Gappy Weak Lensing:
KL interpolation of shear fields

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May 6, 2011

Outline

Challenges of Weak Lensing cluster searches

Shear Peak statistics: a solution?

KL interpolation of shear fields

Results and future prospects

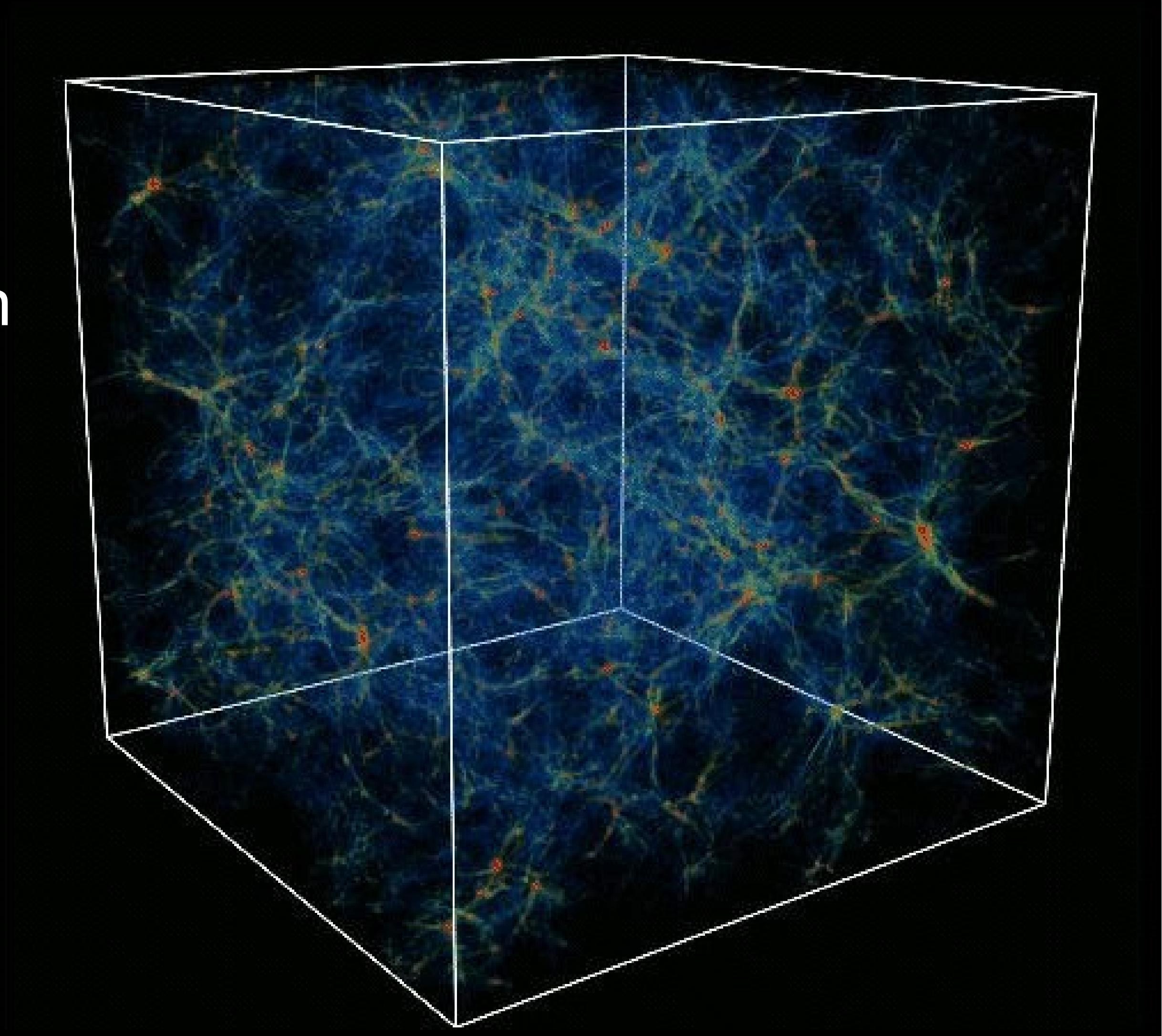
Motivation:

Galaxy Clusters probe cosmology:

Number counts with redshift

2-point correlation function

Mass profiles & Substructure



Finding Clusters: Optical & IR

Most straightforward method: search for groups of galaxies (e.g. SDSS, 2MASS)

Problem: how well do optical/IR sources trace the mass distribution?

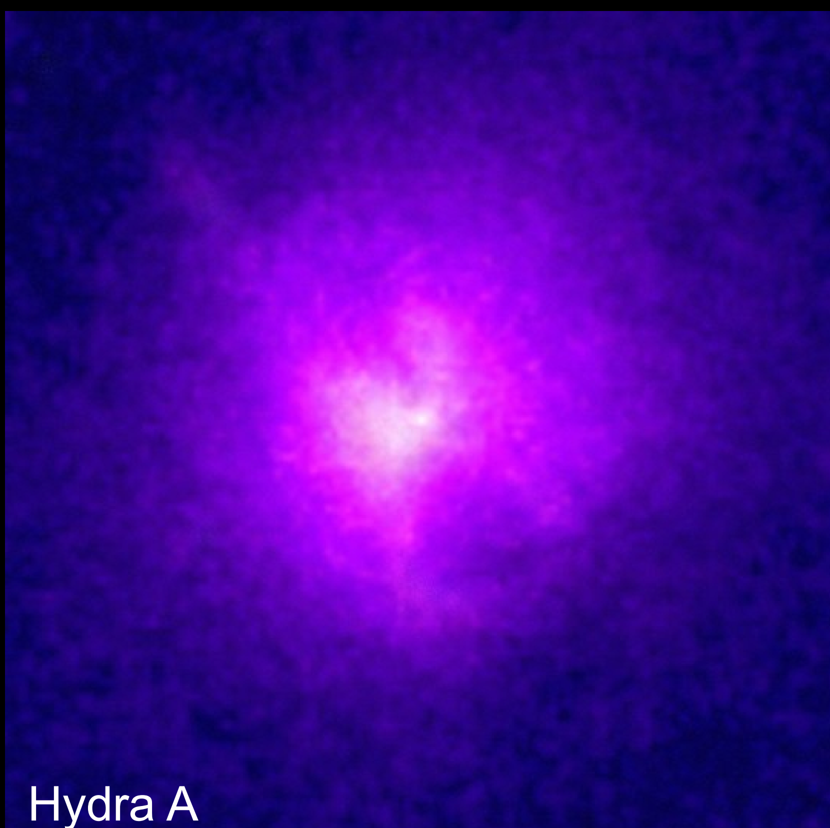
Perseus Cluster: 2MASS

Coma Cluster: SDSS

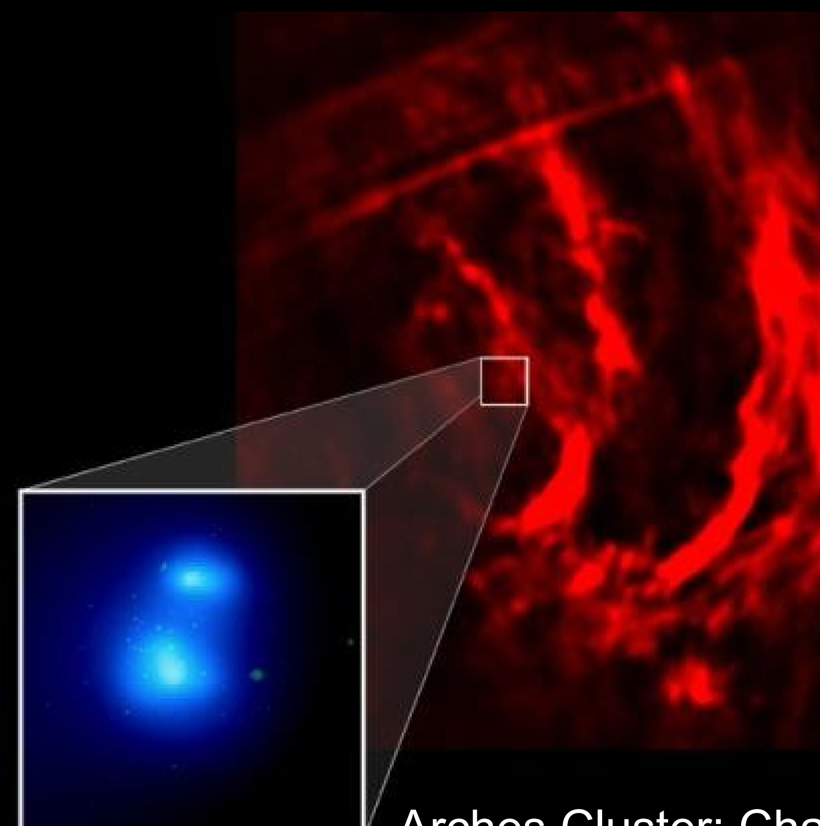
Finding Clusters: X-ray

Look for X-ray signatures of Intracluster gas

Problem: uncertain conversions from X-ray flux to gas density to dark matter density



Hydra A

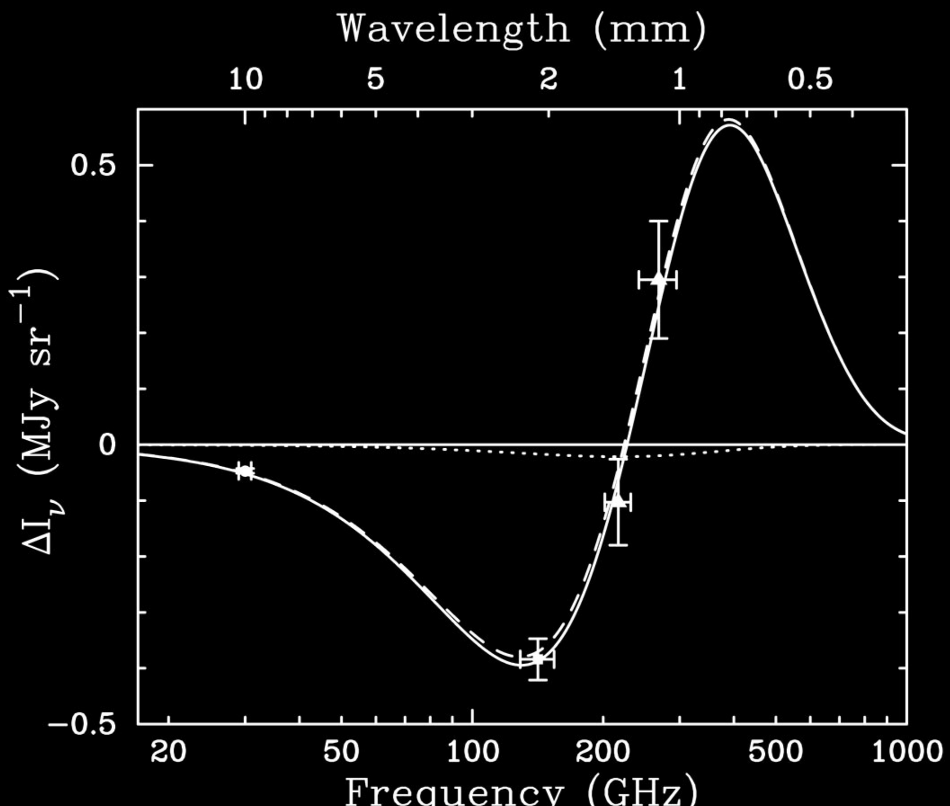


Arches Cluster: Chandra

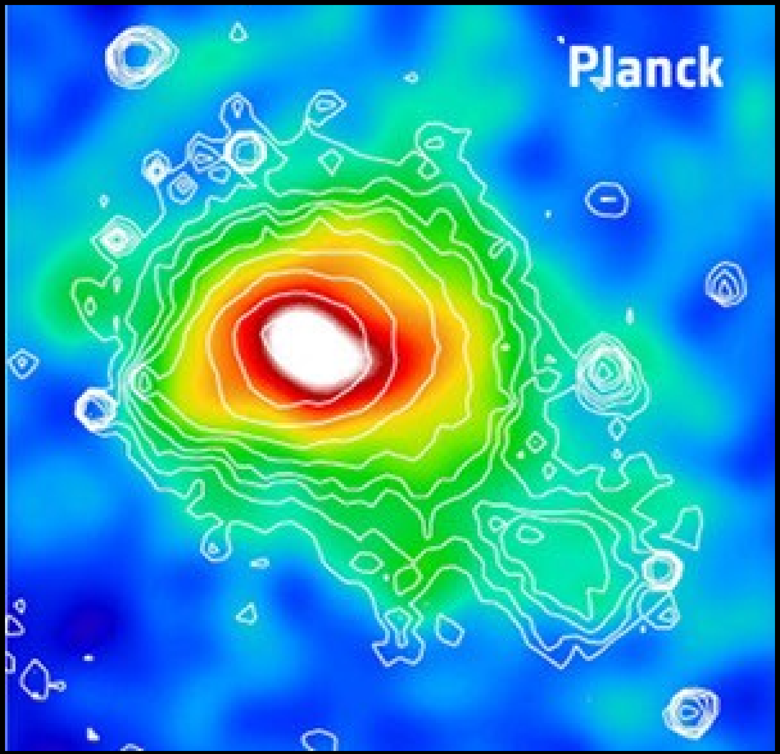
Finding Clusters: SZ

Sunyaev and Zeldovich's bright idea: look for scattering of CMB off hot gas!

Problem: gas density to mass conversion



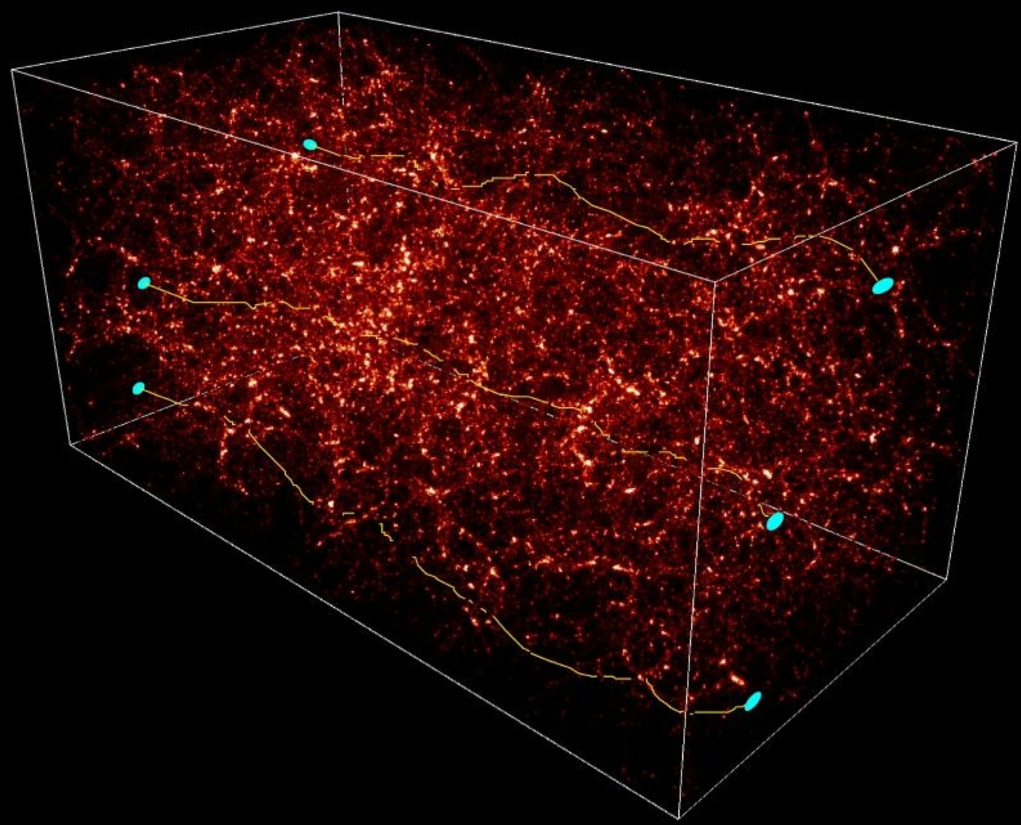
Erik Reese



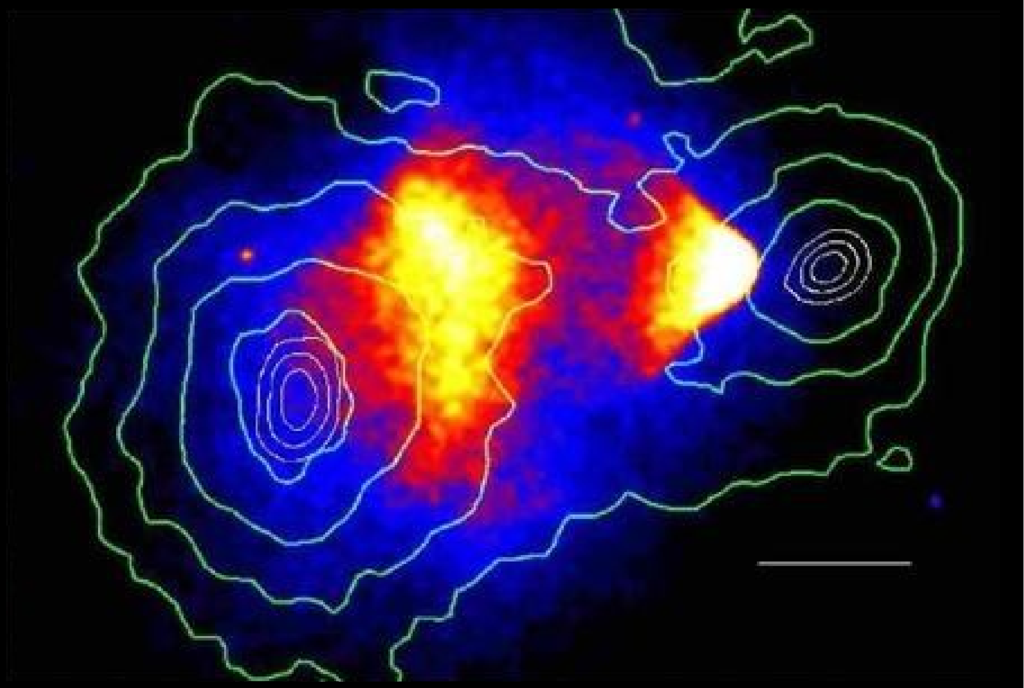
Planck & ROSAT: Coma Cluster

Finding Clusters: Weak Lensing

Look for gravitational distortion of background galaxies: towards a robust mass-selected cluster catalog?



Ray tracing: S. Colombi, CFHT

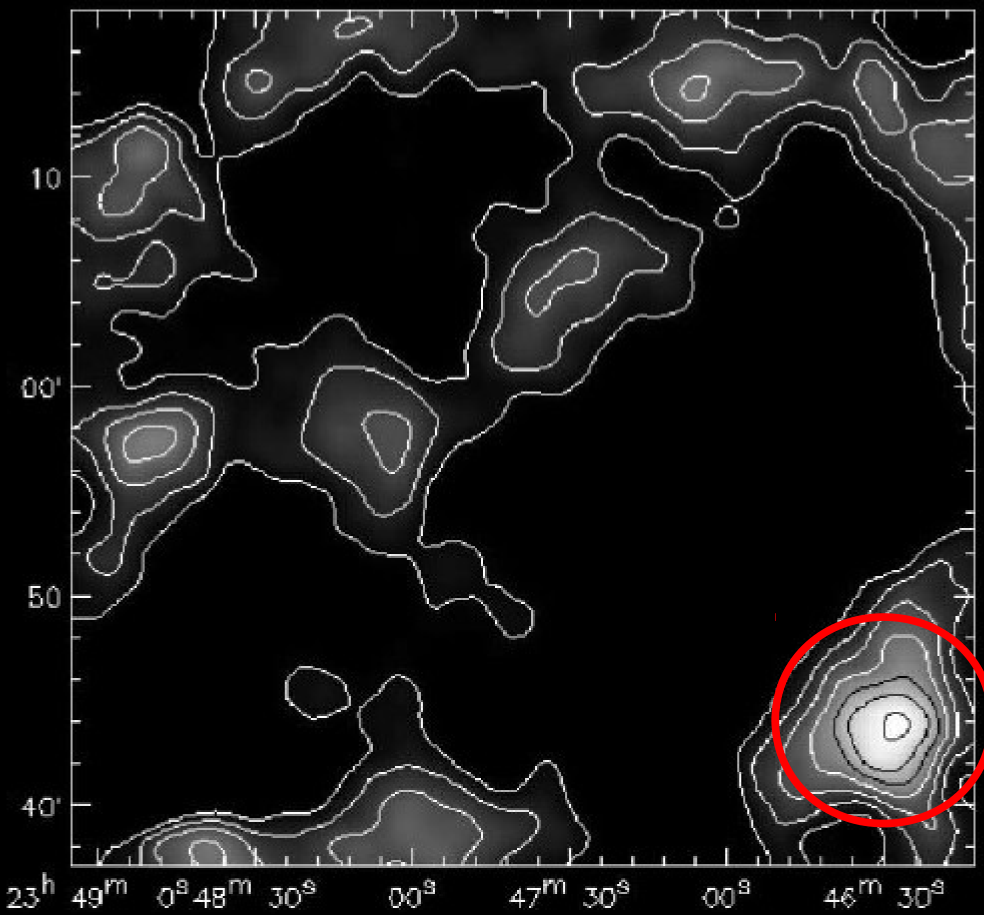


Bullet Cluster: Clowe *et al.*

Finding Clusters in 3D

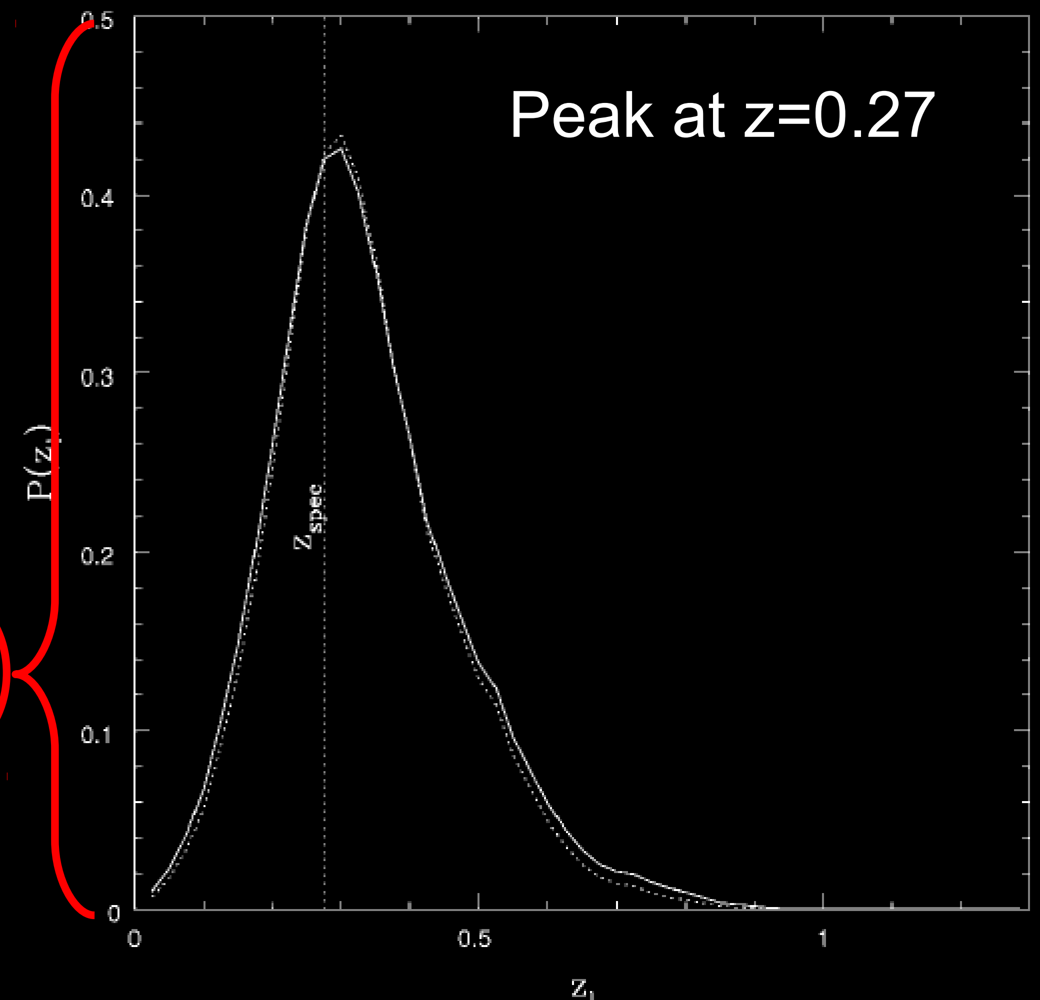
Parametric methods:

e.g. Wittman *et al.* 2001



Data: CTIO

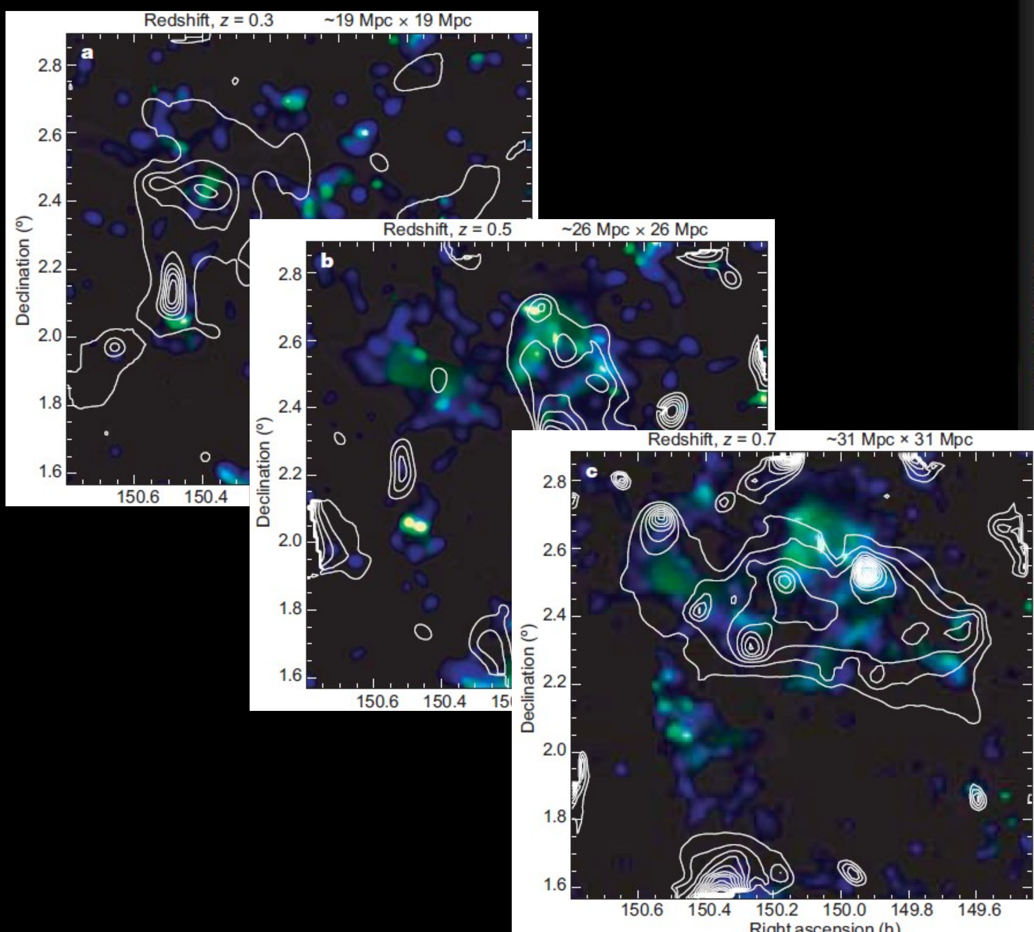
Fit SIS and NFW profiles at different redshifts

