Non-linear evolution of the BAO scale

in alternative theories of gravity

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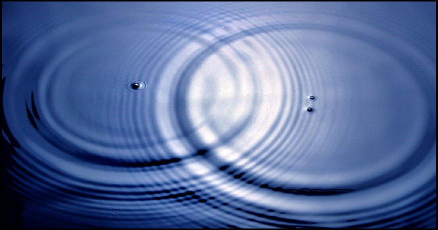
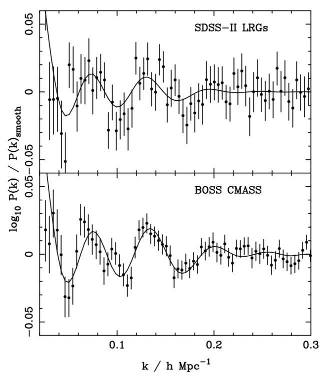
Lawrence Berkeley National Laboratory - April 2015

with E. Bellini (1504.xxxxx)

Motivation





images from from `www.sdss3.org` , `Aubourg` *et al.* `'14, Crocce & Scoccimarro '08`

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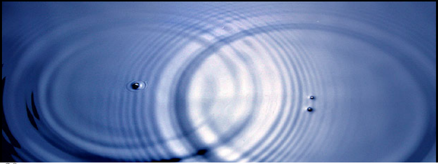

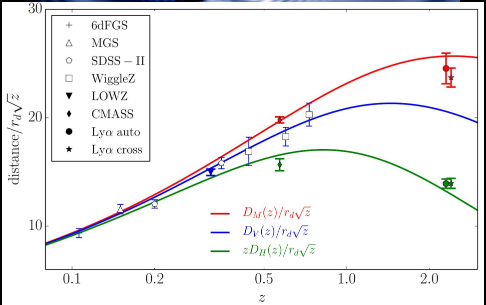

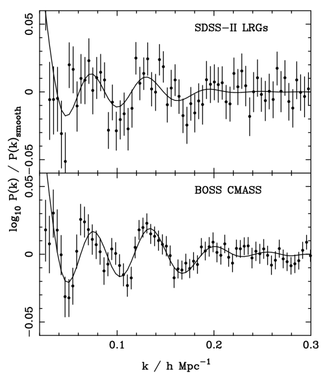

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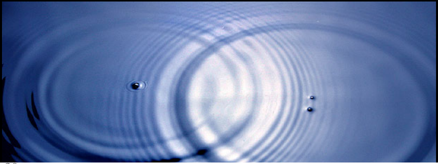

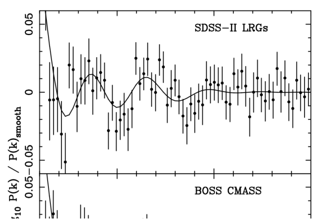

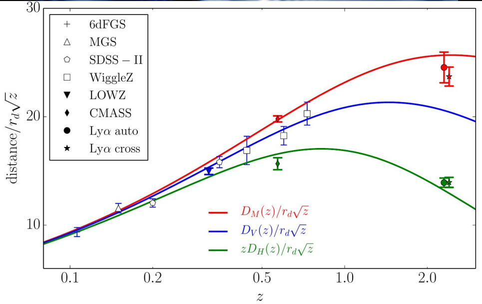

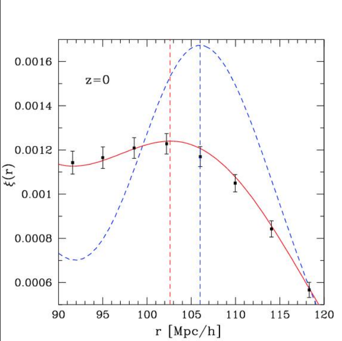

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Menu

1. Alternative theories of gravity

   Scalar-tensor *à la Bellini-Sawicki*

2. BAO shift *à la Sherwin-Zaldarriaga*

   Computation for scalar-tensor theories

Why alternative gravities?

- Inflation again? $n_s \neq 1$

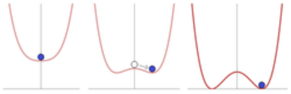
  alternatives to Λ?

- Interesting field-theoretical questions

  find proxy for inflation/quantum gravity?

  viable massive spin-2 particles?

  cosmological constant problems?



- Test gravity on cosmological scales

  effects on cosmological scales?

  model independence of tests?

