New Perspective on Galaxy Clustering as a Cosmological Probe: General Relativistic Effects

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CONTENTS

in collaboration with

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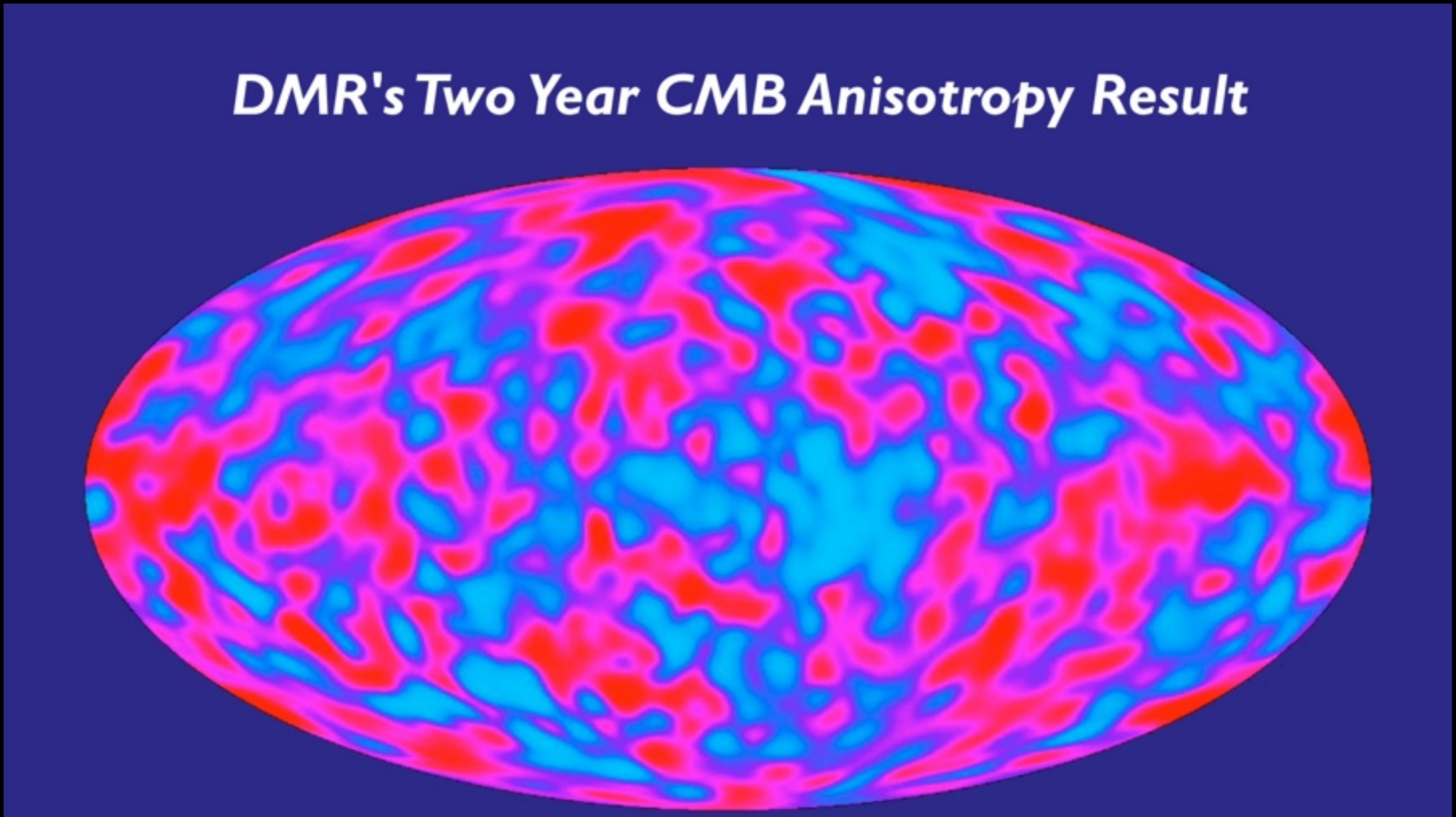
I. INTRODUCTION:

Galaxies as a Cosmological Probe – What is the Problem?

Cosmic Microwave Background

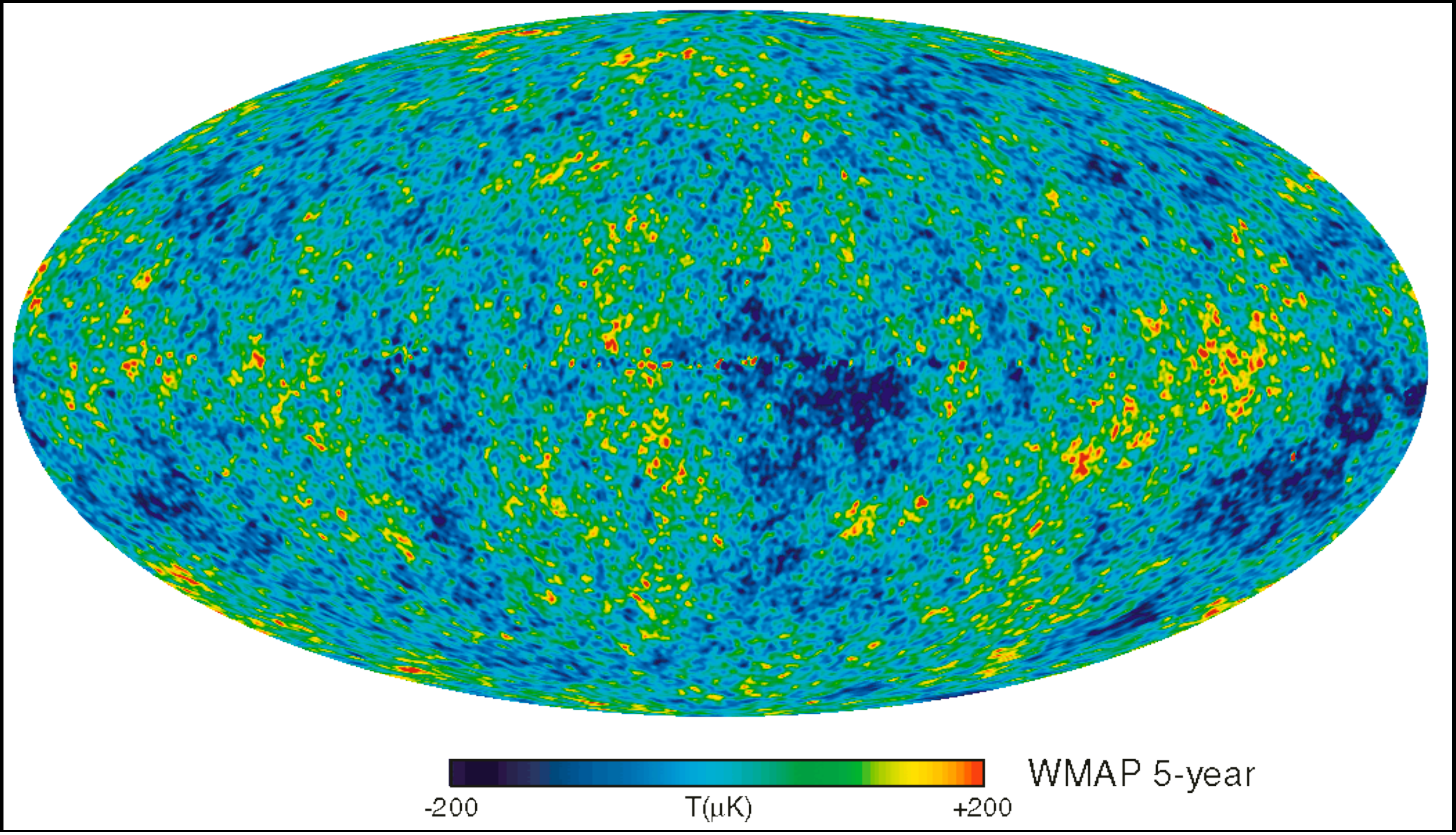
- **Co**smic **B**ackground **E**xplorer in 1989 ~ 1993
  - large-scale anisotropies (up to 6 degree)



Credit: NASA LAMBDA

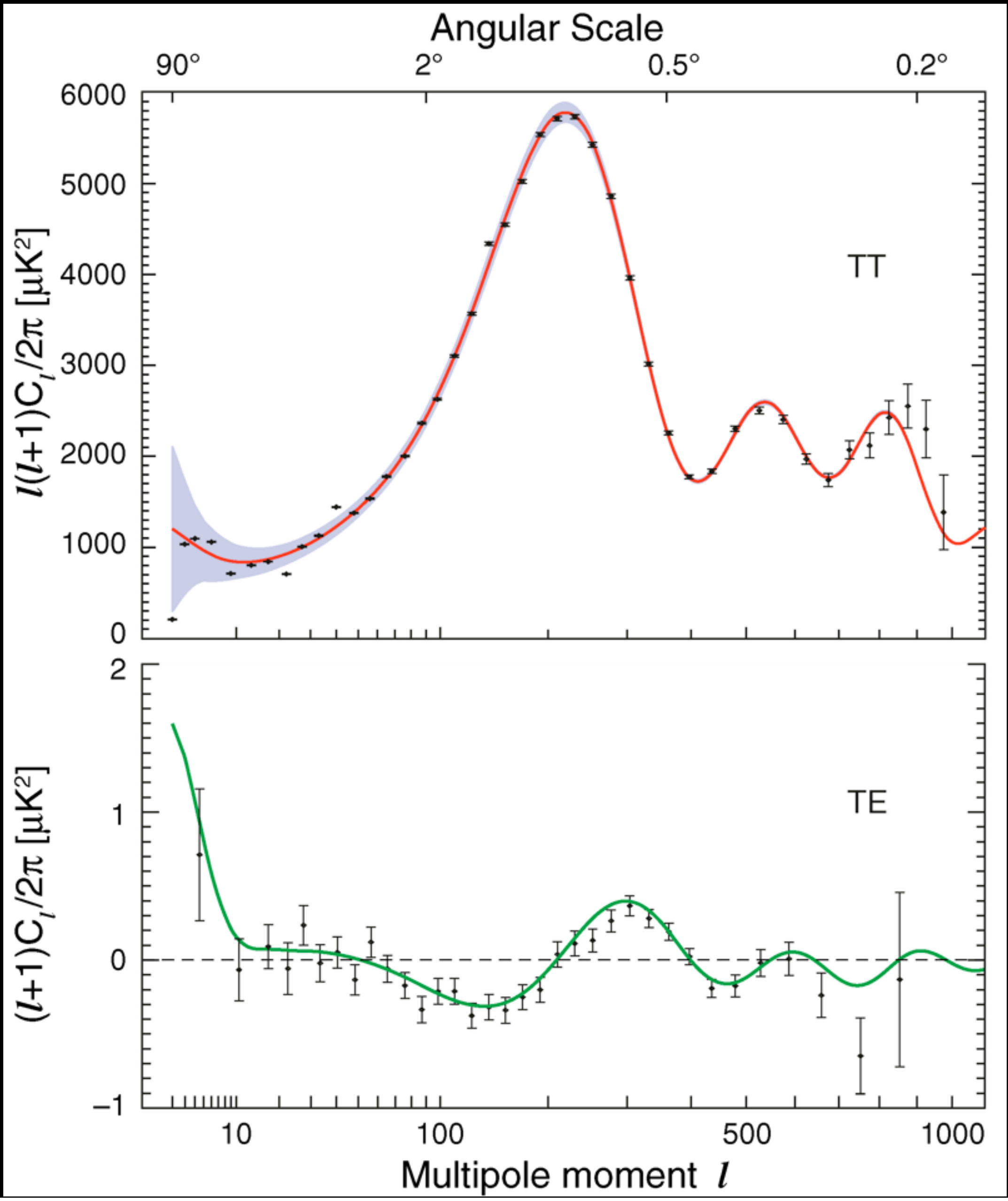
Cosmic Microwave Background

- **Wilkinson Microwave Anisotropy Probe in 2001 ~**
  - **precise measurements of anisotropies (> 10')**



CMB Anisotropies

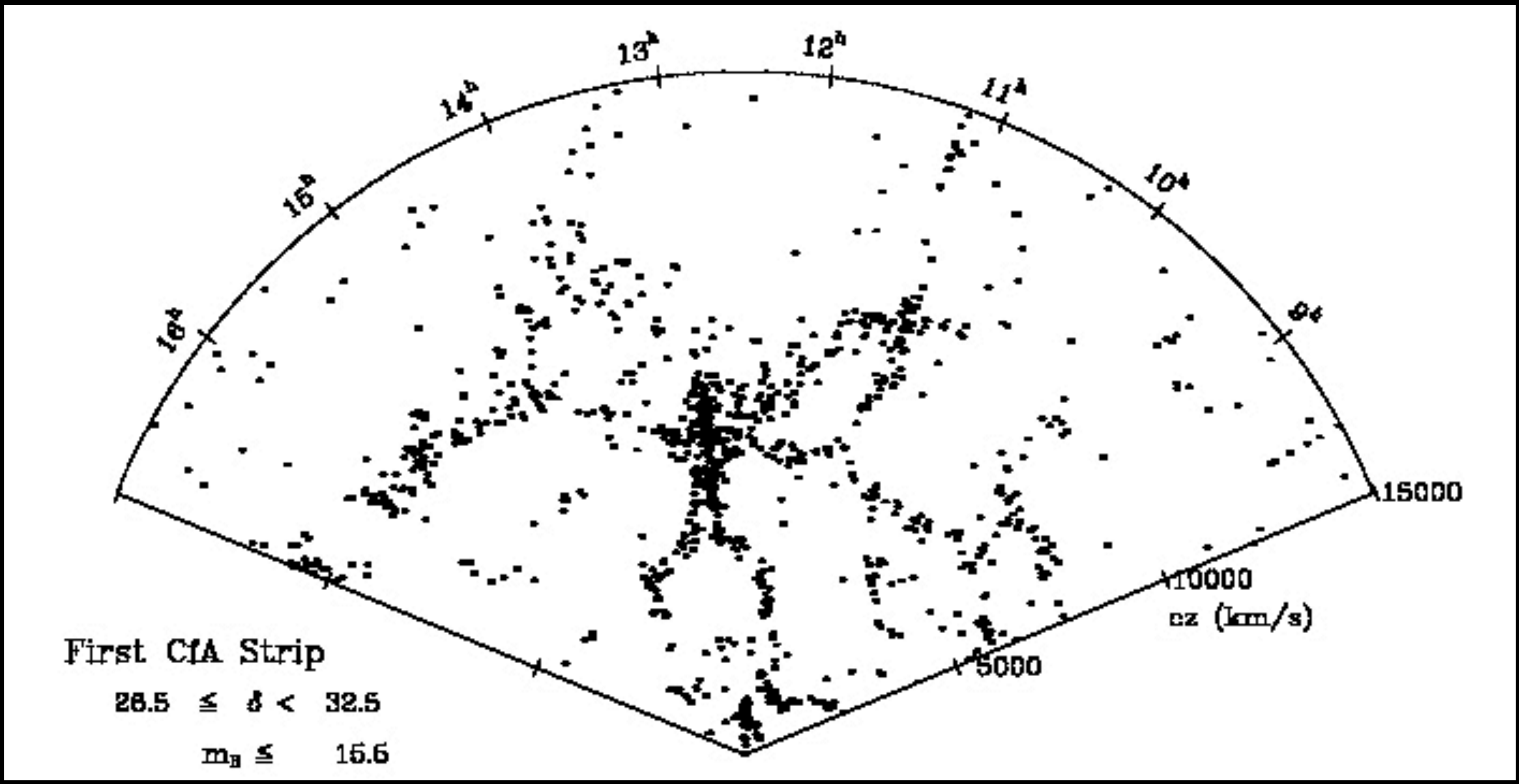
- precise measurements of CMB anisotropies
- physics of CMB: general relativity, linear theory
- good understanding of early universe



