Making you all experts on CMB internal (de)lensing

Marius Millea

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

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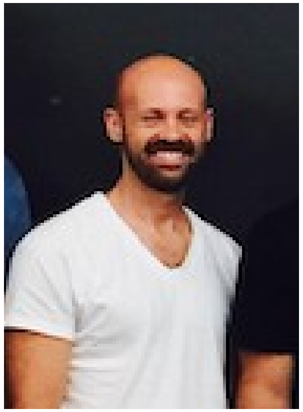
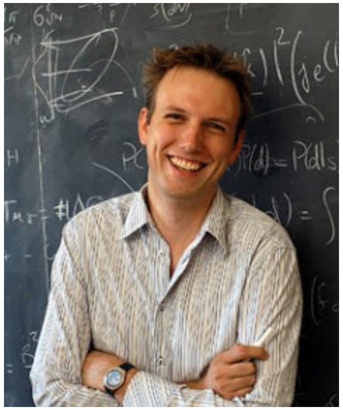
@marius311 @cosmicmar

with



Ethan Anderes



Ben Wandelt

BCCP Lensing Workshop – Jan 16, 2019

Making you all experts on CMB internal (de)lensing

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- Why is this interesting scientifically?

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Making you all experts on CMB internal (de)lensing

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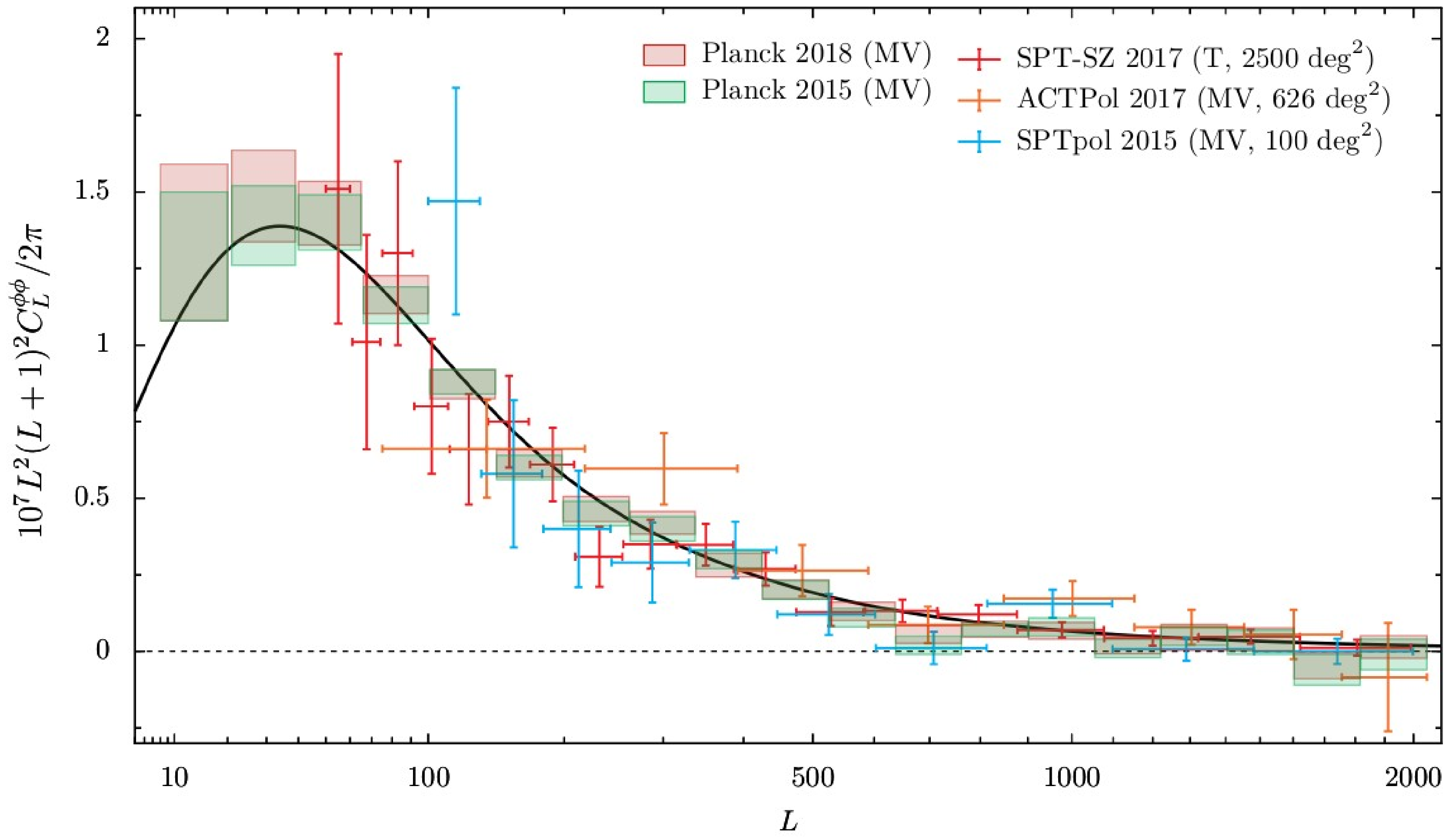
- The quadratic estimate and beyond
- LenseFlow and the Bayesian sampling solution

Measurements of the power spectrum of the lensing potential are becoming increasingly precise and will continue to do so.



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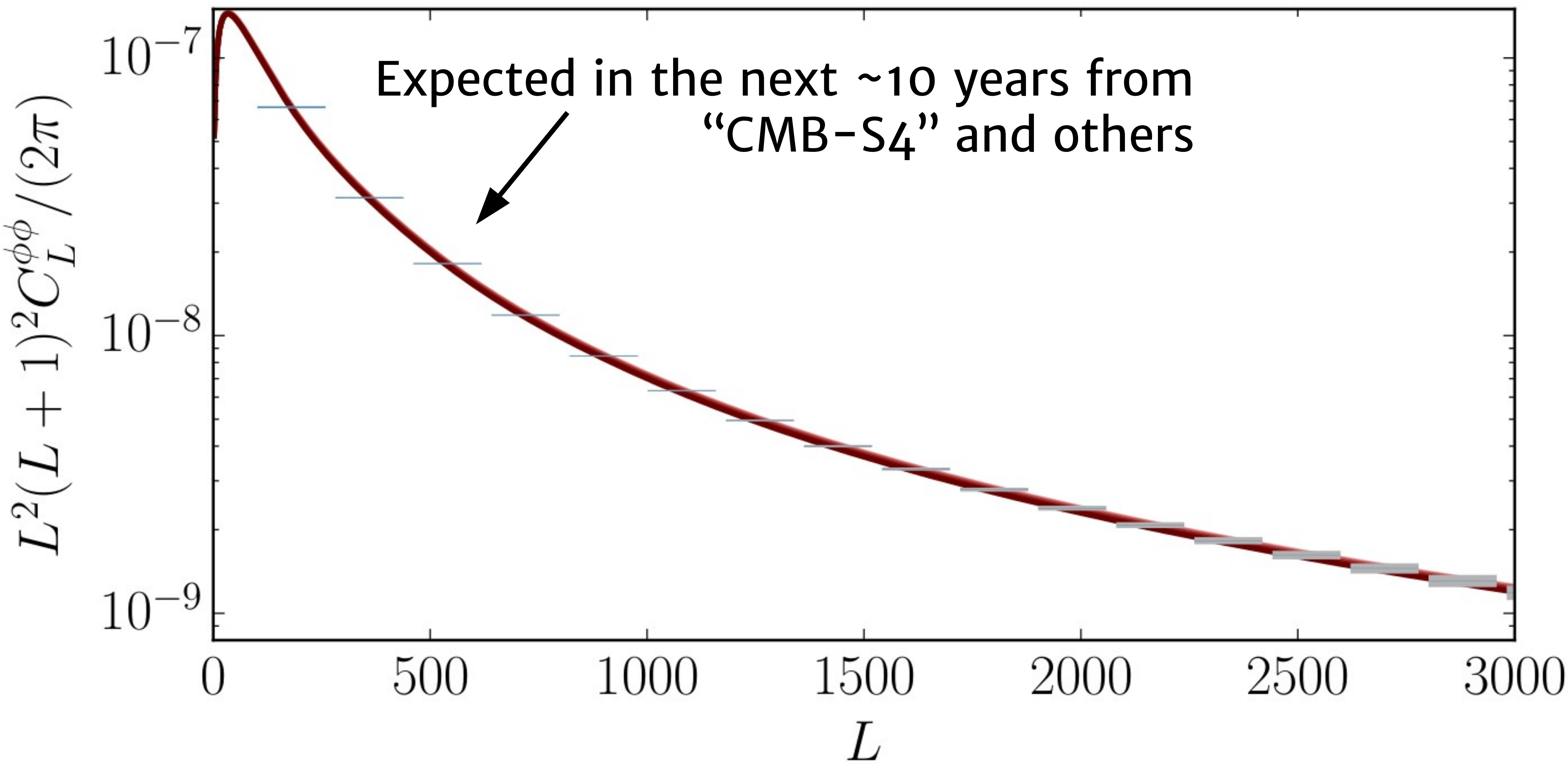

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The most exciting possibility...

r controls amplitude of tensor fluctuations

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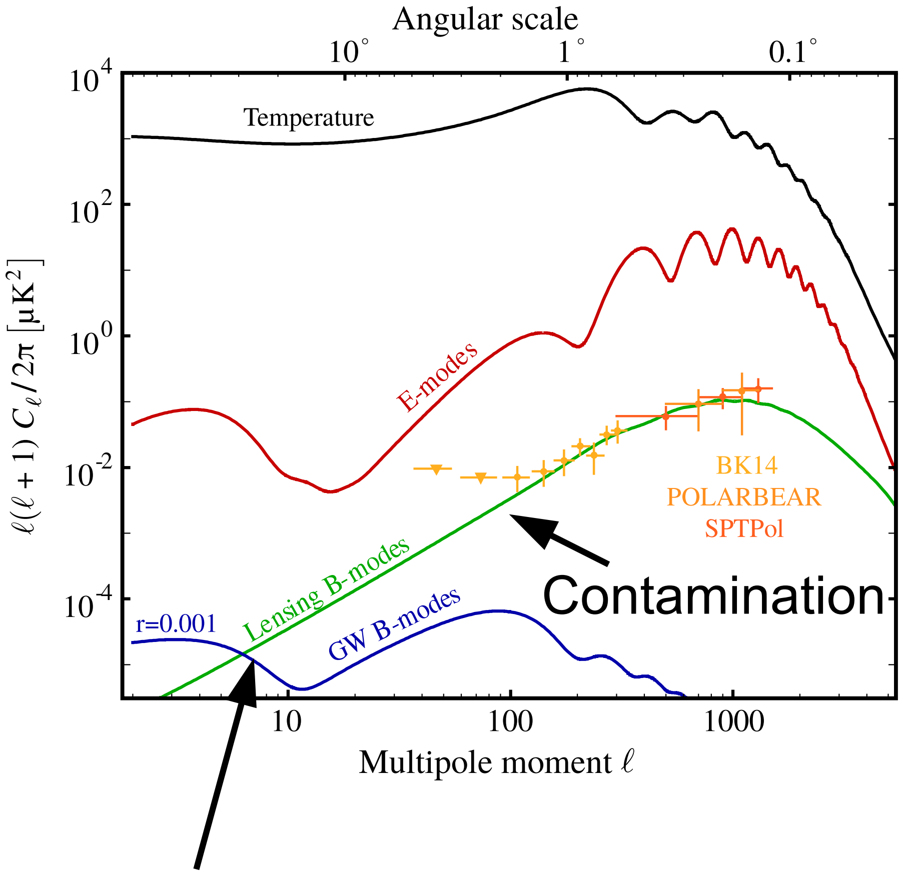

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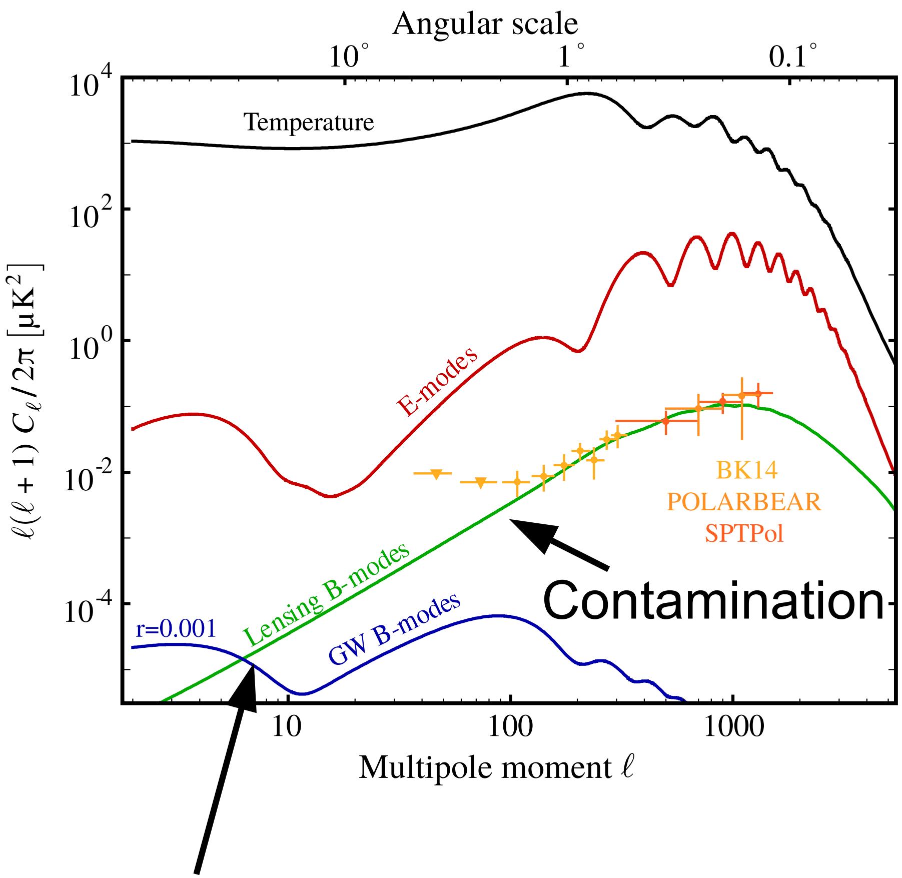

