Large scale structure formation with the Schrödinger method

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& Excellence Cluster Universe

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in collaboration with

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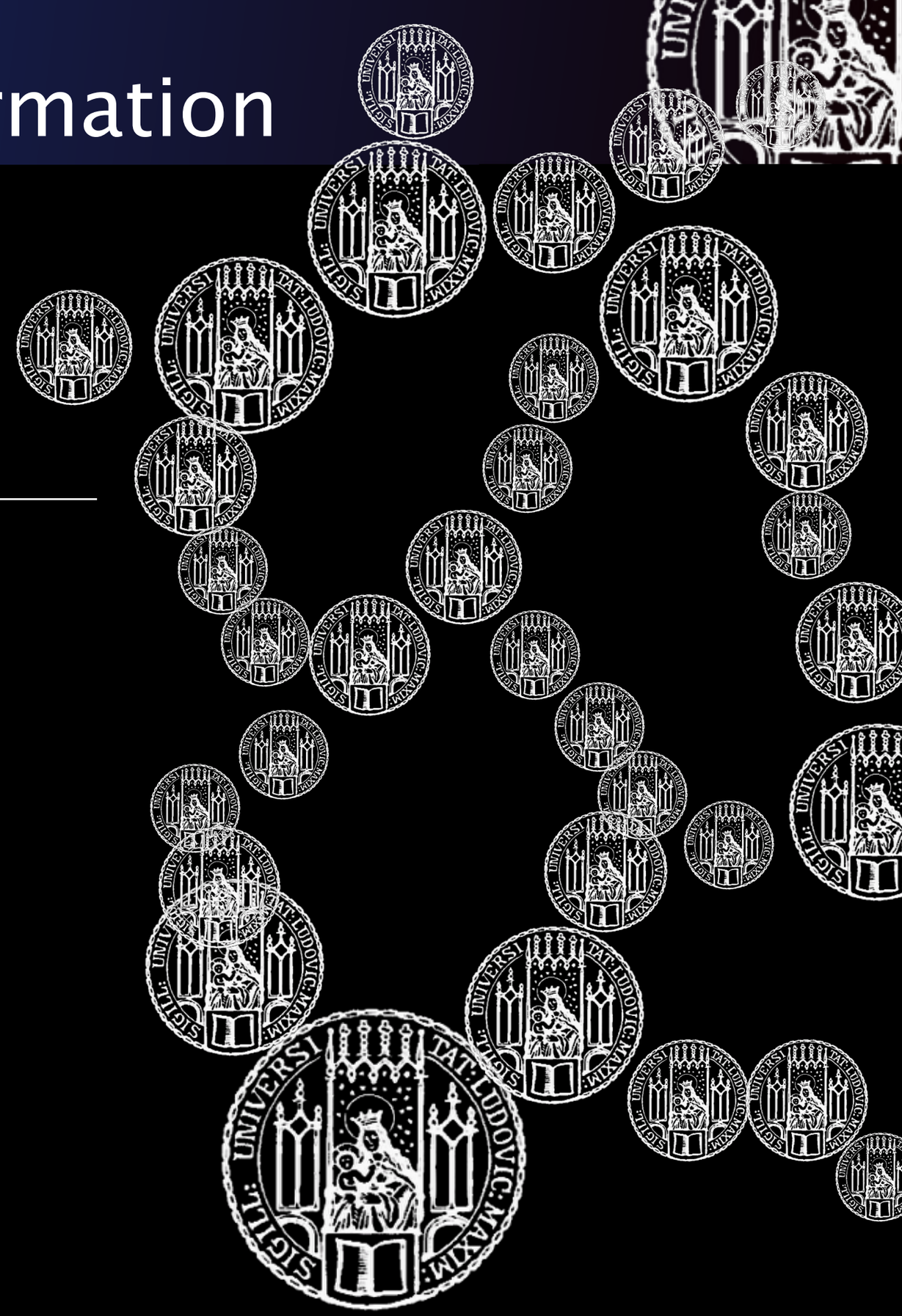
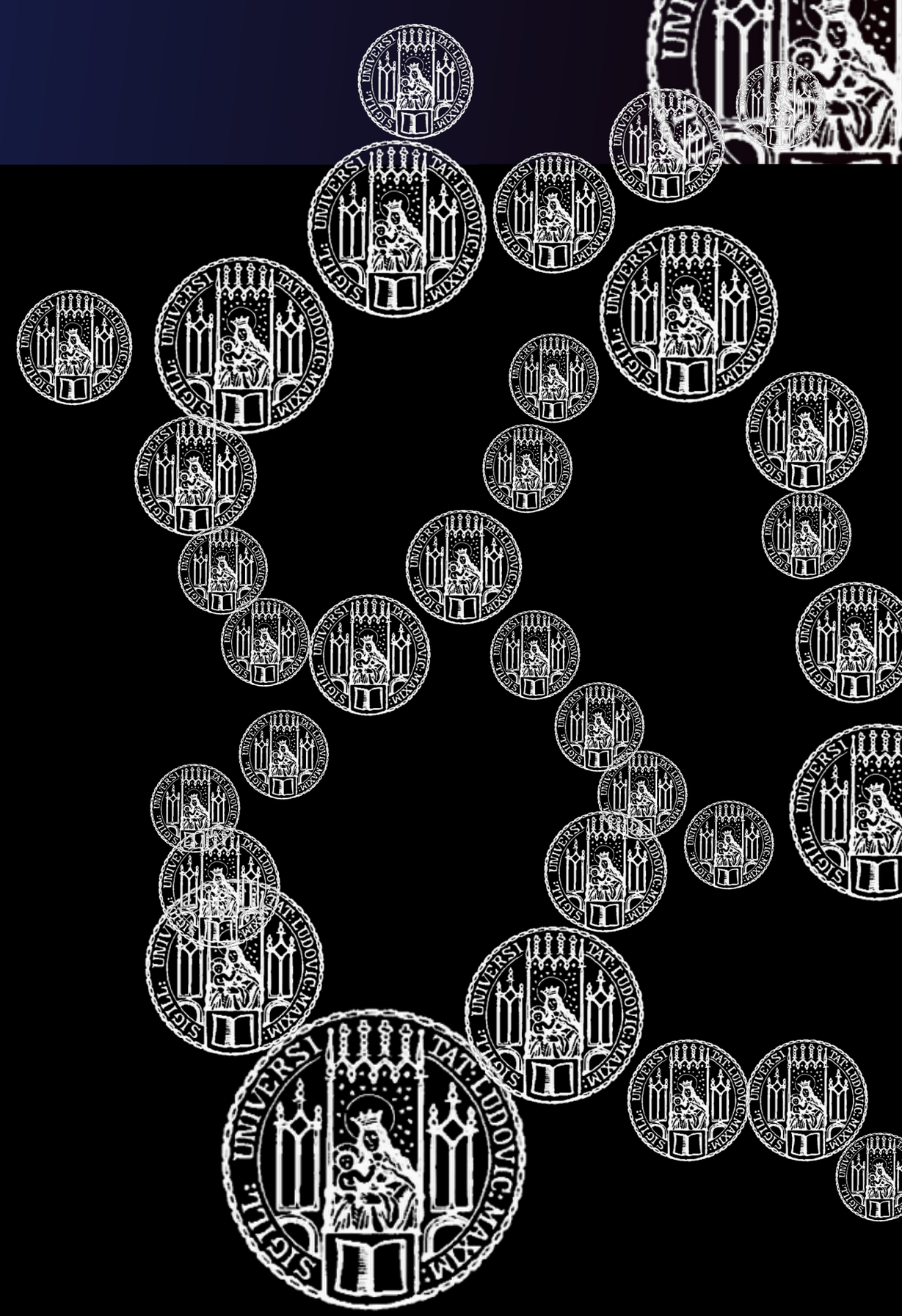


