Nonlinear Reconstruction

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Overview

- Cosmic power dilemma
- Reconstruction algorithm
- Hidden physics
- Implementation and results
- Applications

Cosmic power dilemma

- Power spectrum theoretically cleanest at low k
- Statistically best at high k
- Solutions: Modeling the nonlinearities? Sampling at low density to bypass nonlinear effects? Reducing nonlinearities?

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Reconstruction

- Standard (linear) BAO reconstruction:

$$\boldsymbol{x} \to \delta(\boldsymbol{x}) \to \delta * W(\boldsymbol{x}) \to \boldsymbol{s}(\boldsymbol{x}) \to \delta_r(\boldsymbol{x}) \equiv \delta_d(\boldsymbol{x}) - \delta_s(\boldsymbol{x})$$

- fiducial cosmology, smoothing, linear flows

- Nonlinear reconstruction:

$$\boldsymbol{x} \to \delta(\boldsymbol{x}) \to \delta_r(\boldsymbol{\xi}) \equiv -\nabla^2 \phi(\boldsymbol{\xi})$$

- non-parametrically, no smoothing, nonlinear flows

Algorithm

- Potential iso-mass gauge (constant mass per volume element)

- Define a pure gradient coordinate transformation:

$$x^i = \xi^\mu \delta^i_\mu + \Delta x^i, \ \Delta x^i \equiv \frac{\partial \phi}{\partial \xi^\nu} \delta^{i\nu}$$

- Positive definite, no grid overlapping, moving the grid slowly

$$\sqrt{g} = \det(\partial x^i / \partial \xi^\mu) > 0, \ \partial x^a(\boldsymbol{\xi}) / \partial \xi^a > 0$$

Algorithm

- Continuity equation: $\frac{\partial \rho}{\partial t} + \frac{\partial \rho v^i}{\partial x^i} = 0$
- Time-dependent coordinate transformation: $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{\xi}, t)$
- Continuity equation in curvilinear coordinates:

$$\frac{\partial \sqrt{g}\rho}{\partial t} + \frac{\partial}{\partial \xi^\mu}\left[\sqrt{g}\rho e_i^\mu \left(v^i - \frac{\partial \dot{\phi}}{\partial \xi^\nu}\delta^{\nu i}\right)\right] = 0$$

- Initial state: $\rho(\boldsymbol{\xi}, t_0) = \rho(\boldsymbol{x}),\ v^i(\boldsymbol{x}) = 0,\ \phi(\boldsymbol{\xi}, t_0) = 0$
- Evolution equation for the grid

$$\partial_\mu(\rho\sqrt{g}e_i^\mu \delta^{i\nu}\partial_\nu \Delta\phi) = \Delta\rho,\ \Delta\rho \equiv \bar{\rho} - \rho\sqrt{g}$$

Algorithm

- Perturbative solution: $\phi = \Delta\phi^{(1)} + \Delta\phi^{(2)} + \Delta\phi^{(3)} + \cdots$

- Iterating the evolution equation:

$$\partial_\mu(\rho\sqrt{g}e_i^\mu\delta^{i\nu}\partial_\nu\Delta\phi) = \Delta\rho,\ \Delta\rho \equiv \bar{\rho} - \rho\sqrt{g}$$

- Reconstructed density (potential iso-mass gauge):

$$\delta_r(\boldsymbol{\xi}) \equiv -\nabla^2\phi(\boldsymbol{\xi})$$

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Hidden physics

- Schematic description:

$$\delta(\boldsymbol{x}) = \delta(\boldsymbol{q}) + \boldsymbol{s}(\boldsymbol{q}) \cdot \nabla\delta(\boldsymbol{q})$$

- LPT interpretation:

$$\delta(\boldsymbol{x}) = \delta_L(\boldsymbol{q}) + \frac{17}{21}\delta_L^2(\boldsymbol{q}) + \boldsymbol{s}(q) \cdot \nabla\delta(\boldsymbol{q}) + \frac{2}{7}K_{ij}(\boldsymbol{q})K_{ij}(\boldsymbol{q}) + \cdots$$

- Potential iso-mass gauge eliminates nonlinearities induced by coordinate transformation:

$$\delta_r(\boldsymbol{q}) = \delta_L(\boldsymbol{q}) + \frac{17}{21}\delta_L^2(\boldsymbol{q}) + \frac{2}{7}K_{ij}(\boldsymbol{q})K_{ij}(\boldsymbol{q}) + \cdots$$

Displacement

- Lagrangian displacement:

$$\boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},t)$$

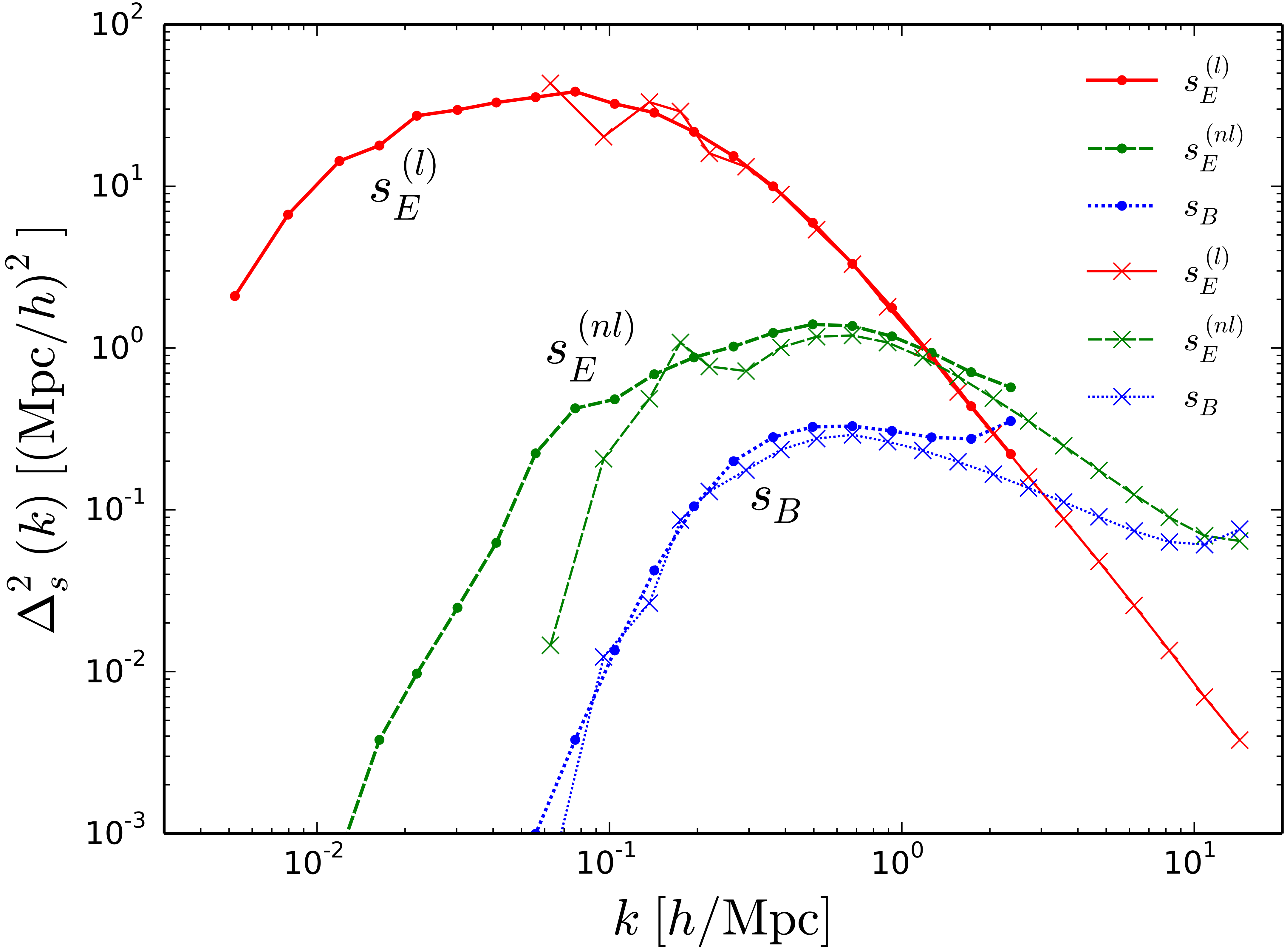
- E/B decomposition: $\boldsymbol{\Psi}(\boldsymbol{q}) = \boldsymbol{\Psi}_E(\boldsymbol{q}) + \boldsymbol{\Psi}_B(\boldsymbol{q})$

