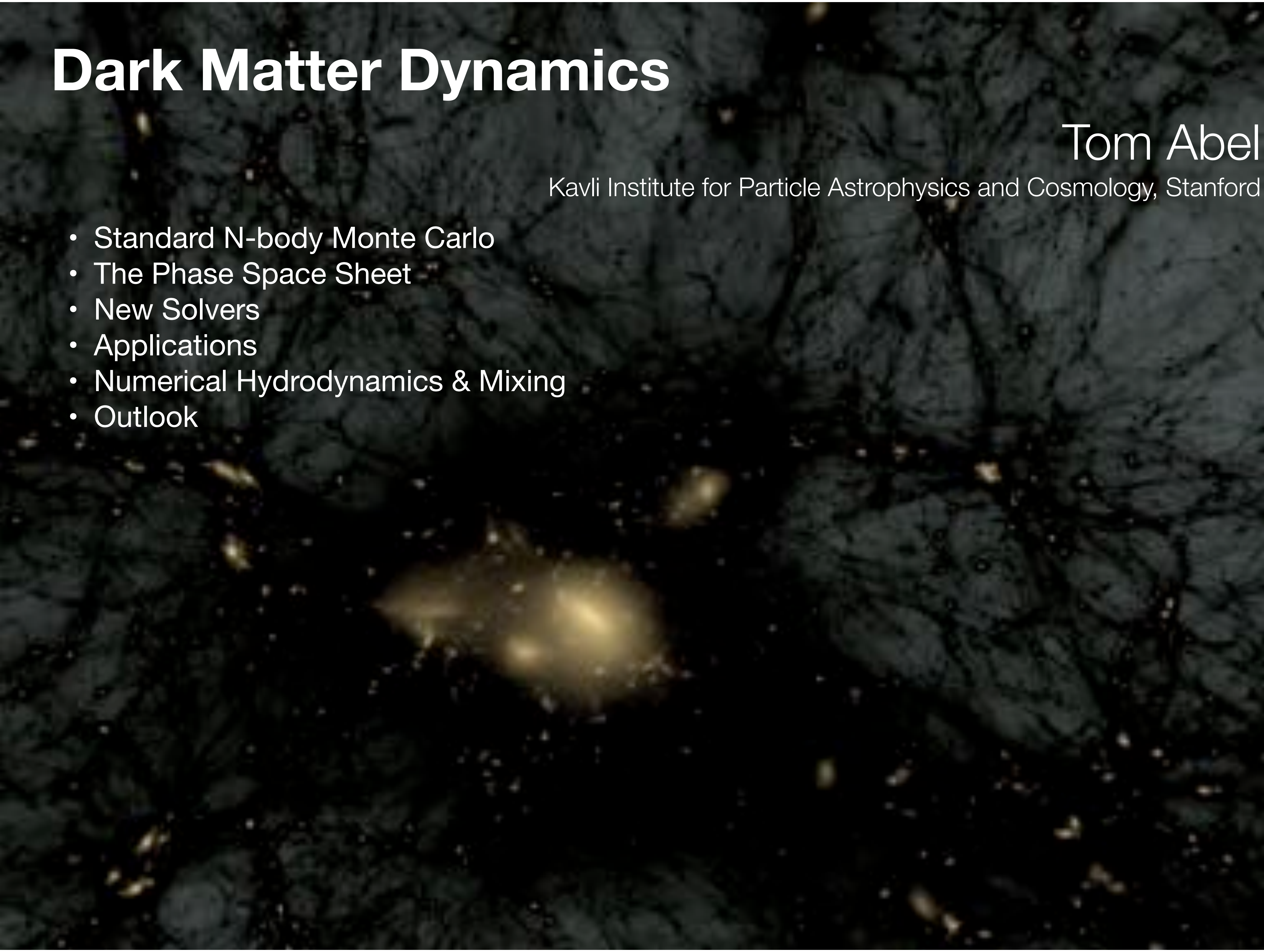
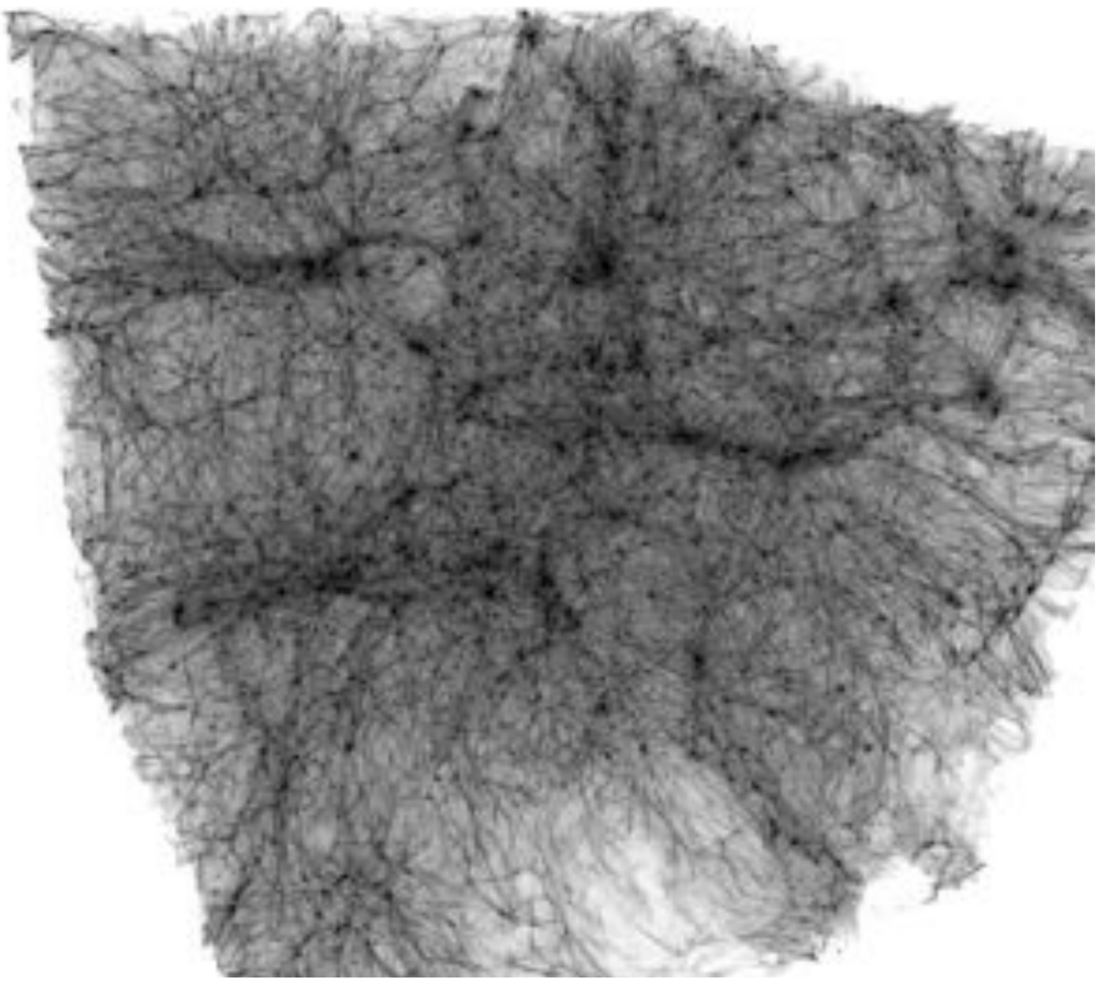
Dark Matter Dynamics

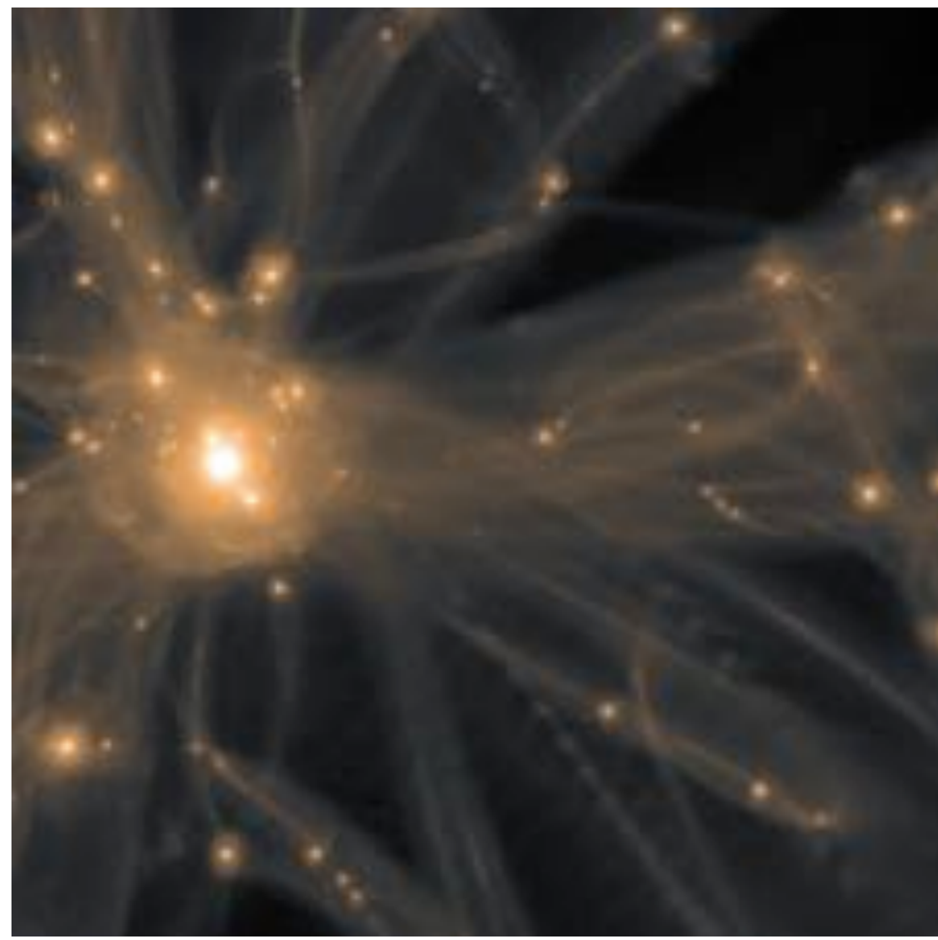
Tom Abel
Kavli Institute for Particle Astrophysics and Cosmology, Stanford

- Standard N-body Monte Carlo
- The Phase Space Sheet
- New Solvers
- Applications
- Numerical Hydrodynamics & Mixing
- Outlook

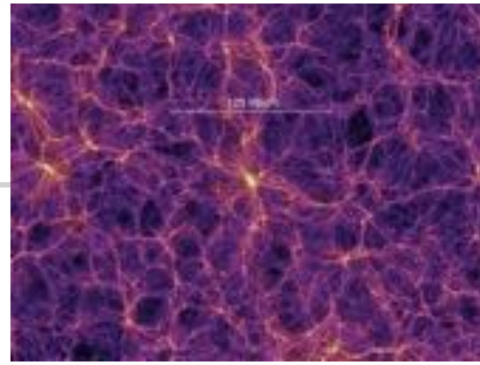


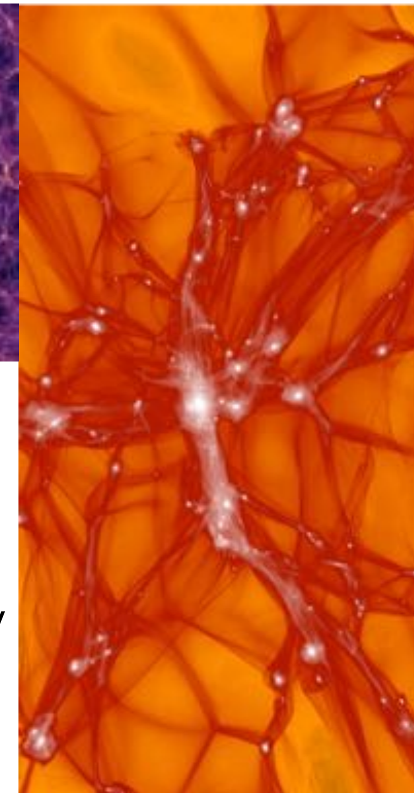
Novel way to interpret and carry out the simulations of dark matter fluids.

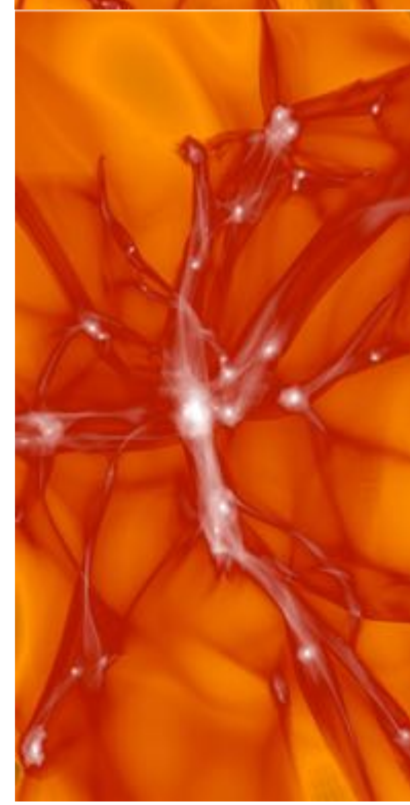




Cosmological N-body simulations







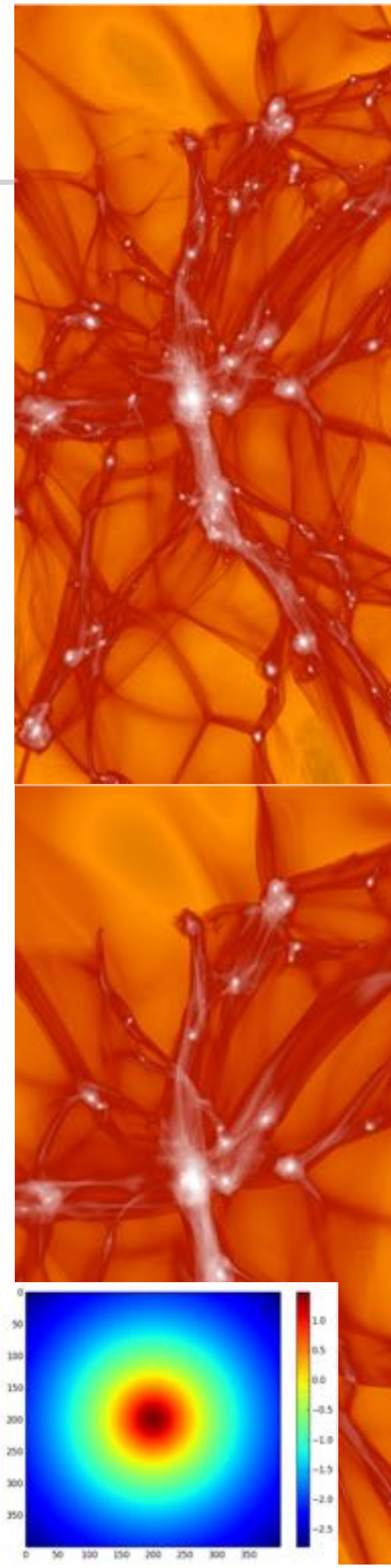
- Used to make predictions about the distribution of dark matter in the Universe
- Key results
  - Galaxies are arranged in cosmic web of voids/sheets/filaments/halos
  - Universal spherical Dark Matter density profile (NFW) [not understood from analytical arguments]
  - Predicted mass functions of halos and their clustering and velocity statistics
- Primary tool to study observational consequences of LCDM
  - initial conditions: warm vs cold DM, Gaussian vs non-Gaussian
  - sensitivity on global cosmological parameters such as the total matter content and amount of dark energy, etc.
  - Gravitational Lensing signatures