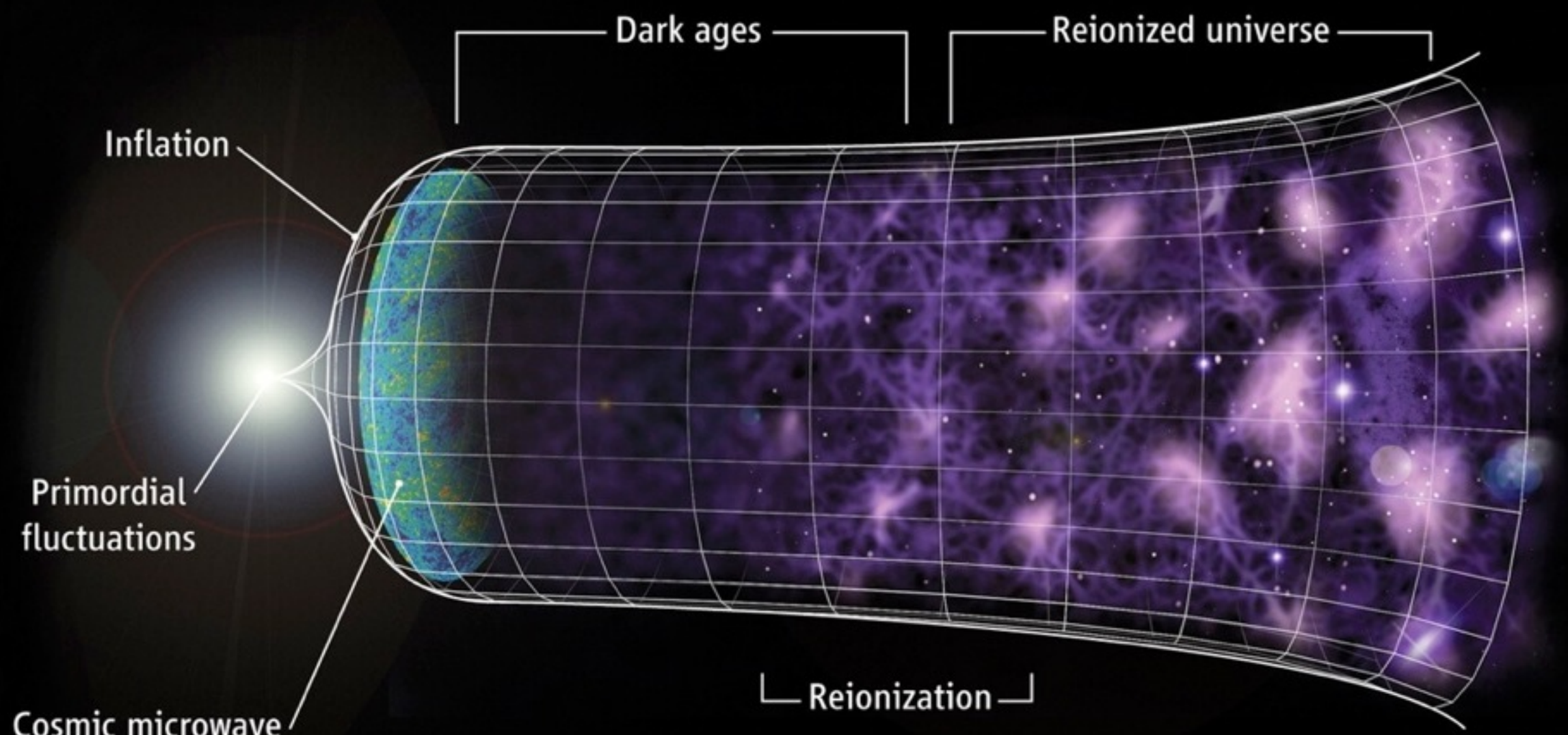
Outline

1. Structure formation
2. Analytical description cold dark matter
   a. dust model
   b. Schrödinger method
3. Correlation functions
   a. coarse-grained dust power spectra
   b. halo correlation incl. redshift space distortions
4. Summary



Cosmological Structure Formation

-13.8 billion years: nearly uniform initial state



Faucher-Giguère et al (2008, Science 319)

today: rich structures in cosmic web

Cosmological Structure Formation

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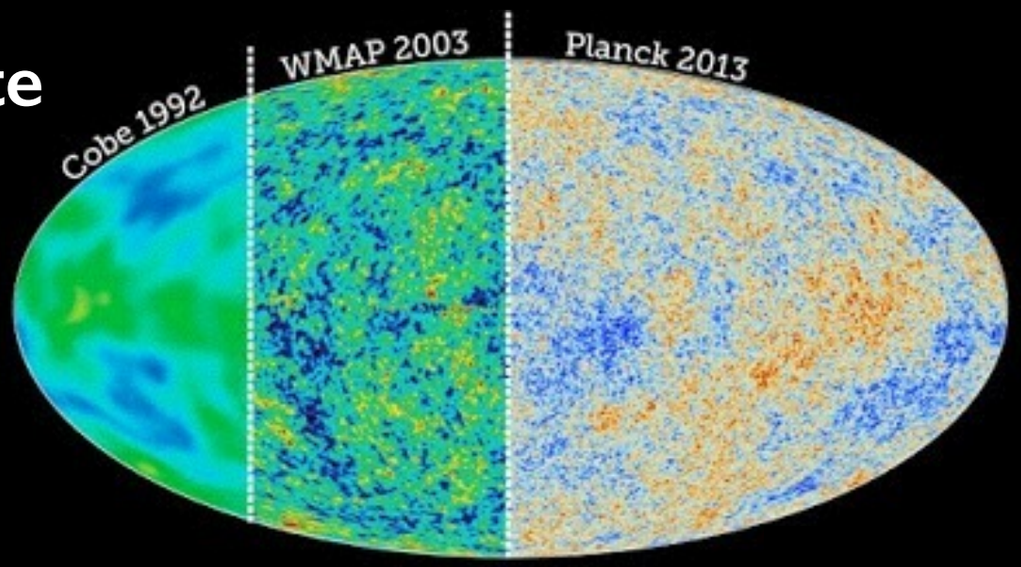


Inflation

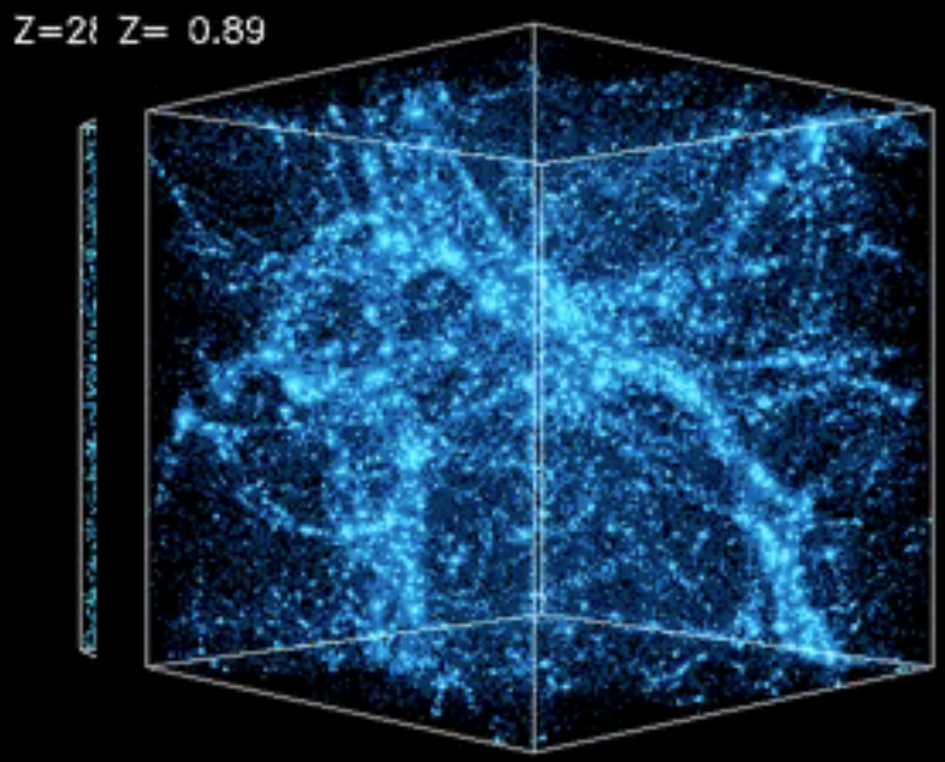
- established `boring` initial conditions
  - quantum fluctuations get amplified
  - primordial plasma cools → recombination → **CMB**