How to modify gravity

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Diet $\rightarrow$ already very hard!

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Lorentz + QM $\Rightarrow$ restrictions on massless graviton interactions!

(Weinberg '64)

Einstein gravity: only covariant metric theory with 2nd order eqs.

(Lovelock '71)

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Need to give up some of the assumptions:

- Add degrees of freedom:
  - Massive gravity: → 5 d.o.f.
  - Scalar-tensor: → 2+1 d.o.f.

  (Decoupling limit)
  - vector-tensor, tensor-vector-scalar (TeVeS), $\cdots$
- Lorentz violation, Non-local interactions, $\cdots$

Massive Gravity

(Reviews: de Rham 2014, Hinterbichler 2011)

- ⋆ Very difficult problem: $\sim 1939 - 2010$ to "solve"!
- ⋆ Degrees of freedom:

  $g_{\mu\nu} \rightarrow 10$ components $\rightarrow 2s + 1 = 5$ d.o.f. + ghosts!

- ⋆ dRGT massive gravity (de Rham, Gabadadze, Tolley 2010)

  5 d.o.f. $\rightarrow$ ✓

  flat FRW $\Rightarrow \dot{a}(t) = 0$:(

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$g_{\mu\nu},\ f_{\mu\nu}$: 5+2 d.o.f $\rightarrow \checkmark$

Cosmic evolution + acceleration :)

Either boring ($\approx \Lambda$) or unstable :(

⋆ Multigravity (Hinterbichler Rosen '12), Lorentz-violating (Blas Sibiryakov '14)

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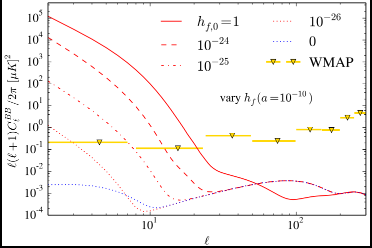

(Amendola, Könnig, Martinelli, Pettorino, MZ '15)

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Scalar-Tensor gravity

⋆ Old-School: $f(\phi)R + K(X,\phi)$ $X \equiv -(\partial\phi)^2/2$

$\supset$ quintessence, $f(R)$, Brans-Dicke (Jordan '59, Brans & Dicke '61)

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⋆ Horndenski's Theory (1974)

$g_{\mu\nu} + \boxed{\phi}$ + Local + 4-D + Lorentz Theory with 2^{nd} order Eqs.

$$G_2(X,\phi) - G_3(X,\phi)\Box\phi + G_4(\phi,X)R + G_{4,X}\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]$$

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$\supset$ all Old-school,

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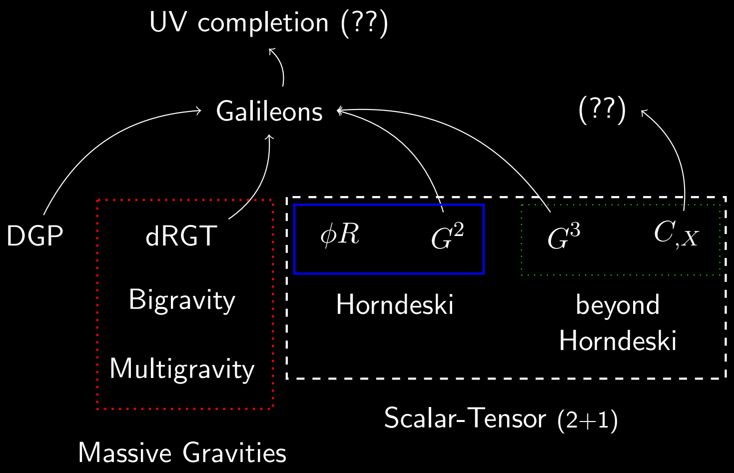
- ⋆ Beyond Horndeski (MZ & Garcia-Bellido '13)
  - $\supset$ General disformal coupling (Bekenstein '92)
  - $\supset$ "Covariantized" galileons (Gleyzes *et al.* '14)

A landscape of theories



Effective theory approach to DE:

$\exists N \sim \infty$ many models of gravity $\Rightarrow$ many papers!

