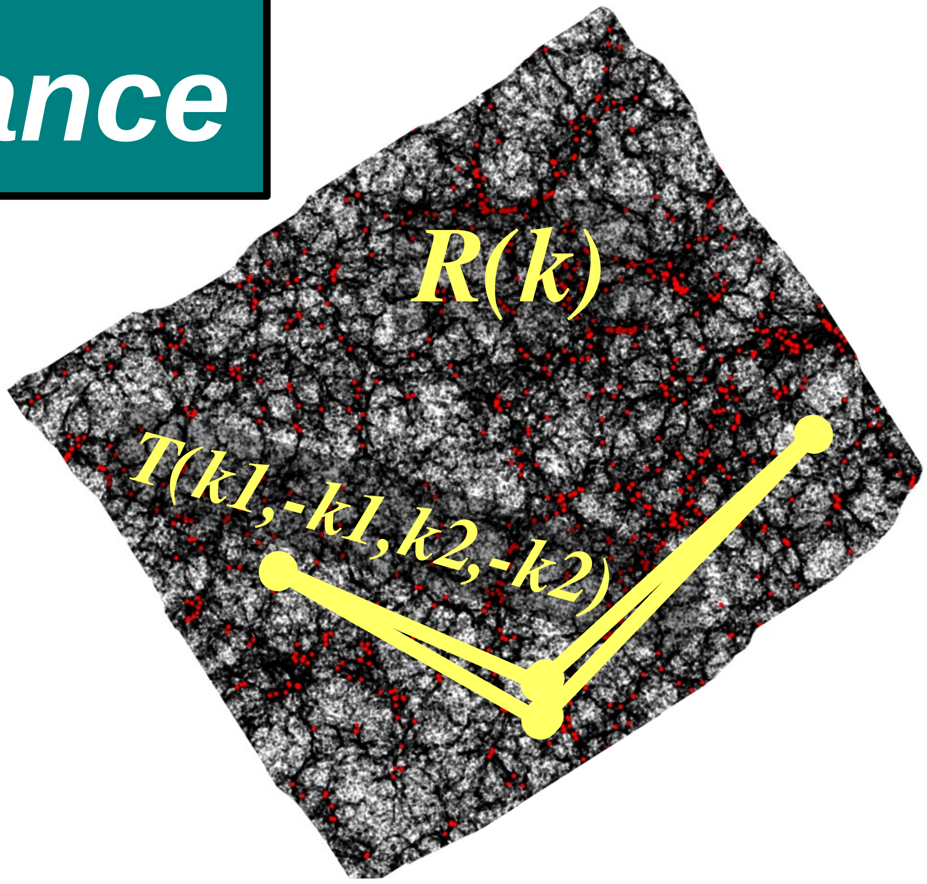
Responses on 2pt-lensing covariance

Alexandre Barreira
MPA



with Elisabeth Krause & Fabian Schmidt

$$\mathrm{Cov}^{G}_{\kappa}(\boldsymbol{\ell}_1,\boldsymbol{\ell}_2) + \mathrm{Cov}^{cNG}_{\kappa}(\boldsymbol{\ell}_1,\boldsymbol{\ell}_2) + \mathrm{Cov}^{SSC}_{\kappa}(\boldsymbol{\ell}_1,\boldsymbol{\ell}_2)$$

G **cNG** **SSC**

Accurate lensing in the era of precision cosmology

Berkeley, CA, USA, Jan 2019

In this talk ...

1) Response Approach to Perturbation Theory

Barreira, Schmidt , 1703.09212
Barreira, Schmidt , 1705.01092

2) Application to lensing spectra covariance

Barreira, Krause, Schmidt, 1711.07467
Barreira, Krause, Schmidt, 1807.04266

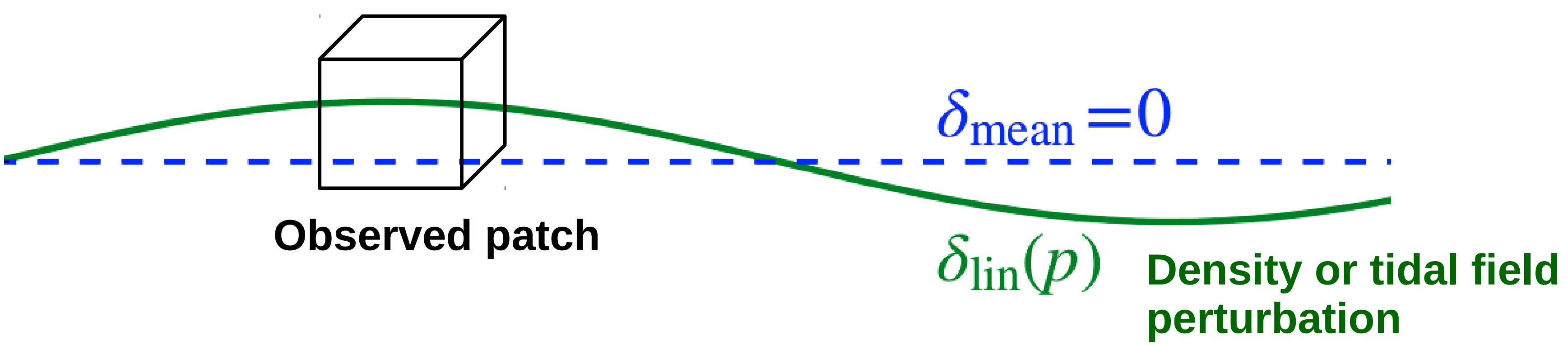
Response Approach to PT

Barreira, Schmidt , 1703.09212
Barreira, Schmidt , 1705.01092

What are responses?

Responses describe how the power spectrum responds to the presence of large-scale perturbations.

$$\mathcal{R}_n \equiv \frac{1}{n!P(k)} \frac{\mathrm{d}^n P(\boldsymbol{k}, \delta_1 \cdots \delta_n)}{\mathrm{d}\delta_1 \cdots \mathrm{d}\delta_n}\bigg|_{\delta_a=0}$$



What are responses?

Responses describe how the power spectrum responds to the presence … turbations.

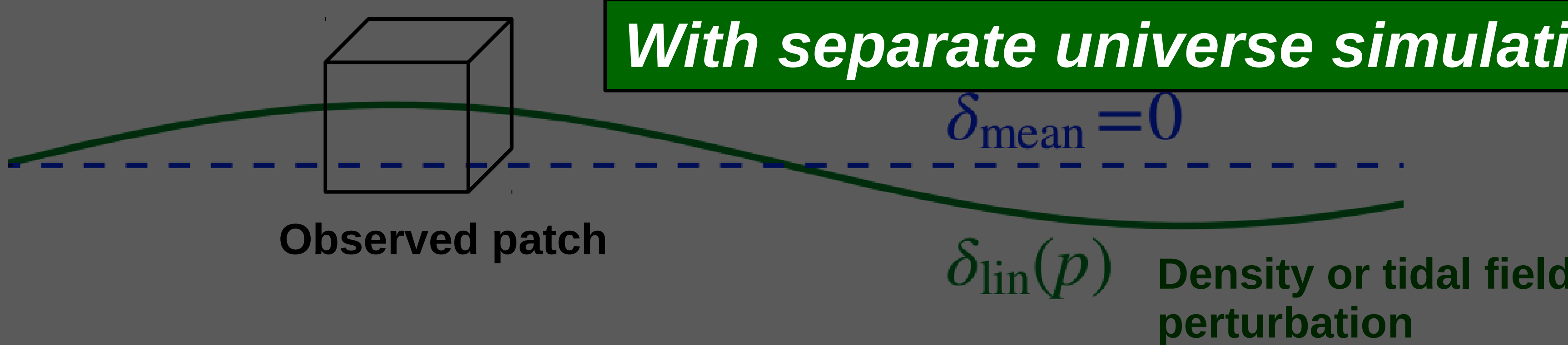
What are they good for?

To describe squeezed N-point functions

$$\mathcal{R}_n \equiv \frac{1}{n!P(k)} \frac{\mathrm{d}^n P(\boldsymbol{k}, \delta_1 \cdots \delta_n)}{\mathrm{d}\delta_1 \cdots \mathrm{d}\delta_n}\bigg|_{\delta_a=0}$$

How do we evaluate them?

