Disconnected Covariance of 2-point Functions in Large-Scale Structure

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Messages

Accurate, **efficient**, and **general** method to **analytically** quantify the **disconnected** (“Gaussian”) covariance of LSS 2-point functions, taking into account the **window** effect.

- Accurate and efficient: tested against mocks; taking minutes on a single core
- Generality: cross correlation, joint probe, 2D & 3D, configuration & Fourier
- Analytical: noiseless (easy to invert), and can use best-fit cosmology/model

Code available at https://github.com/eelregit/mcfit and https://github.com/eelregit/covdisc

What is “disconnected” covariance?

- “Gaussian” ≠ “Disconnected” because tracers are discrete
- Two ways to decompose your covariance
  - Statistically, by cumulant expansion: disconnected (nearly diagonal / full-rank) vs connected (smooth and low-rank, Harnois-Déraps & Pen 12)

Conn. cov. kicks in;
Window is negligible

Disc. cov. dominates;
Window is important

small scale → large scale

  - Theoretically, as a doubly stochastic process: point process (Poisson, ...) sampling a continuous one (**Gaussian** and higher-order stats)
- The conventional terminology “Gaussian covariance” can be confusing, e.g. both disc. and conn. have Gauss+Poisson contributions

Approaches

Internal (from data, e.g. subsample, jackknife, bootstrap)

- Pros
  - Has everything (right cosmology, physic, systematics)
- Cons
  - Lose large-scale modes
  - Noisy, need to inflate the errors on parameters

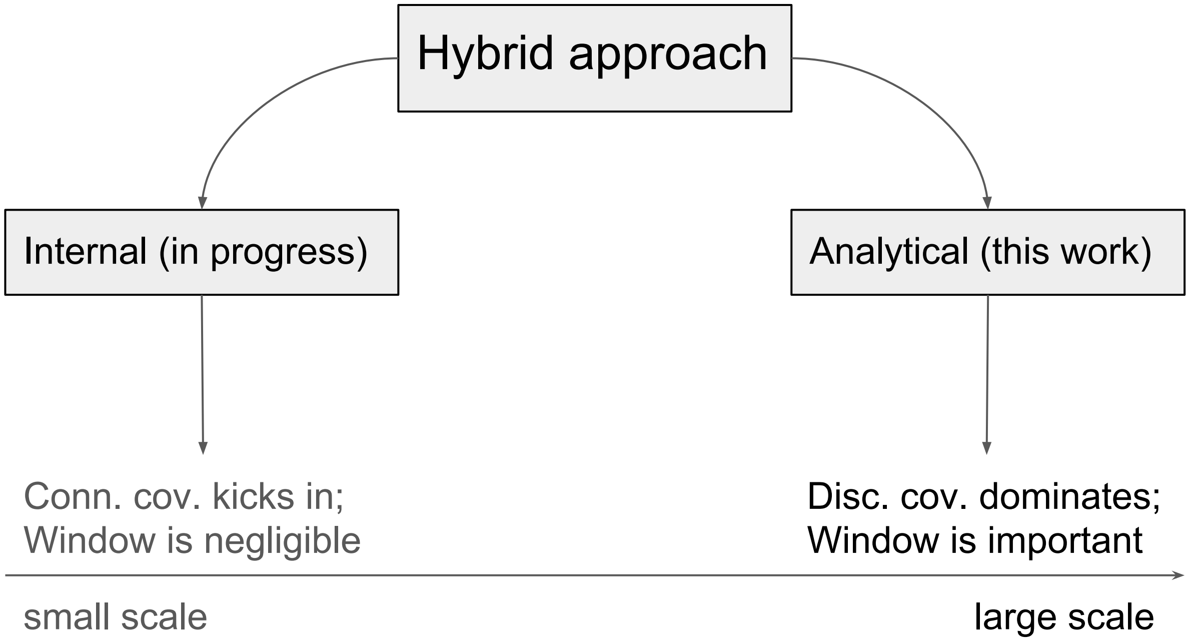
External (from mocks)

- Pros
  - Can in principle include the right physics and systematics
- Cons
  - But difficult in practice, and need a fixed fiducial cosmology
  - Noisy, need to inflate the errors on parameters
  - Costly