- Further decomposition:

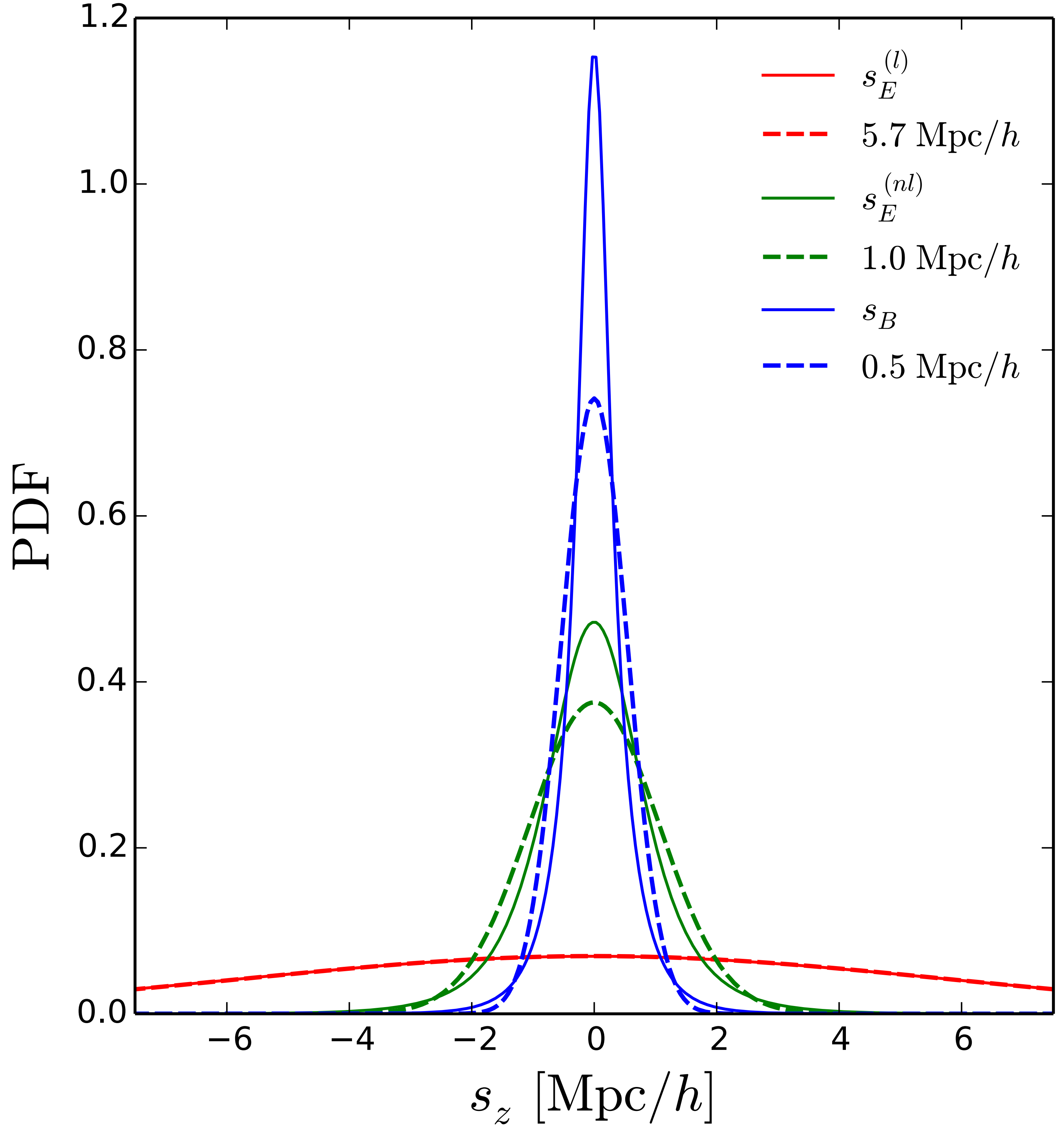
$$\boldsymbol{\Psi}_E(\boldsymbol{q}) = \boldsymbol{\Psi}_E^{(l)}(\boldsymbol{q}) + \boldsymbol{\Psi}_E^{(nl)}(\boldsymbol{q})$$

- Which dominates the shift from Lagrangian to Eulerian coordinates?

Power spectra



PDFs



1.2
1.0
0.8
0.6
0.4
0.2
0.0
PDF
−6
−4
−2
0
2
4
6
s_z [Mpc/h]
$s_E^{(l)}$
5.7 Mpc/h
$s_E^{(nl)}$
1.0 Mpc/h
s_B
0.5 Mpc/h

