{pmatrix} M_t = m + M \\ \mu = mM/M_t \\ \eta = \mu/M_t \\ \mathcal{M} = M_t^{2/5}\mu^{3/5} \\ \Omega = M_t^{1/2} r^{-3/2} \end{pmatrix}$$

$\downarrow$

$$\mathcal{M}^{5/3} dt = \frac{5}{96}\pi^{-8/3} f^{-11/3}\left(1 + \frac{5}{48}A\frac{(m^2+M^2)l^2}{\mu^3 M_t}(\pi M_t f)^{-8/3}\right) df$$

Constrain l by DECIGO/BBO

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$$\mathcal{M}^{5/3}dt = \frac{5}{96}\pi^{-8/3}f^{-11/3}\left(1+\frac{5}{48}A\frac{(m^2+M^2)l^2}{\mu^3 M_t}(\pi M_t f)^{-8/3}\right)df$$

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Integrating each side, we find

$$t(f) \simeq -\frac{5}{256}\mathcal{M}_0(\pi\mathcal{M}_0 f)^{-8/3}\left(1 - AC_t L(\pi M_{t0} f)^{-8/3}\right)$$

$$\begin{pmatrix} C_t = \frac{5}{1536}\frac{3(m_0^4+M_0^4)-14\mu_0 M_{t0}(m_0^2+M_0^2)}{\mu_0^4} \\ L = \ell^2/M_{t0}^2 \end{pmatrix}$$

Constrain l by DECIGO/BBO

[Yagi, NT & Tanaka, in prep.]

1. Solve the binary motion, and write down the GW phase in terms of physical parameters.

- Phase of GW Fourier component $\Psi(f)$:

$$\Psi(f) \equiv 2\pi f t(f) - \phi(t(f))$$

$t(f)$: Time measured from the coalescence

$$\phi(t) = \int 2\pi f dt$$

Constrain l by DECIGO/BBO

[Yagi, NT & Tanaka, in prep.]

1. Solve the binary motion, and write down the GW phase in terms of physical parameters.

Calculating $\phi(f) = \int 2\pi f\, dt$ in a similar way, we get

$$
\begin{aligned}
\Psi(f) &= 2\pi f t(f) - \phi(t(f)) \\
&\approx \frac{3}{128}(\pi \mathcal{M}_0 f)^{-5/3}\left(1 - A C_L L x^{-4}\right)
\end{aligned}
$$

$$
\begin{aligned}
M_t &= m + M \\
\mu &= mM/M_t \\
\eta &= \mu/M_t \\
\mathcal{M} &= M_t^{2/5}\mu^{3/5} \\
\Omega &= M_t^{1/2} r^{-3/2}
\end{aligned}
$$

GW

correction by BH evaporation

$$
x \equiv v^2 = (\pi M_0 f)^{2/3}, \quad C_L = \frac{25}{19968}\frac{1}{\eta_0^4}(3 - 26\eta_0 + 34\eta_0^2)
$$

$$
L = \ell^2/M_{t0}^2
$$

Constrain l by DECIGO/BBO

[Yagi, NT & Tanaka, in prep.]