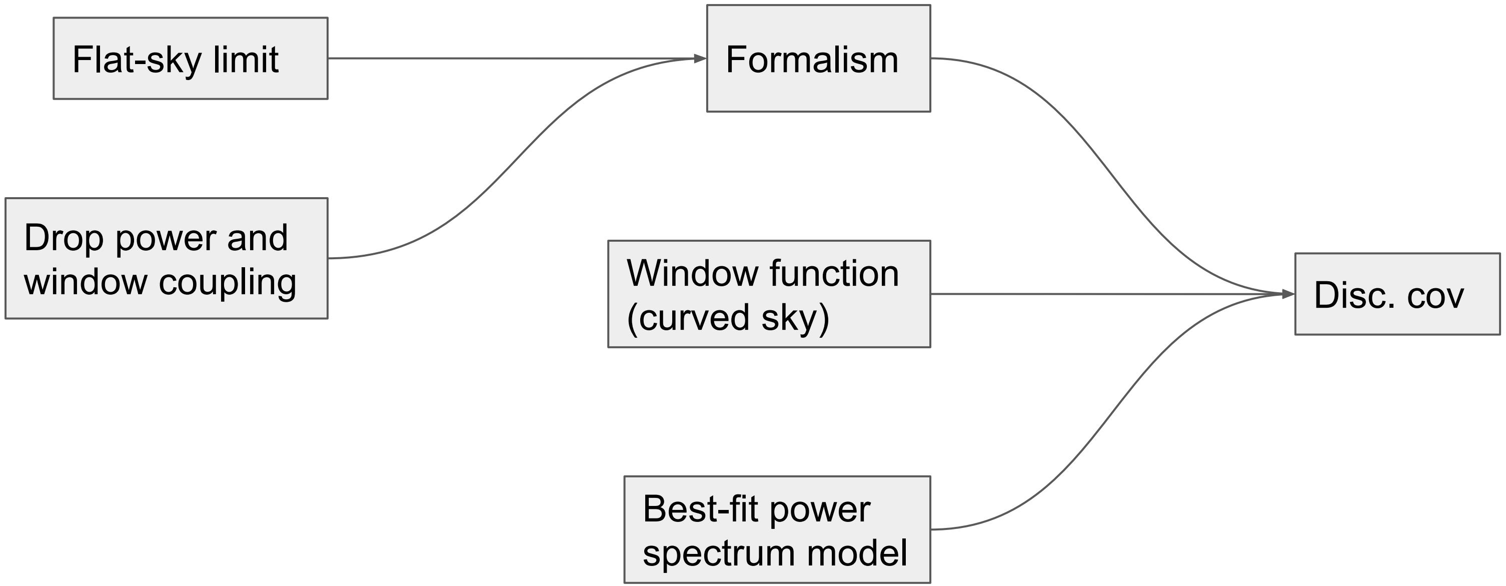
Analytic

- Pros
  - Noiseless
  - Can be based on the best-fit cosmology
- Cons
  - Intractable, especially in the connected part, to accurately model the nonlinear mode coupling, super-sample covariance, baryonic effects, etc.
- Talks by Tim & Alexandre

Our approach



Disconnected Covariance: Modeling the Window



Disconnected Covariance: Modeling the Window

- Usual analytic cov. ignores the shape of the window, by using a diagonal covariance, that only captures the size of the window
- We model the window effect in the flat-sky limit (FKP94)
- Numerical integration to predict disc. cov. given power spectrum and window
  - power spectrum constrained by the data given a model
  - curved-sky window measured from a random catalog
- So both power and window inputs are curved-sky entities
- Will show later validation against mocks on power spectrum multipoles
- Corollaries: correlation function, wedge, angular, projected, projected x multipole

Q Windows (you need three of them)

$$\mathrm{Cov}^{\mathrm{disc}}\left[\hat{P}(\boldsymbol{k}), \hat{P}(\boldsymbol{k}')\right] \approx \frac{1}{\mathcal{W}_0^2}\Big\{P(\boldsymbol{k})P(\boldsymbol{k}')\mathcal{Q}_{\mathcal{W}}(\boldsymbol{k}-\boldsymbol{k}') + \left[P(\boldsymbol{k})+P(\boldsymbol{k}')\right]\Re\left[\mathcal{Q}_{\times}(\boldsymbol{k}-\boldsymbol{k}')\right] + \mathcal{Q}_{\mathcal{S}}(\boldsymbol{k}-\boldsymbol{k}')\Big\} + (\boldsymbol{k}' \leftrightarrow -\boldsymbol{k}').$$

$$\mathcal{Q}_{\mathcal{W}}(\boldsymbol{q}) \equiv \mathcal{W}(\boldsymbol{q})\mathcal{W}(\boldsymbol{q})^* = \int_{\boldsymbol{s}} \mathcal{Q}_{\mathcal{W}}(\boldsymbol{s})e^{-i\boldsymbol{q}\cdot\boldsymbol{s}},$$

$$\mathcal{Q}_{\mathcal{S}}(\boldsymbol{q}) \equiv \mathcal{S}(\boldsymbol{q})\mathcal{S}(\boldsymbol{q})^* = \int_{\boldsymbol{s}} \mathcal{Q}_{\mathcal{S}}(\boldsymbol{s})e^{-i\boldsymbol{q}\cdot\boldsymbol{s}},$$

$$\mathcal{Q}_{\times}(\boldsymbol{q}) \equiv \mathcal{W}(\boldsymbol{q})\mathcal{S}(\boldsymbol{q})^* = \int_{\boldsymbol{s}} \mathcal{Q}_{\times}(\boldsymbol{s})e^{-i\boldsymbol{q}\cdot\boldsymbol{s}},$$

$$W(\boldsymbol{x}) \equiv \bar{n}(\boldsymbol{x})w(\boldsymbol{x}).$$

$$\mathcal{W}(\boldsymbol{q}) = \int_{\boldsymbol{x}} \mathcal{W}(\boldsymbol{x})e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \equiv \int_{\boldsymbol{x}} W(\boldsymbol{x})^2 e^{-i\boldsymbol{q}\cdot\boldsymbol{x}},$$

$$\mathcal{S}(\boldsymbol{q}) = \int_{\boldsymbol{x}} \mathcal{S}(\boldsymbol{x})e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \equiv (1+\alpha)\int_{\boldsymbol{x}} \bar{n}(\boldsymbol{x})w(\boldsymbol{x})^2 e^{-i\boldsymbol{q}\cdot\boldsymbol{x}},$$