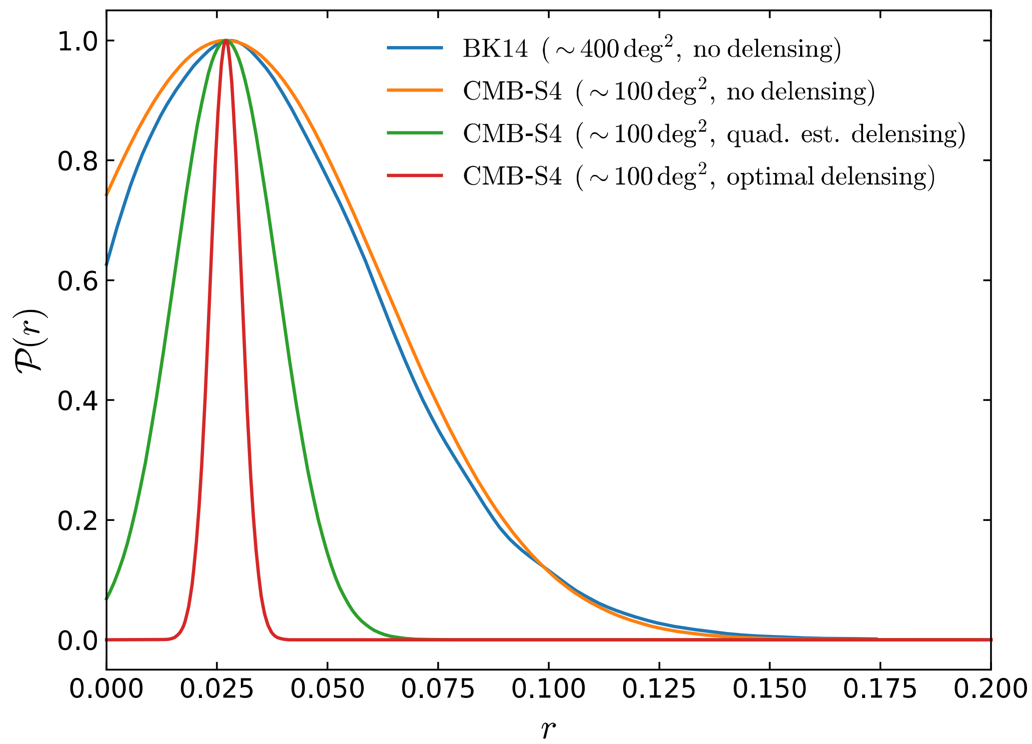

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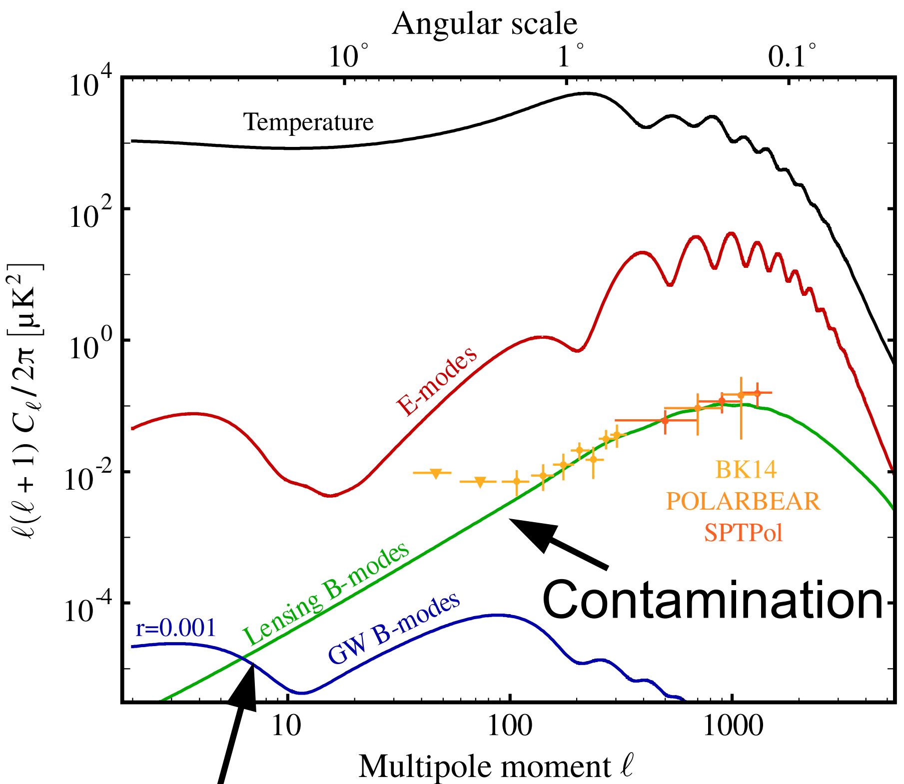
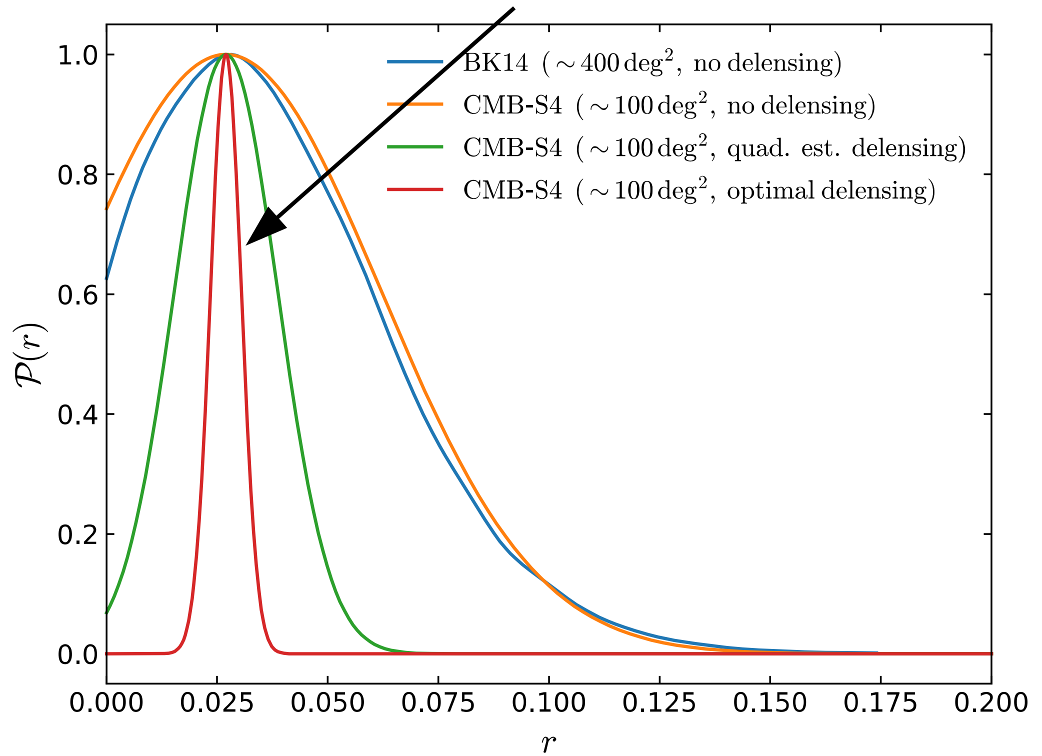
How to achieve this Fisher forecast performance in practice is an open question.





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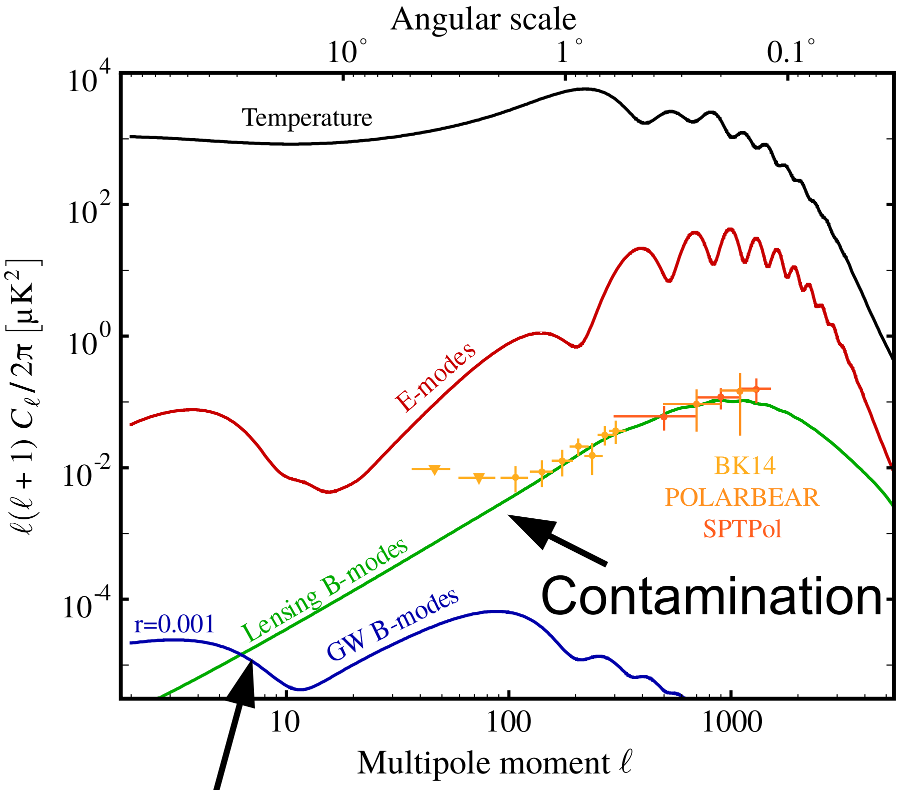

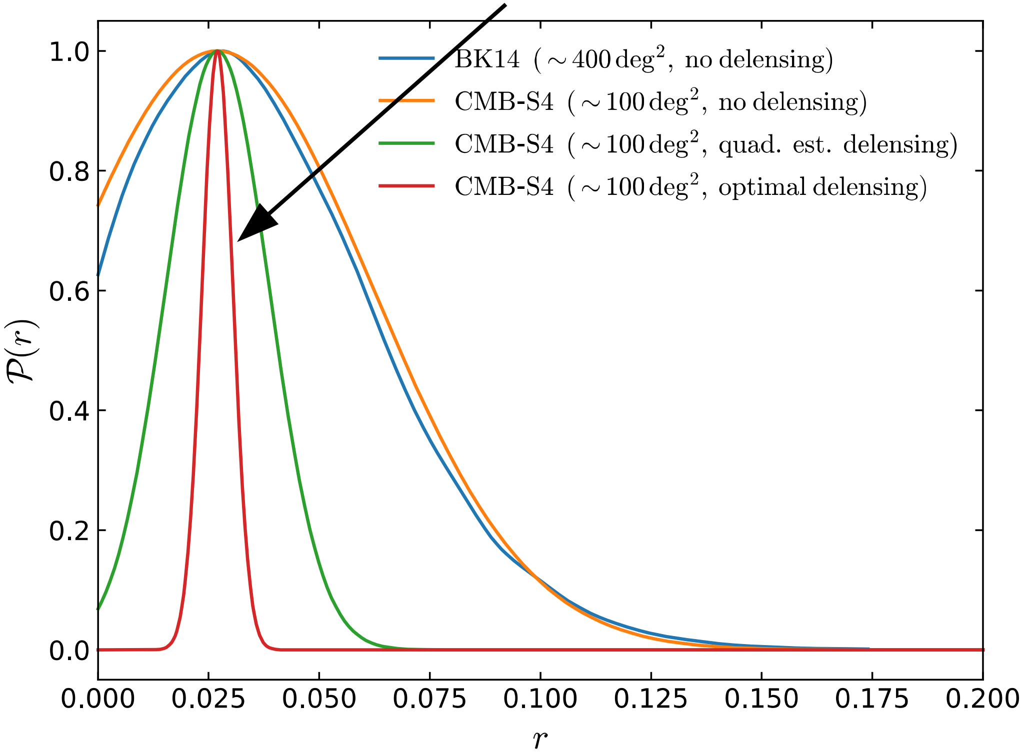

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The detailed accuracy of the “Fisher forecast” itself is also an open question.