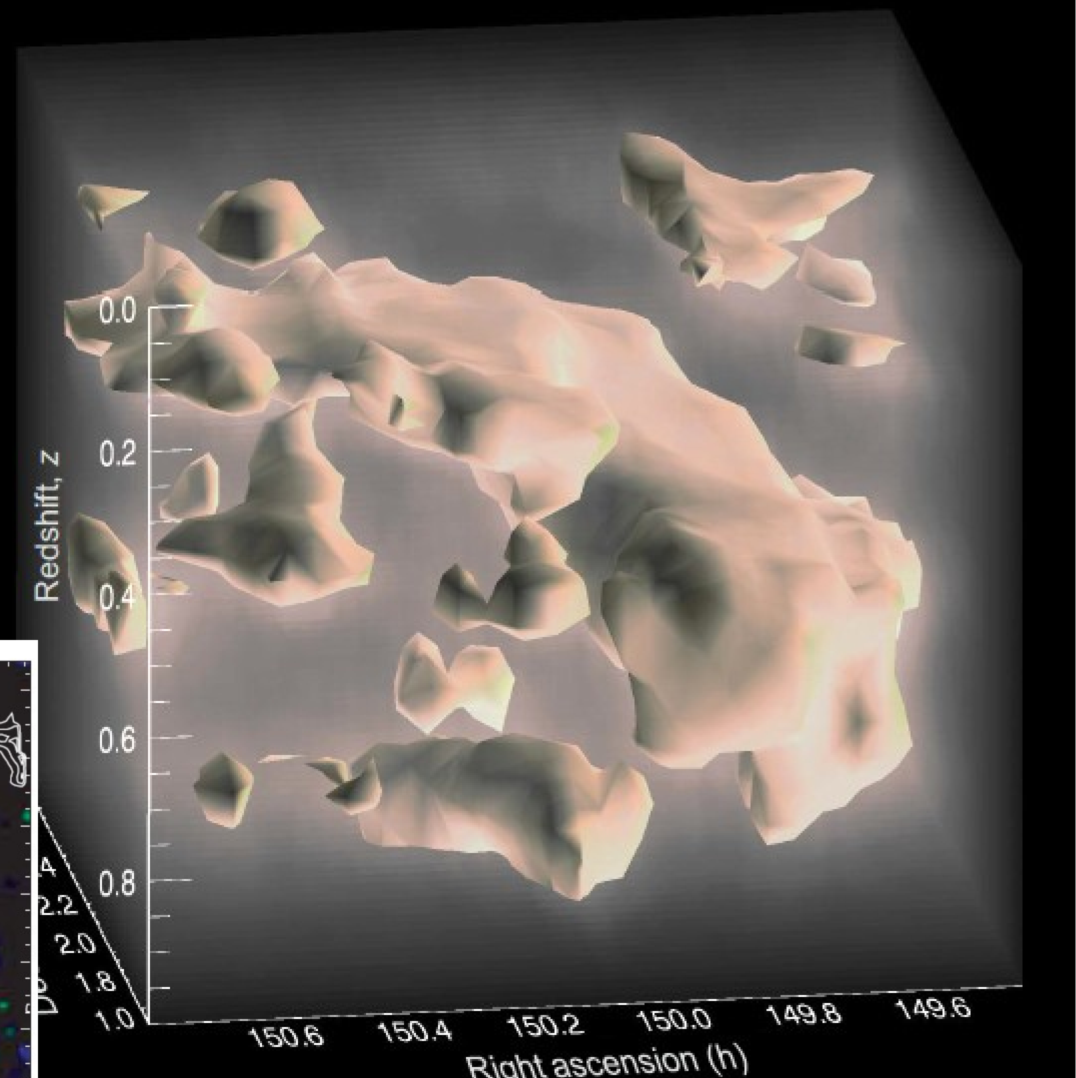


Finding Clusters in 3D

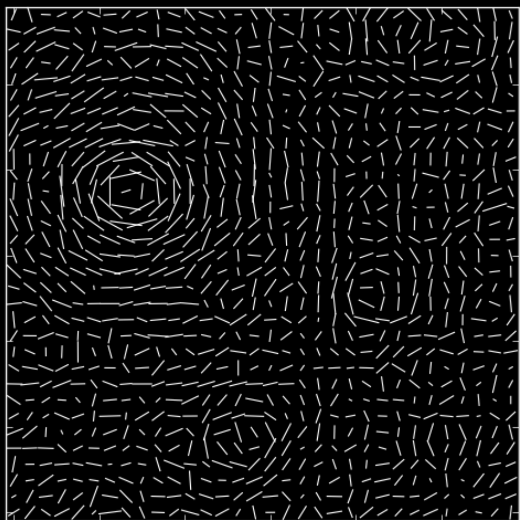
Nonparametric methods:

"2½ Dimensional" approach:
Massey 2007 (COSMOS)

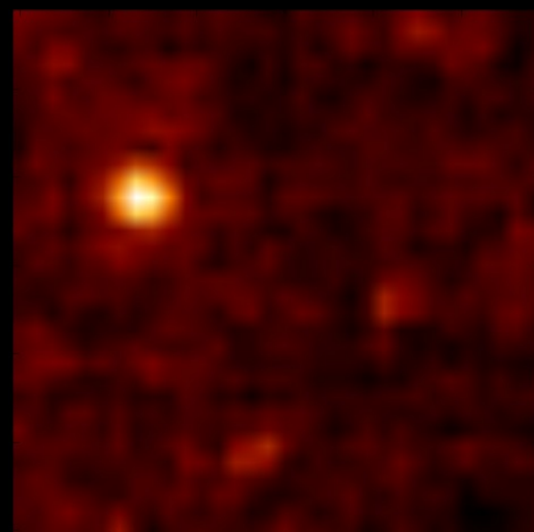




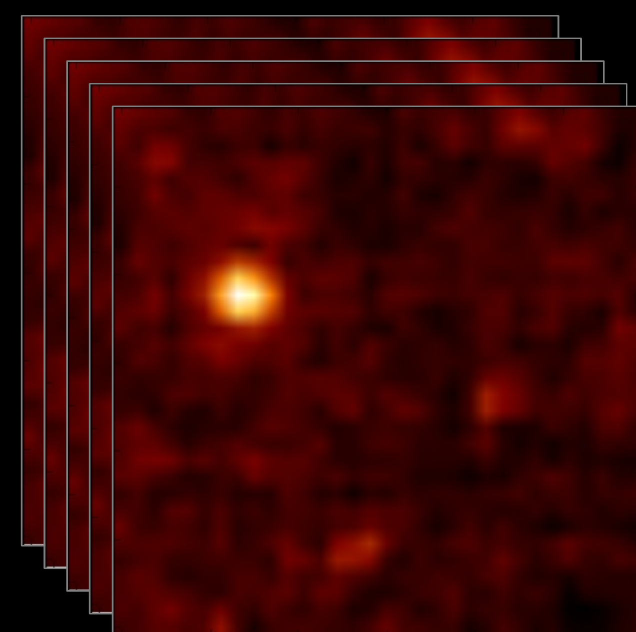
Toward a full 3D reconstruction:



$$\gamma \rightarrow \kappa$$



$$\kappa \rightarrow \delta$$



Toward a full 3D reconstruction:

Vanderplas *et al* 2011: SVD filtering

$\gamma \rightarrow \kappa$:

$$\gamma(\vec{\theta}) = \int d\vec{\theta}'^2 \kappa(\vec{\theta}') D(\vec{\theta} - \vec{\theta}')$$

$\vec{\gamma} = P_{\gamma\kappa} \vec{\kappa}$: operates in each source plane

$\kappa \rightarrow \delta$:

$$\kappa(\chi_S) = \frac{3 H_0^2 \Omega_M}{2} \int_0^{\chi_S} \frac{\chi(\chi_S - \chi)}{\chi_S} \frac{1 + \delta(\chi)}{a(\chi)} d\chi$$

$\vec{\kappa} = Q_{\kappa\delta} \vec{\delta}$: operates in each line-of-sight

Final Result: $\longrightarrow$ $\vec{\gamma} = M \vec{\delta}$

Hu&Keeton 2002
Simon et al 2009

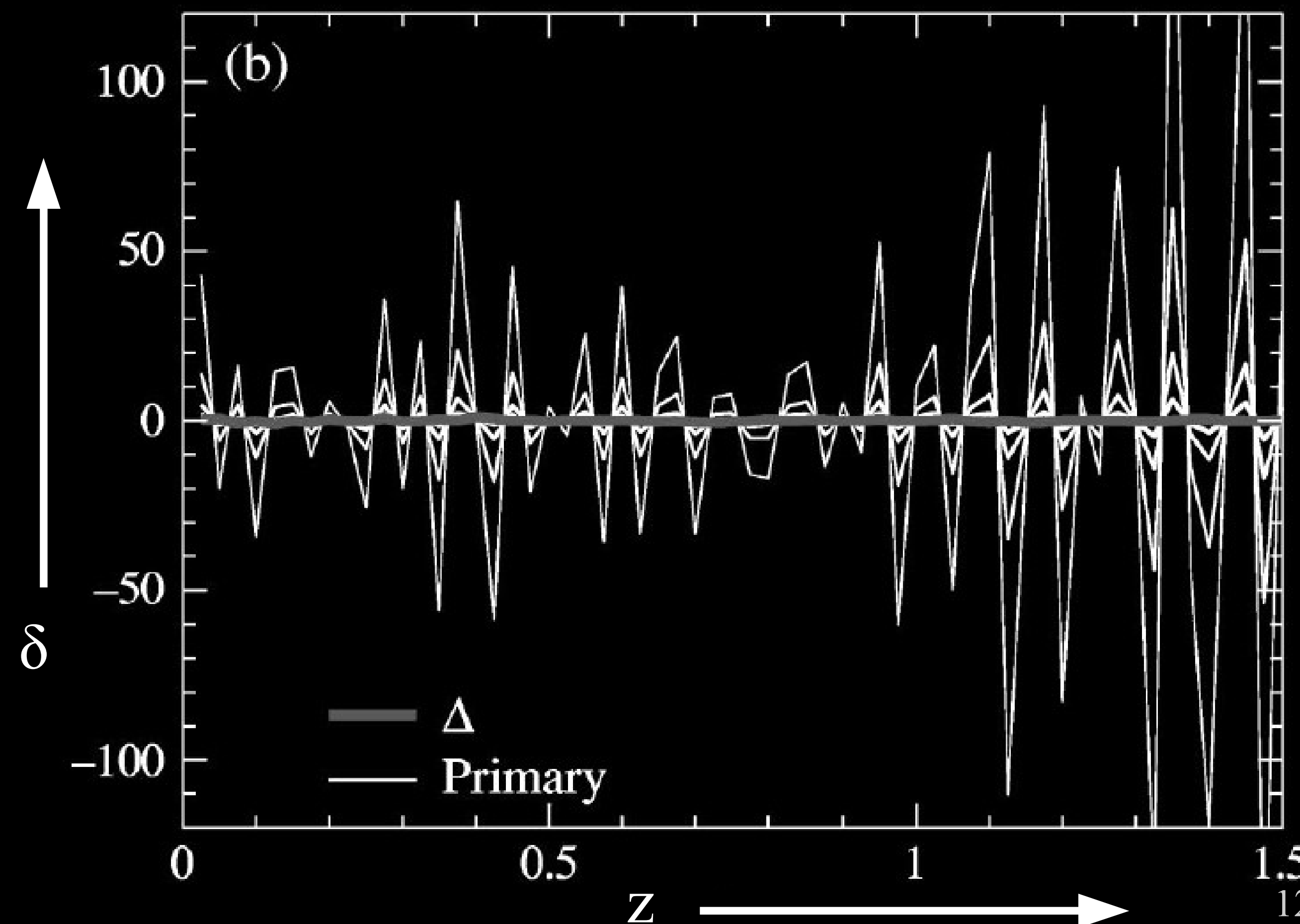
3D Lensing is simply a linear inversion:

$$\vec{\gamma} = M\,\vec{\delta} + \vec{n}_\gamma$$

Best estimator, via Aitken (1935):

$$\hat{\delta} = \left(M^T N_{\gamma\gamma}^{-1} M\right)^{-1} M^T N_{\gamma\gamma}^{-1} \vec{\gamma}$$

Problem: Noise can obscure the signal by several orders of magnitude!



(Hu & Keeton 2002)

Tracing the source of the problem:

$$\vec{y} = M\,\vec{\delta} + n_\gamma$$

Singular Value Decomposition (SVD)

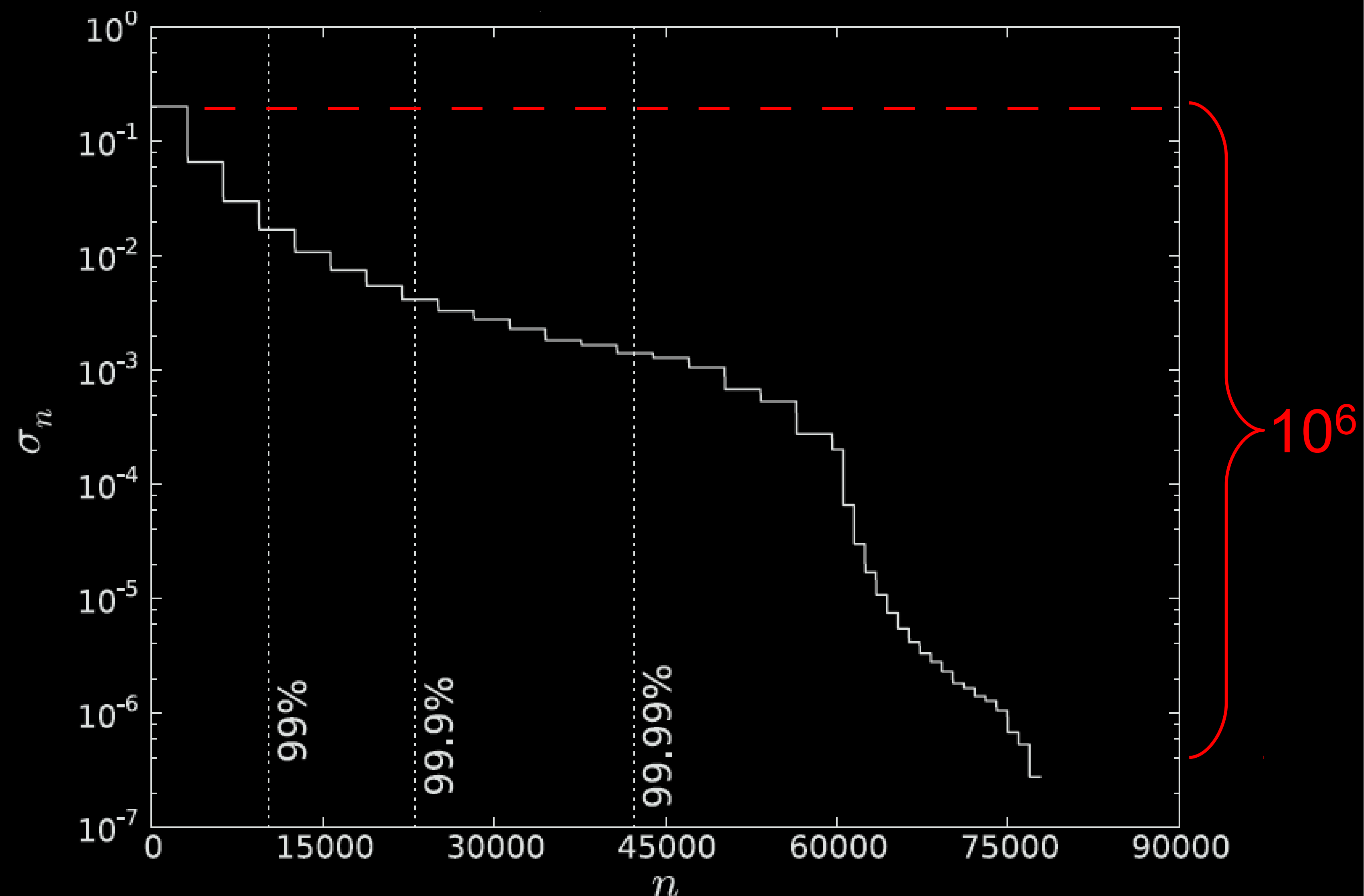
$$N_{\gamma\gamma}^{-1/2} M = U\,\Sigma\,V^T \qquad \begin{cases} U^T U = V^T V = I \\ \Sigma = \text{diagonal} \end{cases}$$

Aitken estimator becomes:

$$\hat{\delta} = V\,\Sigma^{-1}\,U^T N_{\gamma\gamma}^{-1/2}\,\vec{y}$$

Small singular values lead to large noise in δ!

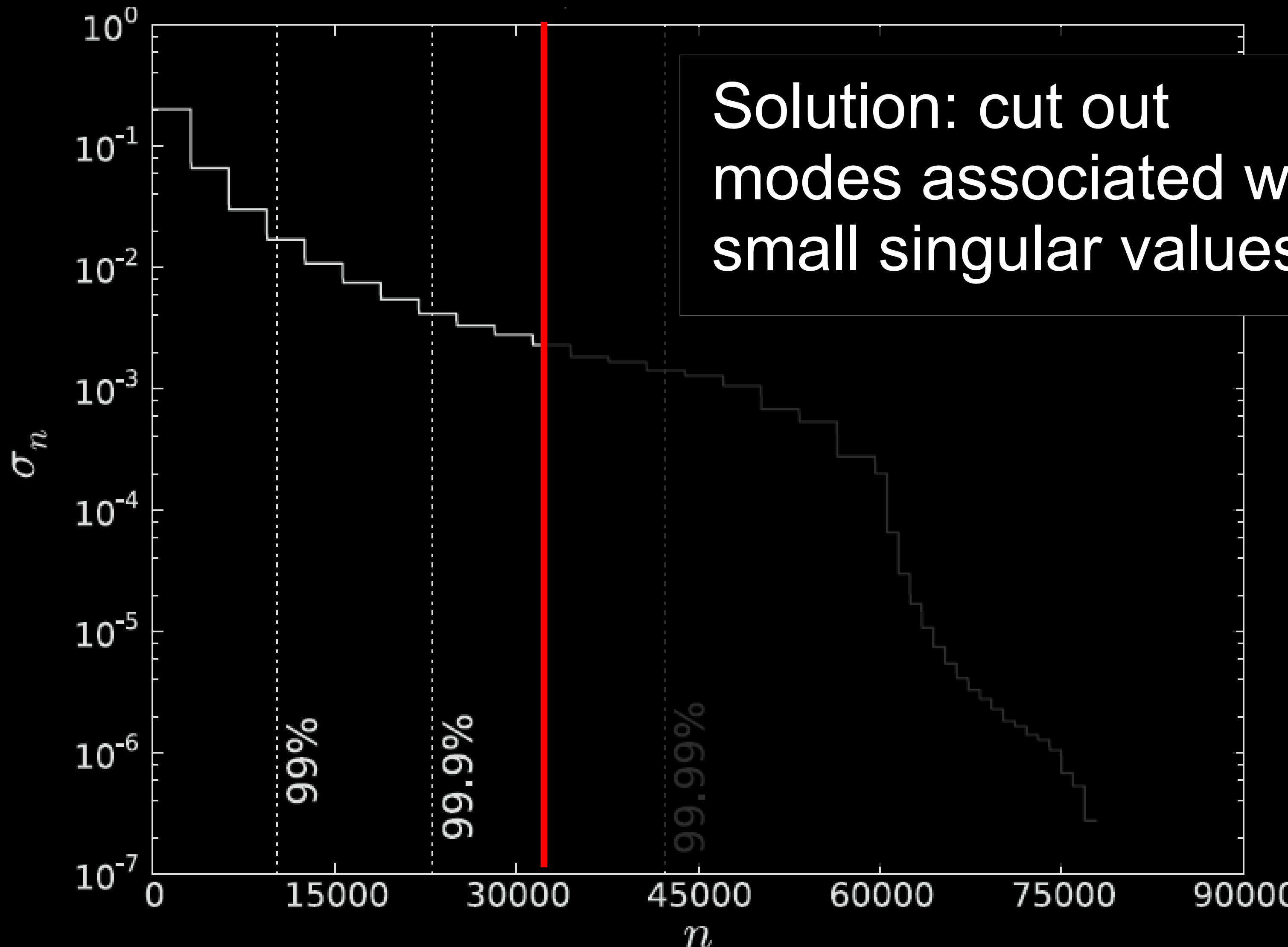
Singular Value Profile:



$$\hat{\delta} = V \Sigma^{-1} U^T N_{\gamma\gamma}^{-1/2} \vec{\gamma}$$

Noise amplified by a factor of 10^6!

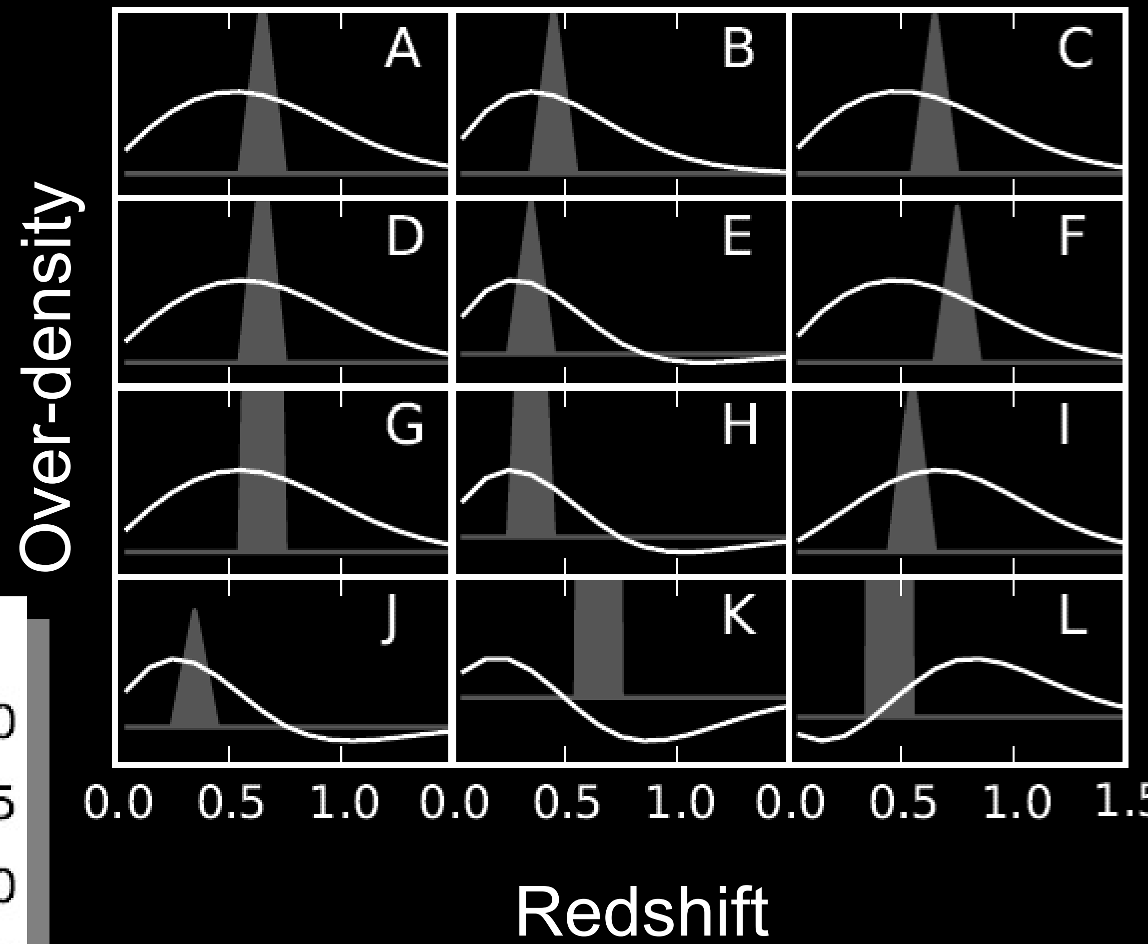
Singular Value Profile:

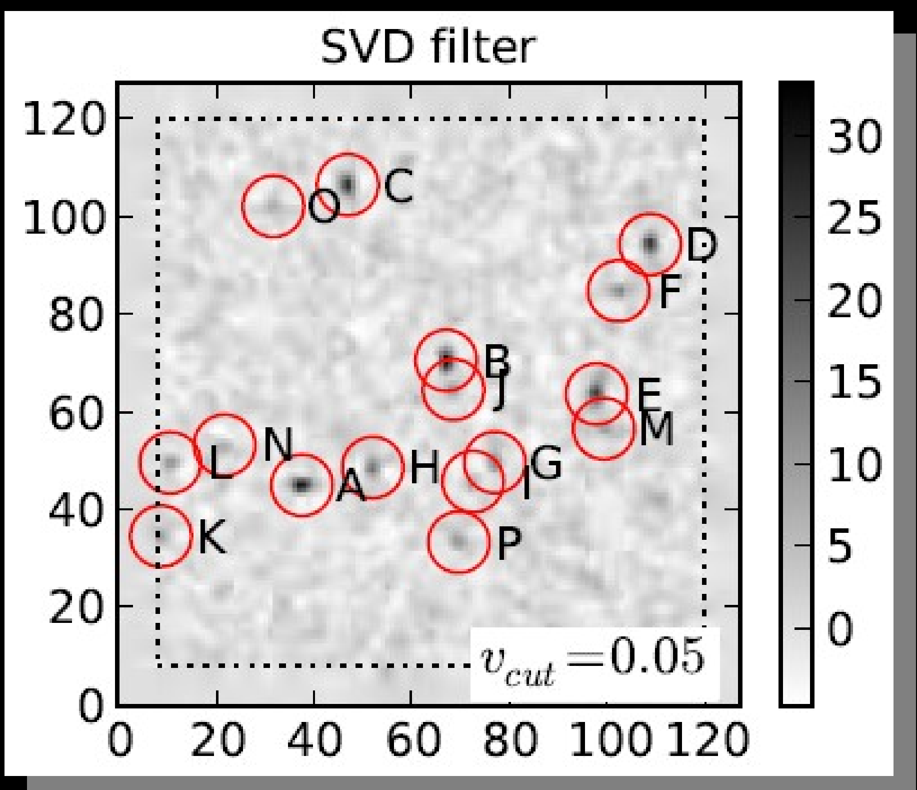


Solution: cut out modes associated with small singular values

$$\hat{\delta} = V \Sigma^{-1} U^{T} N_{\gamma\gamma}^{-1/2} \vec{\gamma}$$

Testing SVD reconstruction:





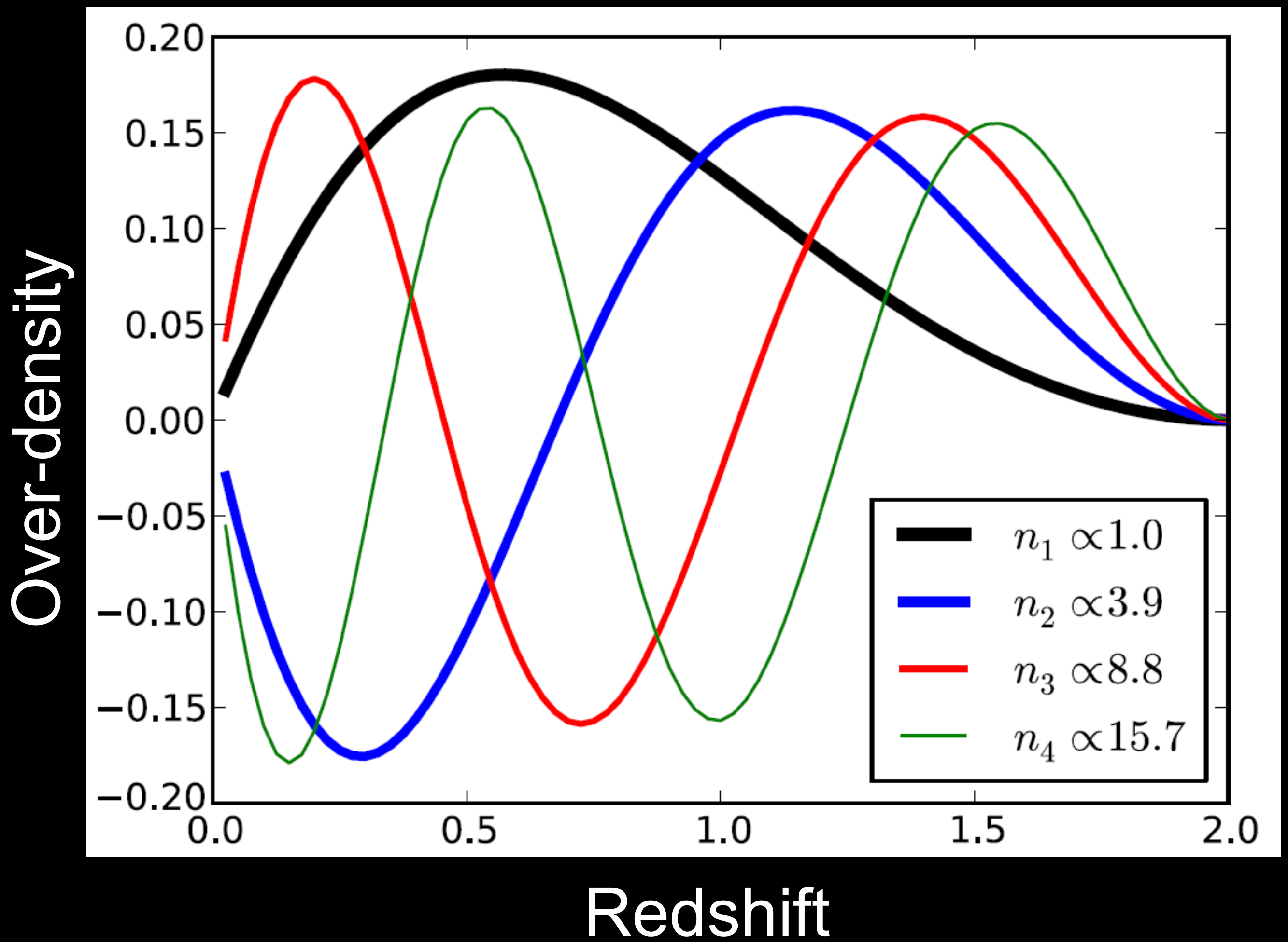
Spread and bias in redshift direction

Vanderplas *et al.* 2011

3D non-parametric weak lensing: Fundamentally Limited?