Diagonal Limit

$$\mathrm{Cov}^{\mathrm{diag}}\big[\hat{P}(\boldsymbol{k}), \hat{P}(\boldsymbol{k}')\big] = (2\pi)^3\, \delta^{\mathrm{D}}(\boldsymbol{k}-\boldsymbol{k}')\left\{\frac{P(\boldsymbol{k})^2}{V_{\mathcal{W}}} + \frac{2P(\boldsymbol{k})P_{\mathrm{shot}}}{V_{\times}} + \frac{P_{\mathrm{shot}}^2}{V_{\mathcal{S}}}\right\} + (\boldsymbol{k}' \leftrightarrow -\boldsymbol{k}'),$$

Just like there are three windows, now you have three different effective volumes

$$\mathrm{Cov}^{\mathrm{diag}}\big[\hat{P}(k_i), \hat{P}(k_j)\big] = 2\delta^{\mathrm{K}}_{ij} \int_{k_{i-\frac{1}{2}}}^{k_{i+\frac{1}{2}}} \frac{4\pi k^2 \mathrm{d}k}{V_{k_i}} \left\{\frac{P(k)^2}{N_{\mathcal{W}}} + \frac{2P(k)P_{\mathrm{shot}}}{N_{\times}} + \frac{P_{\mathrm{shot}}^2}{N_{\mathcal{S}}}\right\},$$

After binning in k, they become three different effective number of modes

Double Bessel Quadrature

After integrating out the angular part of the integrals analytically, we are left with some radial integrals that we have to do numerically. The tricky ones look like the following

$$G(y, y') = \int_0^\infty x \mathrm{d}x \; F(x) J_\nu(xy) J_{\nu'}(xy'),$$

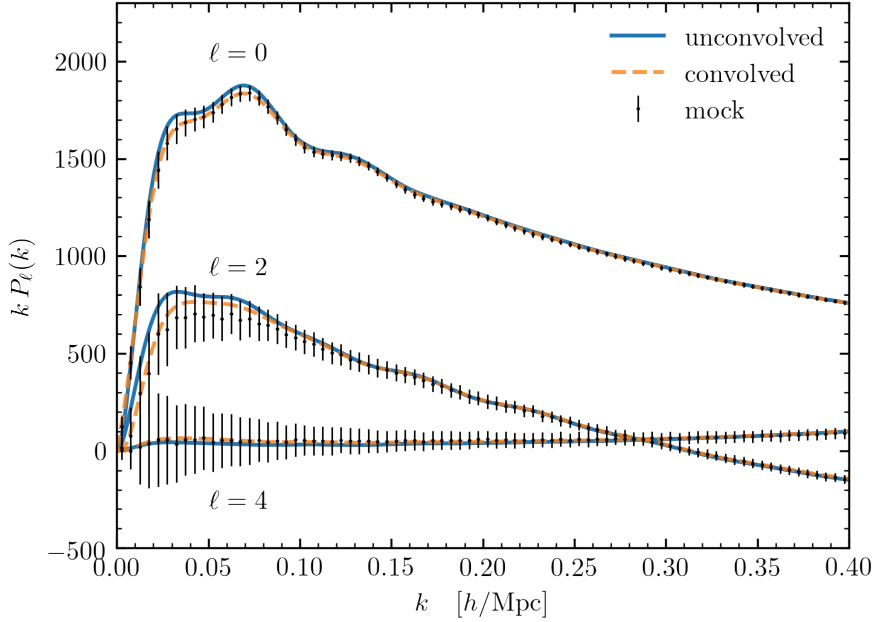
which we solved with a novel algorithm using Hankel transform (integral involving only a single Bessel function) implemented with `mcfit`