Structure formation

- hierarchical
- tiny over-densities act as seeds
  - congregation via gravitational instability
  - collapse into bound structures

Large scale structure: Dark Matter

- linear regime
  ✓ analytically understood
- nonlinear stage
  ?! **N-body simulations inevitable**

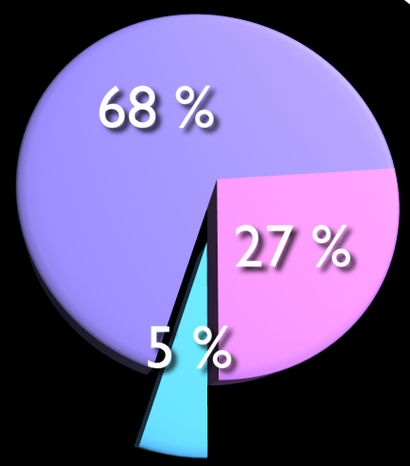


today: rich structures in cosmic web

Kravtsov & Klypin (simulations @NCSA)

Cosmological Standard Model

ΛCDM



Dark Energy

cosmological constant

baryons

standard model
of particle physics

Cold Dark Matter

collisionless

cold (initially)

↓

Large Scale Structure (LSS)

dark matter halos

↓

galaxy surveys

constrain parameters

halo density profiles

understand universality

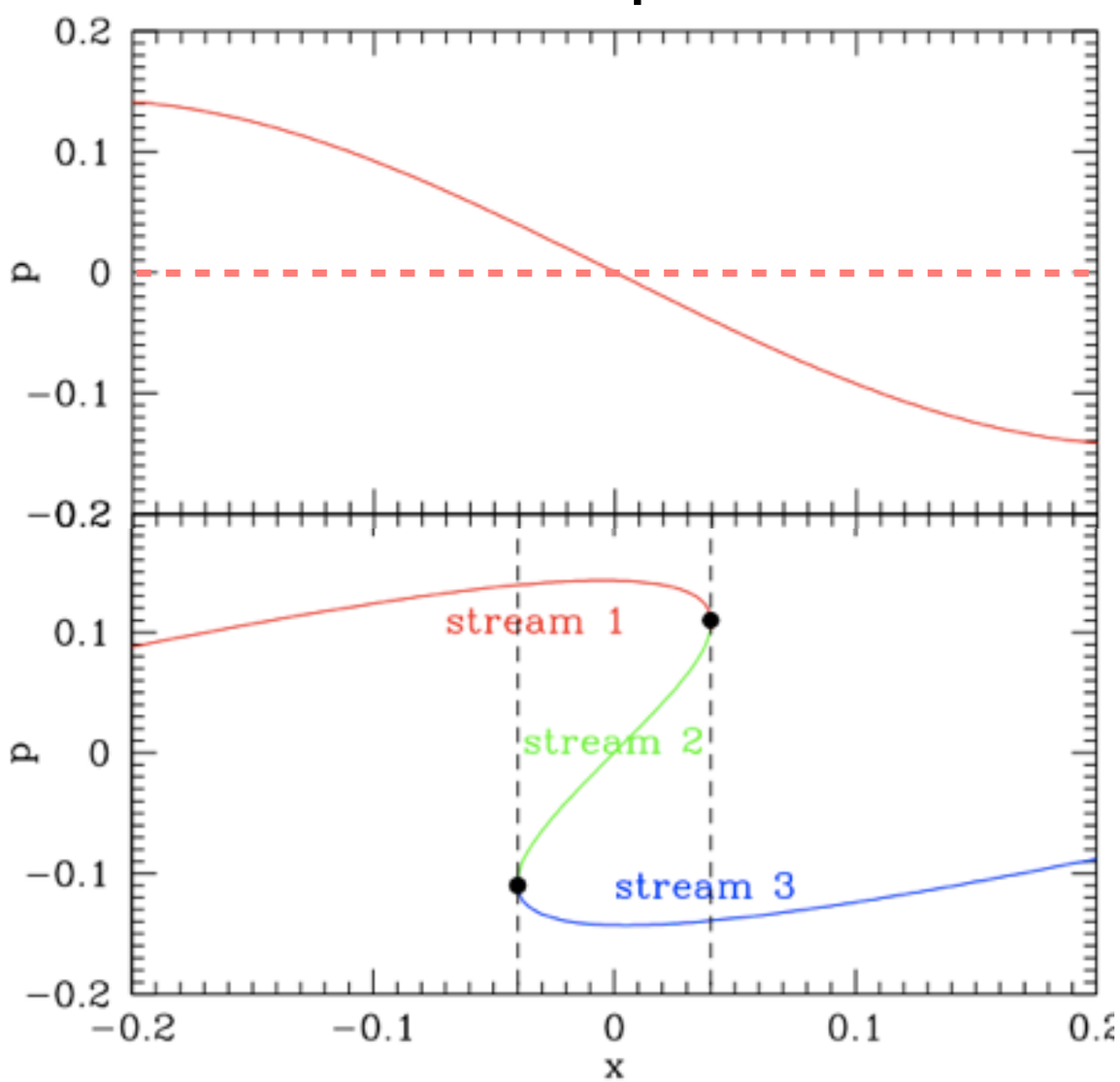
Describing Cold Dark Matter with the Schrödinger method

Describing Cold Dark Matter

phase space distribution function f(t,x,p)

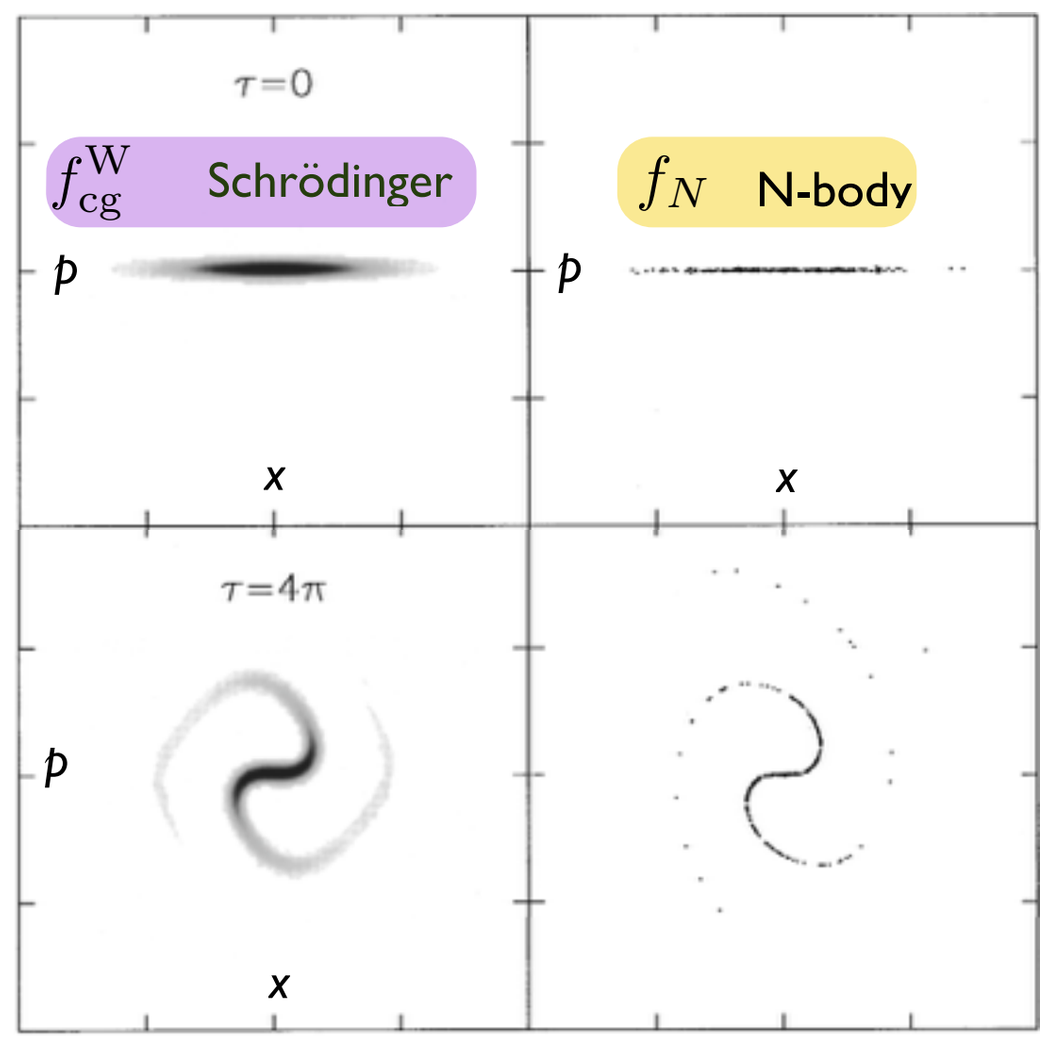
- describes number density & distribution of momenta *p*