Noise in mode n scales as $\sim n^2$

Typical surveys can constrain first ~ few modes



Vanderplas *et al.* 2011

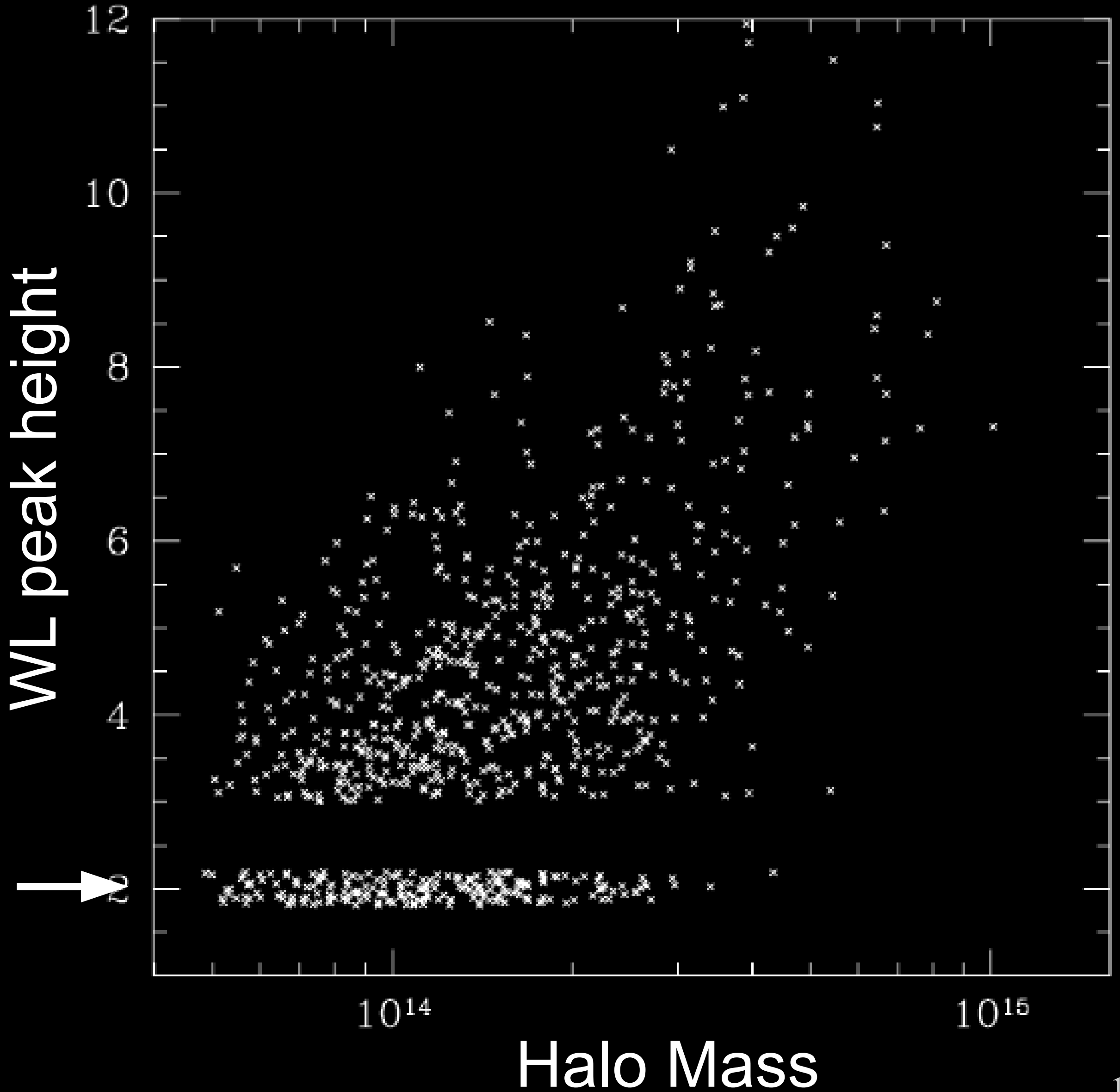
Cluster Mass Calibration

Difficulty with WL mass calibration

- Bias from shape noise

Also...
- Correlated Projections
- Unresolved Substructure
- Halo asymmetry
- Redshift dependence

Undetected peaks here



Hamana *et al.* 2004

Problem:

Data and theory are difficult to compare

Weak lensing yields projected mass,
Theory gives 3D mass.

Solution:

Rather than force-fitting data to theory, let's work toward a theory that naturally fits the data.

Shear Peak Statistics

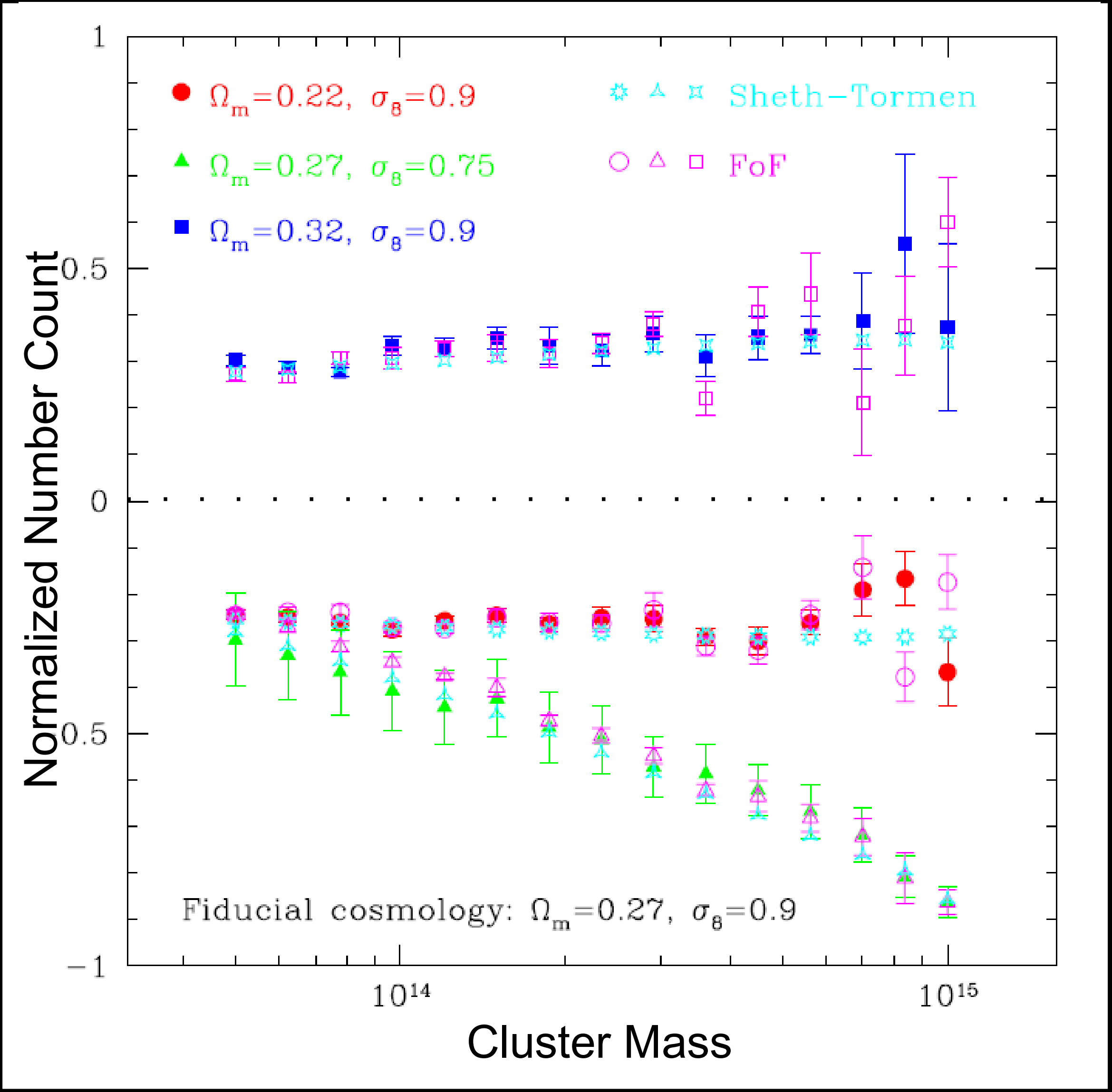
Pioneering Work: Marian *et al.* 2009, Wang *et al.* 2009

Marian *et al:*

How do *projected* mass peaks scale with cosmology?

Slab Thickness: 50Mpc
(Δz~0.1 at z=0.6)

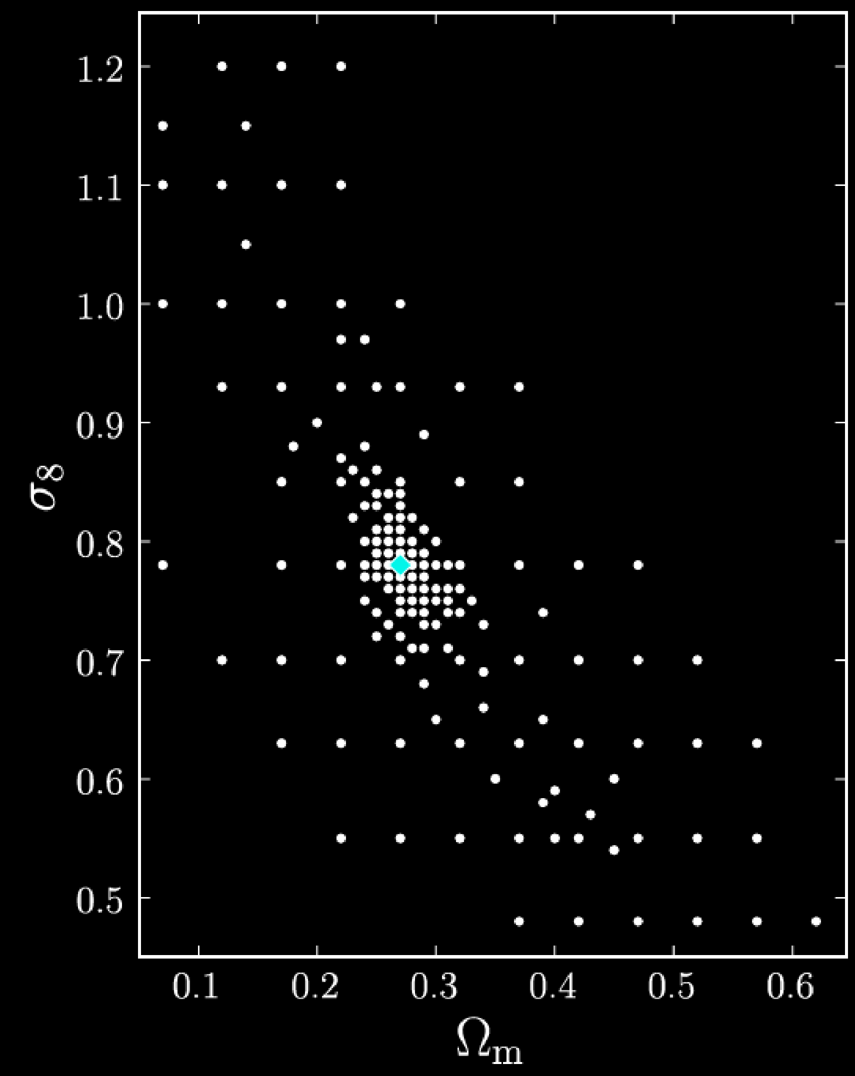
Marian *et al* 2010: results extended to different thicknesses



Shear Peak Statistics

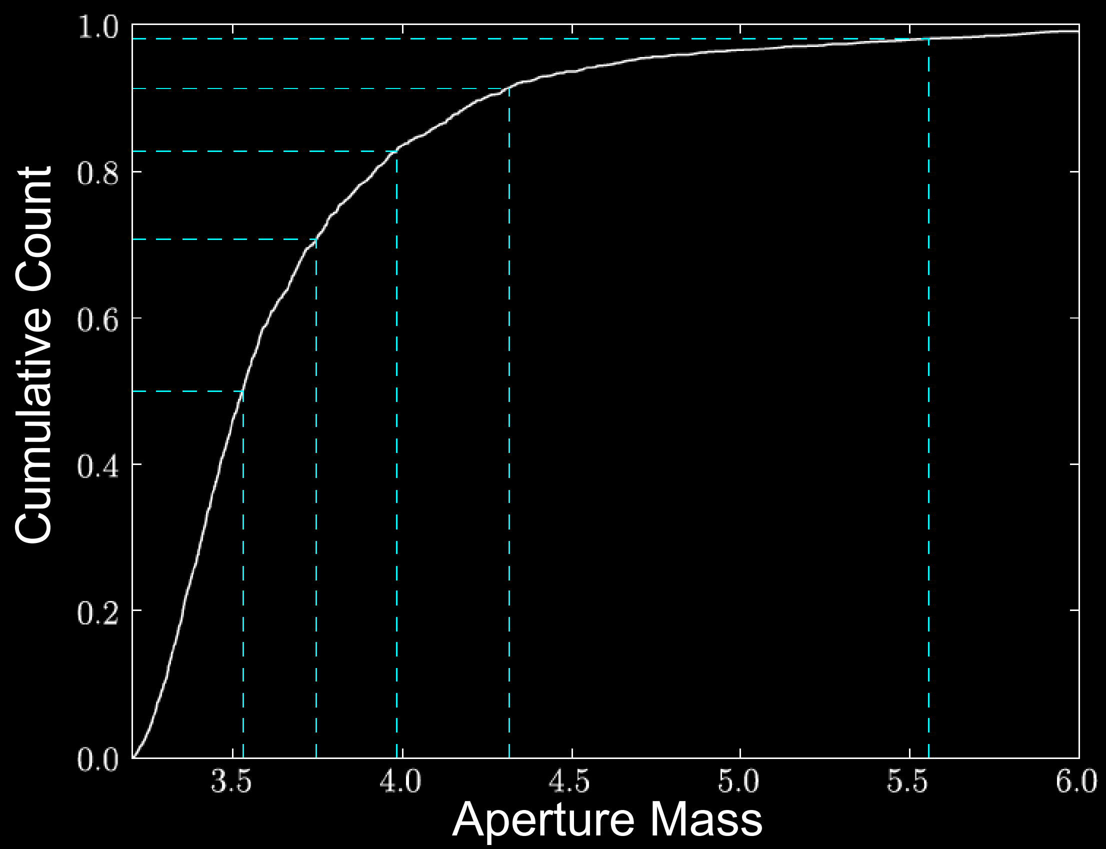
What information is in the correlated projections?

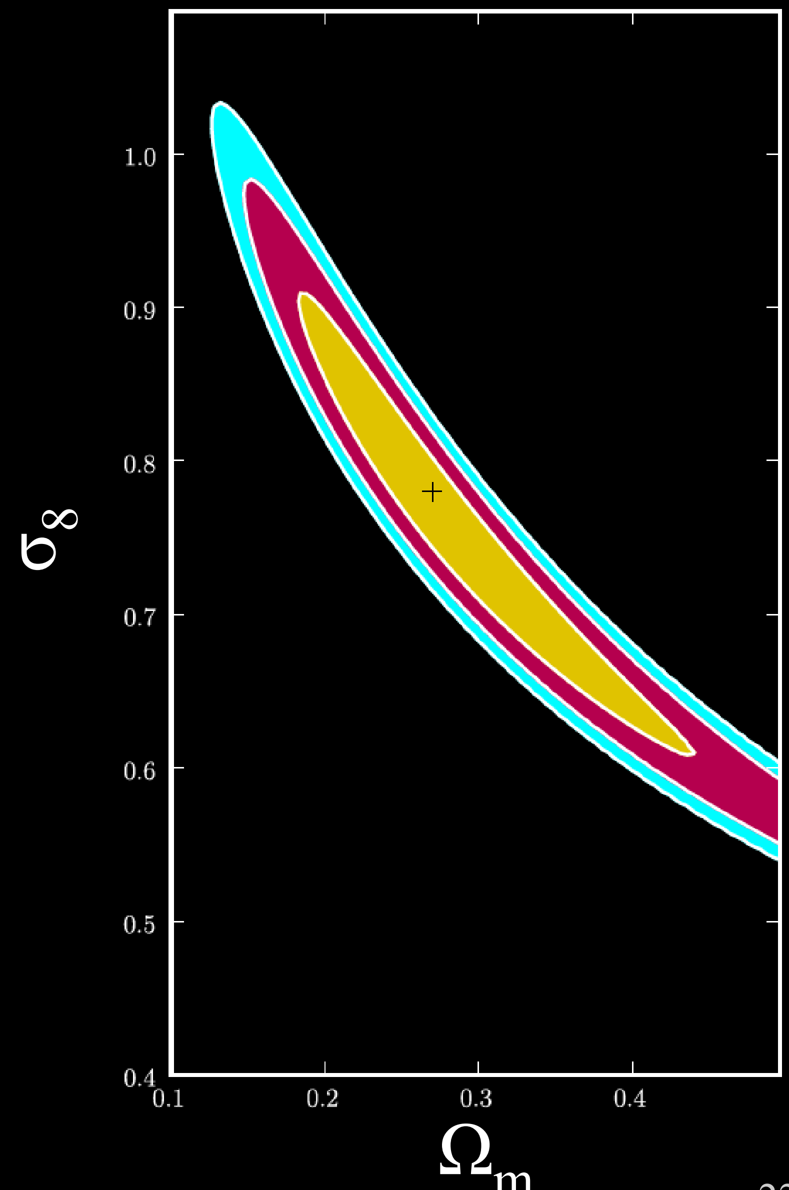
Dietrich & Hartlap 2010:
Test cosmological information content of shear peaks



Cosmology Constraints

Cumulative distribution of peak heights can be used to constrain cosmology

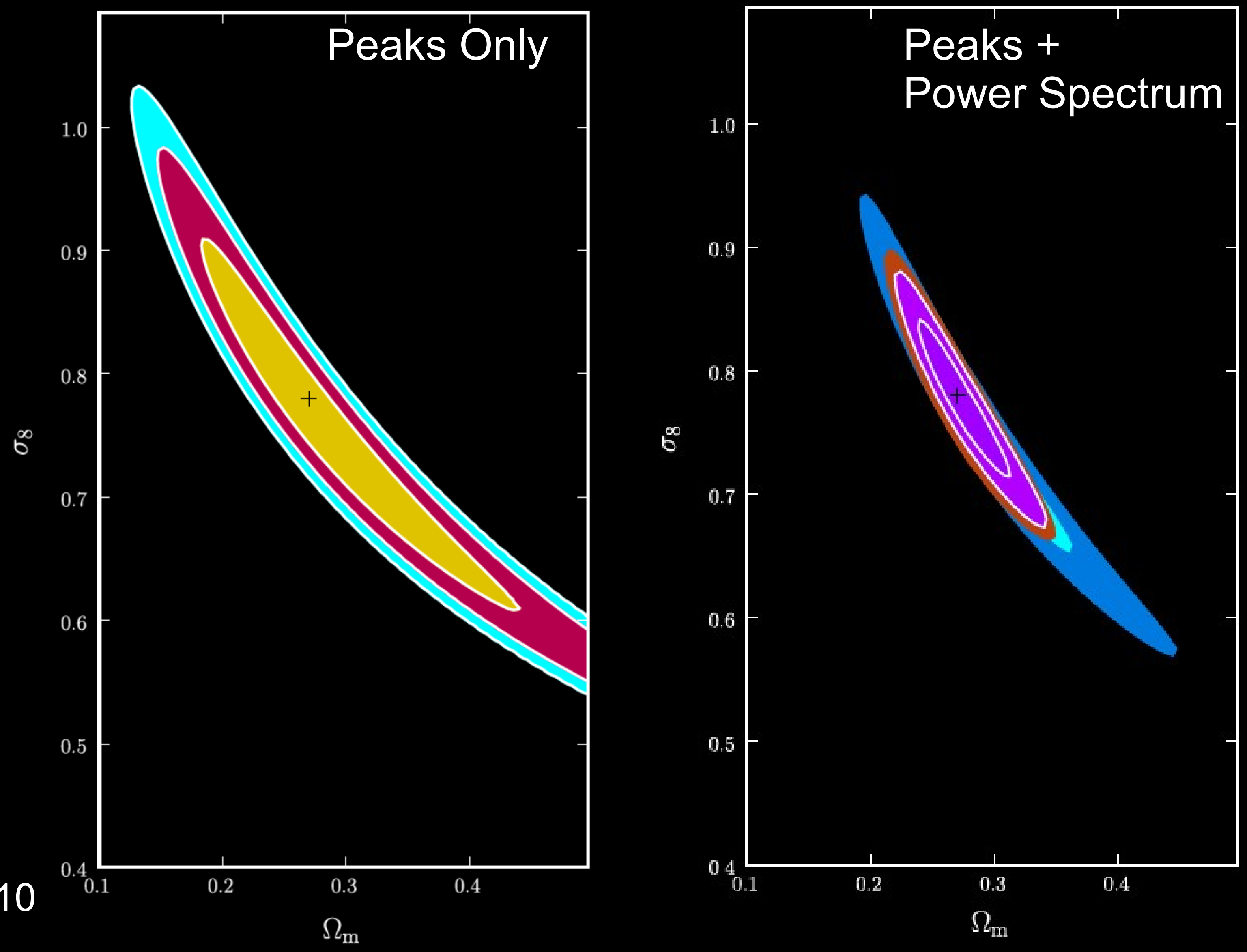




Dietrich & Hartlap 2010

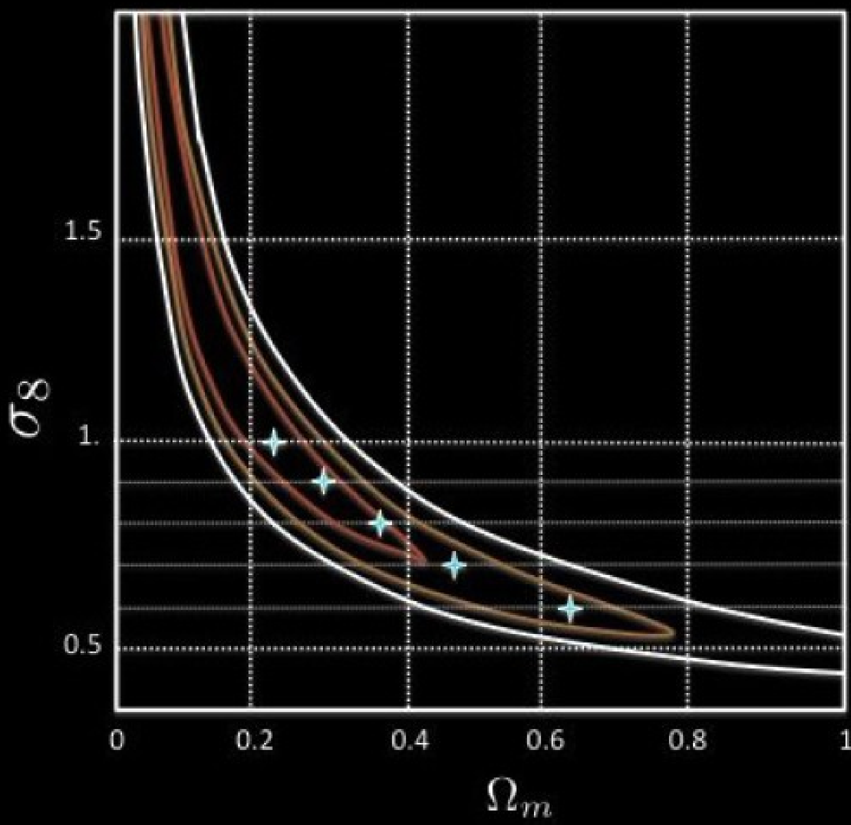
Shear Peak Statistics

Peak functions probe nonlinear structure: higher-order information!

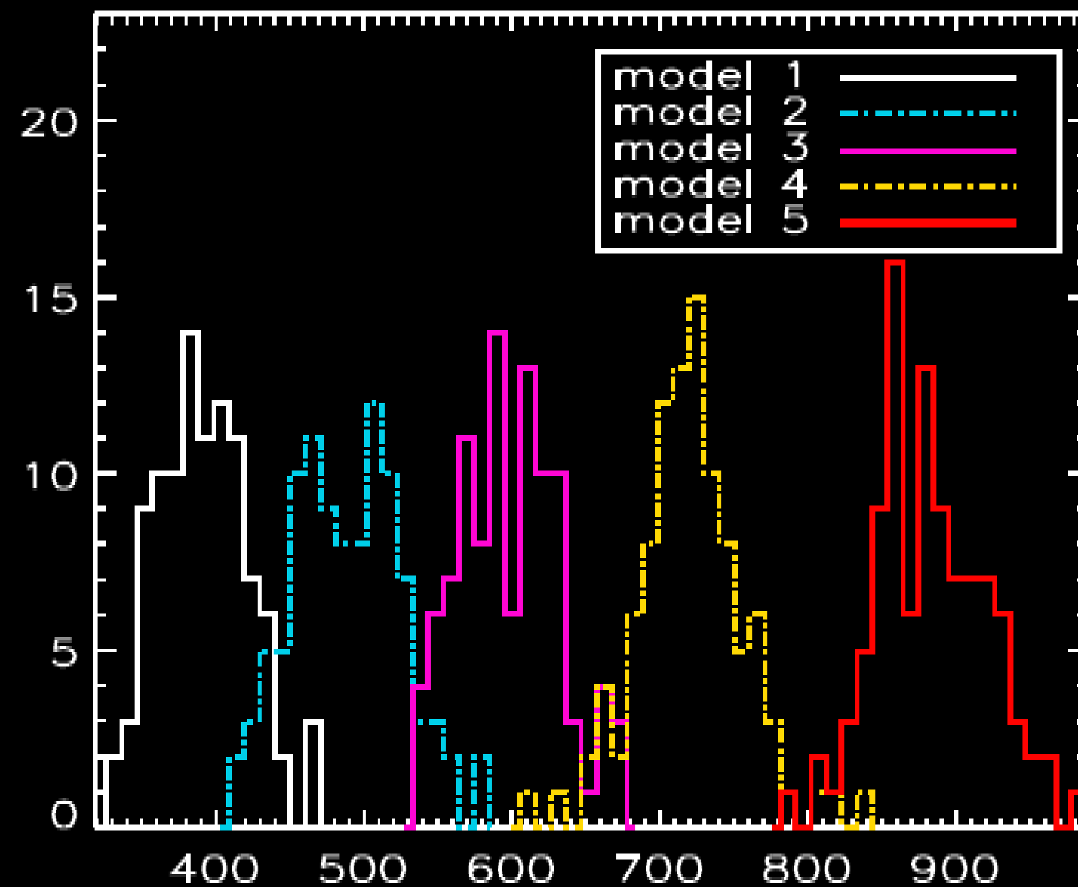


Dietrich & Hartlap 2010

Shear Peaks: Higher order information



Using a "wavelet transform" filter gives similar discriminatory power



Pires *et al.* 2009

Shear Peak Statistics

Summary:

Shear peak counts scale with cosmology in predictable ways *(Marian 2009,2010)*

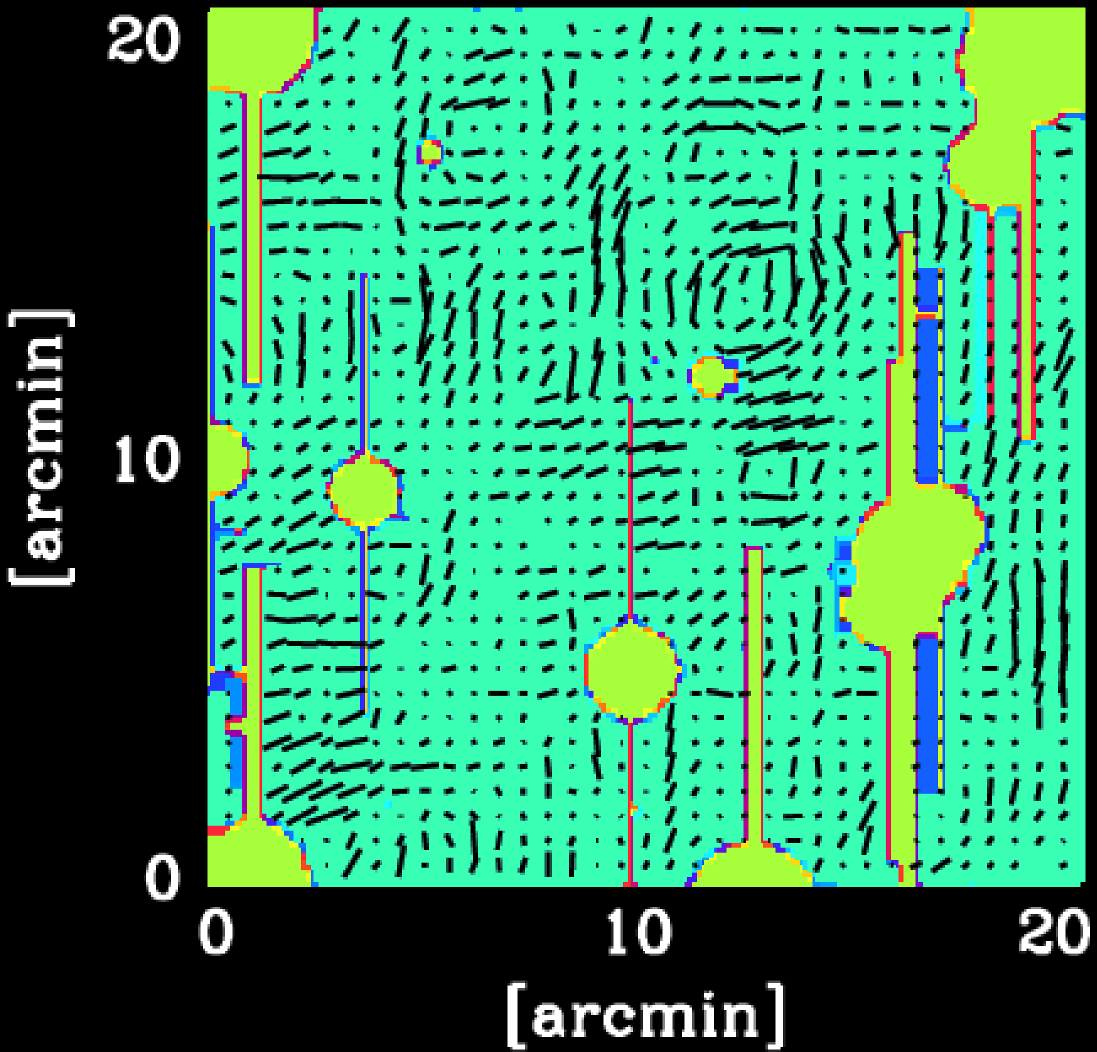
Projected structure encodes cosmological information *(Wang 2009, Kratochvil 2010)*

Peak distributions contain complementary information to 2-point analyses
(Dietrich 2010, Pires 2009. See also Maturi 2011)

Methodology is in its infancy: the ideal filtering and peak-finding method needs to be explored
(but see Pires 2009, Schmidt & Rozo 2010)

Shear Peaks: The Problem

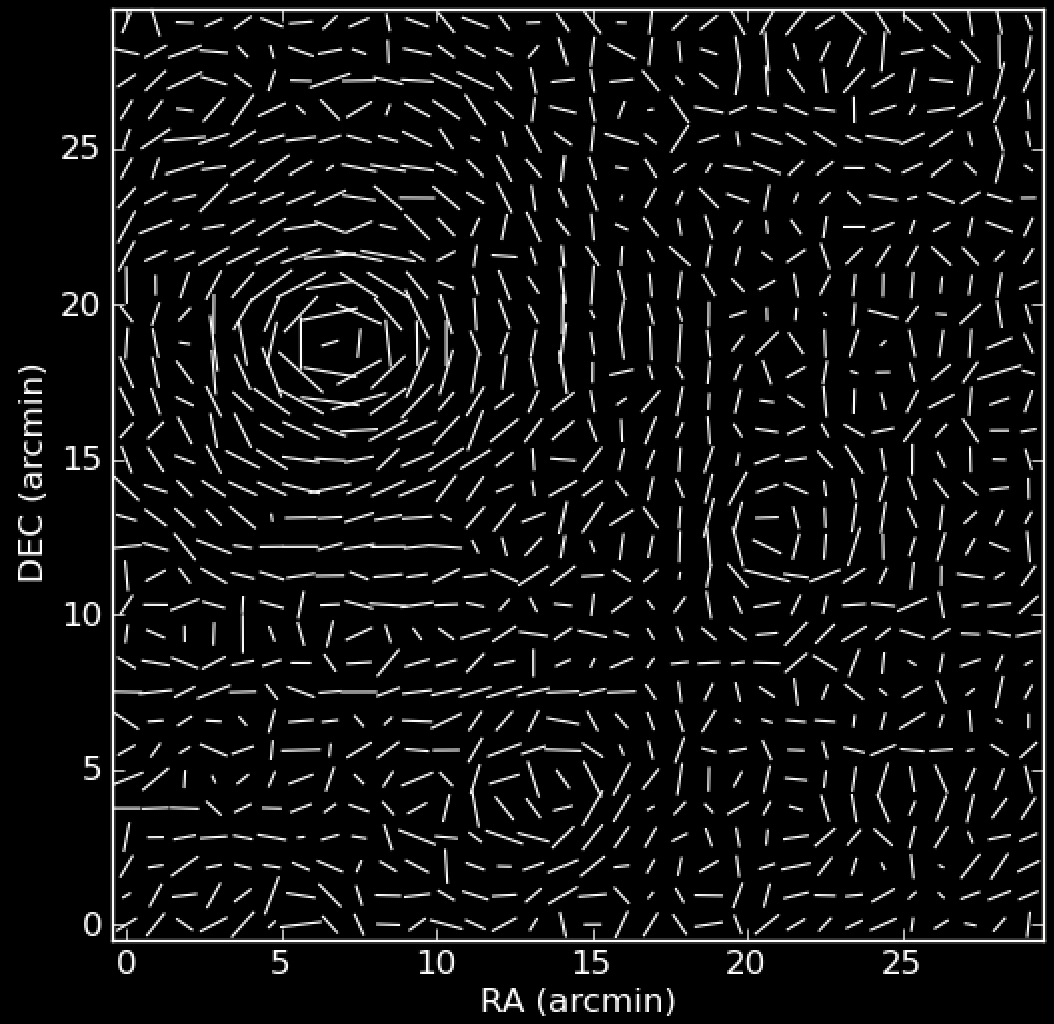
Shear surveys are subject to masking effects: what sort of bias will this create?