Credit: WMAP team

Large-Scale Structure

- The CfA galaxy redshift survey in 1977 ~ 1985
  - 18,000 galaxies, $z \sim 0.05$ *CfA great wall*

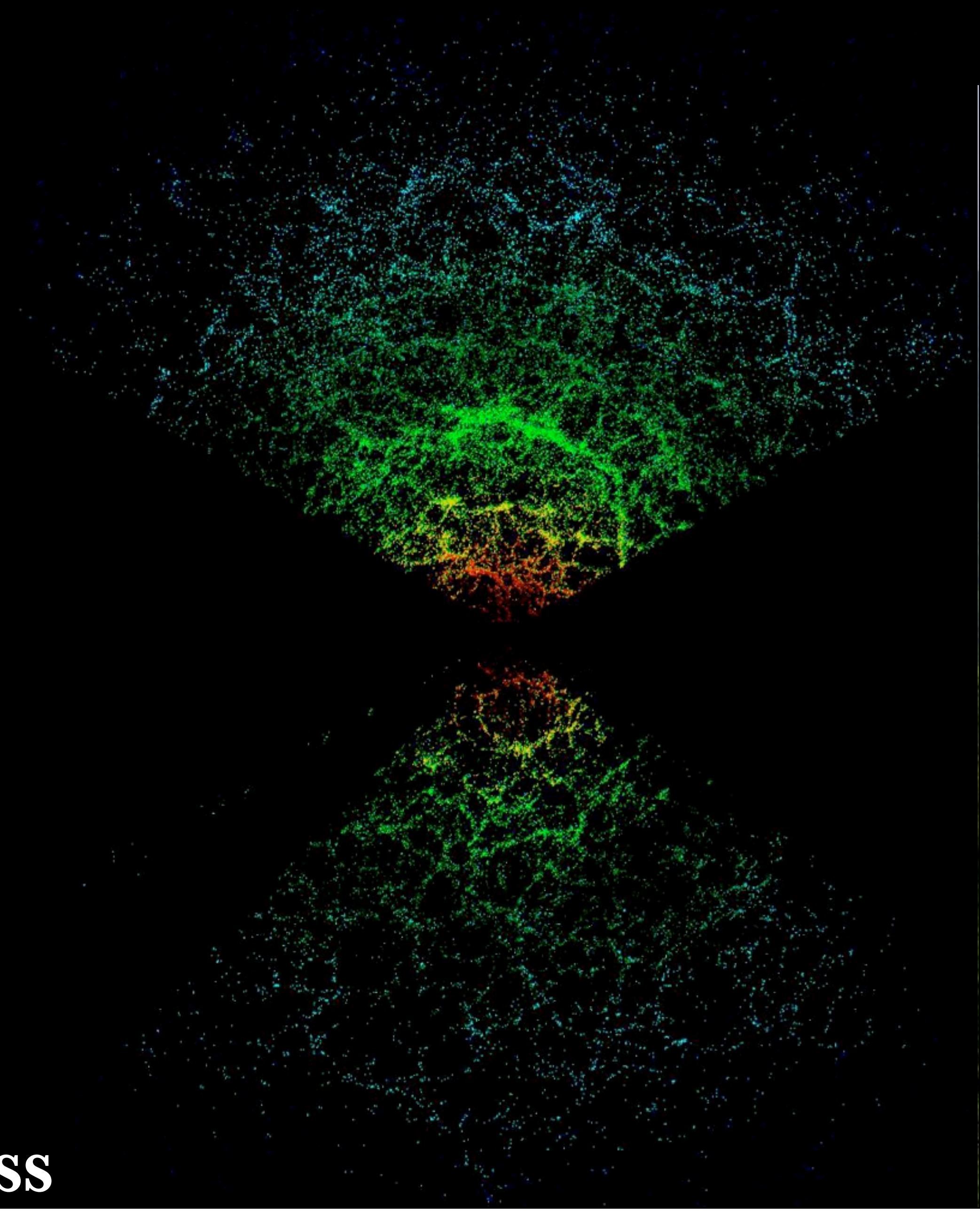


Huchra, Davis, Latham, Tonry, ApJS, 1983

de Lapparent, Geller, Huchra, ApJ, 1991

Large-Scale Structure

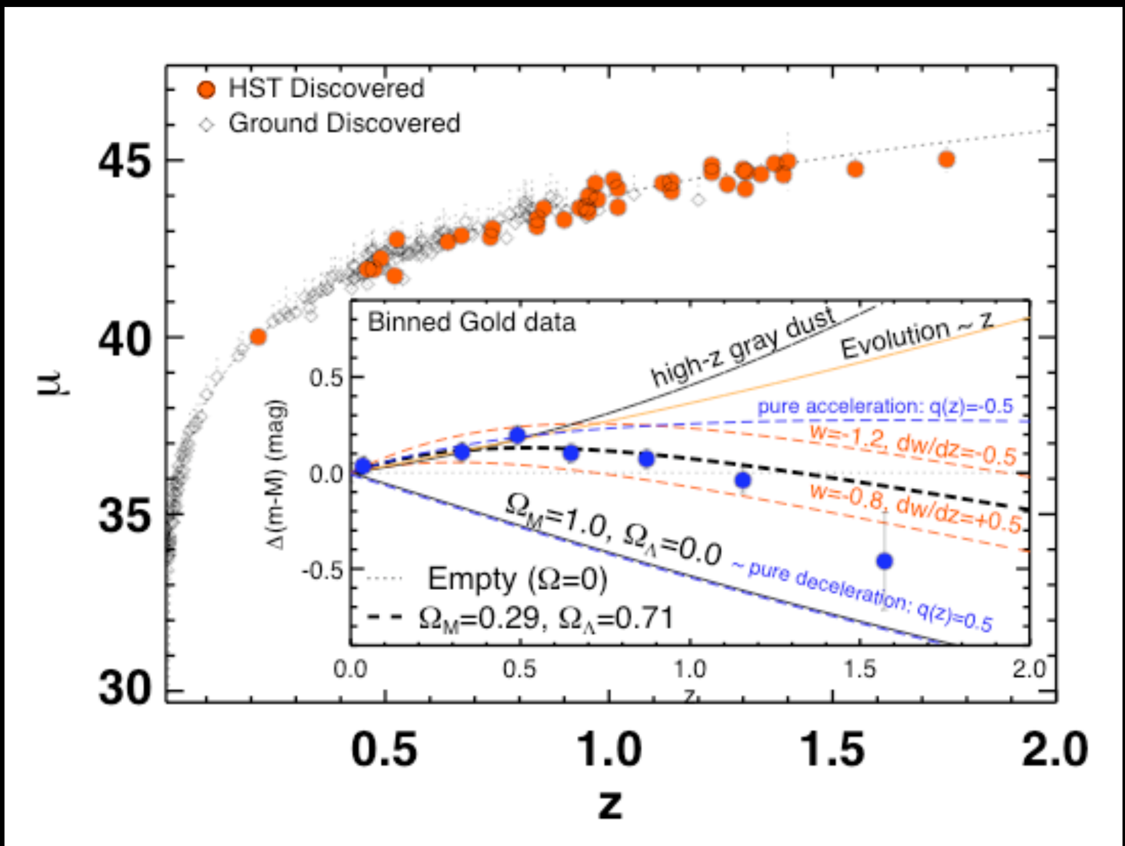
- Sloan Digital Sky Survey
  - 2000 ~ 2008
  - 1/4 sky
  - 1 million galaxies
  - 120,000 quasars
  - ~ 24 magnitude
  - $z \sim 0.5$



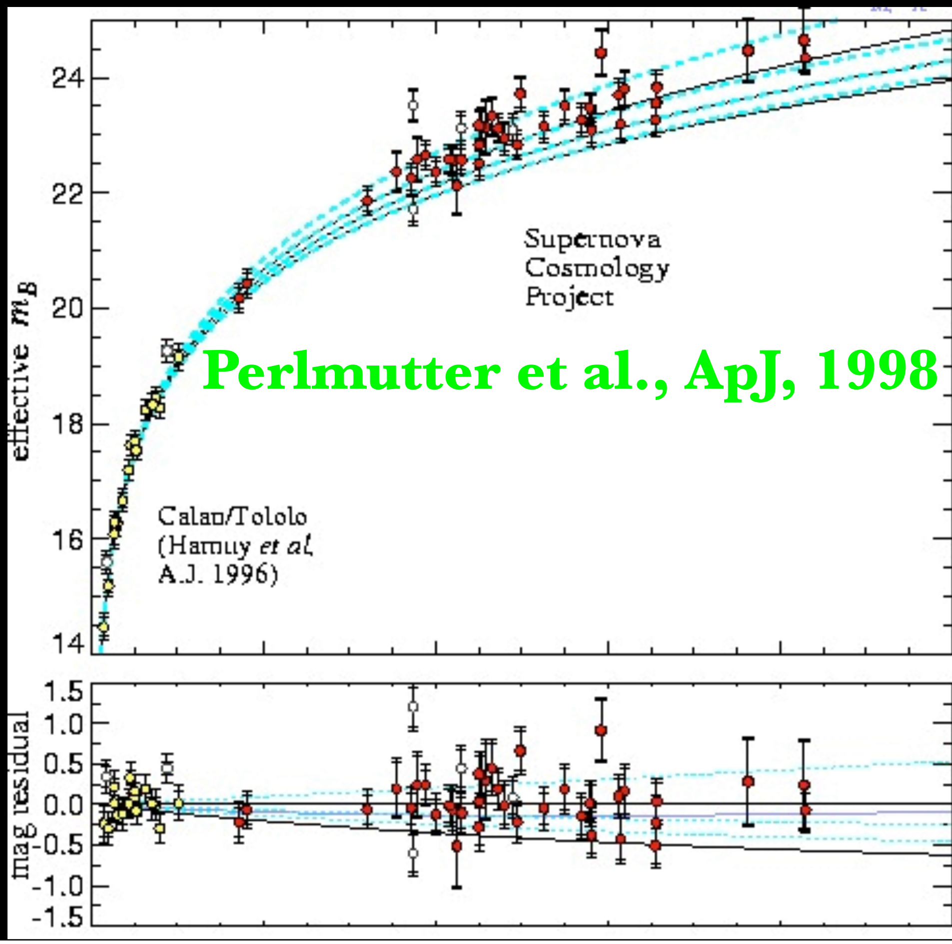
Credit: SDSS

Unfinished Story

- 2nd phase of cosmic acceleration (*dark energy*)
  - need probes of late time universe



Riess et al., ApJ, 1998, 2007



Perlmutter et al., ApJ, 1998

Dark Energy Surveys

- current and future surveys:
  - Baryonic Oscillation Spectroscopic Survey
  - Dark Energy Survey
  - Panoramic Survey Telescope & Rapid Response System
  - Hobby-Eberly Telescope Dark Energy Experiment
  - Wide Field Multi-Object Spectroscoph
  - Large Synoptic Survey Telescope
- future space missions:
  - EUCLID, Supernova Acceleration Probe
  - Cosmic Inflation Probe
  - Advanced Dark Energy Physics Telescope

Legend of Galaxies

- **current and future dark energy surveys:**
  - ***better precision and larger scales!***

- **galaxies as cosmological probes:**
  - **BAO signature: D_A, Ω_b/Ω_m, k~0.06 h/Mpc**
  - **galaxy power spectrum: n_s, Ω_k, $\Omega_m h$, k~0.01 h/Mpc**
  - **primordial non-Gaussianity: f_{NL}, n_s, k~0.001 h/Mpc**

a "clean" cosmological probe in linear regime

BUT is our faith well founded?

Cosmological Probe

- *precision cosmology!*
- **galaxies trace underlying matter**
  - **biased tracer:** $\delta_g = b\ \delta_m$
  - **z-space distortion:** $\delta_g = b\ \delta_m - \frac{1+z}{H}\frac{\partial V}{\partial r}$
  - **gravitational lensing:** $\delta_g = b\ \delta_m + (5p-2)\ \kappa$
  - **contributions are added in *adhoc* manner!**

is this everything? or are there more contributions?

we need unified treatments!

Relativistic Perspective

- **theoretical inconsistency**
  - **standard description:** $\delta_g = b\,\delta_m - \frac{1+z}{H}\frac{\partial V}{\partial r}$
  - **synchronous gauge (e.g., CMBFAST, CAMB)**
    - **free falling frame** $\psi = V = 0$
  - **Poisson equation** $\nabla^2\psi \sim \delta_m \sim 0$?

- **theoretical quantities vs observables**
  - **perturbations are *gauge-dependent***
  - **often *infinite* (e.g., bare mass vs observed mass)**

they are* NOT *observables!