theoretical expectation



Pueblas & Scoccimarro (2009, PRD 80)

numerical realization



Schrödinger method: Widrow & Kaiser (1993, ApJ 416)
Widrow (1997, PRD 55)

Describing Cold Dark Matter

phase space distribution function f(t,x,p)

- **N-body**: non-relativistic, only gravitationally
- **continuous**: ensemble average, no collisions

$$f_N = \sum_i \delta_D(\boldsymbol{x} - \boldsymbol{x}_i)\delta_D(\boldsymbol{p} - \boldsymbol{p}_i)$$

$$f$$

Vlasov - Poisson equation

$$\partial_\tau f(\boldsymbol{x}, \boldsymbol{p}, \tau) = -\frac{\boldsymbol{p}}{am}\boldsymbol{\nabla}_x f + am\boldsymbol{\nabla}_x V \boldsymbol{\nabla}_p f$$

3+3+1 variables

partial

nonlinear

gravitational potential

$$\Delta V(\boldsymbol{x}, \tau) = \frac{4\pi G m}{a}\left(n(\boldsymbol{x}, \tau) - \langle n \rangle\right)$$

integro

number density

$$n = \int \mathrm{d}^3 p \;\; f$$

Solving is hard!

have to choose a special ansatz
for phase space distribution f(x,p)

Describing Cold Dark Matter

phase space distribution function f(t,x,p)

- **N-body**: non-relativistic, only gravitationally
- **continuous**: ensemble average, no collisions

$$f_N \rightarrow f$$

Vlasov - Poisson equation

$$\partial_\tau f(\boldsymbol{x}, \boldsymbol{p}, \tau) = -\frac{\boldsymbol{p}}{am} \boldsymbol{\nabla}_x f + am \boldsymbol{\nabla}_x V \boldsymbol{\nabla}_p f$$

gravitational potential

$$\Delta V(\boldsymbol{x}, \tau) = \frac{4\pi G m}{a} \left(n(\boldsymbol{x}, \tau) - \langle n \rangle \right)$$

Hierarchy of Moments $M^{(n)}(\boldsymbol{x}) = \int d^3p \; p_{i_1} \ldots p_{i_n} f$

- density n(x): $M^{(0)} = n(\boldsymbol{x})$, velocity **v**(x): $M^{(1)} = n\boldsymbol{v}(\boldsymbol{x})$
- velocity dispersion ***σ***(x): $M^{(2)} = n\left(\boldsymbol{v}\boldsymbol{v} + \boldsymbol{\sigma}\right)(\boldsymbol{x}), \ldots$

cumulant

$$\partial_t M^{(n)} = -\frac{1}{a^2 m} \nabla \cdot M^{(n+1)} - m \nabla V \cdot M^{(n-1)}$$

infinite coupled hierarchy

Dust model

dust model

- only consistent truncation of hierarchy
- pressureless fluid: density and velocity

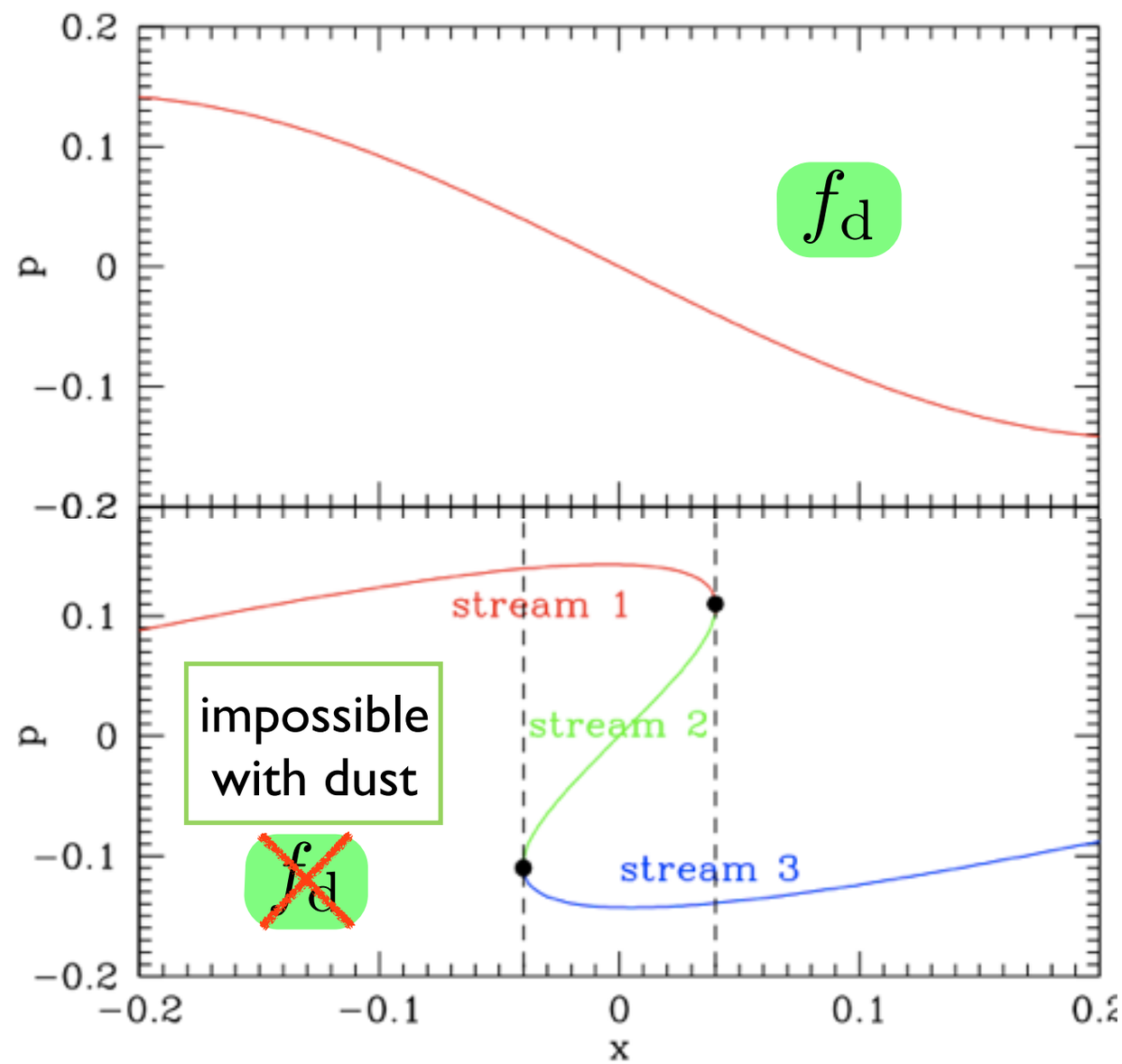
$$f_{\rm d}(\boldsymbol{x},\boldsymbol{p},\tau) = n(\boldsymbol{x},\tau)\delta_D^{(3)}(\boldsymbol{p} - \boldsymbol{\nabla}\phi(\boldsymbol{x},\tau))$$