Hikage *et al.* 2010

Our Solution: KL analysis

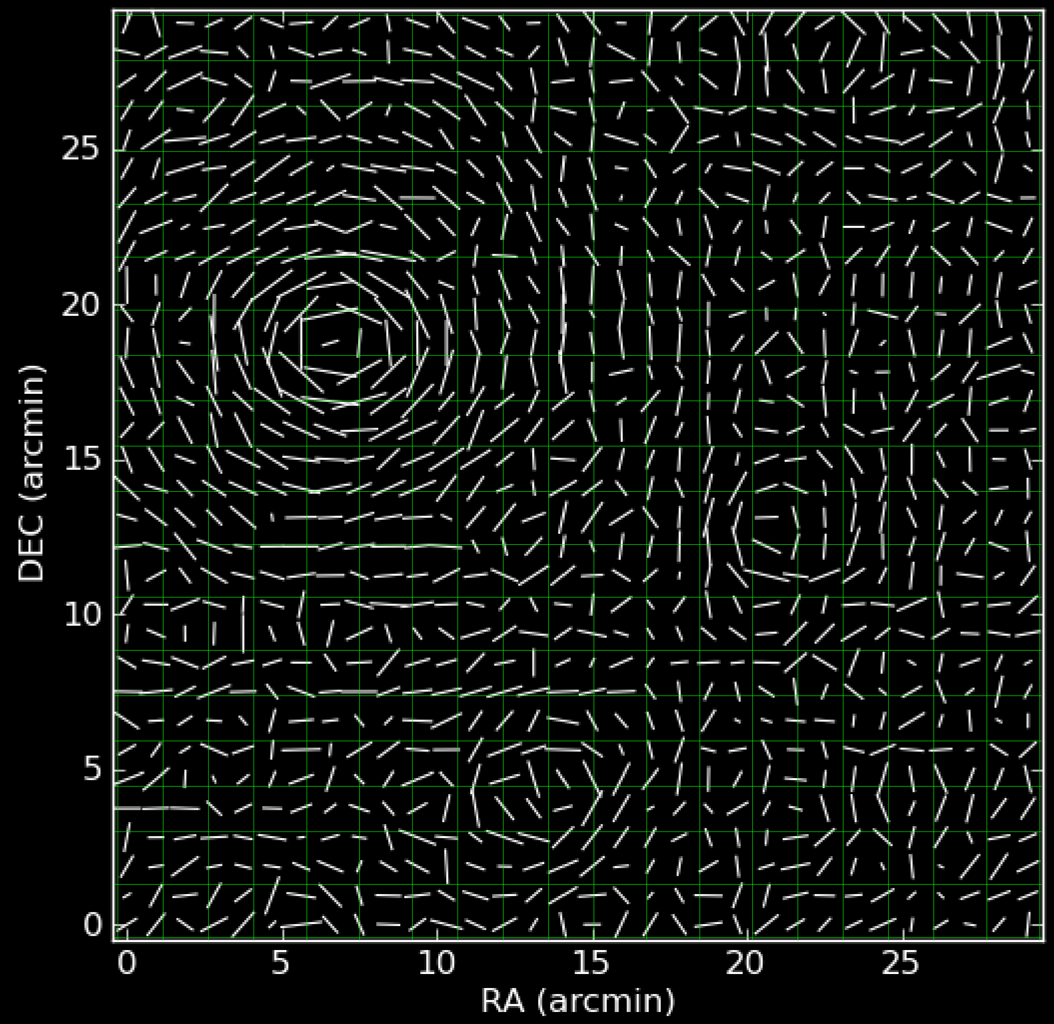
Use the *theoretical* 2-point correlation function to reconstruct the missing information



Our Solution: KL analysis

Use the *theoretical* 2-point correlation function to reconstruct the missing information

1. Pixelize the shear



Our Solution: KL analysis

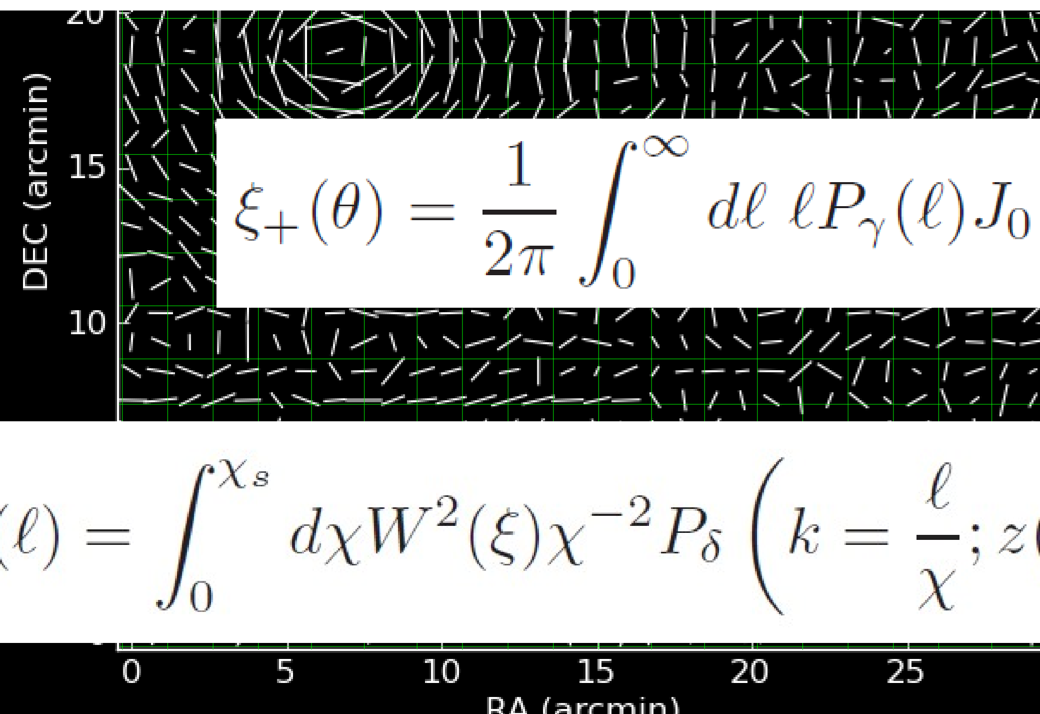
Use the *theoretical* 2-point correlation function to reconstruct the missing information

1. Pixelize the shear

2. Compute the correlation of the shear between pixels, using the expected nonlinear matter power spectrum.

(We use Smith *et al* 2003)

$$\boldsymbol{\xi}_{ij} = \langle \gamma_i \gamma_j^* \rangle + \langle n_i n_j^* \rangle$$
$$= \left[\int_{A_i} d^2x_i \int_{A_j} d^2x_j \xi_+(|\boldsymbol{x_i} - \boldsymbol{x_j}|) \right] + \delta_{ij} \frac{\sigma_\epsilon^2}{\bar{n}}$$



$$\xi_+(\theta) = \frac{1}{2\pi} \int_0^\infty d\ell \; \ell P_\gamma(\ell) J_0(\ell\theta)$$

$$P_\gamma(\ell) = \int_0^{\chi_s} d\chi W^2(\xi) \chi^{-2} P_\delta \left(k = \frac{\ell}{\chi}; z(\chi) \right)$$

KL Decomposition:

Now that we have the theoretical correlation matrix ξ, we can compute the KL basis via an eigenvalue decomposition:

$$\xi = \mathbf{\Psi \Lambda \Psi}^\dagger$$

These eigenvectors Ψ are an orthogonal basis, and give the *optimal* low-rank reconstruction of a shear vector.

$$\gamma = \mathbf{\Psi} \boldsymbol{a}$$

$$\boldsymbol{a} = \mathbf{\Psi}^\dagger \gamma$$

$$\hat{\gamma}^{(n)} \equiv \sum_{i=1}^{n<N} a_i \mathbf{\Psi}_i$$

KL Decomposition:

We can use the unmasked region to constrain the coefficients

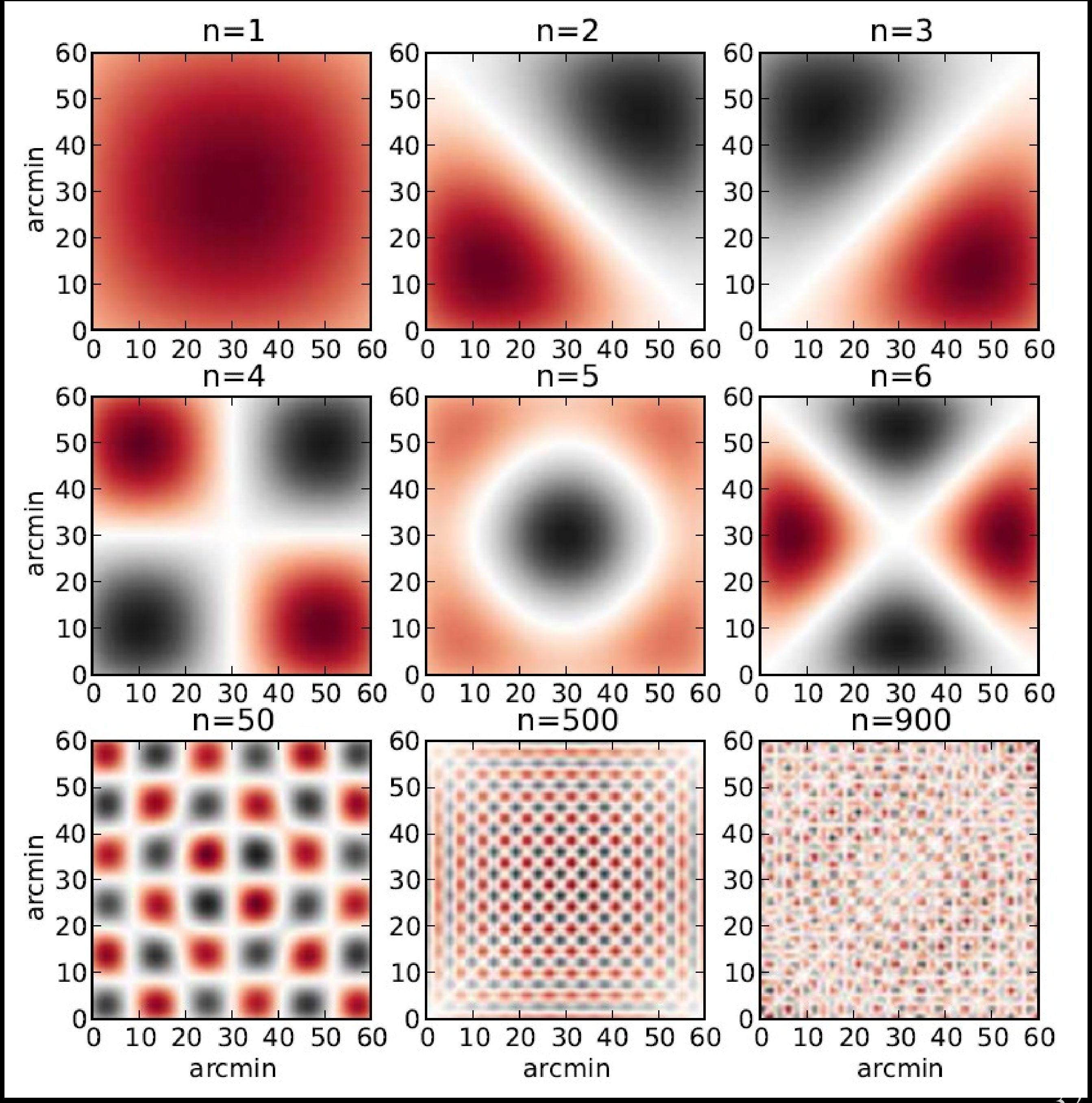
$$\hat{\boldsymbol{a}}_{(n)} = \boldsymbol{M}_{(n)}^{-1} \boldsymbol{\Psi}_{(n)}^{\dagger} \boldsymbol{W} \boldsymbol{\mathcal{N}}_{\gamma}^{-1/2} \gamma^{o}$$

$$\boldsymbol{M}_{(n)} \equiv \boldsymbol{\Psi}_{(n)}^{\dagger} \boldsymbol{W} \boldsymbol{\Psi}_{(n)}$$

These coefficients can then be used to estimate the masked-out shear, and additionally filter noise from the entire field

$$\hat{\gamma}^{(n)} = \boldsymbol{\mathcal{N}}_{\gamma}^{1/2} \boldsymbol{\Psi}_{(n)} \hat{\boldsymbol{a}}_{(n)}$$

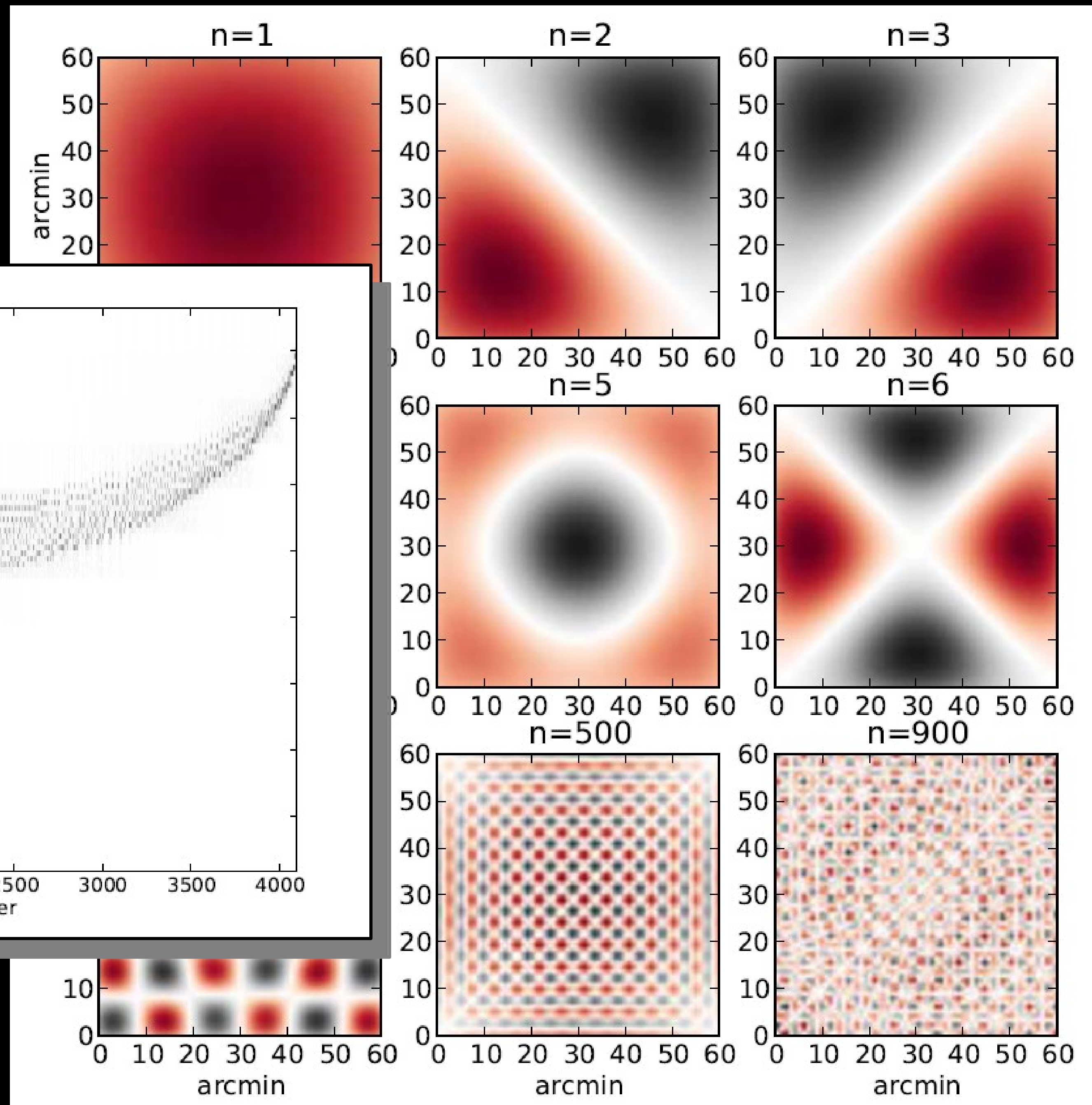
KL Eigenmodes

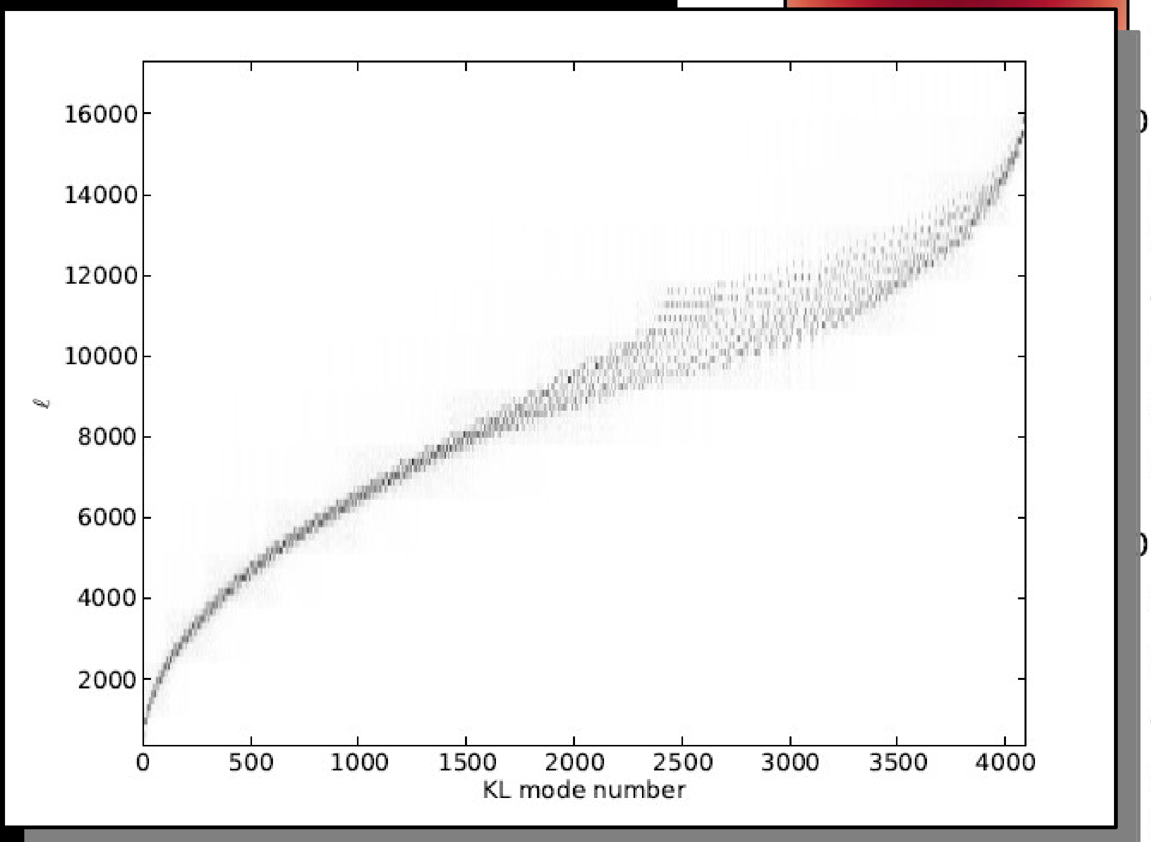


Vanderplas *et al. (in prep)*

KL Eigenmodes

Similar to Fourier modes

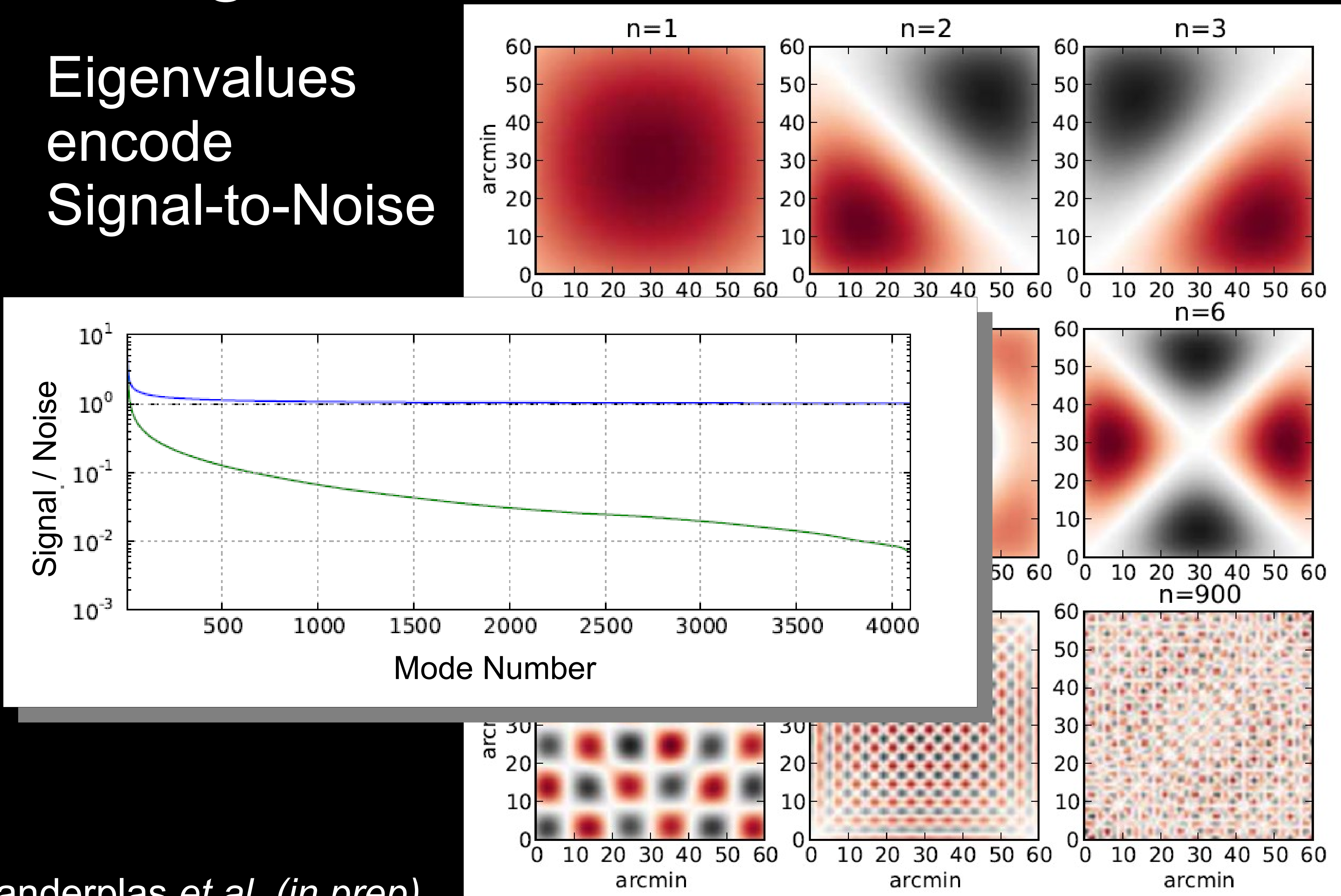




Vanderplas *et al. (in prep)*

KL Eigenmodes

Eigenvalues encode Signal-to-Noise



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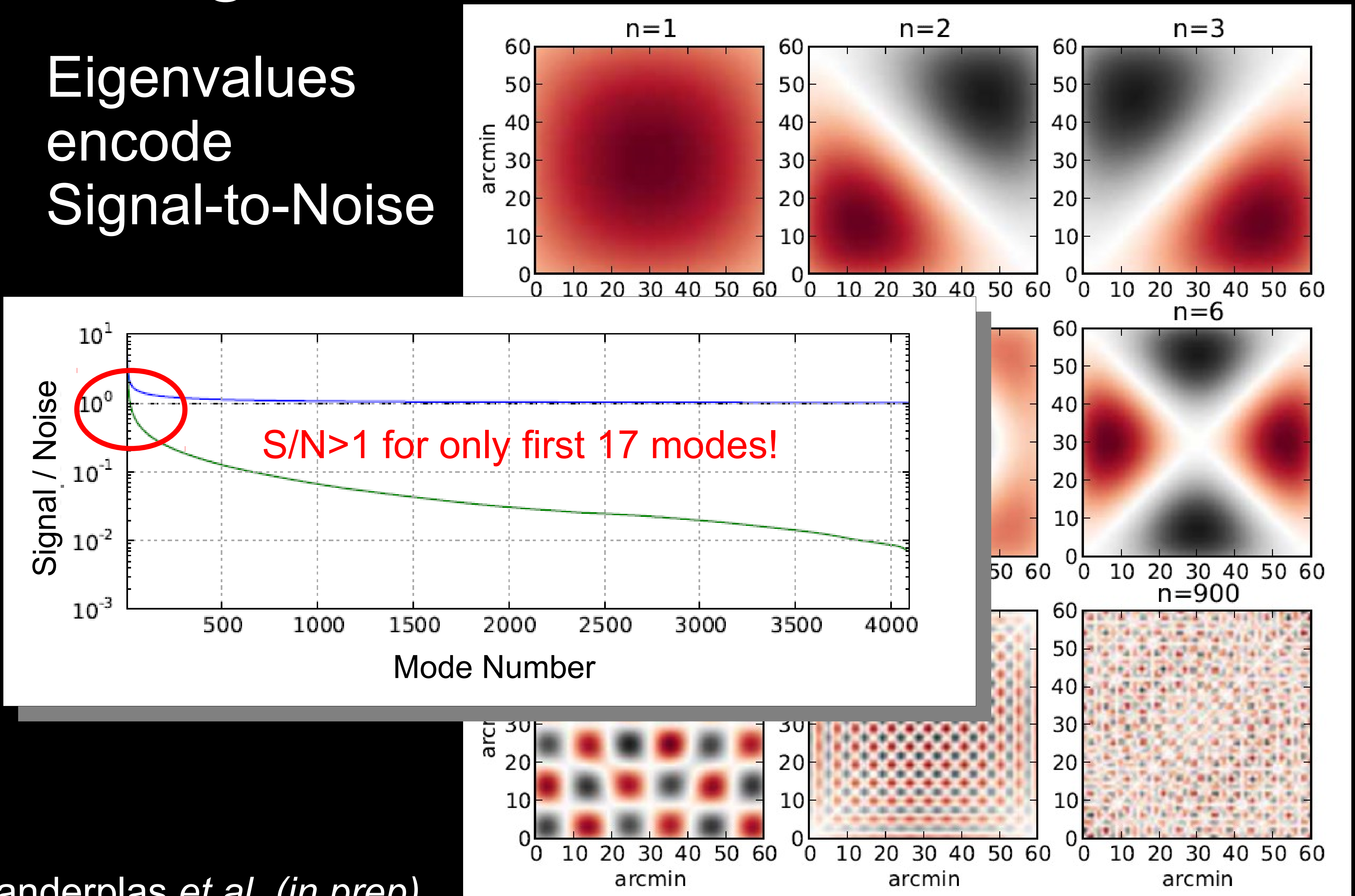