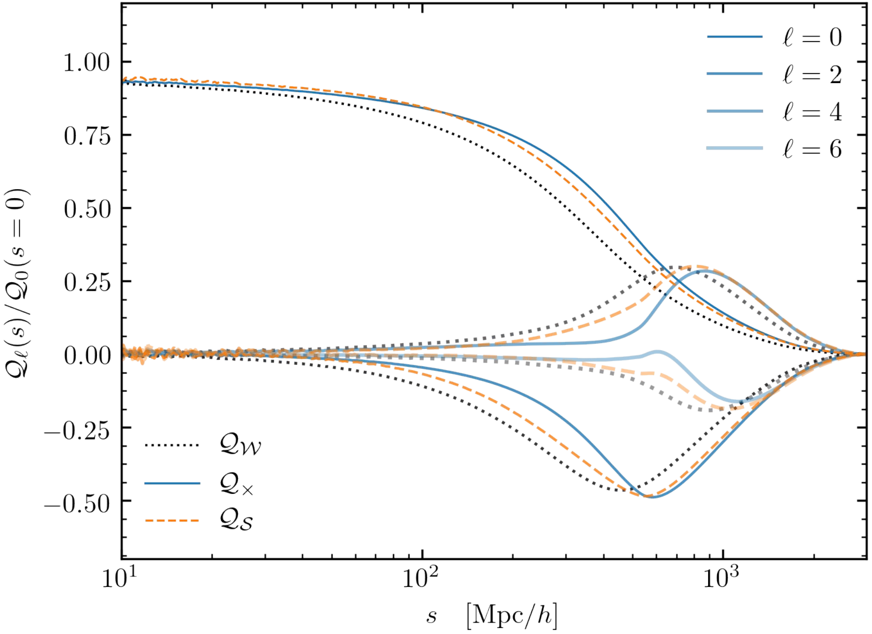
Other application includes exact 3D-to-angular projection without Limber approx.

Test case: 3D galaxy clustering

- The 2-point functions are the power spectrum multipoles
- Validation against Patchy mocks (BOSS DR12)
- Formalism is general. See the paper for corollaries: correlation function, wedge, angular, projected, projected x multipole, etc.