MM, Anderes, Wandelt (2017)

Lensing potential

CMB "fields"

$f \equiv (T, E, B)$

Cosmo params

Data

$$\mathcal{P}(f, \phi, \theta \,|\, d) \;=$$

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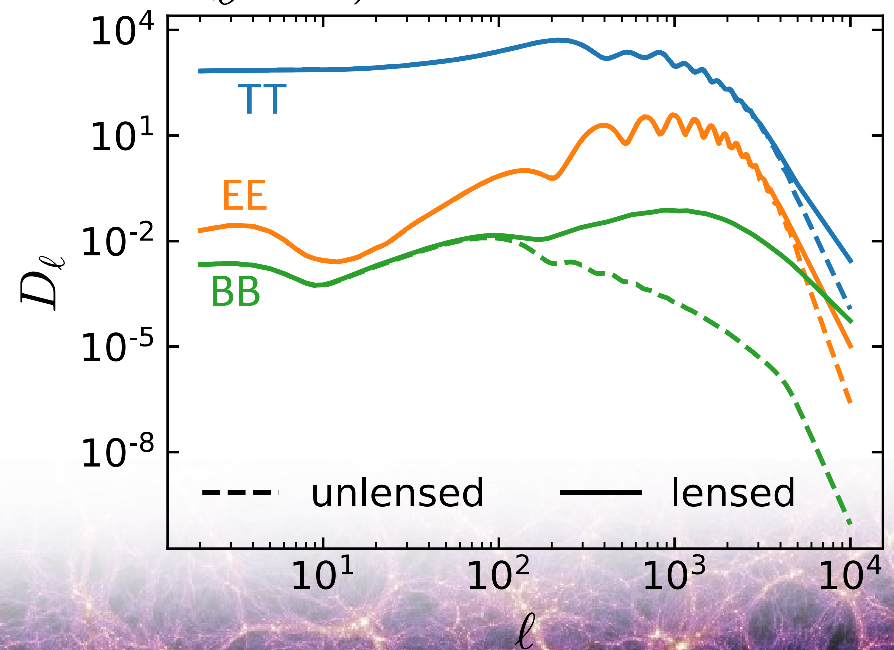

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In practice, this heurestic procedure has only been used for forecasting

Iterated quadratic estimate forecasting:

$$N_\ell^{\phi\phi} = \left[\frac{1}{2\ell+1} \sum_{\ell_1 \ell_2} |f^{EB}_{\ell_1 \ell_2 \ell}|^2 \left(\frac{1}{C_{\ell_1}^{B_{\rm res}} + N_{\ell_1}^{BB}} \right) \left(\frac{(C_{\ell_2}^{EE})^2}{C_{\ell_2}^{EE} + N_{\ell_2}^{EE}} \right) \right]^{-1}$$

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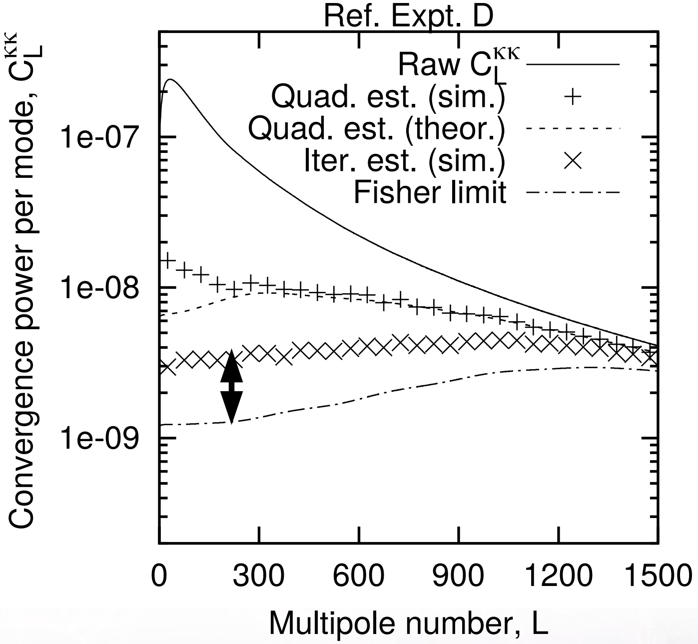

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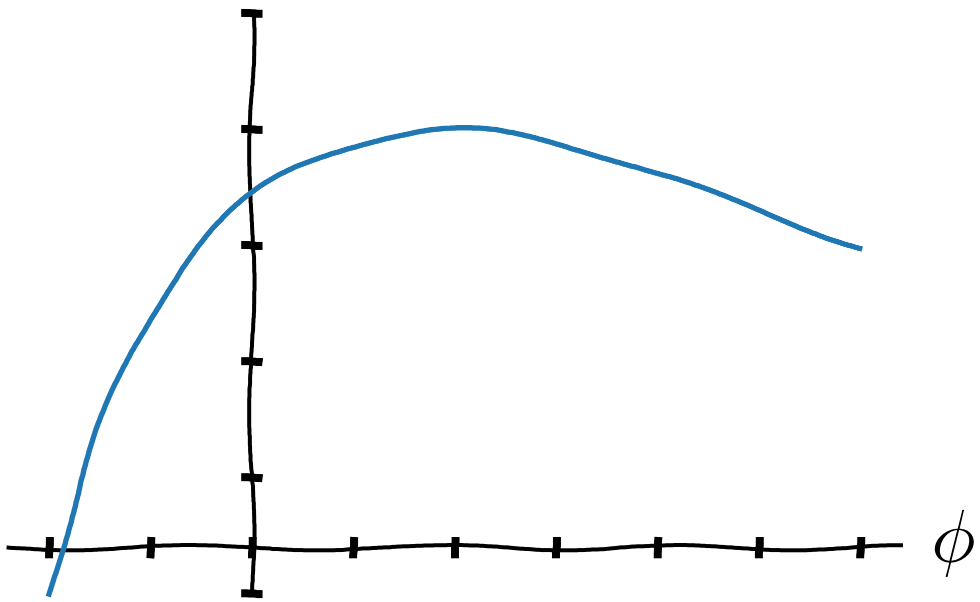
$$\phi' = \phi - H^{-1} \frac{\delta}{\delta\phi} \log \mathcal{P}(\phi, \theta \,|\, d)$$

$\log \mathcal{P}(\phi \,|\, \theta, d)$



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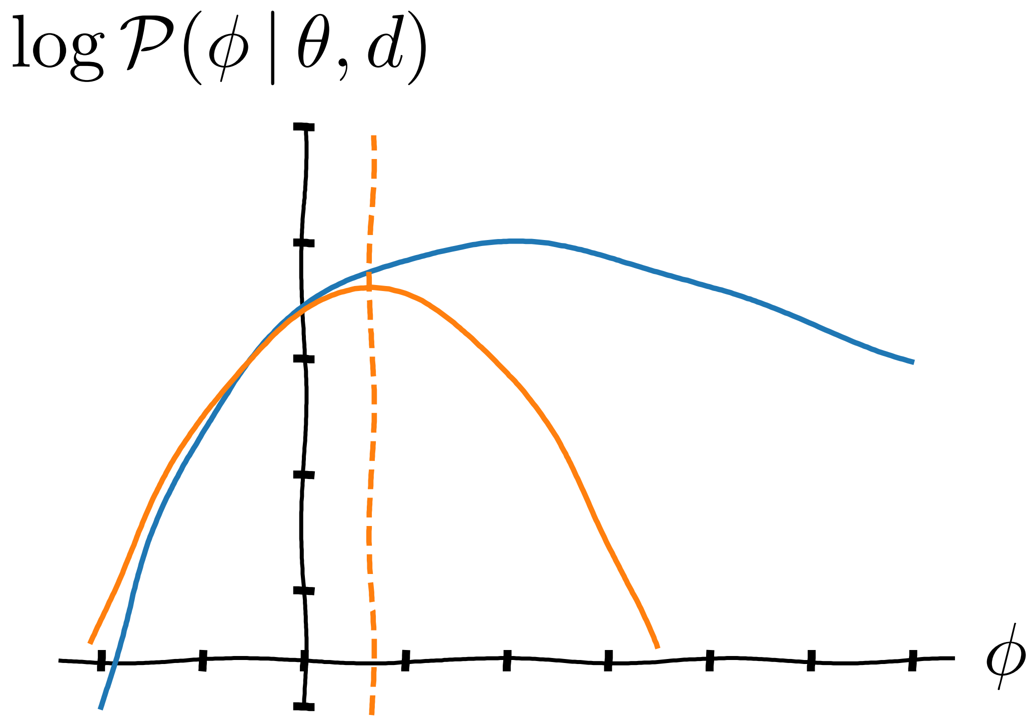

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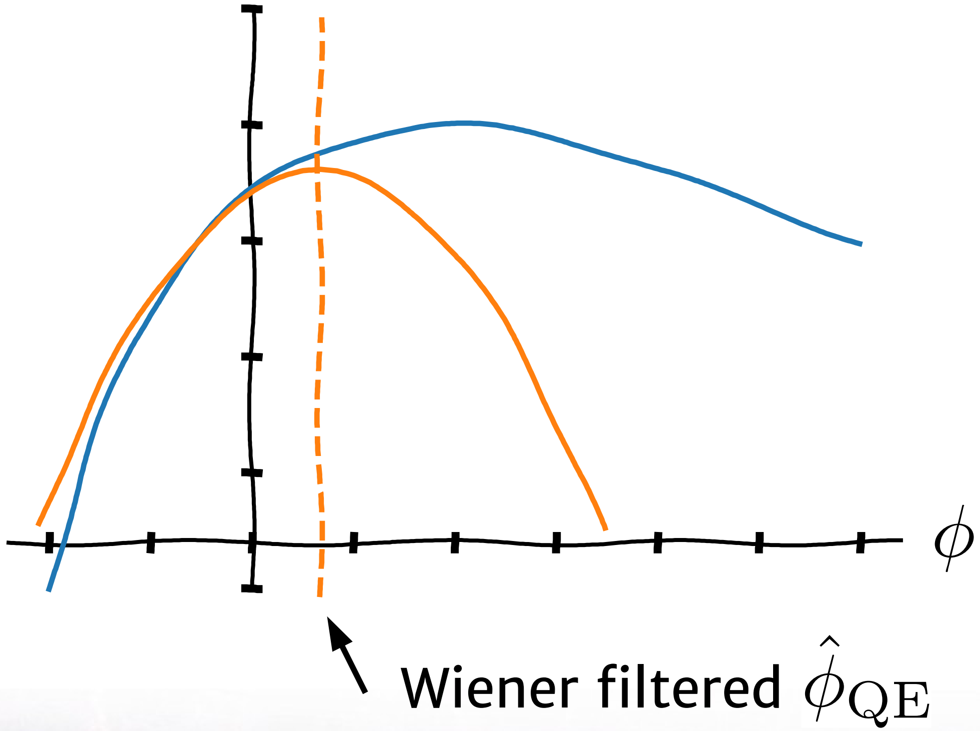

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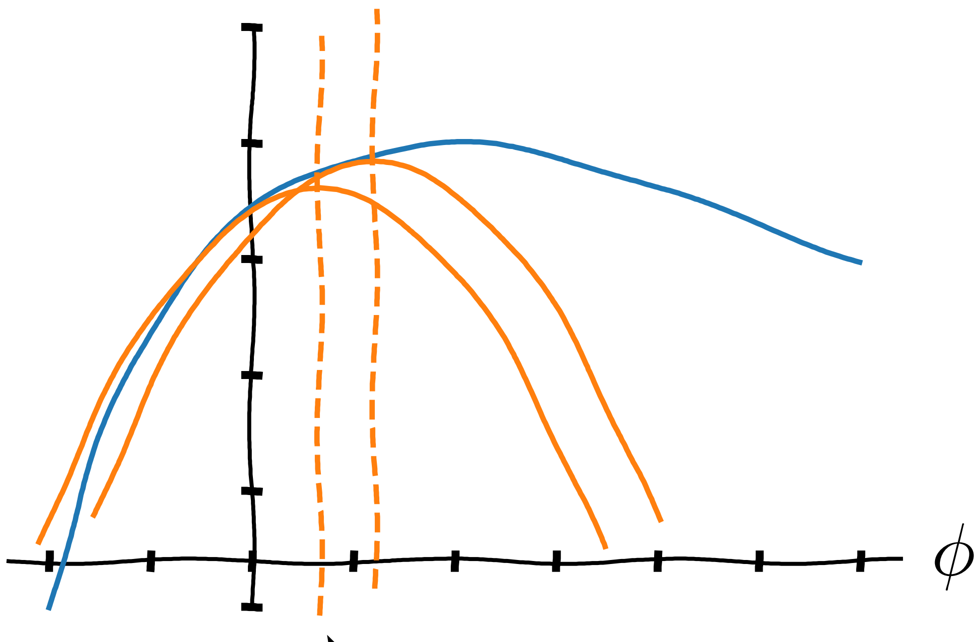

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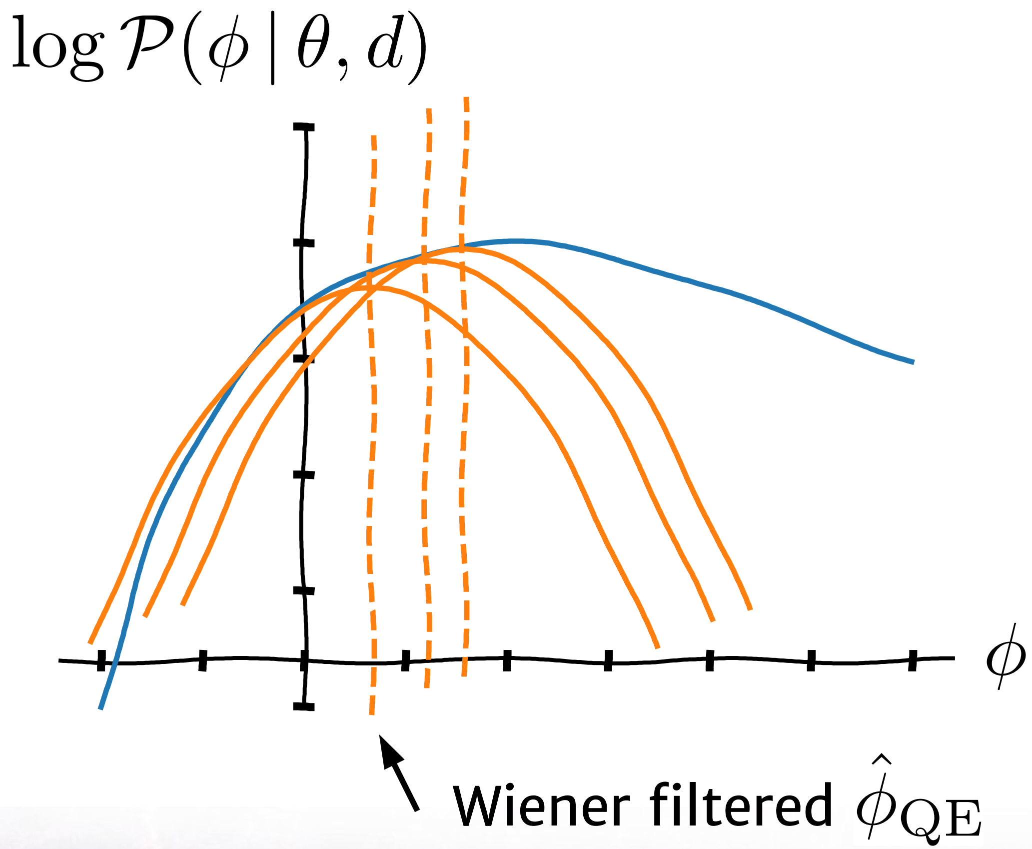