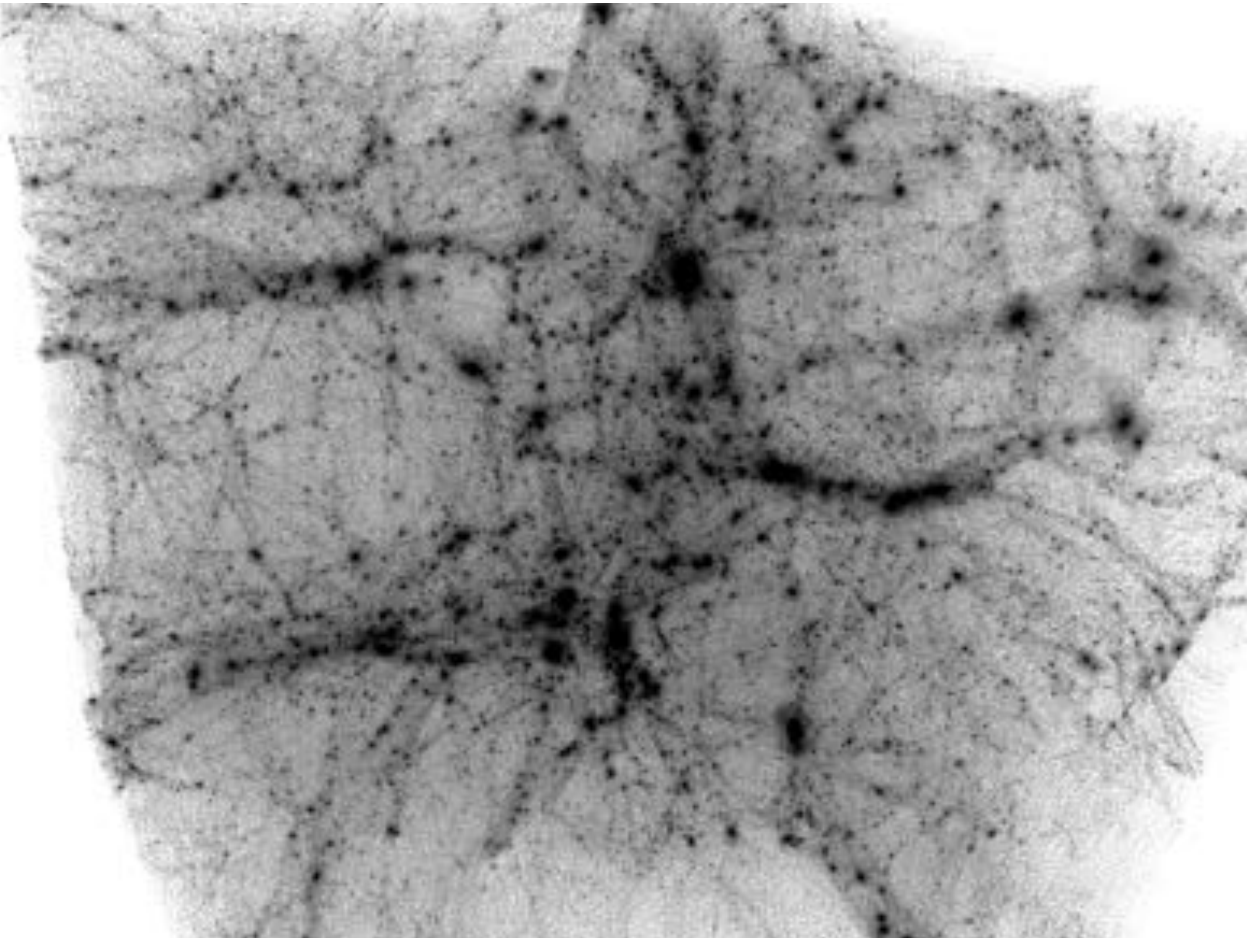
Cosmological N-body simulations

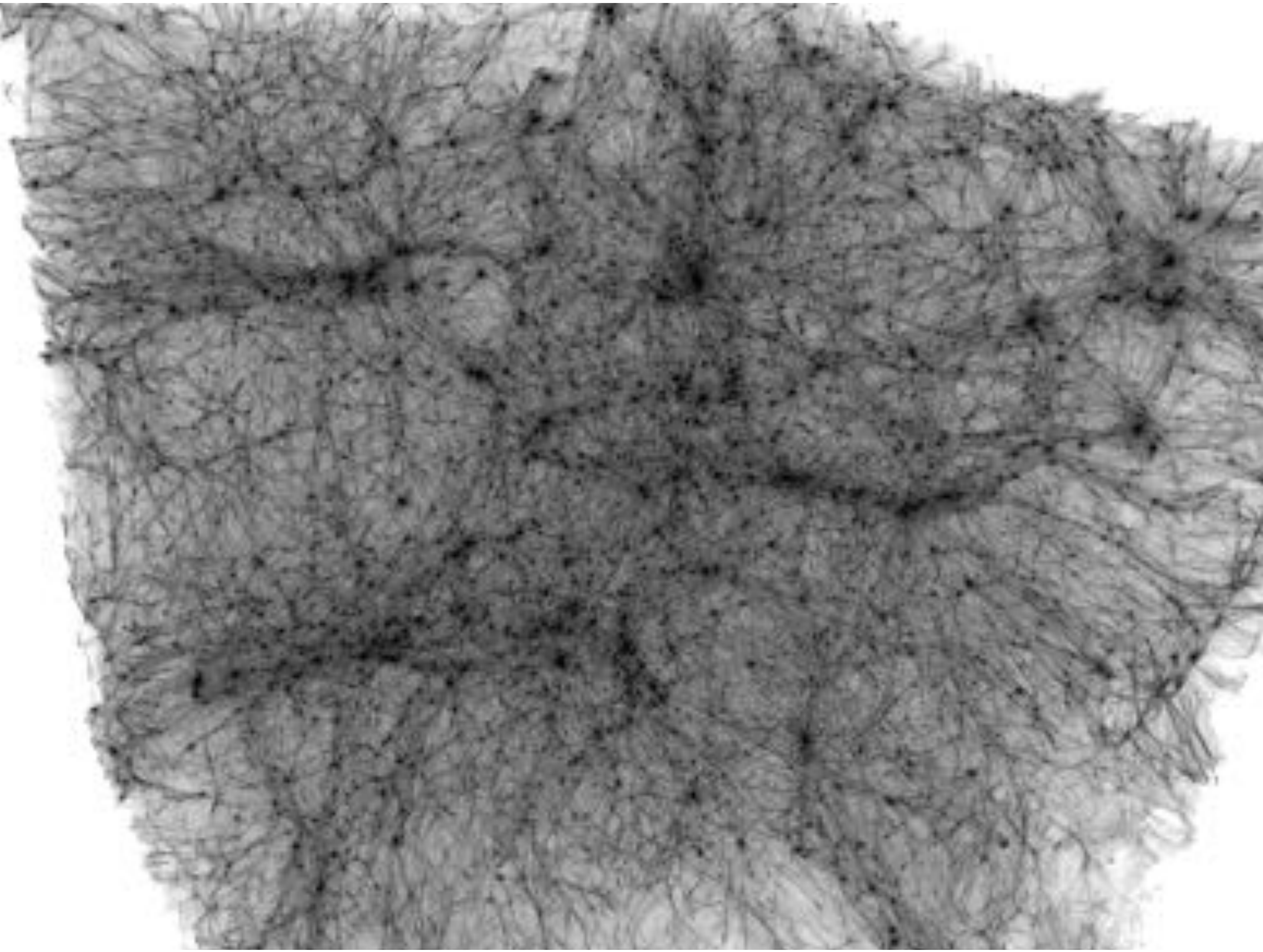
$$\dot{\mathbf{x}} = \mathbf{v}(t) \qquad \dot{\mathbf{v}}_\mathbf{i} = -\sum_{i \neq j}^{N} G m_i m_j \frac{(\mathbf{x_j} - \mathbf{x_i})}{|\mathbf{x_j} - \mathbf{x_i}|^3}$$

- All modern cosmological simulation codes only differ in how they accelerate the computation of the sum over all particles to obtain the net force
- End result are simply the positions and velocities of all particles
- Softening of forces (add epsilon^2 in denominator) avoids singularities.
- Limit N goes to infinity must give correct answer, right?
- Plummer

$$\dot{\mathbf{v}}_i = -\sum_{i \neq j}^{N} G\, m_i\, m_j \frac{(\mathbf{x}_j - \mathbf{x}_i)}{(|\mathbf{x}_j - \mathbf{x}_i|^2 + \epsilon^2)^{3/2}}$$







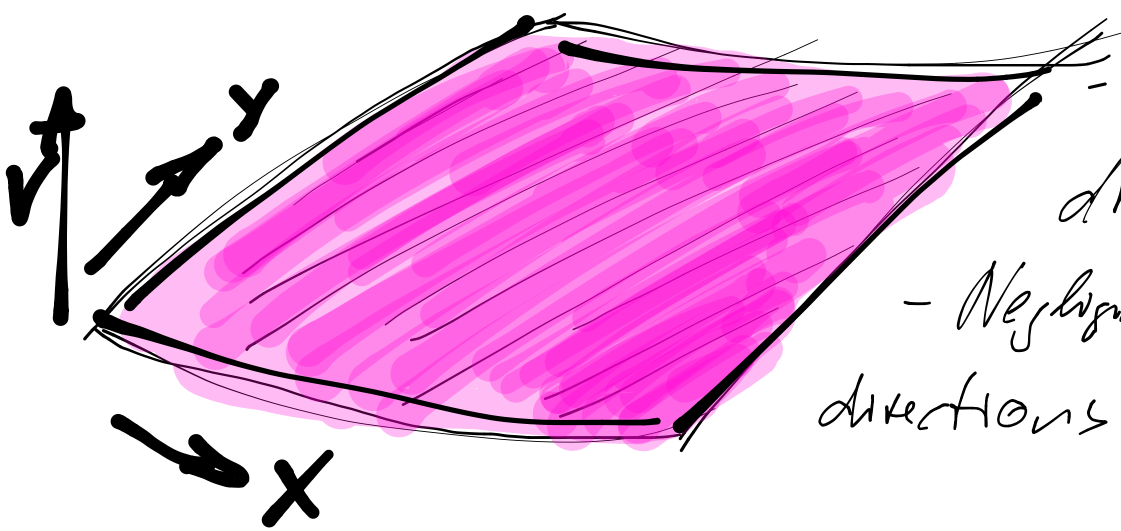
The Dark Matter Sheet?

COOL WIMPS

Dark Matter is commonly hypothesized to originate within seconds after the BIG BANG. If it were moving relativistically today, galaxies and other structures would not exist. We speak of **COLD DARK MATTER**.

Working HYPOTHESIS:

- Weakly interacting massive particle (say ≈ 100 GeV).
- Very cold. Even keV particles would only have $\sim \frac{m}{s}$ speeds today.



- Almost perfectly uniformly distributed initially.
- Negligible extent along velocity directions in phase space.

The Dark Matter Sheet ?

Fluid

OF DARK MATTER PARTICLES IN THE MILKY WAY :

$N_{DM} \approx 10^{67} \left(\frac{100\,GeV}{m_{DM}}\right) \gg$ # OF STARS IN THE UNIVERSE

$\gg$ # OF PARTICLES THAT FIT ON A COMPUTER

USING ALL THE COMPUTERS IN THE WORLD : $\lesssim 10^{17}$ particles

SOLVE VLASOV-POISSON SYSTEM INSTEAD.

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_x f + \vec{a} \cdot \vec{\nabla}_v f = 0$$

$$\vec{a} = -\vec{\nabla}\phi$$

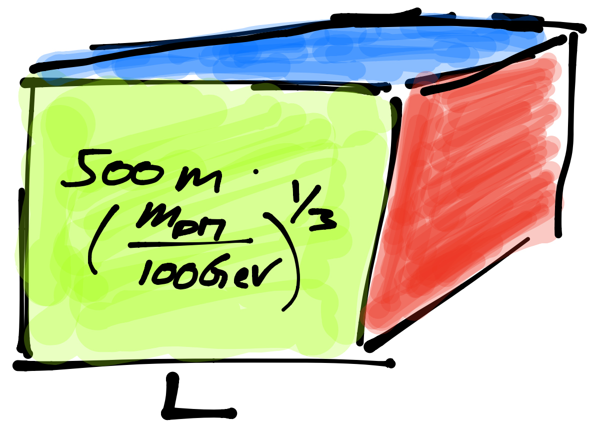
$$\nabla^2 \phi = 4\pi G \rho$$

f: distribution function IN PHASE SPACE

ϕ: potential

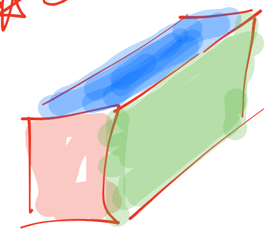
FOR PHASE SPACE ELEMENT TO CONTAIN 10^6 PARTICLES @ MEAN DENSITY IT HAS TO BE LARGER THAN

$$L \sim 500\,m \left(\frac{m_{DM}}{100\,GeV}\right)^{1/3}$$



The Dark Matter Sheet ?

3D MANIFOLD MOVING IN SIX DIMENSIONAL PHASE SPACE



Phase space volume is conserved

$$\Delta V \cdot (\Delta v)^3 = \text{const}$$

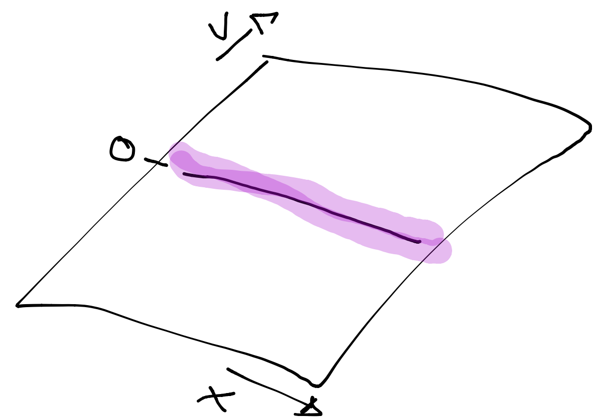
spatial volume — volume in velocity space

redshift when $E \approx 100\,\text{GeV}$: $1\,\text{eV}$ @ $z \sim 1000$

$100\,\text{GeV}$ @ $z \sim 10^{14}$

Matter density dropped by a factor $\sim 10^{42}$ since then.

→ YES. VERY COLD.



Tiny initial peculiar velocities

⇒ distribution function $f(\vec{x},\vec{v}) = f_0\,\delta(\vec{v})$

is single valued at every $\vec{x}$ — DIRAC DELTA

GRAVITY:

TOTAL MASS DENSITY

POISSON EQUATION: $\nabla^2 \phi = 4\pi G \rho$

CONTINUUM DESCRIPTION

$$F/m = -\nabla \phi$$

VLASOV EQUATION $\frac{\partial f}{\partial t} + v \nabla_x f - \nabla \phi \nabla_v f = 0$

FOR $N \rightarrow \infty$ IDENTICAL

N POINT MASSES: $\vec{a}_j = -\sum_{i \neq j} \frac{G m_i}{|\vec{x}_j - \vec{x}_i|^2 + \varepsilon^2} \frac{(\vec{x}_j - \vec{x}_i)}{|\vec{x}_j - \vec{x}_i|}$

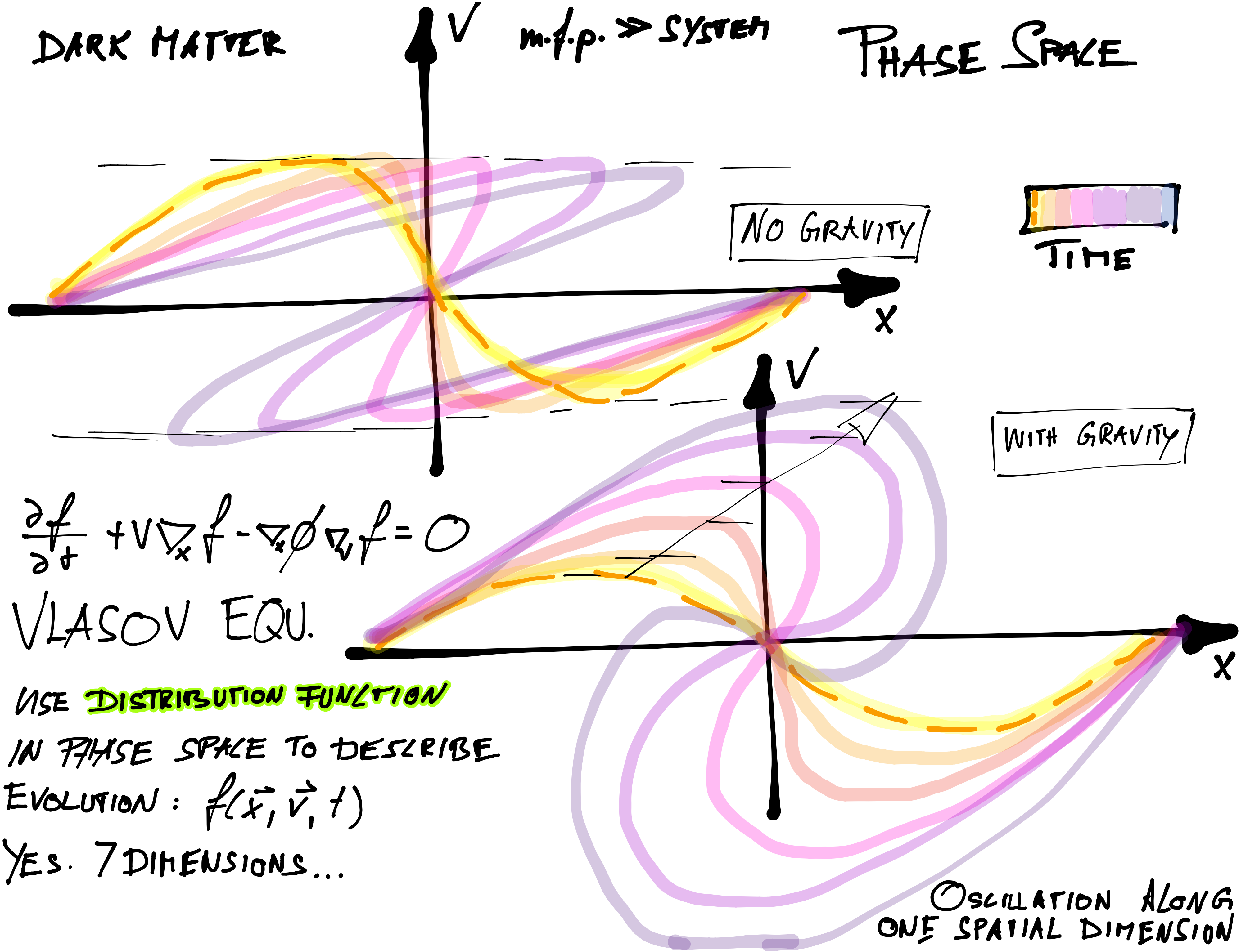
PARTICLE PICTURE

Particle "advection": $\frac{\partial \vec{x}_j}{\partial t} = \vec{v}$; $\frac{\partial \vec{v}_j}{\partial t} = \vec{a}_j$

DARK MATTER

m.f.p. ≫ SYSTEM

PHASE SPACE



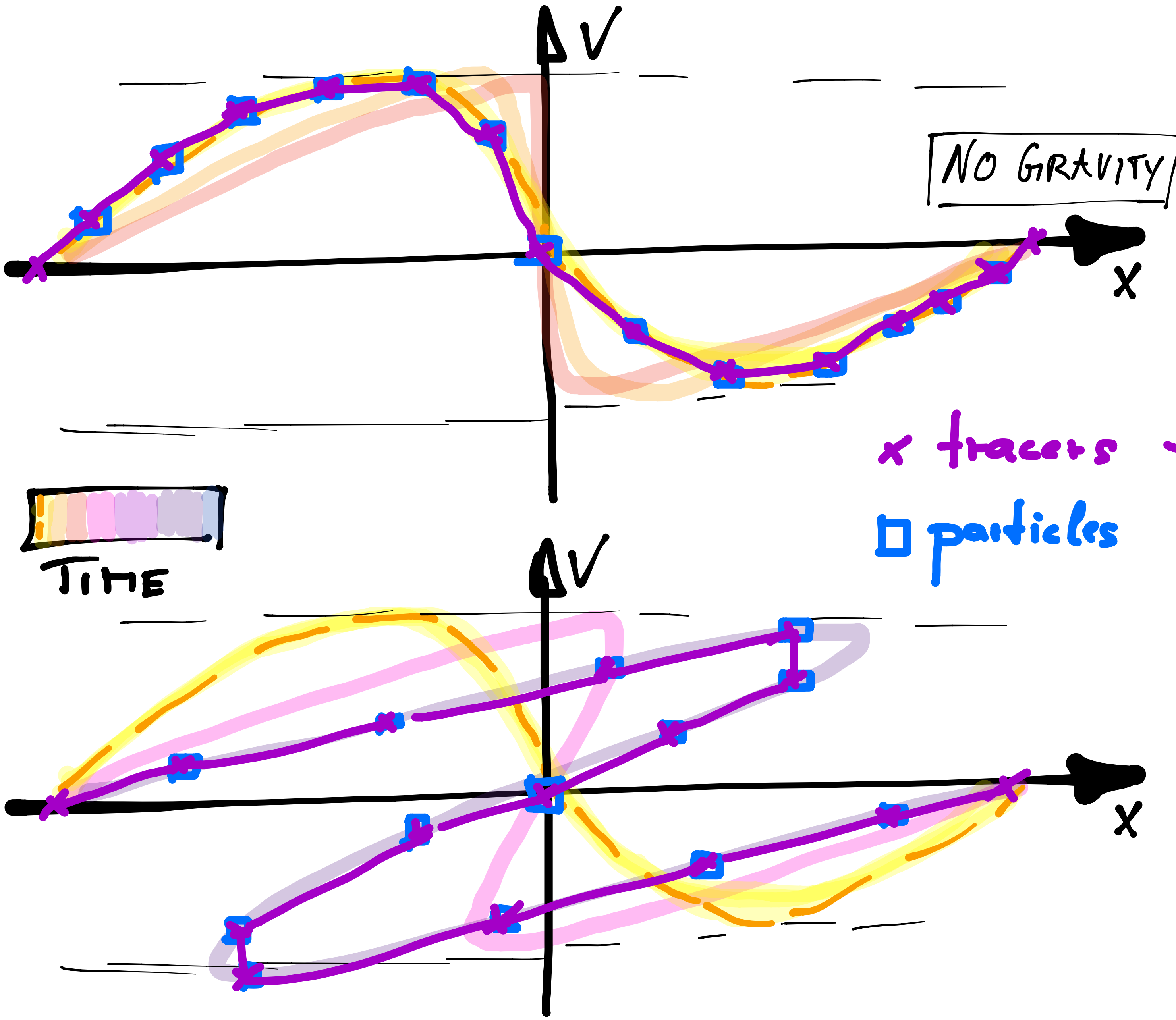
$$\frac{\partial f}{\partial t} + v \nabla_x f - \nabla_x \phi \nabla_v f = 0$$

VLASOV EQU.

USE **DISTRIBUTION FUNCTION** IN PHASE SPACE TO DESCRIBE EVOLUTION : $f(\vec{x}, \vec{v}, t)$

YES. 7 DIMENSIONS...

DISCRETIZE DARK MATTER DISTRIBUTION: Mass or Volume?