II. LARGE-SCALE STRUCTURE:

Galaxies as a Cosmological Probe – A New Perspective

Cosmology

- **modern cosmology: general theory of relativity**
- **cosmological framework:**
  - **described by Einstein equations**
  - **homogeneous & isotropic FLRW universe**
  - **inflation, big bang nucleosynthesis, CMB**

- ***galaxies in cosmological framework?***
  - **Newtonian, no GR description**
  - **automatic disqualification?**

Galaxies in General Relativity

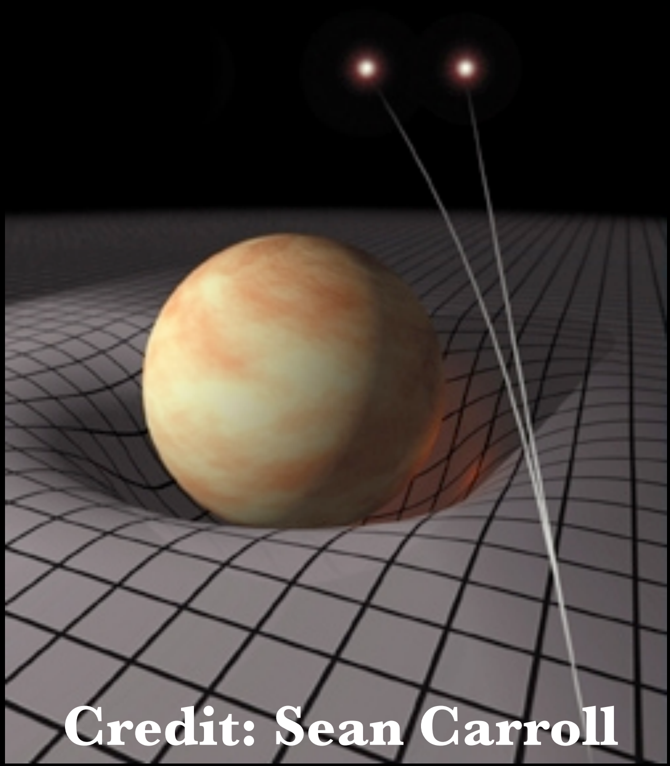
- ***what are observables?***
- **geodesic equations of photons from galaxies**
- **time component:** ***Sachs-Wolfe*** **effect**
  - **observed redshift**

$$1+z_{\rm obs}=(1+z)\left[1+V(z)-V(0)-\psi(z)+\psi(0)-\int_0^r dr'~(\dot\psi-\dot\phi)\right]~.$$

- **spatial component:** ***gravitational lensing*** **effect**
  - **observed position** $\hat n=(\theta,\phi)$
  - **lensing displacement** $(\delta r,~\delta\theta,~\delta\phi)$
  - **magnification** $\mu\simeq 1+2\kappa$

Sachs-Wolfe effect

- **time component:** ***Sachs-Wolfe*** **effect**
  - **observed redshift**



$$1 + z_{\rm obs} = (1+z)$$ **true redshift**

$$\times\Big[1 + V(z) - V(0)$$ **Doppler effect**

$$-\psi(z) + \psi(0)$$ **gravitational redshift**

$$- \int_0^r dr' \; (\dot\psi - \dot\phi)\Big].$$ **integrated Sachs-Wolfe effect**

Sachs & Wolfe, ApJ, 1967

Gravitational Lensing

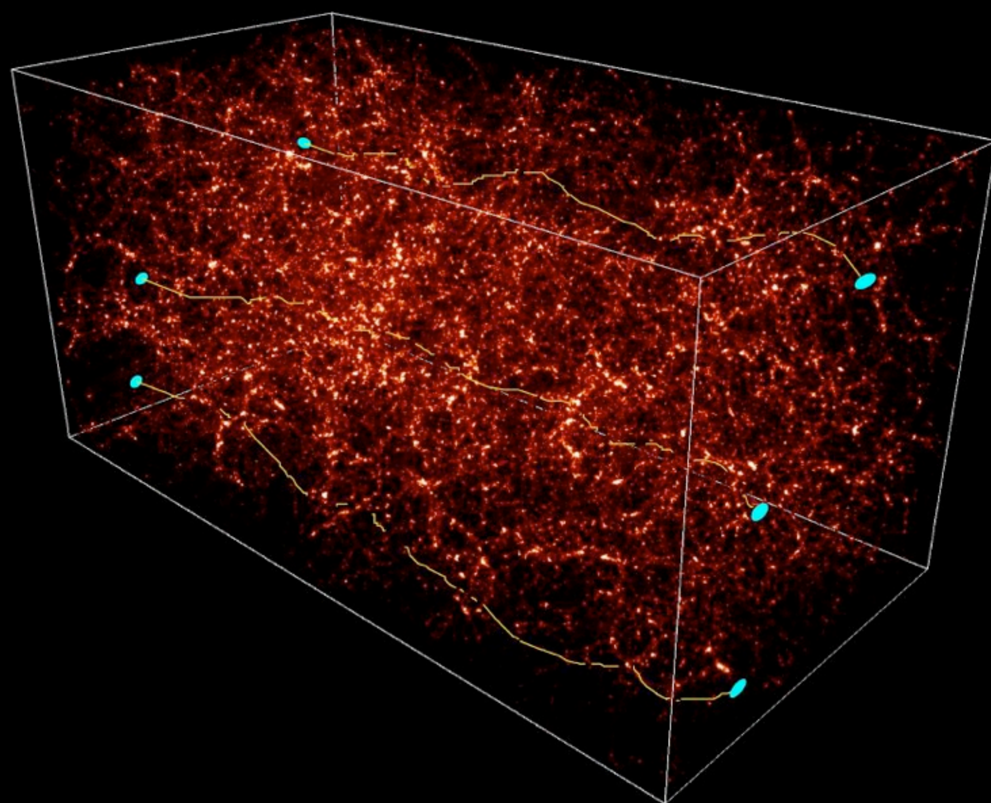
- **spatial component:** ***gravitational lensing*** **effect**
  - **true source position**

$$\hat{s} = (\theta + \delta\theta,\ \phi + \delta\phi)$$

  - **magnification**

$$\mu \simeq \left| \frac{d^2 \hat{n}}{d^2 \hat{s}} \right| = 1 + 2\ \kappa$$

  - ***not observable!***



Credit: Stephane Colombi

Effects on Galaxies

- **construct a galaxy fluctuation field:**
  - **total number of observed galaxies** $N_{\rm tot}$
  - **observed volume** $dV_{\rm obs}$ **given** $(z_{\rm obs}, \hat{n})$
  - **fluctuation field** $\delta_{\rm obs} = \frac{n_{\rm obs}}{\langle n_{\rm obs}\rangle} - 1$

- **relation to *<u>true</u>* number density:**
  - **number conservation** $N_{\rm tot} = n_{\rm true}\, dV_{\rm true} = n_{\rm obs}\, dV_{\rm obs}$
  - **volume element** $dV = \frac{r^2(z)}{H(z)} dz\, d\Omega$
  - **note** $z_{\rm true} \neq z_{\rm obs}$, $d\Omega_{\rm true} \neq d\Omega_{\rm obs}$

Unified Treatment

- **observable:** **total number of observed galaxies**
- **volume effects:**
  - **redshift-space distortion:** $\frac{\partial z}{\partial z_{\rm obs}} \simeq \frac{1+z}{H}\frac{\partial V}{\partial r}$
  - **lensing magnification:** $\frac{\partial \Omega}{\partial \Omega_{\rm obs}}\frac{\partial f}{\partial f_{\rm obs}} \simeq \frac{1}{\mu^2} = 1 - 4\,\kappa$
- **source effects:**
  - **magnification bias:** $\bar{n}_{\rm obs}(f_{\rm obs}) \simeq \bar{n}(f_{\rm obs}/\mu)$
- ***complete description of different effects***

Subtle Issues

- ***what are "true" redshift, volume …?***
  - **"*true*" just means quantities in homogeneous & isotropic FLRW universe**
- **Newtonian description:**
  - **at what coordinate system (*gauge*)?**

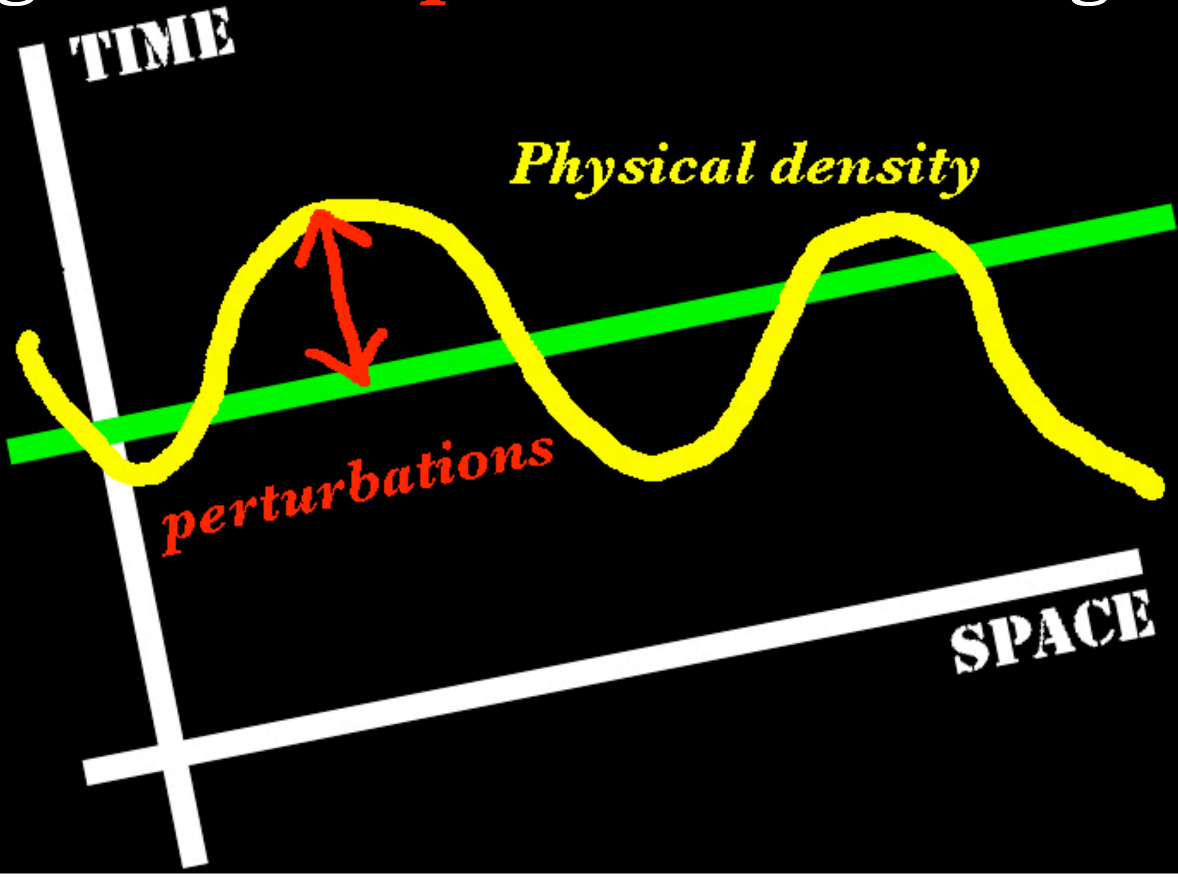
    e.g., $z_{\rm true} = z(t)$ **,** $\bar{\rho}_m = \bar{\rho}_m(t)$
  - **local inertial frame defines** $n_{\rm phy}$
- **fully relativistic theory:**
  - **no gauge ambiguity**
  - **expressed in terms of observables**

Correspondence

- **cosmological perturbations:**
    - **inhomogeneous physical spacetime**
    - **homogeneous *fictitious* background**

Gauge Freedom

- general covariance in GR:
  - free to choose a coordinate system
  - change in *correspondence* to background

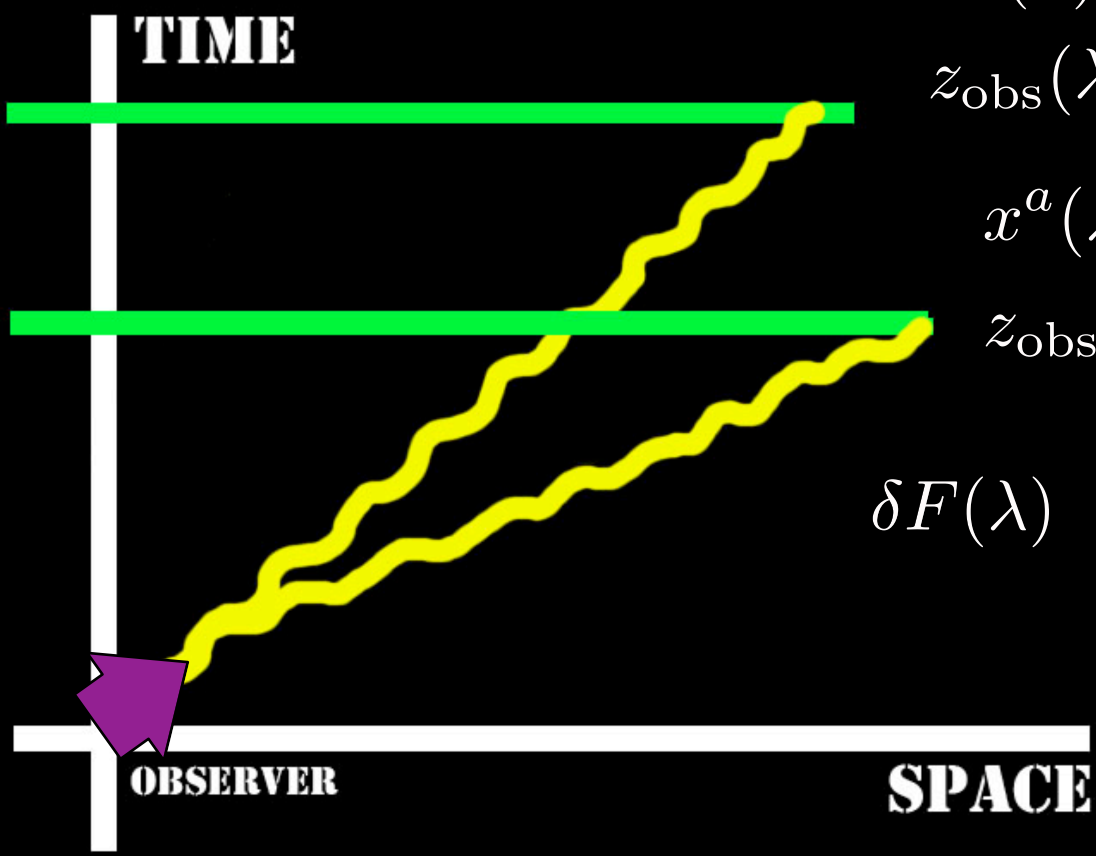


Gauge Issues

- **problems:**
  - **perturbations are gauge-dependent**
  - ***no physical meaning!***
  - **gauge freedom creates unphysical solution**
- **observables:**
  - **should be gauge-invariant**
- **Newtonian limit:**
  - **unambiguous hypersurface of simultaneity**

Complications

- **perturbations along photon geodesic:**
  - **observed angle and redshift**



$$\bar{x}^a(\lambda)$$

$$z_{\rm obs}(\lambda) = z_{\rm src} = z_h$$

$$x^a(\lambda) = \bar{x}^a(\lambda) + \delta x^a(\lambda)$$

$$z_{\rm obs}(\lambda) \neq z_{\rm src} \neq z_h$$

$$\begin{aligned} \delta F(\lambda) &= F[x^a(\lambda)] - \bar{F}[\bar{x}^a(\lambda)] \\ &\neq \delta F[x^a(\lambda)] \end{aligned}$$

Observed Redshift

- **observed redshift:**

$$1+z_{\rm obs}=(1+z)\left[1+V(z)-V(0)-\psi(z)+\psi(0)-\int_0^r dr'~(\dot\psi-\dot\phi)\right]~.$$

- **observer's point of view:**
  - **true redshift *z* from observed redshift $z_{\rm obs}$**

- **new perspective:**
  - **true redshift is *fictitious & gauge-dependent***
  - **in another coordinate** $z,\quad V,\quad \psi$ ***change!***

Observed Redshift

- **observed redshift:**

$$1+z_{\rm obs}=(1+z)\left[1+V(z)-V(0)-\psi(z)+\psi(0)-\int_0^r dr'~(\dot\psi-\dot\phi)\right]~.$$

$$=(1+z^s)\left[1+\frac{1}{6}\int_0^r dr'~(\dot h+3~\dot h^\parallel_{\alpha\beta}e^\alpha e^\beta)\right]$$ **(synchronous)**

- **gauge transformation:**
  - **true redshift** $z~\neq~z^s$
  - **true volume** $\frac{r^2(z)}{H(z)}~\neq~\frac{r^2(z^s)}{H(z^s)}$
  - **observed redshift is *gauge-invariant*** $z_{\rm obs}~=~z^s_{\rm obs}$
  - **full gauge-invariant expression (lengthy!)**

Gravitational Lensing

- **observed position:** $\hat{n} = (\theta, \phi)$
- **lensing displacements:** $(\delta r,\ \delta\theta,\ \delta\phi)$ **&** $\delta\tau$
- **true position:** $\hat{s} = (\theta + \delta\theta,\ \phi + \delta\phi)$
- **magnification:** $\mu \simeq \left| \frac{d^2 \hat{n}}{d^2 \hat{s}} \right| = 1 + 2\ \kappa$

- **gauge-dependent quantities:** $\delta r,\ \delta\theta,\ \delta\phi,\ \hat{s},\ \kappa$
  - ***standard formalism is Newtonian!***
  - **coordinate dependent descriptions**

magnification is observable or gauge-dependent?