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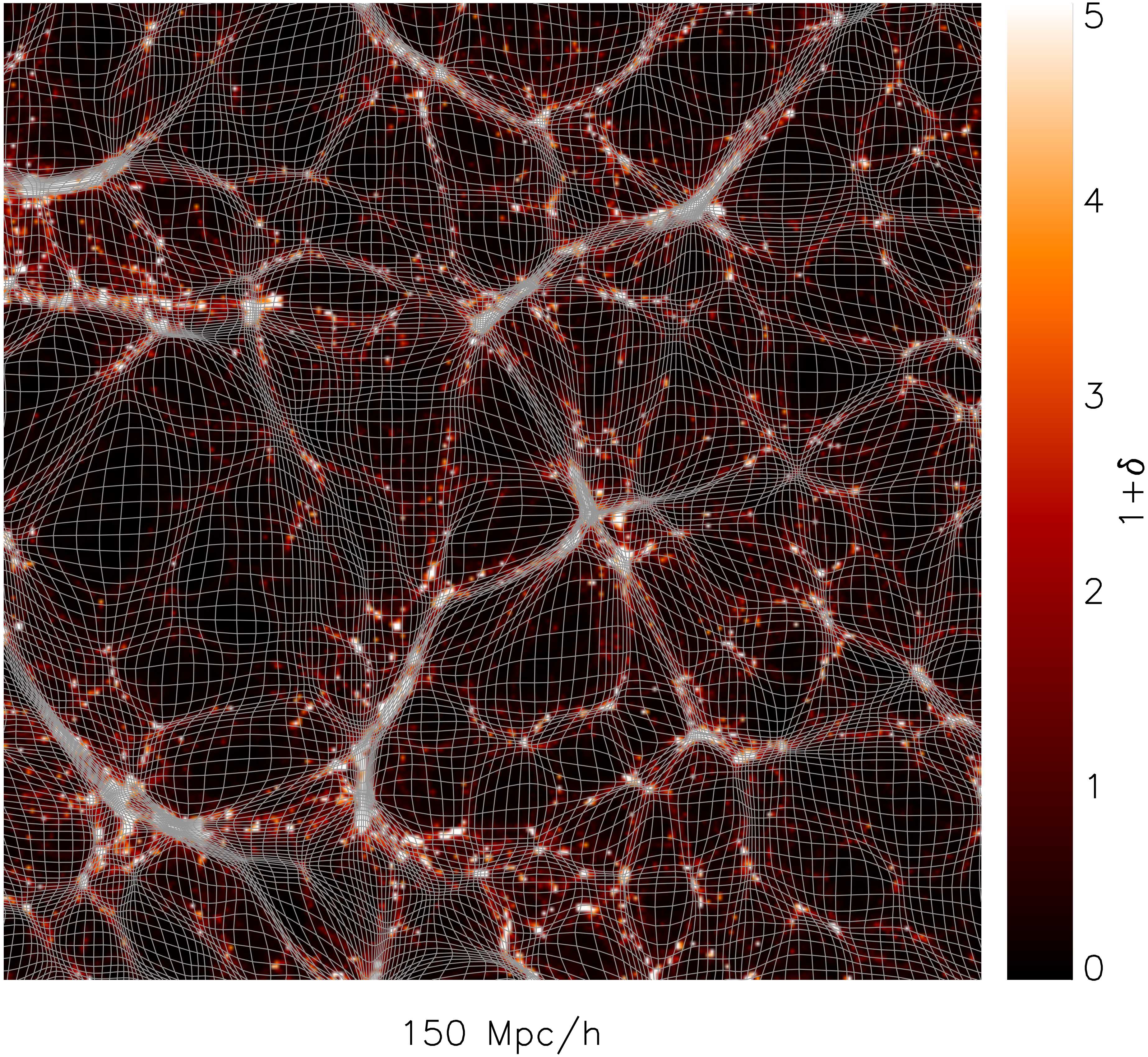
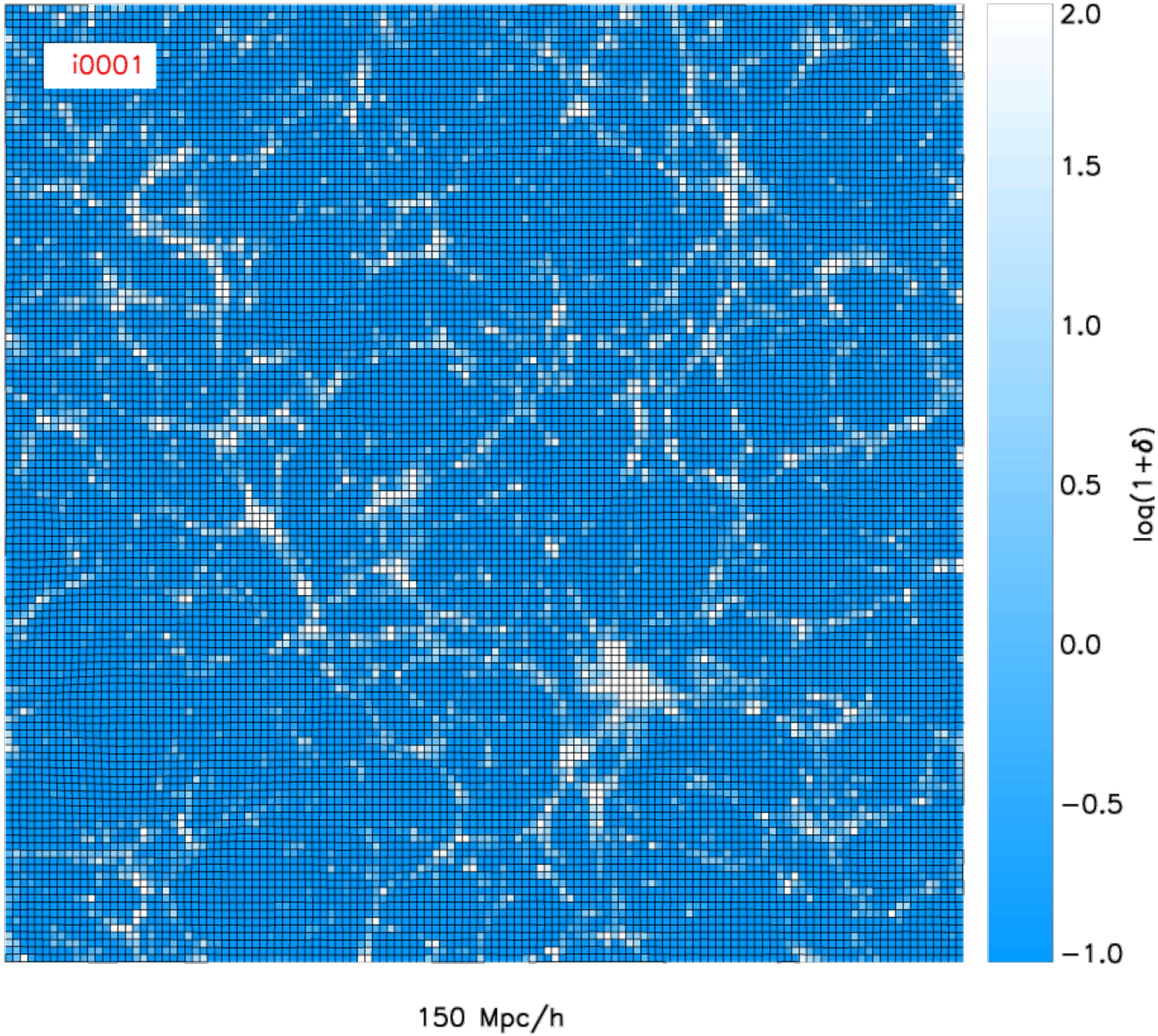
Implementation

- N-body simulation: 2048^3 particles, 600 Mpc/h
- Densities on 512^3 grids as multi-grid input
- Reconstructed density field: $\delta_r(\boldsymbol{\xi}) \equiv -\nabla^2\phi(\boldsymbol{\xi})$

Densities and grids



Movie



Linear signal and noise

- Description of the density modes:

$$\delta(\boldsymbol{k}) = C(\boldsymbol{k})\delta_L(\boldsymbol{k}) + n(\boldsymbol{k})$$

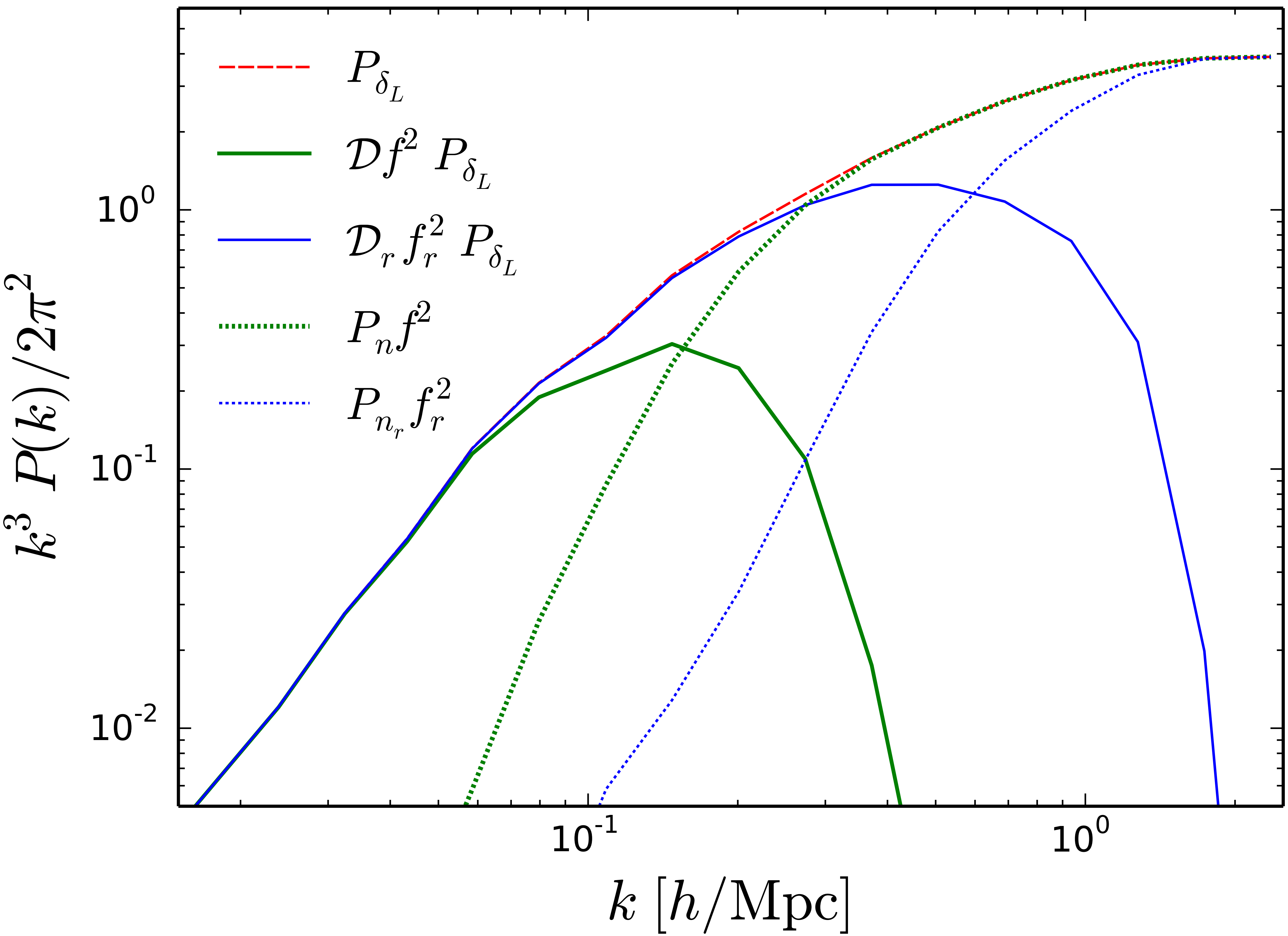
- Correlating with the linear density field:

$$\langle\delta(\boldsymbol{k})\delta_L(\boldsymbol{k})\rangle = C(\boldsymbol{k})\langle\delta_L(\boldsymbol{k})\delta_L(\boldsymbol{k})\rangle$$

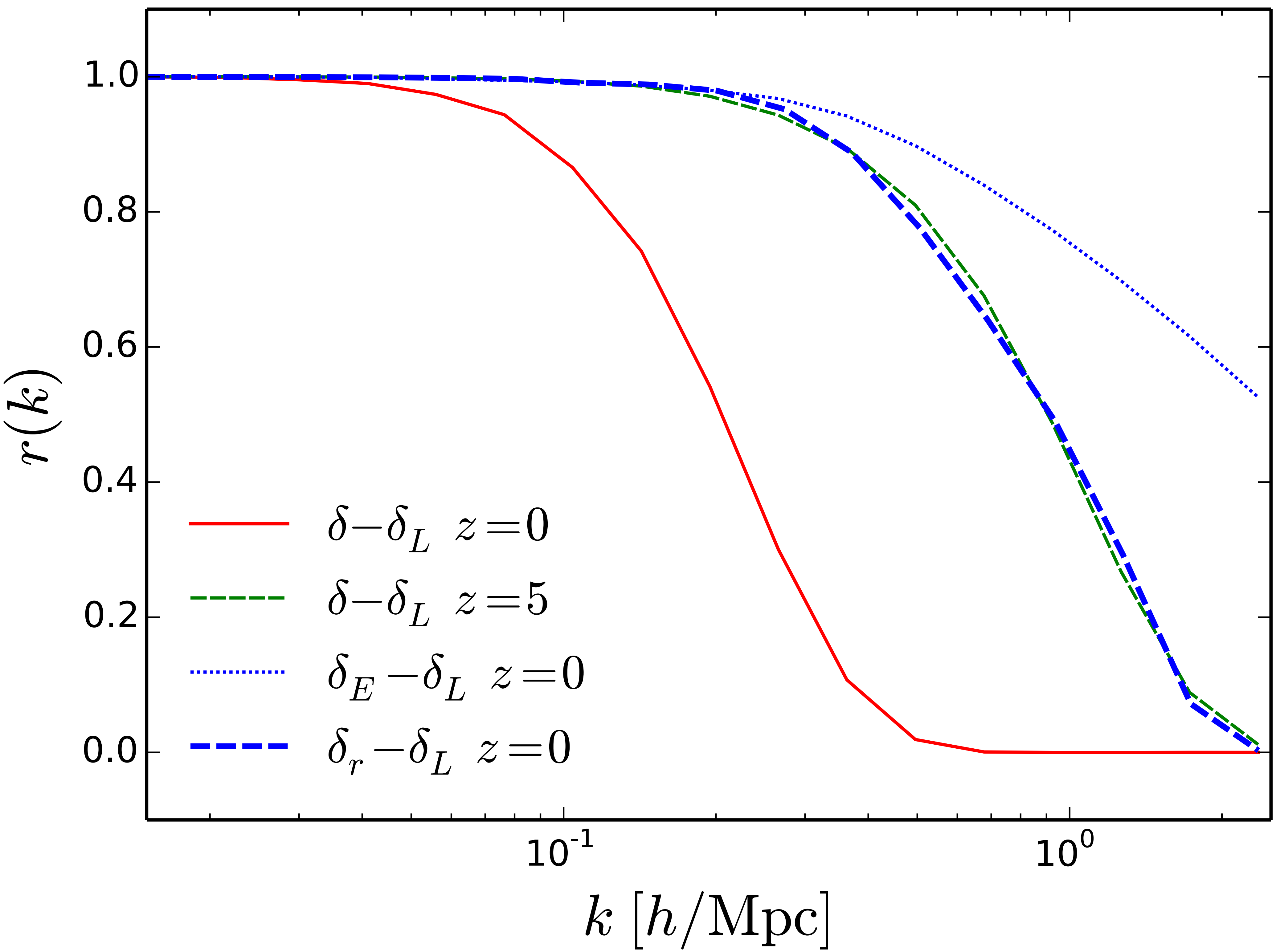
- The propagator: $C(\boldsymbol{k}) = C(k) = P_{\delta\delta_L}(k)/P_{\delta_L}(k)$