Need systematic approach:

Post Friedman	15-22 $\times f_i(t)$	Baker *et al.* '12
EFT for DE	7-9 $\times f_i(t)$	Gubitosi *et al.*, Bloomfield *et al.* '12
		Gleyzes et al. '13 & '14, $\cdots$
max freedom, min cost	$\boxed{5 \times f_i(t)}$	Bellini & Sawicki '14

- Parameterizations only for linear perturbations
- Beyond linear order: Need underlying theory $\Rightarrow$ Horndeski

Horndeski in four/six words (Bellini & Sawicki JCAP '14)

Background $\longrightarrow H(t)$

$$G_2 - G_3\Box\phi + G_4 R + G_{4,X}\left[\nabla\nabla\phi\right]^2 + G_5 G_{\mu\nu}\phi^{;\mu\nu} - \frac{G_{5,X}}{6}\left[\nabla\nabla\phi\right]^3$$

Linear $\longrightarrow$ 4$\times$ $\alpha_i(t)$

Non-linear: 4x new functions, 2 relevant for $k \gg H$

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Kinetic Mixing of $g_{\mu\nu}$ & ϕ

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Variation rate of effective M_p

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Tensor speed excess: α_T

Gravity waves $c_T^2 = 1 + \alpha_T$

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Non-linear: 4x new functions, 2 relevant for $k \gg H$

Quartic coupling α_4

fromm $G_{4,X}$, G_5

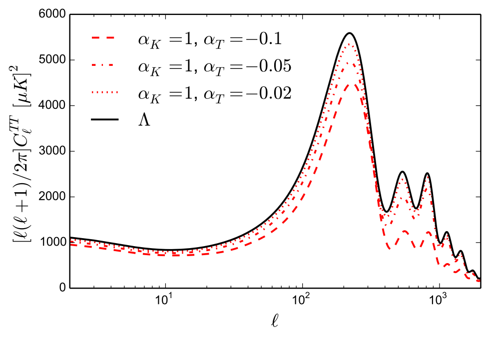
Quintic coupling α_5

from $G_{5,X}$

Testing gravity

- Kineticity $\alpha_K = 1$
- Braiding $\alpha_B = 0$
- M_p running $\alpha_M = 0$
- Tensor speed $1 + \alpha_T$

$\Rightarrow$ observable predictions



Soon in a Boltzmann code: *Hi-CLASS* (Bellini, Lesgourgues & MZ)

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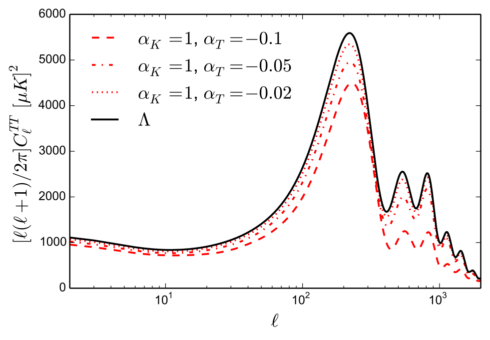

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Theory specific relations:

- Quintessence: only $\alpha_K \propto \Omega_{\rm DE}$
- JBD theories: only $\alpha_K,\ \alpha_B = -\alpha_M$, $\quad f(R) \Rightarrow \alpha_K = 0$
- Galileon-like: $\boxed{\alpha_B + \alpha_M,\ \alpha_T,\ \alpha_4, \alpha_5 \neq 0}$

$\Rightarrow$ Stronger non-linear corrections

Menu

1. Alternative theories of gravity ✓

   Scalar-tensor *à la Bellini-Sawicki* ✓

2. → BAO shift *à la Sherwin-Zaldarriaga*

   Computation for scalar-tensor theories

Non-linear evolution of the BAO scale

BAO scale in the galaxy distribution: comoving standard ruler:

$$r_{\rm phys} = a(t) \cdot r_{\rm coord}$$

Non-linear BAO evolution $(z{=}0)$

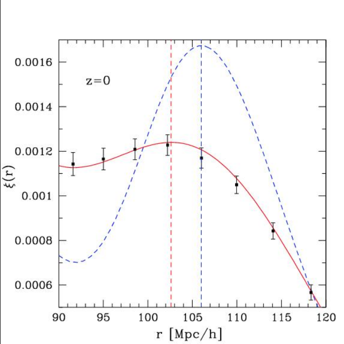
- Shift $\sim 0.3\%$ smaller
- Broadening ~ 8 Mpc$/h$

(Prada *et al.* 1410.4684)

Adjust to a template:

$$P(k) = P_{11}(k/\alpha) \approx P_{11}(k) - \boxed{(\alpha-1)kP'_{11}(k)}$$

(Padmanabhan & White '08)