With separate universe simulations



Responses and N-point functions

n+2 squeezed point function given by n-th order response

$$\left\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)\right\rangle = \mathcal{R}_n \; P_m(k)P_m(p_1)\cdots P_m(p_n)$$

Barreira & Schmidt , 1703.09212

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Small scale

Barreira & Schmidt , 1703.09212

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Small scale **Large scale**

Barreira & Schmidt , 1703.09212

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Small scale **Large scale** **Power spectra**

"Standard" simulations

Barreira & Schmidt , 1703.09212

Responses and N-point functions

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$$\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)\rangle = \mathcal{R}_n \, P_m(k)P_m(p_1)\cdots P_m(p_n)$$

Small scale — **Large scale** — **Response** — **Power spectra**

Separate Universe simulations → Response

"Standard" simulations → Power spectra

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Responses and N-point functions

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Small scale — **Large scale** — **Response** — **Power spectra**

Separate Universe simulations → Response

"Standard" simulations → Power spectra

Predictive for nonlinear values of k, k'

Barreira & Schmidt , 1703.09212

Responses and N-point functions

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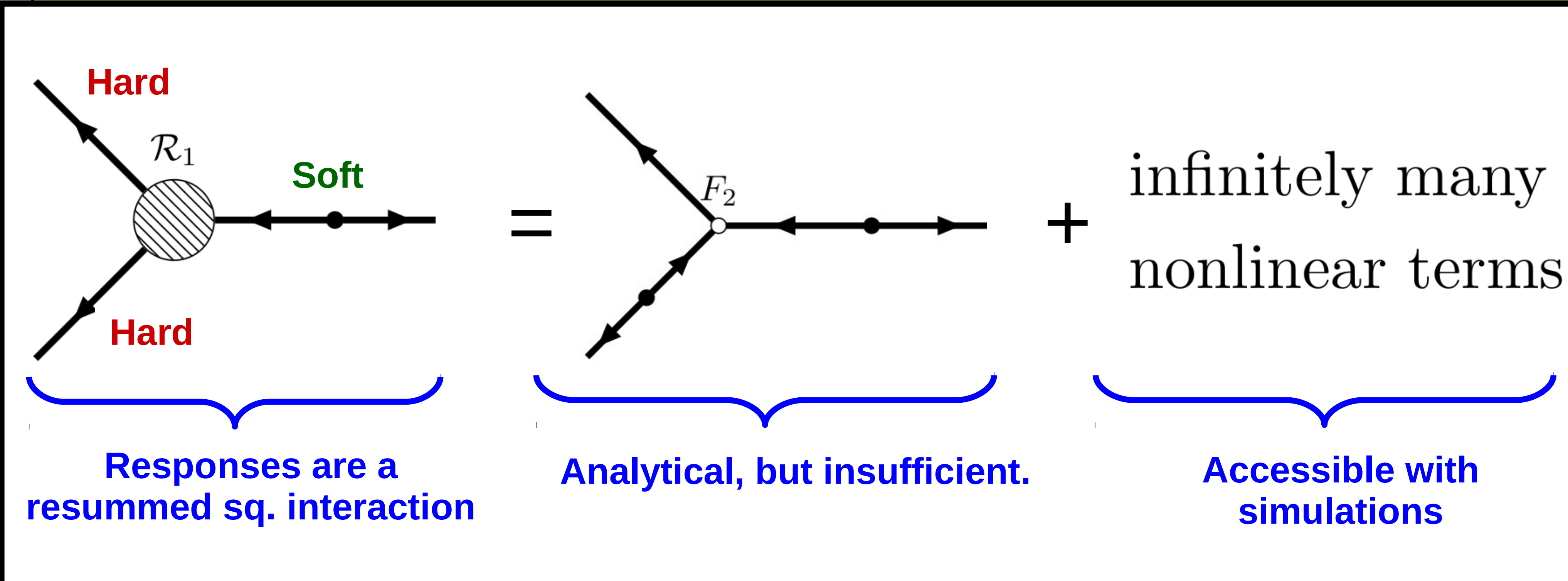

Predictive for nonlinear values of k, k'

Barreira & Schmidt , 1703.09212

Response decomposition

Write the response in terms of all possible local gravitational observables

$$\mathcal{R}_n = \sum_{\mathcal{O}} R_{\mathcal{O}}(k)\ \mathcal{K}^{(n)}_{\mathcal{O},\mathrm{large\ scale}}$$

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$$\mathcal{R}_n = \sum_{\mathcal{O}} R_{\mathcal{O}}(k)\, \mathcal{K}^{(n)}_{\mathcal{O},\mathrm{large\ scale}}$$

All possible configurations of large-scale density/tidal fields;

Given by perturbation theory.

Response decomposition

Write the response in terms of all possible local gravitational observables

$$\mathcal{R}_n = \sum_{\mathcal{O}} R_{\mathcal{O}}(k)\, \mathcal{K}^{(n)}_{\mathcal{O},\mathrm{large\ scale}}$$

Measure the response to each specific large-scale configuration;

What we will get from simulations.

All possible configurations of large-scale density/tidal fields;

Given by perturbation theory.

Response decomposition

Large-scale overdensity

Large-scale tidal field

$$\mathcal{R}_1 \longrightarrow R_1(k)\delta(\boldsymbol{p}) + R_K(k)\hat{k}^i\hat{k}^j K_{ij}(\boldsymbol{p})$$

Response to overdensity

Response to tidal field

Response decomposition

$$
\begin{aligned}
\mathcal{R}_2 \longrightarrow\ & R_1(k)\left[\delta^{(2)}(\boldsymbol{p}_1,\boldsymbol{p}_2)\right] + R_K(k)\left[\hat{k}^i\hat{k}^j K^{(2)}_{ij}(\boldsymbol{p}_1,\boldsymbol{p}_2)\right] \\
+\ & \frac{1}{2}R_2(k)\left[\delta(\boldsymbol{p}_1)\delta(\boldsymbol{p}_2)\right] + R_{K\delta}(k)\left[\hat{k}^i\hat{k}^j K_{ij}(\boldsymbol{p}_1)\delta(\boldsymbol{p}_2)\right] \\
+\ & R_{K^2}(k)\left[K_{ij}(\boldsymbol{p}_1)K^{ij}(\boldsymbol{p}_2)\right] + R_{K.K}(k)\left[\hat{k}^i\hat{k}^j K_{il}(\boldsymbol{p}_1)K^l{}_j(\boldsymbol{p}_2)\right] \\
+\ & R_{KK}(k)\left[\hat{k}^i\hat{k}^j\hat{k}^l\hat{k}^m K_{ij}(\boldsymbol{p}_1)K_{lm}(\boldsymbol{p}_2)\right] + R_{\hat{\Pi}}(k)\left[\hat{k}^i\hat{k}^j\hat{\Pi}_{ij}(\boldsymbol{p}_1,\boldsymbol{p}_2)\right]
\end{aligned}
$$

Response coefficients

All 2nd order large-scale operators

Generalizations to any order are always straightforward, just more cumbersome.

Separate Universe simulations

Nitty-gritty: Li et al (1401.0385) ; Wagner et al (1409.6294); Schmidt et al (1803.03274);

$$\mathcal{R}_n = \sum_{\mathcal{O}} R_{\mathcal{O}}(k) \, \mathcal{K}^{(n)}_{\mathcal{O},\text{large scale}}$$

Response to specific perturbations

All possible configurations of large-scale density/tidal fields;