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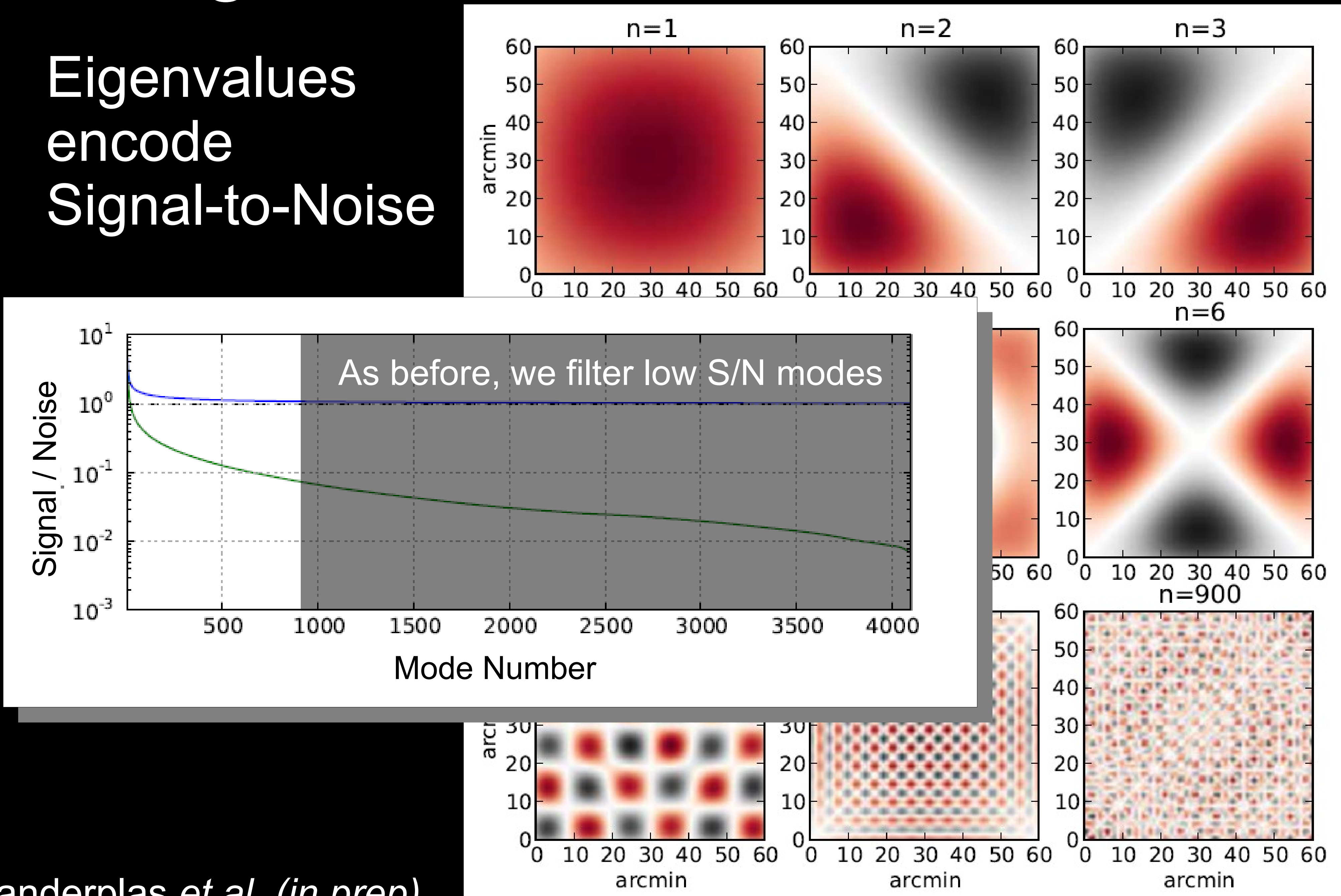
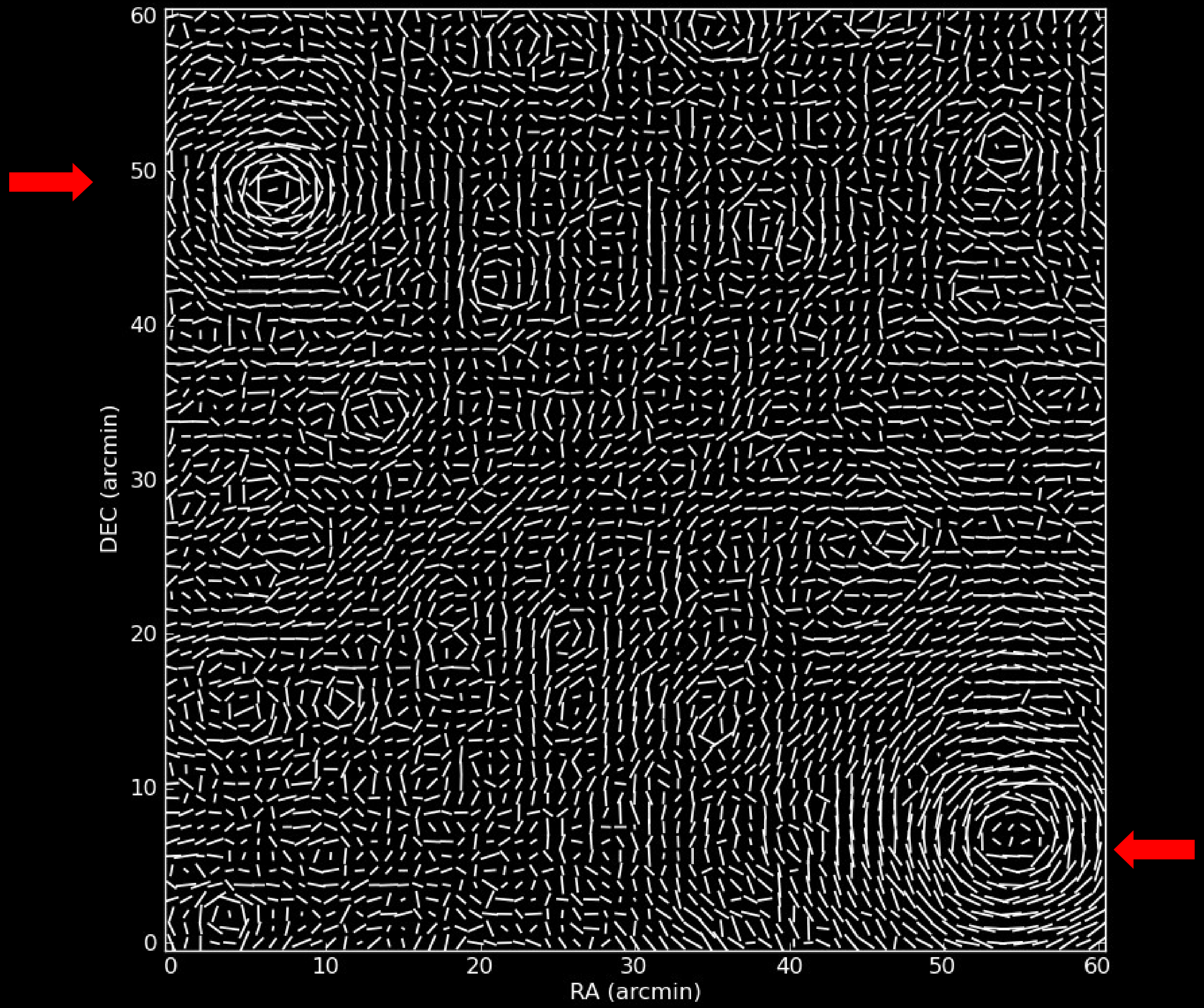