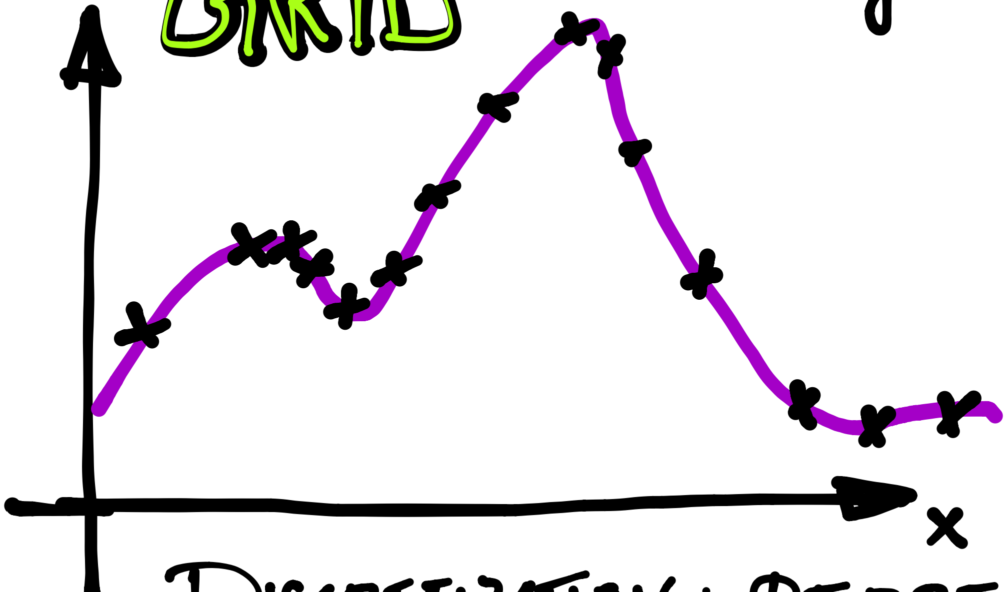


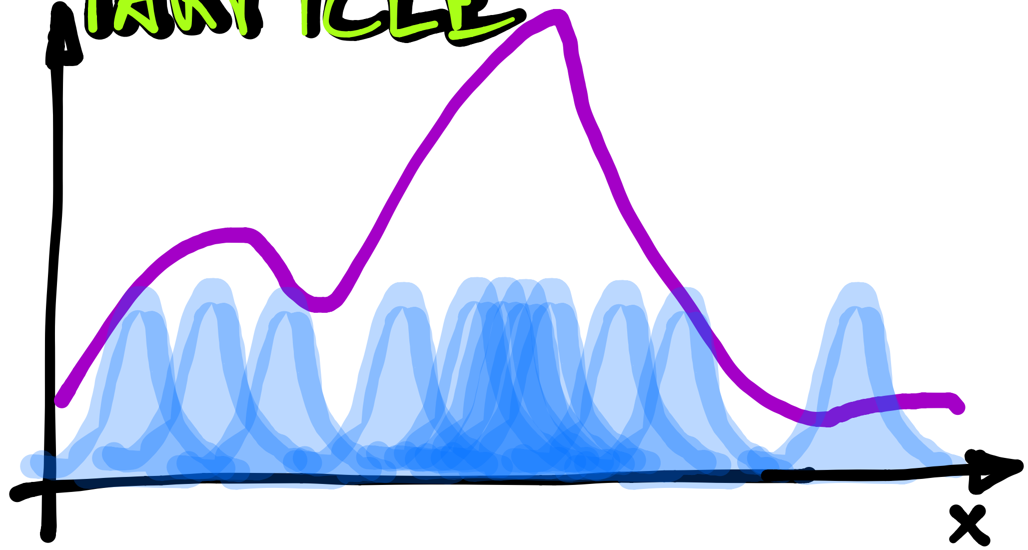
COUNT PARTICLES IN RANDOM VOLUMES GIVES AVERAGE DENSITIES

THINK MASS BETWEEN PARTICLES !

⟹ DENSITY KNOWN EVERYWHERE !

GRID e.g. PARTICLE





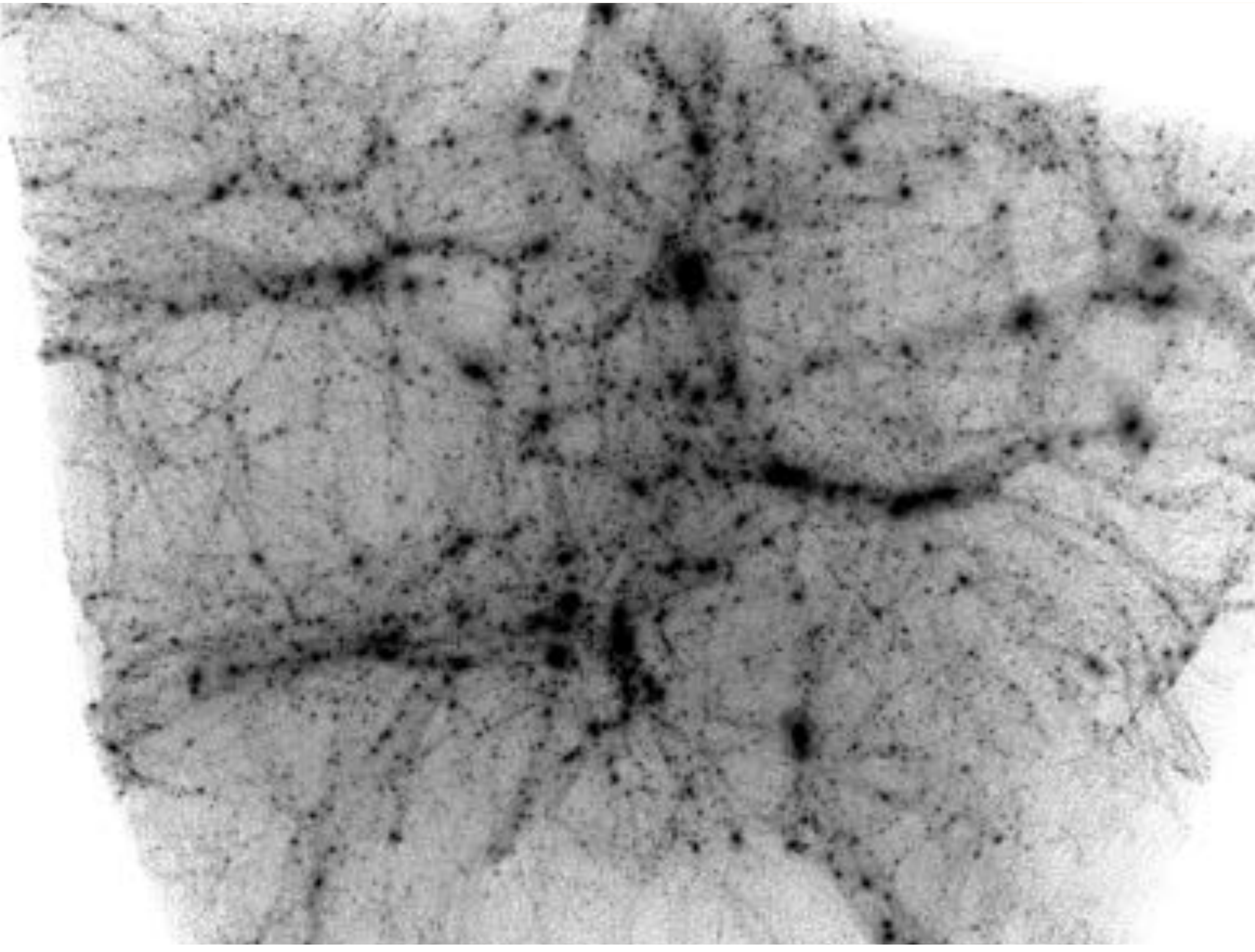
DISCRETIZATION : REPRESENT CONTINUUM SOLUTION WITH FINITE NUMBER OF ELEMENTS
("COMPRESSION")

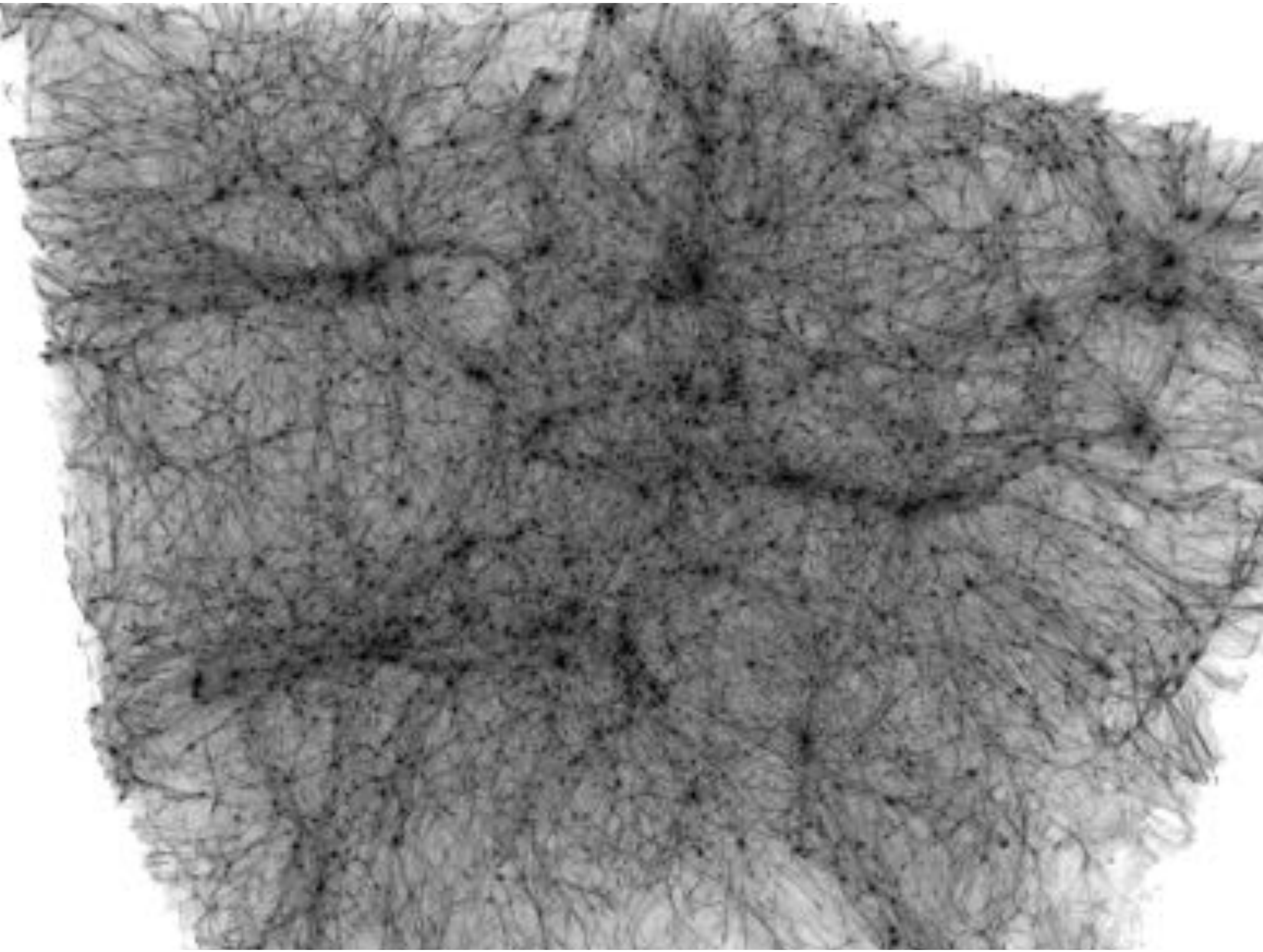
uniform, adaptive, moving, structured, unstructured, tetrahedral, cartesian, cylindrical

MESH

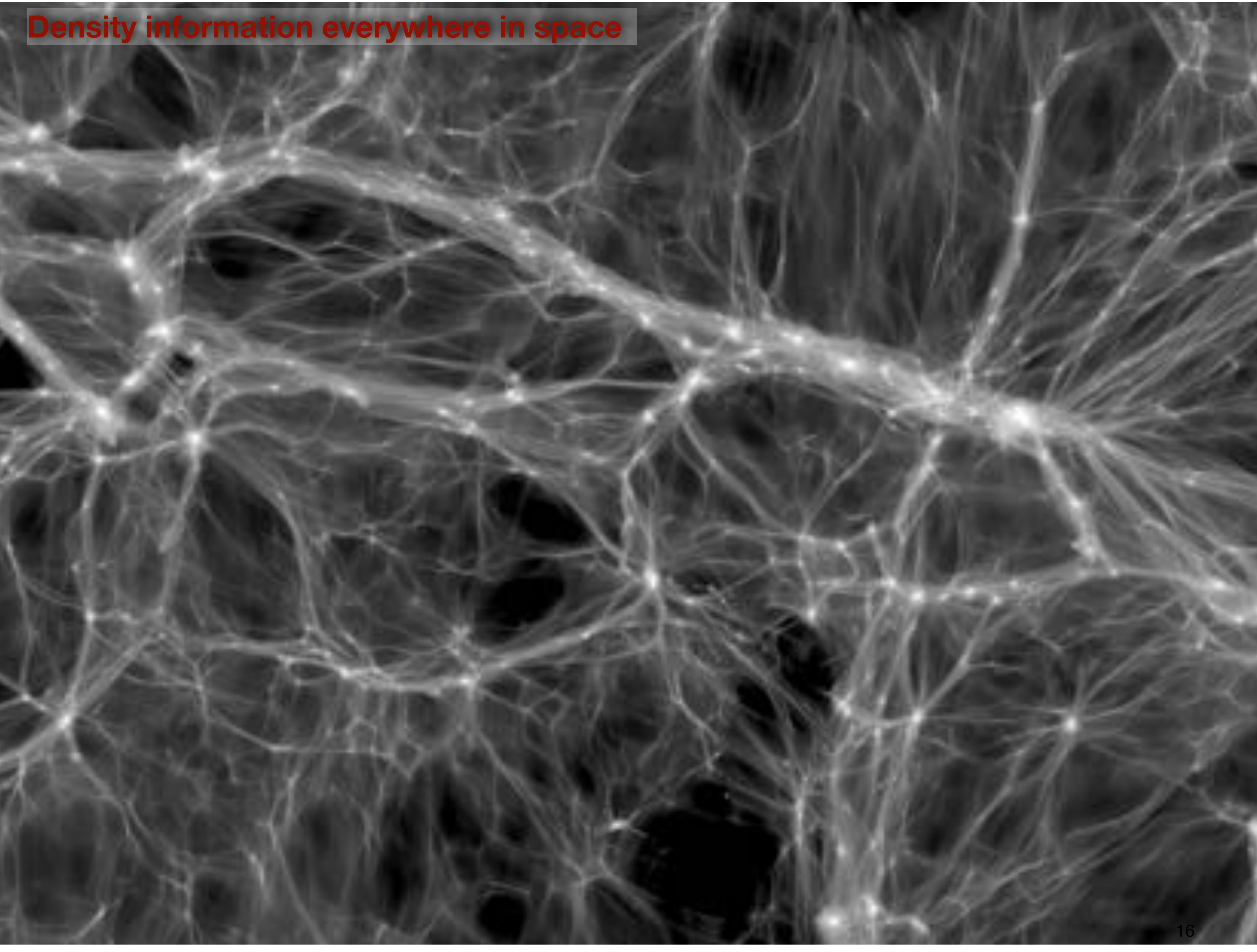
fixed, adaptive, high order, assymmetric, ...

KERNEL



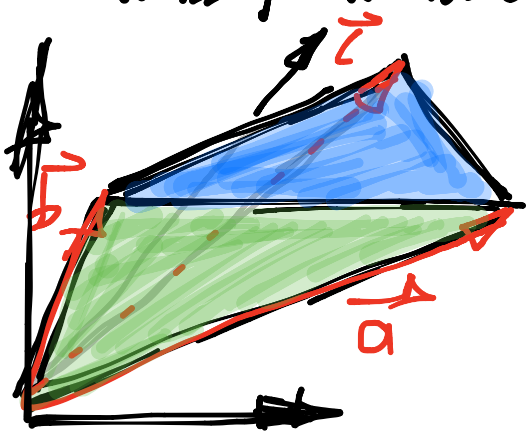


Density information everywhere in space

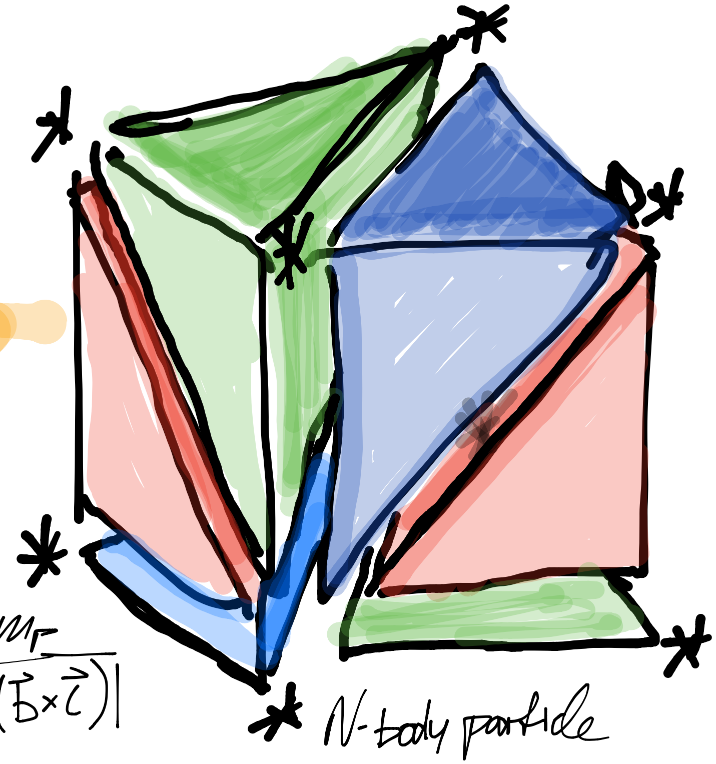


TESSELATE 3D MANIFOLD & TRACK IN 6D PHASE SPACE

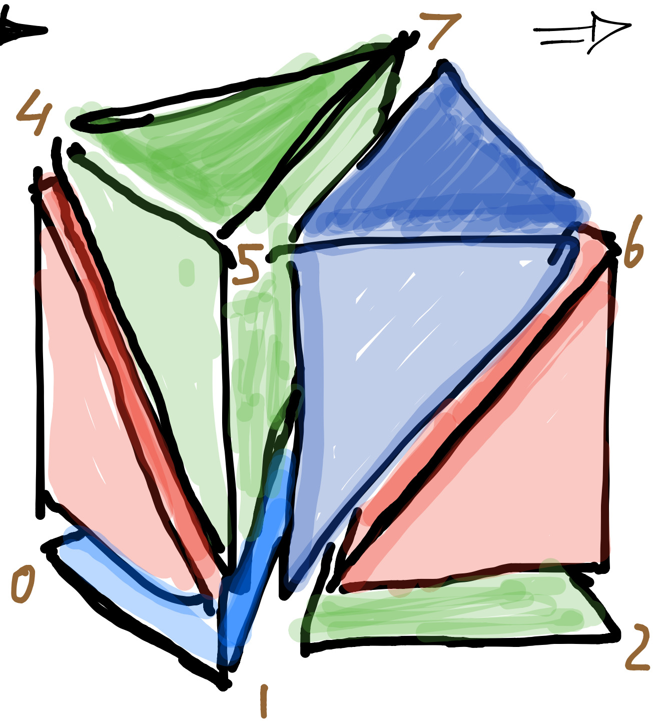
- NATURAL TESSELATION SPLITS CUBE INTO 6 EQUAL SIZED TETRAHEDRA
- mass per tetrahedron = 1/6 of DM particle mass.