Two Inputs: power and window



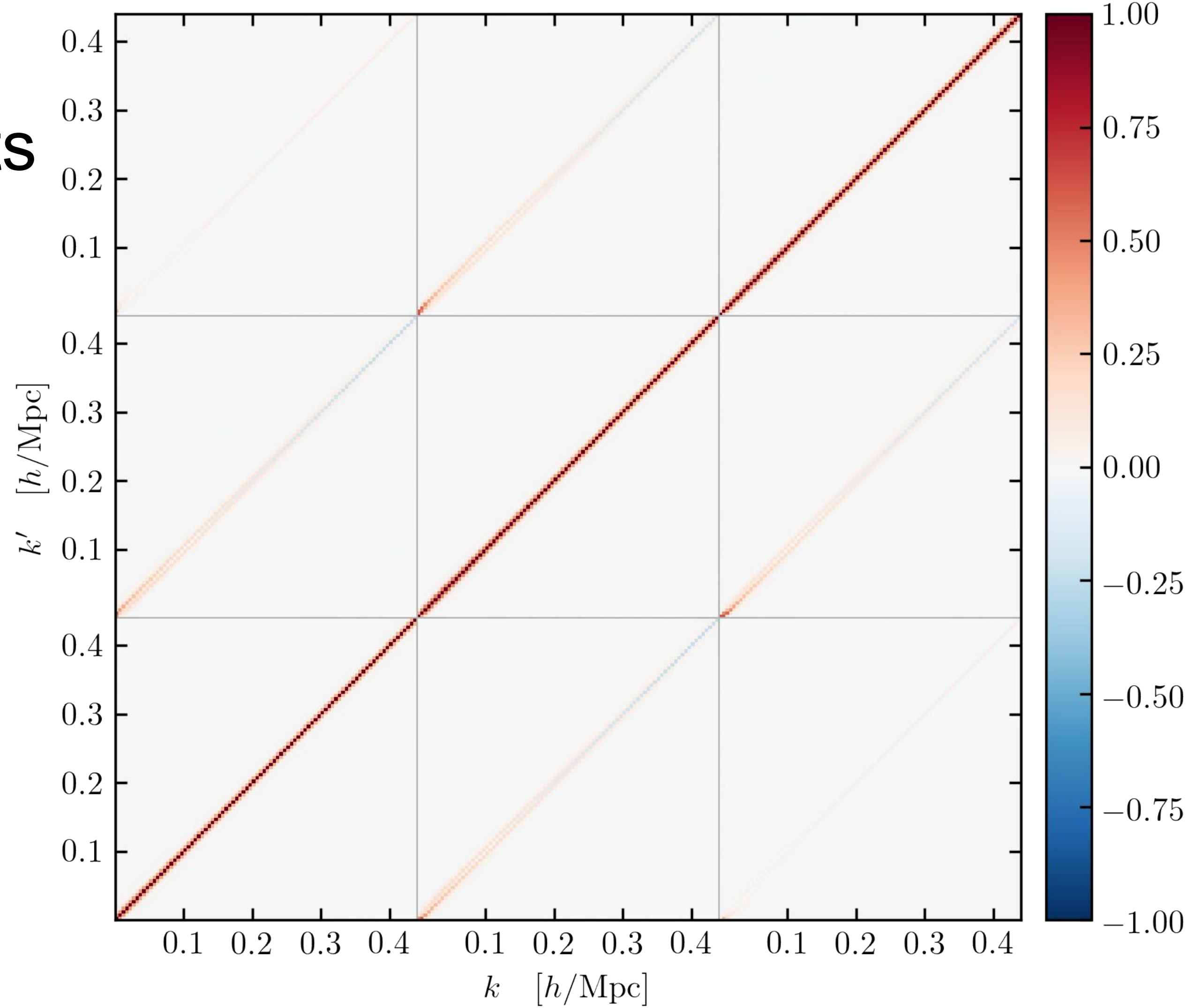


Analytic Results

correlation matrix of power spectrum multipoles $\{P_0, P_2, P_4\}$

disconnected only

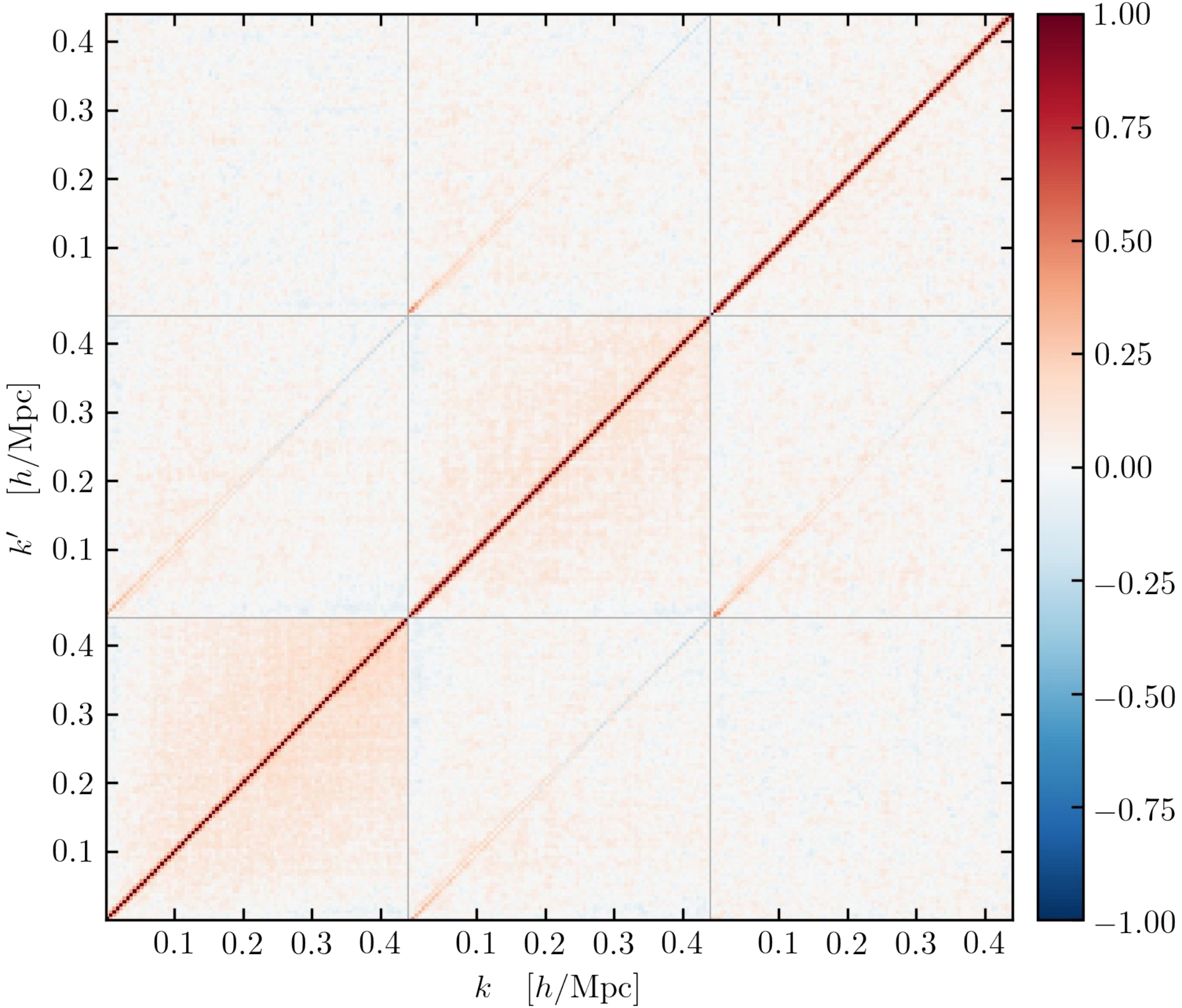
Performance: 2 mins



Patchy Mocks

sample covariance
from 1000 mocks

