

The Non-Gaussian Likelihood of WL Correlation Functions

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*This work is currently under the internal
review of the LSST DESC collaboration.

Likelihood analysis

Likelihood function

- Bayes' rule:

$$P(\vec{\pi} \mid \vec{\xi}) = \frac{P(\vec{\xi} \mid \vec{\pi}) P(\vec{\pi})}{N}$$

- Gaussian Likelihood function:

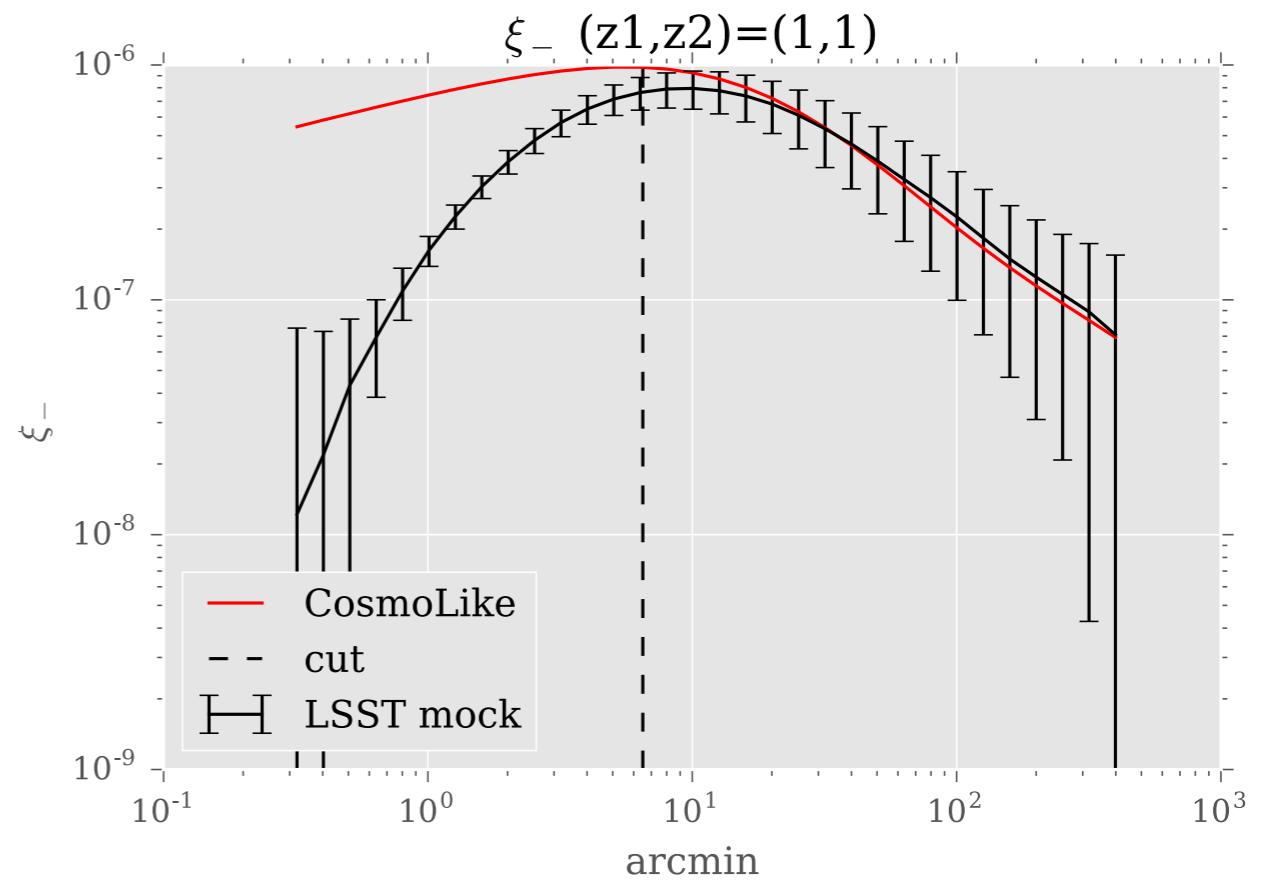
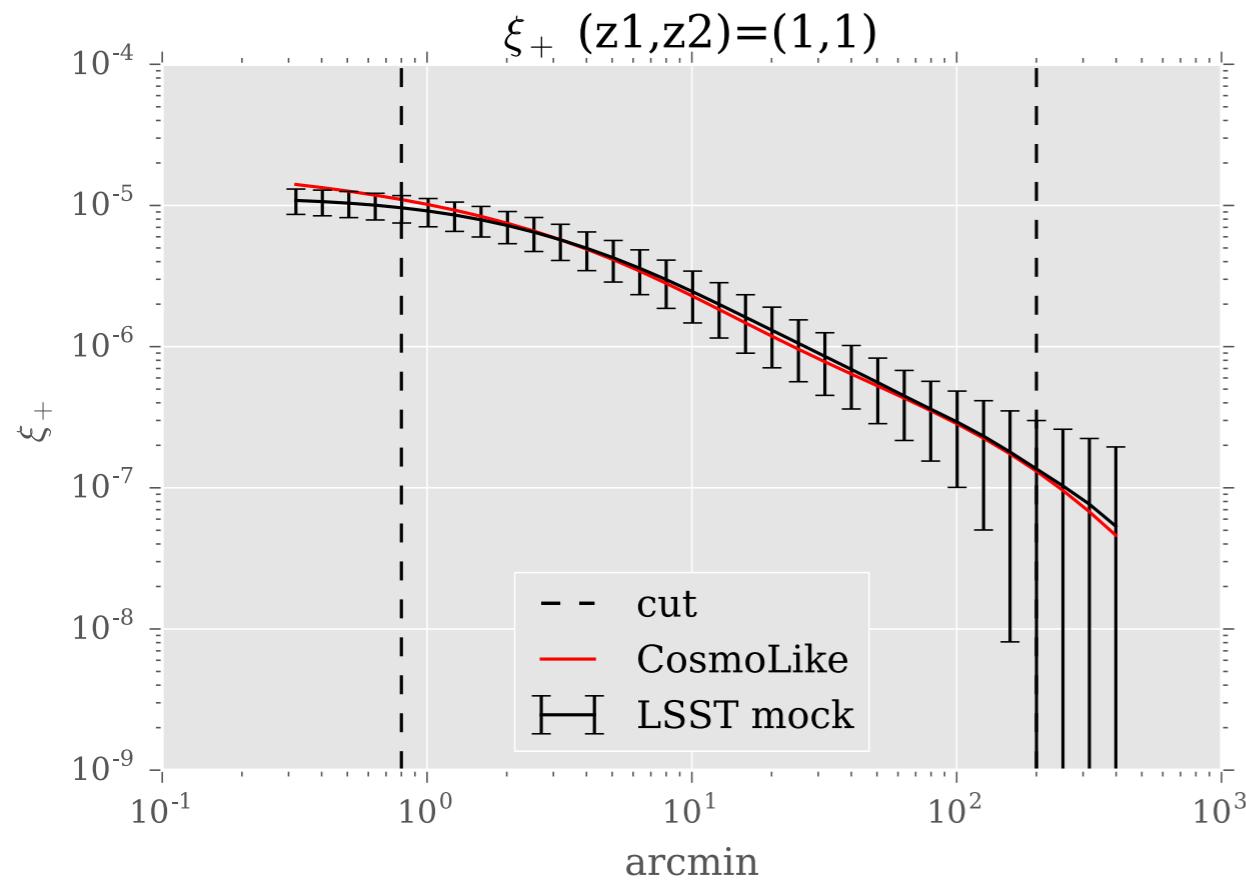
$$P(\vec{\xi} \mid \vec{\pi}) = A \cdot e^{-\frac{1}{2}(\vec{\xi} - \vec{\xi}_\pi)^T C^{-1} (\vec{\xi} - \vec{\xi}_\pi)}$$

