Multi-Epoch Galaxy Modeling

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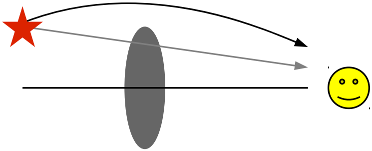
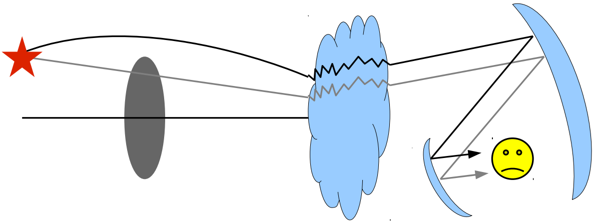
Cosmology in Northern California 2010

Gravitational lensing: the dirty truth.



- Mass deflects light - we can infer mass!

Gravitational lensing: the dirty truth.



- Mass deflects light - we can infer mass!
- Atmosphere and telescope also deflect light, but we can infer the properties of the atmosphere and telescope from stars.

Defining the shear measurement problem

- We have many exposures of each patch of sky, with different orientations and observing conditions. We might even want to combine observations from multiple telescopes.
- For each exposure, we have a model of the PSF and the geometric distortion.
- We want to measure an ellipticity for each galaxy, in a way that is
  1. unbiased,
  2. fast,
  3. and accurate

  (in that order of importance).

Handling multi-epoch data: other options

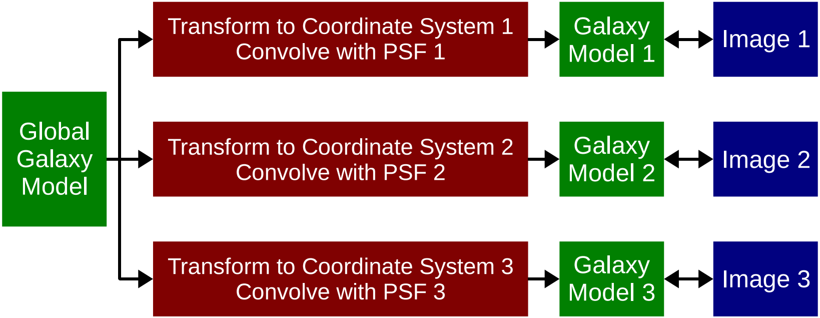
	supports non-modeling analyses	works in short-exposure limit	optimal PSF weights	supports calibration marginalization	exact treatment of per-pixel errors
analysis on coadd	✓	✓	?*	✗	✗
analysis on exposures	✓	✗	✓	✓	✓
simultaneous multifit	✗	✓	✓	✓	✓
serial likelihood sampling	✗	✓**	✓	✓	✓

* requires special coadd algorithm (Kaiser), which assumes stationary noise, non-spatially varying PSF, no missing pixels

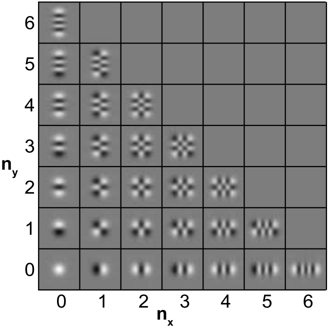
** only optimal if there's no galaxy parameter marginalization before combining likelihoods (e.g. LENSFIT, Miller et al 2007)

Multifit

- Fit a transformed galaxy to each image, convolving with the PSF for that image.
- It's crucial to fit *simultaneously*, or the fit isn't robust.



Introduction to shapelets



$$\Phi_{n_x n_y}(x, y) = \frac{H_{n_x}(\mu) H_{n_y}(\nu) e^{-\frac{\mu^2+\nu^2}{2}}}{2^{n_x+n_y} \beta \sqrt{\pi n_x! n_y!}}$$

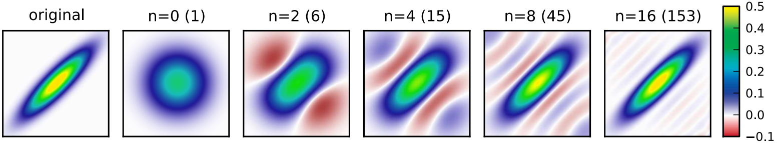
$$\mu = \frac{x - x_0}{\beta}$$

$$\nu = \frac{y - y_0}{\beta}$$

- Eigenfunctions of the 2-d quantum harmonic oscillator (orthonormal, complete).
- Zeroth order is a circular Gaussian.
- *Analytic convolution!*