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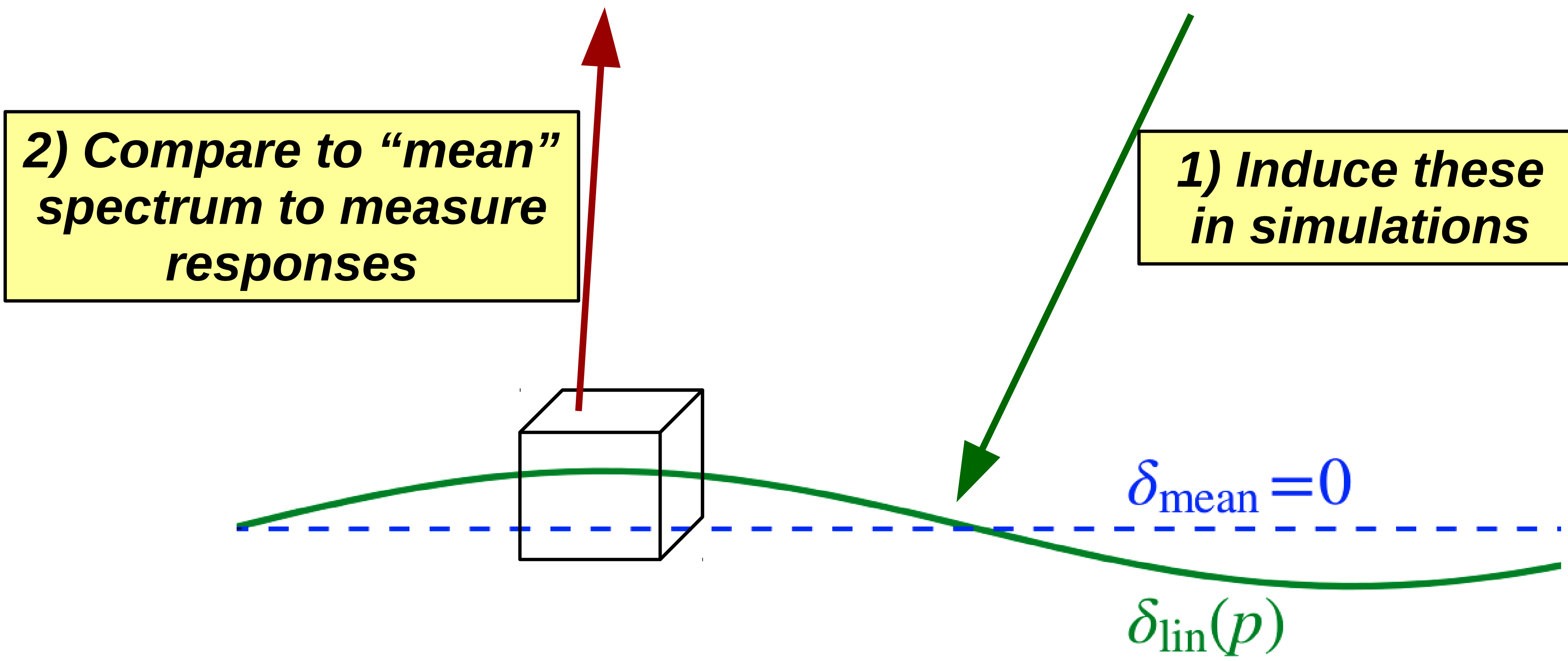


Separate Universe simulations

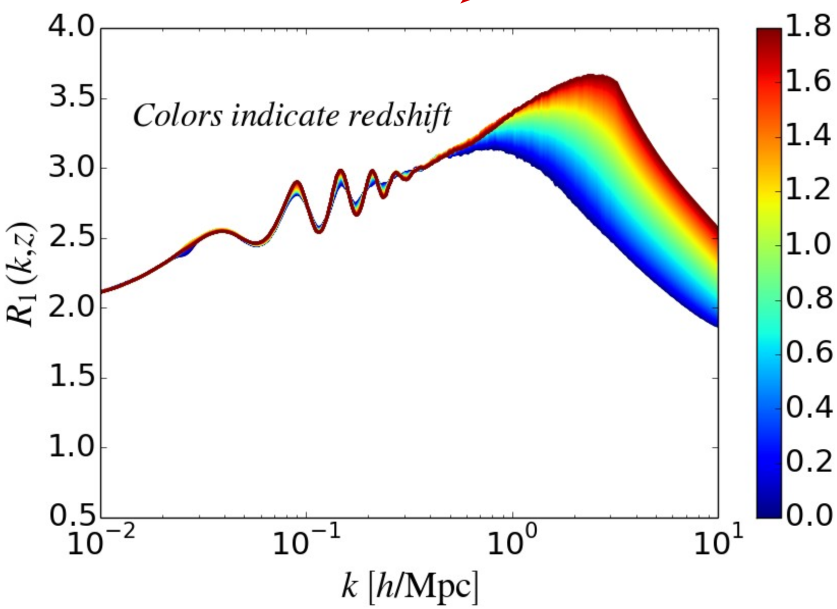
$$P_m(\boldsymbol{k}, \boldsymbol{x}) = P_m(k)\left[1 + R_1(k)\delta(\boldsymbol{x}) + R_K(k)\hat{k}^i\hat{k}^j K_{ij}(\boldsymbol{x})\right]$$

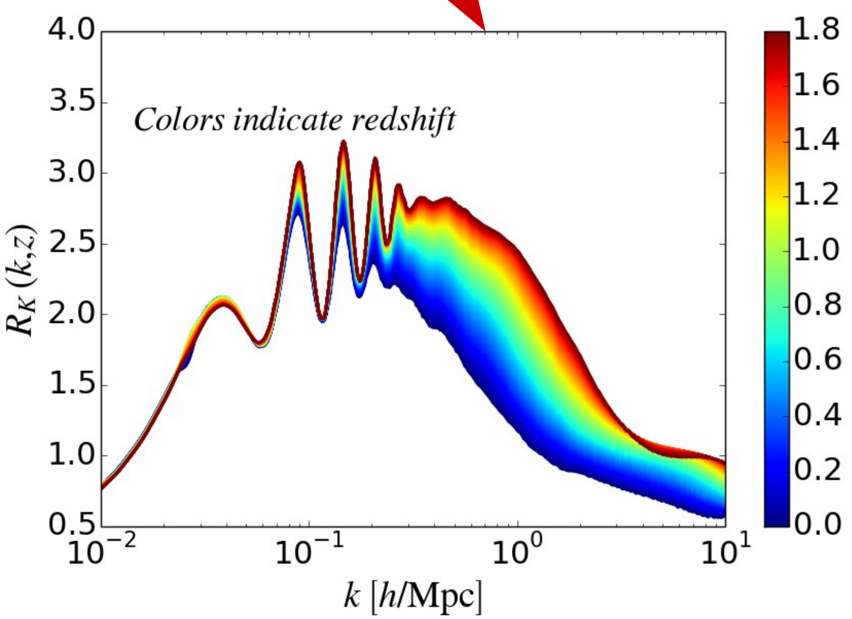
Response to overdensity

Li et al (1401.0385) ; Wagner et al (1409.6294)

Response to tidal field

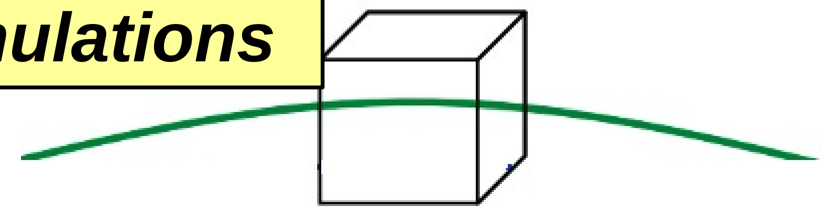
Schmidt et al (1803.03274)





To keep in mind ...

Separate Universe Simulations



give us

Responses

give us

Squeezed mode-coupling interactions in the nonlinear regime

$$\left\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)\right\rangle_{c,\mathcal{R}_n} \propto \mathcal{R}_n(k,\mathrm{angles})\; P_m(k)$$

Small scale — Large scale — Response

Covariances with Responses

Barreira, Krause, Schmidt, 1711.07467
Barreira, Krause, Schmidt, 1807.04266

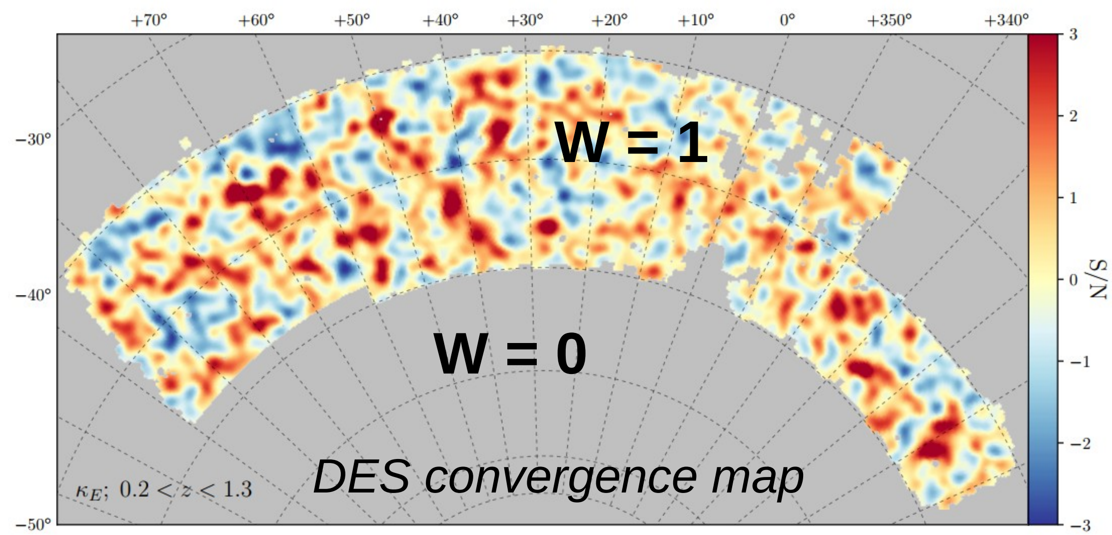
The covariance decomposition

- **Windowed lensing convergence**

$$\kappa_{\mathcal{W}}(\boldsymbol{\theta}) = \mathcal{W}(\boldsymbol{\theta})\kappa(\boldsymbol{\theta})$$