Continuity $\partial_\tau n = -\frac{1}{am}\boldsymbol{\nabla}(n\boldsymbol{\nabla}\phi)$

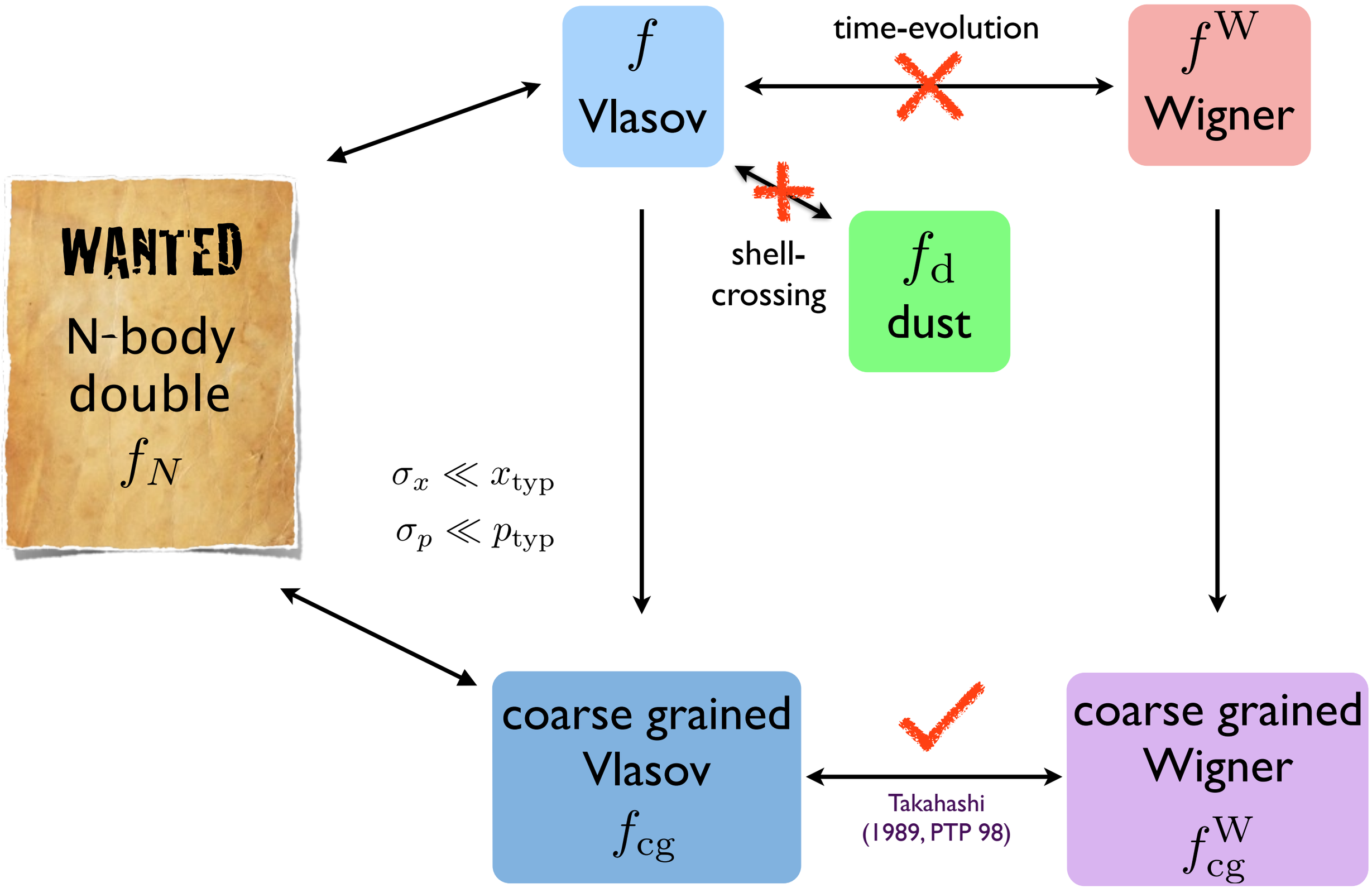
Euler $\partial_\tau \phi = -\frac{1}{2am}(\boldsymbol{\nabla}\phi)^2 - amV$

- limited to **single-stream**
- no velocity dispersion, …

- shell-crossing singularities
- **no virialization**



Schrödinger method at a glance



Schrödinger method

Schrödinger method

- Coarse-grained Wigner function, constructed from self-gravitating field

$$\psi = \sqrt{n}\exp\left(\frac{i}{\hbar}\phi\right)$$

$$f^W_{cg}(\boldsymbol{x},\boldsymbol{p}) = \int \frac{d^3x' d^3p'}{(\pi\sigma_x\sigma_p)^3}\exp\left[-\frac{(\boldsymbol{x}-\boldsymbol{x'})^2}{2\sigma_x^2} - \frac{(\boldsymbol{p}-\boldsymbol{p'})^2}{2\sigma_p^2}\right]\int \frac{d^3\tilde{x}}{(2\pi\hbar)^3}\exp\left[2\frac{i}{\hbar}\boldsymbol{p'}\cdot\tilde{\boldsymbol{x}}\right]\psi(\boldsymbol{x'}-\tilde{\boldsymbol{x}})\bar{\psi}(\boldsymbol{x'}+\tilde{\boldsymbol{x}})$$

Schrödinger - Poisson equation

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2}{2a^2m}\Delta + mV\right]\psi$$

$$\Delta V = \frac{4\pi G\rho_0}{a}(|\psi|^2 - 1)$$

degrees of freedom

- 2: amplitude n & phase ϕ

parameters

- coarse-graining σ_x, σ_p
  - fundamental resolution $\sigma_x\sigma_p \gtrsim \hbar/2$
- Schrödinger scale $\hbar$
  - degree of restriction
  - dust as special case

Continuity $\partial_\tau n = -\frac{1}{am}\boldsymbol{\nabla}(n\boldsymbol{\nabla}\phi)$