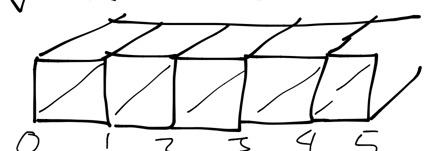
$$V = \frac{|\vec{a} \cdot (\vec{b} \times \vec{c})|}{6}$$



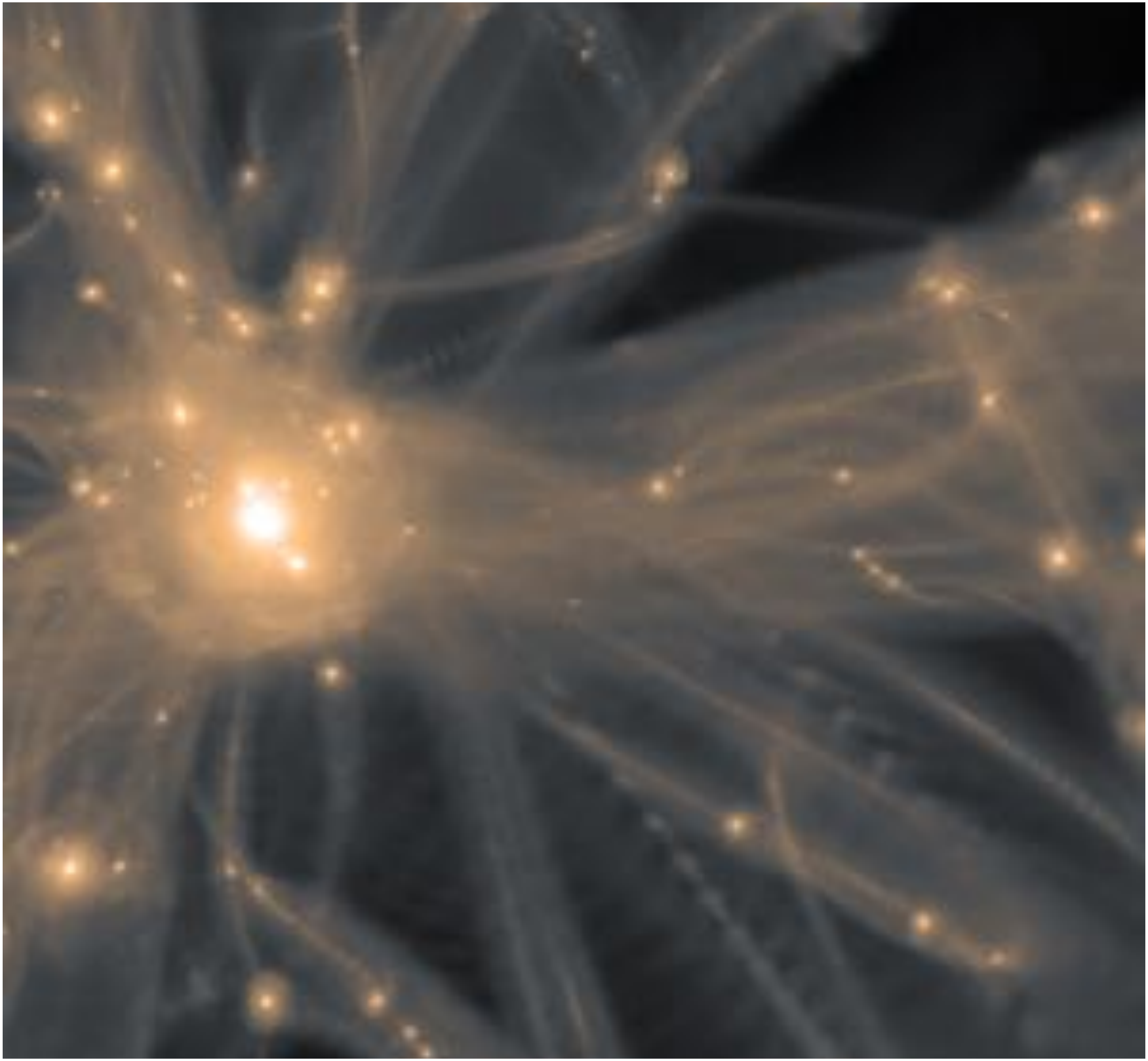
$$\Rightarrow \rho = \frac{m_p}{6V} = \frac{m_p}{|\vec{a} \cdot (\vec{b} \times \vec{c})|}$$

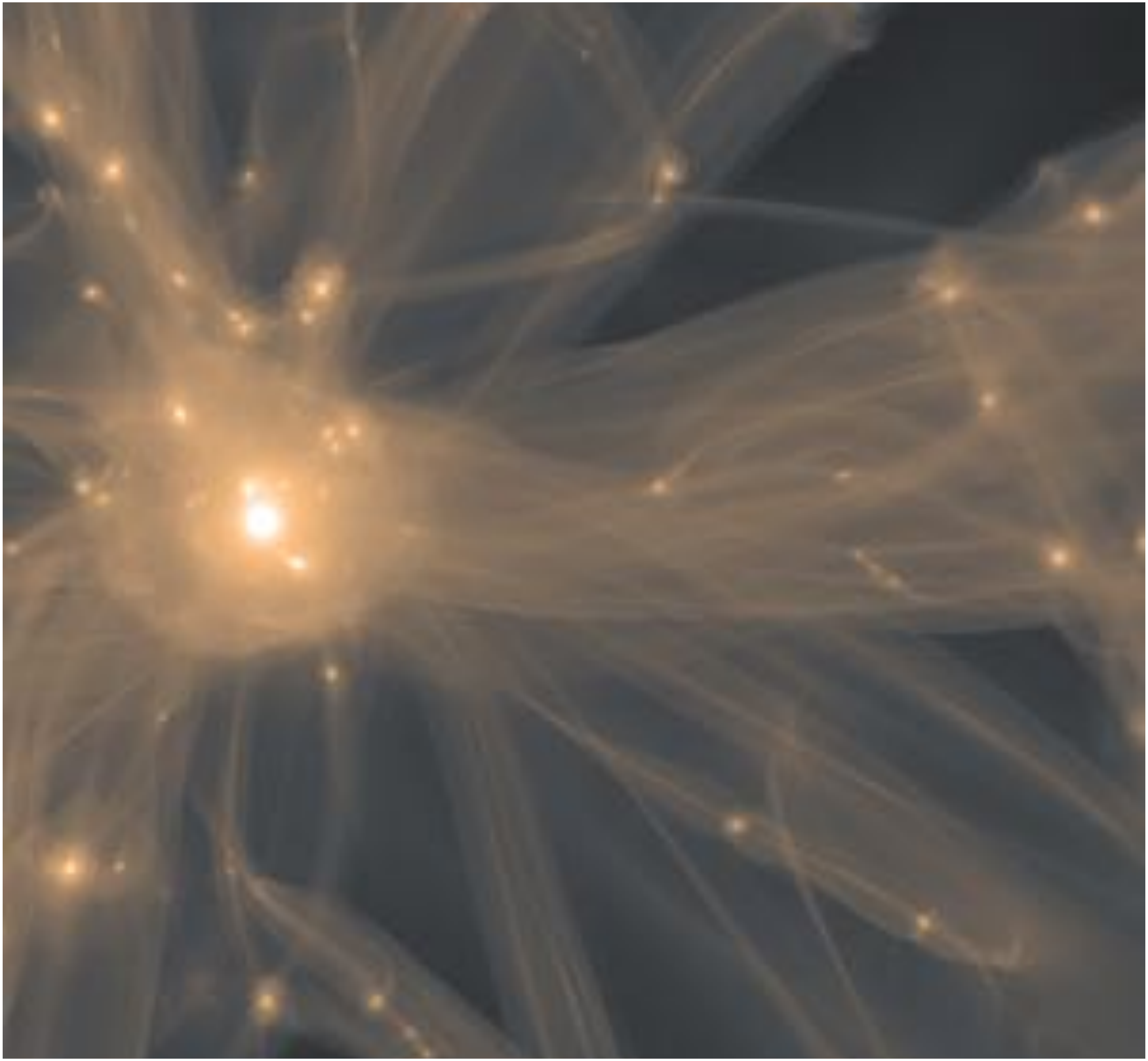


- Number the edges of the cube
- think of lattice
- Looping over



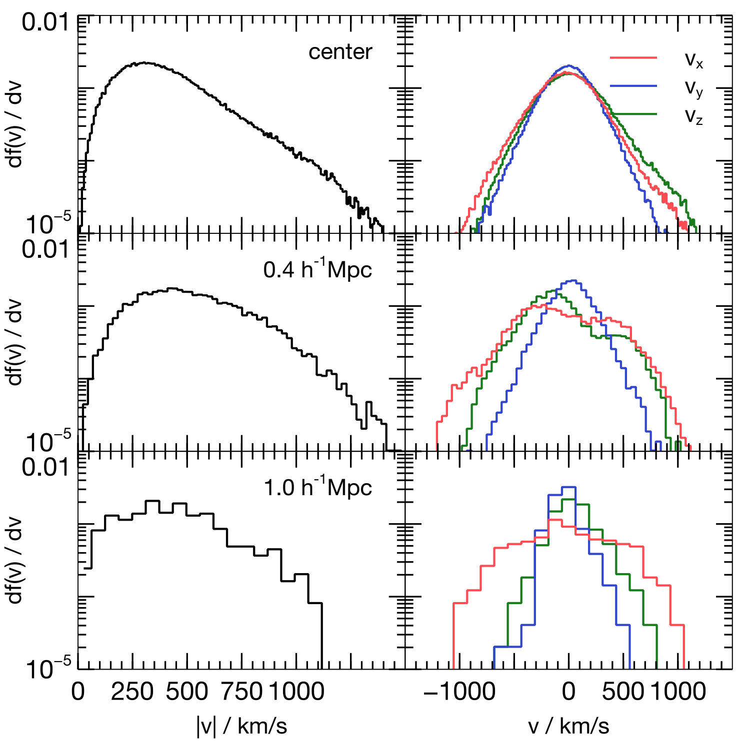
The initial cartesian (LAGRANGIAN lattice generates the 6N tetrahedra.

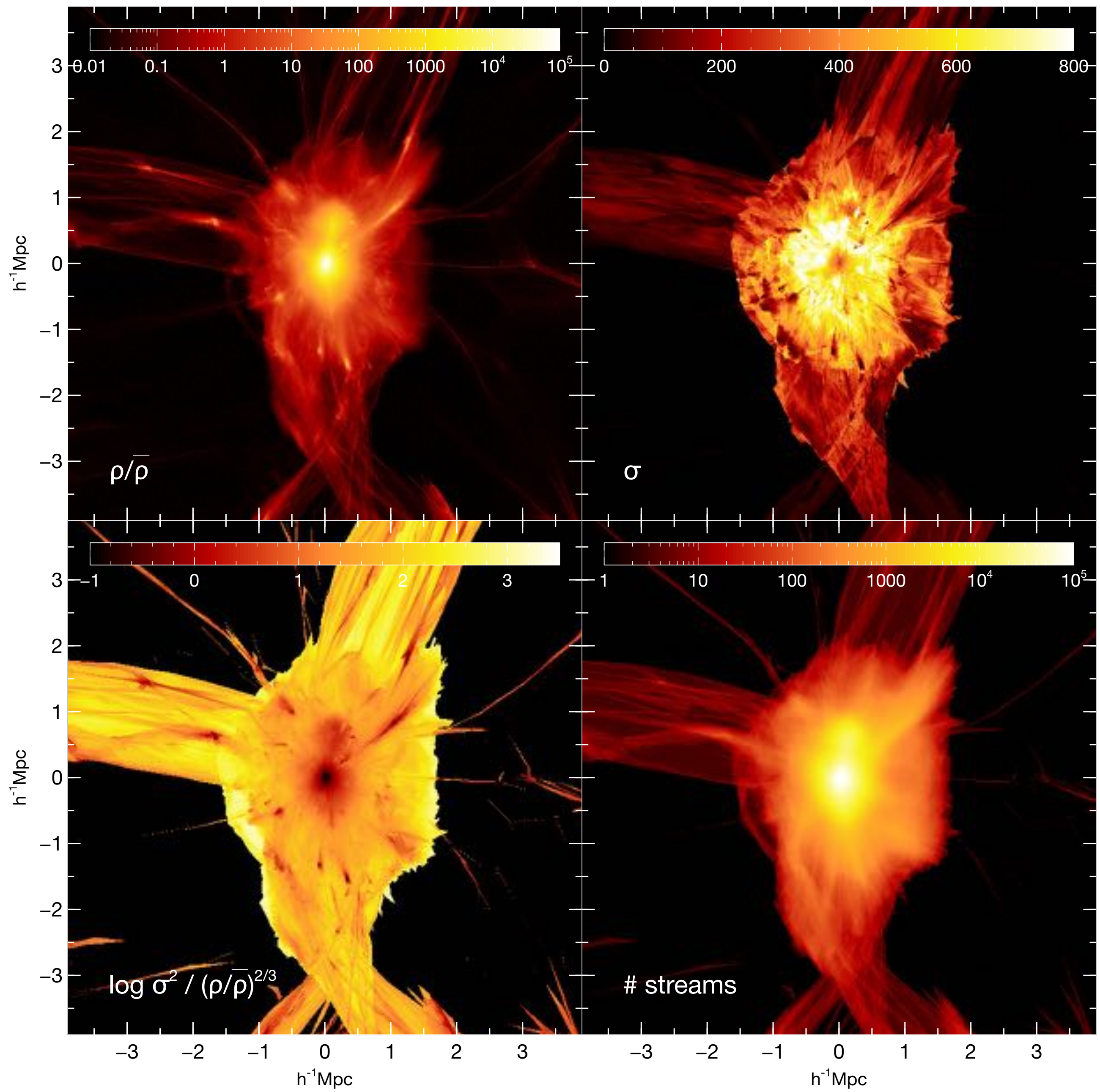


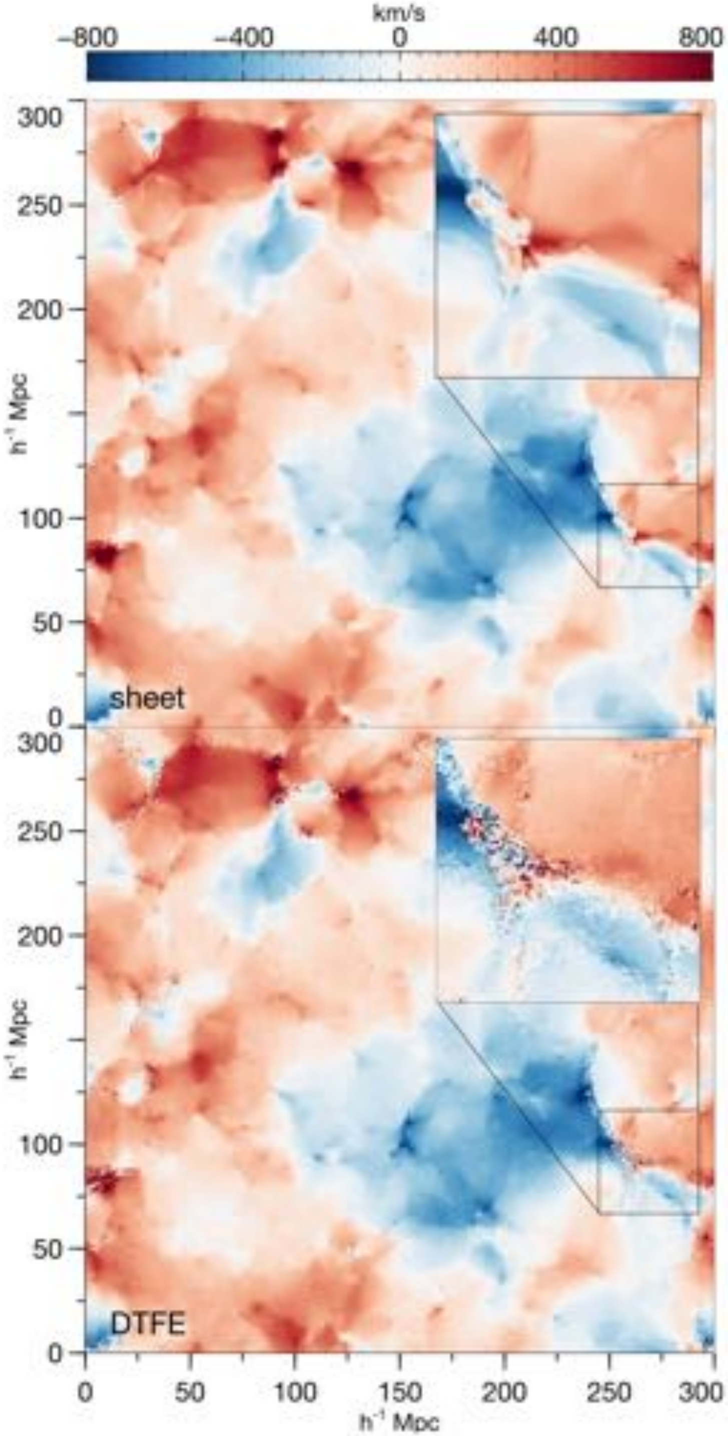


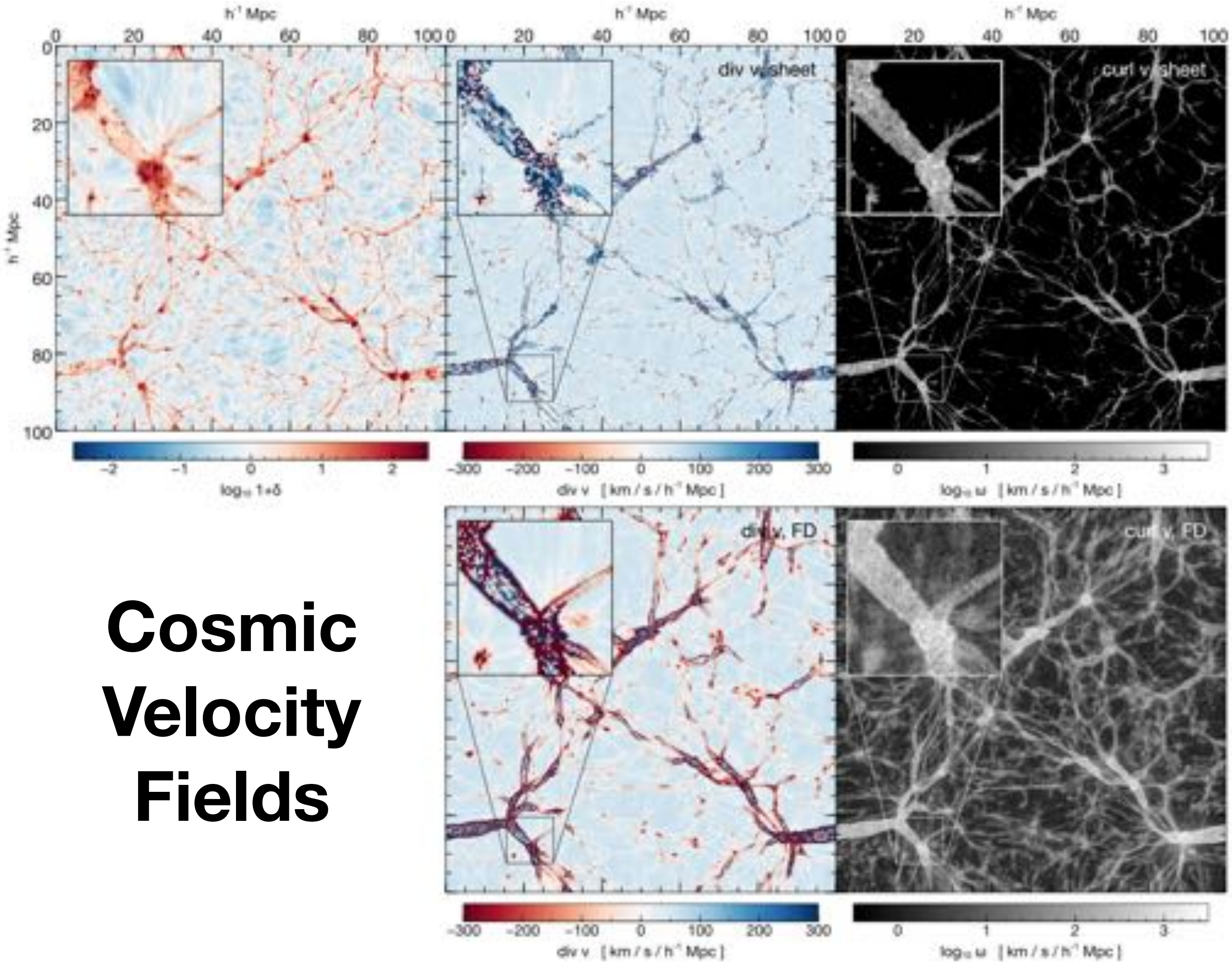
All microphysical phase space information available

can probe fine-grained phase space structure.