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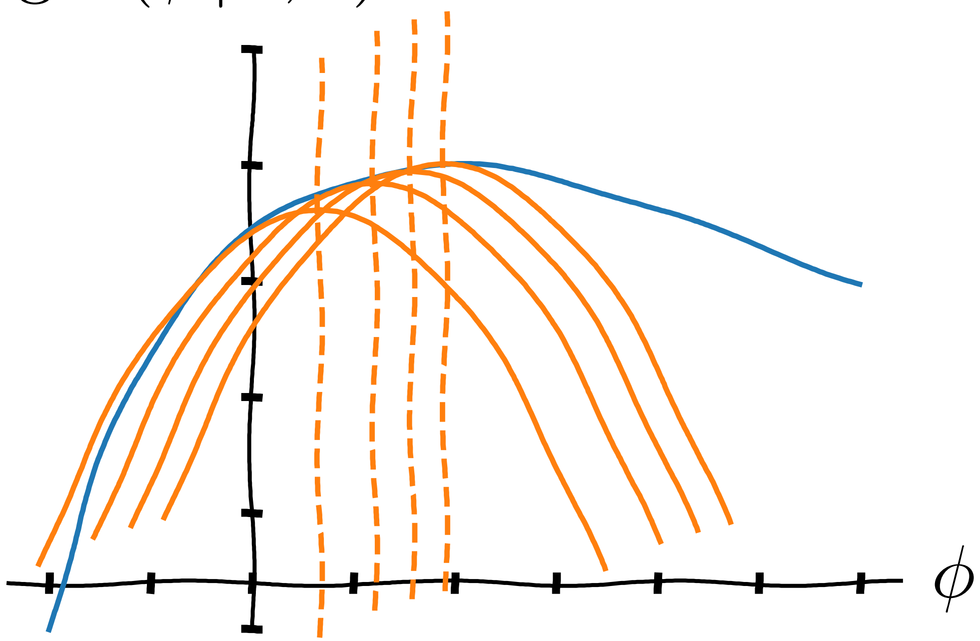

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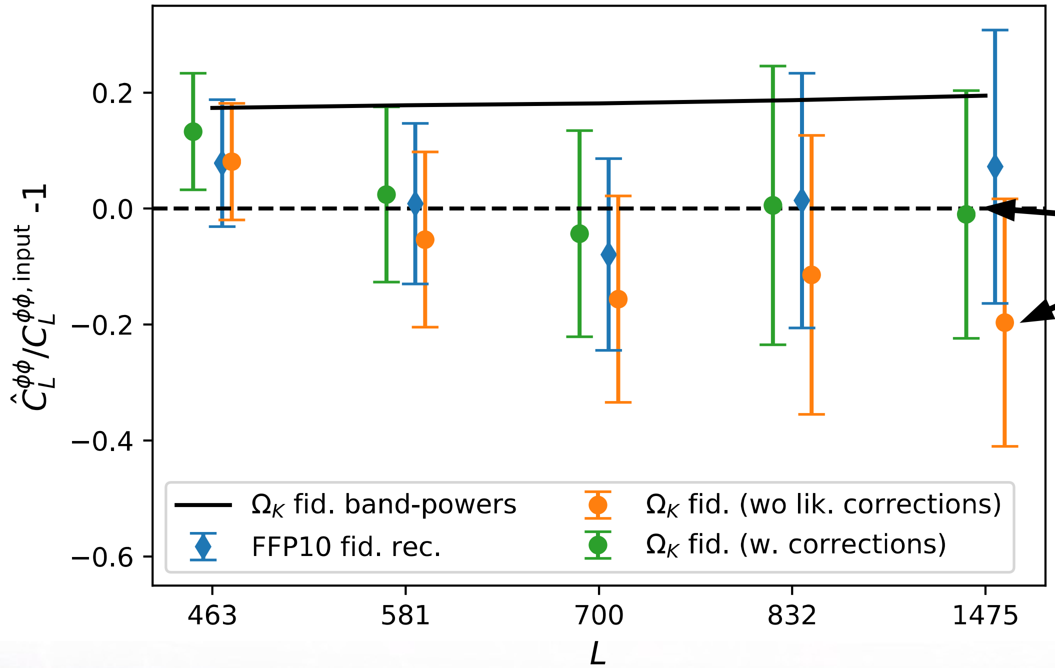

Shift due to accounting for cosmology dependence in Planck quadratic estimator analysis.

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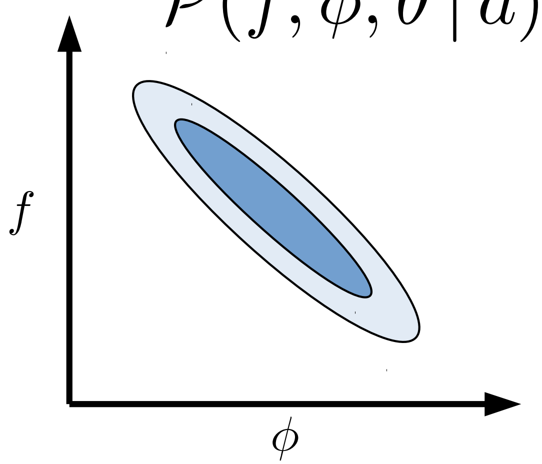

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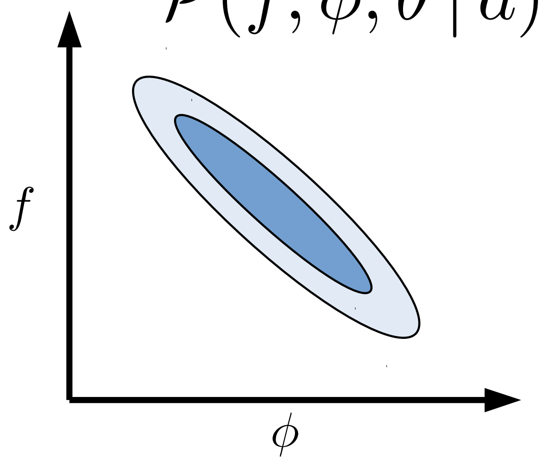

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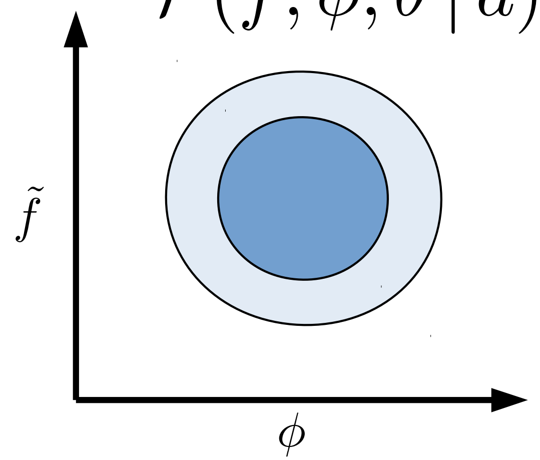

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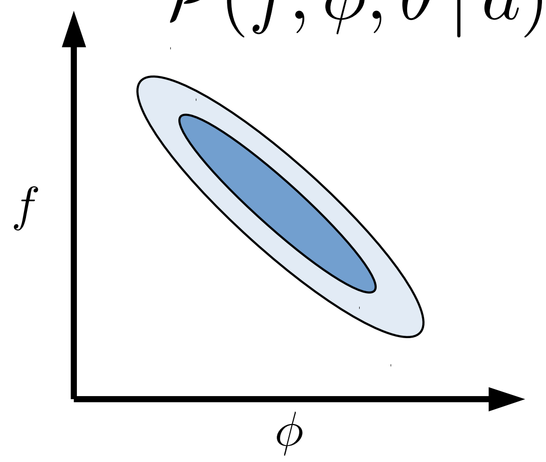
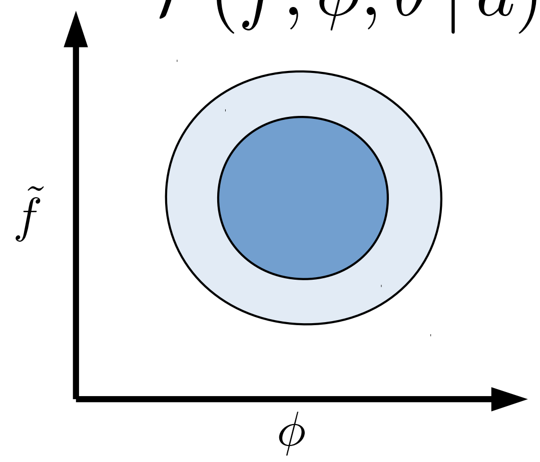


$$\mathcal{P}(\tilde{f}, \phi, \theta \,|\, d) = \mathcal{P}(f(\tilde{f}, \phi), \phi, \theta \,|\, d) \times \left|\det \mathcal{L}(\phi)\right|^{-1}$$



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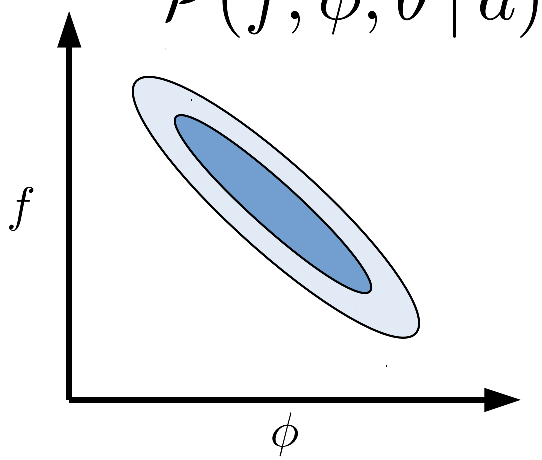

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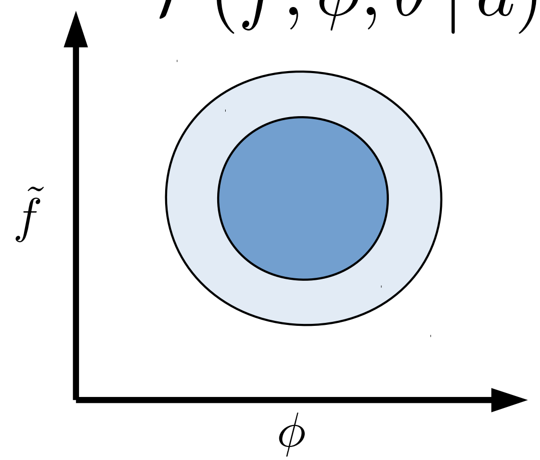

We need the lensing determinant, $\det \mathcal{L}(\phi)$ to do this change-of-variables.

What is the determinant of lensing?

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Consider the usual Taylor series lensing approximation:

$$\tilde{f}(x) = f(x + \nabla\phi(x)) \approx f(x) + \nabla f(x) \cdot \nabla\phi(x) + \ldots$$

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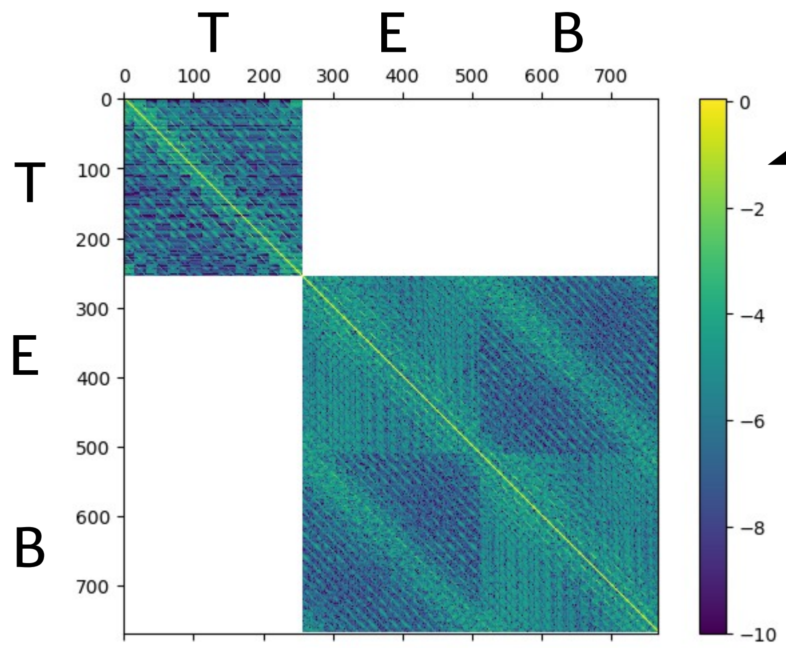

Matrix representation of $\mathcal{L}(\phi)$ for 16x16 1' pixel TEB maps for 7th order Taylor series approximation

$\log(\mathrm{abs}(\mathcal{L}(\phi)_{ij}))$

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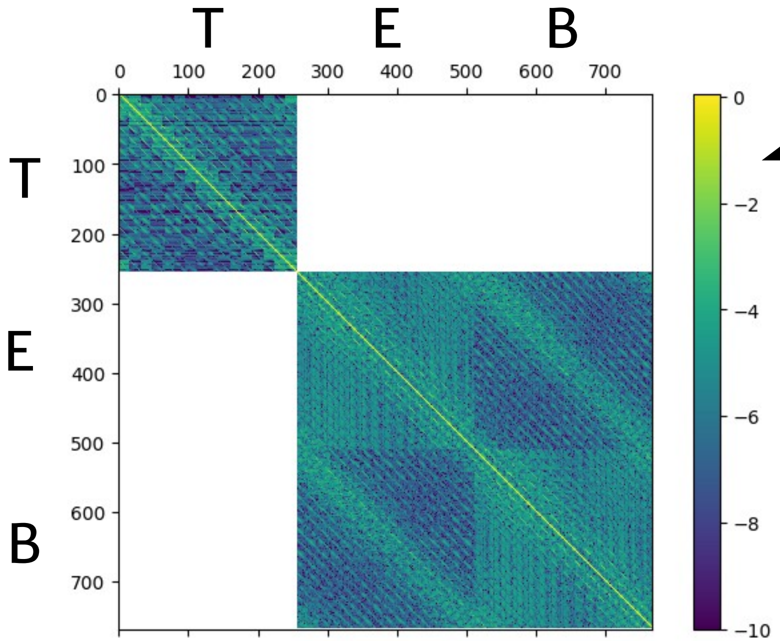

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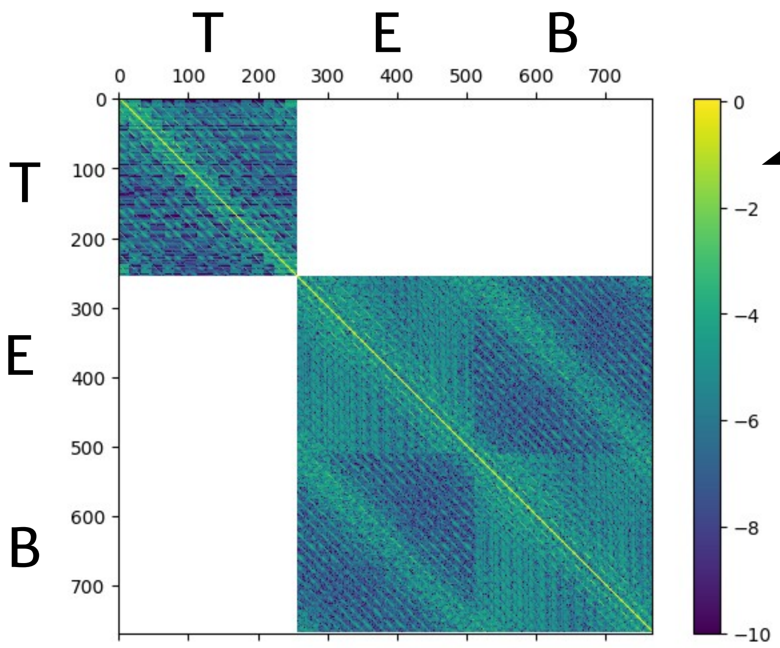

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Additionally, the variation of the determinant with is significant.