- Assumptions in standard likelihood analysis:

Gaussian likelihood assumption

Simulations

- Scinet Light Cone Simulations (**SLICS**) Covered in Joachim's talk yesterday
- 932 lines-of-sight realizations of **100 deg²**
- N-body simulation: 1536^3 particles in a box size of 505 Mpc h^{-1}
- Reproduce redshift distribution of LSST (10 tomographic redshift bins with $n_{\text{gal}} = 2.6 \text{ gal/arcmin}^2$ in each bin).
- Intrinsic shape $\sigma_e = 0.29$ (shape noise can be switched off)

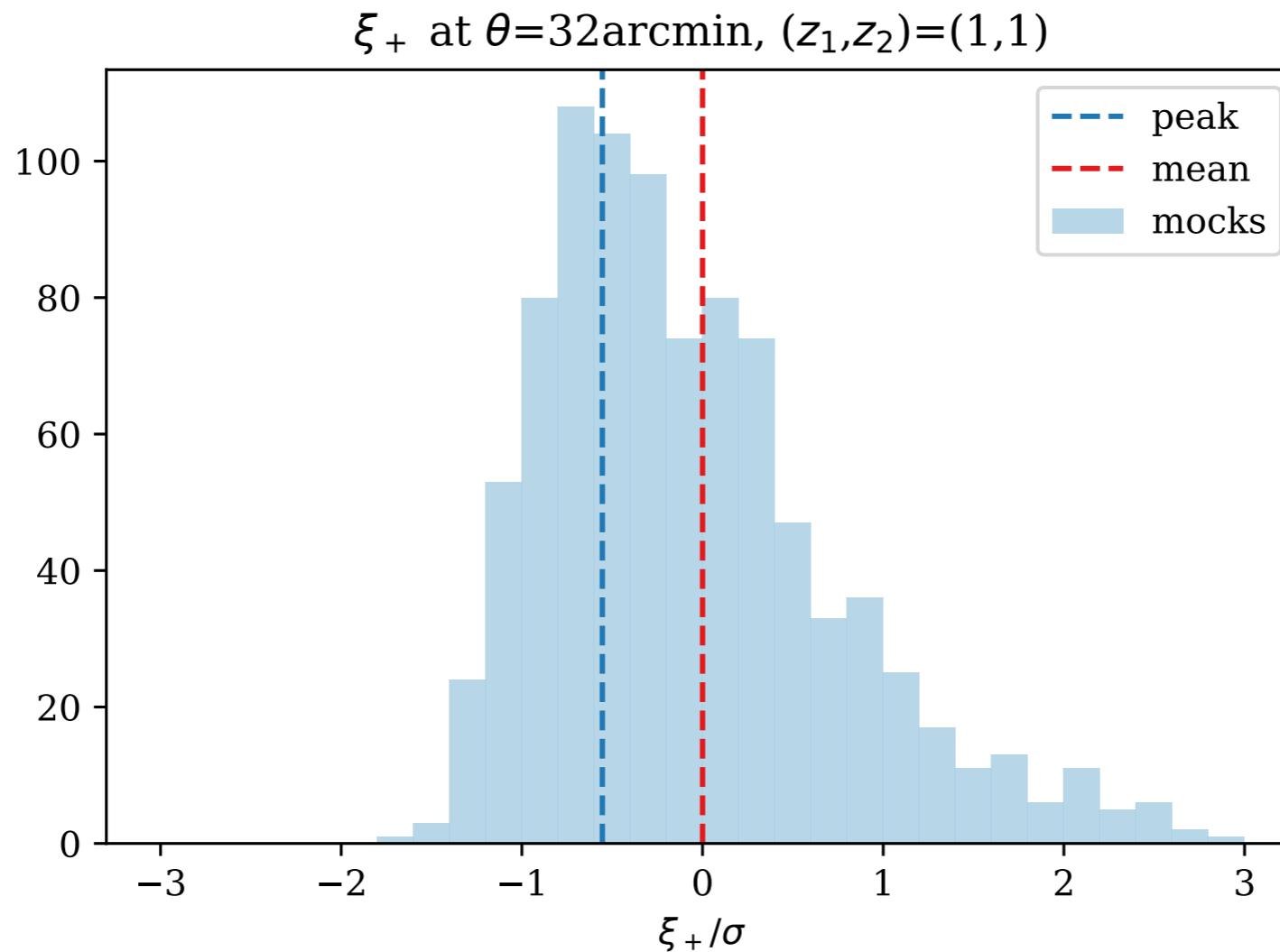


- To avoid biases due to the mismatch between the mocks and the theory, we rescale the mock ξ data vector with compensating ratios: $\xi_{\text{theory}} / \langle \xi_{\text{mocks}} \rangle$