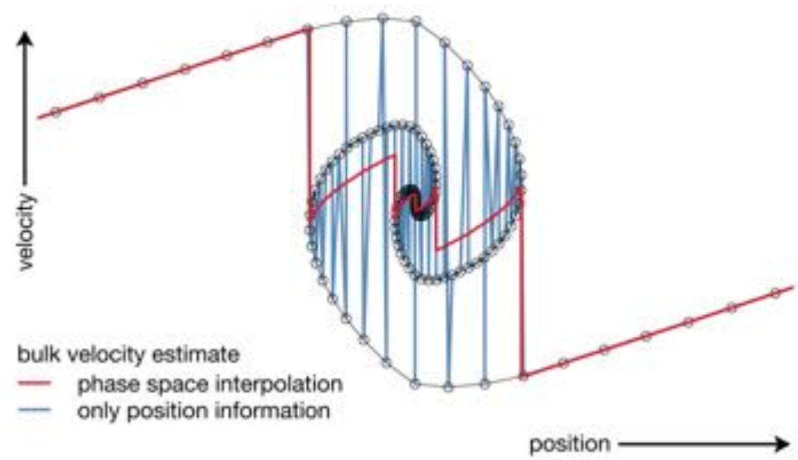


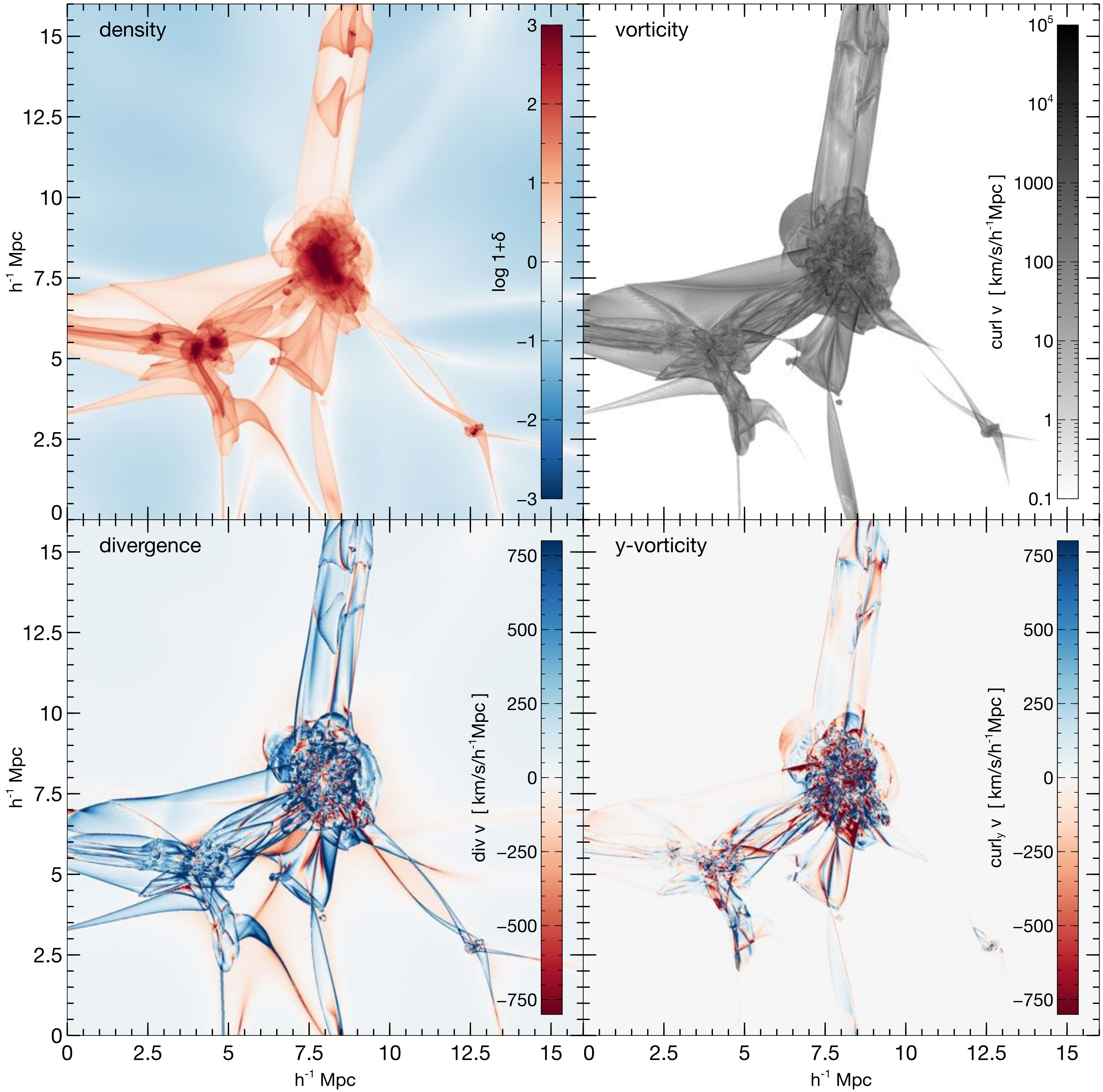






Cosmic Velocity Fields





density
vorticity
divergence
y-vorticity
log 1+δ
curl v [km/s/h⁻¹Mpc]
div v [km/s/h⁻¹Mpc]
curl_y v [km/s/h⁻¹Mpc]
h⁻¹ Mpc

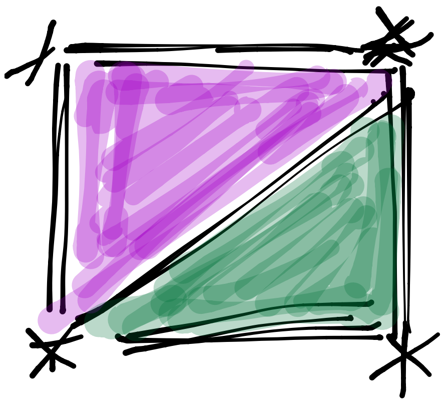
TET-PM

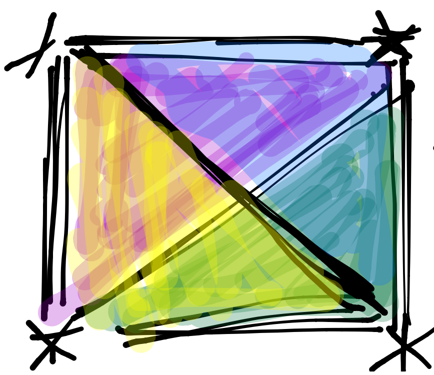
- NEW WAY TO DO N-BODY SIMULATIONS
- Massless TRACERS moving along characteristics.
- These span tetrahedra.
- FIRST IMPLEMENTATION: Monopole approximation
  - → MASS OF TET DEPOSITED AS POINT PARTICLE @ CENTROID LOCATION
- OTHERWISE IDENTICAL TO A PARTICLE MESH CODE

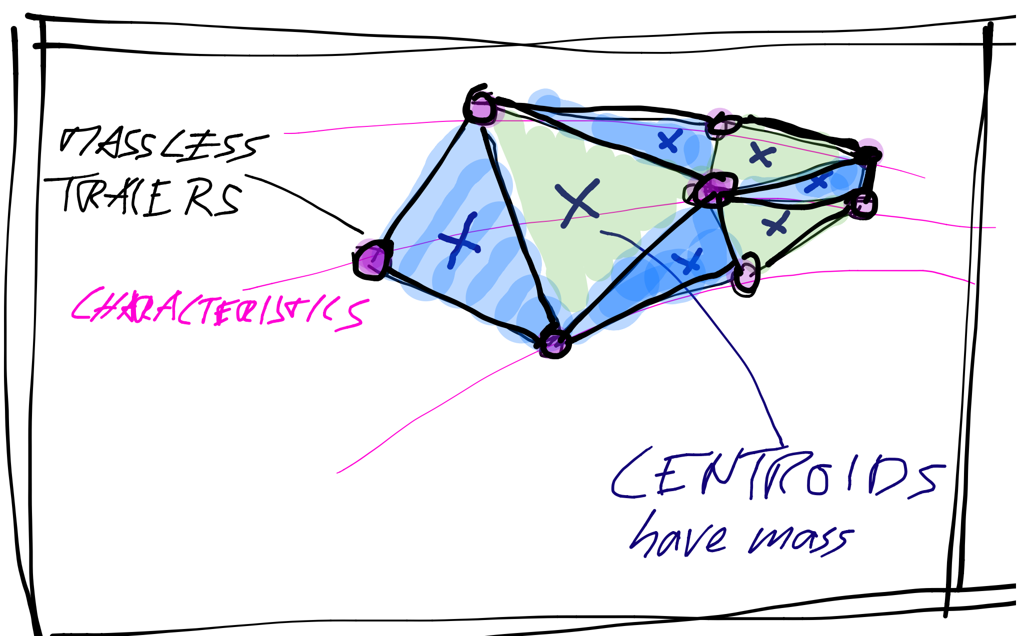
NEW

HAHN, KAEHLER
ABEL 2012 in prep.

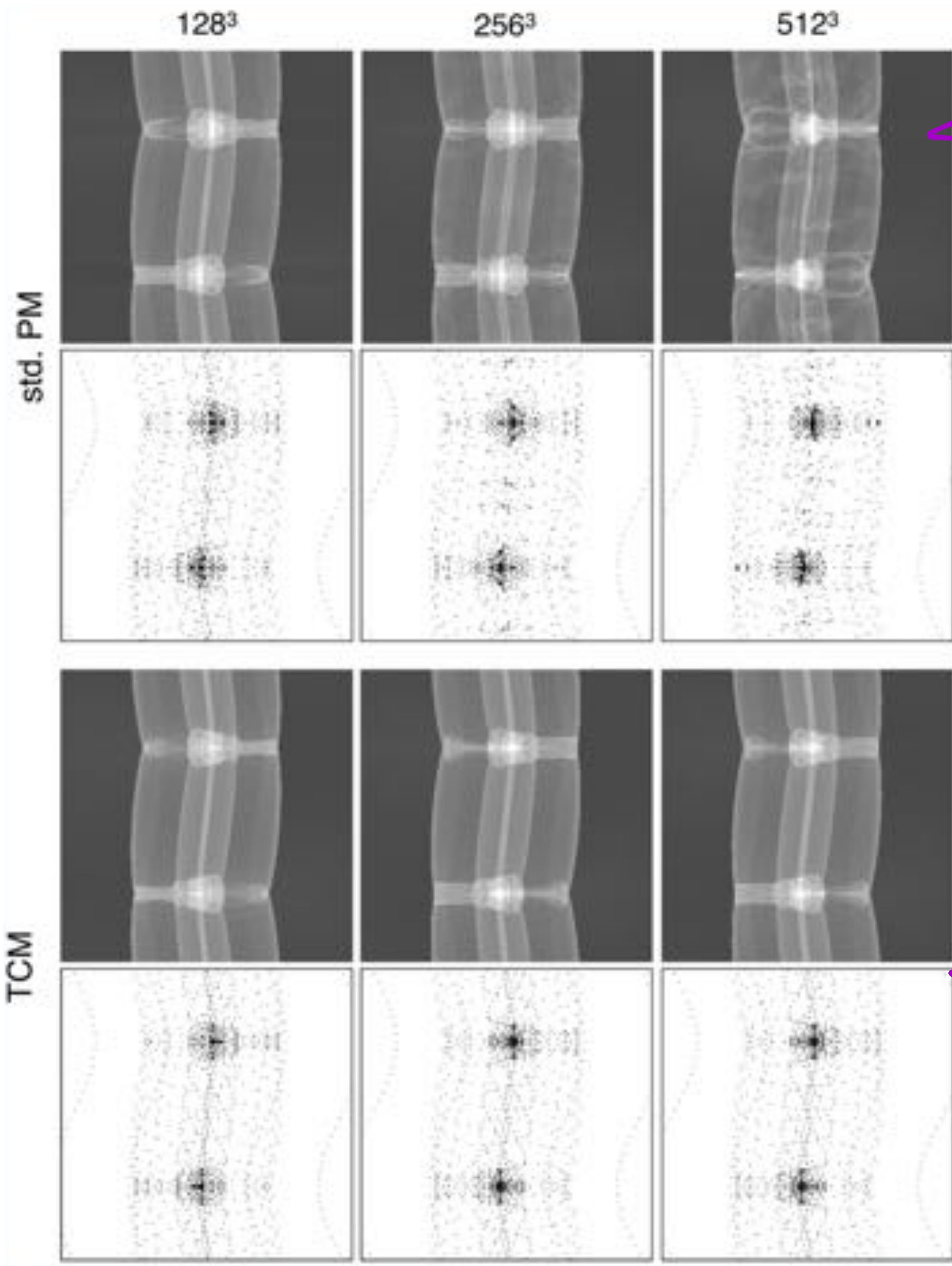
2 TYPES OF PARTICLES







Tesselation & particle arrangement in TET-PM



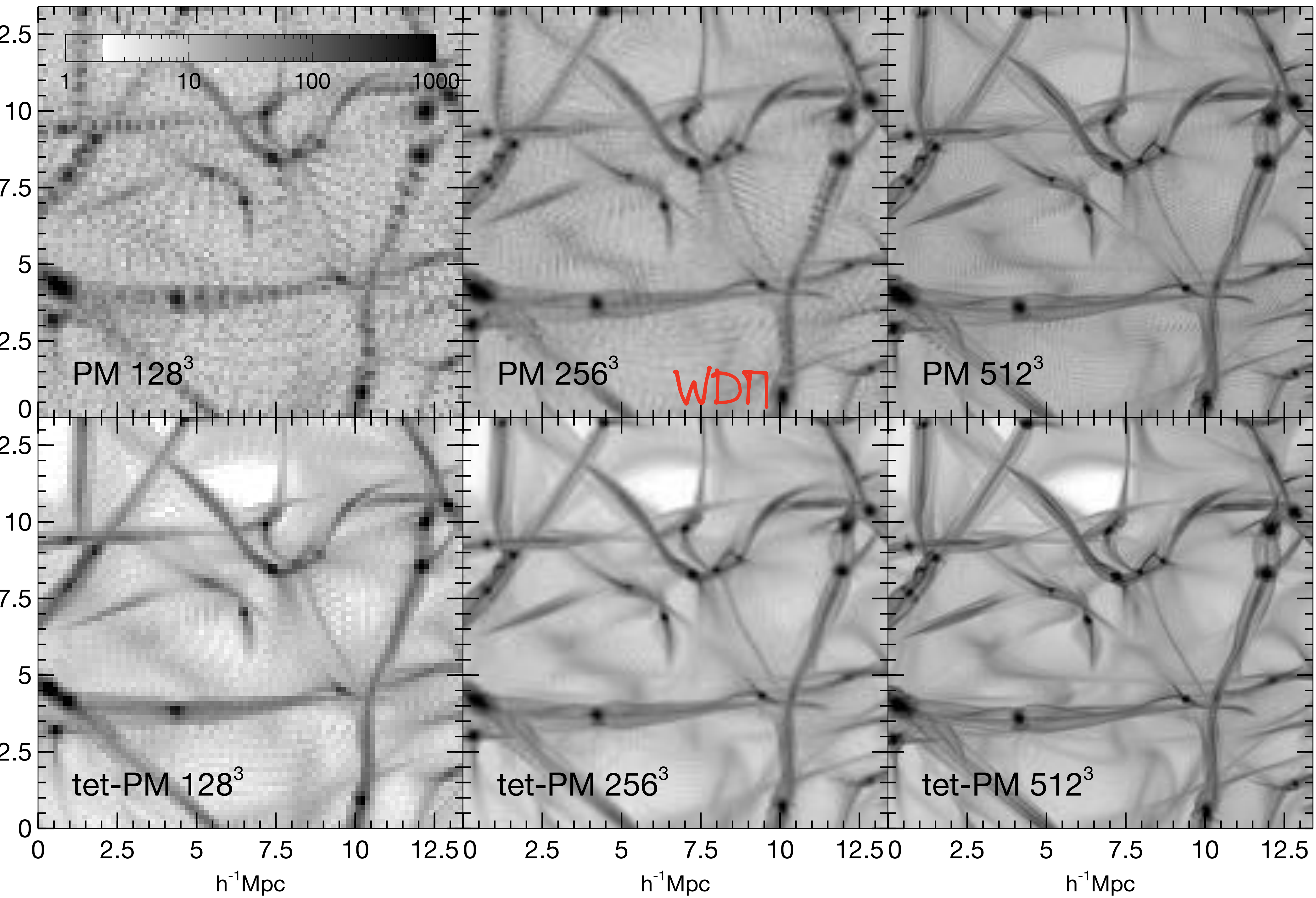
two body scattering undesirable ...

Note how the new visualization technique helps in spotting errors in the N-body integration

Constant mass resolution
Vary force resolution

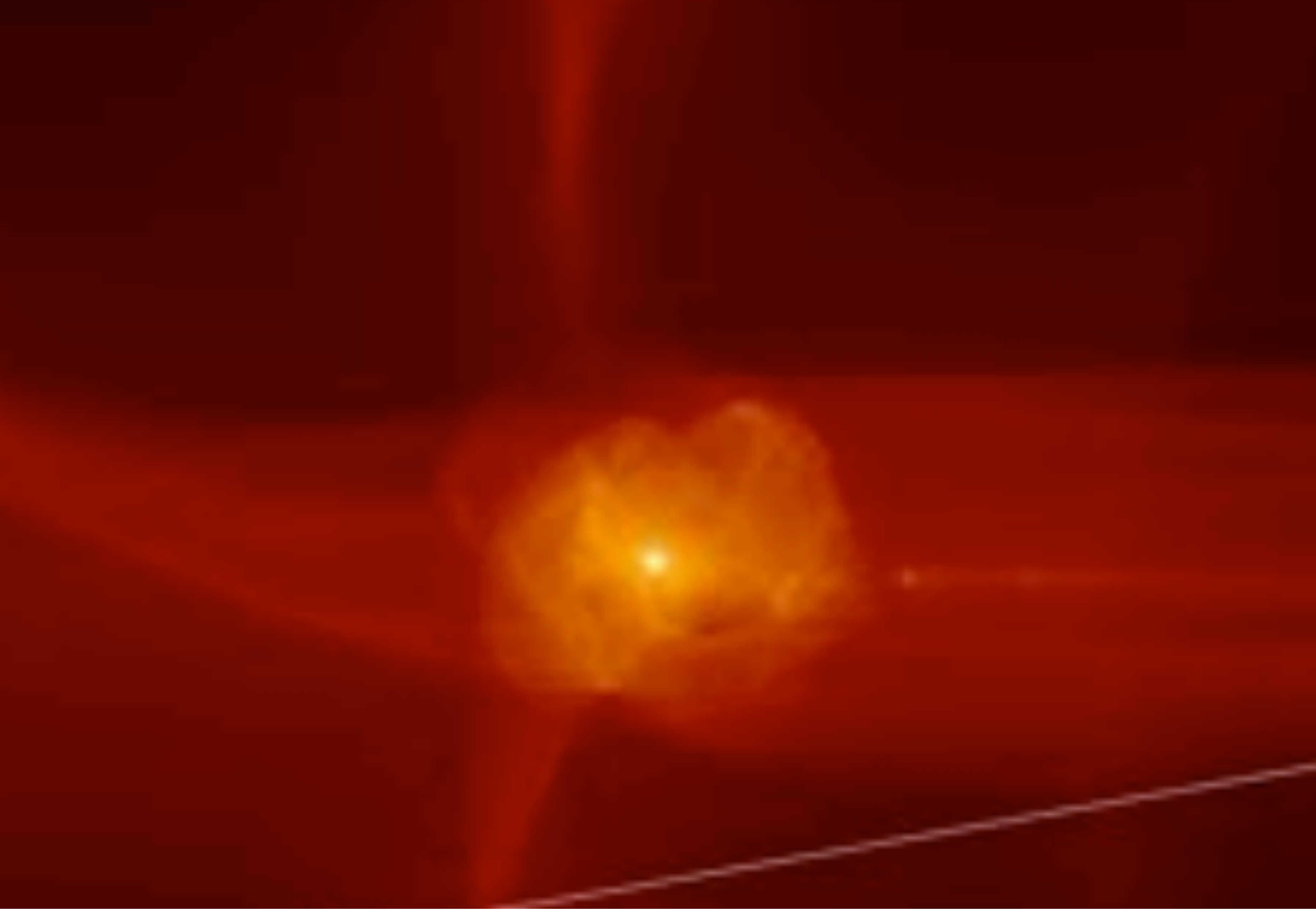
excellent convergence behavior of our new method.