LenseFlow

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Define $f_t(x) \equiv f(x + t\nabla\phi(x))$

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Define $f_t(x) \equiv f(x + t\nabla\phi(x))$

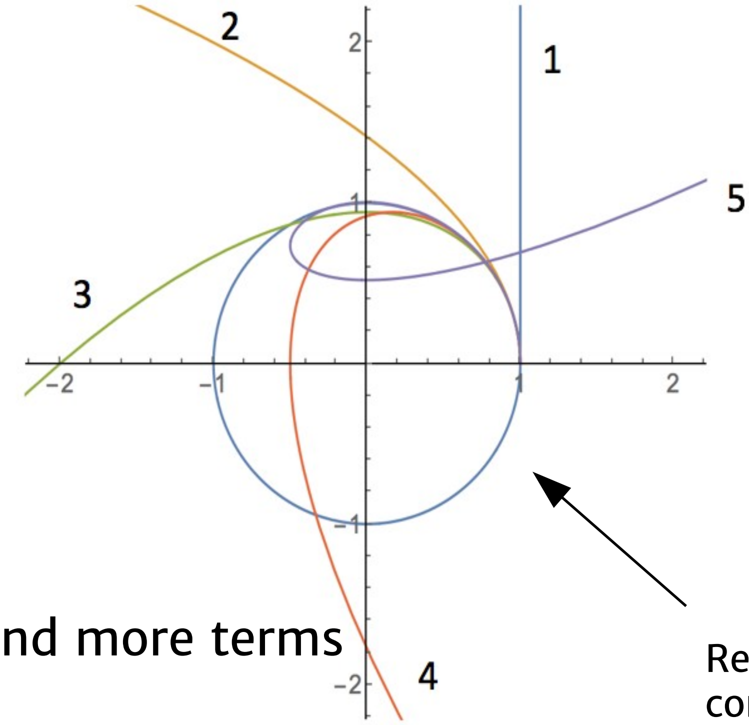
One can show f_t obeys an ODE "flow" equation

$$\frac{df_t(x)}{dt} = \nabla\phi(x) \cdot \left[\mathbb{1} + t\nabla\nabla\phi(x)\right]^{-1} \cdot \nabla f_t(x)$$

This allows easy inversion, gradients, transposes, and the determinant can be made arbitrarily close to 1.

LenseFlow Conceptually

Taylor series lensing



More and more terms

Remaining on-circle
corresponds to determinant=1

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix} = \begin{bmatrix} 1 - t^2/2 + \ldots \\ t - t^3/6 + \ldots \end{bmatrix}$$

LenseFlow Conceptually

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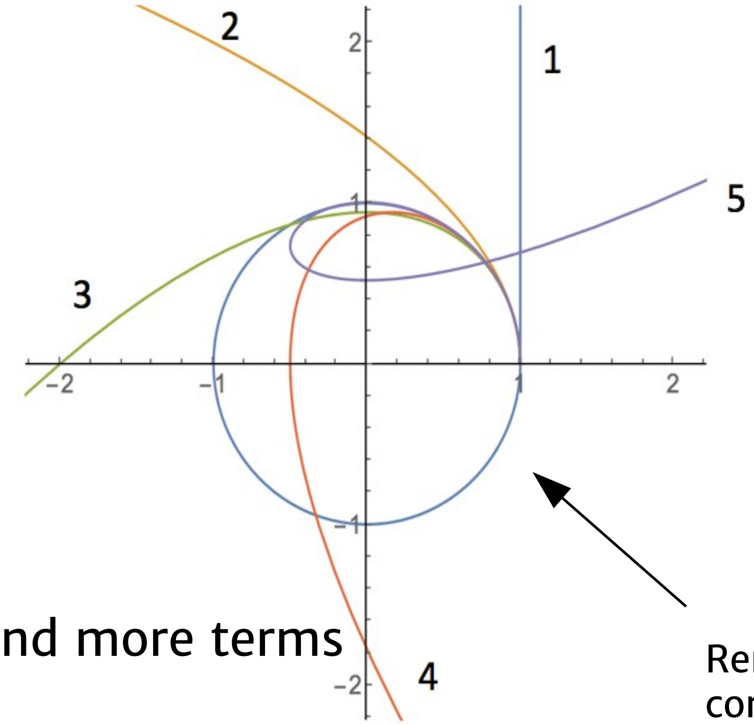

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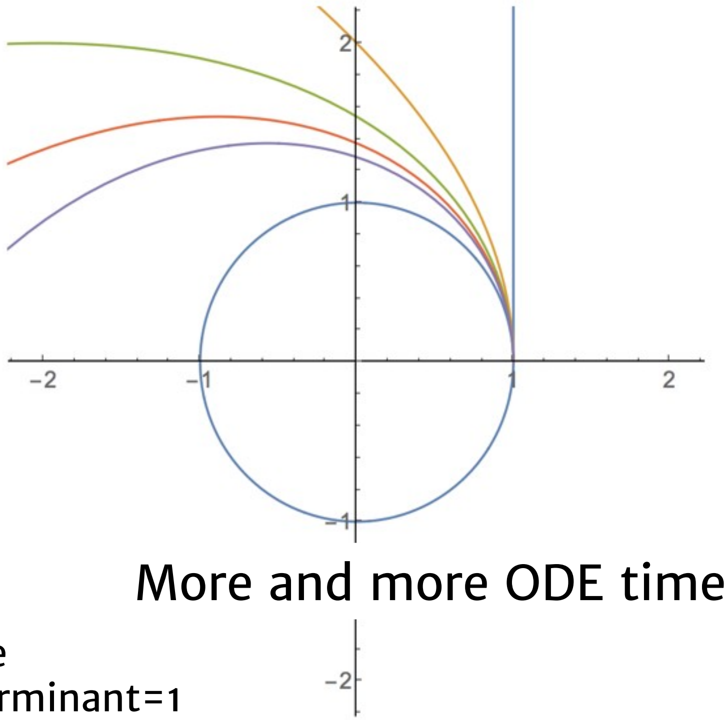

More and more ODE timesteps

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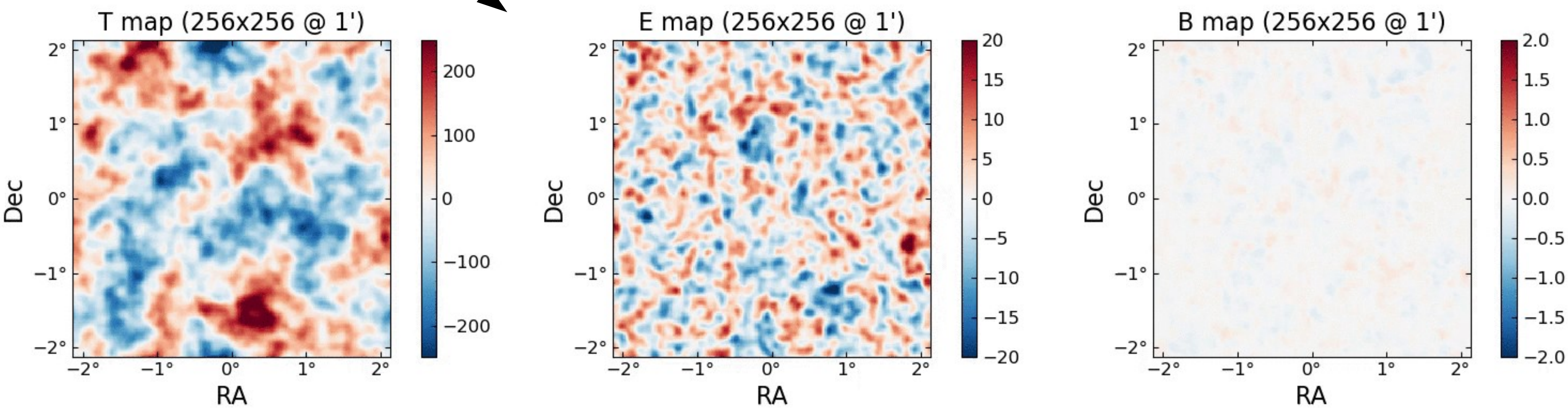
$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

LenseFlow In Action

f_t during ODE integration



LenseFlow In Action

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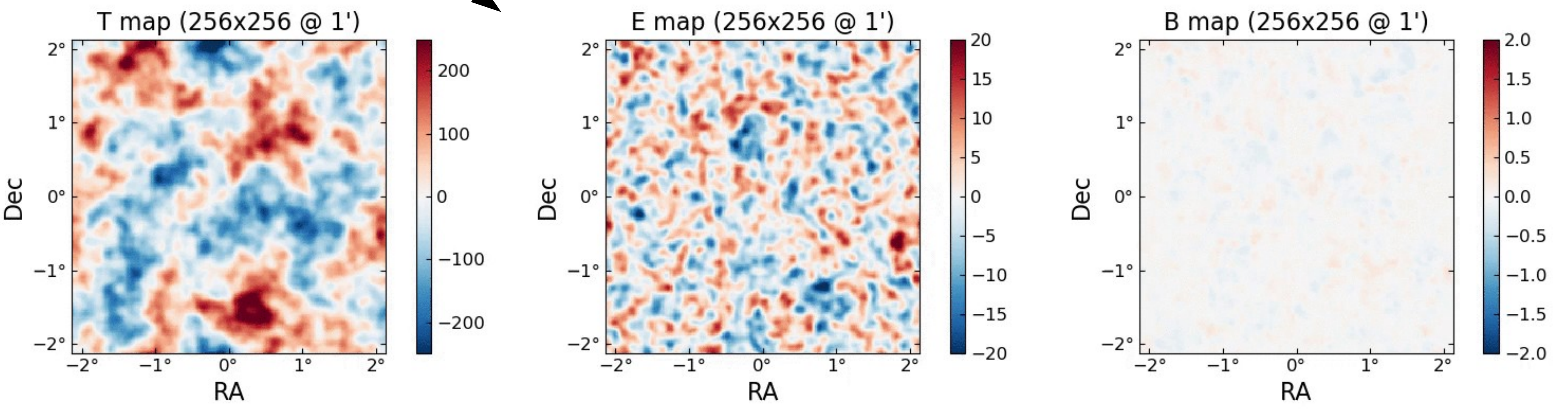
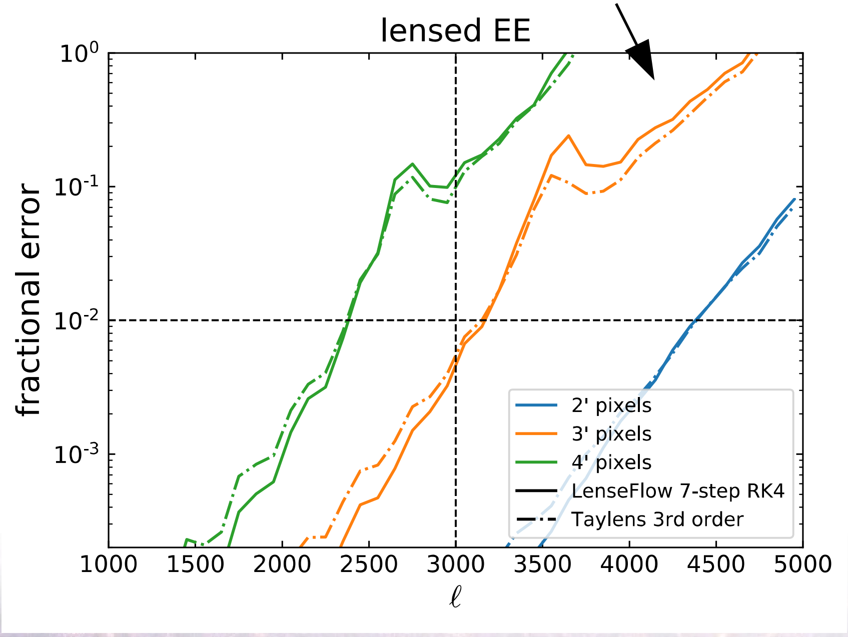


Errors are comparable to other methods



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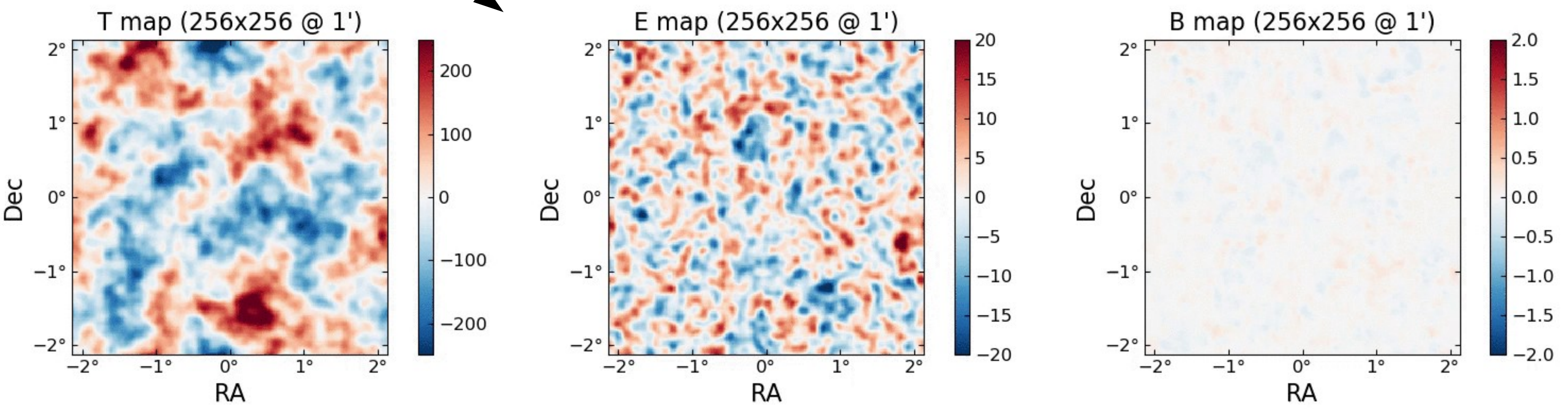