$$P_\delta(\boldsymbol{k}) = C^2(k)P_{\delta_L}(k) + P_n(k)$$

Reconstruction results: power spectrum



Reconstruction results: correlation



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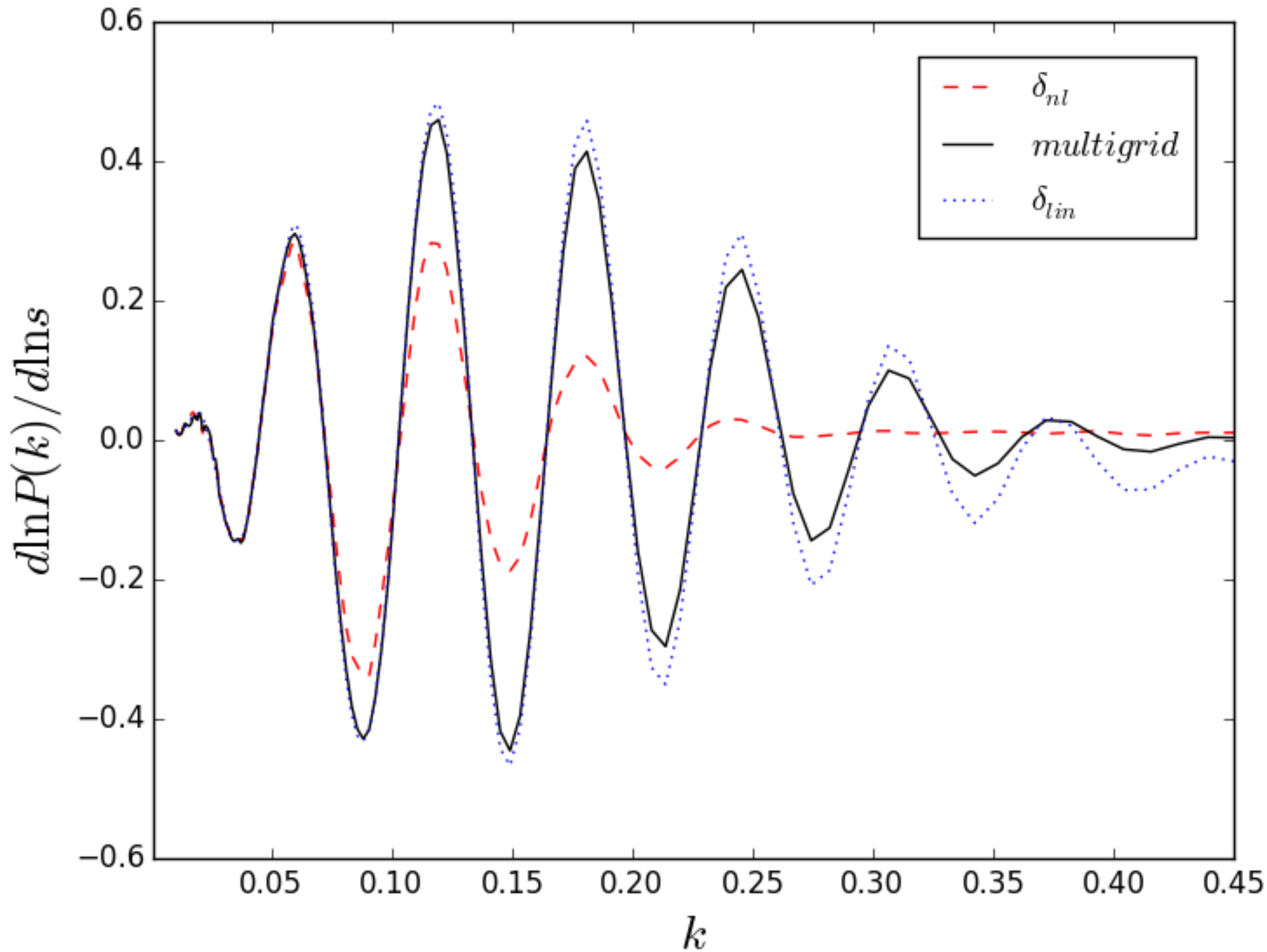
Applications

- BAO, RSD

- peculiar velocity reconstruction

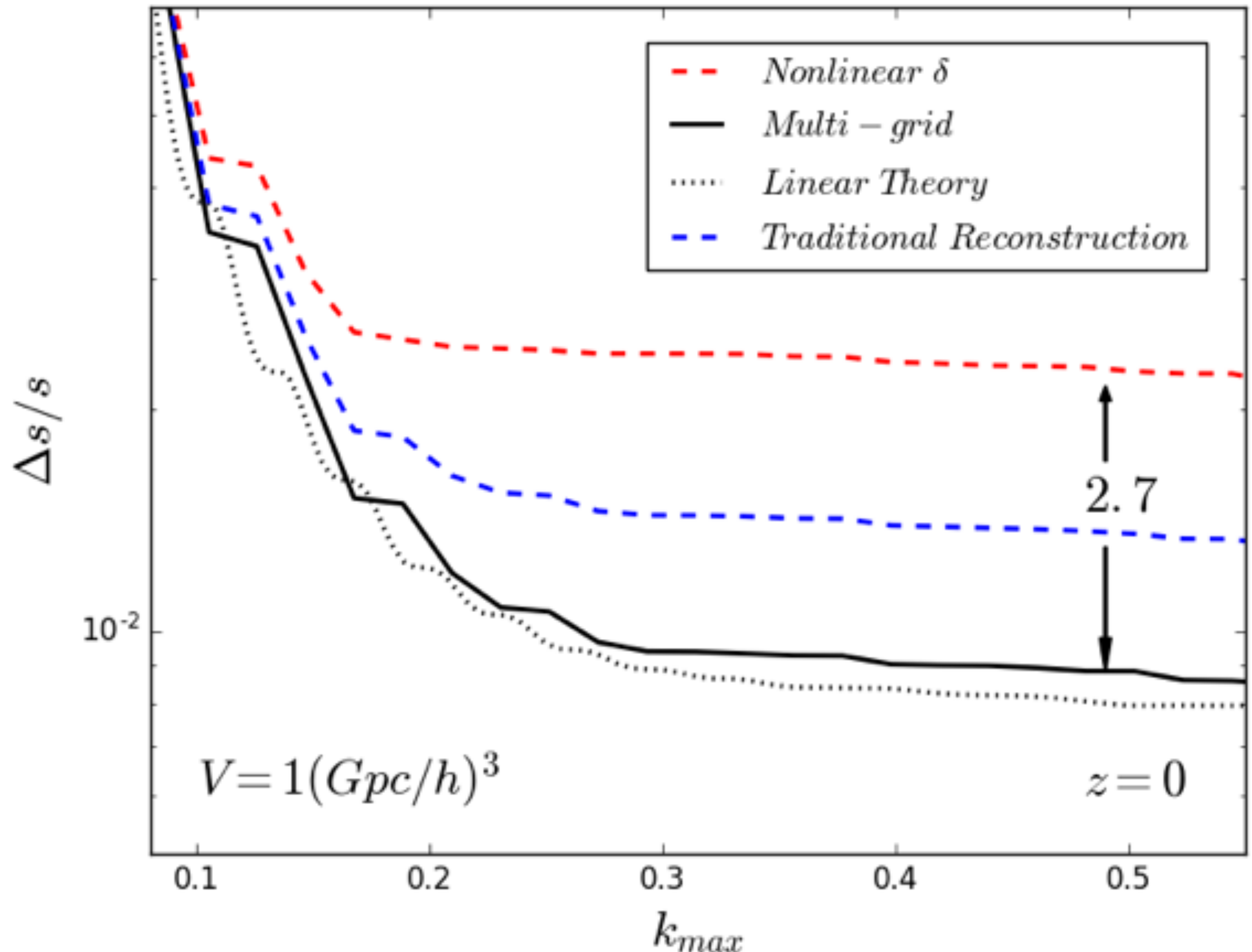
- neutrino mass, modify gravity, primordial non-Gaussianity, and etc