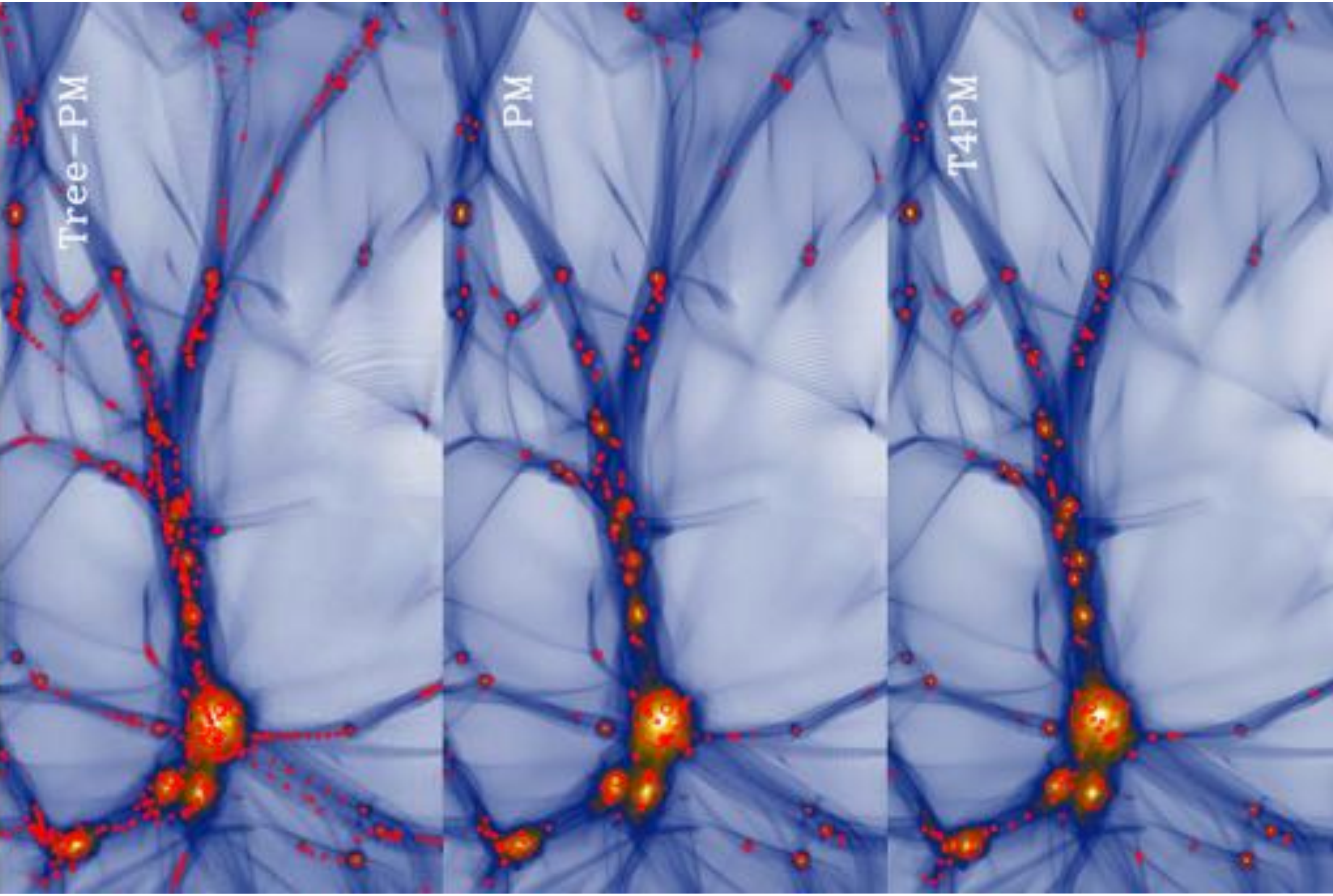
Solution to 30 yr old artificial fragmentation problem in warm and hot DM models



Formation of a Single Halo

Hahn, Kaehler & Abel



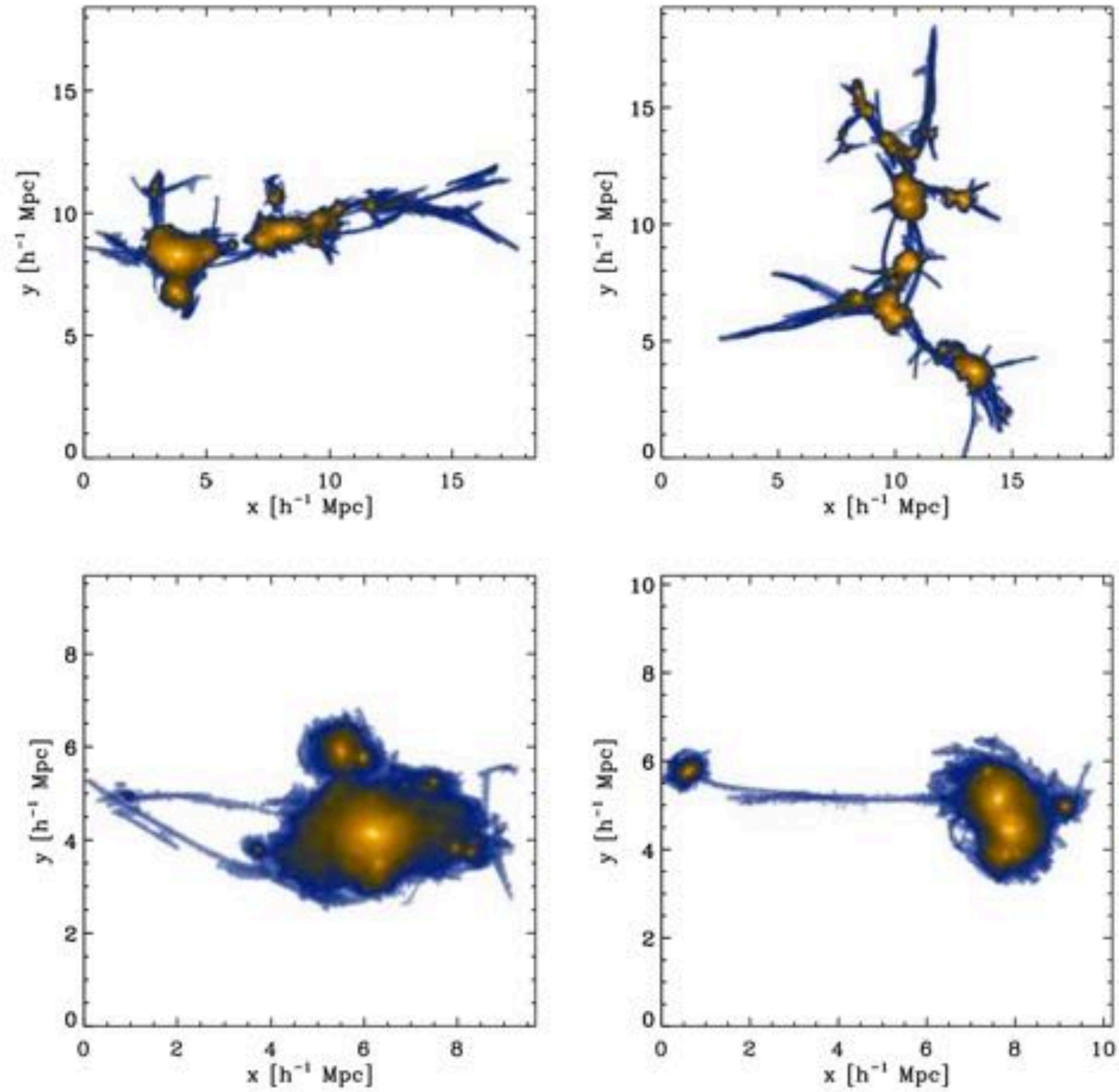


arXiv:1304.2406

The Warm DM halo mass function below the cut-off scale

Raul E. Angulo (1), Oliver Hahn (1 and 2), Tom Abel (1) ((1) KIPAC, Stanford University, (2) ETH, Zurich)

(Submitted on 8 Apr 2013)



Angulo, Hahn & Abel 2013

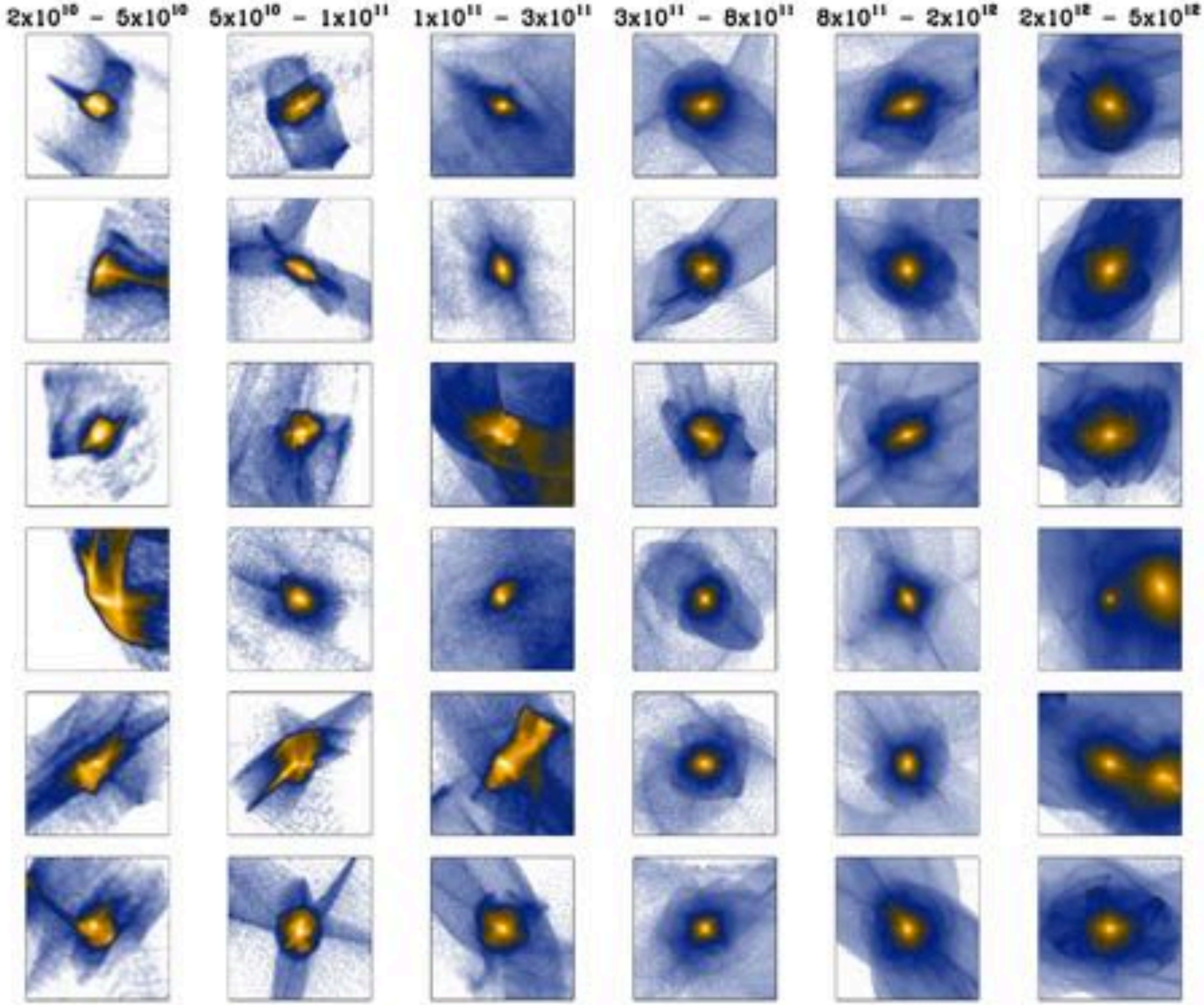
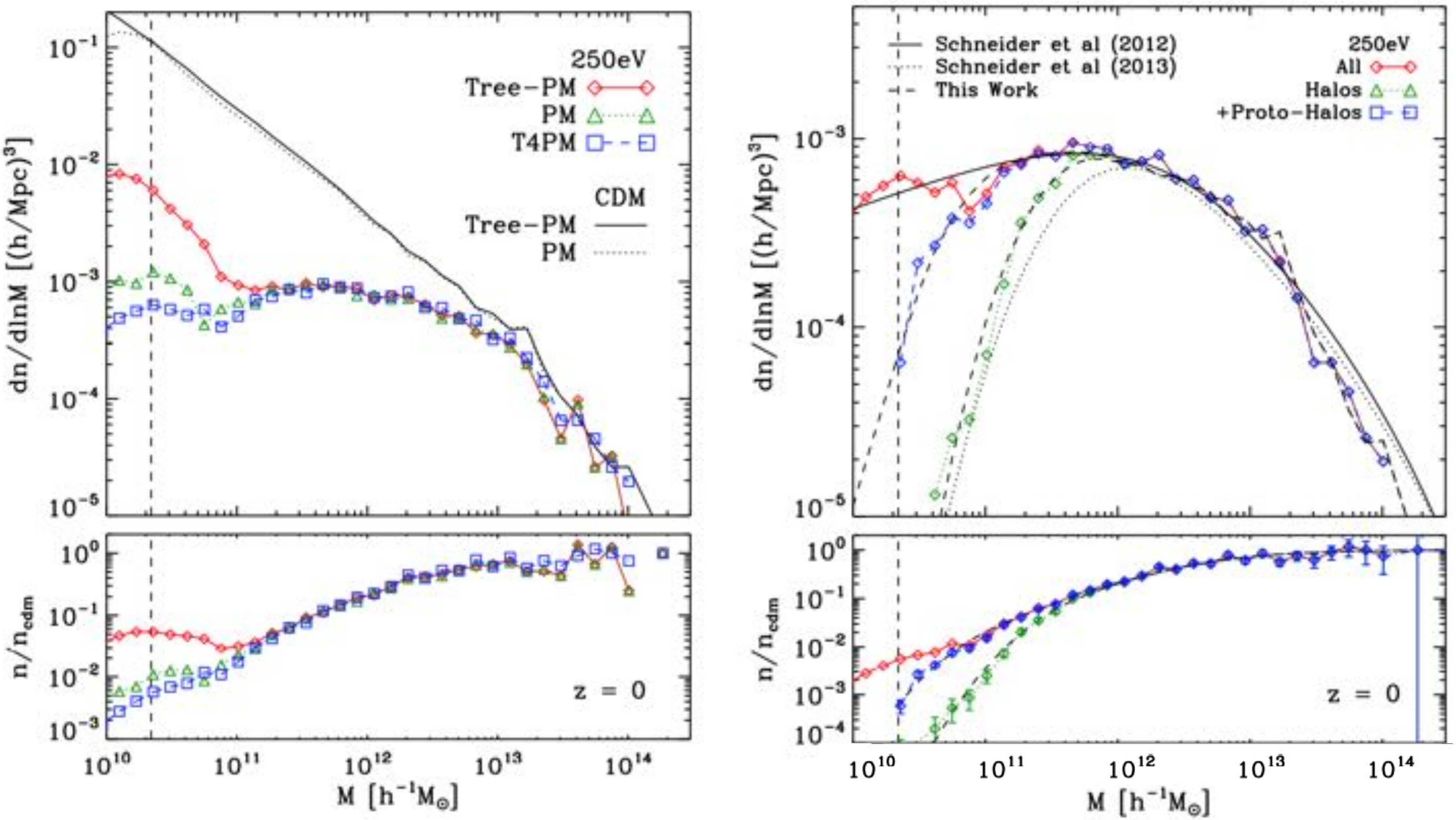
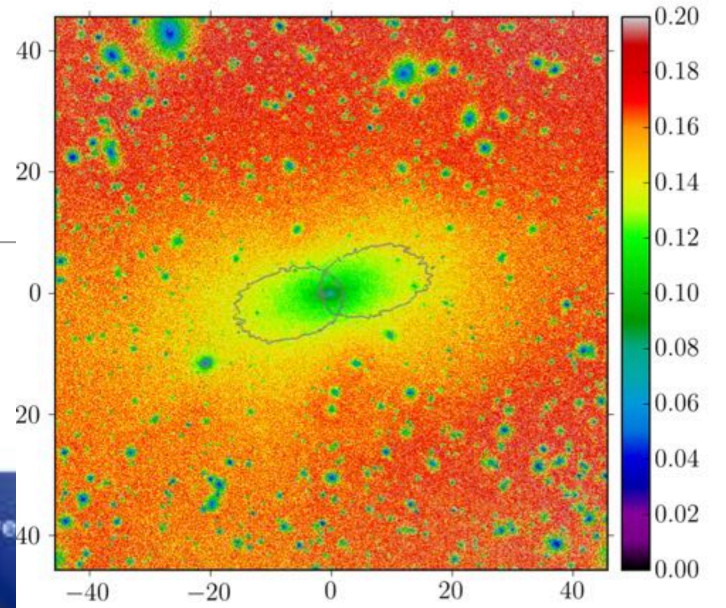


Figure 4. Images of randomly-chosen objects in six disjoint mass bins. These mass bins are equally spaced in log M over the mass range $2 \times 10^{10} < M_{200}/(h^{-1}M_{\odot}) < 5 \times 10^{12}$. Each image displays the logarithmic projected density field computed using the T4PM method. The extent of each image is equal to $2 \times R_{200}$, i.e. twice the virial radius of the respective halo.

Warm Dark Matter Mass functions

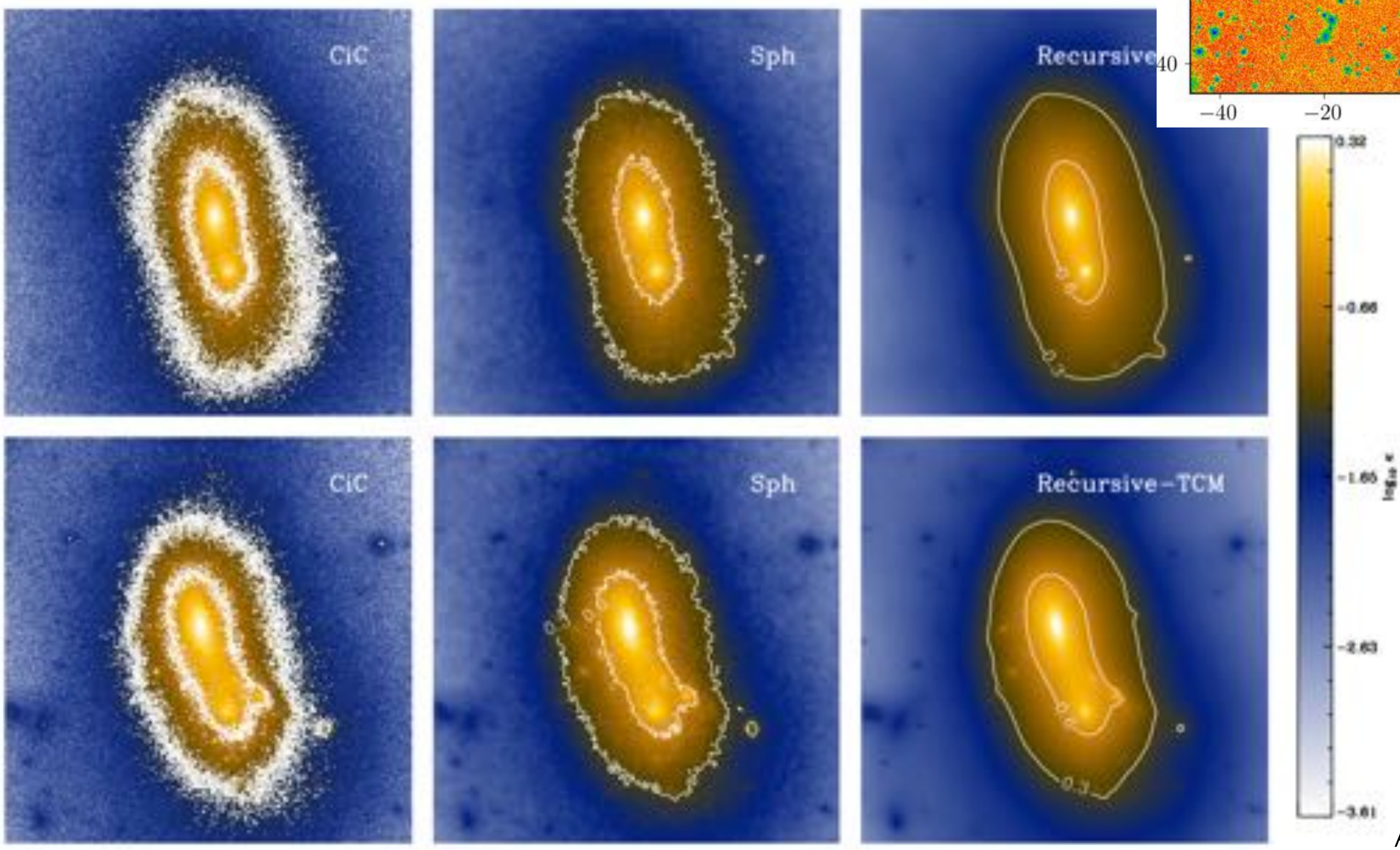


Stefan Rau, Simona Vegetti, Simon White



Particle Noise in Gravitational Lensing Predictions

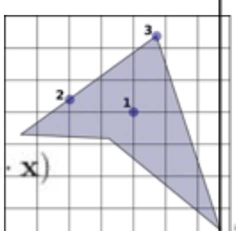
- Noiseless but biased estimator



Angulo, Chen, Hilbert & Abel, MNRAS (2013)