- **Estimator of its power spectrum**

$$\hat{C}_{\kappa}(\boldsymbol{\ell}) = \frac{\tilde{\kappa}_{\mathcal{W}}(\boldsymbol{\ell})\tilde{\kappa}_{\mathcal{W}}(-\boldsymbol{\ell})}{\Omega_{\mathcal{W}}}$$



e.g. Takada&Hu (1302.6994)

- **Covariance matrix of the estimator**

$$\mathrm{Cov}_{\kappa}\left(\hat{C}_{\kappa}(\boldsymbol{\ell}_1), \hat{C}_{\kappa}(\boldsymbol{\ell}_2)\right) = \left\langle \hat{C}_{\kappa}(\boldsymbol{\ell}_1)\hat{C}_{\kappa}(\boldsymbol{\ell}_2)\right\rangle - \left\langle \hat{C}_{\kappa}(\boldsymbol{\ell}_1)\right\rangle\left\langle \hat{C}_{\kappa}(\boldsymbol{\ell}_2)\right\rangle$$

$$= \mathrm{Cov}^{G}_{\kappa}(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2) + \mathrm{Cov}^{cNG}_{\kappa}(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2) + \mathrm{Cov}^{SSC}_{\kappa}(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2)$$

Gaussian | **Connected non-Gaussian** | **Super-sample**

The Gaussian term : G

- It is the only contribution during the linear regime of structure formation
- It is diagonal

Trivially given by power spectrum squared.

$$\mathrm{Cov}_\kappa^G(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2) = \frac{(2\pi)^2}{\Omega_{\mathcal{W}}} \left[C_\kappa(\boldsymbol{\ell}_1)\right]^2 \left[\delta_D(\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2) + \delta_D(\boldsymbol{\ell}_1 - \boldsymbol{\ell}_2)\right]$$

Well understood !

Connected non-Gaussian term : cNG

- Describes the **coupling of different Fourier modes due to nonlinear structure formation** (given by the parallelogram matter trispectrum).

$$\mathrm{Cov}_{\kappa}^{cNG}(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2) = \frac{1}{\Omega_{\mathcal{W}}} \int \mathrm{d}\chi \frac{[g(\chi)]^4}{\chi^6} T_m(\boldsymbol{k}_{\ell_1}, -\boldsymbol{k}_{\ell_1}, \boldsymbol{k}_{\ell_2}, -\boldsymbol{k}_{\ell_2})$$

$$\boldsymbol{k}_{\ell} = \frac{\boldsymbol{\ell} + 1/2}{\chi}$$

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Small scale Large scale

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$$\boldsymbol{k}_{\ell} = \frac{\boldsymbol{\ell} + 1/2}{\chi}$$

$$\mathcal{R}_2(k_{\ell_1}, \mu_{12}) P_m(k_{\ell_1}) \left[P_L(k_{\ell_2})\right]^2$$

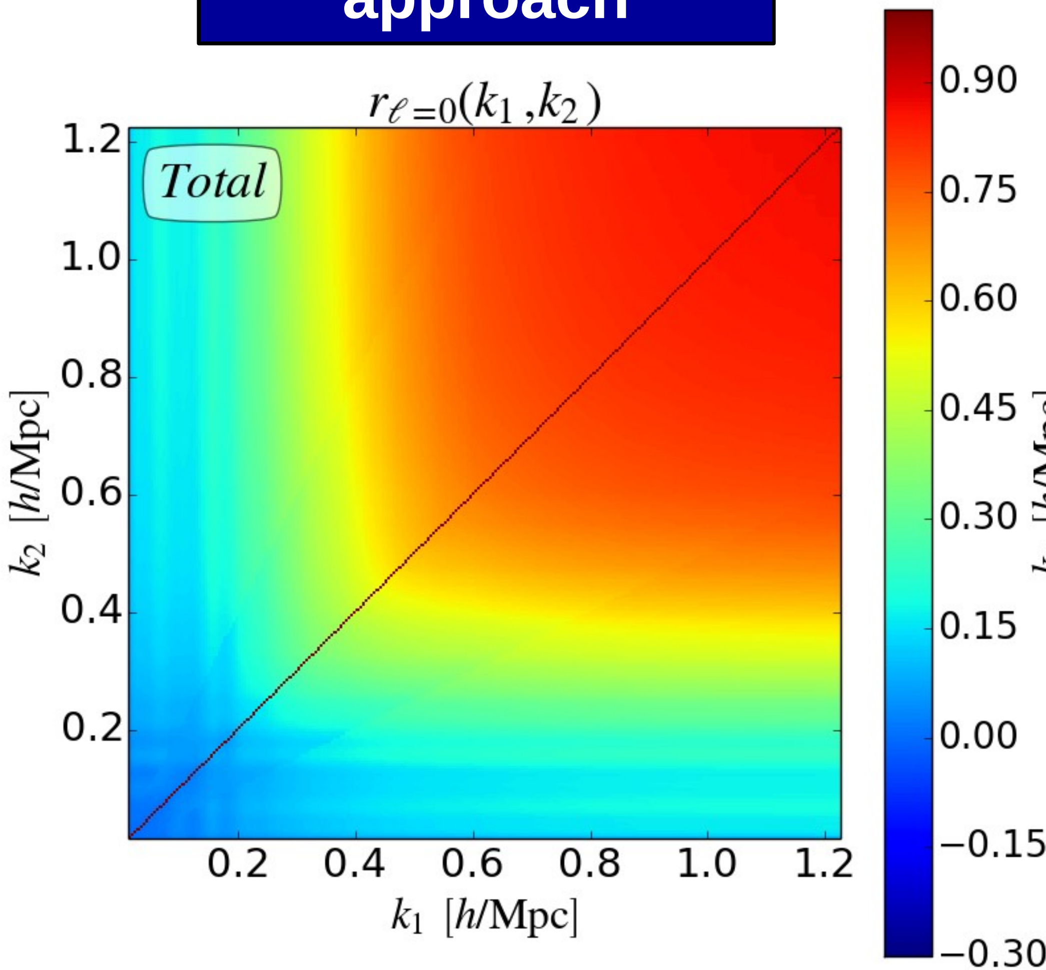
2nd order response

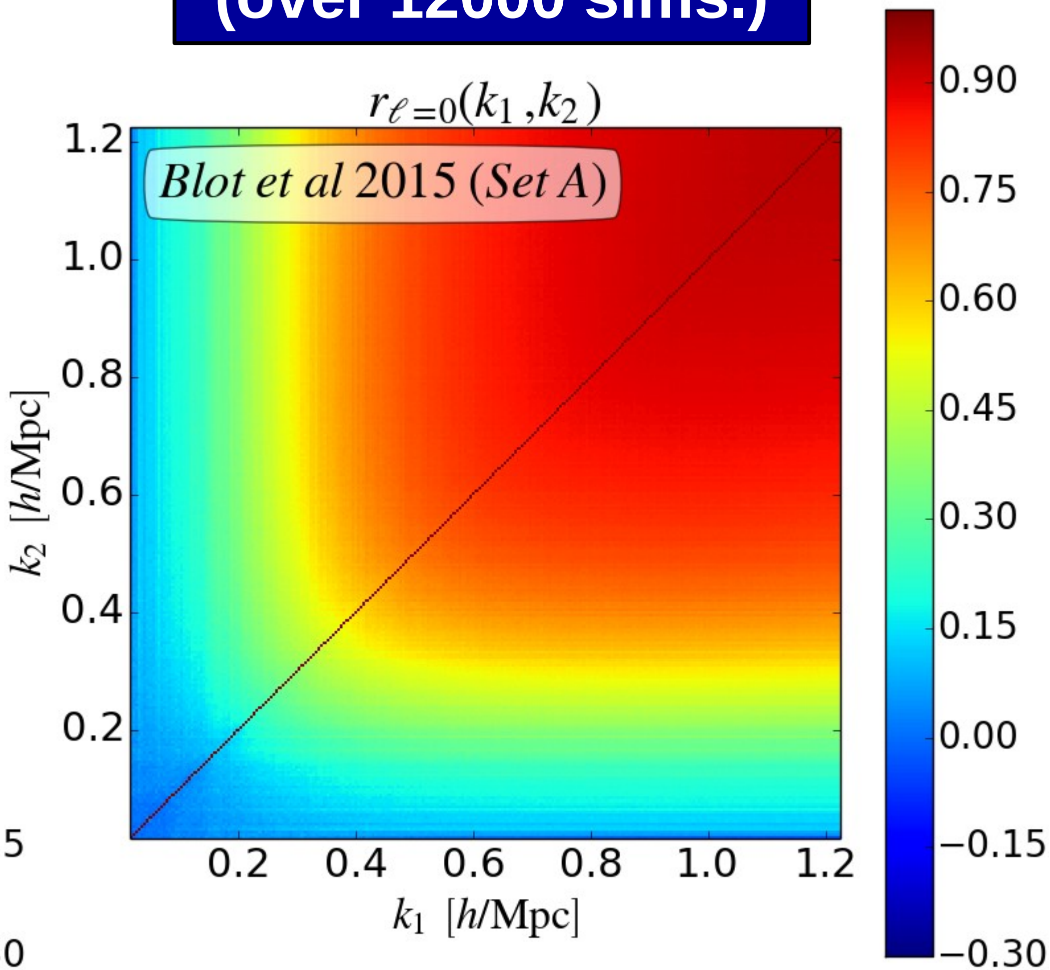
Easy evaluation of the 4-pt function for any value of the small scale mode !

cNG : response vs simulations

Response approach

Ensemble method (over 12000 sims.)