Magnification

- **definition is *inaccurate!***
- **coordinate-independent definition:**
  - **luminosity in local inertial frame** L
  - **measure flux & redshift** $f_{\rm obs}$ & $z_{\rm obs}$
  - **magnification is *physical!***

$$\mu = f_{\rm obs} \left(\frac{L}{4\pi D_L^2(z_{\rm obs})} \right)^{-1}$$

  - **in a homogeneous universe using** $z_{\rm obs}$**, not** $z_{\rm true}$

$$D_L(z_{\rm obs}) = (1 + z_{\rm obs})\, r(z_{\rm obs})$$

what about observed flux and intrinsic luminosity?

Luminosity Distance

- **observed flux and intrinsic luminosity:**
  - **includes Sachs-Wolfe and lensing effects**

$$\mathcal{D}_L(z_{\rm obs}) = \sqrt{\frac{L}{4\pi f_{\rm obs}}} \equiv D_L(z_{\rm obs})(1+\delta\mathcal{D}_L)$$

Sasaki, MNRAS, 1987

$$= D_L(z_{\rm obs})\left[1+\frac{\delta\lambda}{r_s}-\frac{1}{2}\int_0^{r_s}dr\delta\theta\right]$$

- **shift in affine parameter** $\delta\lambda \sim \delta z$
- **distortion in wave vector expansion** $\delta\theta \sim \kappa$
- **gauge-invariant** $\delta\mathcal{D}_L$

Luminosity Distance

- **relation to magnification:**

$$\mu = f_{\rm obs}\left(\frac{L}{4\pi D_L^2(z_{\rm obs})}\right)^{-1} = \left(\frac{D_L(z_{\rm obs})}{\mathcal{D}_L(z_{\rm obs})}\right)^2$$

$$= 1 - 2\,\delta\mathcal{D}_L \simeq 1 + 2\,\kappa$$

- **new perspective:**
  - **observed magnification is *gauge-invariant***
  - ***but* usual parametrization is *gauge-dependent***
  - **source effect** $\bar{n}_{\rm obs} = \bar{n}[f_{\rm obs}(1 + 2\delta\mathcal{D}_L)]$

Observed Number of Galaxies

- ***we still need "true" volume!***
- **total number of observed galaxies:**
  - **observables** $N_{\rm tot}\,,~~\hat n=(\theta,\phi)\,,~~z_{\rm obs}$

$$N_{\rm tot}=\int~dz_{\rm obs}d\Omega_{\rm obs}~n_{\rm obs}\,\frac{r^2(z_{\rm obs})}{(1+z_{\rm obs})^3H(z_{\rm obs})}$$

$$=\int n_{\rm phy}dV_{\rm phy}$$

- **physical volume:** $dV_{\rm phy}$
  - **occupied by observed galaxies**
  - ***trace backward photon geodesic!***

Covariant Expression

- **integral of p-forms in orientable manifolds:**
  - **manifold of dim. *p* in *n*-dim. space**

$$x^a = x^a(u^1, \cdots, u^p) \quad a = 1, \cdots, n$$

  - **volume element**

$$dV_p = \frac{\partial x^{a_1}}{\partial u^1} \cdots \frac{\partial x^{a_p}}{\partial u^p} du^1 \cdots du^p$$

  - **integral of p-form**

$$\int_{\mathcal{M}} t \ dV_p = \int t_{a_1 \cdots a_p} \ dV_p$$

Weinberg, 1972

Matias's Magic

- **integral of 3-form in 4D spacetime manifold:**
  - **observables** $z_{\rm obs}\,, \ \theta_{\rm obs}\,, \ \phi_{\rm obs}$
  - **photon geodesic path** $x^a(\lambda) = \bar x^a(\lambda) + \delta x^a(\lambda)$
  - **Sachs-Wolfe and gravitational lensing effects**
  - **distortion in local Lorentz frame**
  - **manifestly gauge-invariant**

$$N_{\rm tot} = \int \sqrt{-g}\; n_{\rm phy}\; \varepsilon_{abcd}\; u^d\; \frac{\partial x^a}{\partial z_{\rm obs}} \frac{\partial x^b}{\partial \theta_{\rm obs}} \frac{\partial x^c}{\partial \phi_{\rm obs}}\; dz_{\rm obs}\; d\theta_{\rm obs}\; d\phi_{\rm obs}$$

Levi-Civita symbol ε_{abcd} **, comoving velocity** u^a

Observed Number of Galaxies

- ***fun*** **and/or** ***pain*** **in perturbation expansion!**

$$N_{\rm tot} = \int \sqrt{-g}\; n_{\rm phy}\; \varepsilon_{abcd}\; u^d\; \frac{\partial x^a}{\partial z}\frac{\partial x^b}{\partial \theta}\frac{\partial x^c}{\partial \phi}\; dz\; d\theta\; d\phi$$

$$= \int n_{\rm phy}\; \frac{r^2 \sin\theta}{(1+z)^3 H}\; dz\; d\theta\; d\phi\; \Bigg[1 + 3D + V + 2\,\frac{\delta r}{r}$$

$$+ H\frac{\partial}{\partial z}\;\delta r + \left(\cot\theta + \frac{\partial}{\partial\theta}\right)\delta\theta + \frac{\partial}{\partial\phi}\;\delta\phi + \frac{\bar r^2}{r^2}\; H\; \frac{\partial \bar r}{\partial z}\Bigg]$$

$$\equiv \int \; n_{\rm obs}\; \frac{r^2 \sin\theta}{(1+z)^3 H}\; dz\; d\theta\; d\phi$$

- **subscript “*obs*” is omitted!**

Observed Number Density

- **so far, we have**
  - **volume effects:** $n_{\rm obs} = n_{\rm phy} \left(1 + \sum_\mu \delta_\mu \right)$
  - **source effects:** $\bar{n}_{\rm obs} \rightarrow \bar{n}_{\rm phy} \left[f_{\rm obs} (1 + 2\delta\mathcal{D}_L) \right]$

$dn_{\rm phy}/dL \propto L^{-s}$ $\quad = \bar{n}_{\rm phy}(L_{\rm thr})(1 - 5p\ \delta\mathcal{D}_L)$

$p = 0.4\ (s - 1)$

BUT ***why do we care about galaxies?***

Galaxy Bias

- **galaxies trace underlying matter!**
  - **standard relation** $\delta_{\rm gal} = b\ \delta_m$ **(** δ_m **: perturbation)**
  - ***but in what gauge?***

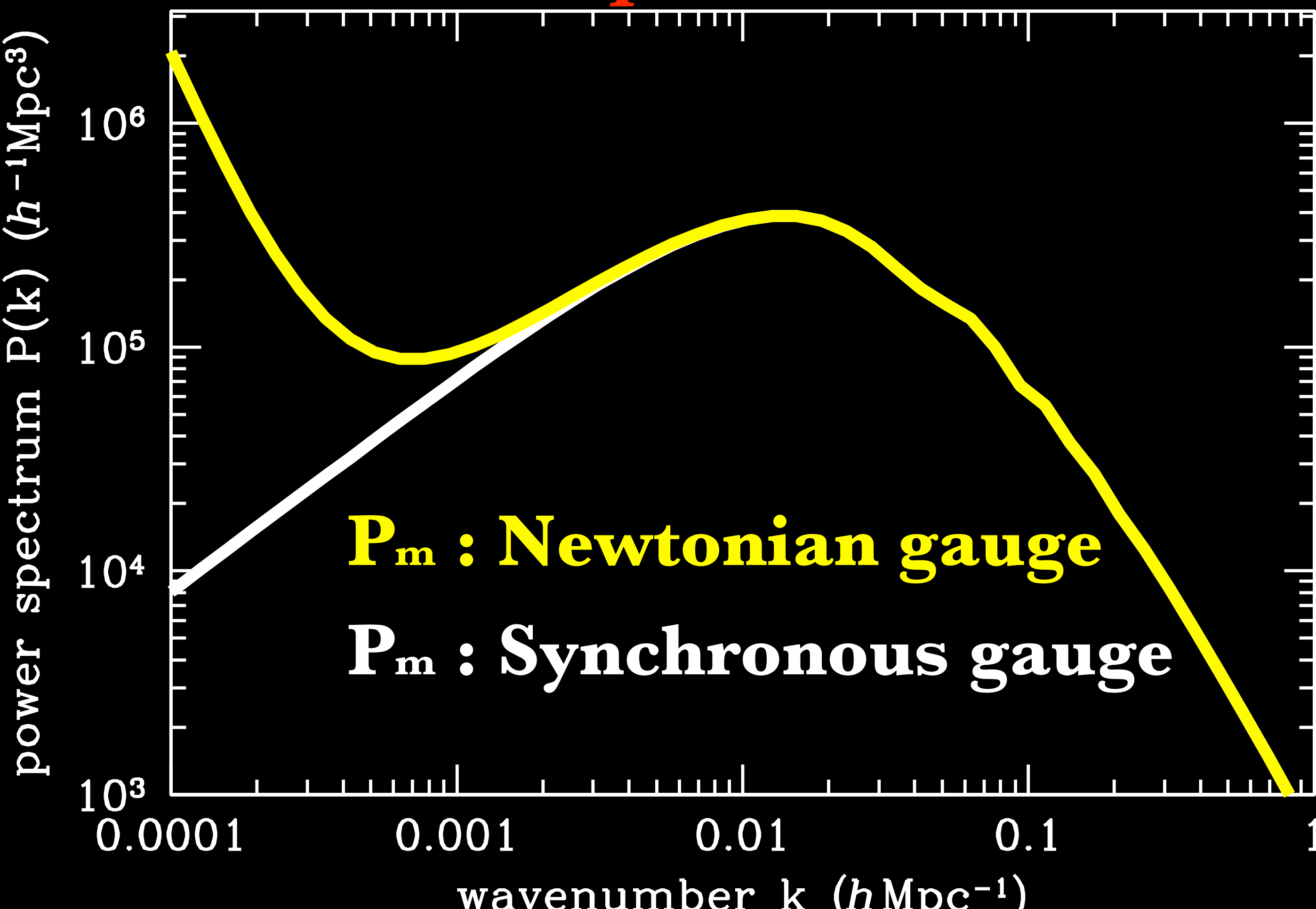
- **matter density, *NOT* matter perturbation** $\rho_{\rm gal} \propto \rho_m$
  - **galaxies NOT at observed redshift $z_{\rm obs}$**
  - **matter density** $\rho_m(x^a) = \bar\rho_m(z)(1+\delta_m)$

    at galaxy position $= \bar\rho_m(z_{\rm obs})(1+\delta_m - 3\ \delta z)$
  - **gauge-invariant combination**

$$n_{\rm phy} = \bar n_{\rm phy}(z_{\rm obs})\,[1 + b\ (\delta_m - 3\ \delta z)]$$

Galaxy Bias

- **perturbations are *gauge-dependent!***
- ***constant bias* vs *scale-dependent bias***



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    at galaxy position $= \bar\rho_m(z_{\rm obs})(1+\delta_m - 3\ \delta z)$
  - **gauge-invariant combination**

$$n_{\rm phy} = \bar n_{\rm phy}(z_{\rm obs})\left[1 + b\ (\delta_m - 3\ \delta z)\right]$$

Cosmological Probe

- ***accurate relation to underlying matter***
  - **most cases :** $\delta_g = b\ \delta_m$
  - **prudent work :** $\delta_g = b\ \delta_m - \frac{1+z}{H}\frac{\partial V}{\partial r}$
  - **best efforts so far :** $\delta_g = b\ \delta_m - \frac{1+z}{H}\frac{\partial V}{\partial r} + (5p-2)\ \kappa$
  - **this work :**

$$\begin{aligned}\delta_{\rm obs} &= b\ (\delta_m - 3\ \delta z) + A + 2D + V - \delta z - 5p\ \delta\mathcal{D}_L - 2\ \kappa \\ &- (1+z)\frac{\partial}{\partial z}\ \delta z - 2\ \frac{1+z}{Hr}\ \delta z + \frac{1+z}{H}\frac{dH}{dz}\ \delta z + 2\ \frac{\delta r}{r}\end{aligned}$$

written in *Newtonian gauge,* but it can be written in *any gauges*, and it is *gauge-invariant!*

Yoo, Fitzpatrick, Zaldarriaga (2009)

Correspondence

- **in Newtonian limit** $k\ \tau_0 \gg 1$
  - **full equation:** $$\delta_{\rm obs} = b\ (\delta_m - 3\ \delta z) + A + 2D + V - \delta z - 5p\ \delta\mathcal{D}_L - 2\ \kappa - (1+z)\frac{\partial}{\partial z}\ \delta z - 2\ \frac{1+z}{Hr}\ \delta z + \frac{1+z}{H}\frac{dH}{dz}\ \delta z + 2\ \frac{\delta r}{r}$$
  - **order of magnitude:** $\delta \gg V > \psi \simeq \phi \simeq \mathrm{iSW}$
  - **distortion in z:** $\delta z \simeq V$ $\qquad (1+z)\frac{\partial}{\partial z}\delta z \simeq \frac{1+z}{H}\frac{\partial V}{\partial r}$