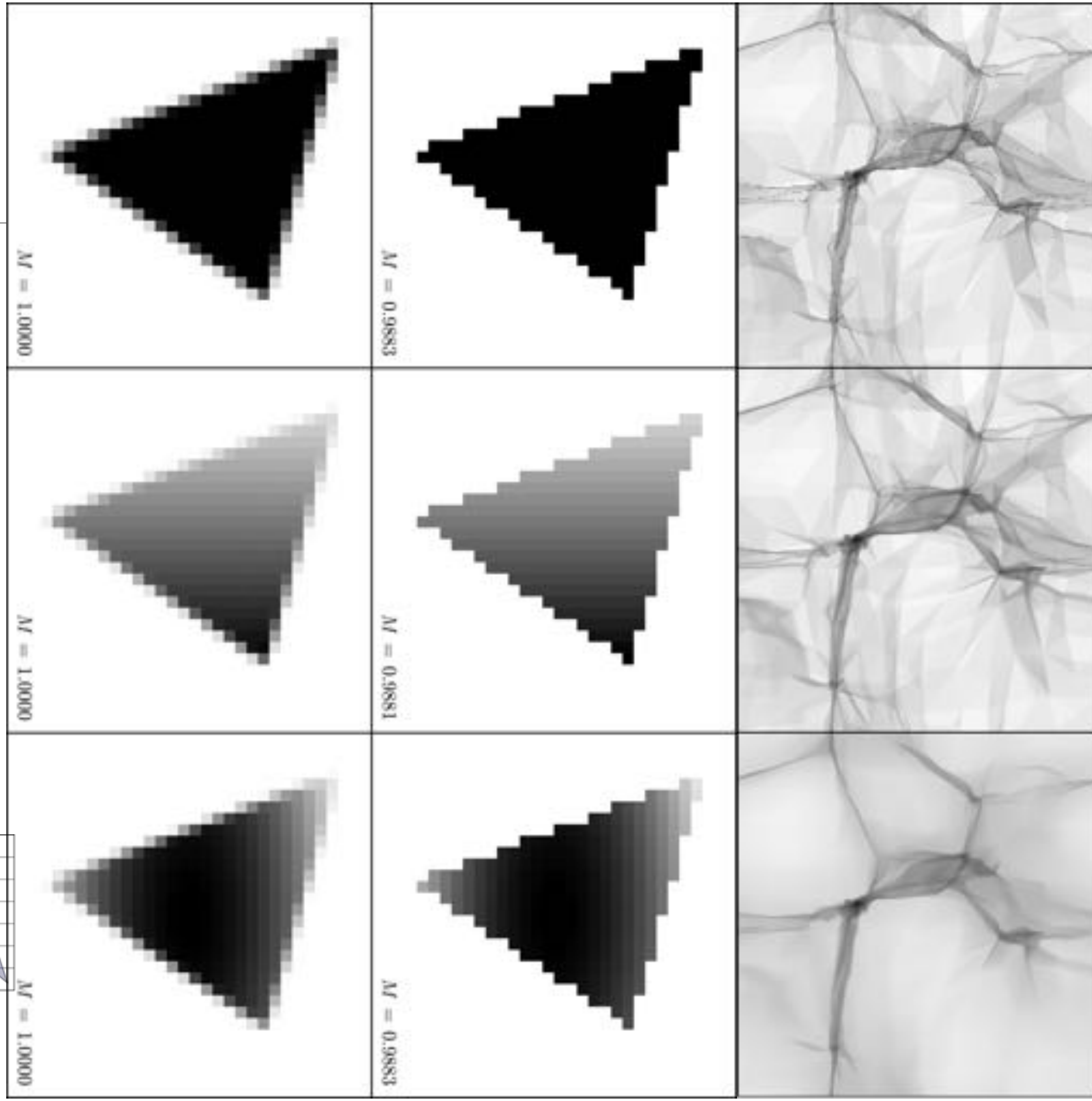
Exact Deposit

- any polyhedra intersections without constructing the overlap
- linear and quadratic function defined over polyhedra
- fundamental building block for many novel algorithms
- 30 times faster than a recursive algorithm
- Computational Geometry - Patent?
- Powell and Abel (2014) submitted to JComP

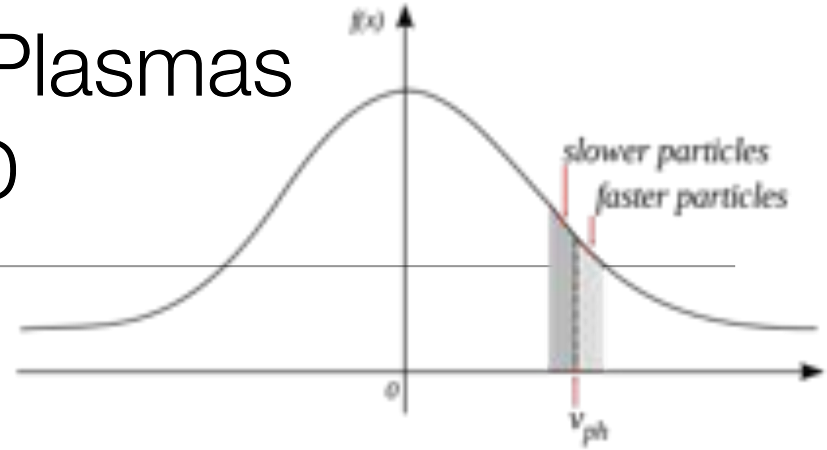


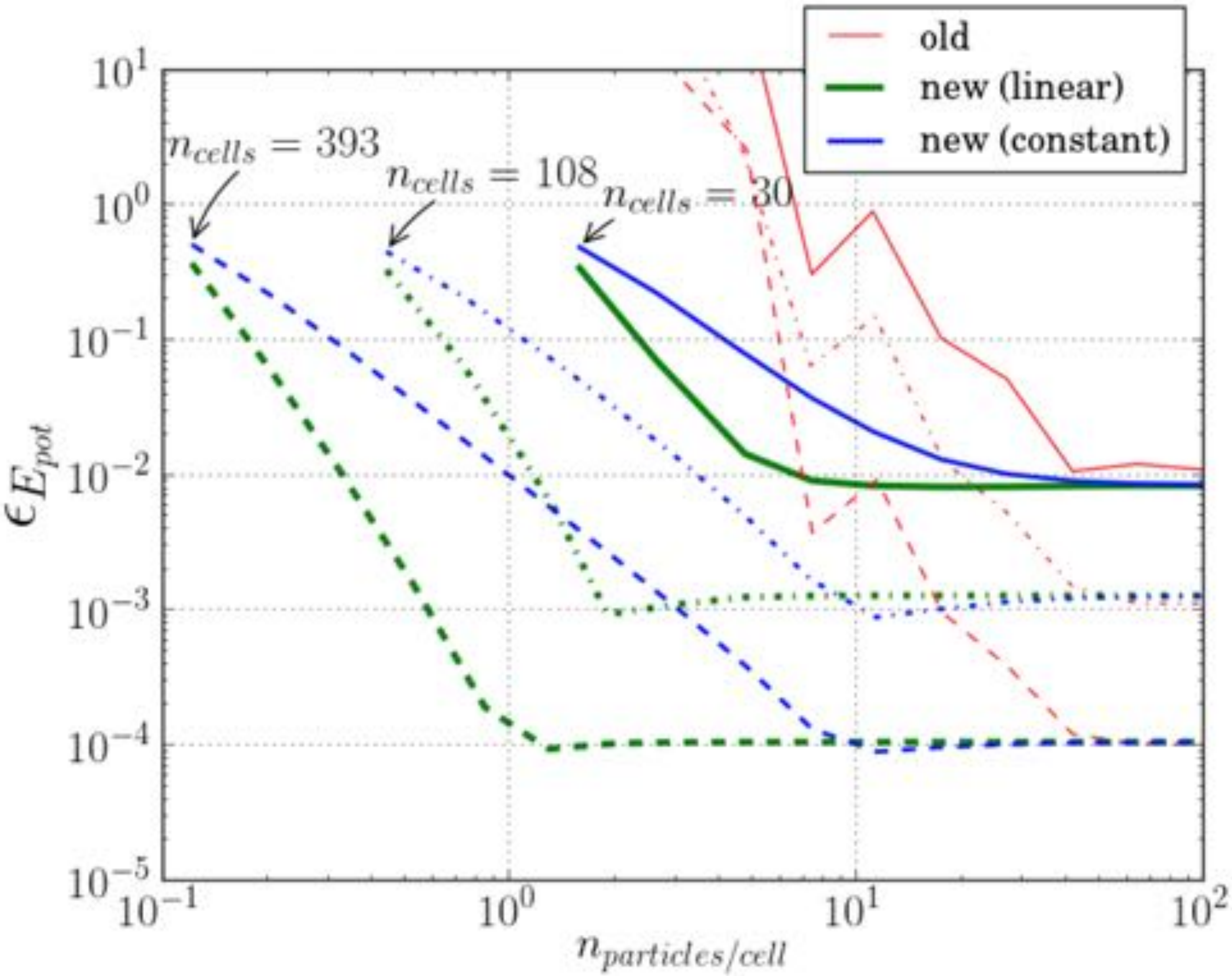
$$A = \frac{1}{2}\sum_{e,p}(\hat{\mathbf{n}}_e \cdot \mathbf{x})(\hat{\mathbf{n}}_p \cdot \mathbf{x})$$

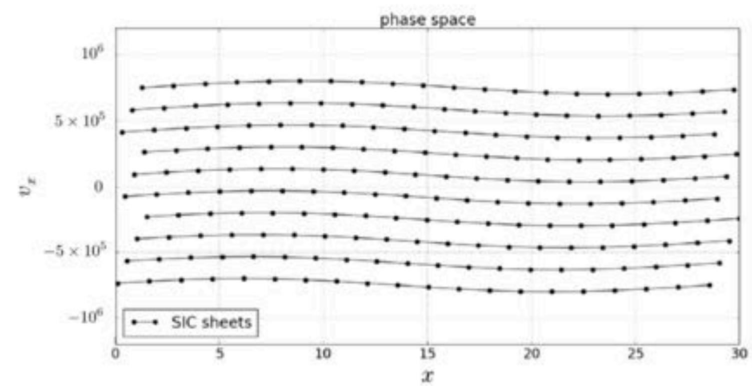
$$V = -\frac{1}{6}\sum_{f,e,p}(\hat{\mathbf{n}}_f \cdot \mathbf{x})(\hat{\mathbf{n}}_e \cdot \mathbf{x})(\hat{\mathbf{n}}_p \cdot \mathbf{x})$$



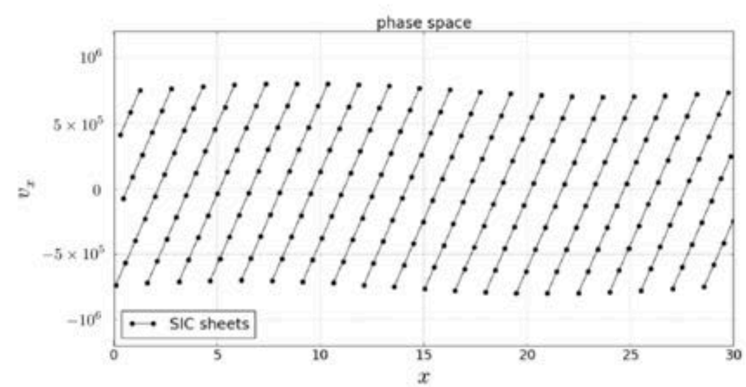
Also applicable to Collisionless Plasmas
Example: Landau Damping in1D







(a) horizontal streams

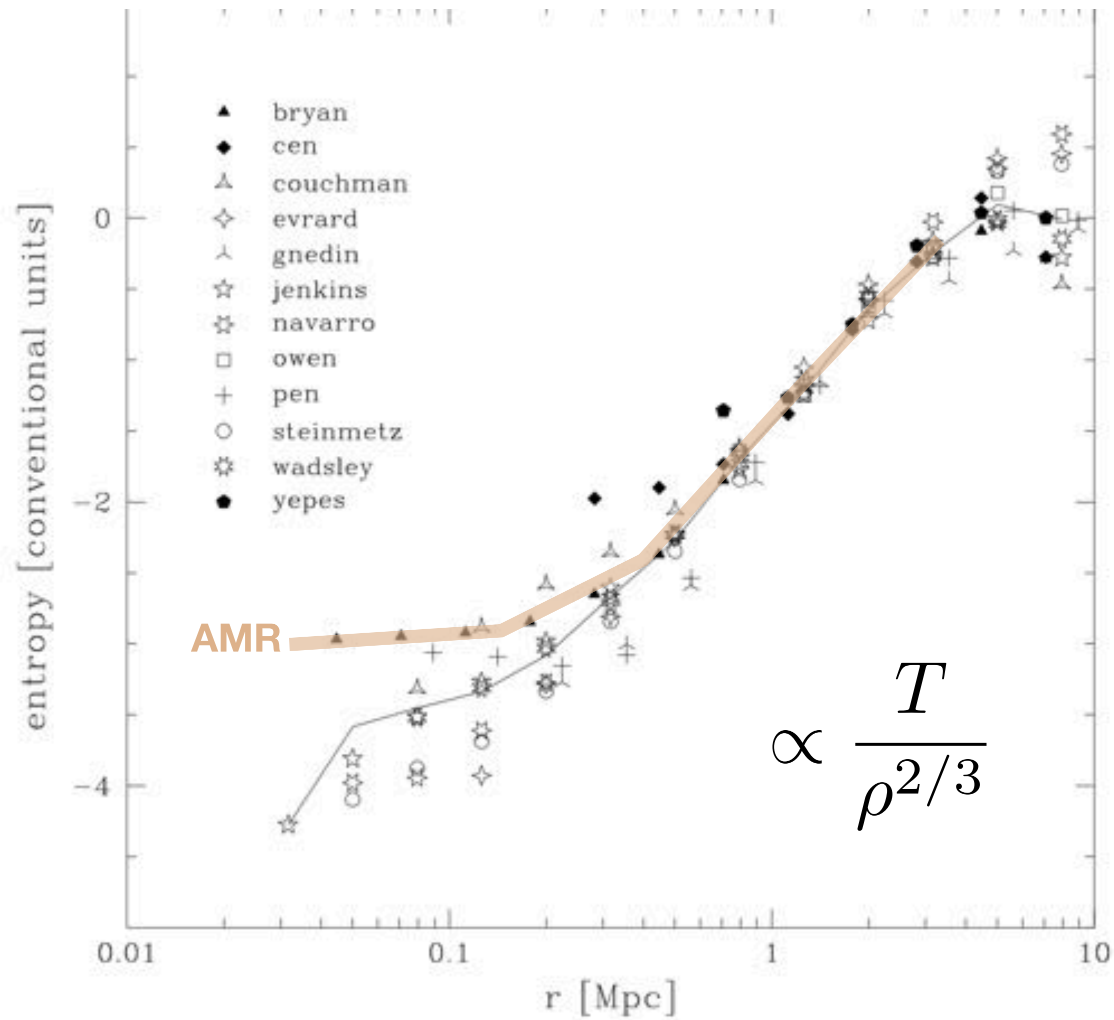


(b) vertical streams

Kates-Harbeck, Totorica, Zrake & Abel 2014, in prep.

Numerical Hydrodynamics & Mixing

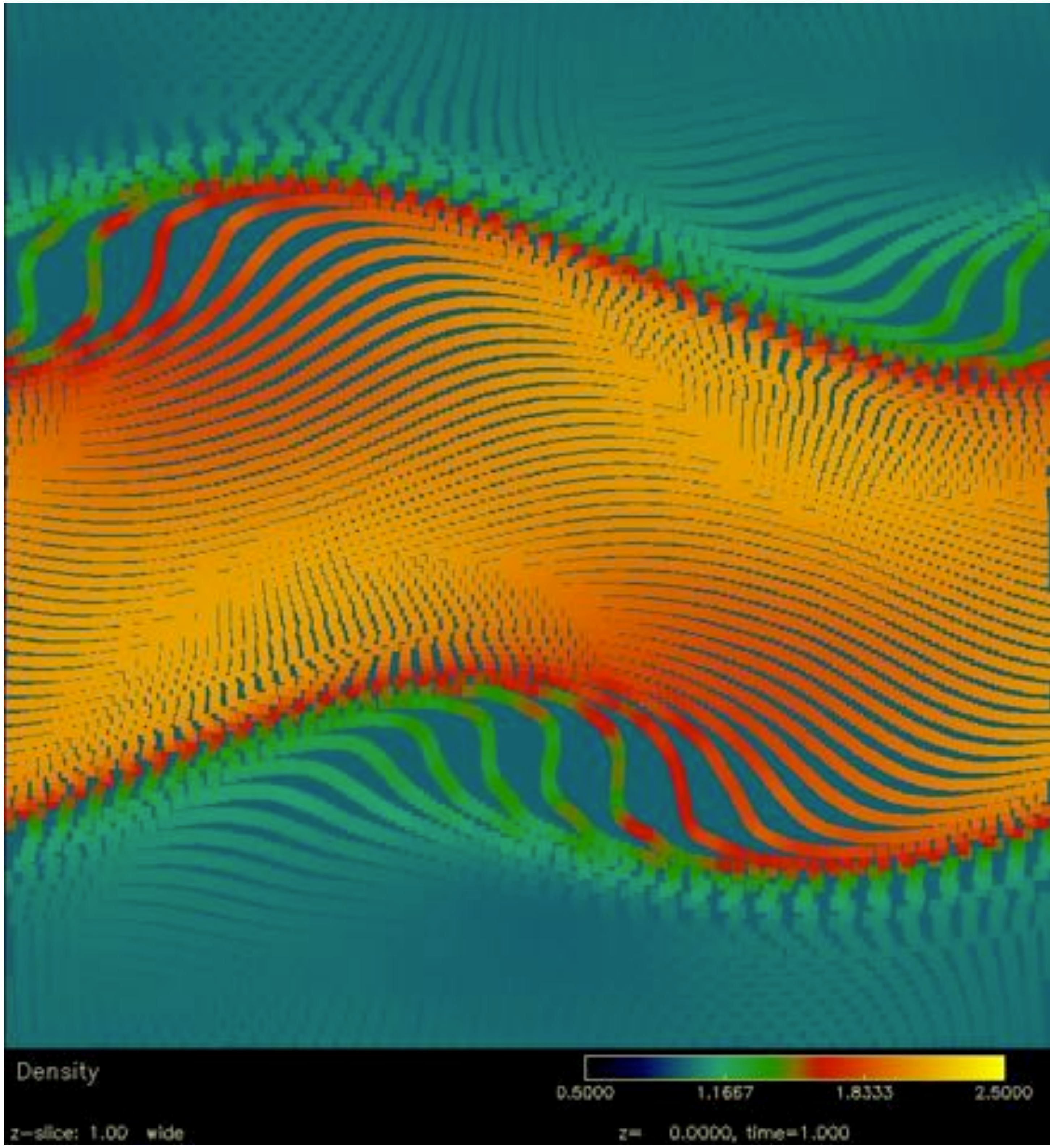
- In comparison of 12 cosmological hydrodynamics codes on simple non-radiative simulations of galaxy clusters, the only grid based result looked significantly divergent in the entropy profile
- Over many years detailed follow up studies concluded that the inability of traditional Smoothed Particle Hydrodynamics techniques was the culprit
- Discretized mass elements which cannot shear and mix with surrounding fluid stay trapped in cluster centers



$$\propto \frac{T}{\rho^{2/3}}$$

Frenk 1999, Santa Barbara
Galaxy Cluster Comparison Project

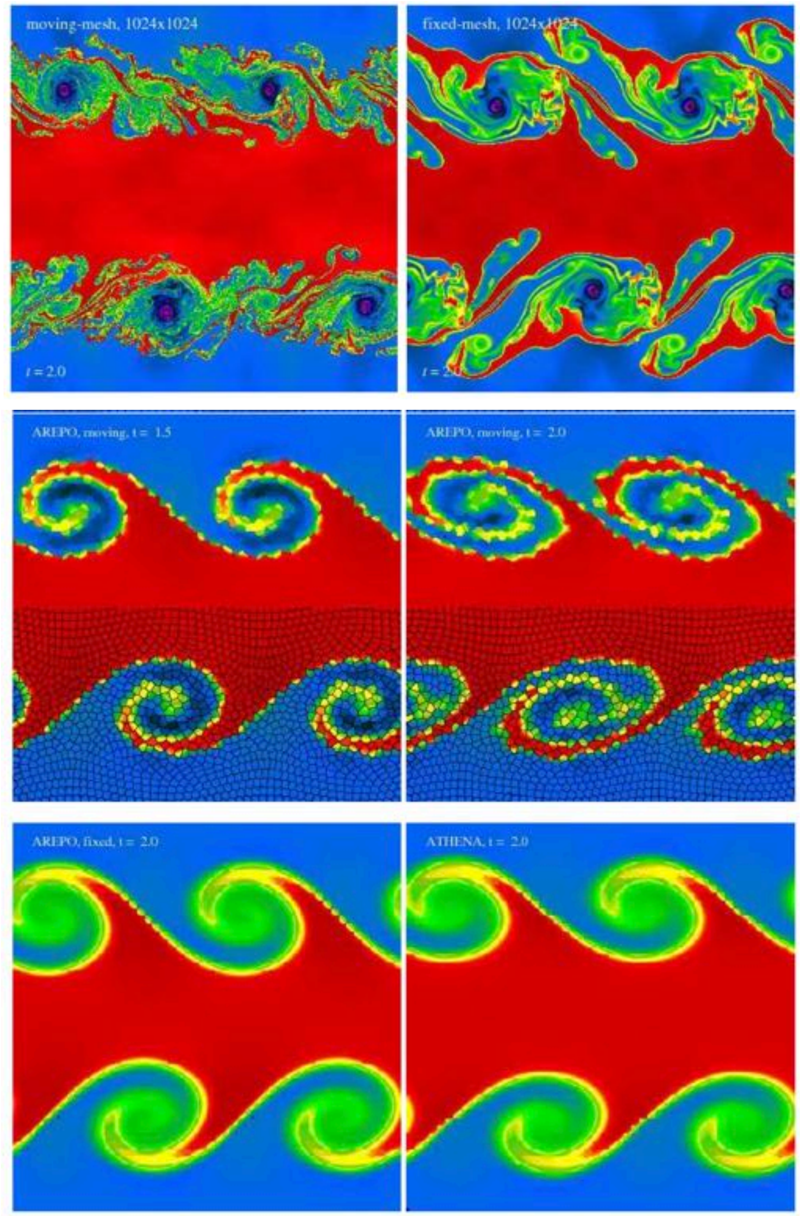
No fully Lagrangian method is possible for compressible flow.