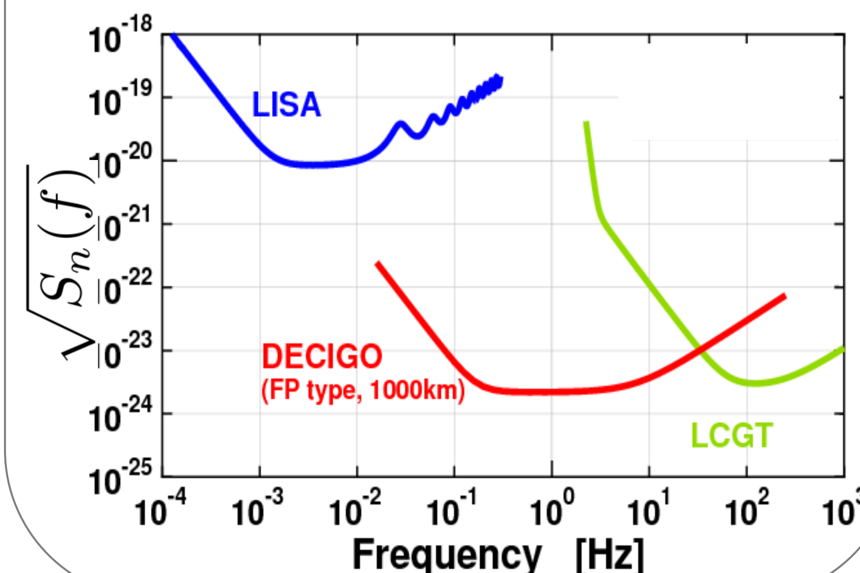
2. Apply the matched filtering analysis to estimate the parameters.

- Binary parameters: θ^i
- GW wave form : $h(\theta) \propto e^{i\Psi}$
- signal: $s(f) = h(f, \theta) + n(f)$
- Probability that binary parameter = θ when you got a signal s :

$$p(\theta|s)$$

- $p(\theta|s) \propto \exp\left[(h(\theta)|s) - \frac{1}{2}(h(\theta)|h(\theta))\right]$

$$(A|B) = \mathrm{Re}\int_{-\infty}^{\infty} df \frac{\tilde{A}^*(f)\tilde{B}(f)}{\frac{1}{2}S_n(f)}$$



Constrain l by DECIGO/BBO

[Yagi, NT & Tanaka, in prep.]

2. Apply the matched filtering analysis to estimate the parameters.

- Probability that binary parameter = $\boldsymbol{\theta}$ for signal s :

$$p\left(\theta|s\right) \propto \exp\left[\left(h(\theta)|s\right) - \frac{1}{2}\left(h(\theta)|h(\theta)\right)\right]$$

- Estimated $\boldsymbol{\theta}$ = $\boldsymbol{\theta}$ at the maximum of $p(\boldsymbol{\theta}|s)$

  = solution of $\left(\frac{\partial h}{\partial \theta^i}\middle| s\right) - \left(\frac{\partial h}{\partial \theta^i}\middle| h\right) = 0$

- Estimation error = deviation of $p(\boldsymbol{\theta}|s)$ around estimated $\boldsymbol{\theta}$

$$p\left(\theta|s\right) \propto \exp\left[-\frac{1}{2}\Gamma_{ij}\Delta\theta^i\Delta\theta^j\right], \qquad \Gamma_{ij} = \left(\frac{\partial h}{\partial \theta^i}\middle|\frac{\partial h}{\partial \theta^j}\right)$$

Error of θ^i : Γ_{ii}^{-1}

Constrain l by DECIGO/BBO

[Yagi, NT & Tanaka, in prep.]

- Results:

1. Constraint from a single NS/BH binary
2. statistical analysis using 10^4 binaries

Assuming that we got a signal that obeys pure GR,
we give a constraint on l as

(Upper bound on l) = $\Gamma_{ll}^{-1}\Big|_{l=0}$

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[Yagi, NT & Tanaka, in prep.]

- Results:

  1. Constraint from a single NS/BH binary
  2. statistical analysis using 10^4 binaries

  Assumption:

  signal that obeys pure GR ($l=0$) is observed

  → Upper bound on l up to statistical error

Constrain l by DECIGO/BBO

[Yagi, NT & Tanaka, in prep.]

- Results:
  - Take $l = 0$ as fiducial value
  - Upper bound on l = $\Gamma_{ll}^{-1}\big|_{l=0}$
  - Constraint from a single binary:

BBO (4 clusters), (1.4+10)$M_\odot$, **D_L=3Gpc:**

Obs. time	Upper bound on l
1yr	50.9 (μ m)
3yr	24.8 (μ m)
5yr	19.7 (μ m)

Comparable to table-top experiments!

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