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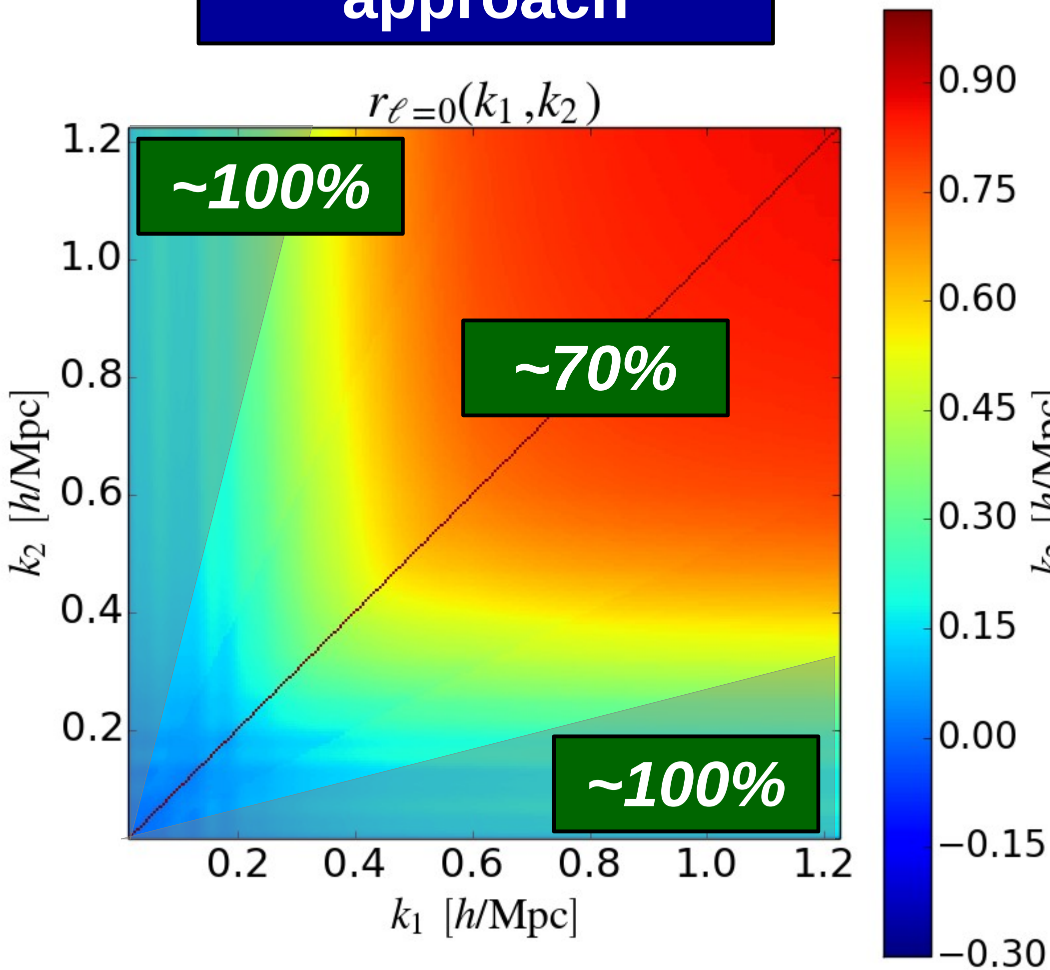

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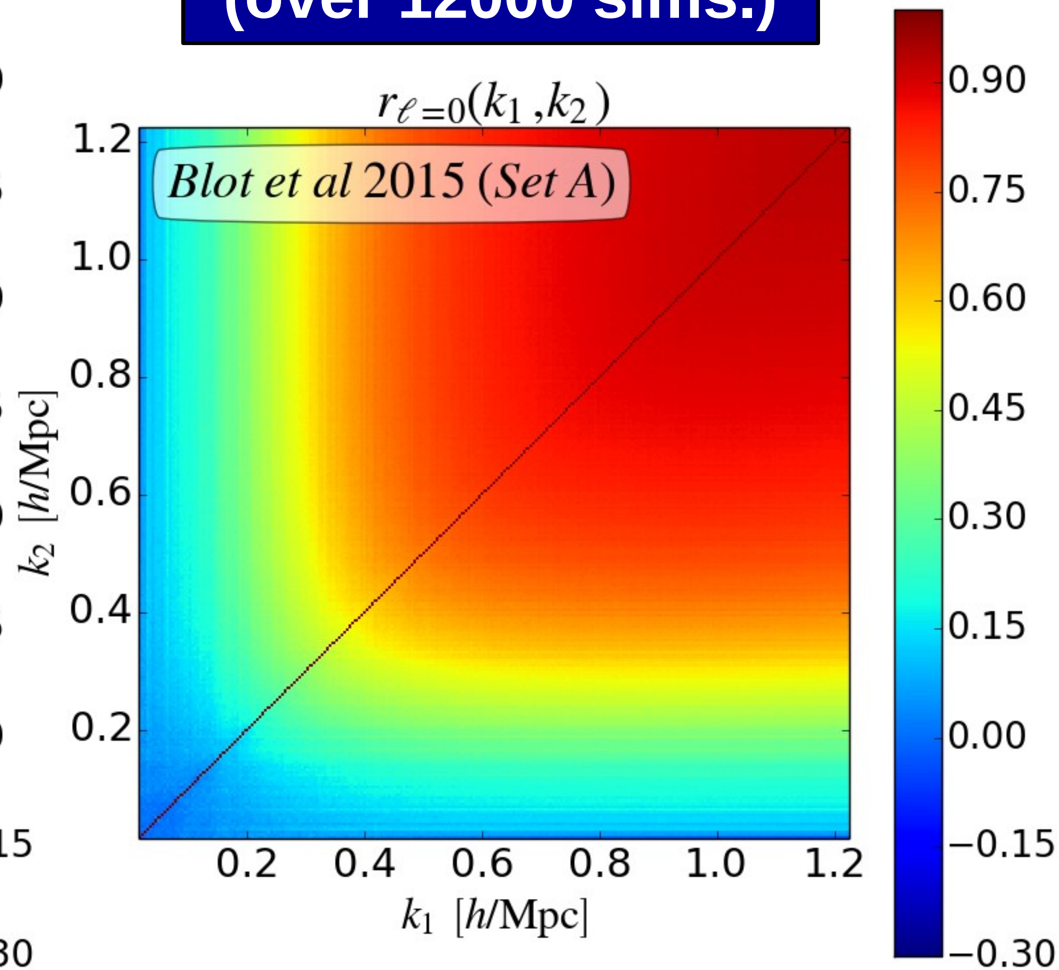

The super-sample term : SSC

- Describes the coupling of modes inside the survey with **unobserved modes outside the survey**.

$$\mathrm{Cov}_{\kappa}^{SSC}(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2) = \frac{1}{\Omega_{\mathcal{W}}^2} \int \mathrm{d}\chi \frac{[g(\chi)]^4}{\chi^6} \int \frac{\mathrm{d}^2\boldsymbol{\ell}}{(2\pi)^2} |\tilde{\mathcal{W}}(\boldsymbol{\ell})|^2 \mathcal{R}_1(\boldsymbol{k}_{\boldsymbol{\ell}_1}, \mu_{\boldsymbol{\ell}_1,\boldsymbol{\ell}}) \mathcal{R}_1(\boldsymbol{k}_{\boldsymbol{\ell}_2}, \mu_{\boldsymbol{\ell}_2,\boldsymbol{\ell}}) \times P_m(\boldsymbol{k}_{\ell_1}) P_m(\boldsymbol{k}_{\ell_2}) P_L(\boldsymbol{k}_{\ell})$$

Note: This expression assumes Limber for the super-survey modes, which is only accurate up to 10% for Euclid/LSST. Beyond Limber SSC expressions exist however (Barreira, Krause, Schmidt , arXiv:1711.07467).