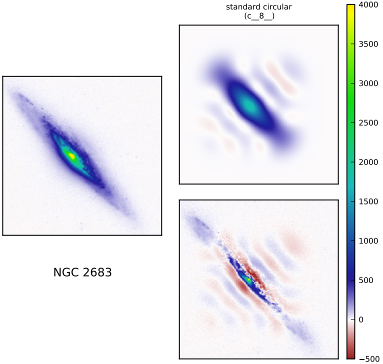
Figures from *Massey & Refregier (2005)*

Shapelets are broken at high ellipticity

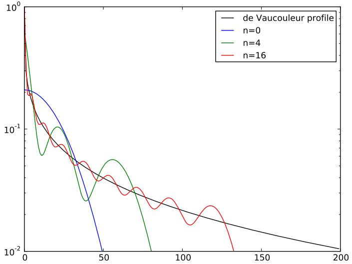


elliptical gaussian ($q = \frac{1}{4}$) $\Uparrow$

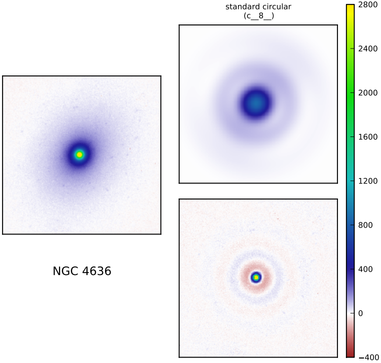
edge-on spiral $\Rightarrow$



Shapelets are broken at high Sérsic index



de Vaucouleur profile ($n = 4$)



elliptical galaxy

Elliptical Shapelets

We can make a shapelet expansion around an elliptical gaussian by transforming the coordinate grid (this actually works for any 2-d function):

$$\Phi_{\mathbf{n}}\left(\mathbf{x}|\mathbf{e}\right) = \Phi_{\mathbf{n}}(\mathbf{S}_e^{-1}\mathbf{x})$$

$$\mathbf{S}_e = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

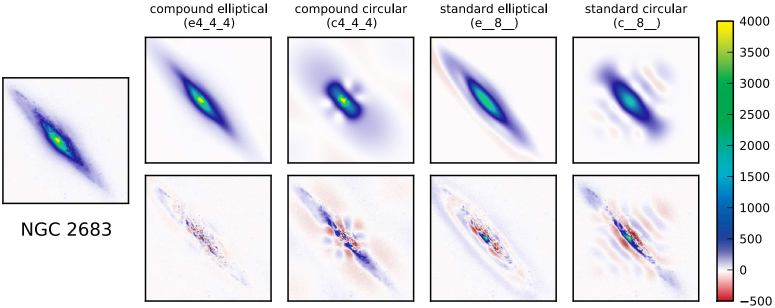
- The most common shapelet convolution relation (Refregier and Bacon 2003) requires galaxy and PSF to have the same ellipticity
- Standard procedure is to use an approximate shear operator on the galaxy or PSF to allow its use.
- It's important to use an exact elliptical convolution relation (Hirata and Seljak (2003) for Gauss-Laguerre, or Bosch (2010) for Gauss-Hermite).

Compound Shapelets

Simple idea: combine multiple low-order shapelet expansions with different radii into a single basis.

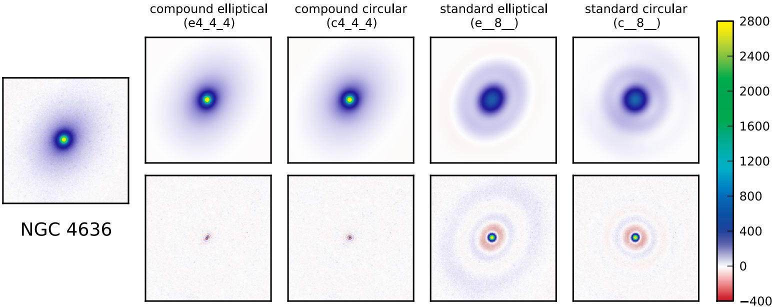
- Shapelet functions with small radii model the core.
- Shapelet functions with large radii model the wings.
- Result is better than a single high-order expansion with the same number of basis functions.