Euler $\partial_\tau\phi = -\frac{1}{2am}(\boldsymbol{\nabla}\phi)^2 - amV + \frac{\hbar^2}{2am}\left(\frac{\Delta\sqrt{n}}{\sqrt{n}}\right)$

quantum potential

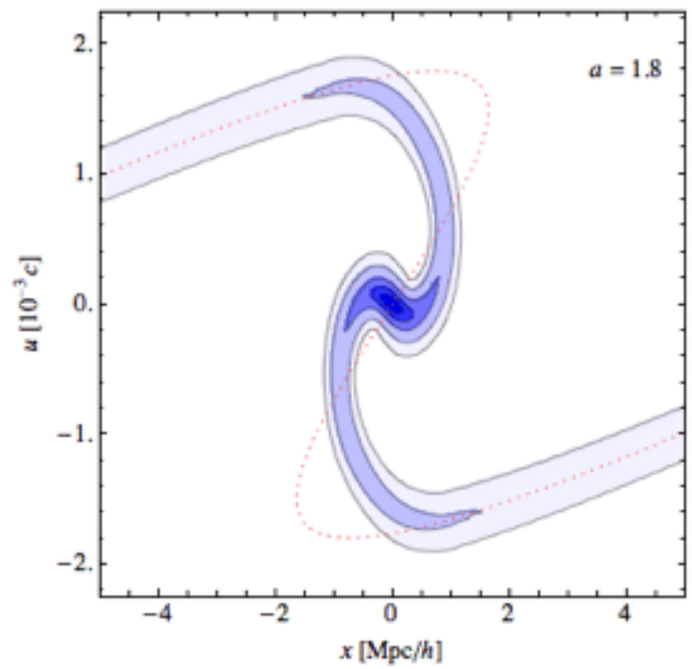
Features of Schrödinger Method

phase space density

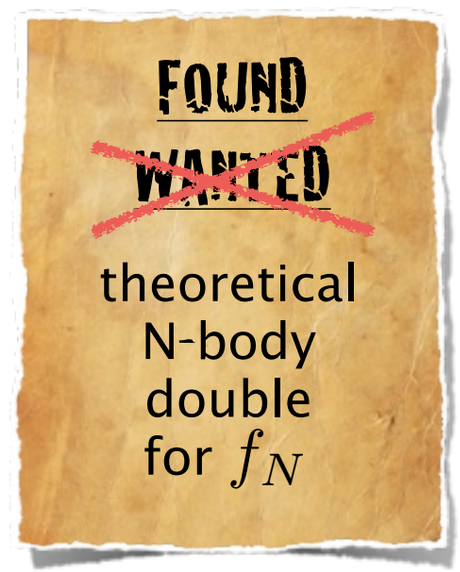
Multi-streaming

❌ dust model: fails at shell-crossing

✅ Schrödinger method: can go **beyond shell-crossing**



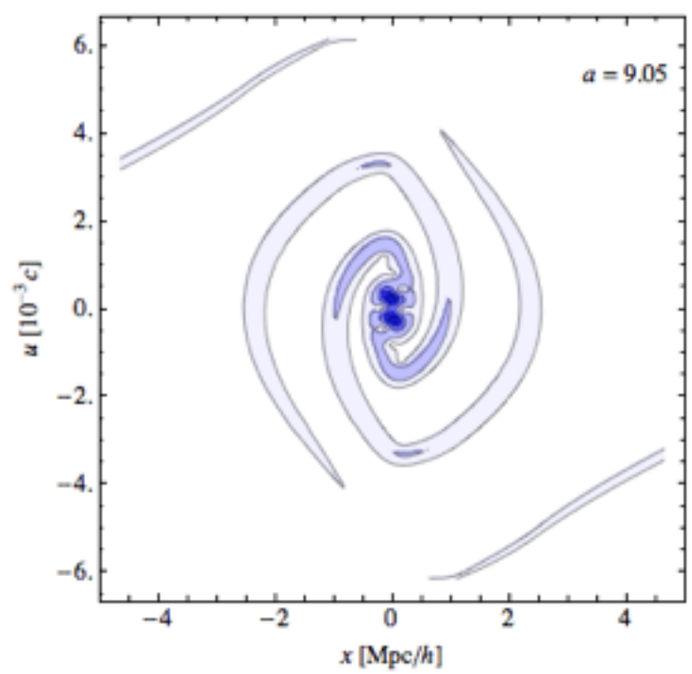
blue S contours: Schrödinger method
red dotted Z line: Zeldovich solution (dust model)



Virialization

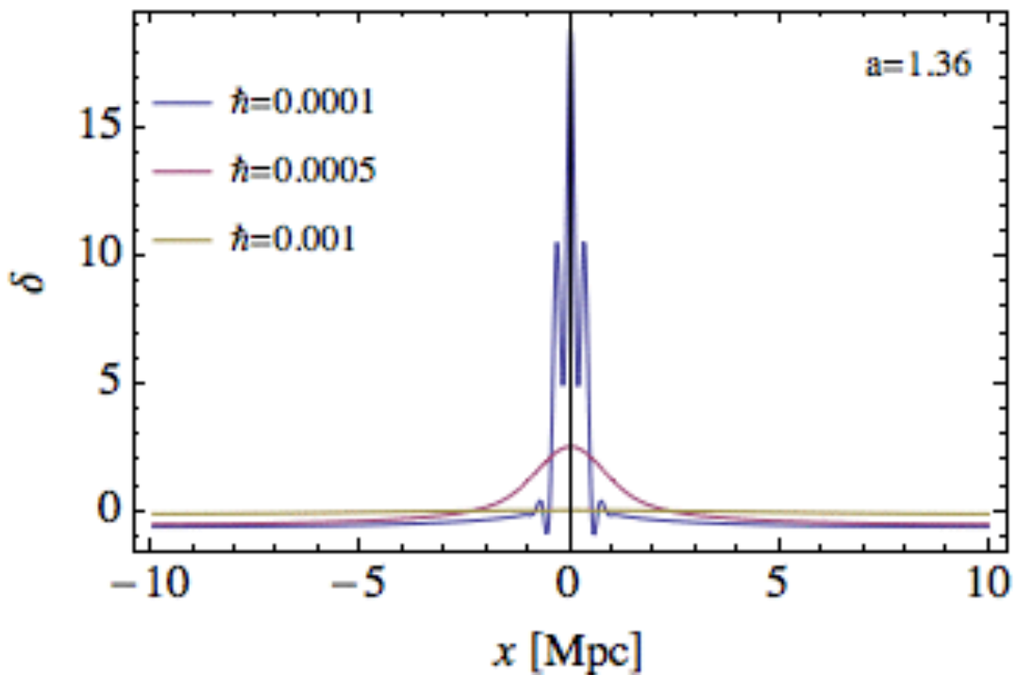
❌ even in extended models: no **virialization**