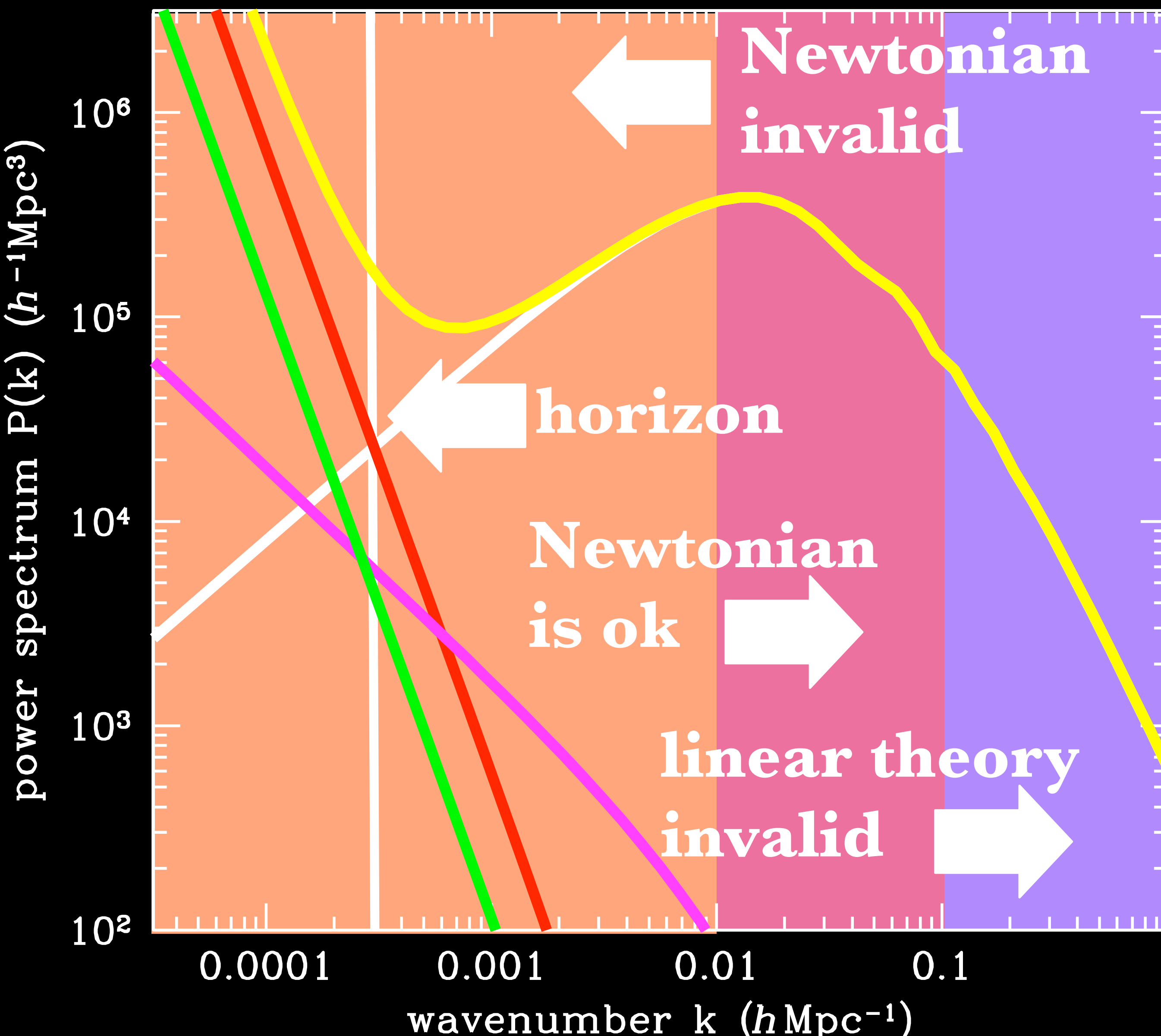
$$\delta z \equiv V(z) - V(0) - \psi(z) + \psi(0) + \int_0^z d\chi\ (\dot\psi - \dot\phi)$$

  - **luminosity distance:** $\delta\mathcal{D}_L \simeq -\kappa$
  - ***standard formula:*** $\delta_{\rm obs} = b\ \delta_m - \frac{1+z}{H}\frac{\partial V}{\partial r} + (5p-2)\ \kappa$

Scales of Interest

- **Synchronous**
  (CMBFast, CAMB)
  $P_\delta(k)$
  $P_\eta(k)$

- **Newtonian**
  $P_\delta(k)$
  $P_\psi(k)$
  $P_v(k)$



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  (CMBFast, CAMB)

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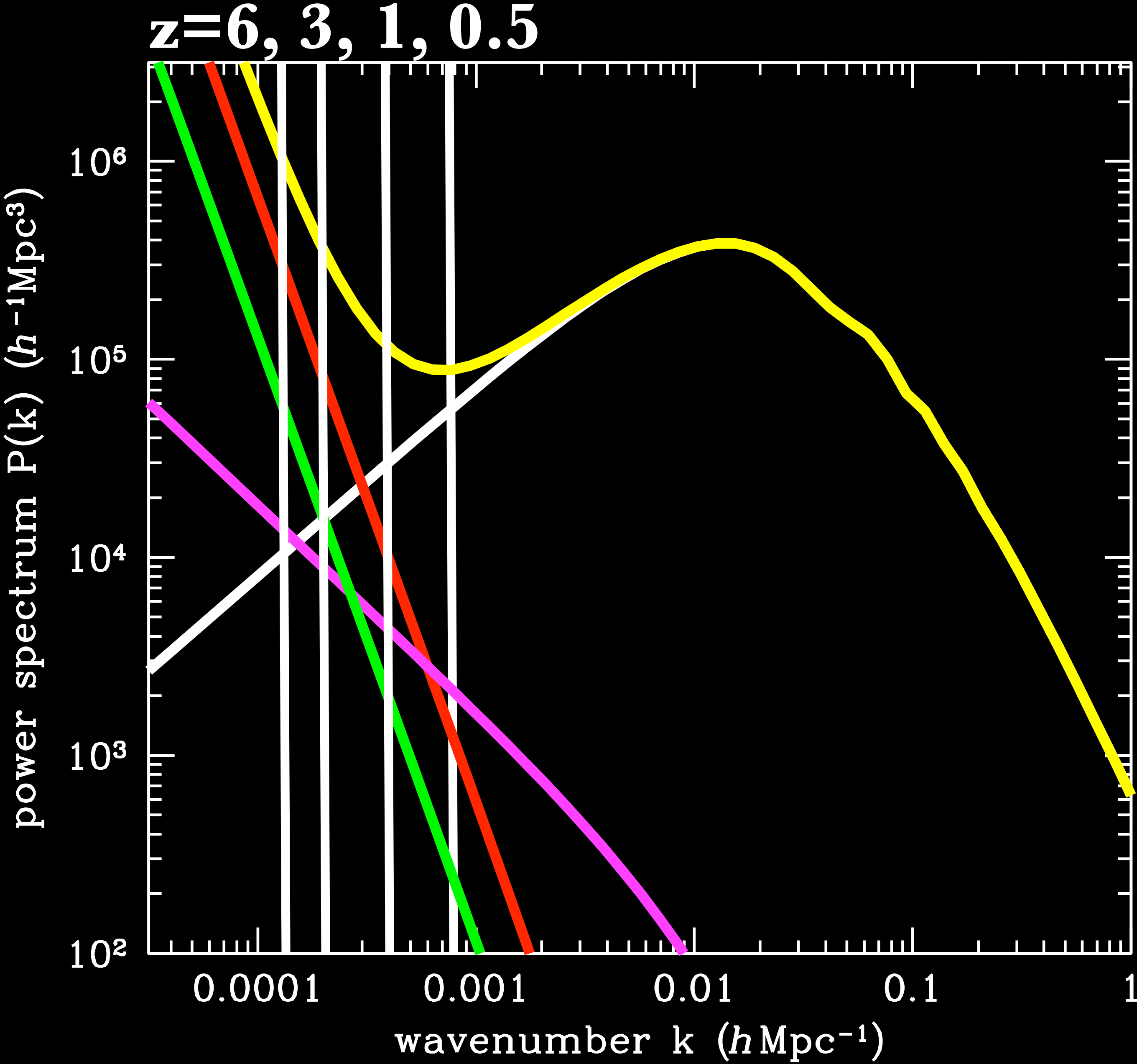
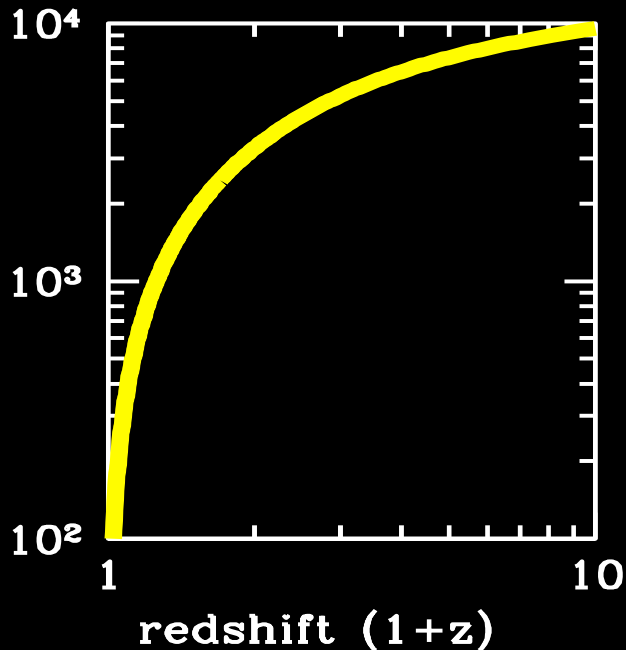


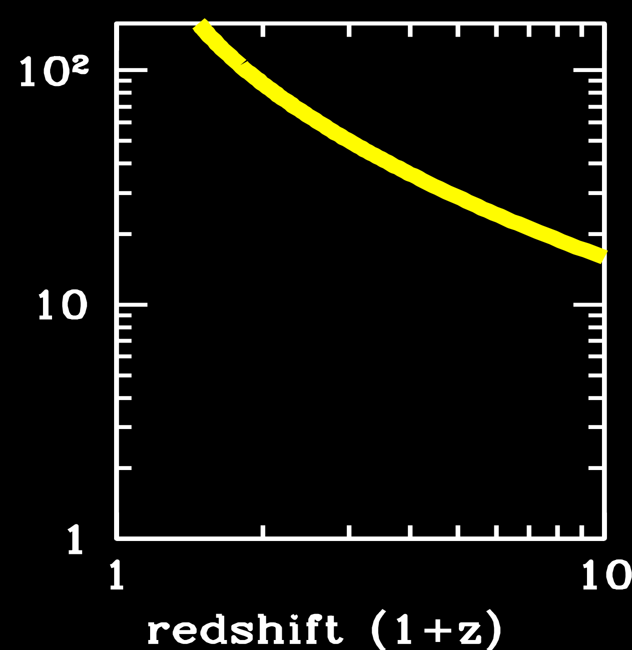
Hubble Horizon

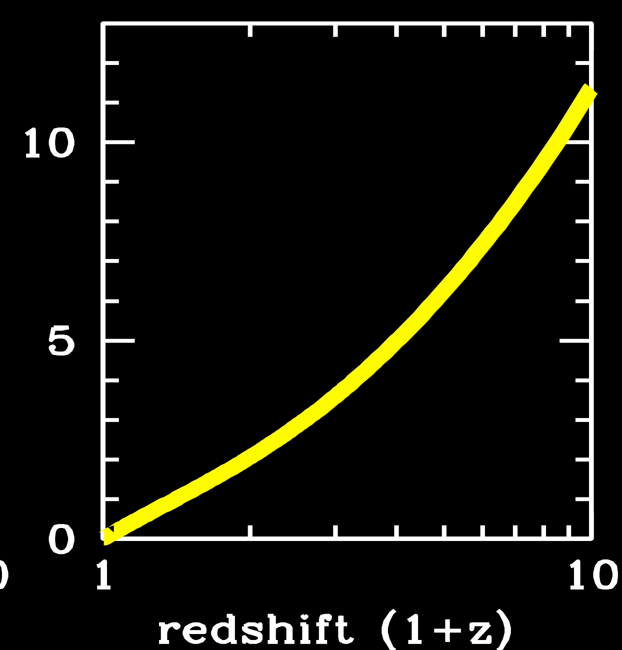
- **horizon scale ~ 3 Gpc/h (today), ~ 3 deg. (recomb.)**

relativistic effects are order one at horizon scale!

angular diameter distance (Mpc/h) | **angle on the sky (degree)** | **angular multipole moment**







Summary

- *fully gauge-invariant general relativistic description*
- **standard method:** *gauge-dependent!*
  - **galaxy clustering**
  - **gravitational lensing**
- **common malpractice:** *Do not mix!*
  - **Newtonian gauge equations**

    e.g., gravitational potential, Poisson equation
  - **synchronous gauge transfer functions**

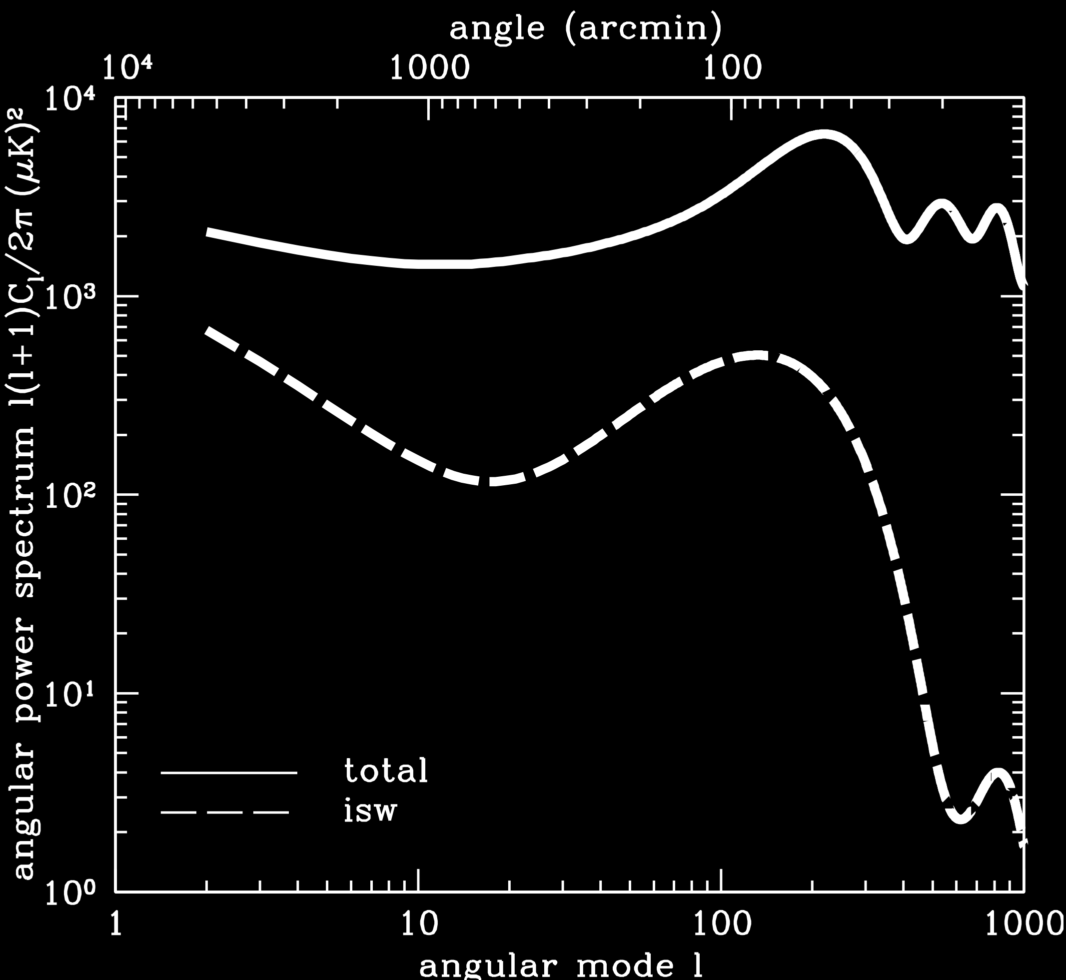
    e.g., CMBFast, CAMB
  - **also in** *numerical simulations!*

III. APPLICATIONS:

Impacts on Current Surveys – Why Bother?