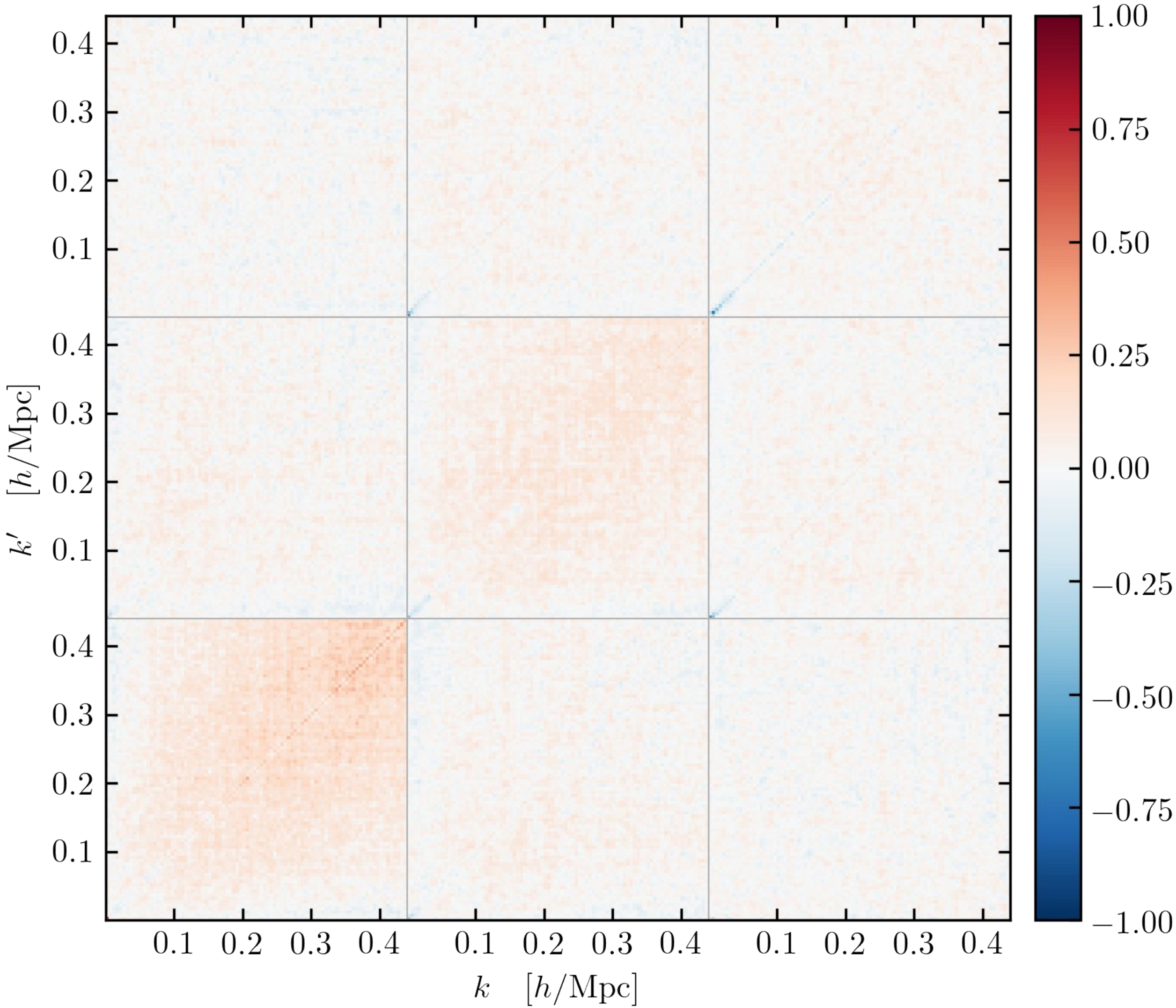
both disconnected
and connected cov



Difference

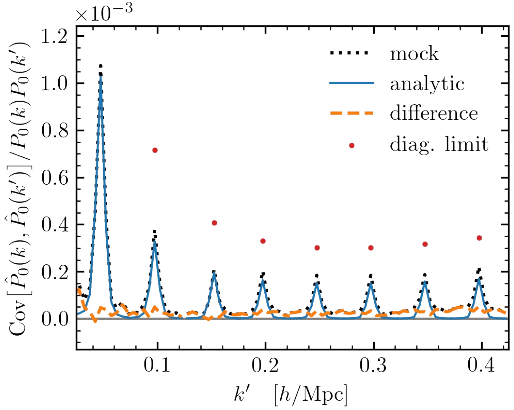
normalized by the diagonal of the analytic covariance

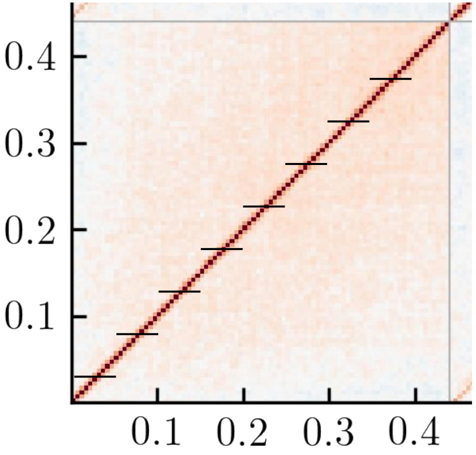
smooth residual, as expected for the connected cov