Theory curves from CosmoLike:
 Krause, E. and Eifler, T., MNRAS 2017 stx1261

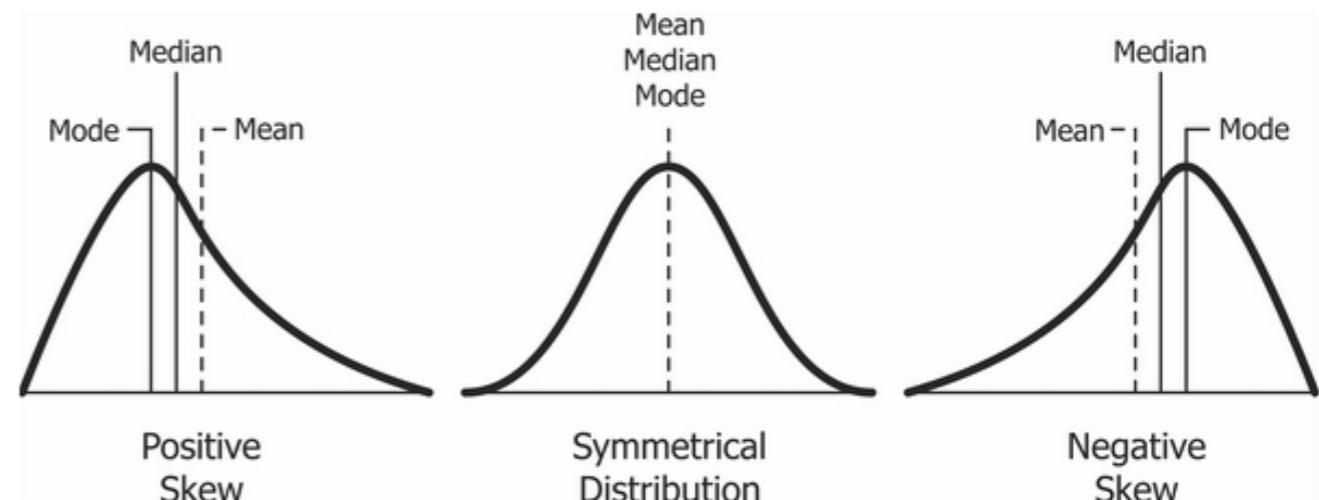
1-D likelihood distributions

- 1-D histograms



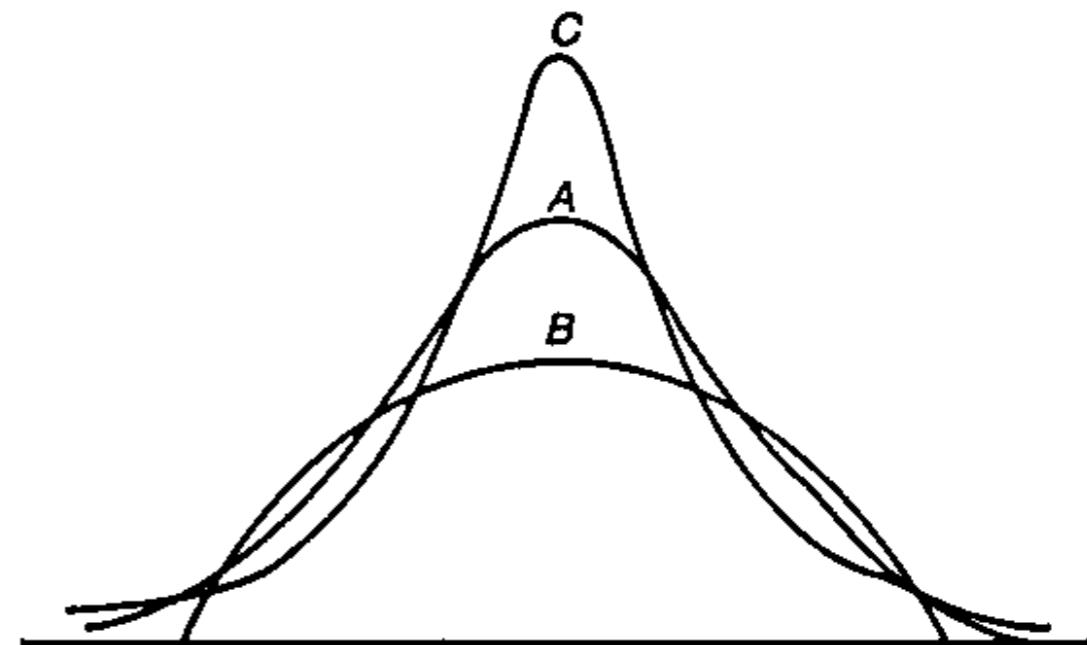
Skewness (asymmetry)