(from Crocce & Scoccimarro - PRD '08)

Eulerian perturbation theory

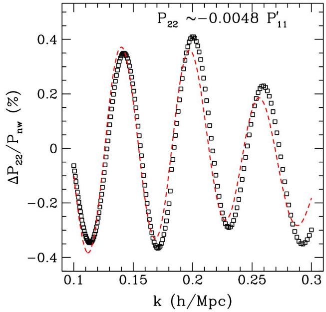
$$P(k) = \underbrace{\underbrace{P_{11}(k)}_{\text{linear}} + \underbrace{\sum_n P_{1n}(k)}_{\text{propagator}}}_{\propto P_{11}(k)} + \underbrace{\sum_{n,m>1} P_{nm}(k)}_{\text{mode coupling}} \qquad (P_{nm} \sim \langle \delta_n \delta_m \rangle)$$

Adjust to a template:

$$\begin{aligned} P(k) &= P_{11}(k/\alpha) \\ &\approx P_{11}(k) - \boxed{(\alpha - 1) k P'_{11}(k)} \end{aligned}$$

- $P_{1n} \propto P_{11}$
- Mode coupling: $\supset (\cdots) k P'_{11}$

$\Rightarrow$ leading order shift from $P_{22}(k)$



(from Padmanabhan & White - PRD'09)

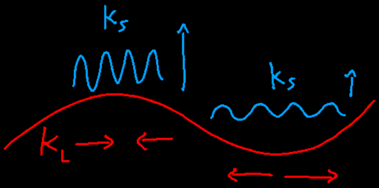
BAO shfit peak-background split (Sherwin & Zaldarriaga - PRD '12)

A simple physical picture

Overdense regions:

- $\downarrow$ expansion $\Rightarrow \downarrow$ physical scales
- $\uparrow$ clustering $\Rightarrow \uparrow$ weight in $\xi(r)$

& opposite for underdense regions



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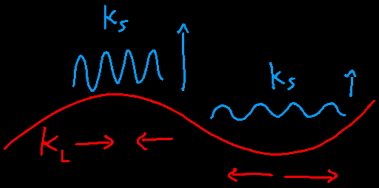

$$P_{22} = \int \frac{d^3q}{(2\pi)^3} \left(F_2(\vec{k}-\vec{q},\vec{q})\right)^2 P_0(\vec{k}-\vec{q})P_0(\vec{q})$$

Expand on $\dfrac{k}{q}$ + angular integral:

$$P_{22}(k) \approx \cdots - \underbrace{\frac{47}{105}\left\langle\delta_L^2\right\rangle}_{\alpha-1} kP'(k) + \cdots \qquad (k \gg q)$$

Shift $\propto$ variance over long modes $\left\langle\delta_L^2\right\rangle \approx \sigma^2_{r_{BAO}}$

Mode coupling & BAO shift in modified gravity

Non-linear gravitational interactions $\Psi = \mathcal{I}_1 \delta\rho + \mathcal{I}_2 \delta\rho^2 + \cdots$

For *sub-horizon* $(p, q \gg H)$ + *quasi-static* $(\dot{\Psi} \ll k\Psi)$ evolution:

$$F_2(p,q,\mu) = C_0(t) + C_1(t)\mu\left(\frac{p}{q} + \frac{q}{p}\right) + C_2(t)\left[\mu^2 - \frac{1}{3}\right]$$

- Galilean invariance $\Rightarrow C_1 = \frac{1}{2}$
- Universal coupling $\Rightarrow C_0 + \frac{2}{3}C_2 = 2C_1$

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Expand P_{22} in $k \gg q$ + angular integral

Generalized Sherwin-Zaldarriaga shift formula:

$$\alpha_k - 1 = \frac{2}{5}\left(2C_0(t) - \frac{1}{2}\right)\langle\delta_L^2\rangle \Rightarrow \begin{cases} \text{Linear growth: } \langle\delta_L^2\rangle \approx \sigma^2_{r_{BAO}} \\ \text{Non-linear gravity: } C_0 \neq \frac{17}{21} \end{cases}$$