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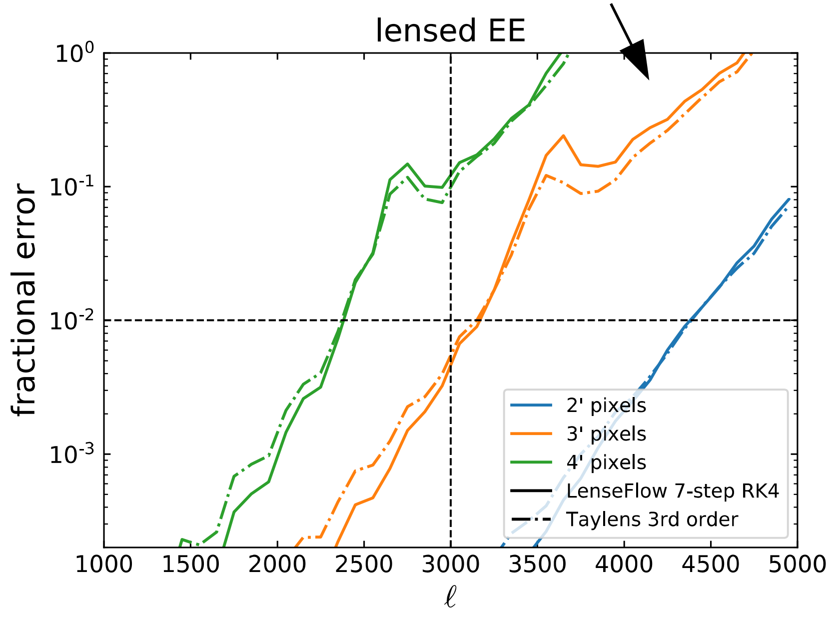
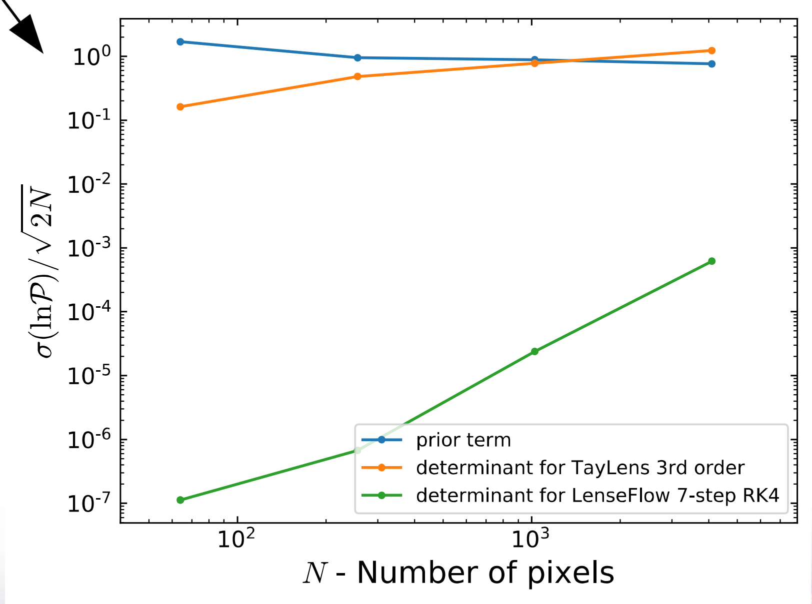


Determinant variation is negligible



With LenseFlow in hand, we can first maximize $\mathcal{P}(\tilde{f}, \phi \,|\, \theta, d)$ which we do in practice by coordinate descent:

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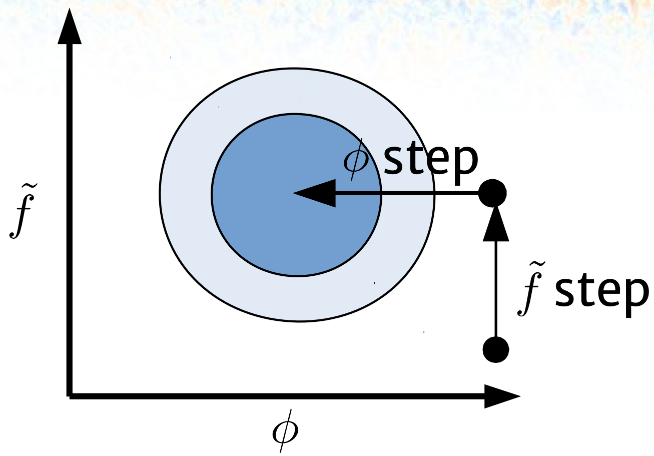

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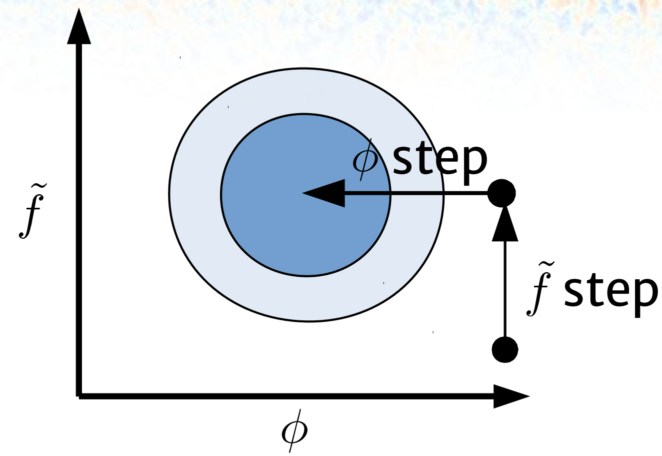

$$\mathcal{P}(\tilde{f}, \phi, \theta \,|\, d) =$$

$$= \exp\left\{ \underbrace{-\tfrac{1}{2}[d - \tilde{f}]^\dagger \mathcal{C}_n^{-1}[d - \tilde{f}] \rlap{$\overbrace{\phantom{- \tfrac{1}{2}\tilde{f}^\dagger[\mathcal{L}(\phi)\mathcal{C}_f\mathcal{L}(\phi)^\dagger]^{-1}\tilde{f} - \tfrac{1}{2}\phi^\dagger \mathcal{C}_\phi \phi}}^{\phi\text{ step}}$} - \tfrac{1}{2}\tilde{f}^\dagger[\mathcal{L}(\phi)\mathcal{C}_f\mathcal{L}(\phi)^\dagger]^{-1}\tilde{f}}_{\tilde{f}\text{ step : a Wiener filter}} - \tfrac{1}{2}\phi^\dagger \mathcal{C}_\phi \phi \right\}$$

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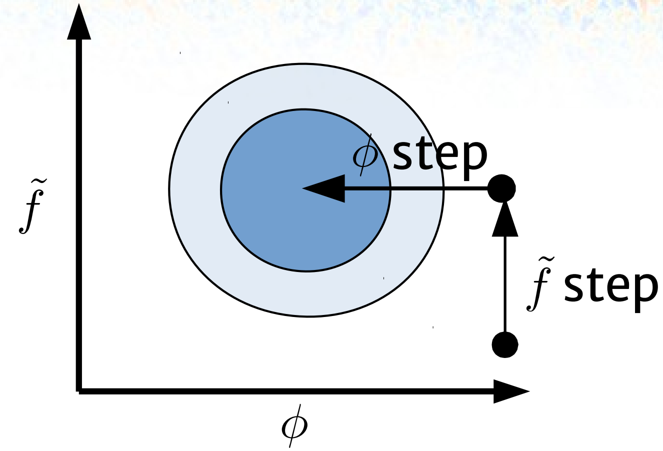

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ϕ step

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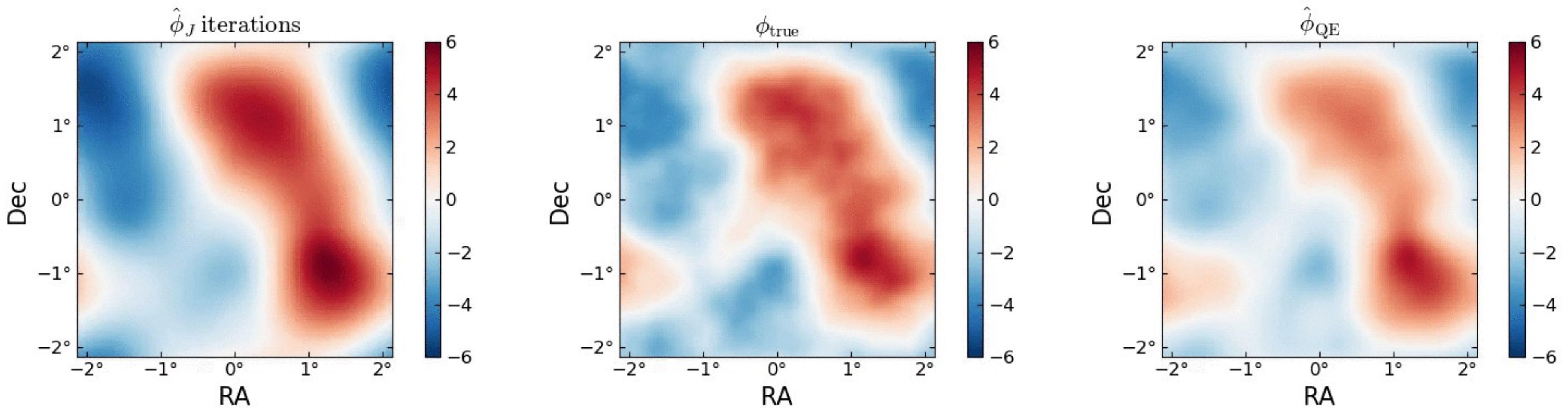
$\tilde f$ step : a Wiener filter



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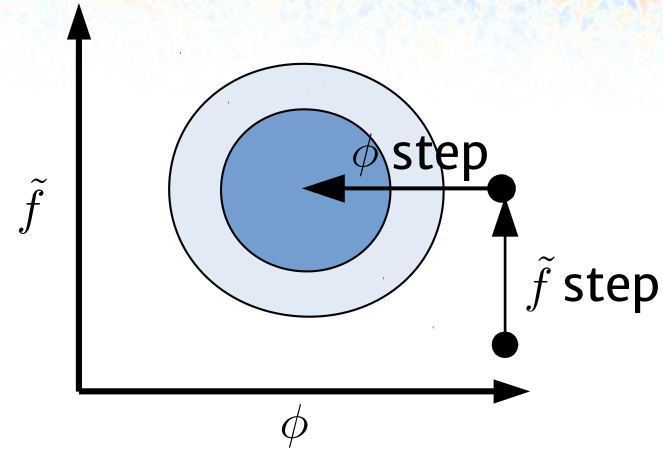

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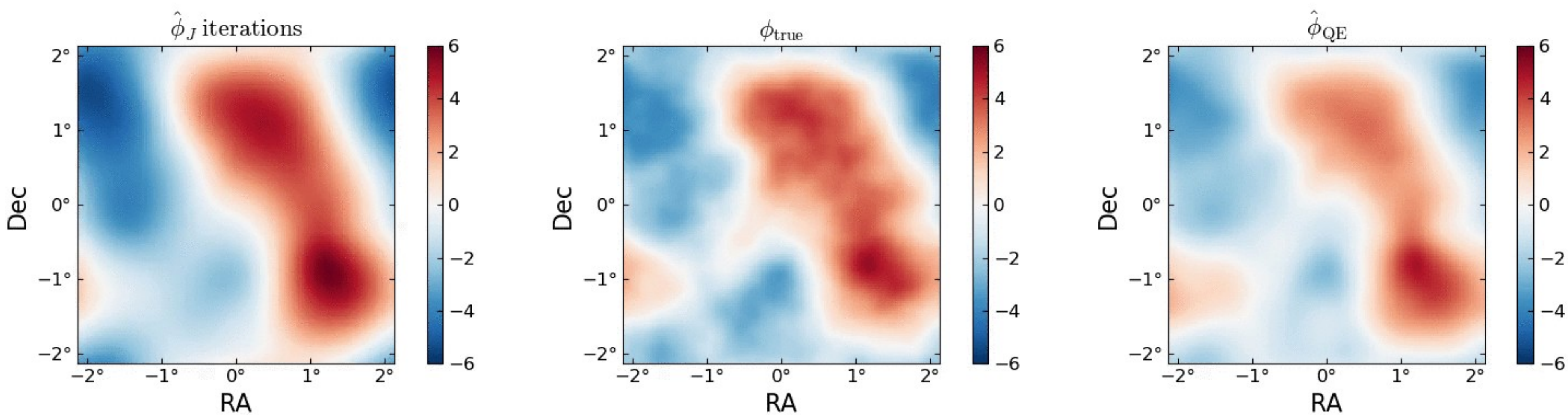

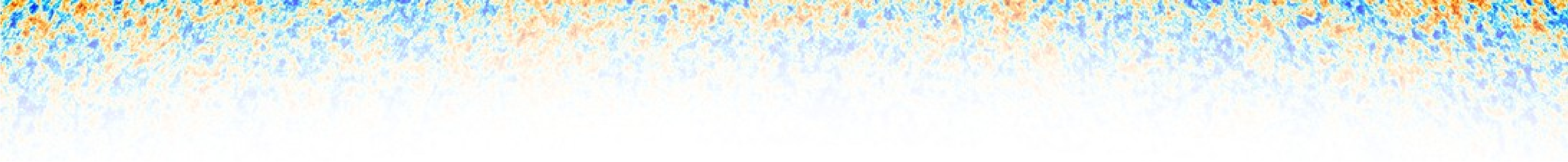

In terms of sampling, the problem breaks up similarly nicely:

$$\text{Gibbs}\begin{cases} \tilde{f} \sim \mathcal{P}(\tilde{f}\,|\,\phi,\theta,d) \\ \phi \sim \mathcal{P}(\phi\,|\,\tilde{f},\theta,d) \\ \theta \sim \mathcal{P}(\theta\,|\,\tilde{f},\phi,d) \end{cases}$$