$$\text{Skew}[X] = \frac{\langle (X - \mu)^3 \rangle}{\sigma^3}$$



Kurtosis (tails, outliers)

$$\text{Kurt}[X] = \frac{\langle (X - \mu)^4 \rangle}{\sigma^4} - 3$$



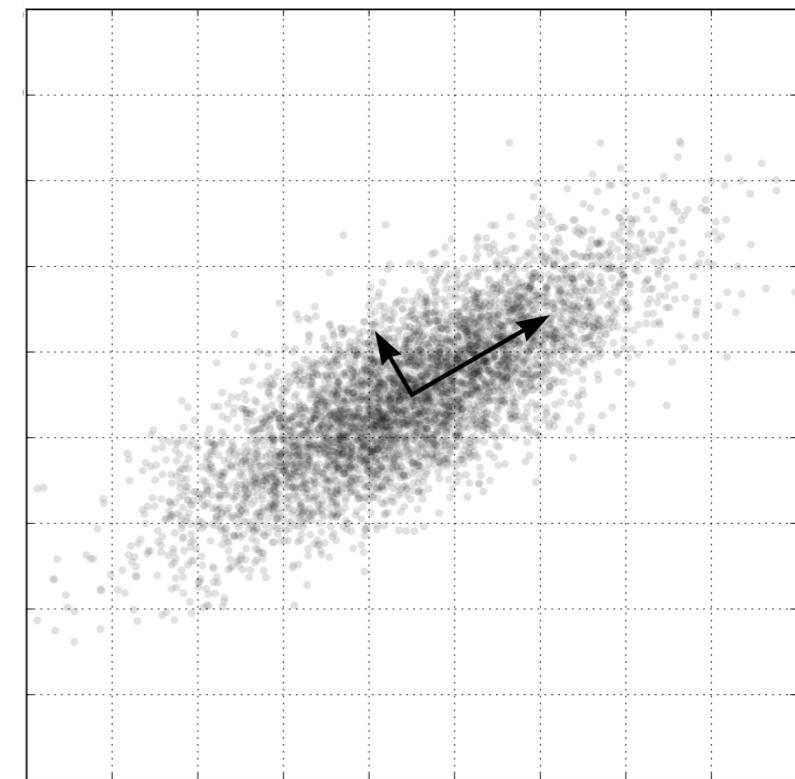
- The plots show the statistically significant non-zero skewness and kurtosis in marginal likelihoods of ξ .

Note that these marginal 1D likelihoods of $\xi \pm$ values do not fully represent the level of non-Gaussianity in the multivariate observable space.

- Data with vs. without shape noise
- The results of selected tomographic bins show the redshift evolution of skewness and kurtosis. Non-Gaussian features decrease as redshift increase.

PCA and MV non-Gaussian likelihood

- Difficulty:
High Dimensionality of the data vectors ~ 770
Number of LOS ~ 932
- **PCA** is a transformation that transforms data points into linearly uncorrelated coordinates.
- Apply Principal Component Analysis (PCA) to approximate the joint distribution as multiple 1-d distributions.
- Model the 1-d distribution with parametric models: **Gaussian, Edgeworth**
- non-parametric multivariate models: kNN, spectral series



Conclusions

- We found strong non-Gaussianity in marginal distributions of ξ .
- We build non-Gaussian multivariate likelihood distributions via PCA. But we do not detect a difference between a Gaussian model and more complicated models.
- We do not detect strong biases in Ω_m and s_8 in the Maximum Likelihood Fitting method or MCMC chains.
- Sims:100 deg² / LSST: 18k deg²

Since the mean-mode difference scales with the survey area as $(\tilde{\xi} - \bar{\xi})/\sigma \propto f_{\text{sky}}^{-1/2}$, the biases would be even smaller for LSST.

- Our results suggest that neglecting the non-Gaussianity of the likelihood for shear-shear correlations is not a significant source of bias for ongoing surveys or even future ones such as LSST.