The super-sample term : SSC

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Fourier transform of survey geometry

$$\mathrm{Cov}_{\kappa}^{SSC}(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2) = \frac{1}{\Omega_{\mathcal{W}}^2} \int \mathrm{d}\chi \frac{[g(\chi)]^4}{\chi^6} \int \frac{\mathrm{d}^2\boldsymbol{\ell}}{(2\pi)^2} |\tilde{\mathcal{W}}(\boldsymbol{\ell})|^2 \mathcal{R}_1(\boldsymbol{k}_{\ell_1}, \mu_{\boldsymbol{\ell}_1,\boldsymbol{\ell}}) \mathcal{R}_1(\boldsymbol{k}_{\ell_2}, \mu_{\boldsymbol{\ell}_2,\boldsymbol{\ell}}) \times P_m(\boldsymbol{k}_{\ell_1}) P_m(\boldsymbol{k}_{\ell_2}) P_L(\boldsymbol{k}_{\ell})$$

Responses capture SSC completely !

Note: This expression assumes Limber for the super-survey modes, which is only accurate up to 10% for Euclid/LSST. Beyond Limber SSC expressions exist however (Barreira, Krause, Schmidt , arXiv:1711.07467).

Lensing covariance summary

$$\mathrm{Cov}_\kappa(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2) \quad = \quad \mathrm{Cov}_\kappa^{G} \quad + \quad \mathrm{Cov}_\kappa^{cNG} \quad + \quad \mathrm{Cov}_\kappa^{SSC}$$

Lensing covariance summary

Solved !

Solved !

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Lensing covariance summary

Solved ! ✓ **Solved !** ✓

$$\mathrm{Cov}_\kappa(\boldsymbol{\ell}_1, \boldsymbol{\ell}_2) = \mathrm{Cov}_\kappa^G + \mathrm{Cov}_\kappa^{cNG} + \mathrm{Cov}_\kappa^{SSC}$$

Responses capture most of it ,

<u>but do we even need it ?</u>

The unimportance of the cNG term for future surveys

Euclid-like lensing setup

Barreira, Krause, Schmidt
1807.04266

- Tomographic convergence power spectrum
  10 tomographic bins
- 20 ell bins in [20, 5000]
- Mask: spherical cap 15000 deg^2
- Source density: 30 / arcmin^2

$$\mathcal{L}(w_0) \propto \exp\left[-\frac{1}{2}\left(\overrightarrow{Theory} - \overrightarrow{Data}\right)\mathbf{Cov}^{-1}\left(\overrightarrow{Theory} - \overrightarrow{Data}\right)\right]$$

What is the impact of different matrices on parameter constraints ?

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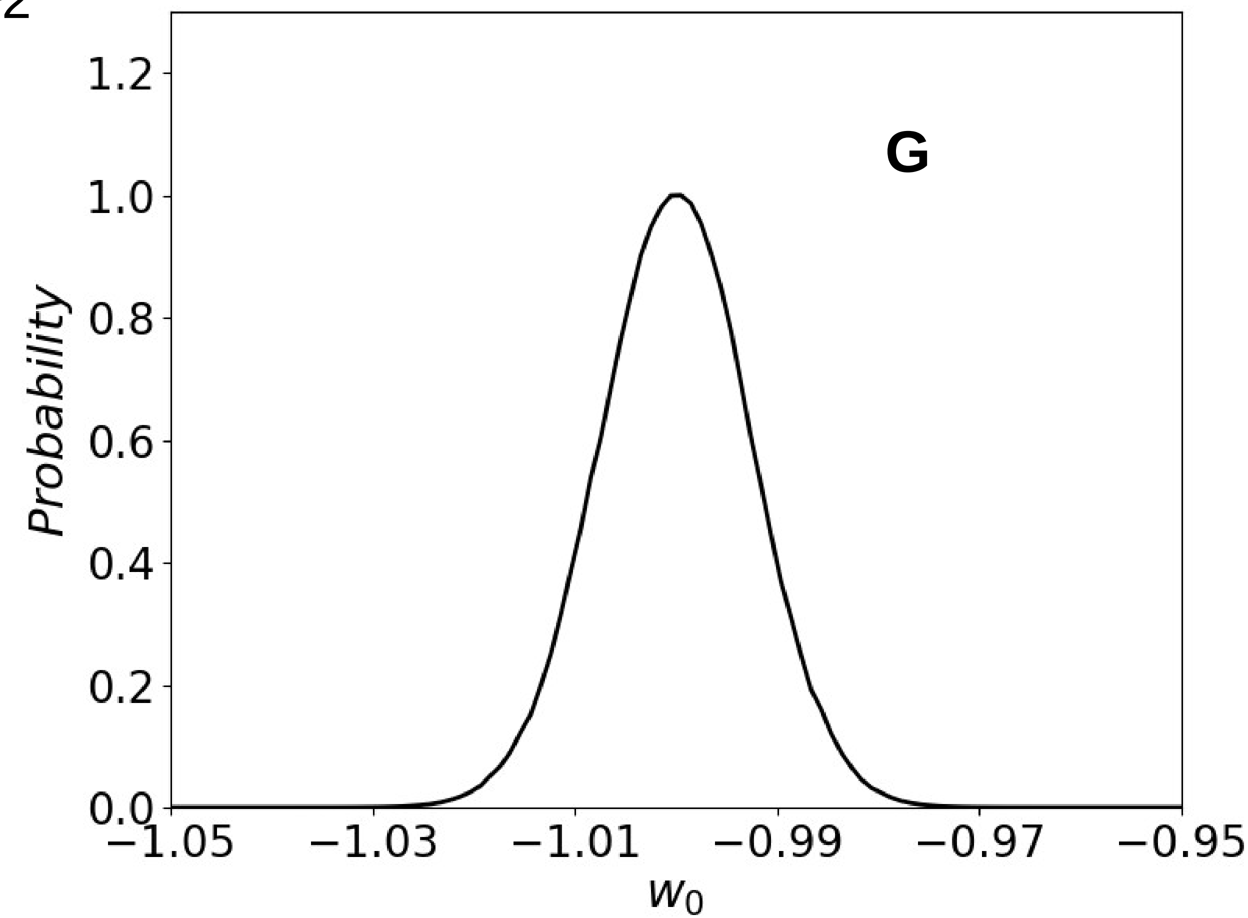

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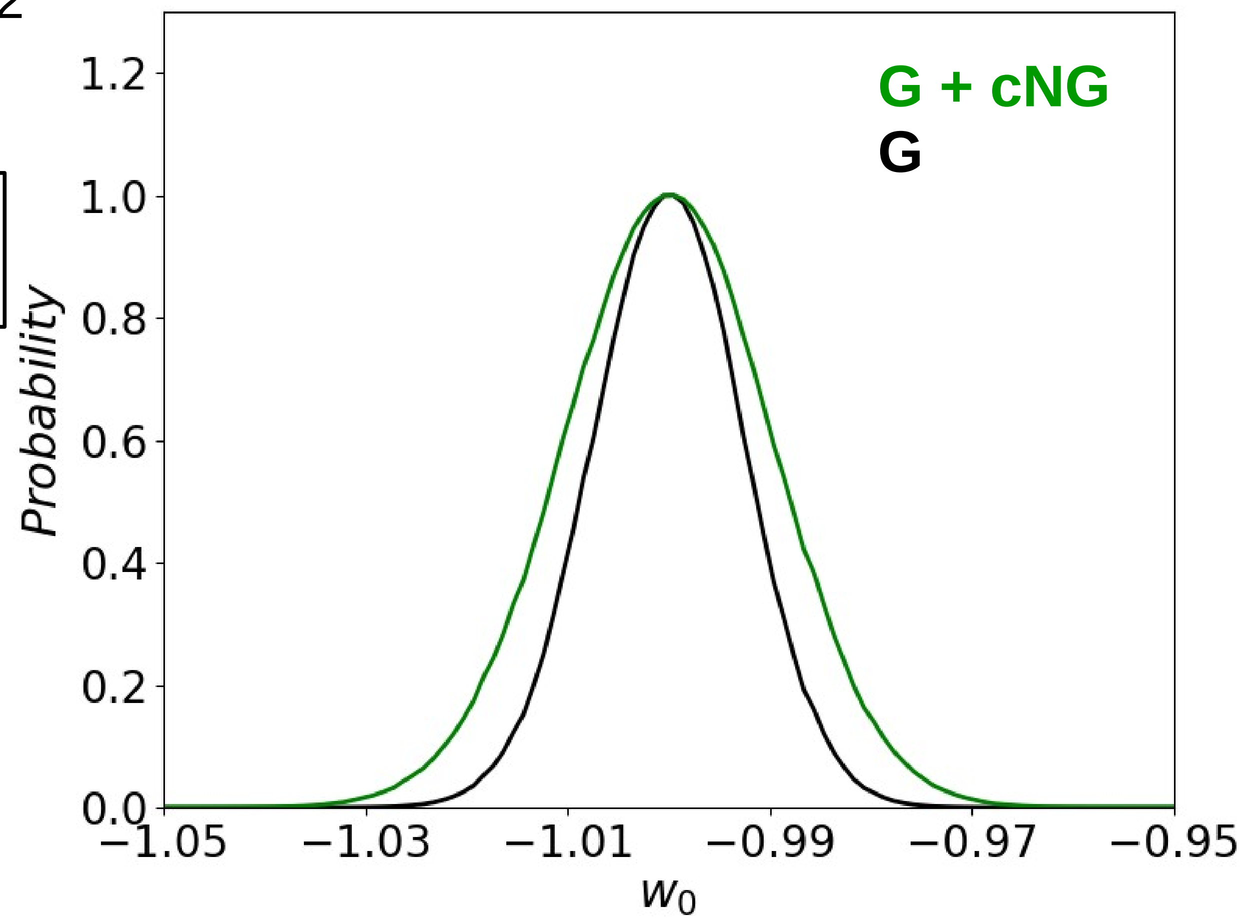
Relative to G, cNG increases error by 38% .