BAO



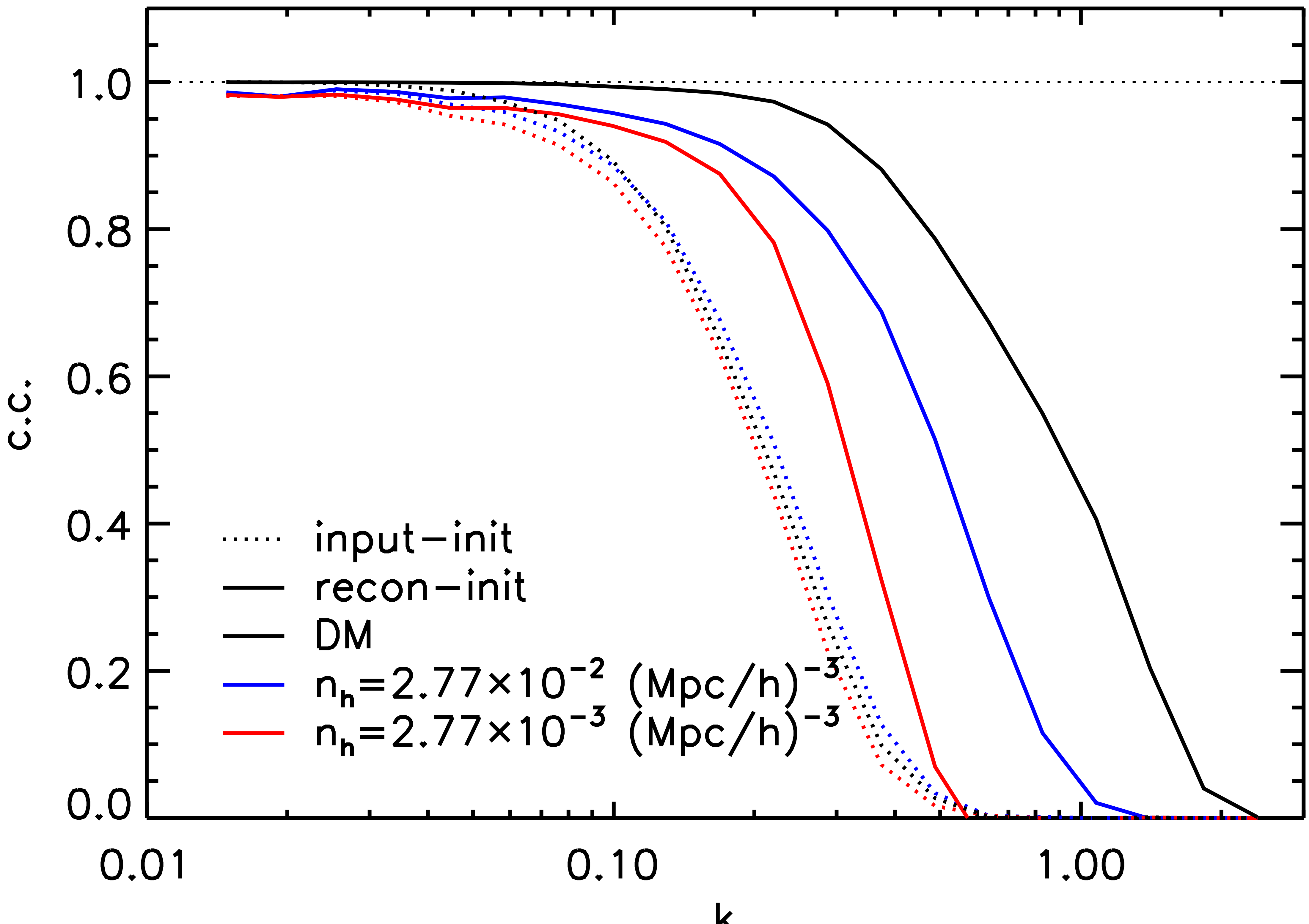
X. Wang et al, in prep

BAO



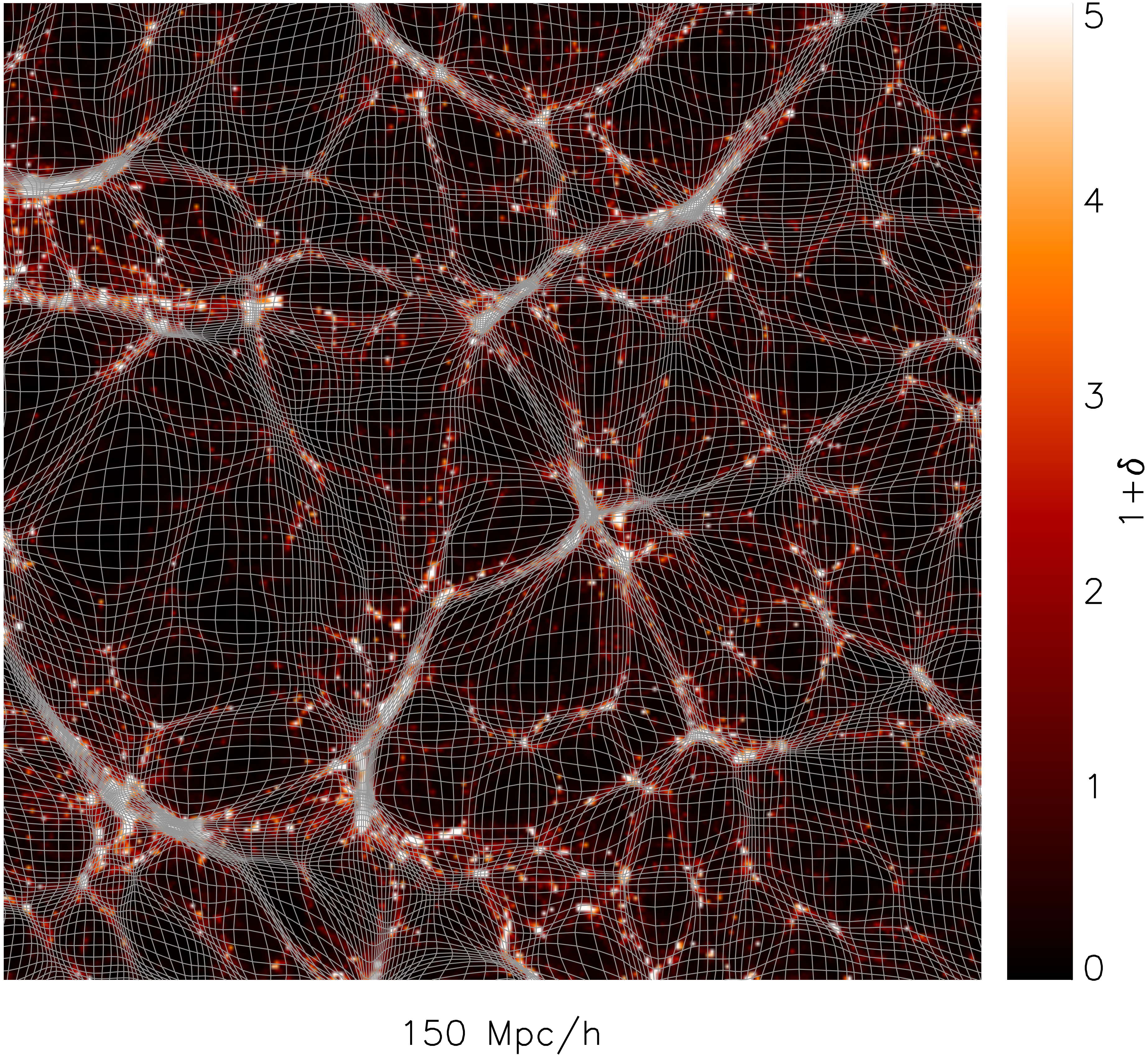
X. Wang et al, in prep

Halos

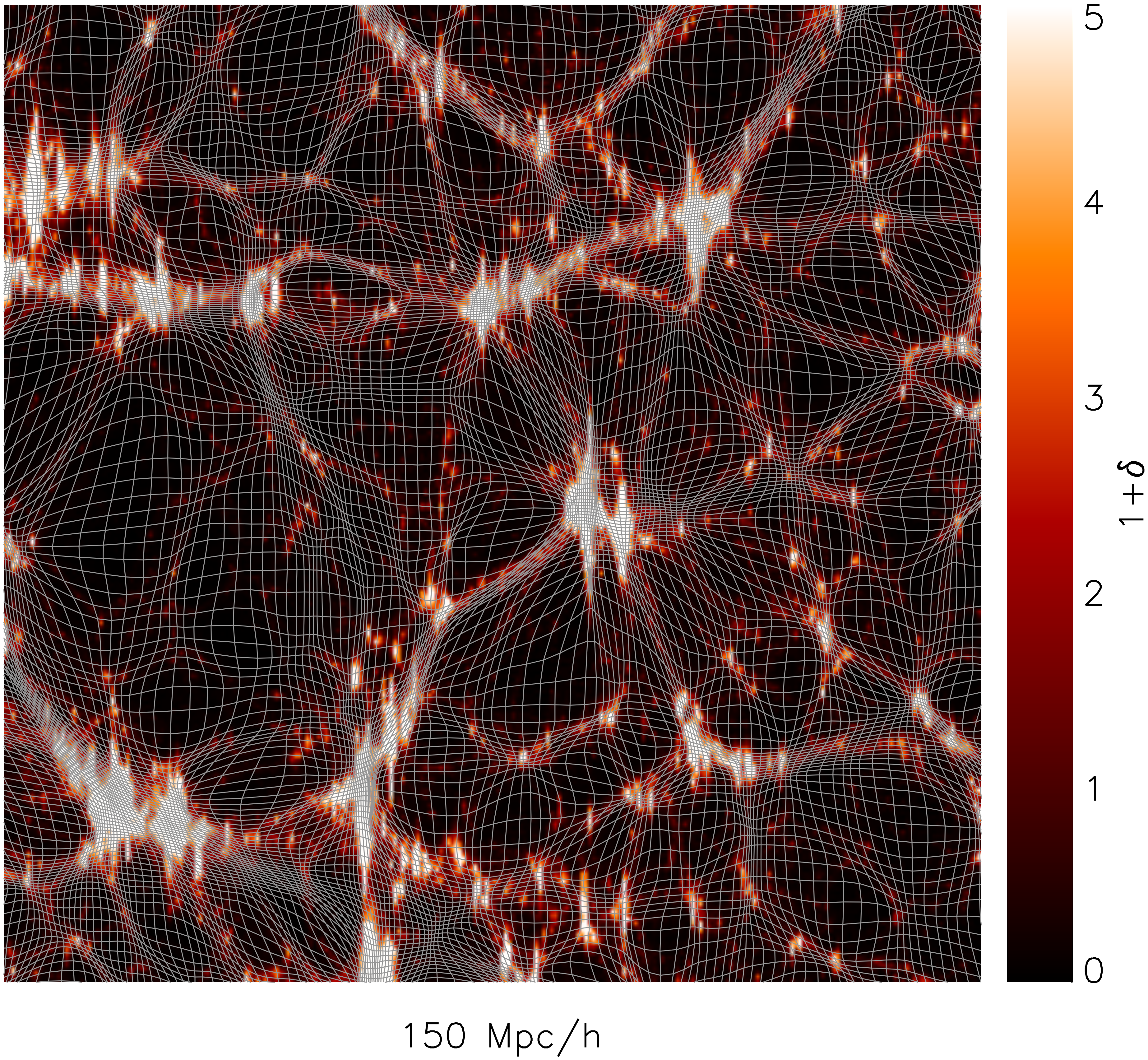


Y. Yu et al, in prep

RSD



RSD



RSD

- Shifts by peculiar velocities: $\boldsymbol{s} = \boldsymbol{x} + \frac{\hat{z} \cdot \boldsymbol{v}(\boldsymbol{x})}{aH} \hat{z}$
- Shifted displacement:

$$\boldsymbol{\Psi}^s(\boldsymbol{q}) = \boldsymbol{\Psi}(\boldsymbol{q}) + \frac{\hat{z} \cdot \boldsymbol{v}(\boldsymbol{q})}{aH} \hat{z}$$

- Reconstructed density field:

$$\delta(\boldsymbol{k}) = C(\boldsymbol{k})\delta_L(\boldsymbol{k}) + n(\boldsymbol{k})$$

- Linear theory: $C(k_\perp, k_{||}) = 1 + f\mu^2$

RSD

- Wiener filtered correlation (isotropic field):

$$\tilde{\delta}(\boldsymbol{k}) = \frac{\delta(\boldsymbol{k})}{C(k_\perp, k_\parallel)} W(k_\perp, k_\parallel)$$

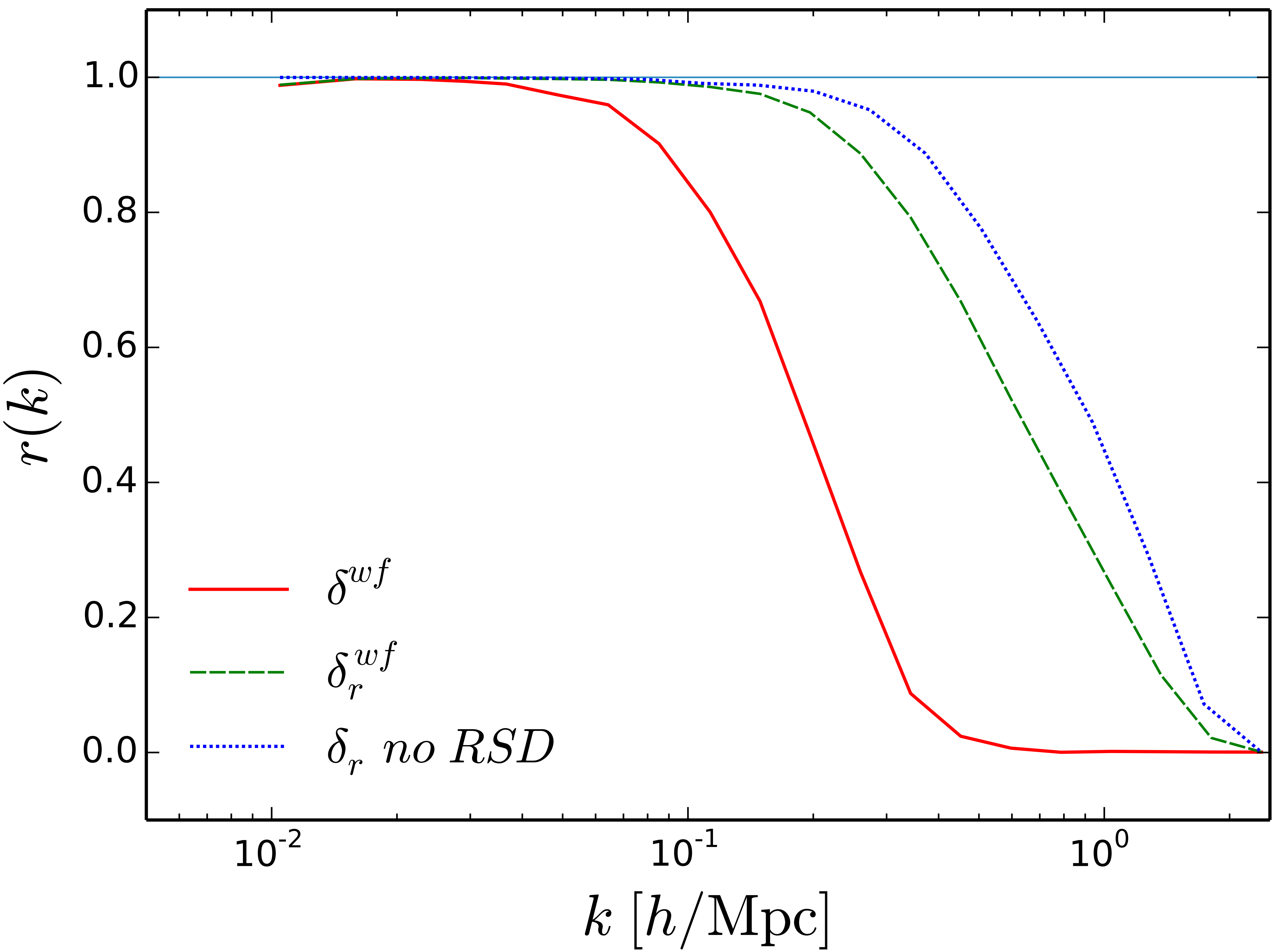
- Raw correlation (anisotropic field):

$$P_{\delta\delta_L}(\boldsymbol{k}) = (1 + f\mu^2) P_{\delta_L}(\boldsymbol{k}), \;\; P_\delta(\boldsymbol{k}) = (1 + f\mu^2)^2 P_{\delta_L}(\boldsymbol{k})$$