Vanderplas *et al. (in prep)*

Noiseless Shear

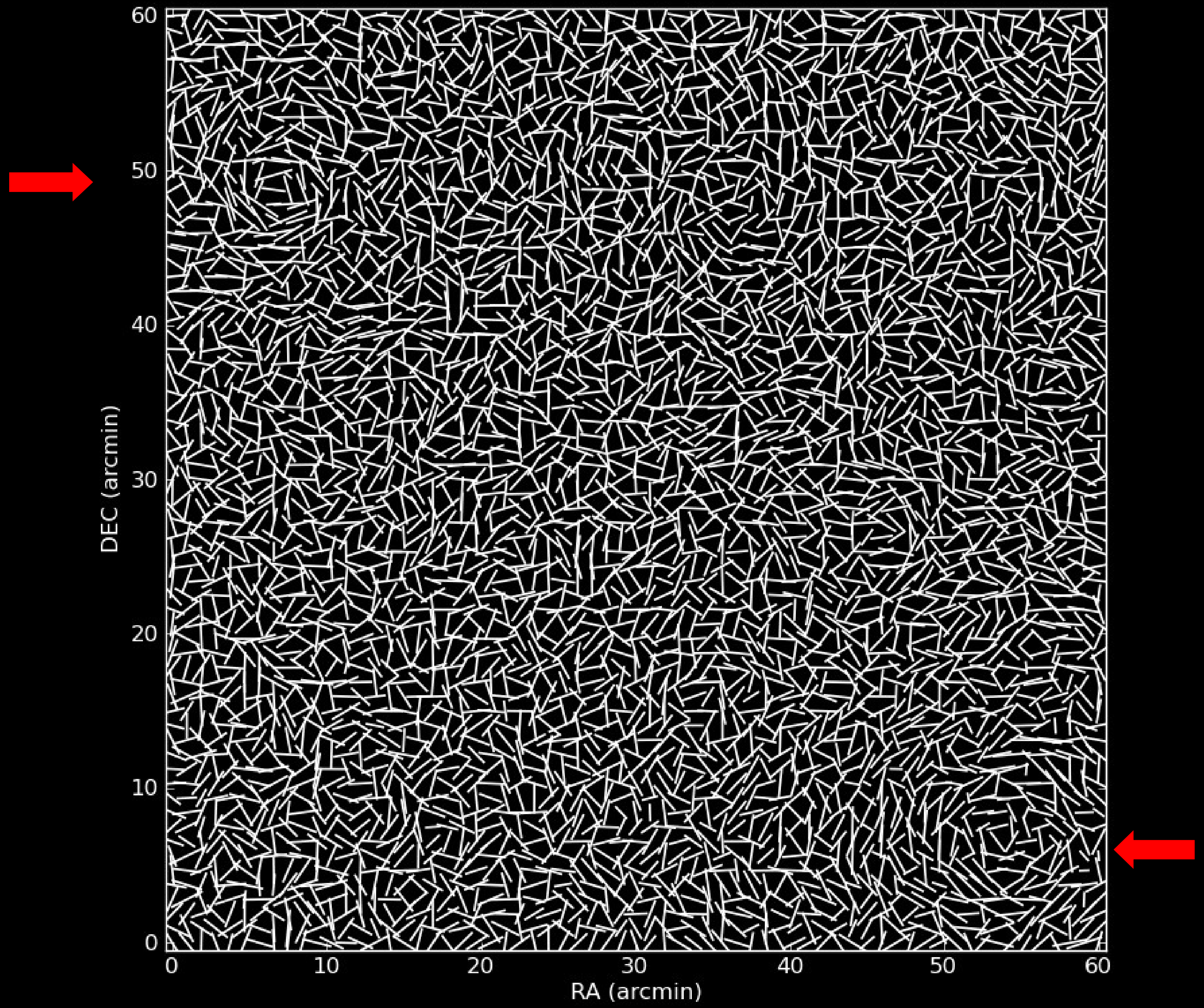
1 square degree, 64x64 pixels



Sims: courtesy of Risa Wechsler, Michael Busha, Matt Becker and Joerg Dietrich

Noisy Shear

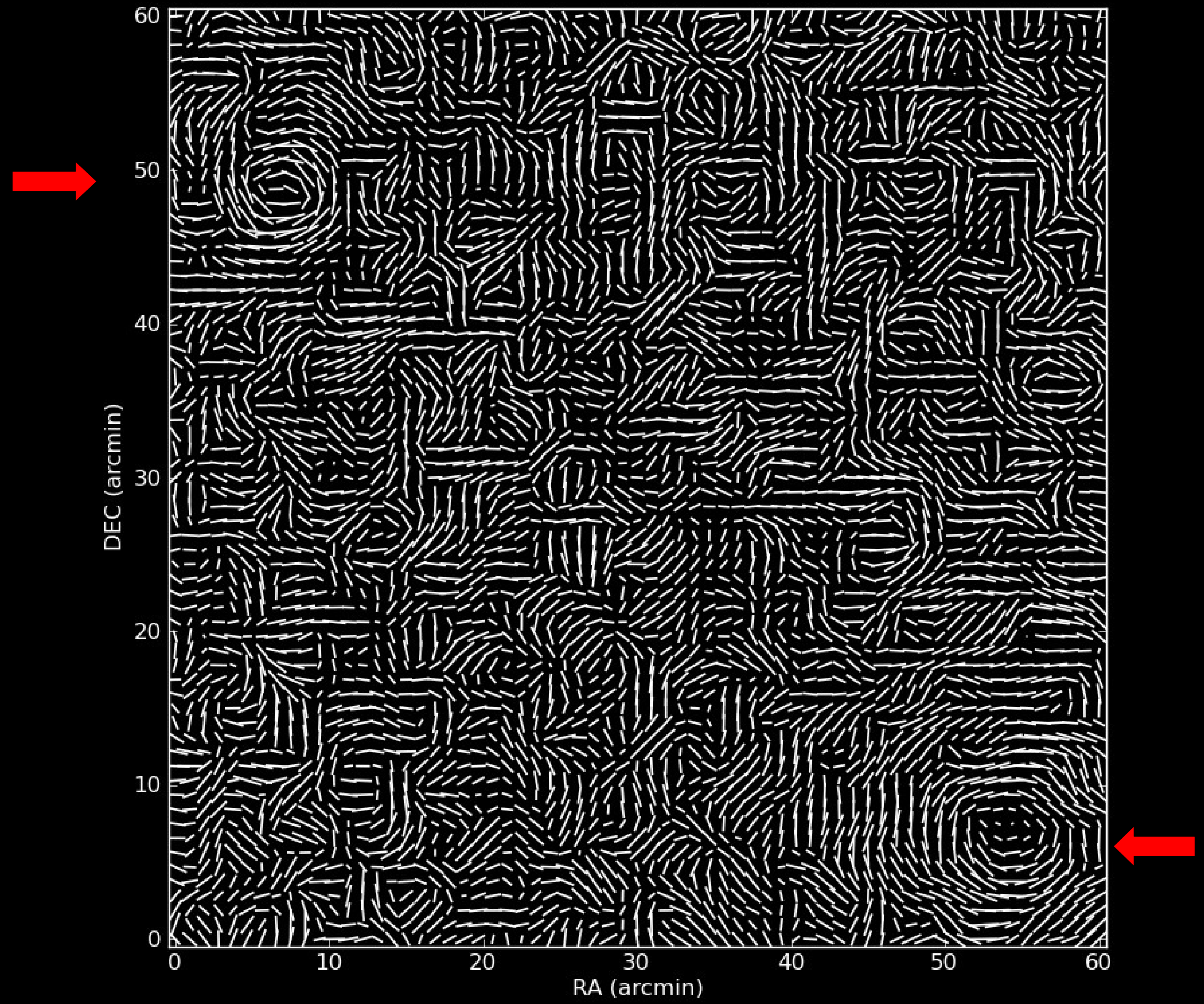
1 square degree, 64x64 pixels



Sims: courtesy of Risa Wechsler, Michael Busha, Matt Becker and Joerg Dietrich

900/4096 KL modes, unmasked

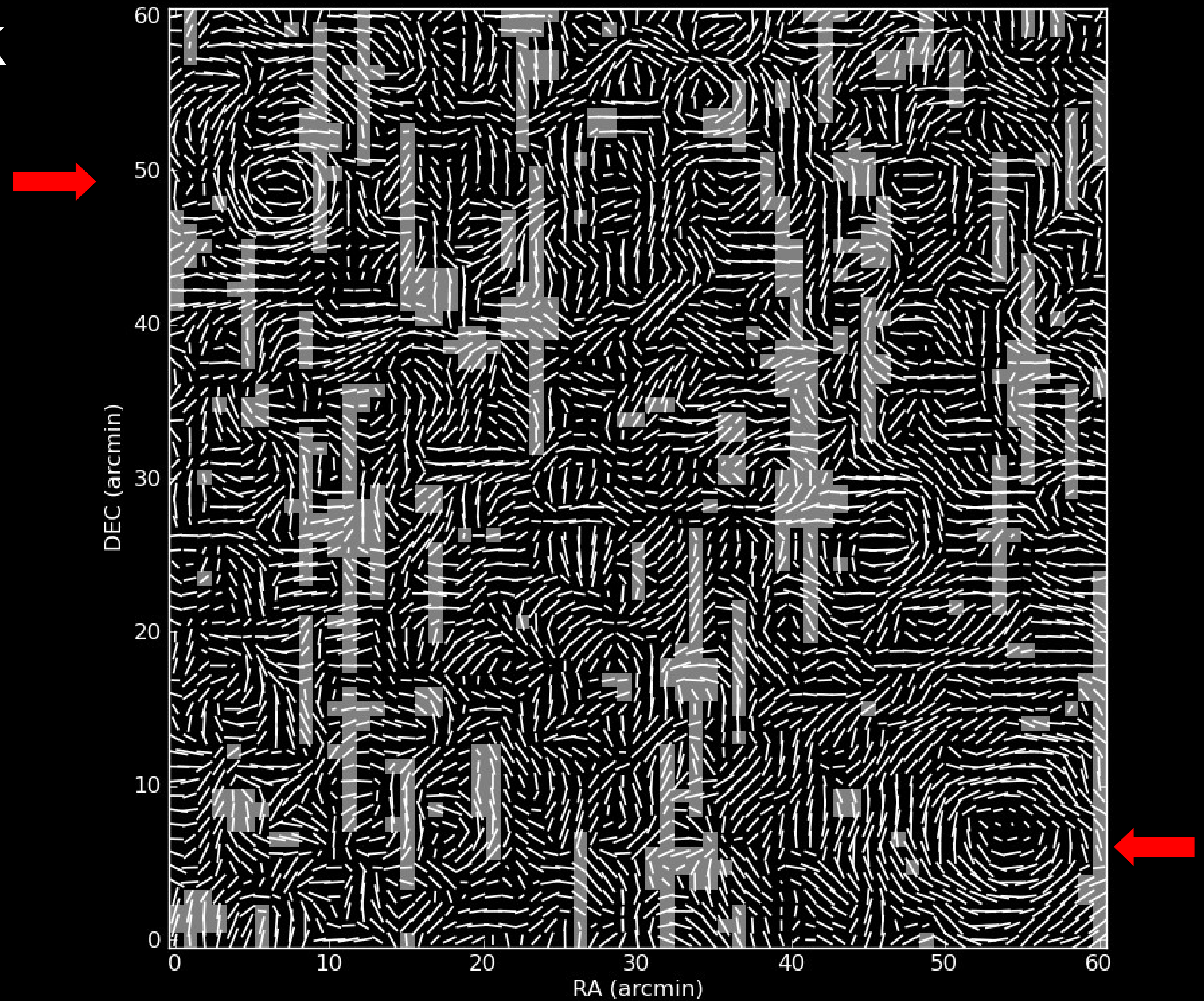
1 square degree, 64x64 pixels



Sims: courtesy of Risa Wechsler, Michael Busha, Matt Becker and Joerg Dietrich

900/4096 KL modes, 20% mask

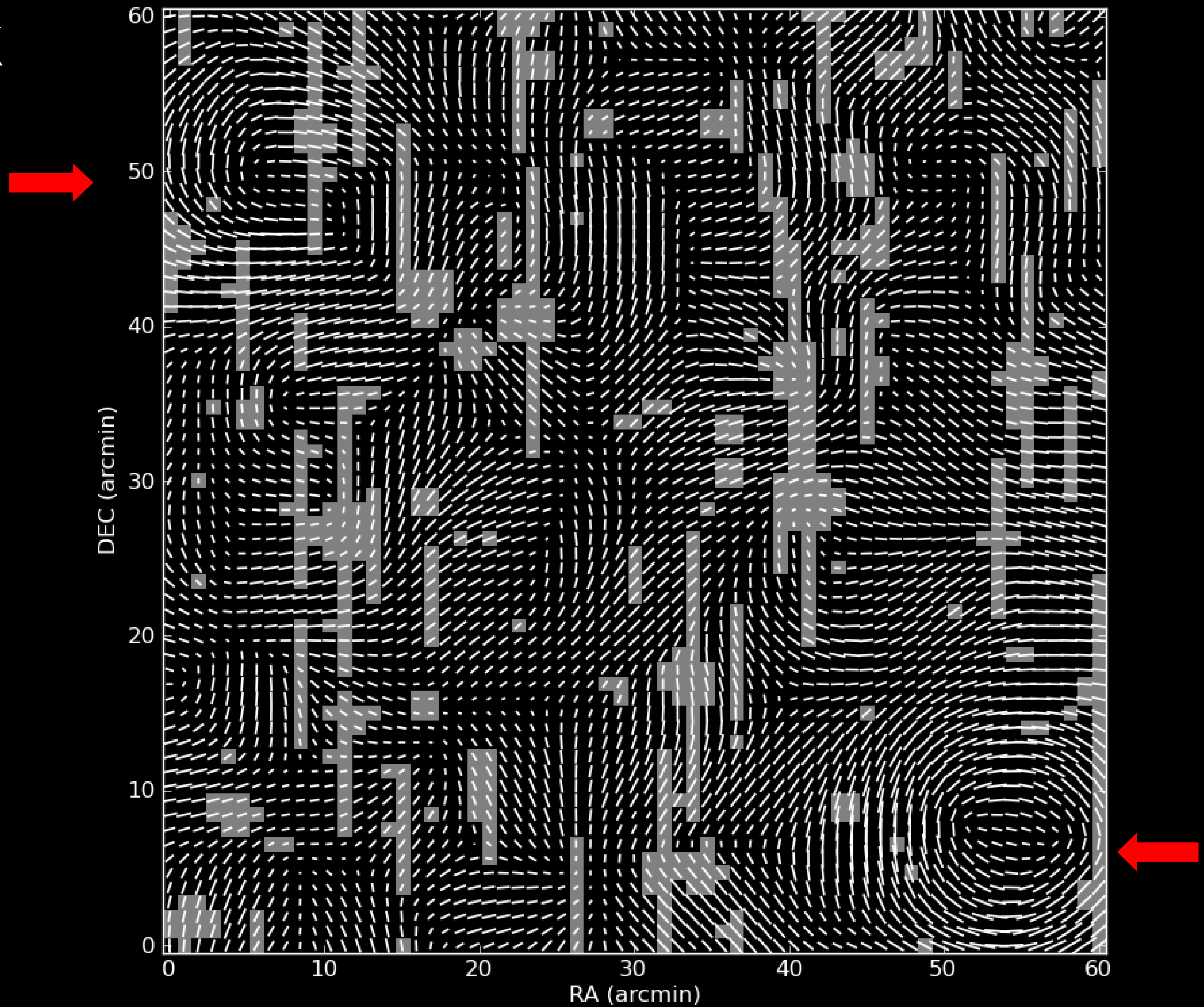
1 square degree, 64x64 pixels



Sims: courtesy of Risa Wechsler, Michael Busha, Matt Becker and Joerg Dietrich

100/4096 KL modes, 20% mask

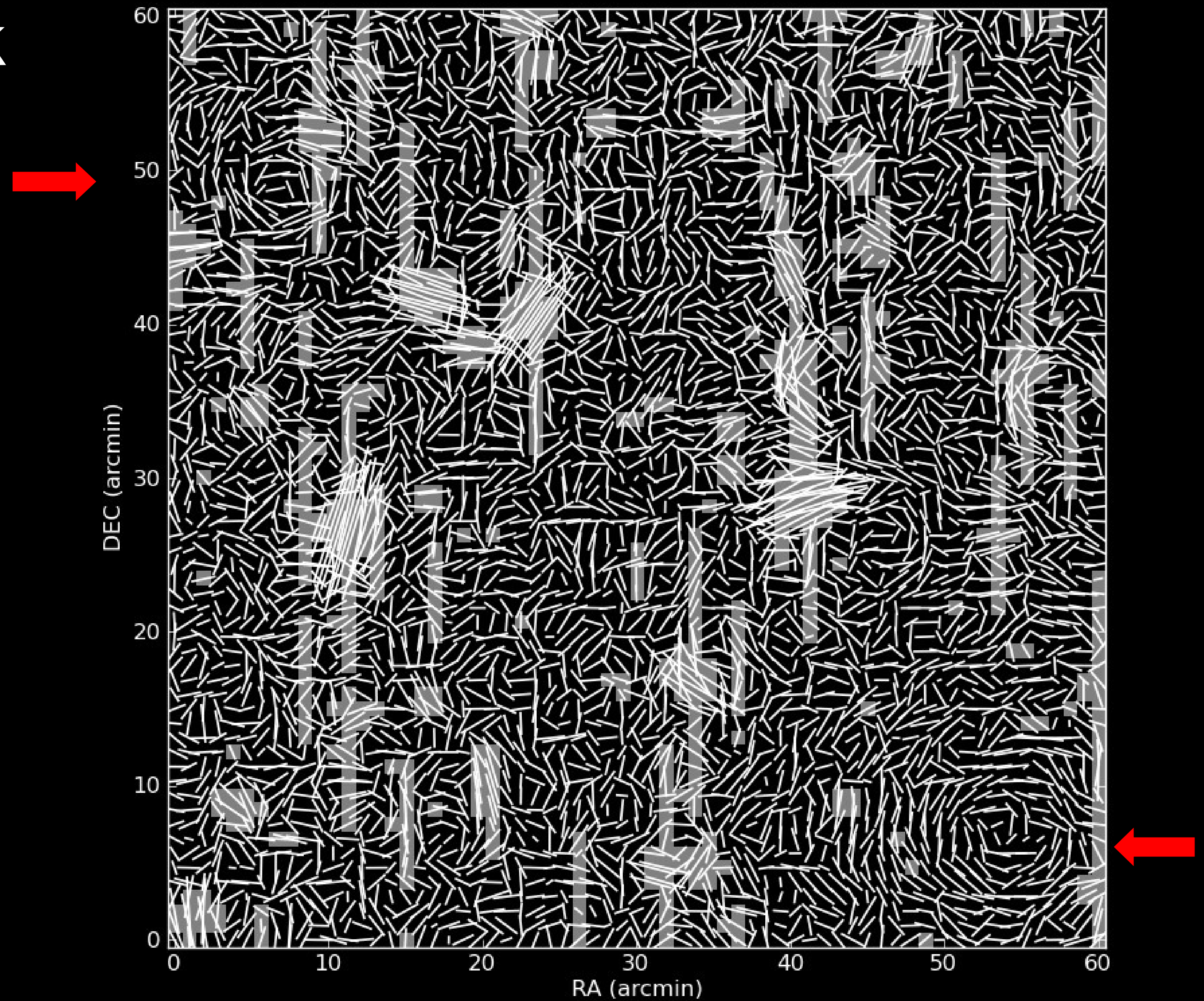
1 square degree, 64x64 pixels



Sims: courtesy of Risa Wechsler, Michael Busha, Matt Becker and Joeg Dietrich

2000/4096 KL modes, 20% mask

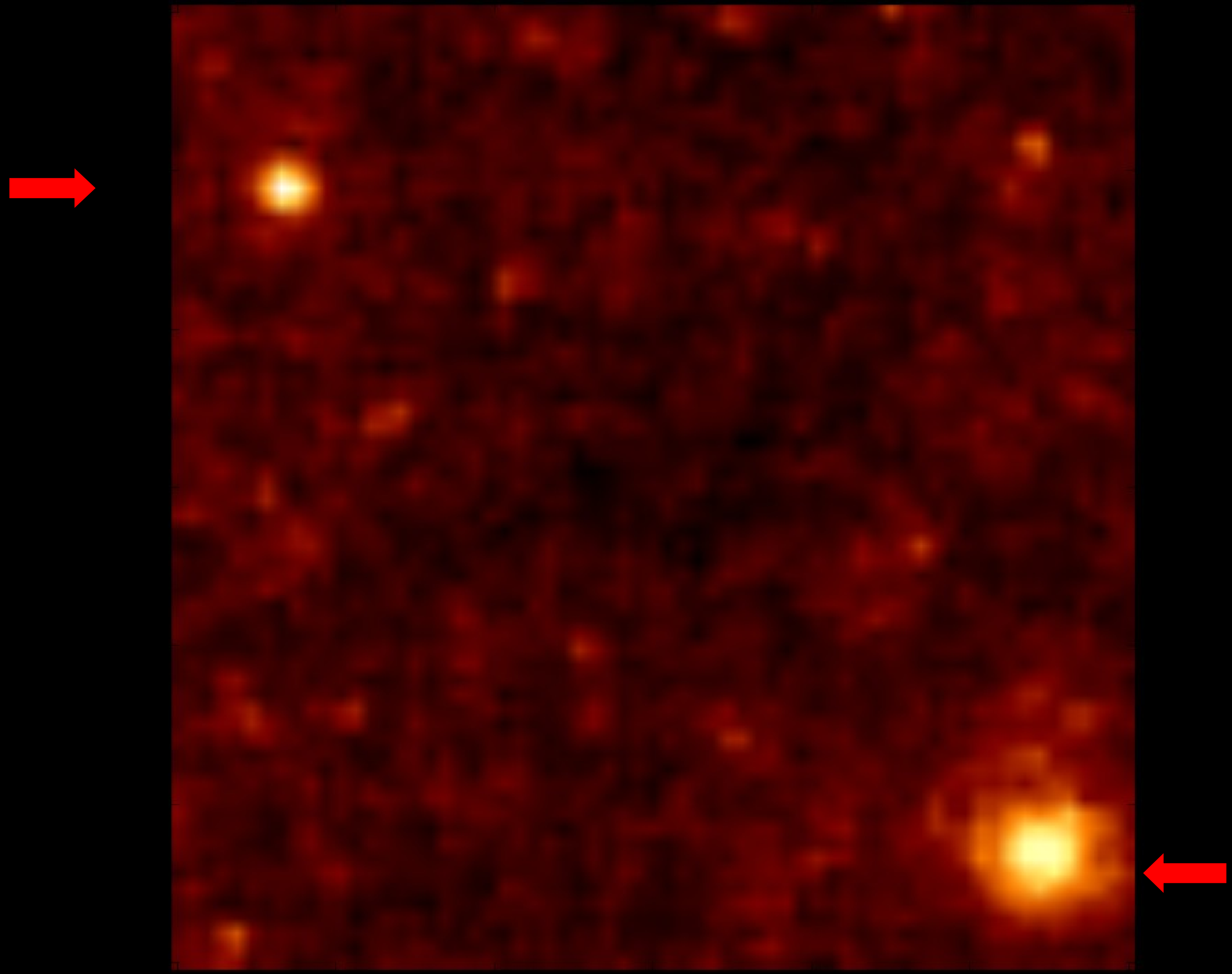
1 square degree, 64x64 pixels



Sims: courtesy of Risa Wechsler, Michael Busha, Matt Becker and Joerg Dietrich

Convergence Map: noiseless

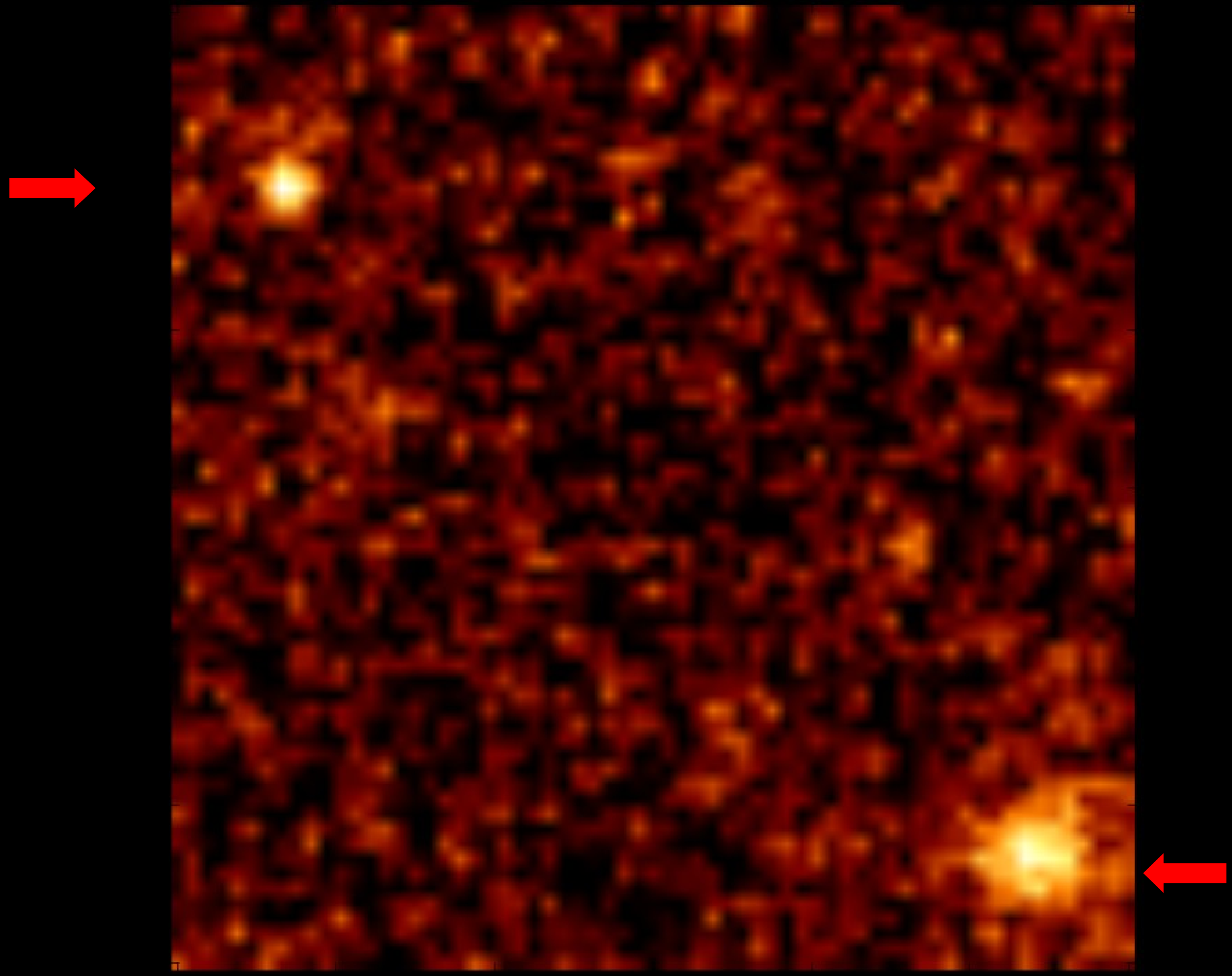
1 square degree, 64x64 pixels



Sims: courtesy of Risa Wechsler, Michael Busha, Matt Becker and Joerg Dietrich

Convergence Map: noisy

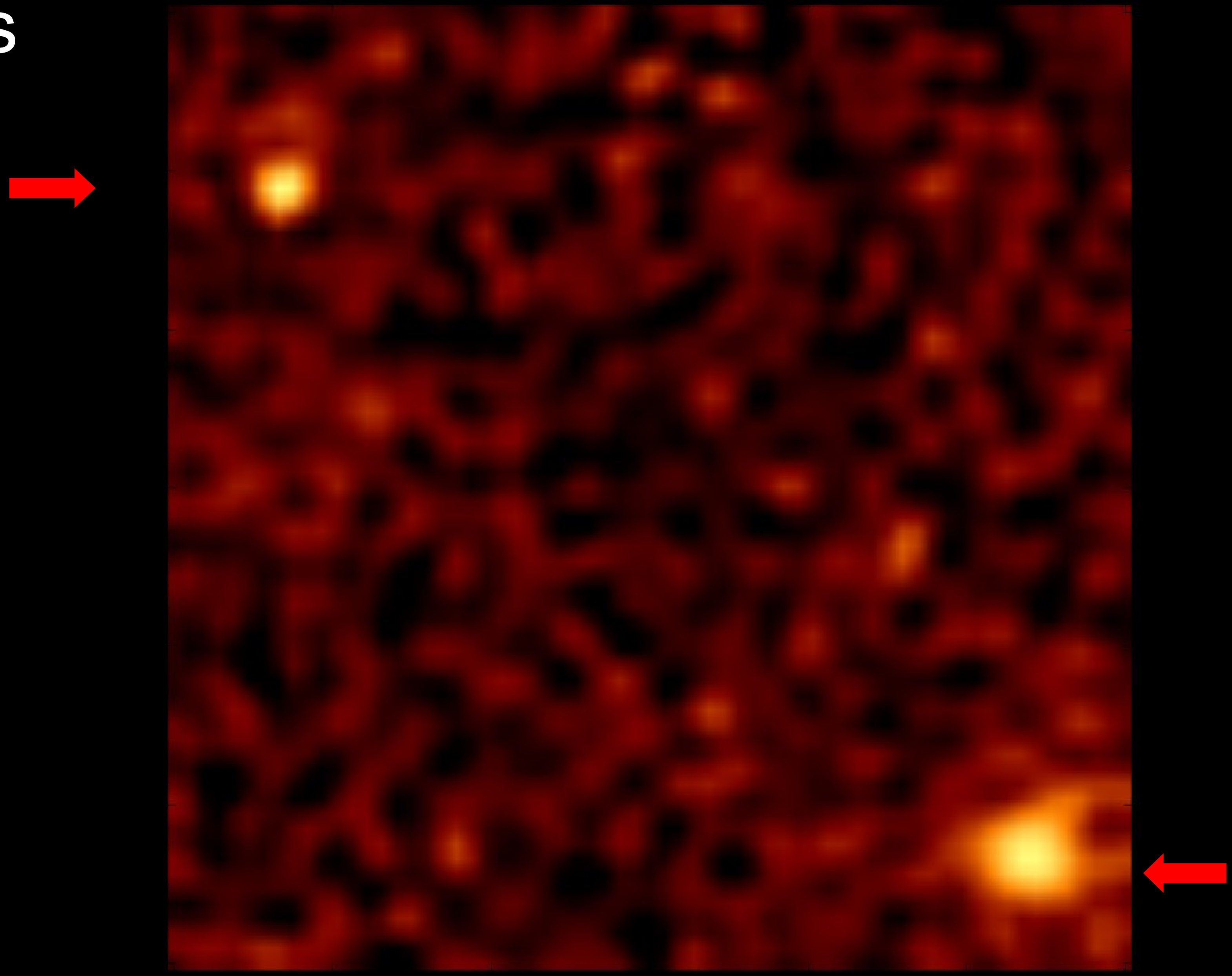
1 square degree, 64x64 pixels



Sims: courtesy of Risa Wechsler, Michael Busha, Matt Becker and Joerg Dietrich

Convergence Map: 900 modes

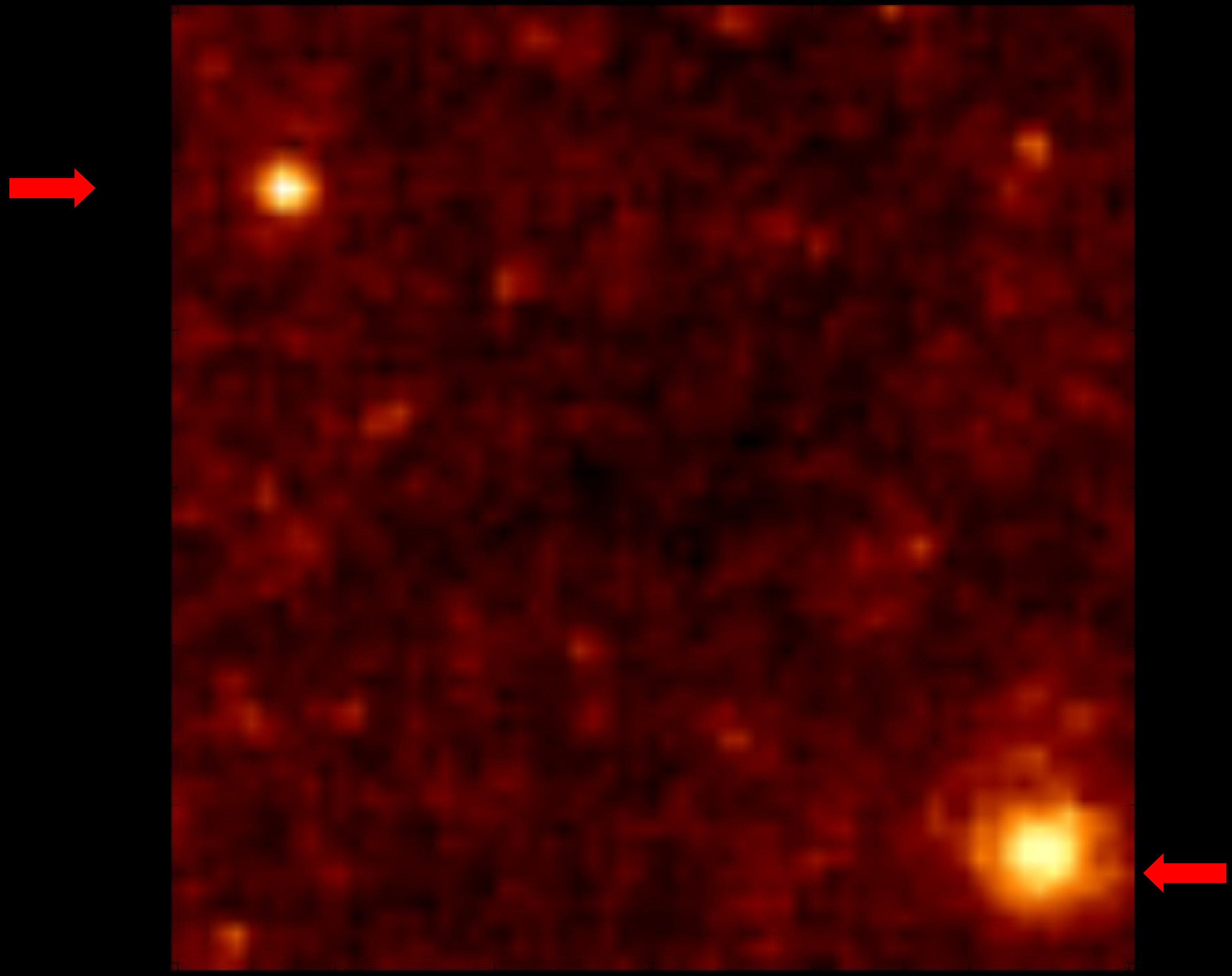
1 square degree, 64x64 pixels



Sims: courtesy of Risa Wechsler, Michael Busha, Matt Becker and Joerg Dietrich

Convergence Map: noiseless

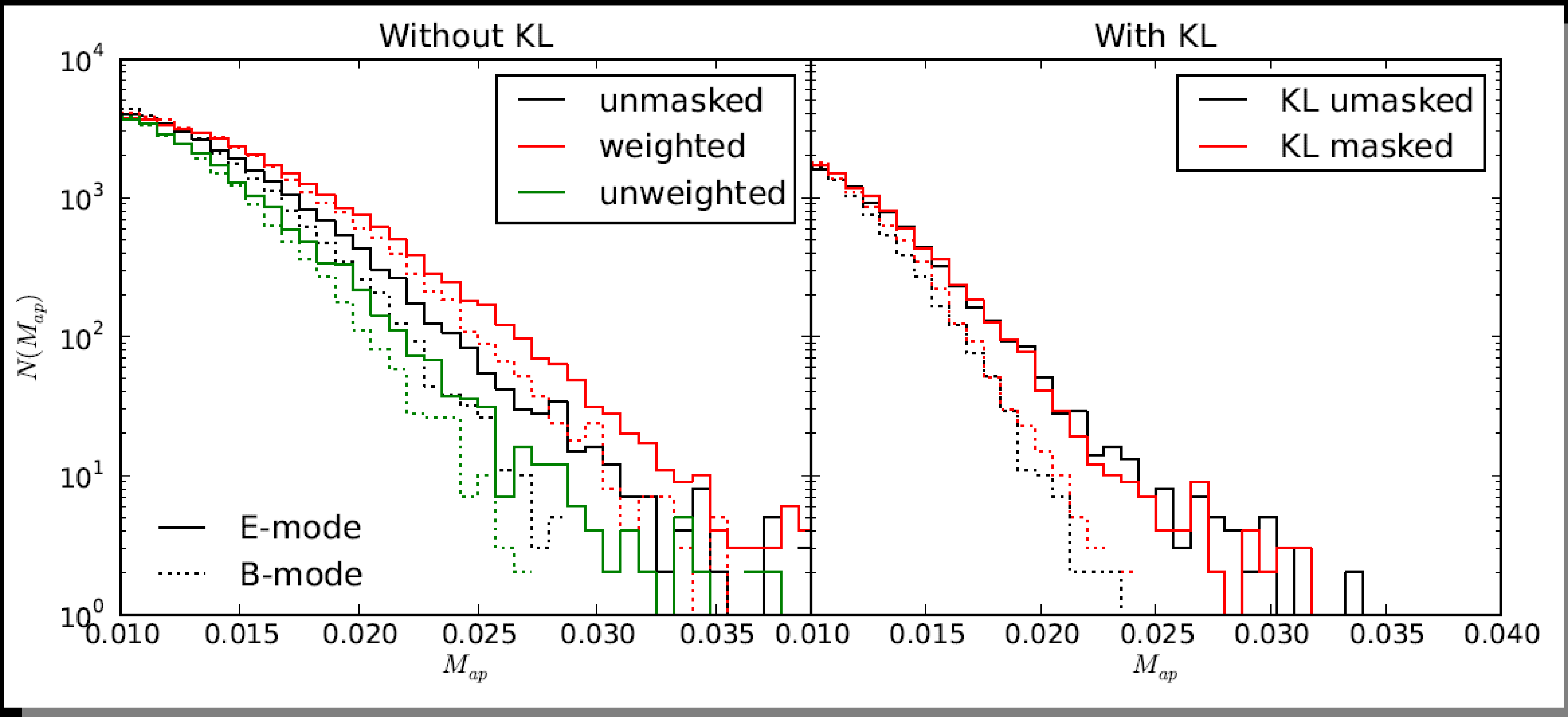
1 square degree, 64x64 pixels



Sims: courtesy of Risa Wechsler, Michael Busha, Matt Becker and Joerg Dietrich

Quantitative comparison:

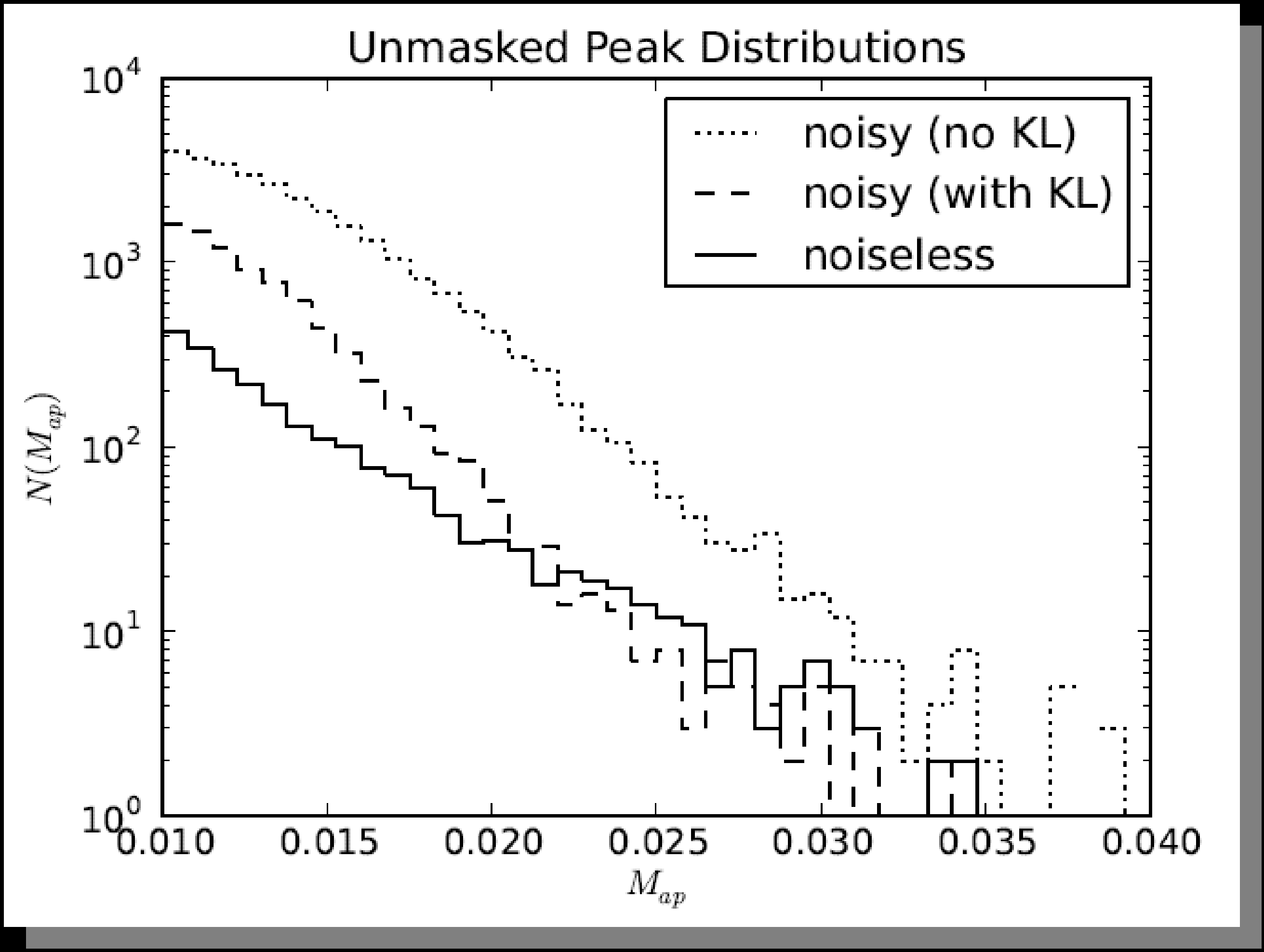
Aperture mass peak distribution



KL recovers the unmasked peak distribution

Quantitative comparison:

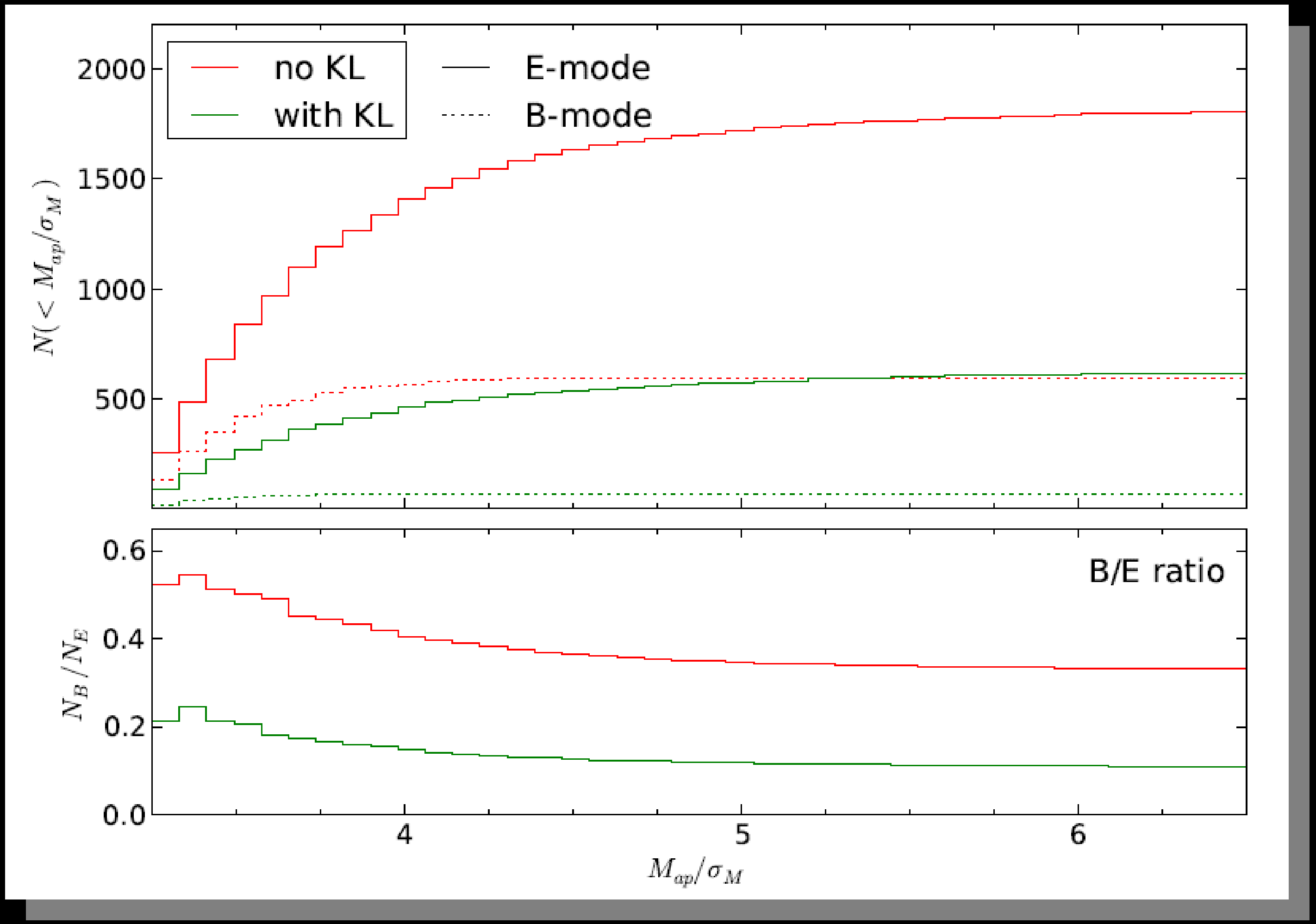
Aperture mass peak distribution



Addition of noise adds a factor of ~3 more peaks.
KL filtering reduces this number.

Quantitative comparison:

Aperture mass peak distribution



KL preferentially filters B-modes, leading to a factor of 3 relative reduction

Quantitative comparison:

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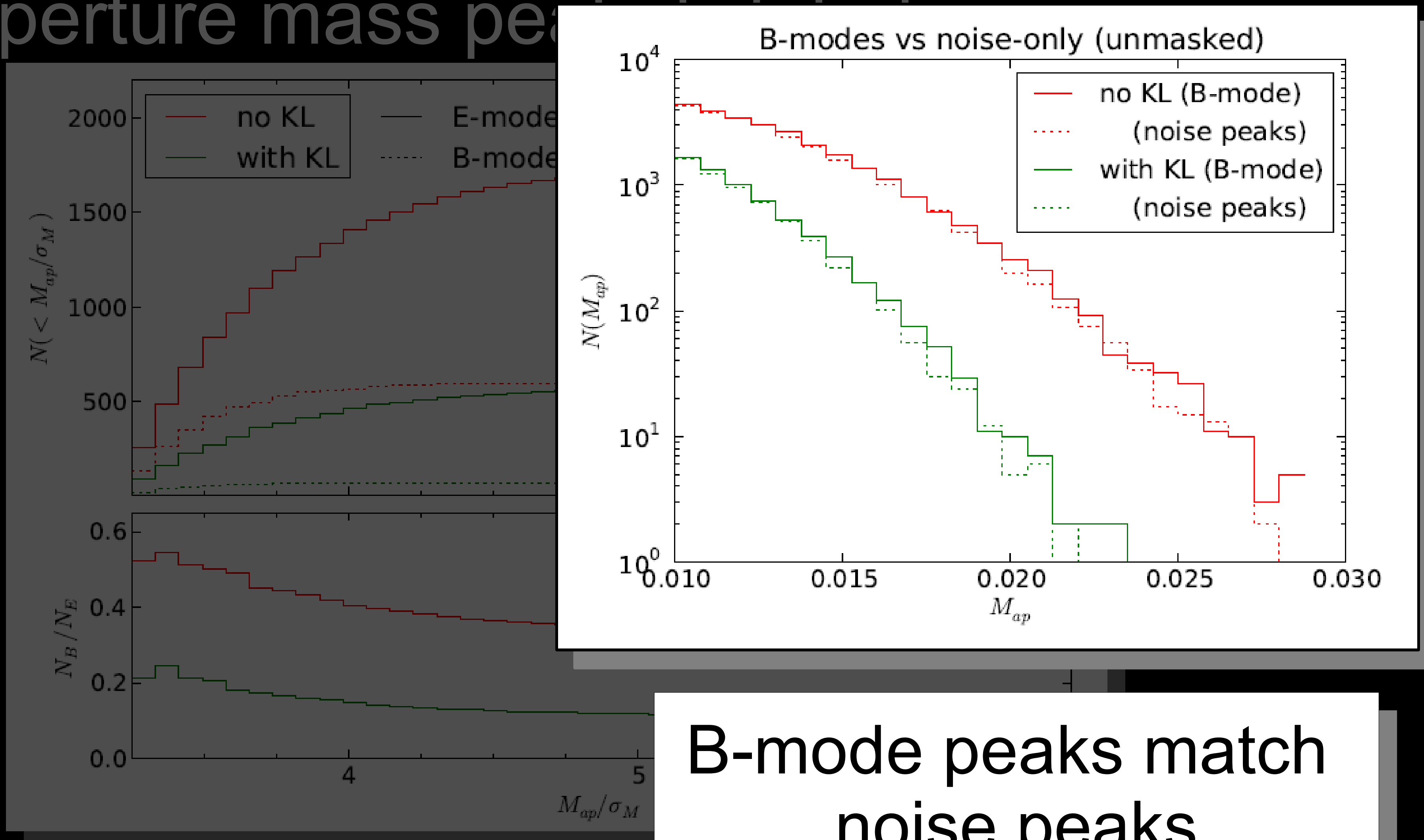


B-mode peaks match noise peaks

KL preferentially filters B-modes, leading to a factor of 3 relative reduction

Conclusions

Using statistics of shear peaks can evade some pitfalls of cluster cosmology.

The KL method robustly interpolates between masked regions of the shear field.

The KL method reduces the statistical error in the peak function: measured by total number of peaks, and B/E ratio.

This suggests that the KL method could improve cosmological constraints from shear peaks: further study is needed to quantify this.

Very Briefly...