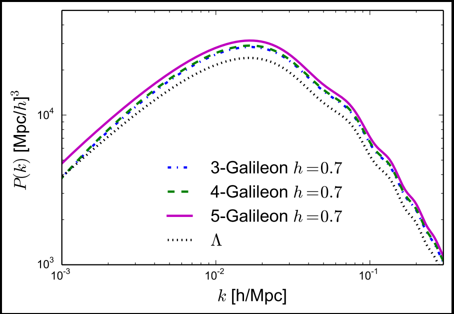
Growth in Horndeski

$$\alpha_k - 1 \approx \frac{2}{5}\left(2C_0(t) - \frac{1}{2}\right)\sigma^2_{r_{BAO}} \Rightarrow \begin{cases} \text{Linear: } \sigma^2_{r_{BAO}} \\ \text{Non-linear: } C_0 \end{cases}$$

- Implement linear equations in a Boltzmann code
- Obtain $\delta_1(z)$, $P_{11}(k)$, & $\sigma_{r_{BAO}}$

Galileons (Barreira *et al.* '14) + Planck '15 → $h, \Omega_m, \cdots$

Mod.	σ_{r_s}	rel. dev.
Λ	0.067	0
Gal-3	0.073	9%
Gal-4	0.074	10%
Gal-5	0.077	14%



Mode coupling in Horndeski

$$\alpha_k - 1 \approx \frac{2}{5}\left(2C_0(t) - \frac{1}{2}\right)\sigma^2_{r_{BAO}} \Rightarrow \begin{cases} \text{Linear: } \sigma^2_{r_{BAO}} \\ \text{Non-linear: } C_0 \end{cases}$$

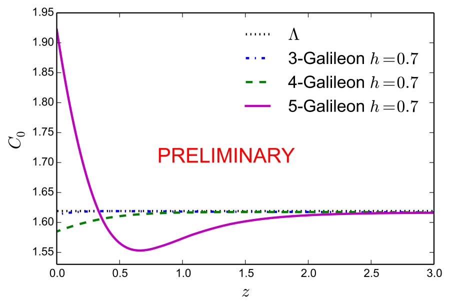
- Expand $\mathcal{L}_H$ over FRW:

  scalar perturbations $\rightarrow \mathcal{O}(\delta^3)$
- Quasi-static + sub-horizon approx.
- Identify inhomogeneous sources:

$$\ddot{\delta}_2 + \cdots = S_2\left[\delta_1(p), \delta_1(q)\right]$$

- Integrate monopole component

$$S_2 \longrightarrow C_0(t)$$



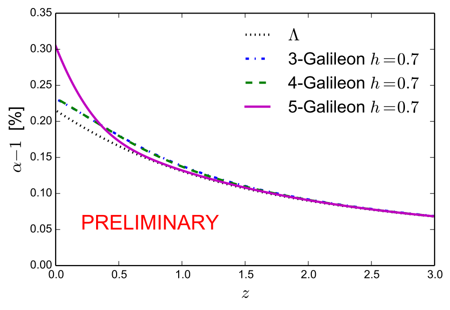
BAO Shift in Horndeski: preliminary results

$$\alpha_k - 1 \approx \frac{2}{5}\left(2C_0(t) - \frac{1}{2}\right)\sigma^2_{r_{BAO}} \Rightarrow \begin{cases} \text{Linear: } \sigma^2_{r_{BAO}} \\ \text{Non-linear: } C_0 \end{cases}$$

- Can have significant enhancement at $z \sim 0$
- Mostly from growth at higher z

Galileons (Barreira *et al.* '14) + Planck '15 $\rightarrow h, \Omega_m, \cdots$

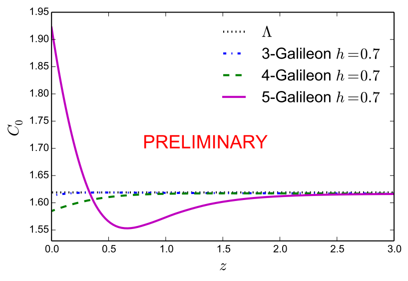
Mod.	$100(\alpha - 1)$	rel. dev.
Λ	0.20	0
Gal-3	0.24	17%
Gal-4	0.24	17%
Gal-5	0.34	66%



Two regimes for gravity

⋆ Mild modifications of gravity

- if $\alpha_i \lesssim 1$
- 3 & 4 Galileon
- all old-school/$f(R)$
  $(\alpha_M + \alpha_B,\, \alpha_T,\, \alpha_4 = 0,\, \alpha_5 = 0)$