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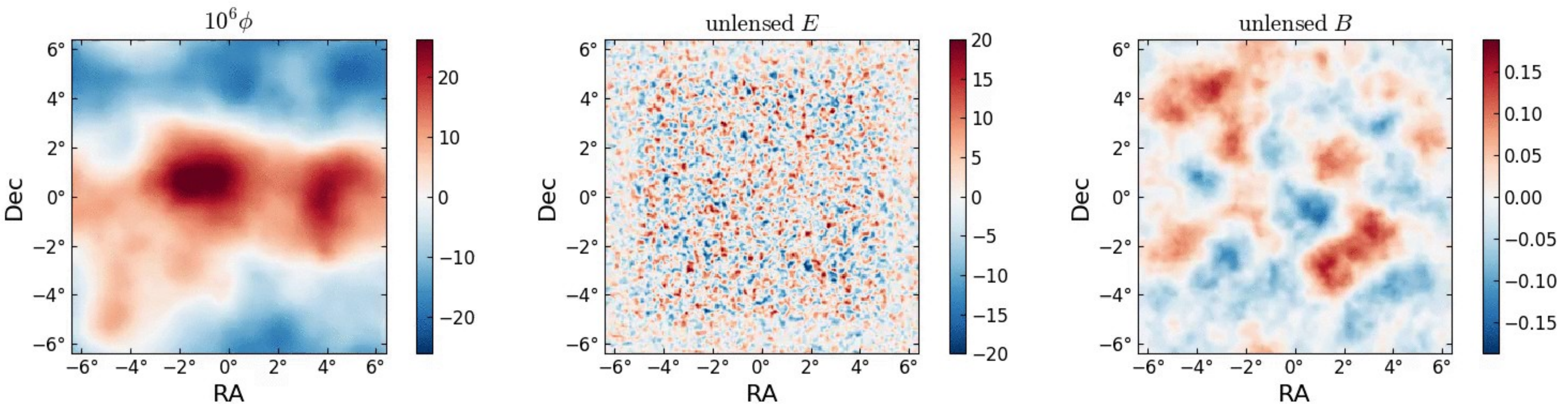
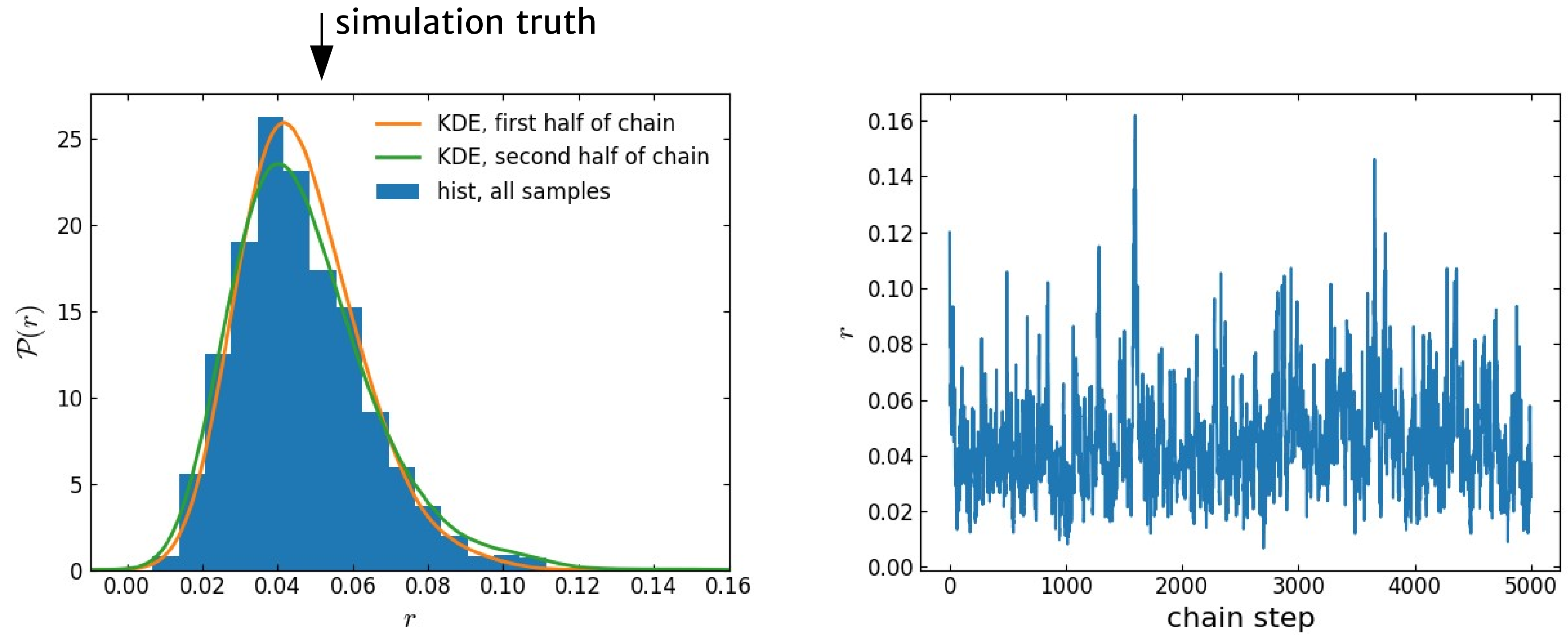


Samples from the posterior of ϕ and unlensed E&B.

r=0.05, EB data, 1μk-arcmin (isotropic, w/ knee), 3' beams

Allowing r to vary in the chain:



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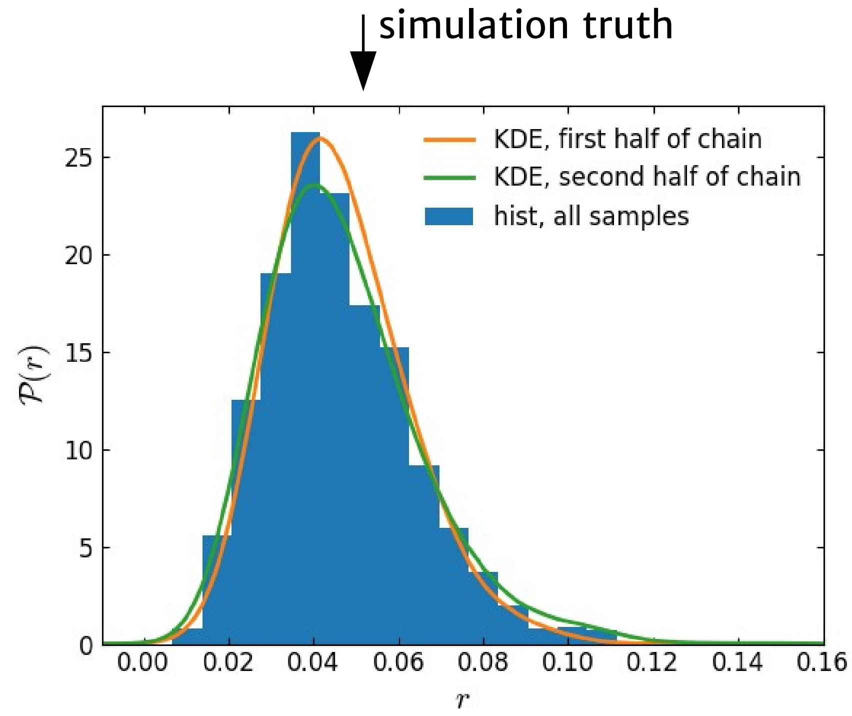
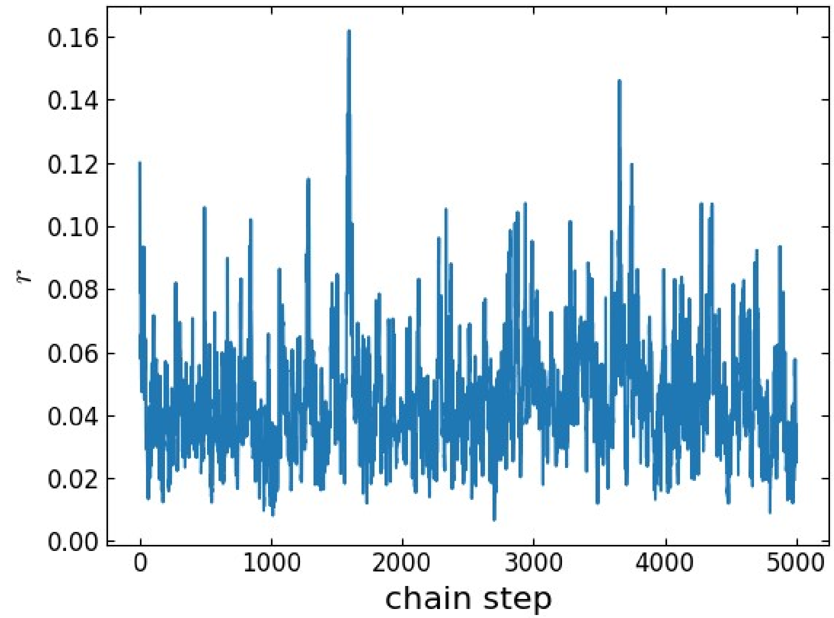
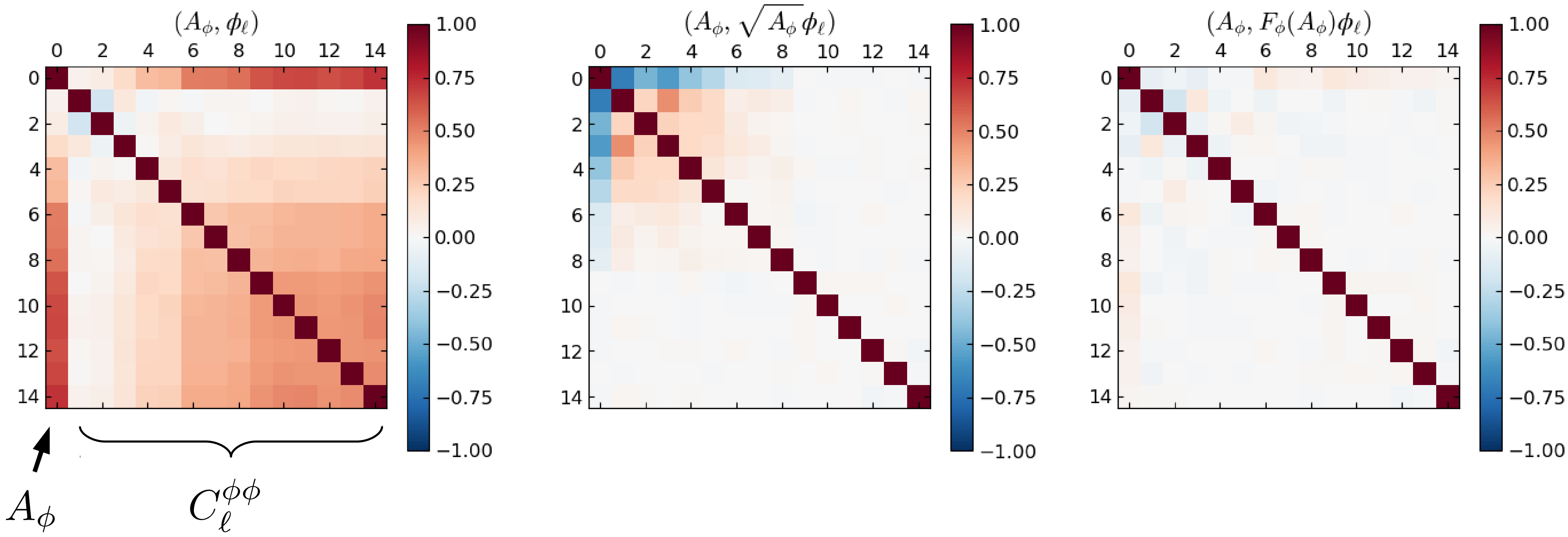




Stepping back and looking at these results:

- Gradient approximation *not* assumed
- No bias terms or covariances needed to be calculated (and none were ignored)
- Ongoing work comparing to Fisher forecasts (see MM+2018 in prep)

We can sample other parameters besides r, for example A_ϕ



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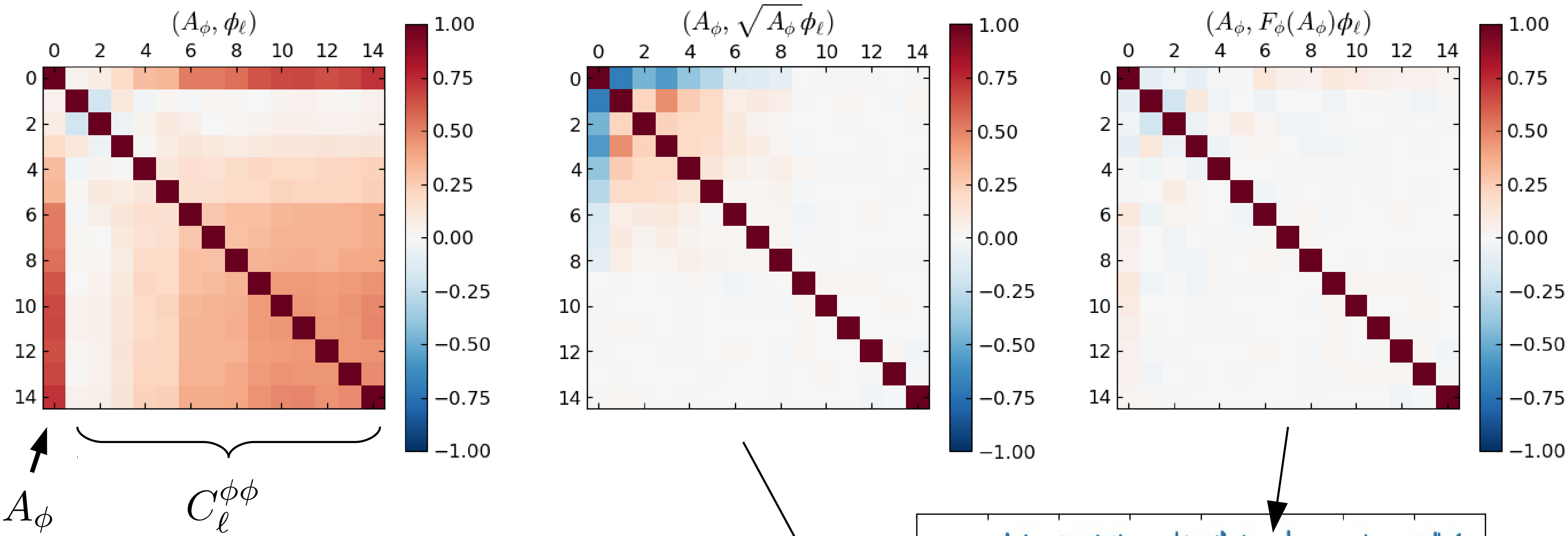