Cross Correlation

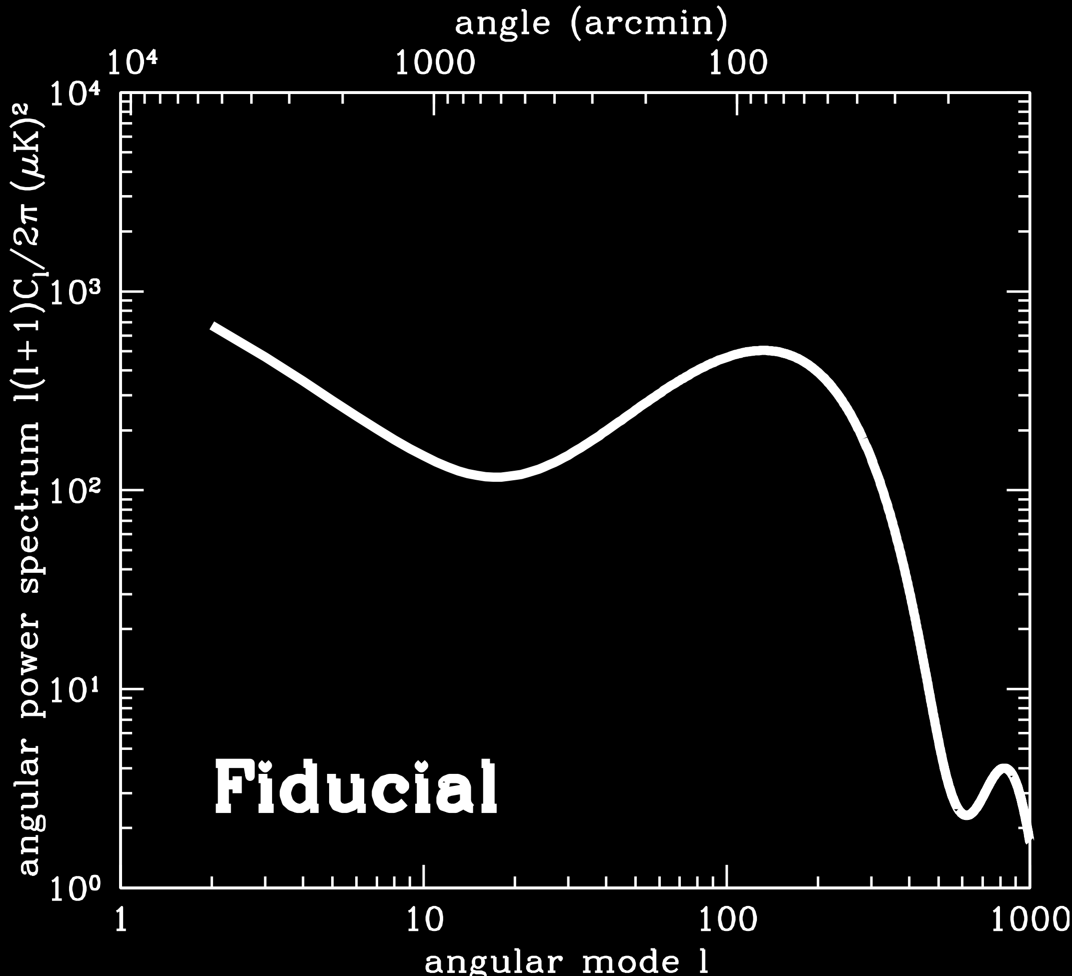
- **anisotropy formation:**
  - **scattering at recomb.**
  - **integrated Sachs-Wolfe effects**
- **cross correlation with**
  - **low redshift tracer: galaxies and quasars**



Integrated Sachs-Wolfe Effect

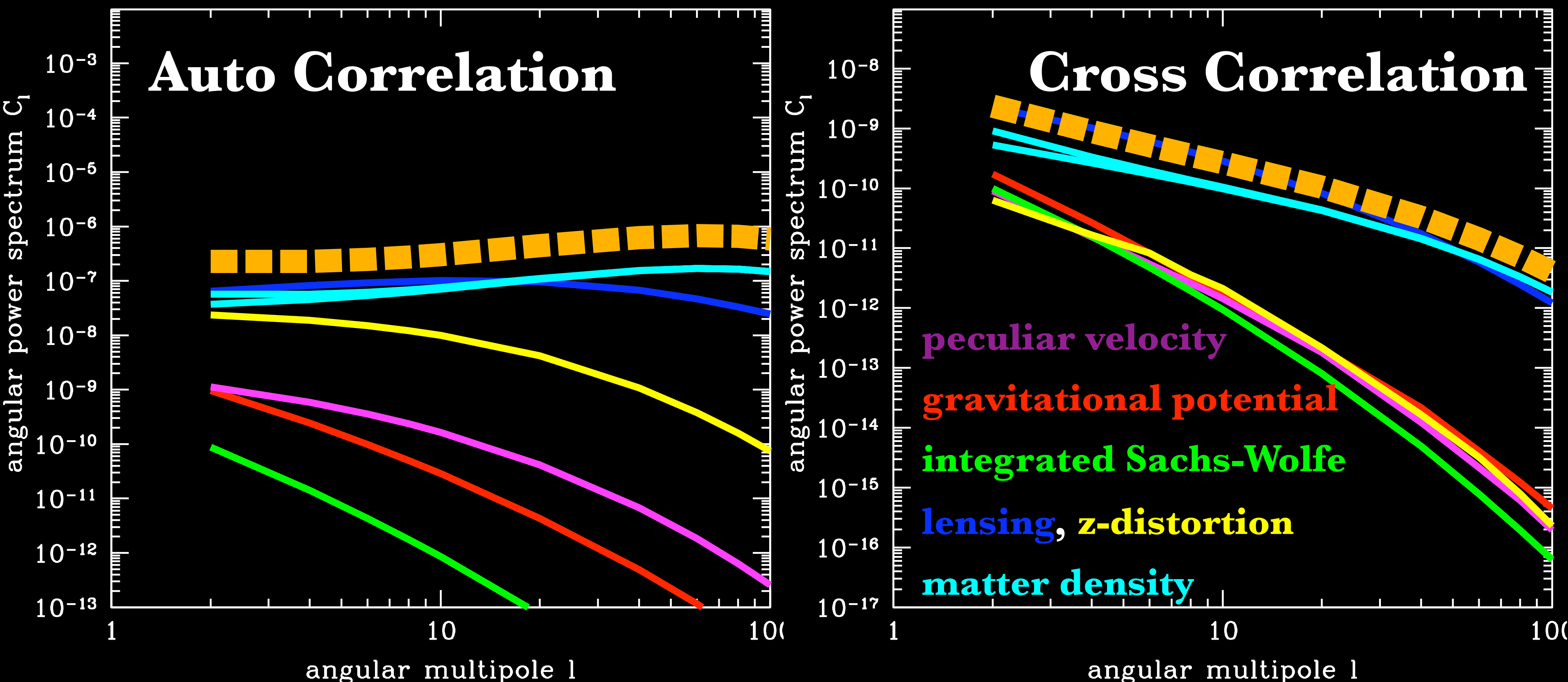
- cosmological sensitivity:
    - fiducial ΛCDM
    - large matter density
    - open universe
    - closed universe
    - dark energy

- angular diameter distance



Auto & Cross Correlations

- SDSS QSO sample with b=2, (5p–2)=0.1
  - *most relativistic contributions are ~ 10% – 30%*



Auto & Cross Correlations

- SDSS QSO sample with b=2, (5p−2)=0.1
  - *most relativistic contributions are ~ 10% – 30%*
- angular power spectrum
  ~ 3D power spectrum
  mode ~ sample distance
- total observed (dashed)

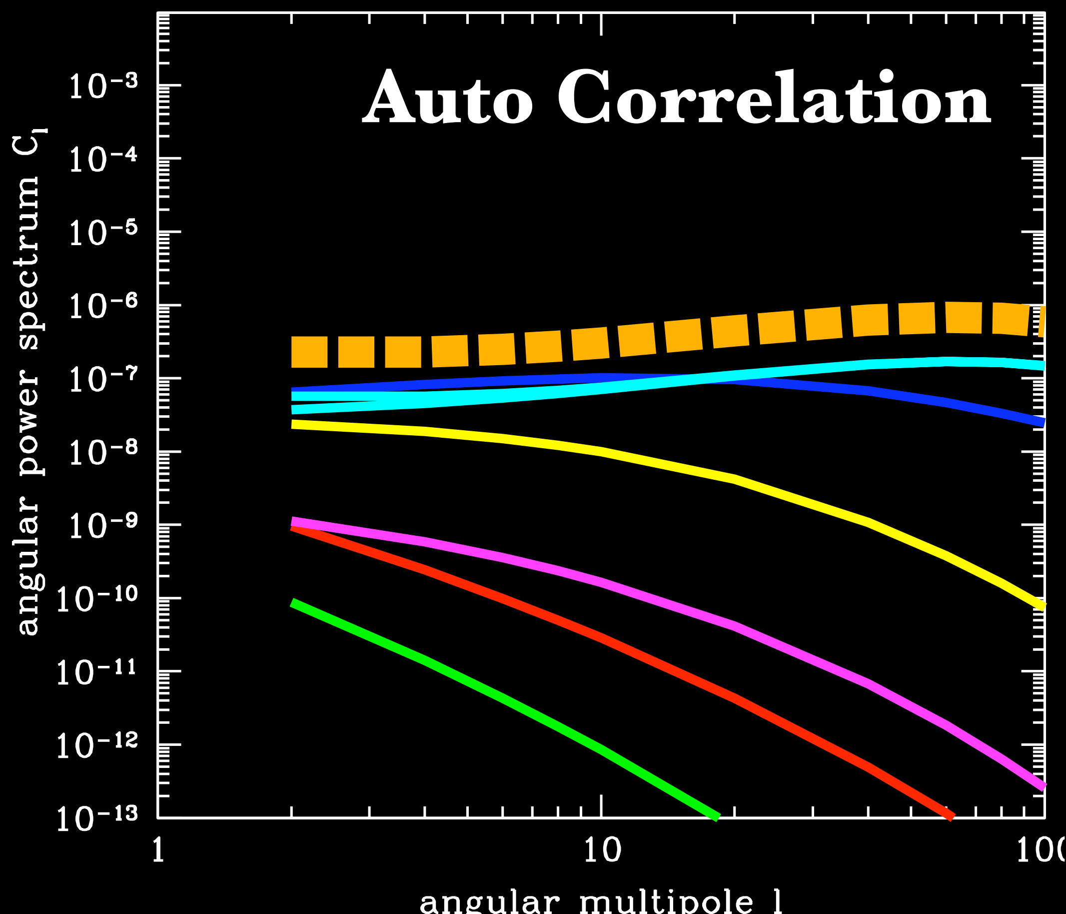
peculiar velocity

gravitational potential

integrated Sachs-Wolfe

lensing, z-distortion

matter density



Auto & Cross Correlations

- SDSS QSO sample with b=2, (5p–2)=0.1
  - *most relativistic contributions are ~ 10% – 30%*
- incoherence of distances:

  distance to CMB

  distance to QSO sample
- total observed (dashed)