✅ Schrödinger method: **bound structures** like halos

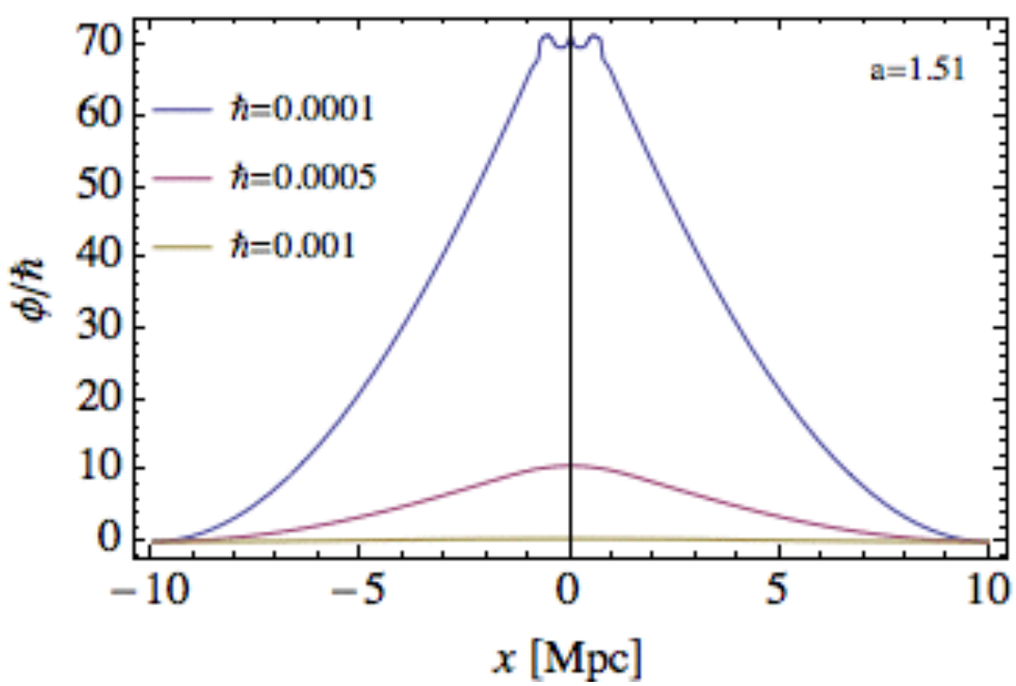


Features of Schrödinger Method

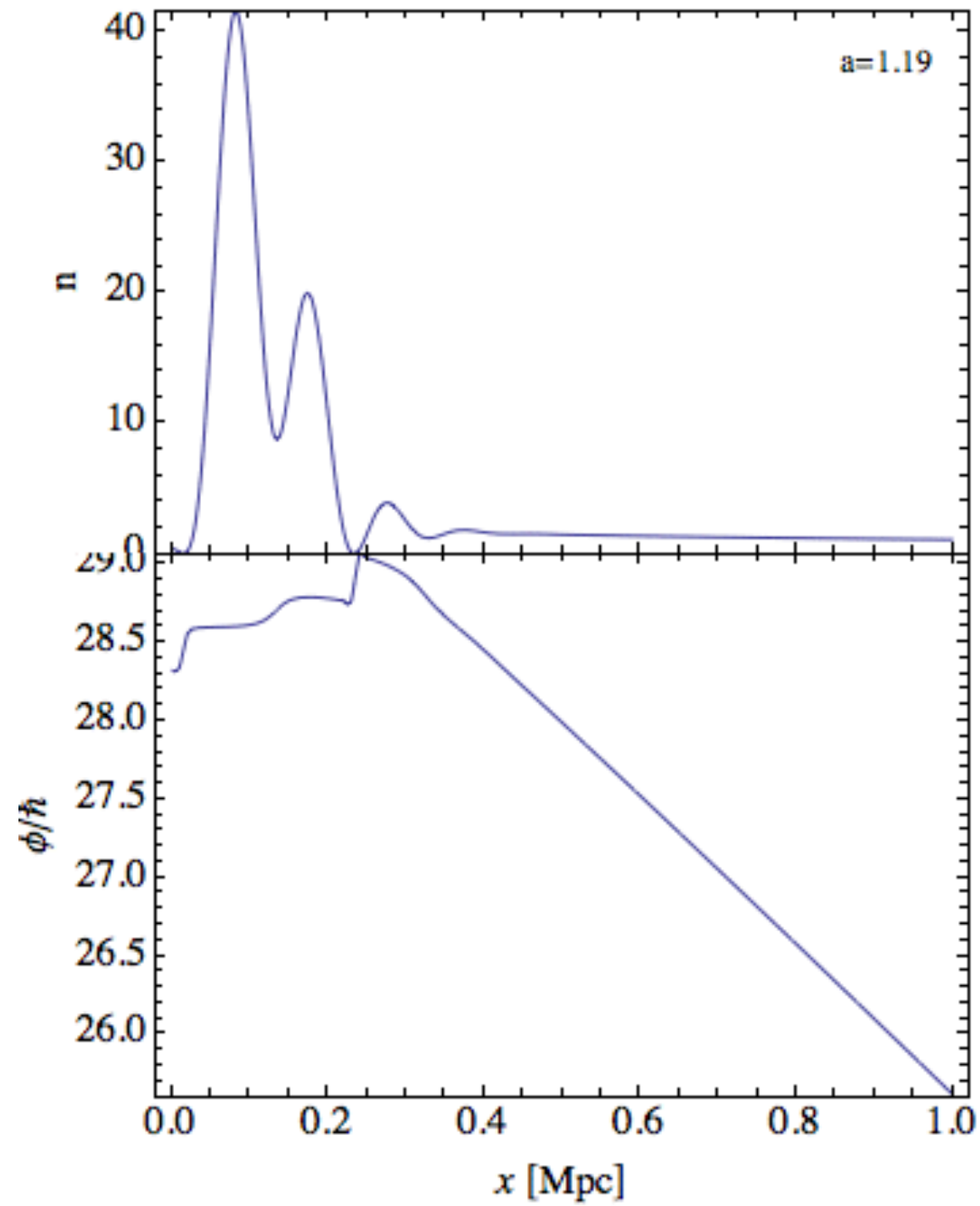
✅ **prevention of shell-crossing singularities**



!? **occurrence of phase jumps**



!! $\psi = \sqrt{n} e^{i\phi/\hbar}$ **free of pathologies**



Features of Schrödinger Method

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special p-dependence
allows to calculate
cumulants analytically

Cumulants

- lowest two: macroscopic density & velocity

$$\bar{n}(\boldsymbol{x}) = \exp\left[\tfrac{1}{2}\sigma_x^2\Delta\right]n(\boldsymbol{x}) \qquad \bar{\boldsymbol{v}}(\boldsymbol{x}) = \frac{1}{am\bar{n}(\boldsymbol{x})}\exp\left[\tfrac{1}{2}\sigma_x^2\Delta\right](n\boldsymbol{\nabla}\phi)(\boldsymbol{x})$$

- higher cumulants given self-consistently
  evolution equations fulfilled automatically