Modified Gravity and Dwarf Galaxies

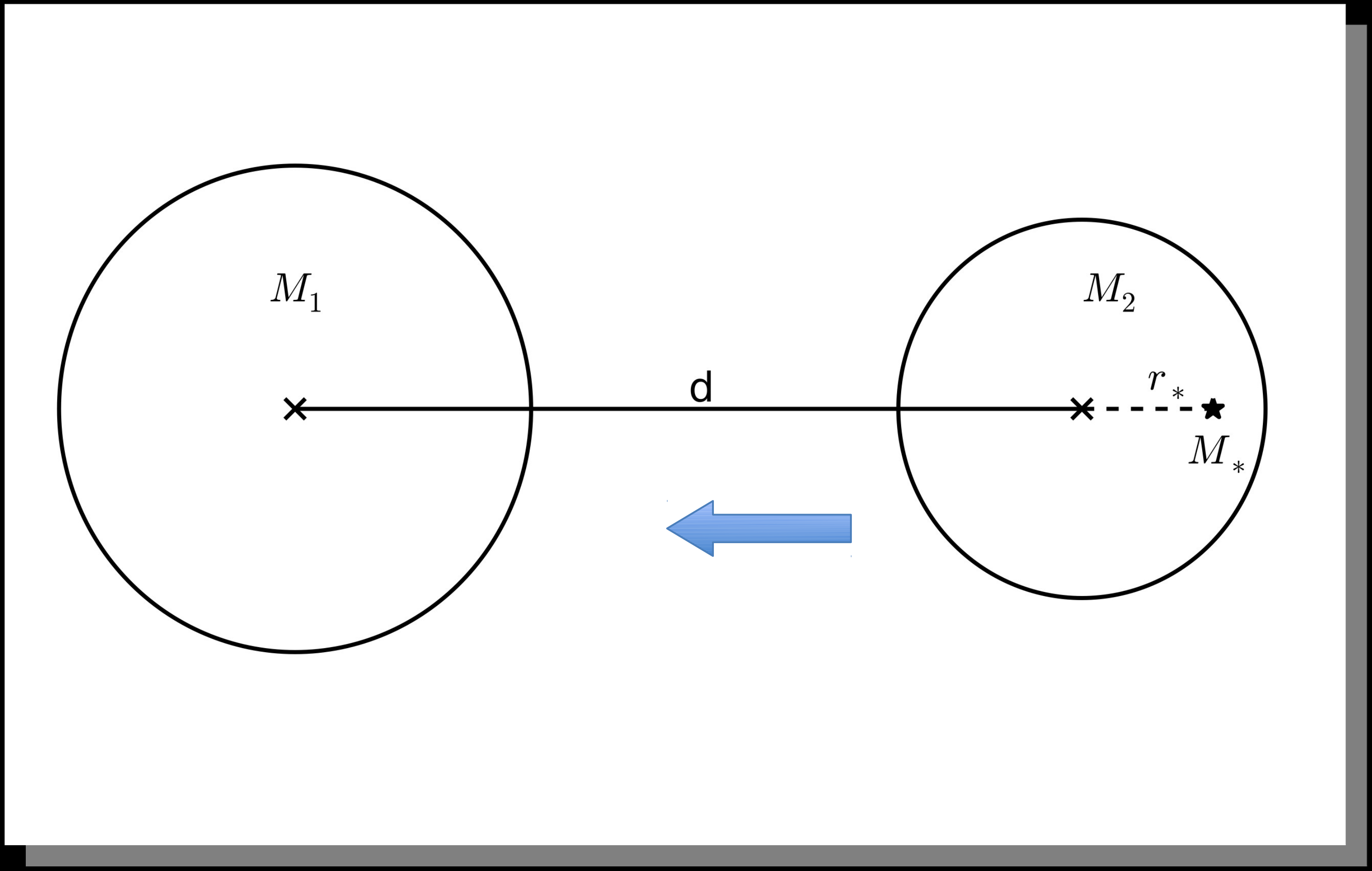
(*preliminary* results of work over the last ~2 weeks with Bhuvnesh Jain)

Many modified gravity theories [e.g. f(R)] involve scalar fields that provide an attractive, fifth-force

BUT… GR must be restored in the Milky Way - via "natural" mechanisms that work for massive/dense objects.
(*Khoury & Weltman 2004; Vainshtein 1972)*

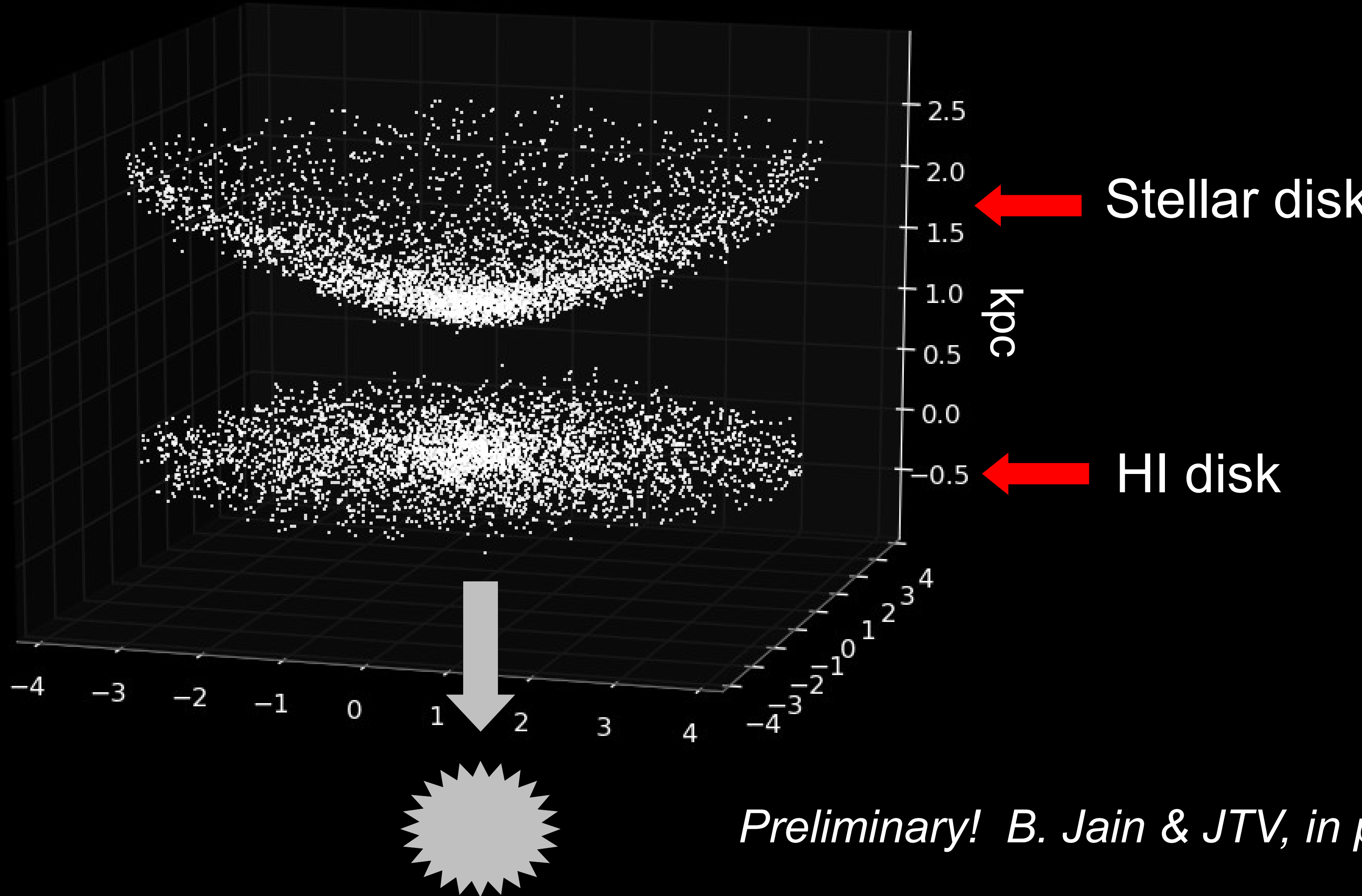
So small galaxies or the outer regions of big galaxy/cluster halos may show deviations from GR.
(Kesden & Kamionkowski 2006; Hui et al 2009; Chang & Hui 2011; Davis et al 2011)

The Situation: Colliding dwarf galaxies:



Preliminary! B. Jain & JTV, in prep

- Unscreened HI disk tracks dark matter
- Self-screened stellar disk can be *offset* and *distorted*



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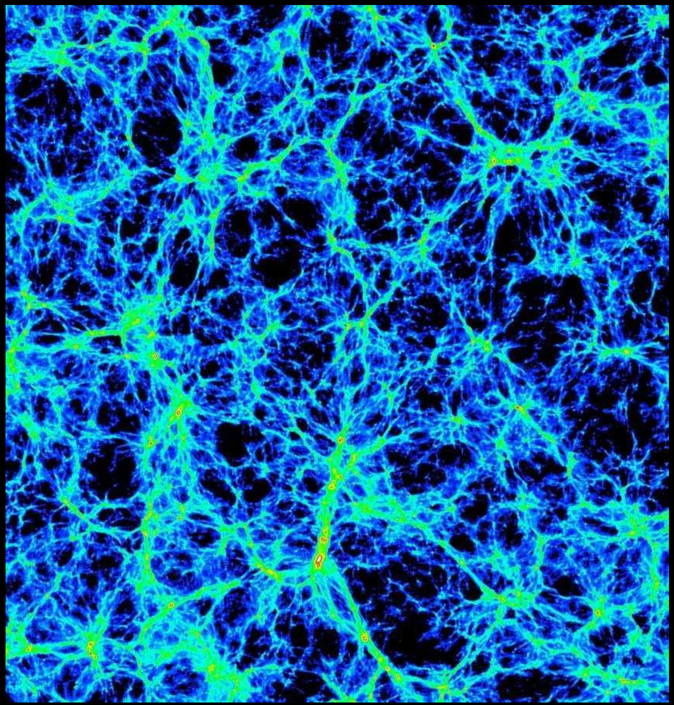
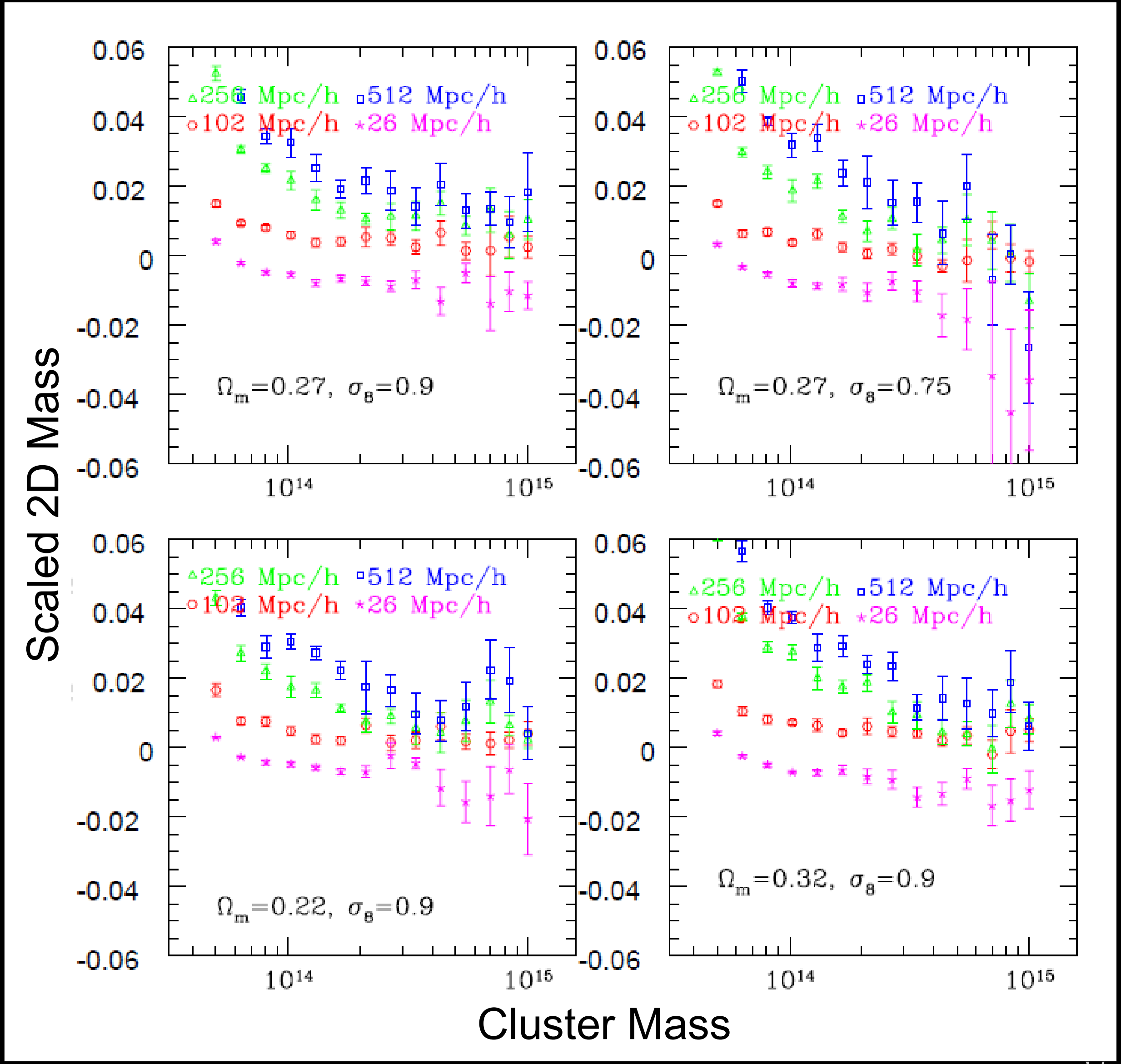
Shear Peak Statistics

Followup: Marian *et al.* 2010

Slab Thickness:
206 Mpc/h 512 Mpc/h
102 Mpc/h 26 Mpc/h

"Correlated Projections" can affect the projected mass





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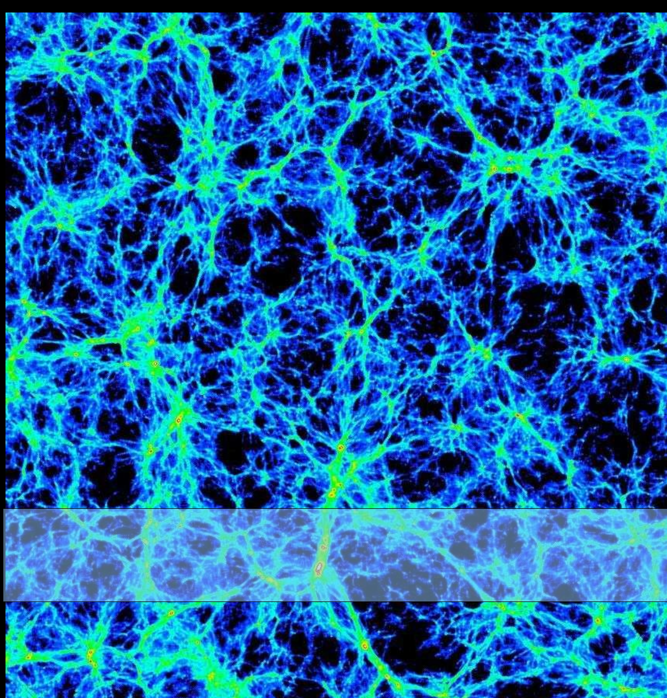

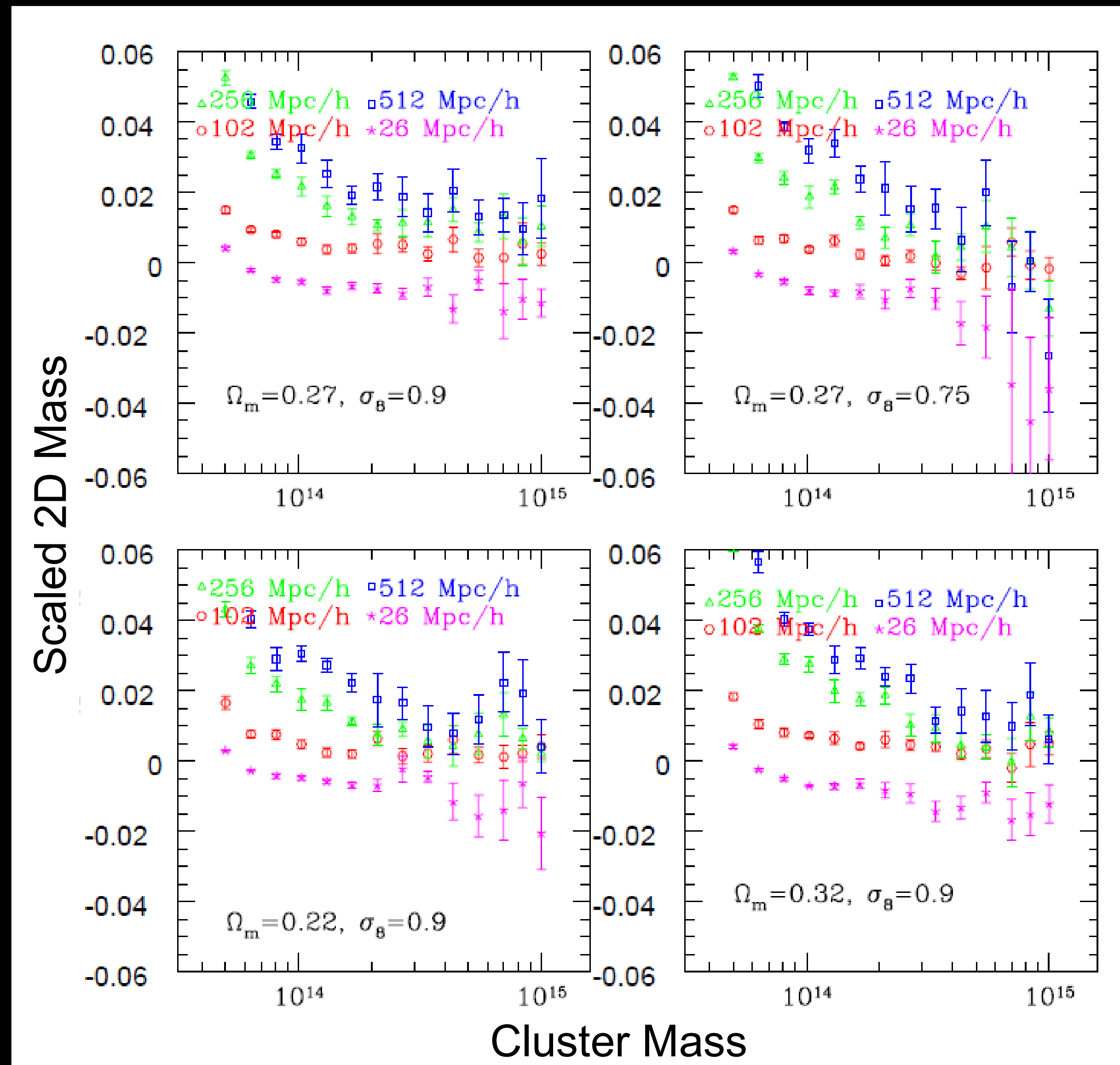

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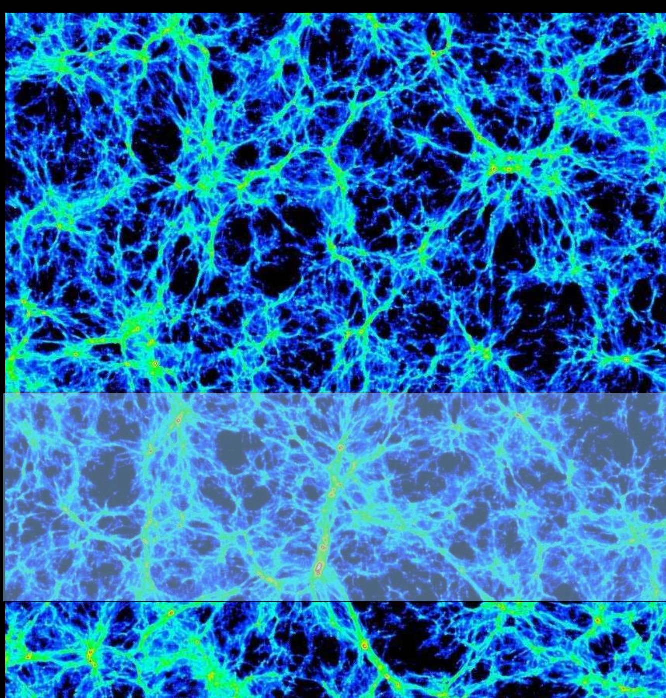
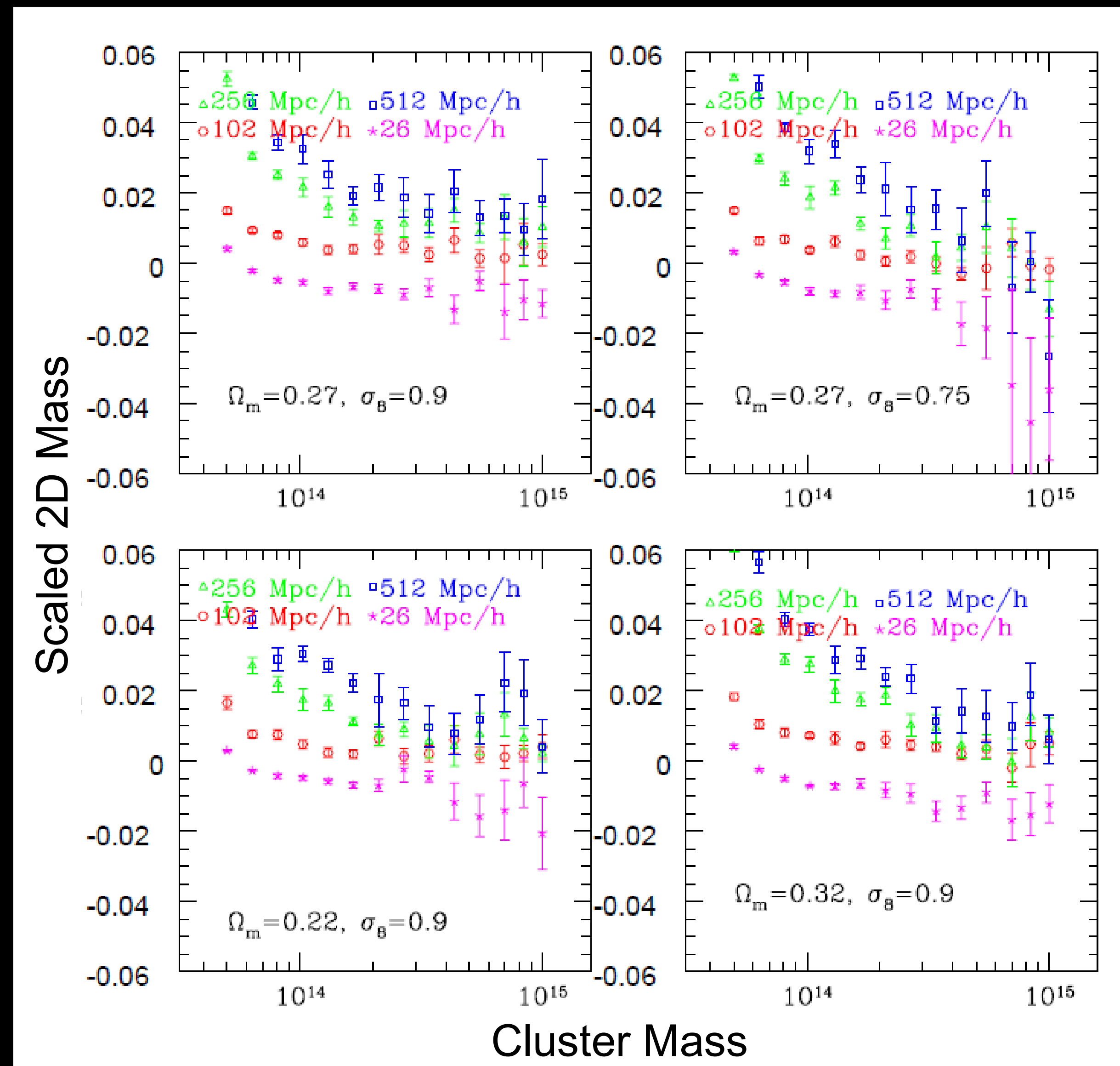
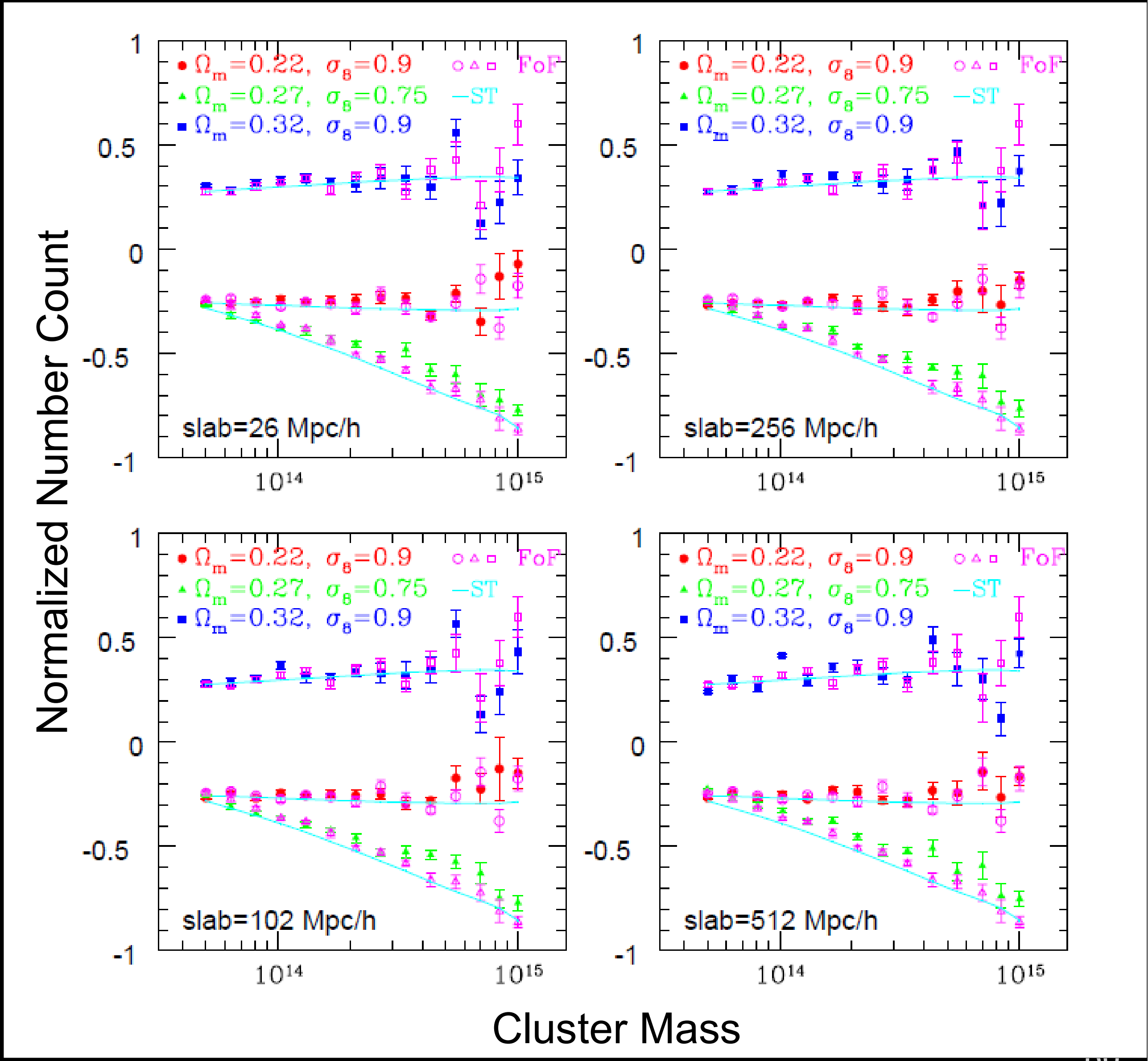




Shear Peak Statistics

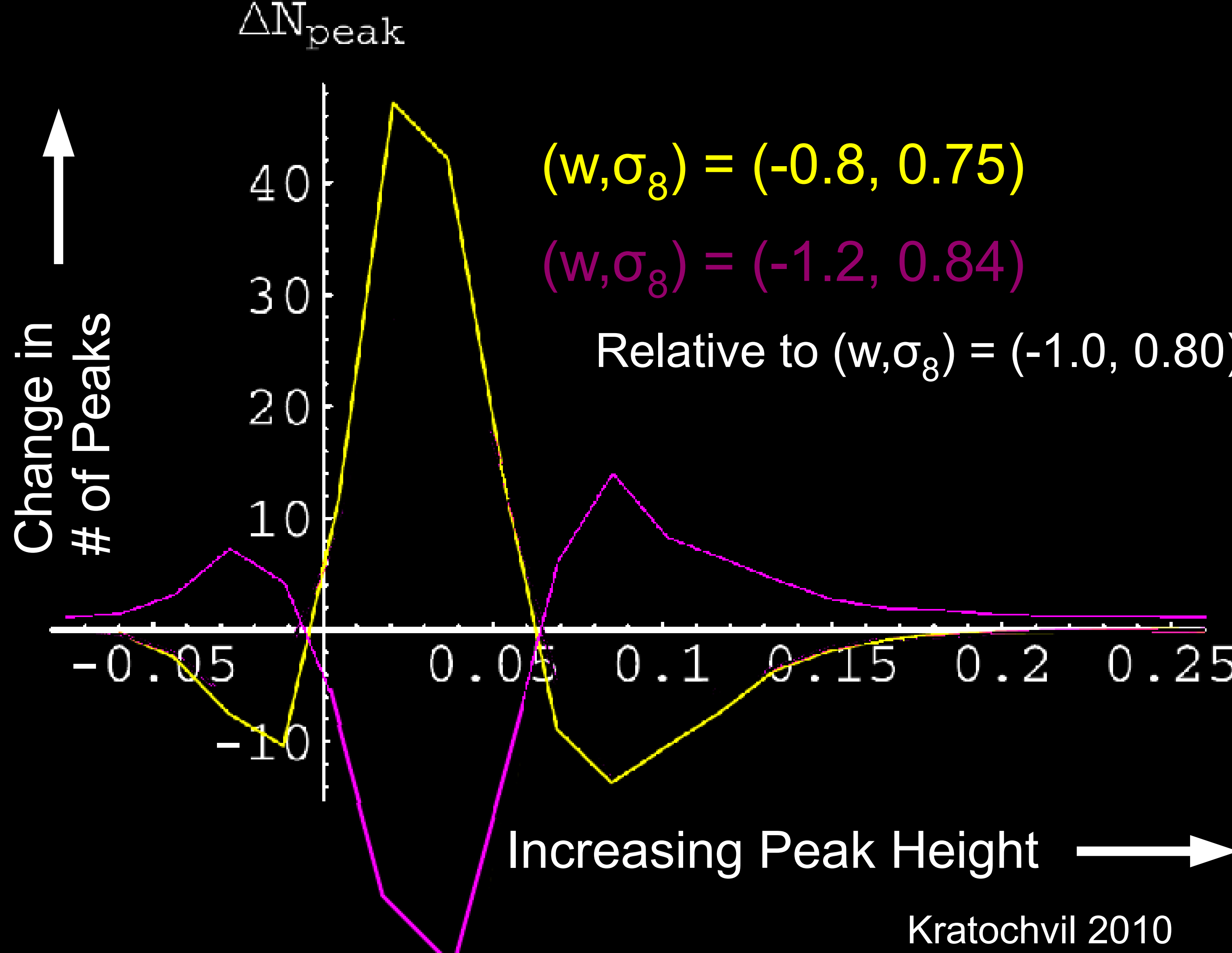
Followup: Marian *et al.* 2010

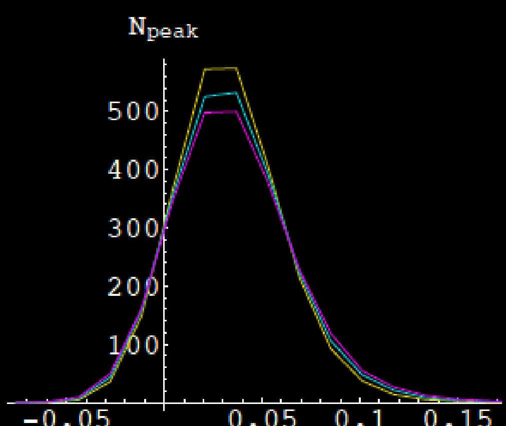
But cosmological scaling is unaffected



Shear Peak Statistics

Is there information in these correlated projections?