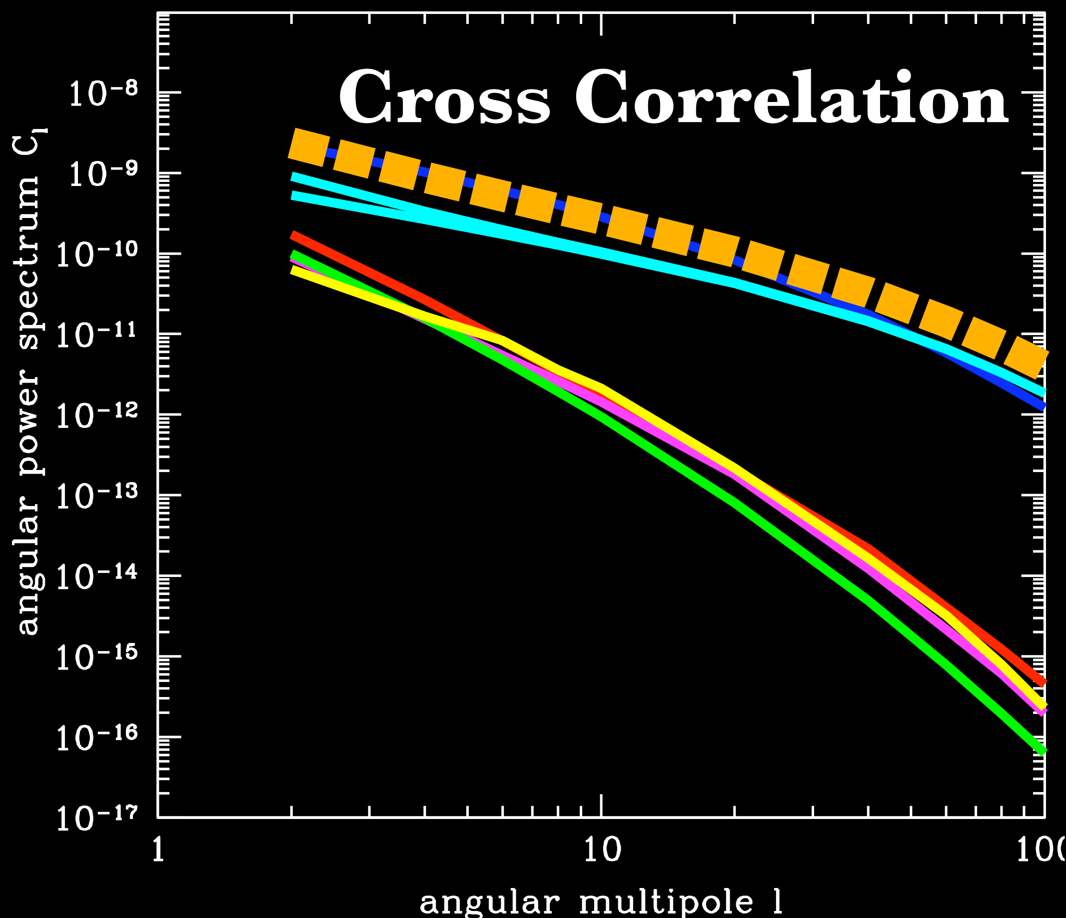
peculiar velocity

gravitational potential

integrated Sachs-Wolfe

lensing, z-distortion

matter density



Auto & Cross Correlations

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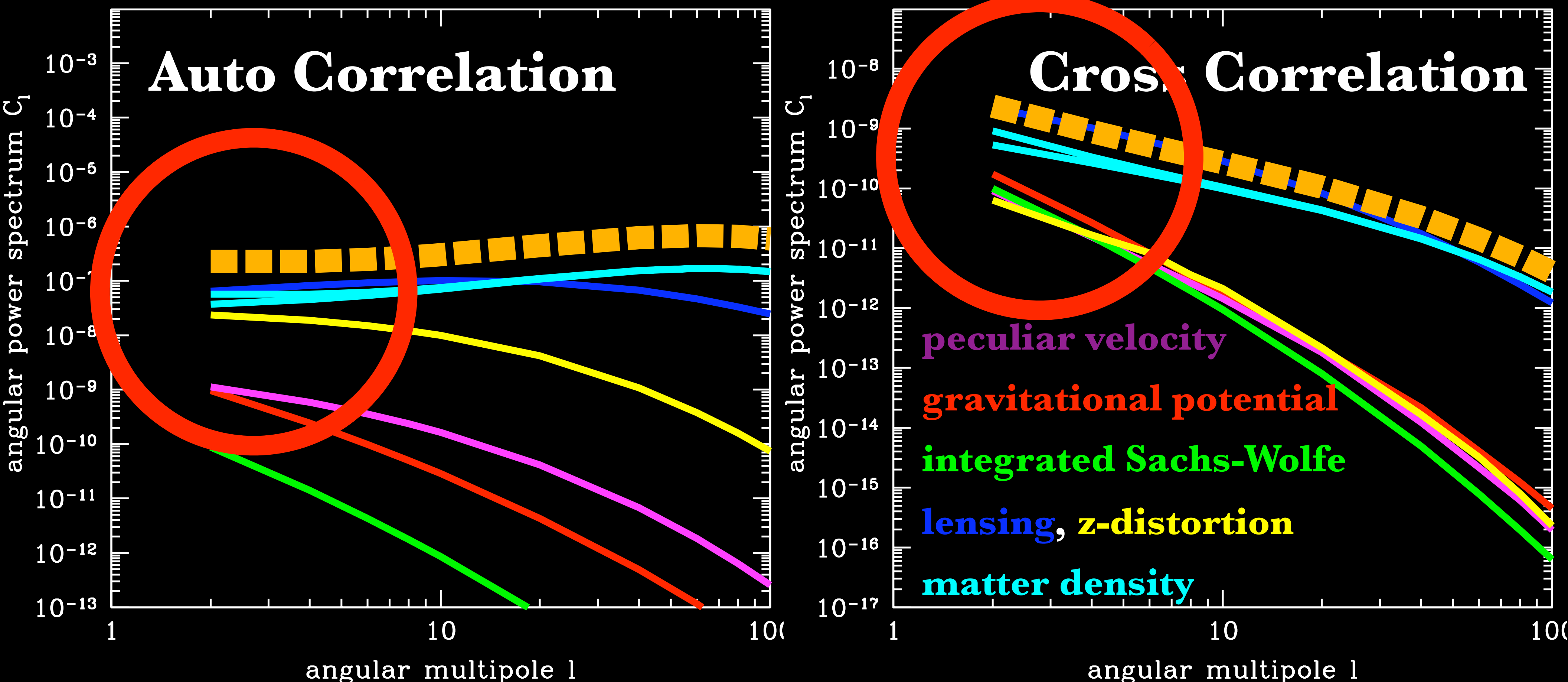
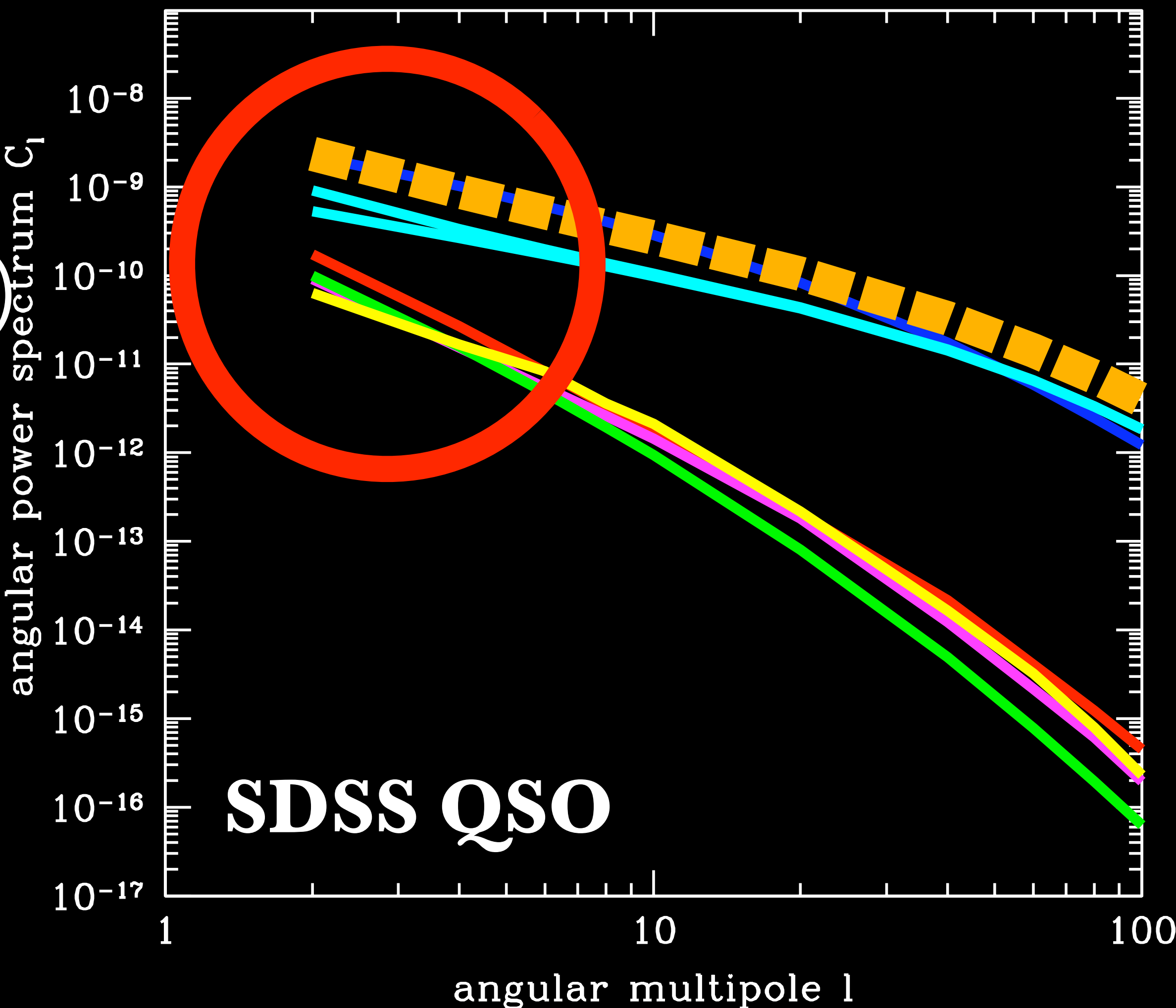


Matter Perturbations

- matter fluctuations
- two gauges:
  - Newtonian (upper)
  - Synchronous (lower)
- *which gauge is right?*
  - observables should be gauge-indep.
  - full expression should be used

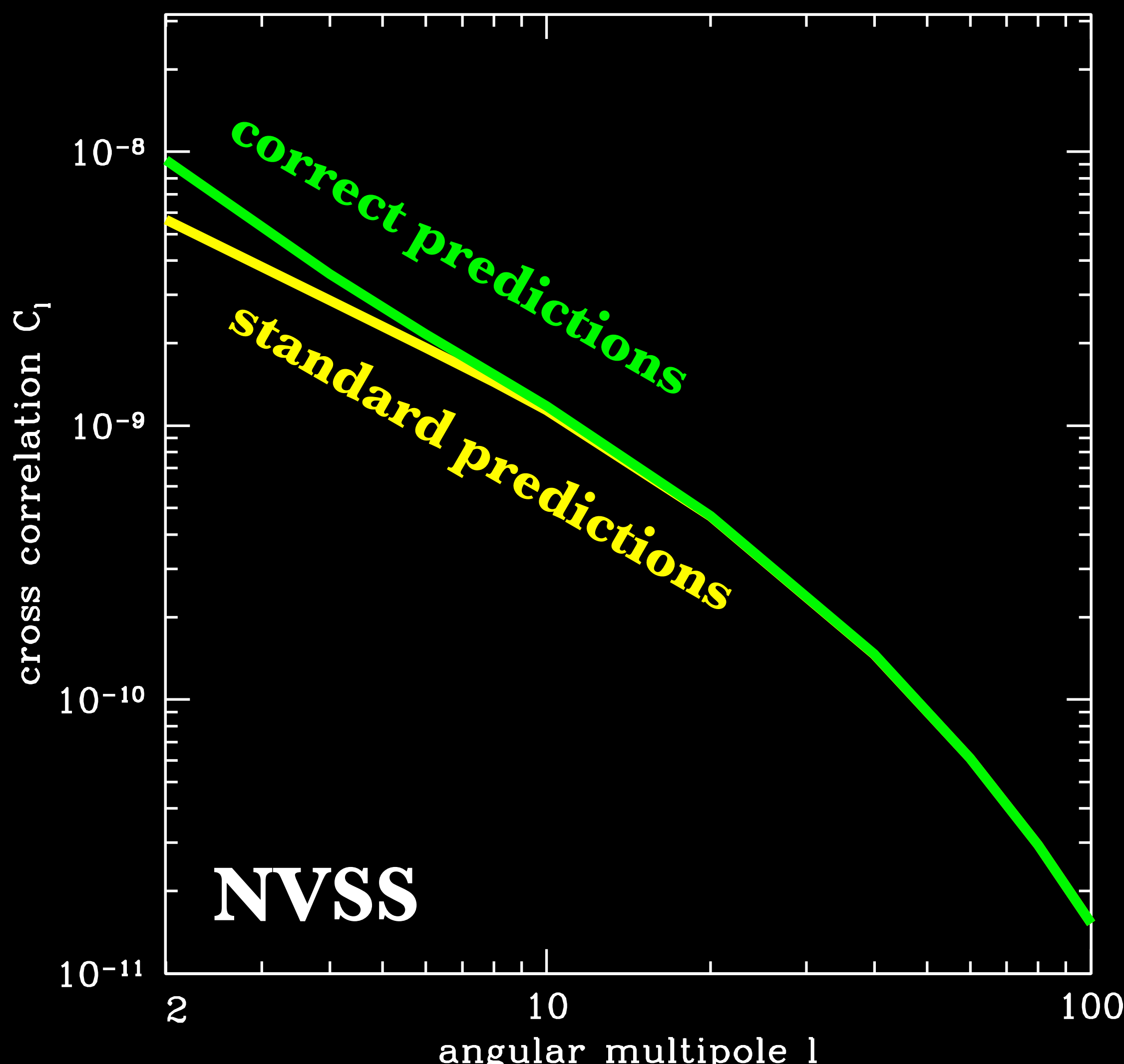


Systematic Errors

- **theoretical predictions:**
  - **new cal.** (***correct***)
  - **standard** (***incorrect***)
- **standard method:**

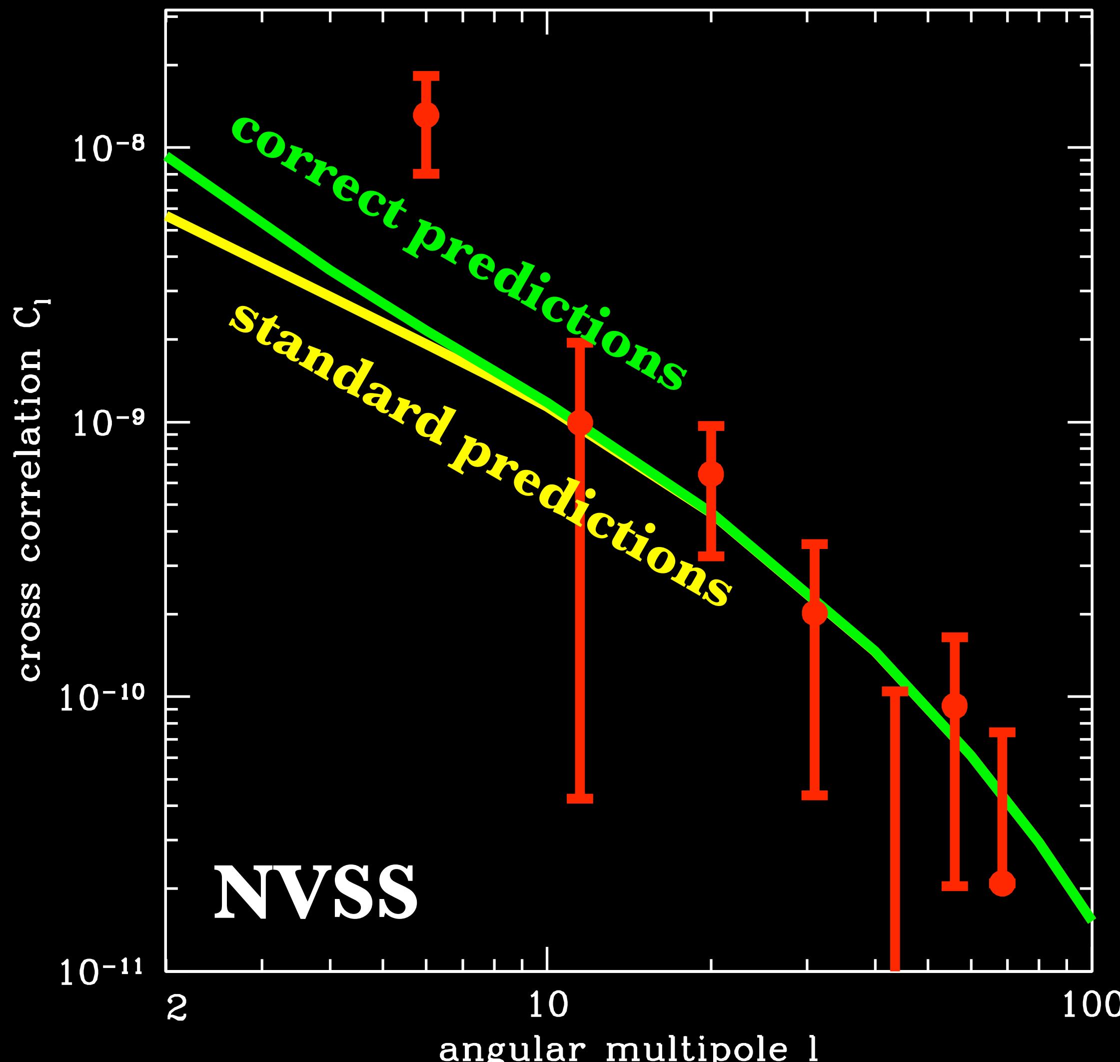
$$\delta_g = b\ \delta_m^{sync} + (5p-2)\ \kappa$$

- ***underestimate*** **the observed signals by *a factor two* at low multipoles**