- Correlation in linear theory

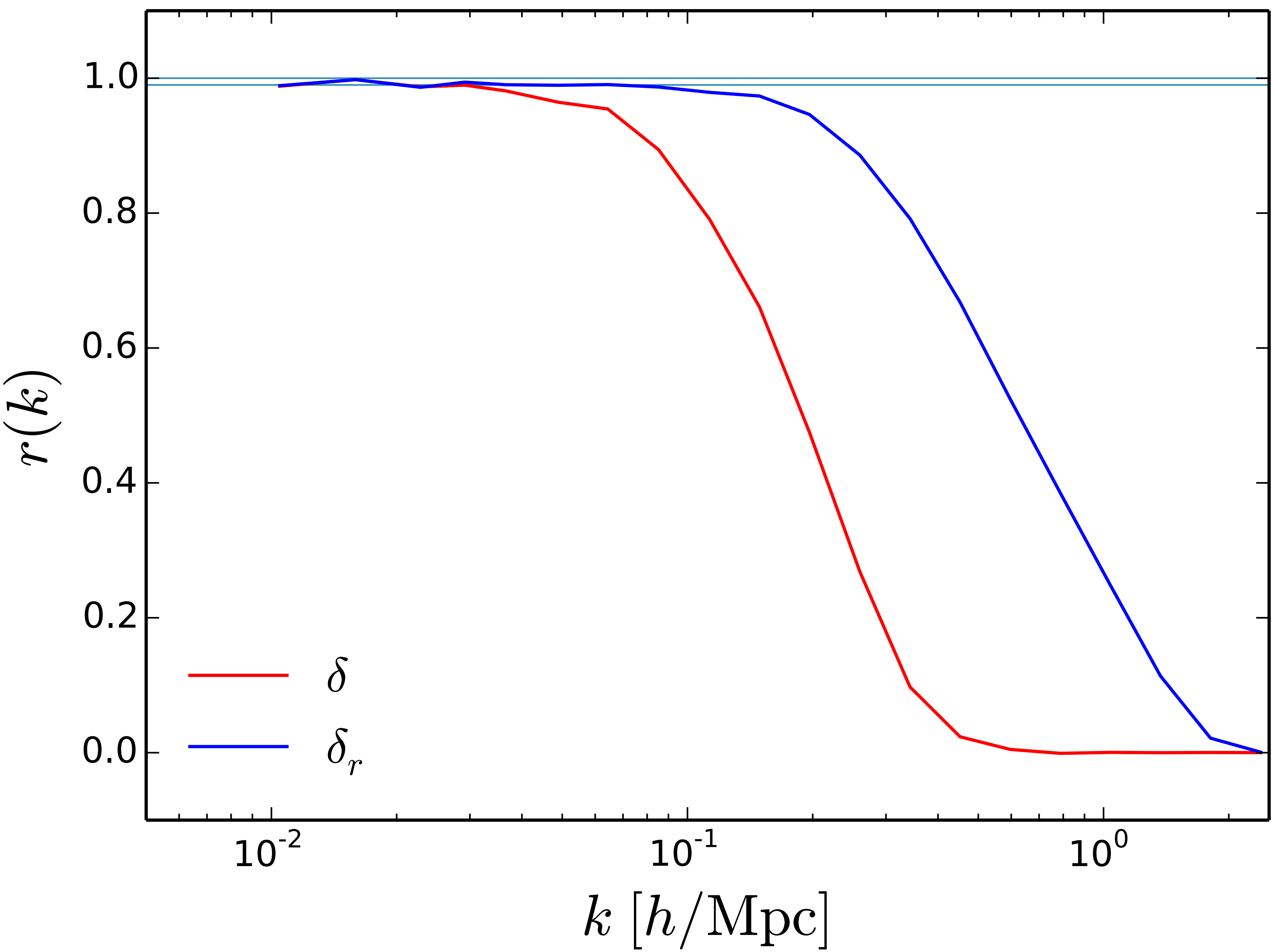
$$r(k) = \frac{1 + \frac{1}{3} f}{1 + \frac{2}{3} f + \frac{1}{5} f^2} \approx 0.99$$

RSD

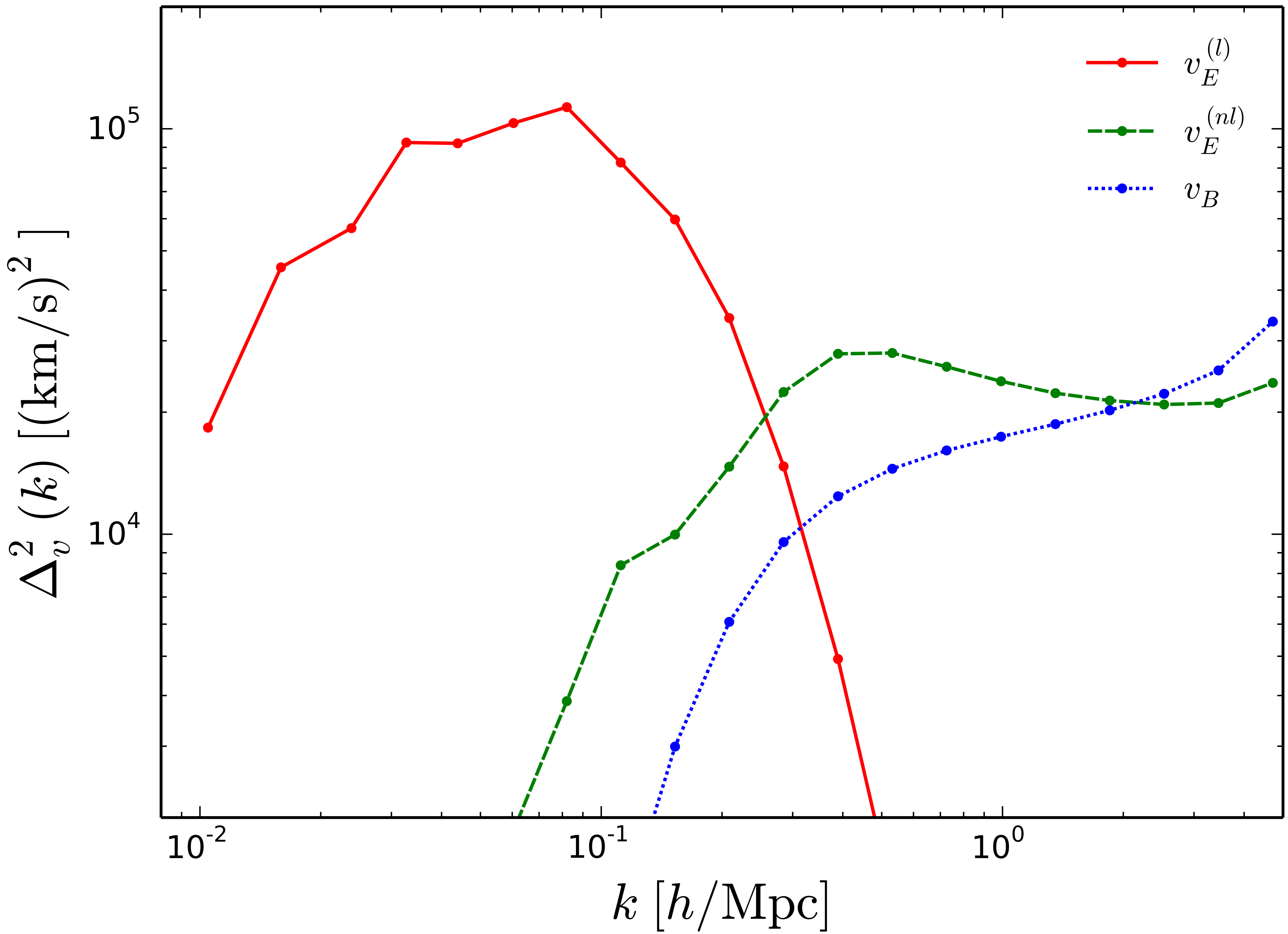


1.0
0.8
0.6
0.4
0.2
0.0
$r(k)$
10^{-2}
10^{-1}
10^{0}
$k\ [h/\mathrm{Mpc}]$
δ^{wf}
δ_r^{wf}
δ_r no RSD

RSD



Peculiar velocity reconstruction



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