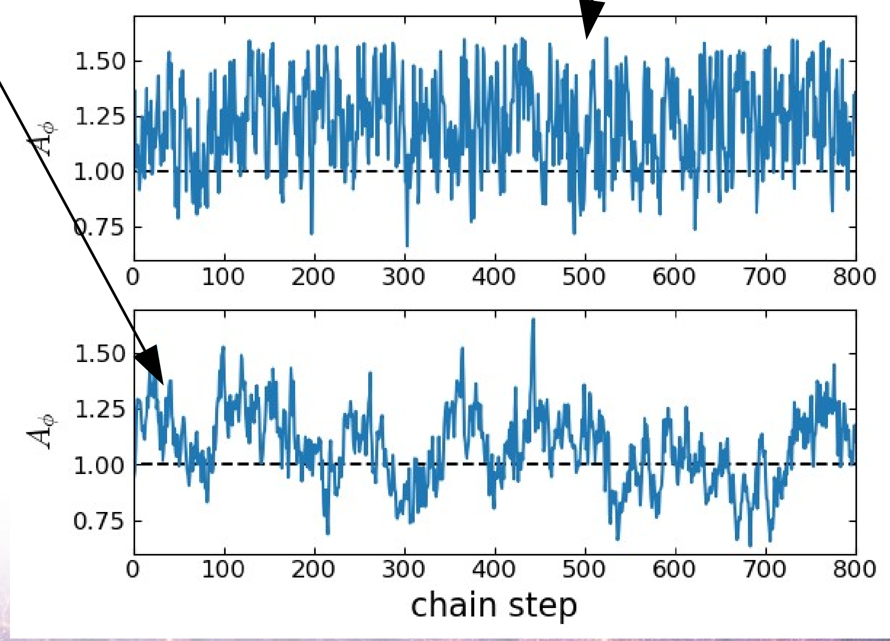

Using a reparametrization inspired by Racine et al. (2016), we can massively decorrelate the chain and improve Gibbs convergence.

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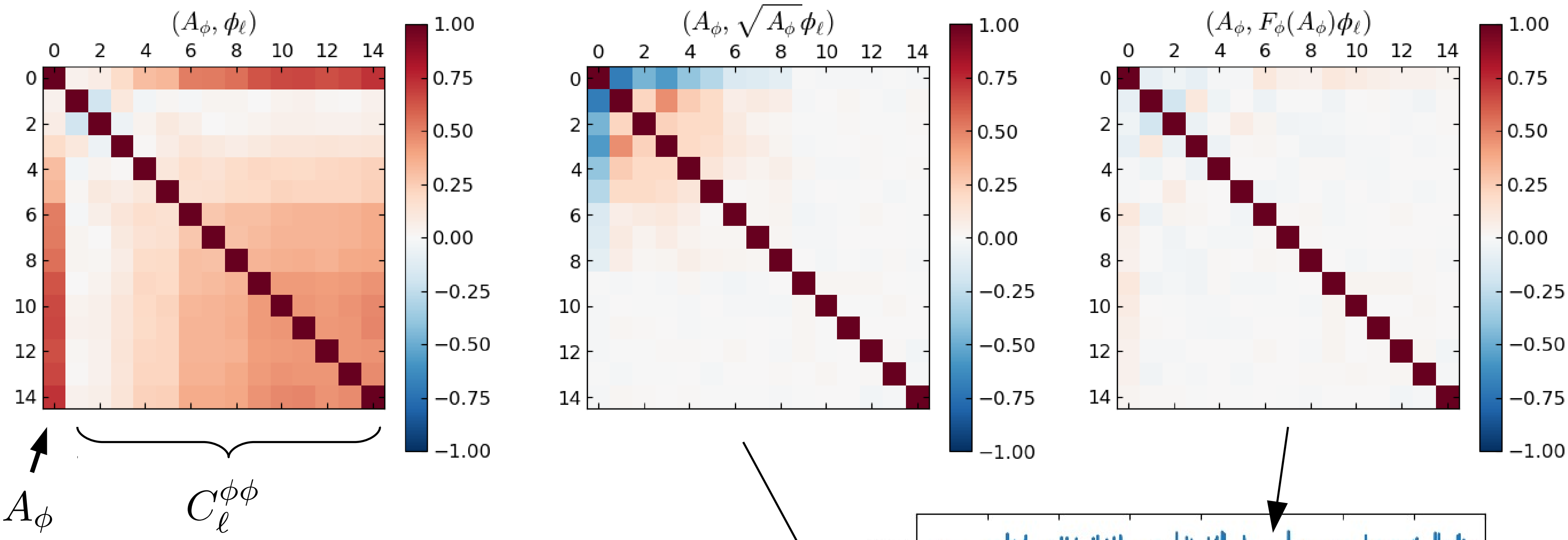

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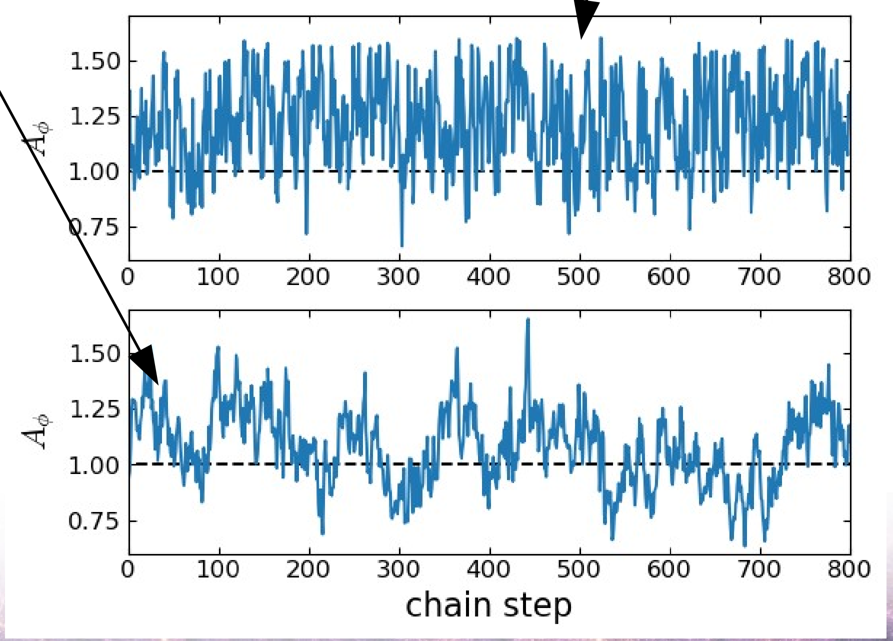

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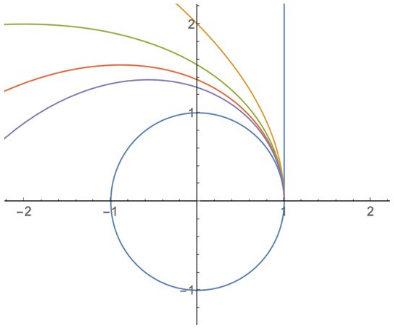
Conclusions and future work

LenseFlow is a new tool in
the cosmologist's toolbox



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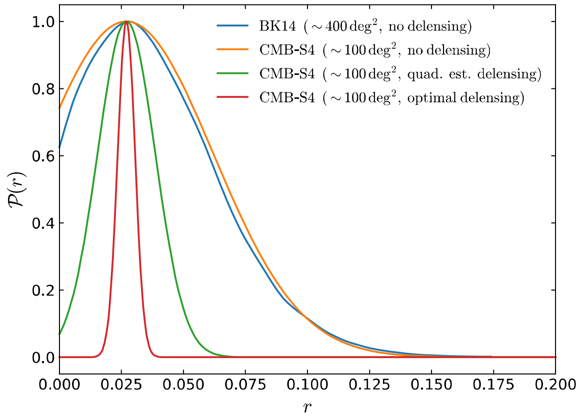

We're currently working on
applying to data from the
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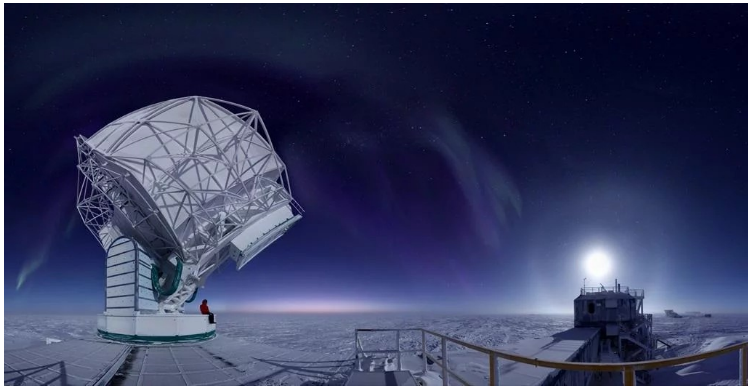

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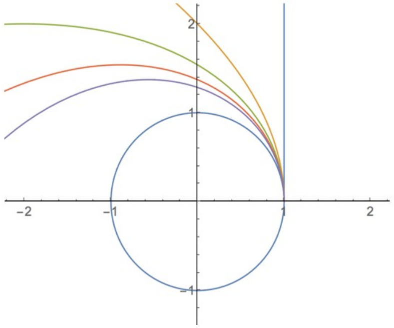

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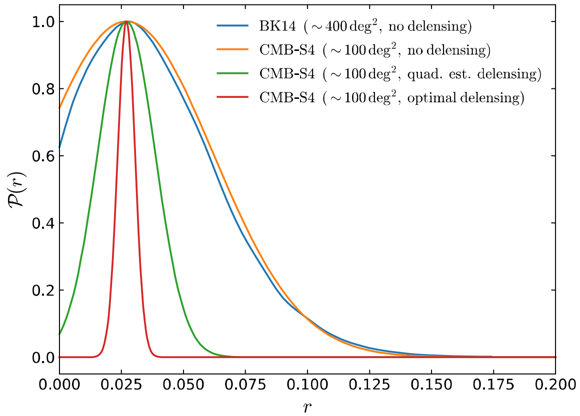

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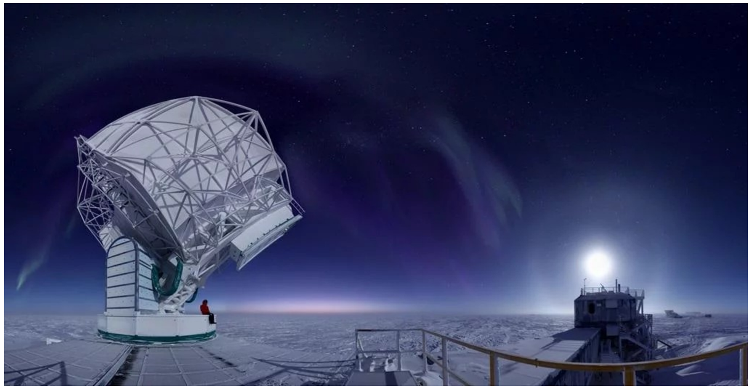

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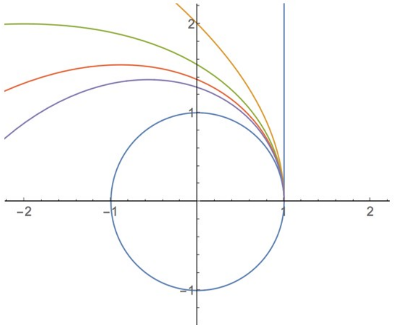

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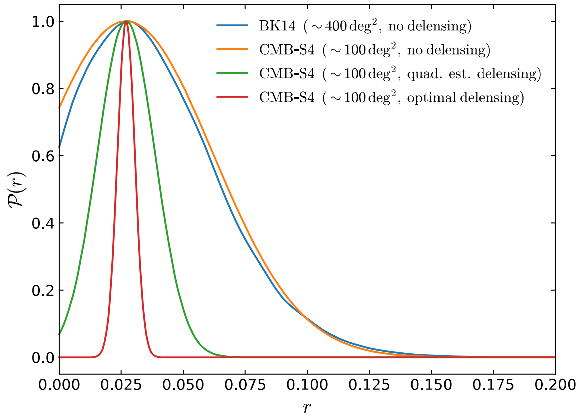

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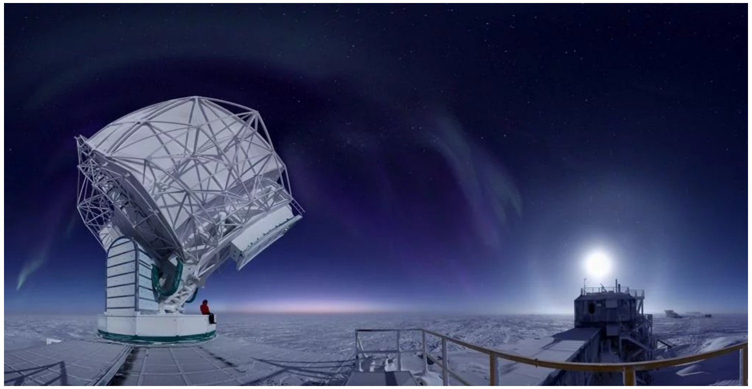

- Future challenges, like including foregrounds, non-Gaussianities, and post-Born effects…
- Check out our code and run a sample Jupyter notebook in your browser:
  https://github.com/marius311/CMBLensing.jl julia