Systematic Errors

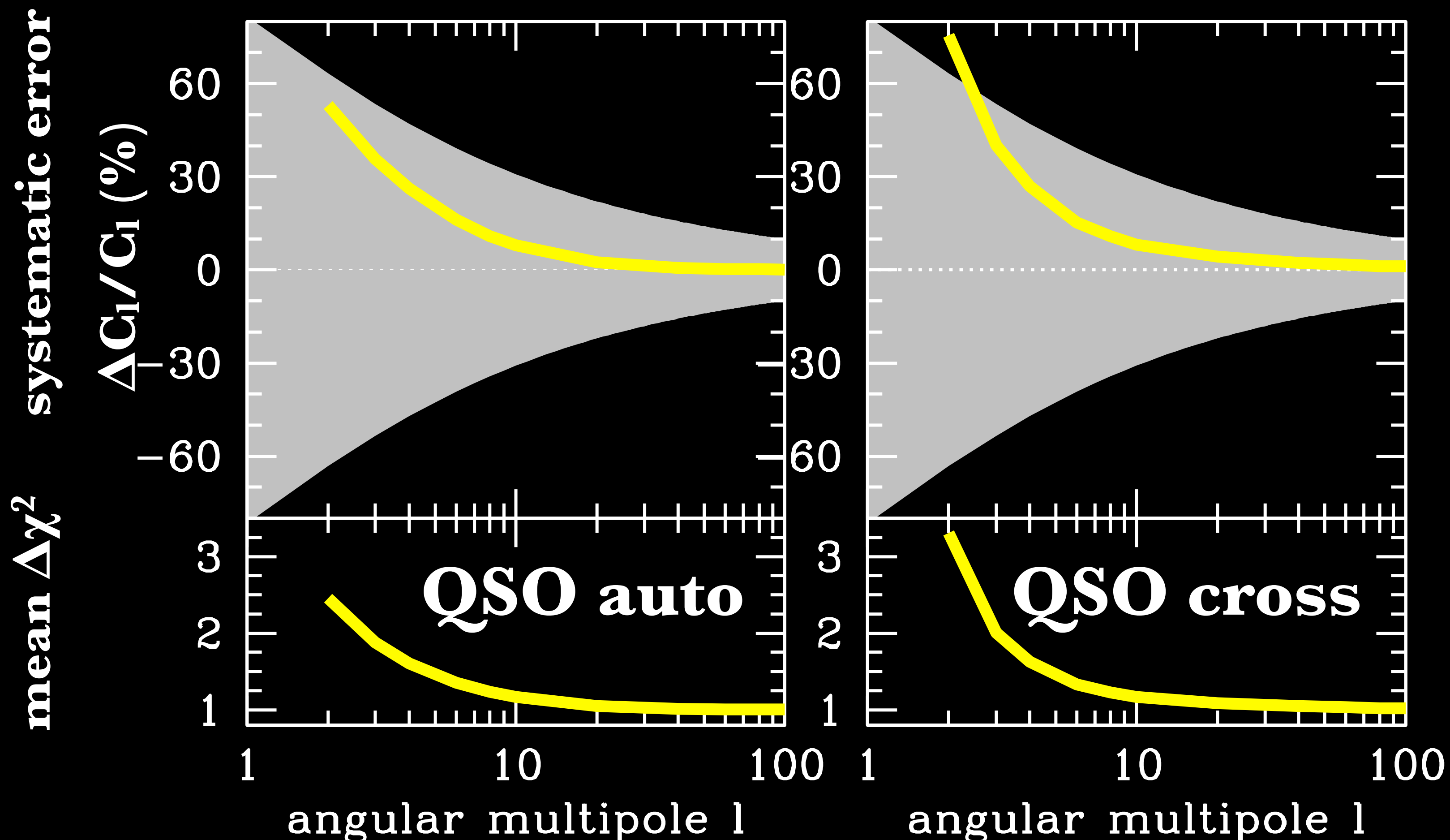
- **theoretical predictions:**
  - **new cal.** (*correct*)
  - **standard** (*incorrect*)
- **3.7-σ detection, but *observed signal is larger by 2 at low multipoles* when all tracers are combined**
- ***anomalous large signal***



Ho et al., PRD, 2008

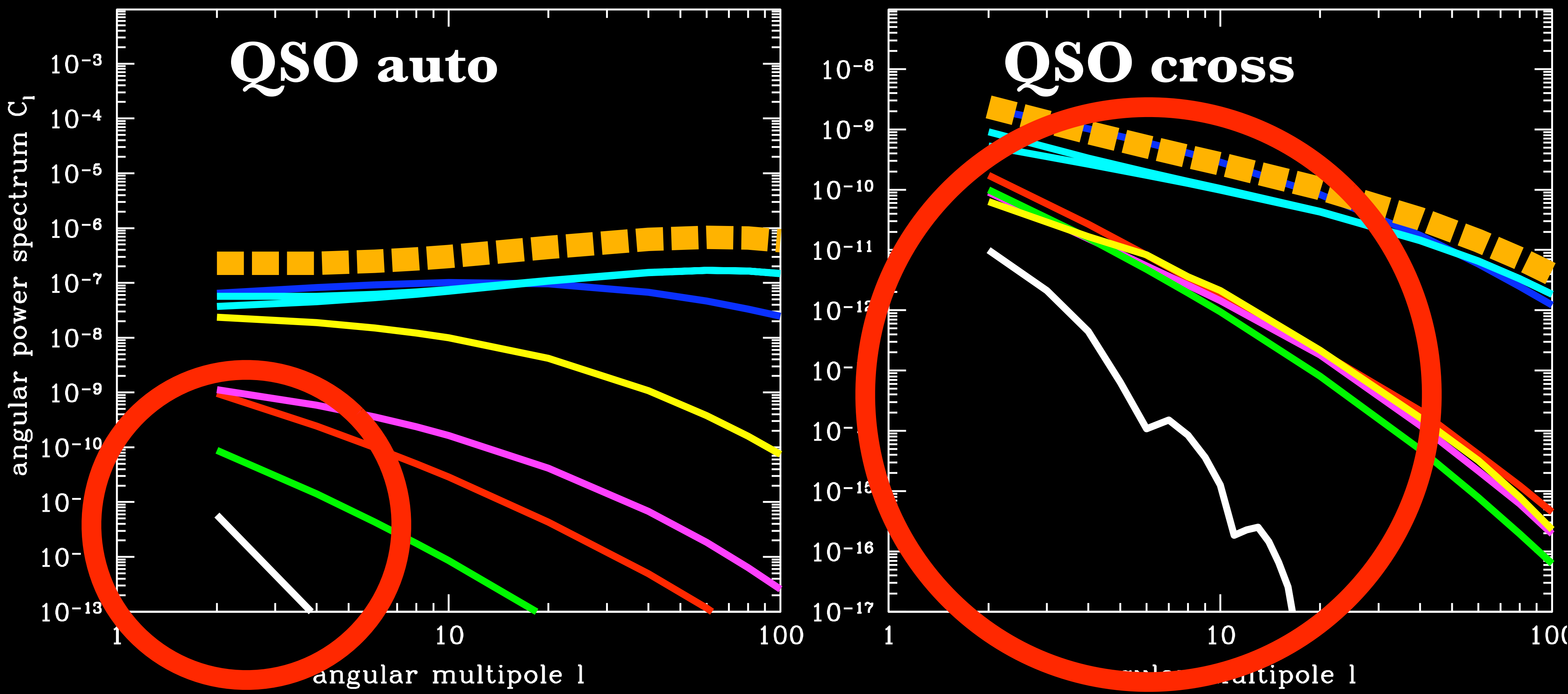
Inferred Cosmology?

- underestimate *signals* & *cosmic variance!*
  - *1.2-σ* away from incorrect cosmic variance



Primordial Gravity Waves

- integrated Sachs-Wolfe effect *for* gravity waves
  - tensor-to-scalar ratio *r=0.1*



Observed Power Spectrum

- **complications:**
  - **local (real-space) vs non-local (Fourier-space)**
  - **different spacetime with angle dependence**
  - **projected quantities (e.g.,** $\kappa, \ \delta z$ **)**
  - ***divergent* quantities (e.g.,** $\phi, \ \psi, \ \eta, \ h, \ \cdots$ **)**
  - ***angle average* vs *gauge-invariance***

- **so what is it,** $P_{\rm obs}(k)$**?**
  - **requires further investigation!**

Primordial non-Gaussianity

- **local type:** $\Phi = \phi + f_{\rm NL}\phi^2$
  - **Poisson equation** $k^2\phi(k) \sim \delta_m(k) \;\; \rightarrow \mathcal{M}(k)\phi(k) \simeq \delta_m(k)$
  - **matter density** $\delta_m^N \simeq \delta_m + \mathcal{M}\phi^2$ **,** $P_m^N(k) \simeq P_m(k)$
  - **galaxies ~ locally averaged matter density**

$$n_g \propto \exp\left[\, b\,\delta_m^N \,\right] = \sum_{n=0}^{\infty} \frac{1}{n!}\left(b\,\delta_m^N\right)^n$$

Kaiser, ApJ, 1984; Politzer & Wise, ApJ, 1984

$$\langle n_g n_g \rangle \propto b^2\langle \delta_m^N \delta_m^N \rangle + b^3 \langle \delta_m^N \delta_m^N \delta_m^N \rangle + \cdots$$

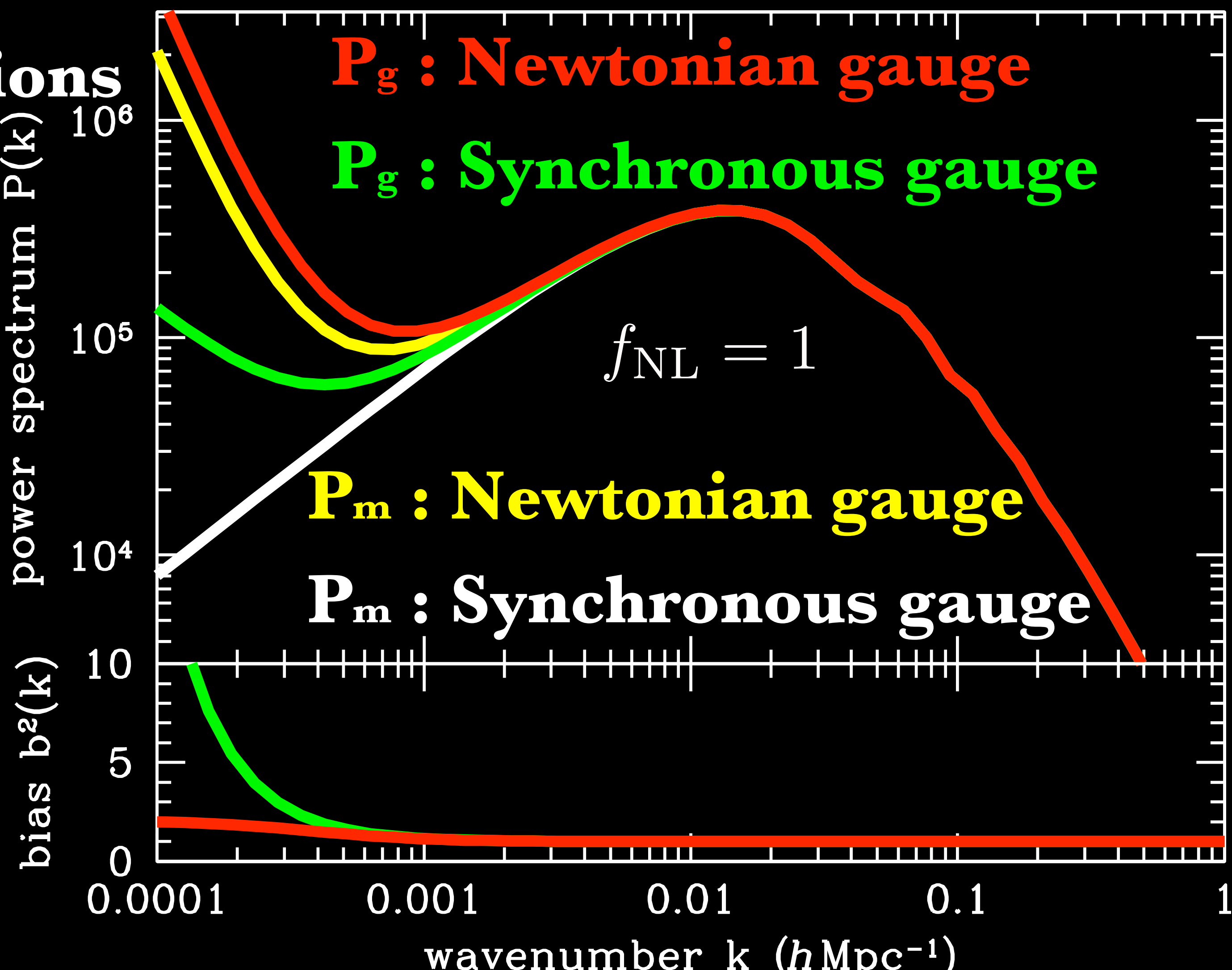
Grinstein & Wise, ApJ, 1986; Matarrese, Lucchin, Bonometto, ApJ, 1986

$$P_g(k) \propto P_m(k)\left[1 + 4 f_{\rm NL} P_{\delta\phi}(k)/P_m(k)\right]$$

Dalal, Dore, Huterer, Shirokov, PRD, 2008

Primordial non-Gaussianity

- **standard method:**
- **different predictions in two gauges (Newtonian & Synchronous)**
- **different scale-dependence of galaxy bias**



Primordial non-Gaussianity

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$$\langle n_g n_g \rangle \propto b^2 \langle \delta_m^N \delta_m^N \rangle + b^3 \langle \delta_m^N \delta_m^N \delta_m^N \rangle + \cdots$$

- **generalization:**
  - **gauge-invariant combination** $\delta_m \to (\delta_m - 3\ \delta z)$
  - **non-Gaussian case** $\delta_m^N \to (\delta_m^N - 3\ \delta z^N)$
  - ***need $P_{\rm obs}(k)$ for Gaussian case***

IV. SUMMARY AND PROSPECTS:

What do We Learn from This?

New Perspective

- ***new perspective*** **on**

  galaxy clustering as a cosmological probe
- **unified treatment:**
  - **redshift-space distortion, magnification bias, …**
- **subtle gauge issues: *observables***
  - **redshift, magnification, galaxy bias**
- **general relativistic description:**
  - **gauge-invariant formalism**
  - ***tensor* as well as scalar contributions**

Cross Correlation with CMB

- **powerful cosmological probe**
- **relativistic effects:**
  - **new contributions ~ 10 – 30%**
  - **density *perturbations* vs *physical* density**
  - **underestimate signals by a factor *two* at low multipole**
- **impact on current surveys:**
  - **1.2-σ away from cosmic variance**
  - **systematic errors (mean $\Delta\chi^2$ ~ a few)**

CONCLUSIONS

- **galaxies in cosmological framework**
  - **classical treatments fail on large scales**
- **measured in current surveys**
  - **higher precision in upcoming surveys**
  - **e.g., baryonic oscillation spectroscopic survey (BOSS), advanced dark energy physics telescope (ADEPT), ...**
- **further discriminatory power**

New Perspective on Galaxy Clustering as a Cosmological Probe: General Relativistic Effects

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Institute for Theory and Computation