⋆ Strong modifications of gravity

- $|\alpha_i| \gtrsim 1$
- 5 Galileon $\alpha_4,\, \alpha_5 \approx 6!$

⟶ See also Bellini, Jimenez & Verde (1504.04341)

Conclusions

- Massive gravity is complicated
- Contemporary scalar-tensor cosmology well understood
- BAO shift for Horndeski (simplest computation):
  - ⋆ rather small $\lesssim 0.3\%$
  - ⋆ $10 - 70\%$ enhancement at $z \sim 0$
- BAO is great standard ruler
  - ⋆ except for extreme gravity in future surveys (?)
- Need to improve computation:
  - ⋆ halos, redshift space, forecasts...
  - ⋆ higher-order, broadening, reconstruction, bispectrum...

Backup Slides

Massive Gravity → Scalar-tensor

Helicity decomposition:

$$m\text{-graviton (5 d.o.f.)} \xrightarrow{E \gg m} \begin{cases} \not{m}\text{-graviton} & \text{(2 d.o.f.)} \\ \text{vector} & \text{(2 d.o.f.)} \\ \boxed{\text{scalar} \quad \phi} & \text{(1 d.o.f.)} \end{cases}$$

Decoupling limit ($M_p \to \infty$, $m^2 M_p = \mathcal{M}$):

$$\mathcal{L} \approx (\partial\phi)^2 + \underbrace{\phi T_m}_{\text{coupling}} + \underbrace{\frac{1}{\mathcal{M}^3}(\partial\phi)^2\partial^2\phi + \cdots}_{\text{Derivative interactions}}$$

Scalar-tensor theories (2+1 d.of.)

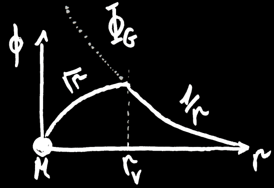
- Easy to reproduce cosmological observations
- Retain features from massive gravity: scalar-force

Need to go cosmological

Massive gravity $\xrightarrow{m\to 0}$ standard gravity (Vainshtein '72)

$$\Rightarrow\ \Box\phi + m^{-2}\left((\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right) = \alpha M\delta(r)$$

$$\phi \propto \begin{cases} r^{-1} & \text{if } r \gg r_V \\ \sqrt{r} & \text{if } r \ll r_V \end{cases}$$



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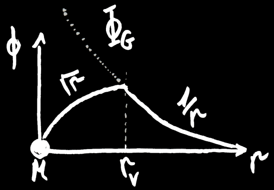

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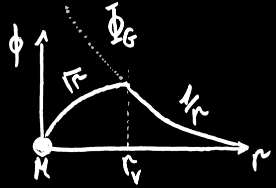

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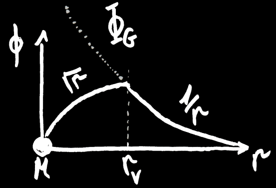

Vainshtein radius $r_V \propto (GM/m^2)^{1/3}$

Small deviations below r_V

(opposite to Yukawa $\Phi_m = \dfrac{e^{-mr}}{r}$)

Need to go cosmological

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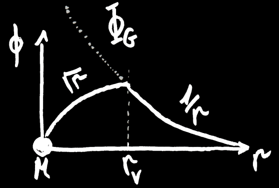


Vainshtein radius $r_V \propto (GM/m^2)^{1/3}$

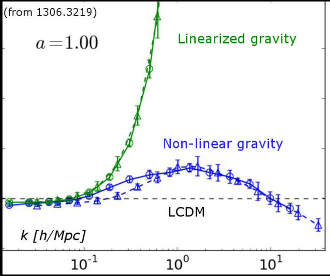
Small deviations below r_V

(opposite to Yukawa $\Phi_m = \dfrac{e^{-mr}}{r}$)

Cosmology: $\to$ test on large scales

- Background expansion
- Baryon acoustic oscillations

$\to \sim 100/h$Mpc



(ΔP_k from Barreira, Li *et al.* '13)

Timeline

	Massive gravity	Scalar-tensor
1910's		Nordstrom Theory
1930's		Kaluza-Klein
1930's	Fierz-Pauli	
1960's	vDVZ discont.	Brans-Dicke
1970's	Vainshtein	
	BD ghost	
		Horndeski's theory
1980's		Inflation
1990's		
2000's		quintessence
	DGP	Galileons
2010's	dRGT	Horndeski redescovered
	bigravity	beyond Horndeski