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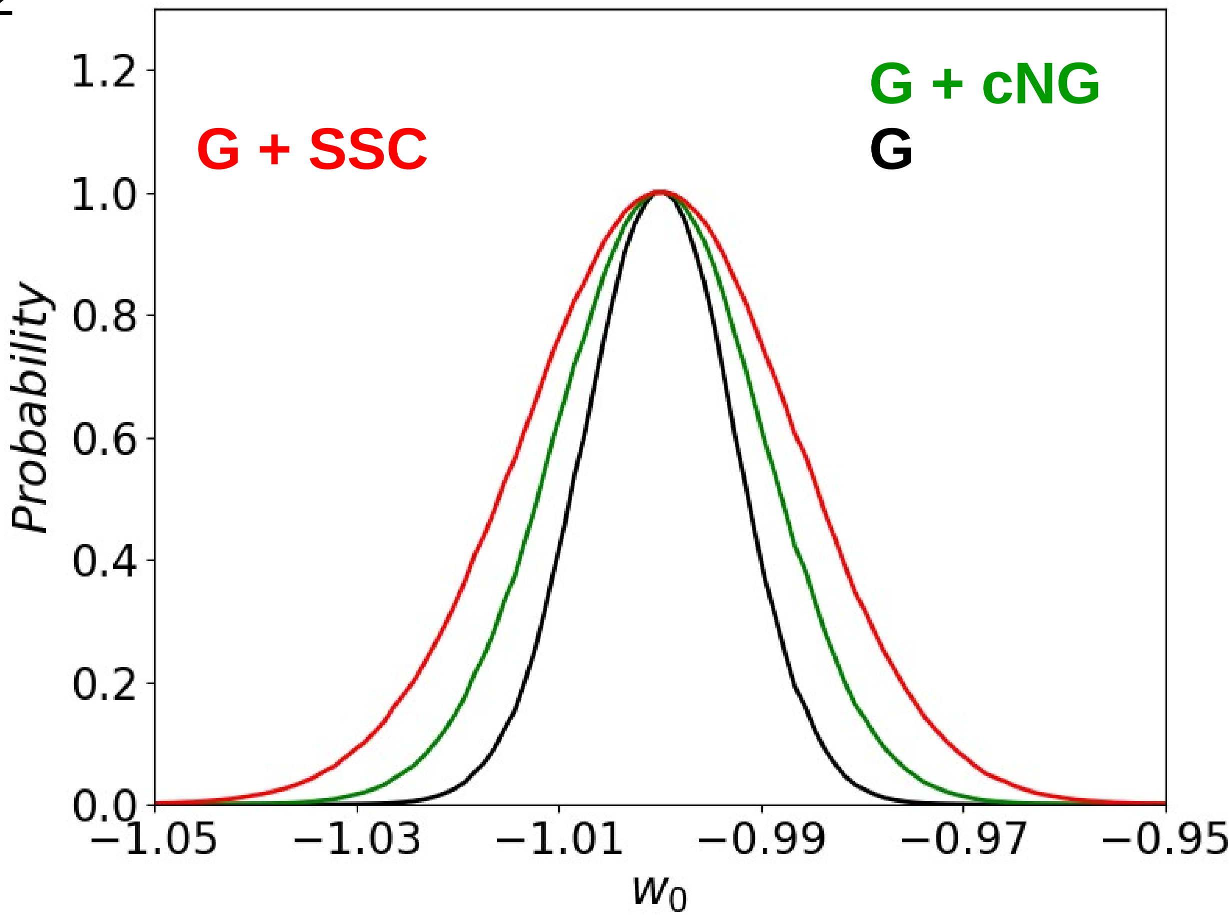

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Barreira, Krause, Schmidt
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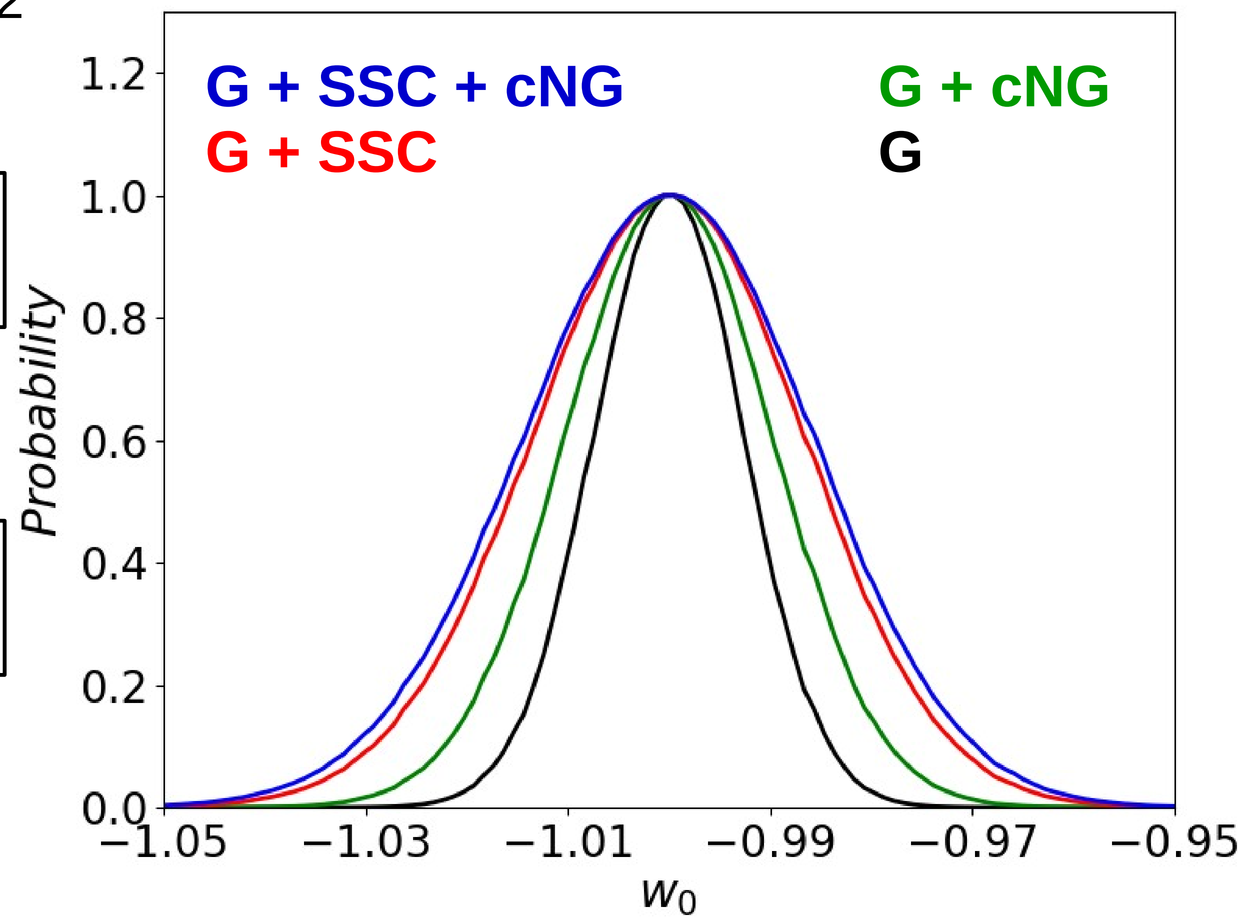
Relative to G, cNG increases error by 38% .