Results



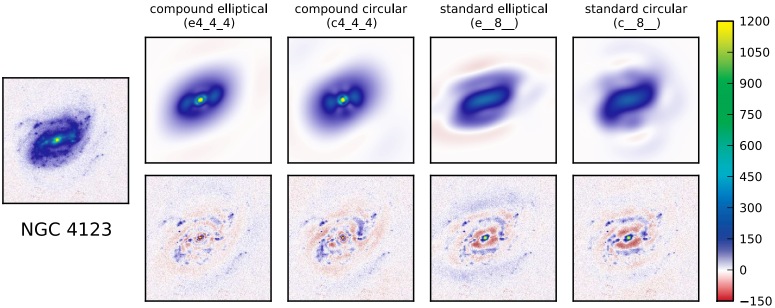
compound elliptical (e4_4_4)
compound circular (c4_4_4)
standard elliptical (e__8__)
standard circular (c__8__)
NGC 2683
4000
3500
3000
2500
2000
1500
1000
500
0
−500

Results

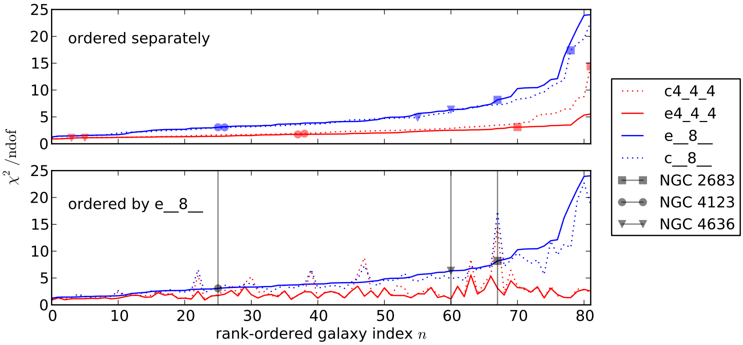


compound elliptical
(e4_4_4)
compound circular
(c4_4_4)
standard elliptical
(e__8__)
standard circular
(c__8__)
2800
2400
2000
1600
1200
800
400
0
−400
NGC 4636

Results


compound elliptical
(e4_4_4)
compound circular
(c4_4_4)
standard elliptical
(e__8__)
standard circular
(c__8__)
1200
1050
900
750
600
450
300
150
0
−150
NGC 4123

Results



ordered separately
ordered by e__8__
χ^2 /ndof
rank-ordered galaxy index n
c4_4_4
e4_4_4
e__8__
c__8__
NGC 2683
NGC 4123
NGC 4636
0
5
10
15
20
25
0
10
20
30
40
50
60
70
80

Training an "eigenmorphology" basis

Given a high S/N, well-resolved sample of galaxies, and an initial compound basis:

1. Fit a compound elliptical shapelet model to each galaxy, obtaining a vector of coefficients $\mathbf{a}_i$ and Fisher information matrix $\mathbf{F}_i$ for each galaxy i.
2. Optimize

$$\min_{\mathbf{M}} \qquad \sum_i \left[(\mathbf{a}_i - \mathbf{M}\mathbf{b}_i)^T \mathbf{F}_i (\mathbf{a}_i - \mathbf{M}\mathbf{b}_i) + \ln \left| \mathbf{M}^T \mathbf{F}_i \mathbf{M} \right| \right],$$

$$\mathbf{b}_i \equiv \left(\mathbf{M}^T \mathbf{F}_i \mathbf{M} \right)^{-1} \mathbf{M}^T \mathbf{F}_i \mathbf{a}_i$$

   subject to the constraint $\mathbf{M}^T\mathbf{M} = \mathbf{I}$, one column at a time.
3. $\mathbf{M}$ maps the original basis to a new basis, ordered such that the first n basis functions are the n most important linear combinations of original basis functions.

Working with a compound basis

It's easy to...

- reorthogonalize it
- convolve it with a PSF expressed in shapelets
- evaluate it on a pixelized grid
- apply a linear coordinate transform
- project out basis functions that are degenerate with the ellipse parameters.

It's hard to...

- ensure the model has positive flux where there's no data
- determine an optimal set of shapelet radii and orders
- interpret best-fit basis coefficients

Summary and Future Work

What you've heard

- Given a good model, simultaneous fitting is the safest way to analyze data from multiple exposures; be careful with coadds!
- Compound elliptical shapelet models are perfect for simultaneous fitting.

On the horizon

- testing the eigenmorphology metric
- morphological classification
- multi-band photometry
- deblending dense fields