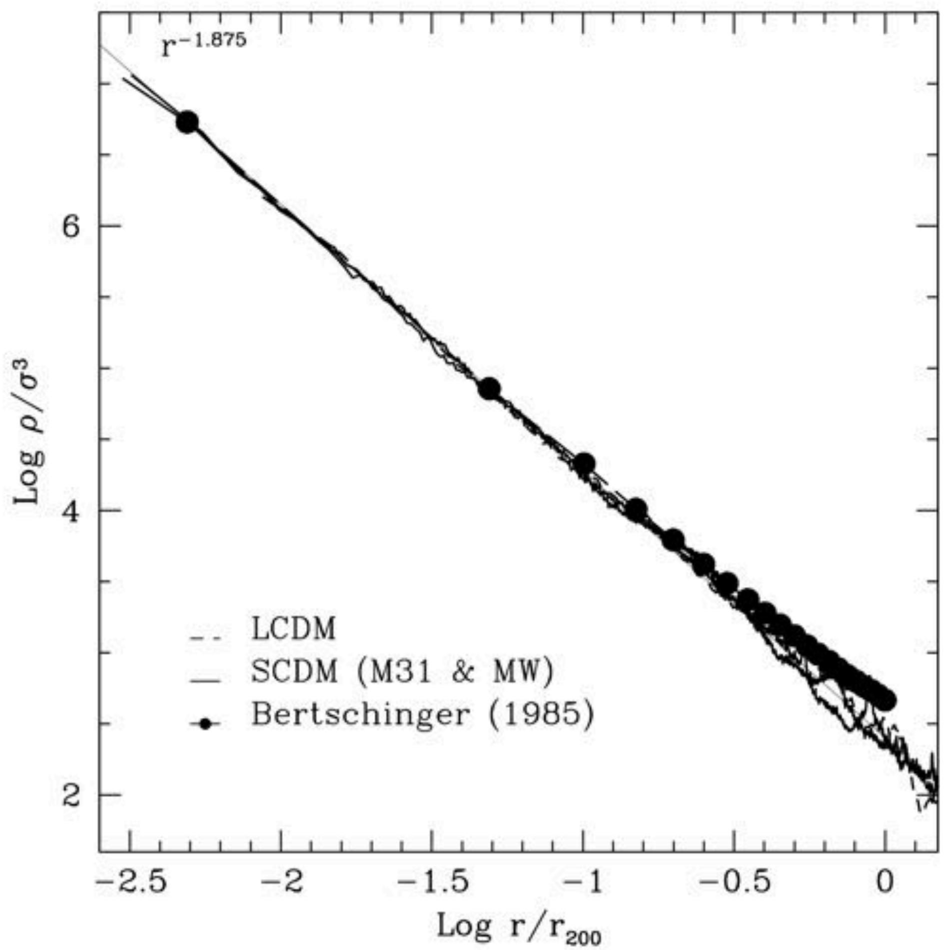
Abel 2011, rpSPH

Numerical Hydrodynamics & Mixing

- Springel 2010, argues that moving mesh is best to avoid diffusion.
- However, this again suppresses mixing and leads to numerical (unphysical) instabilities at shearing contact discontinuities.
- Galaxy Scale Simulations are unlikely to be a testbed for doing subtle code comparisons given that forces on the gas are dominated largely by gravity and Reynolds numbers are much too small.
- How different phases mix is crucial in understanding ISM physics as it affects cooling, energy transport, etc.



Pseudo-Phase Space Density perfect power law in Dark Matter Simulations



THE PHASE-SPACE DENSITY PROFILES OF COLD DARK MATTER HALOS

James E. Taylor and Julio F. Navarro[1]
Department of Physics and Astronomy, University of Victoria, Victoria, BC V8P 1A1, Canada

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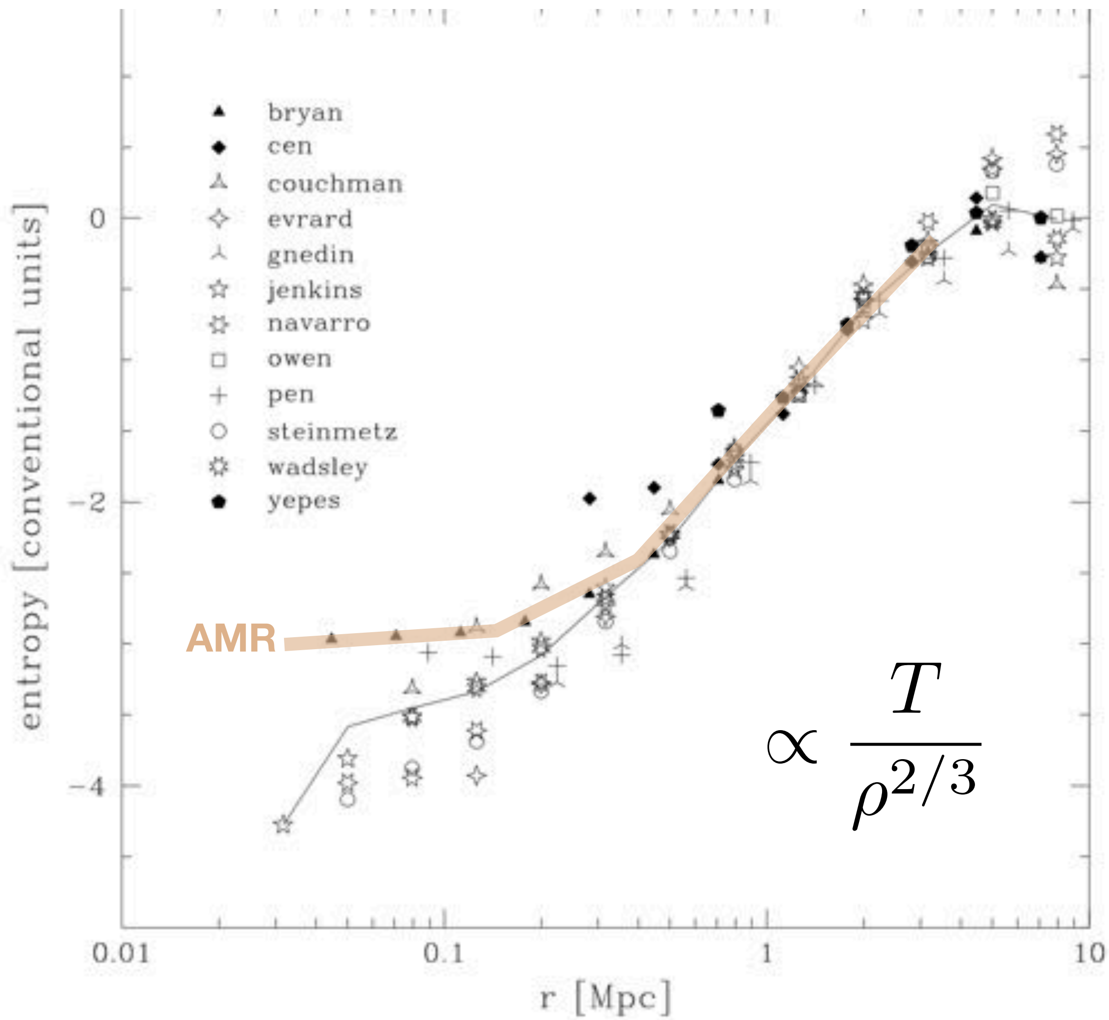
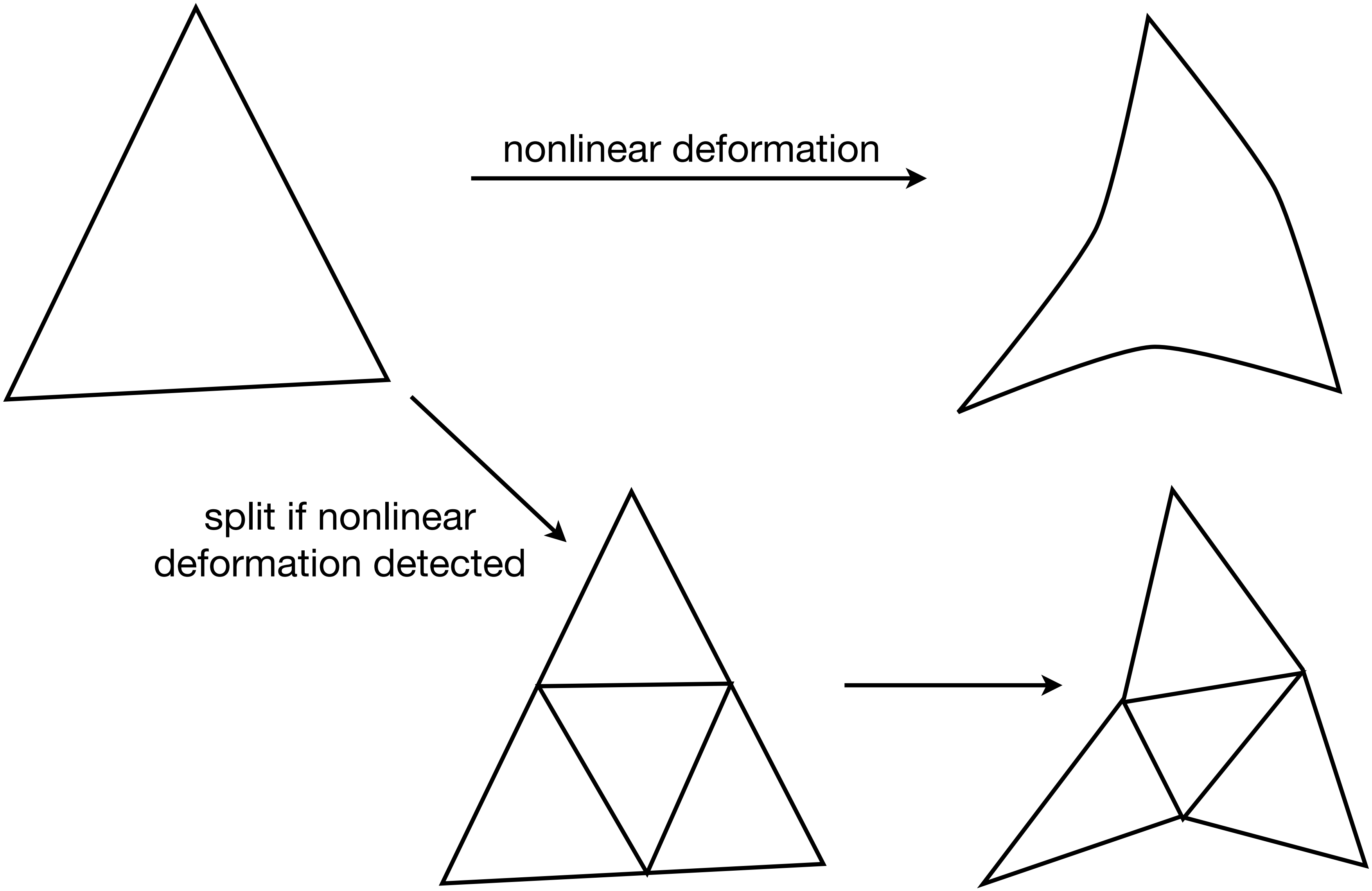


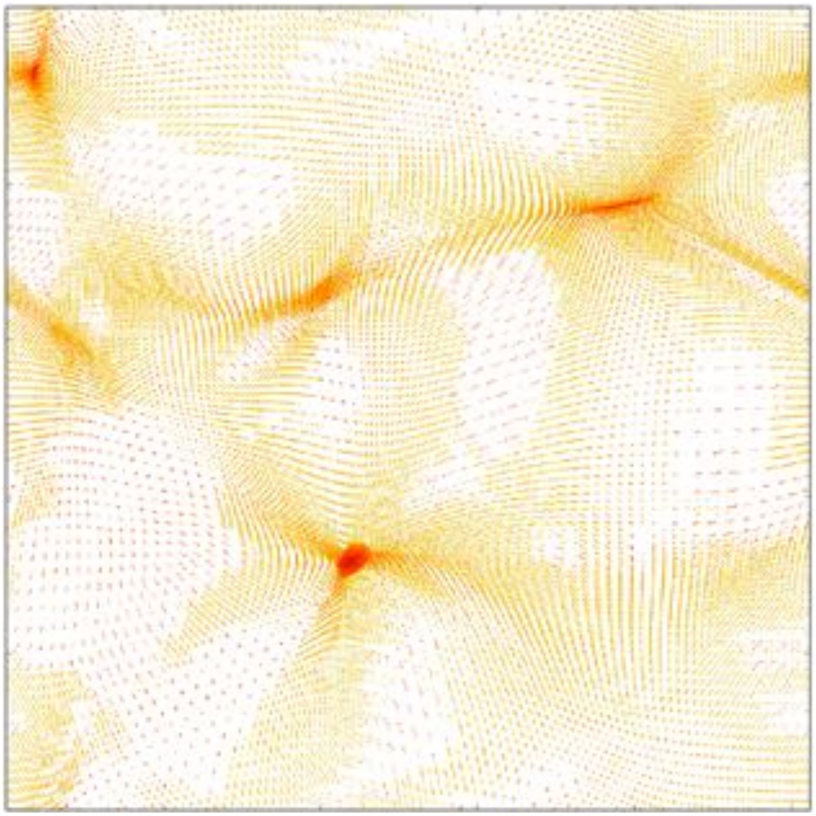
Frenk 1999, Santa Barbara
Galaxy Cluster Comparison Project

Limits of the sheet: need for refinement



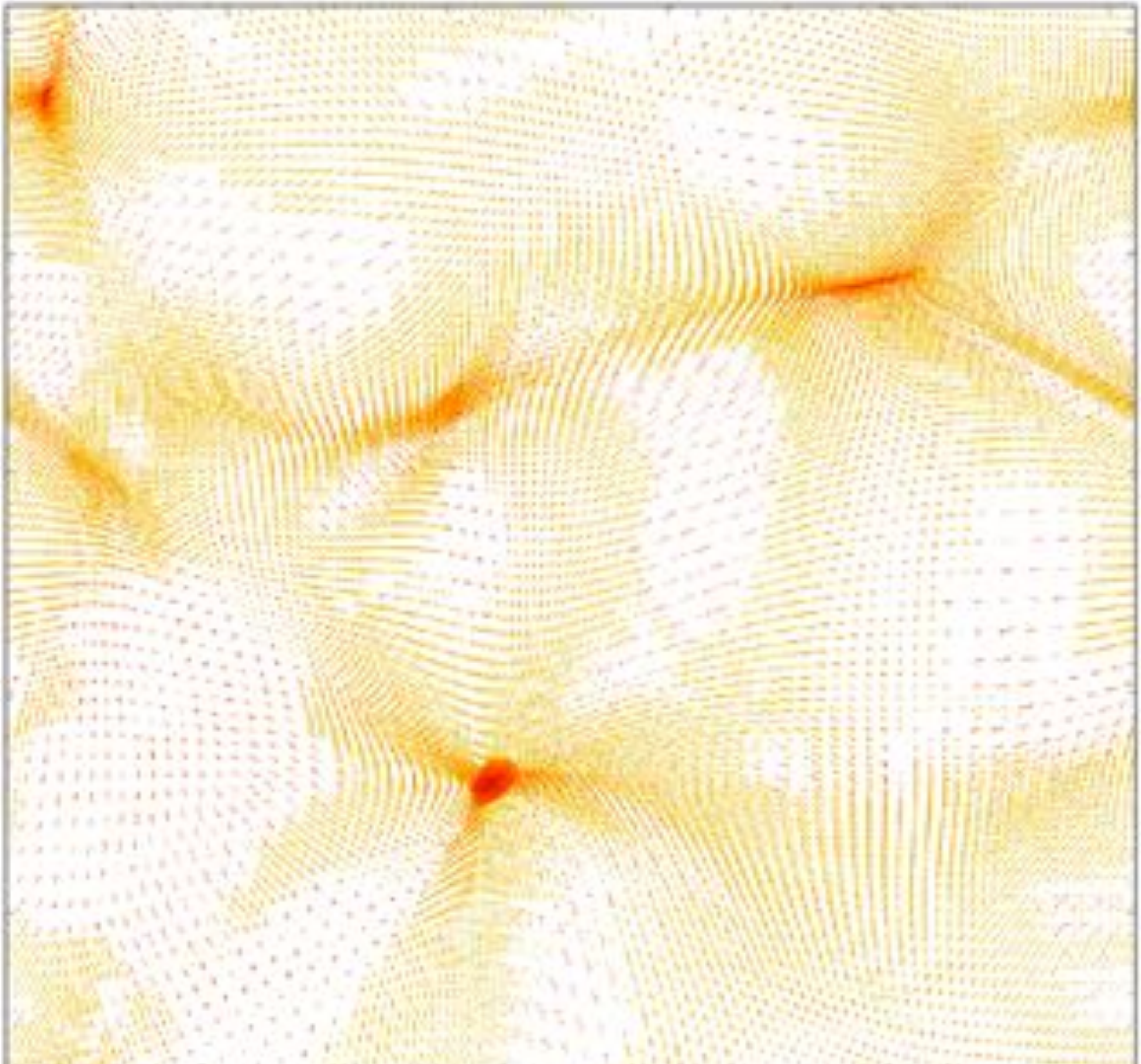
Adaptive Refinement II - work in progress

3 levels of
additional refinement



stay tuned...

Hahn, Angulo & Abel ongoing



N-body

Quadrilaterals + refinement

Quadrilaterals

Linear tets + refinement

Linear tets

Enormous accuracy gains with higher order interpolation schemes.
Shown here in the test case of a cube evolving in a static potential.

Lagrangian Tessellation: What's it good for?

- Analyzing N-body sims, including web classification, velocity dispersion, profiles, resolution study (Abel, Hahn, Kaehler 2012)
- DM visualization (Kaehler, Hahn, Abel 2012)
- Better Numerical Methods (Hahn, Abel & Kaehler 2013, Hahn, Angulo & Abel 2014-)
- Finally reliable WDM mass functions below the cutoff scale (Angulo, Hahn, Abel 2013)
- Gravitational Lensing predictions (Angulo, Chen, Hilbert & Abel 2014)
- Cosmic Velocity fields (Hahn, Angulo, Abel 2014)
- Plasma simulations (Vlasov/Poisson and relativistic Vlasov/Maxwell) (Kates-Harbeck, Totorica, Zrake & Abel 2014, in prep.)
- Exact overlap integrals of Polyhedra (Powell & Abel 2014 submitted.)
- Void profiles, Wojtak et al in prep.
- your application here …

Thank you!

- Questions?