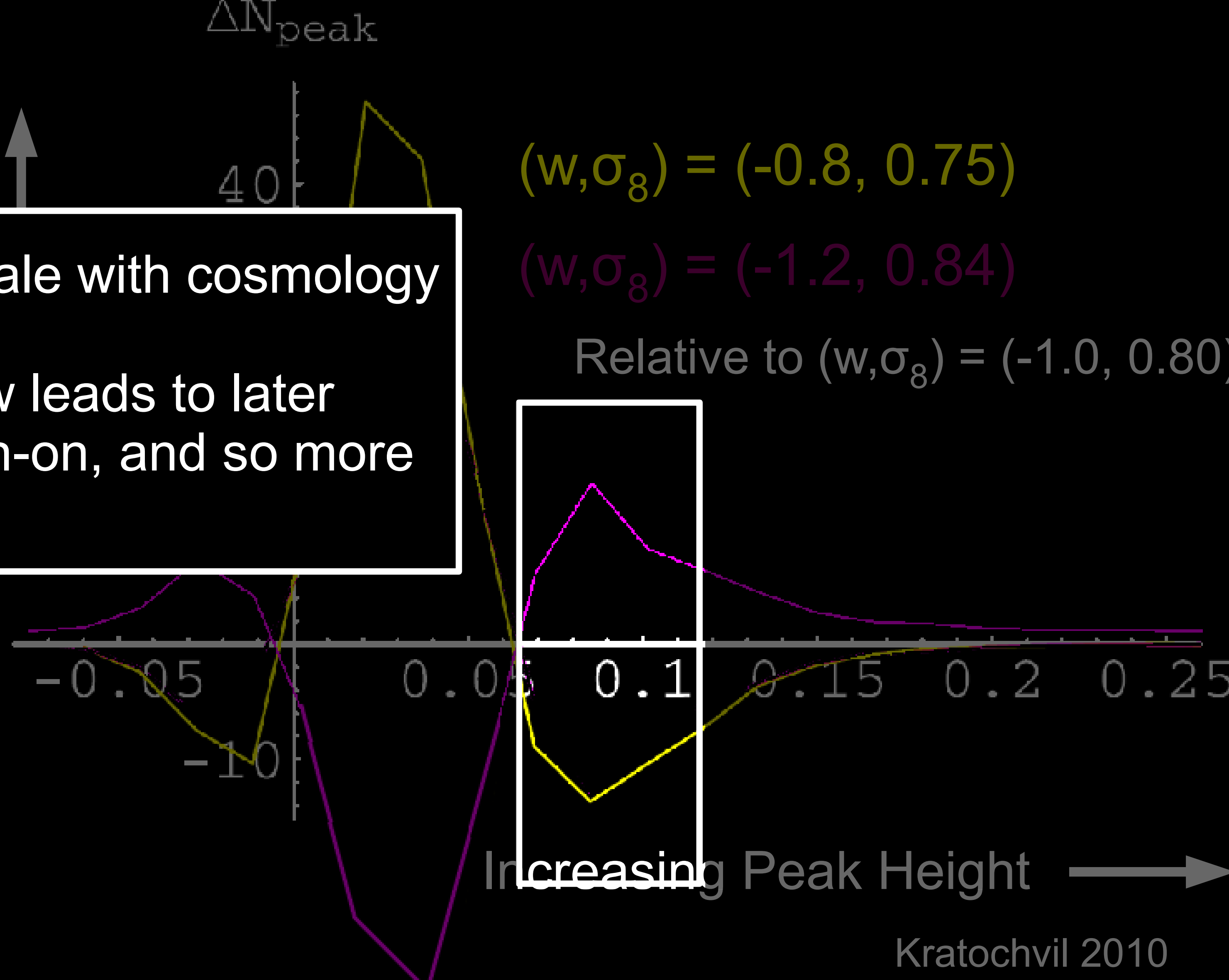




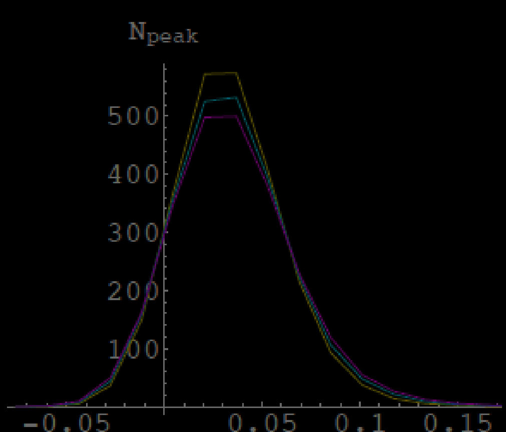
Kratochvil 2010

Shear Peak Statistics

Projections boost the signal of small peaks, and these carry information



Large Peaks scale with cosmology as expected:
More negative w leads to later dark energy turn-on, and so more large peaks

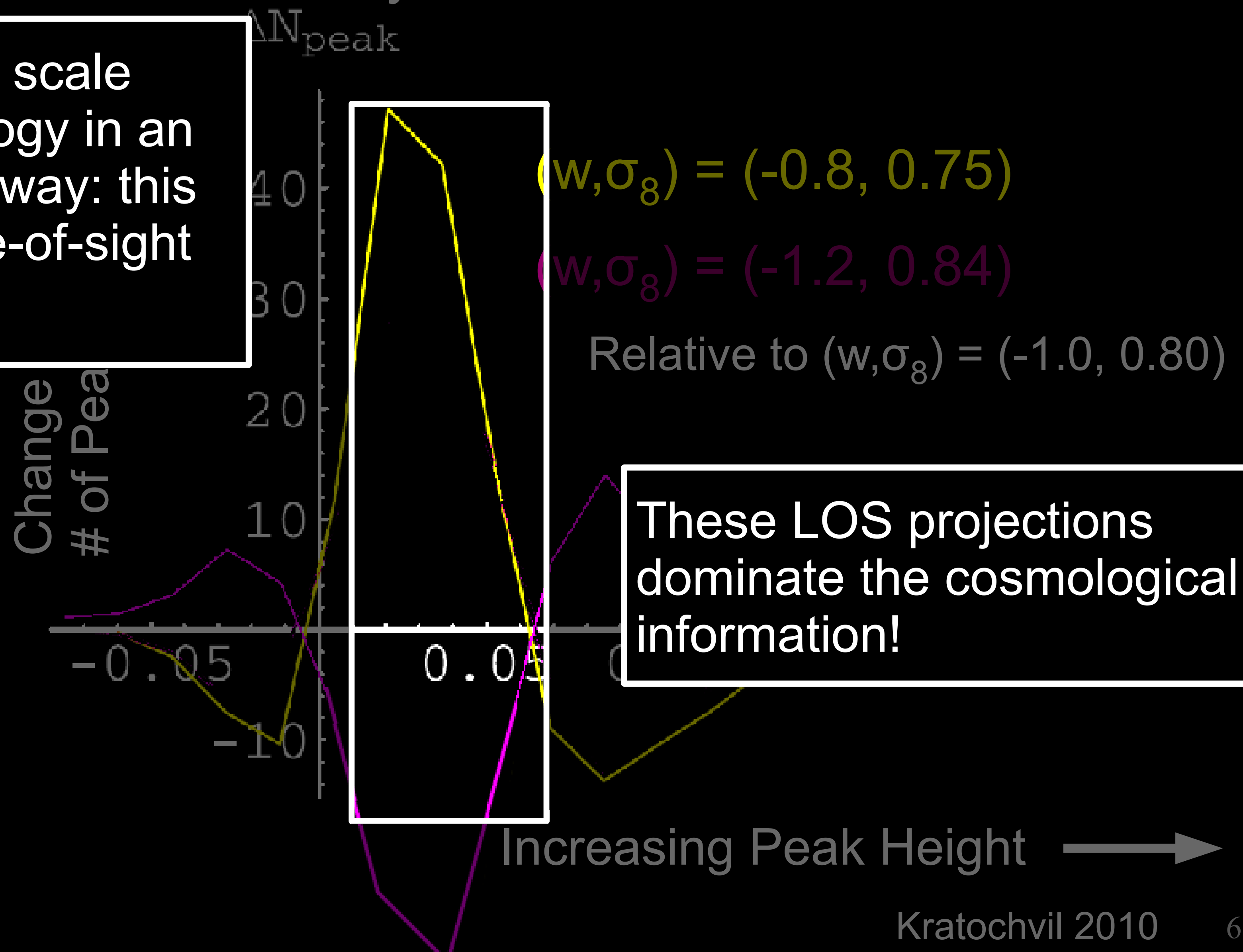


Kratochvil 2010

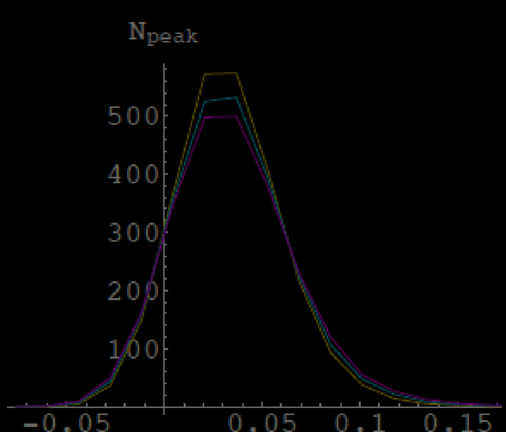
Shear Peak Statistics

Projections boost the signal of small peaks, and these carry information

Small peaks scale with cosmology in an unexpected way: this is due to line-of-sight projections

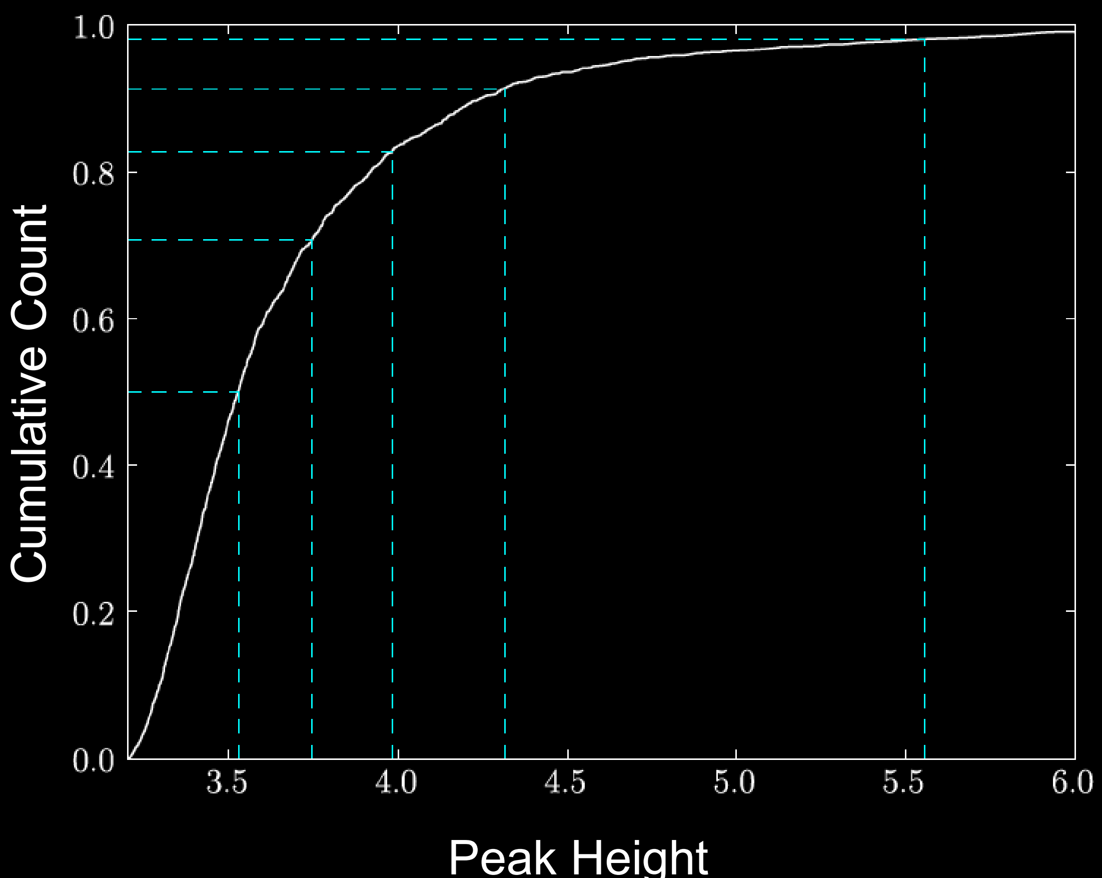


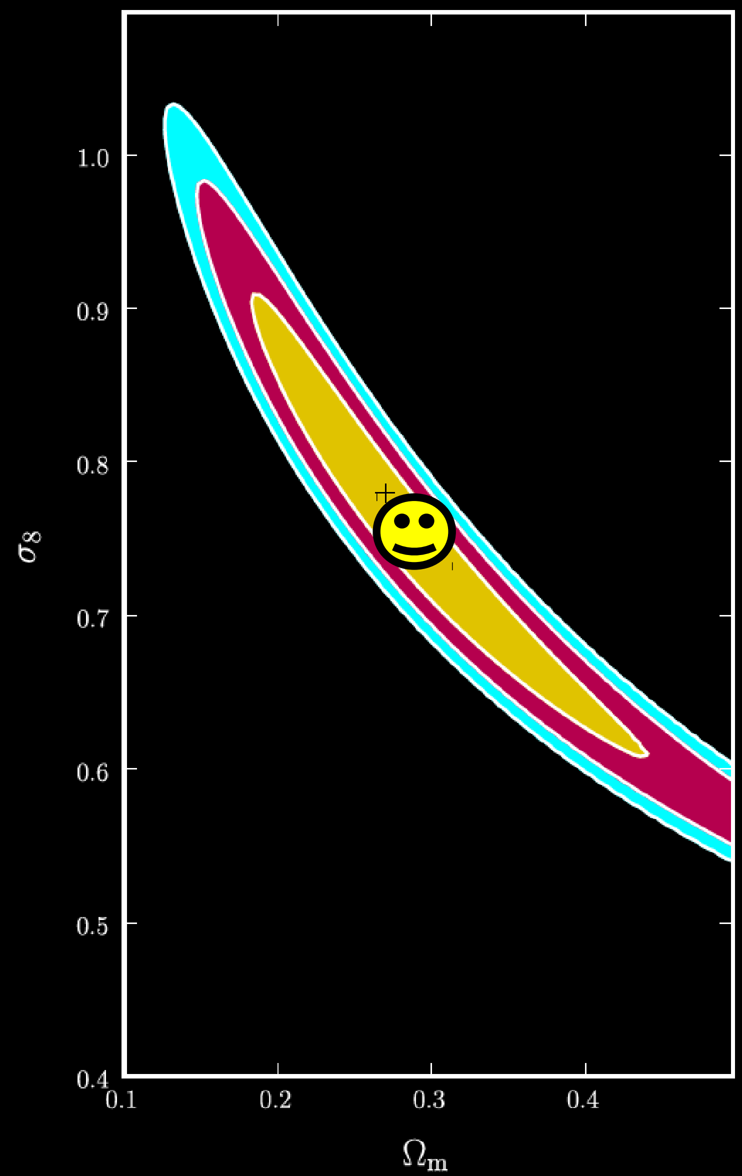
These LOS projections dominate the cosmological information!



Kratochvil 2010

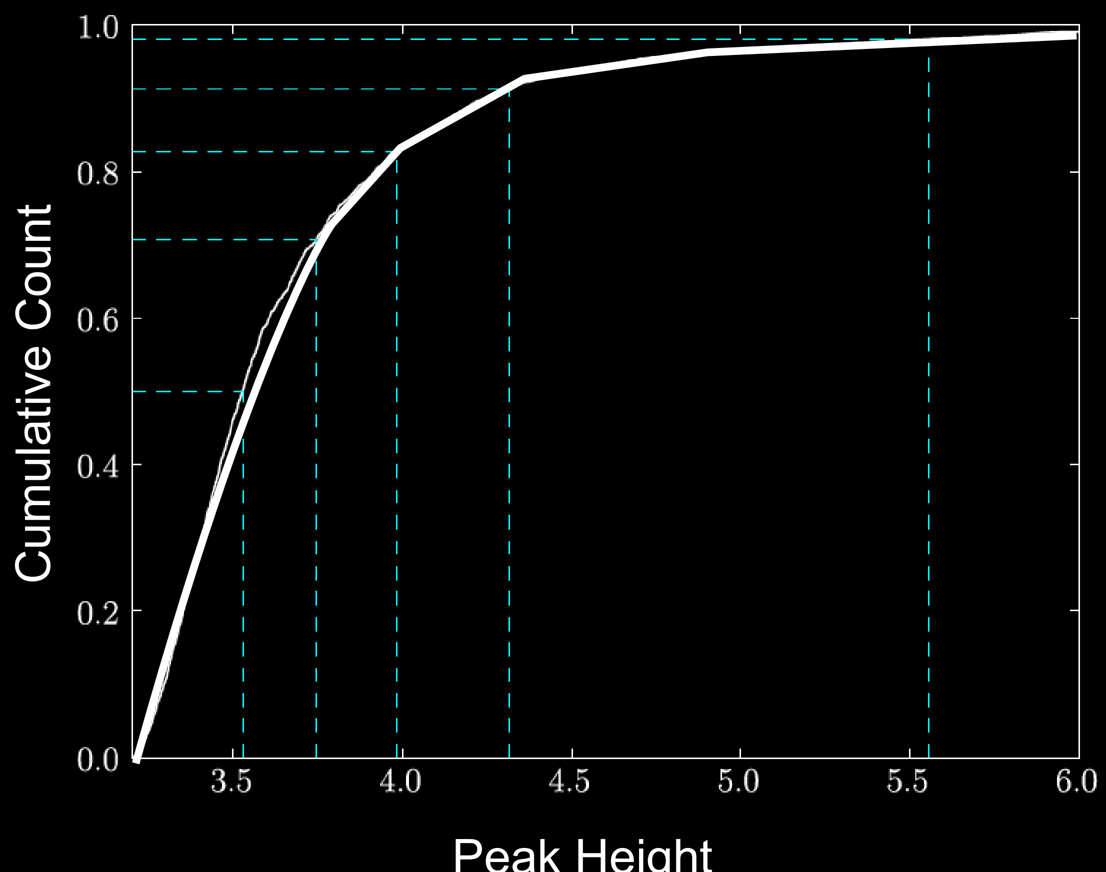
Cosmology Constraints

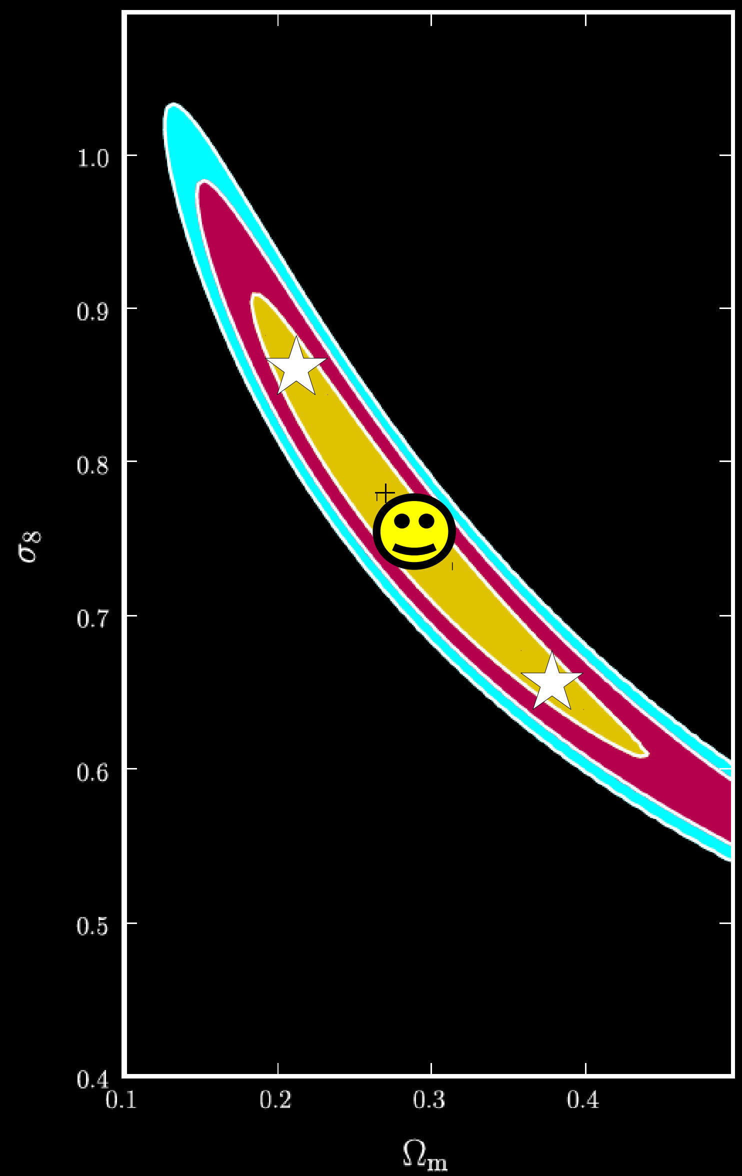




Cosmology Constraints

Increased normalization + less matter
Decreased normalization + more matter





Cosmology Constraints

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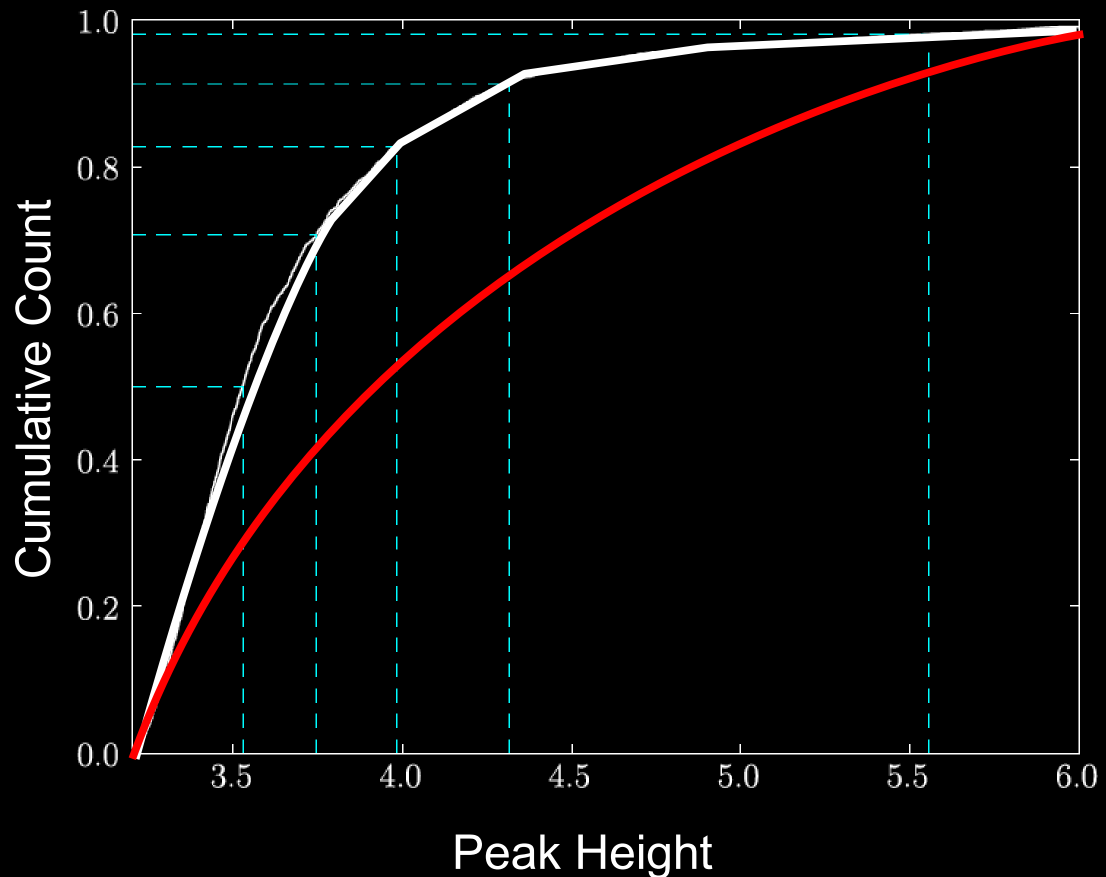
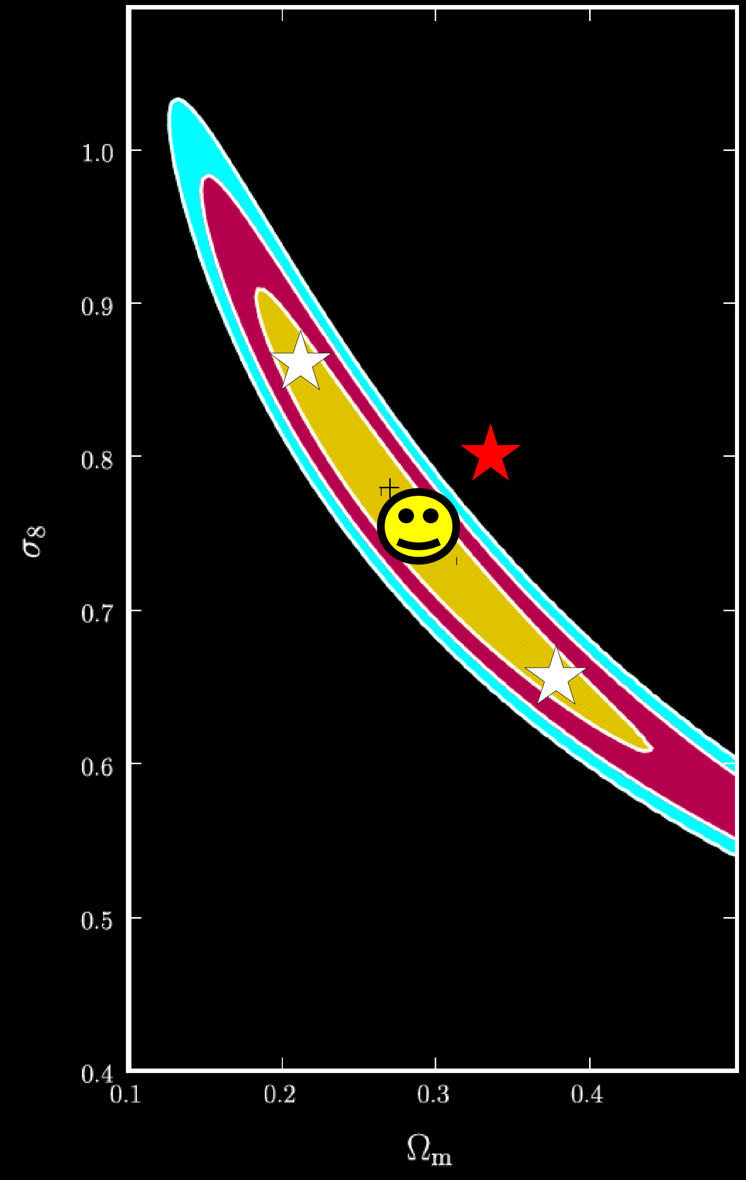
Too many large peaks





Cosmology Constraints

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Too many small peaks