Compare Covariance Matrix Slices

- $P_0 x P_0$, $P_0 x P_2$, $P_2 x P_2$, $P_4 x P_4$,
- Slices at fixed k near the k'=k diagonal: every spike corresponds to one k with variable k' near that k
- Difference between mock and analytic is smooth, as expected for the connected covariance
- Diagonal covariance is very different and only nonzero on top of "peaks"





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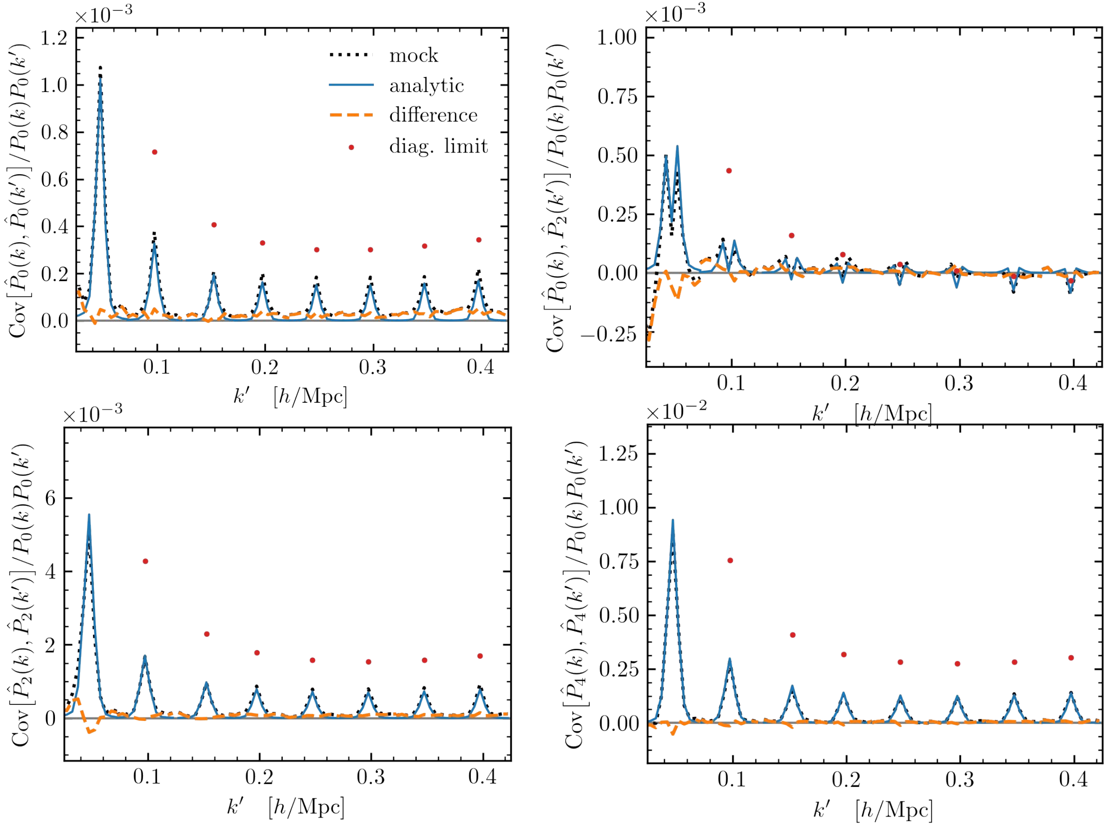
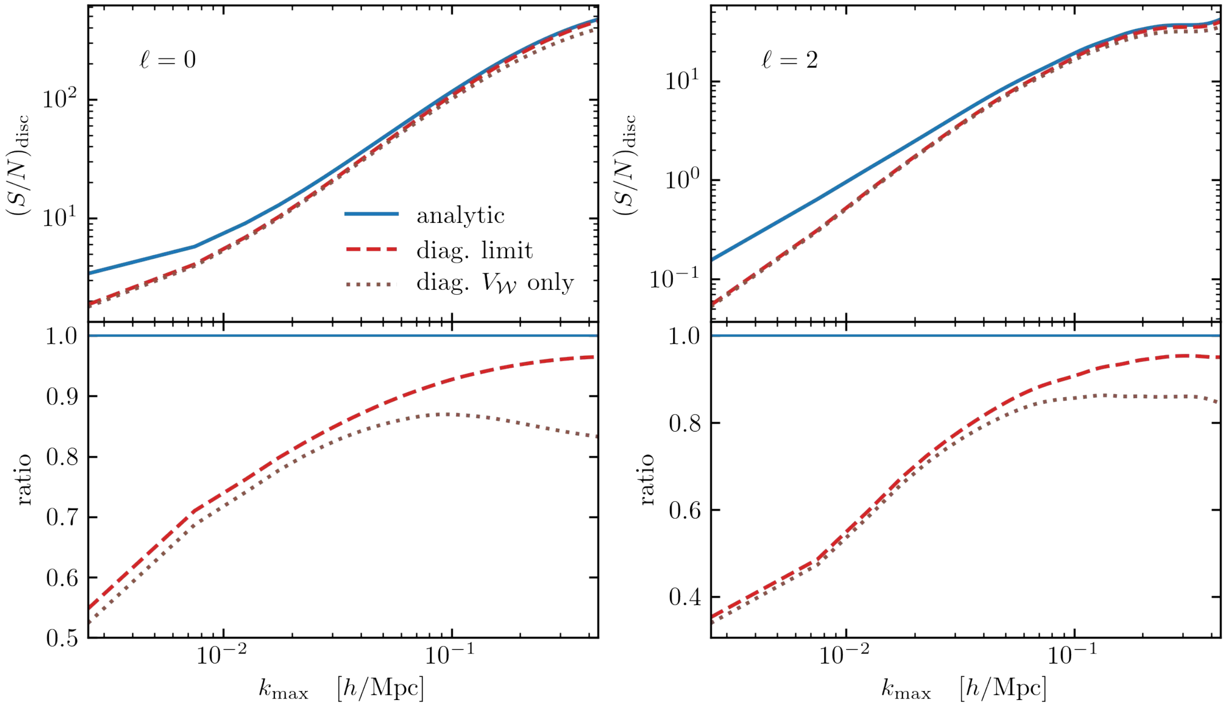