Relative to G+SSC, cNG increases error by only 6% .



The unimportance of the cNG term for future surveys

Euclid-like lensing setup

Barreira, Krause, Schmidt
1807.04266

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  10 t
- 20 e
- Mas
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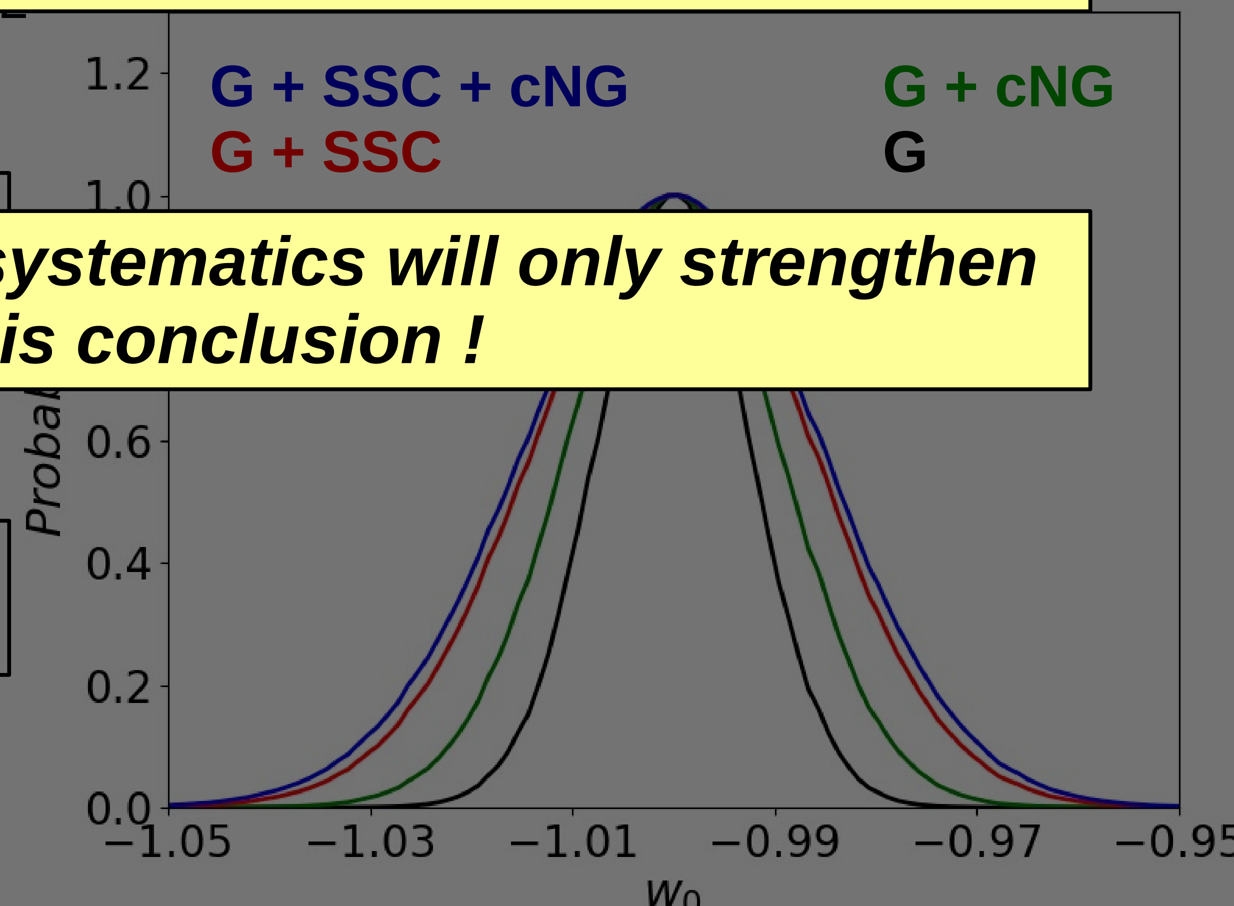
In the presence of the dominant off-diagonal SSC term, cNG contributes only marginally …

Relat

… accounting for systematics will only strengthen this conclusion !

Relative to G+SSC, cNG increases error by only 6% .



Squeezed bispectrum covariance

- **Squeezed bispectrum**

Barreira (arXiv:1901.01243)

$$B_i = B_m(k_i, k_i', s_i), \quad s_i \ll k_i, k_i'$$

- **Covariance decomposition**

$$\mathrm{Cov}\Big(B_1, B_2\Big) = \mathrm{Cov}^{PPP} + \mathrm{Cov}^{BB} + \mathrm{Cov}^{TP} + \mathrm{Cov}^{SSC} + \mathrm{Cov}^{cNG}$$

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Fully given by P(k) and its responses

Squeezed bispectrum covariance

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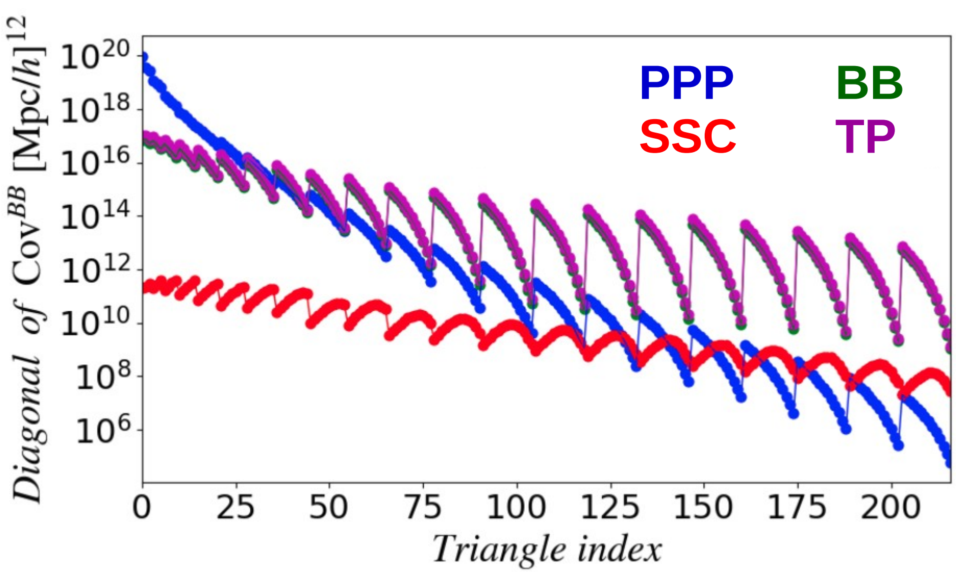


The SSC term is negligible in the squeezed limit.

Responses on Sample Covariance

Off-diagonal covariance is ***dominated by responses*** *.*

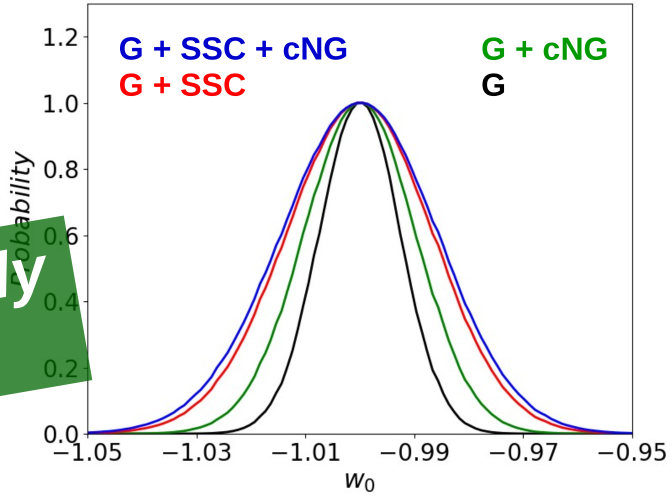
Solved ! Solved !

$$\mathrm{Cov}_\kappa(\boldsymbol{\ell}_1,\boldsymbol{\ell}_2) = \mathrm{Cov}_\kappa^{G} + \mathrm{Cov}_\kappa^{cNG} + \mathrm{Cov}_\kappa^{SSC}$$

Most of it ,
but small anyway!

The unimportance of the cNG term indicates ***its accuracy requirements are much more relaxed*** *that previously thought !*

Accuracy of analytical models is likely sufficient for lensing.



- **Implementation for 3x2pt analyses is underway (stay tuned);**