Compare SNR, Analytical vs Diagonal

- Binning affects the direct comparison of cov, but not the signal-to-noise ratio
- Diagonal limit (with three effective volumes) underestimates SNR due to ignorance of the shape of the window, especially on large scales
- Diagonal cov using only one effective volume (that of the P^2 term) even underestimates SNR on small scales



Summary

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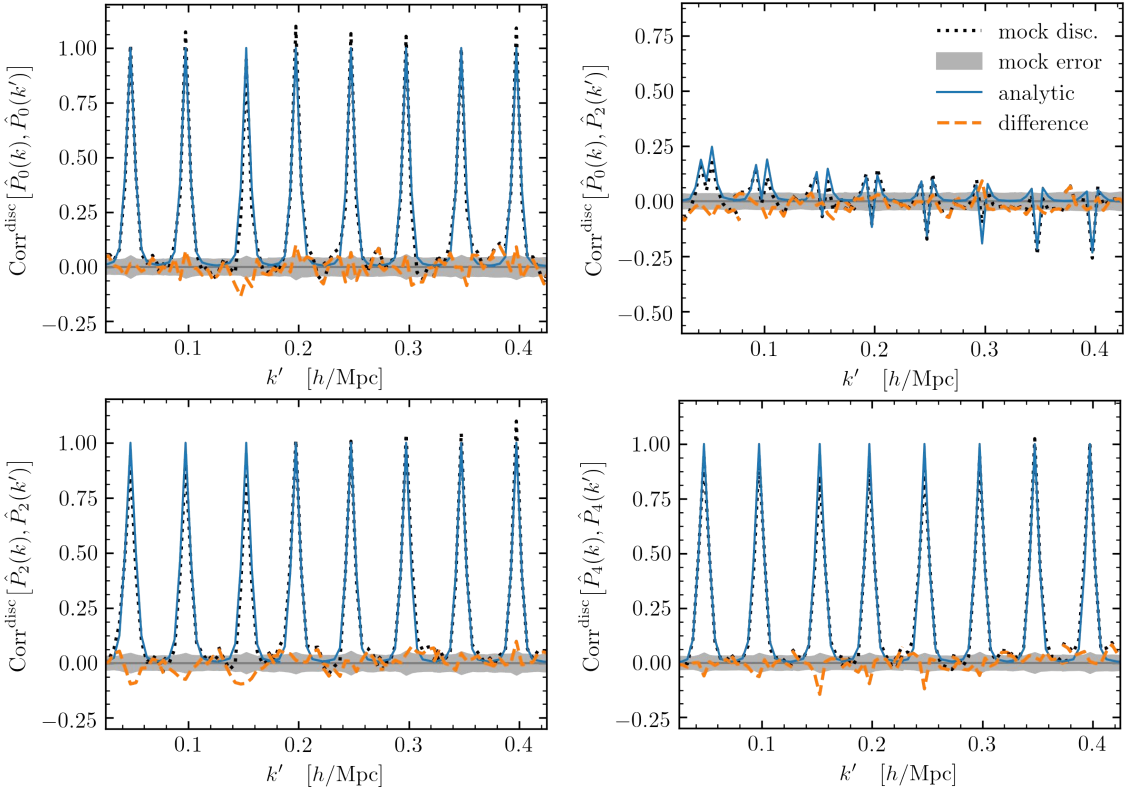
(Backup) Just the Disconnected Part

- The connected covariance is a low-rank matrix
- After subtracting a low-rank approx. of the off-diag. mock cov., we obtain an estimate of the disc. cov. from the mock, which can be directly compared to our analytic results
- All curves are normalized by the diagonal of the analytic cov.
- The residual is consistent with the bootstrap error on the mock covariance: the analytic cov. is accurate



(Backup) Why you need Three Q Windows

Both curves are analytic, so the unbinned covariance is shown for clarity. The prediction using only one window (that of the P^2 term) becomes inaccurate where P_{shot} is important