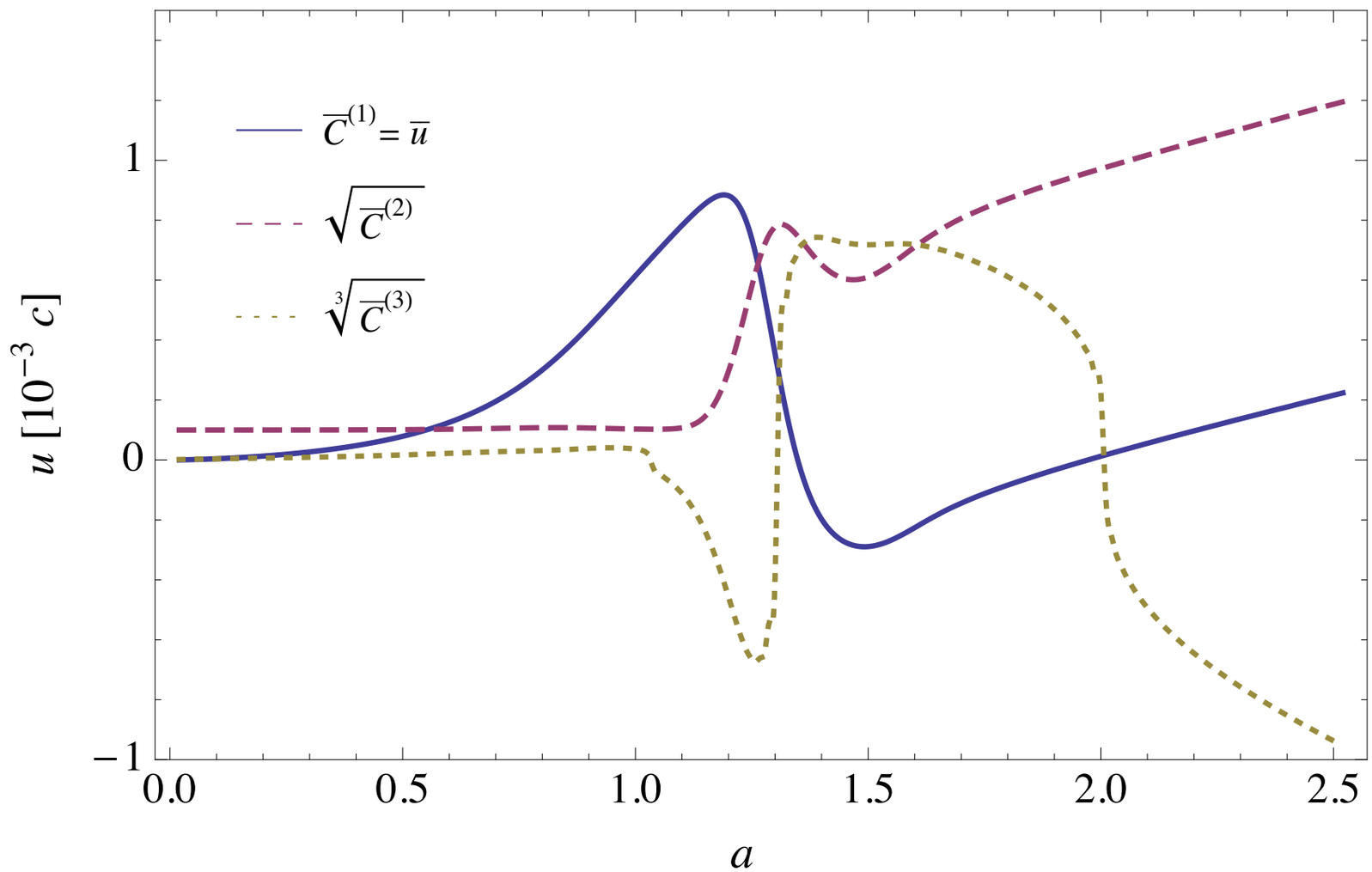
closure of hierarchy

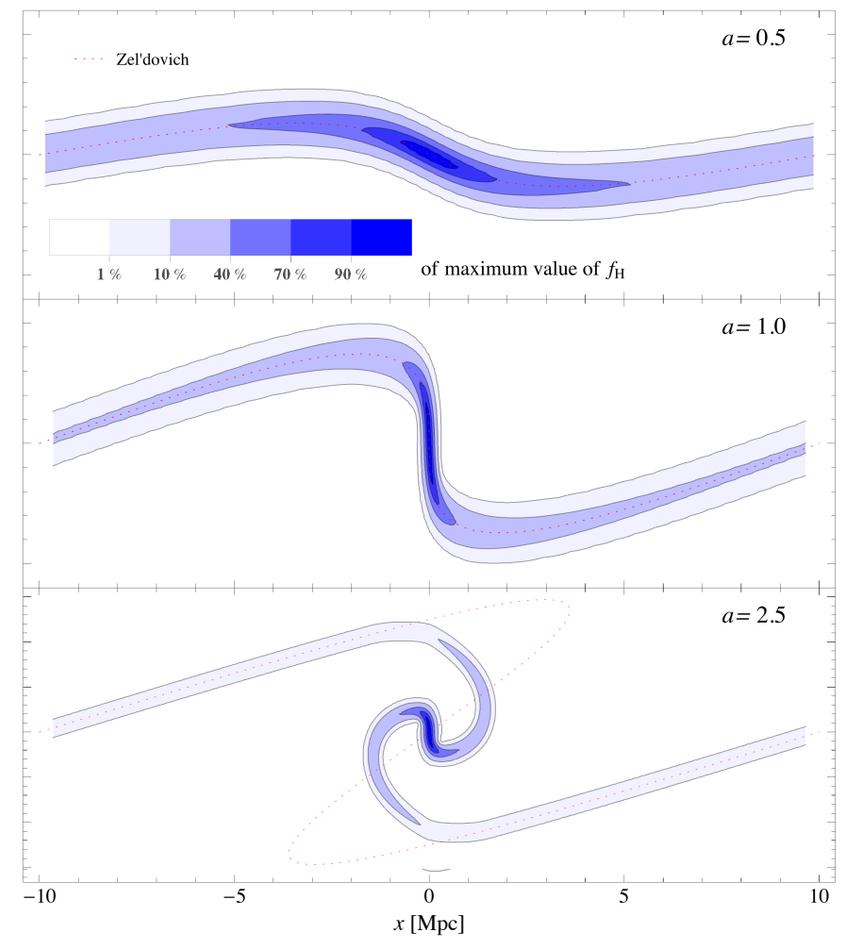
CU, Kopp & Haugg (2014, PRD 90, 023517)

Features of Schrödinger Method

Multi-streaming

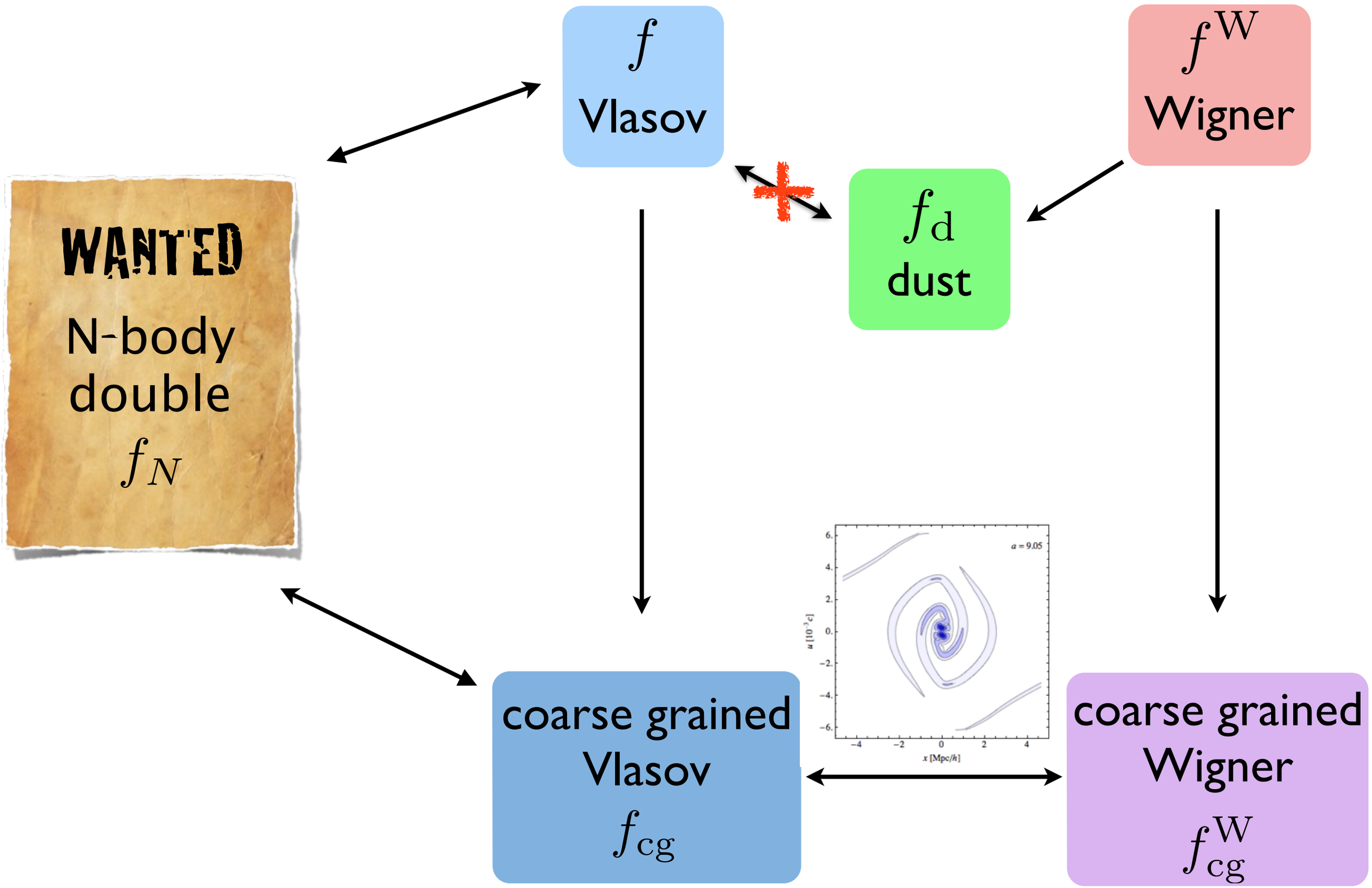
- higher cumulants encode **multi-streaming** effects
- during shell-crossing: **higher moments sourced dynamically**





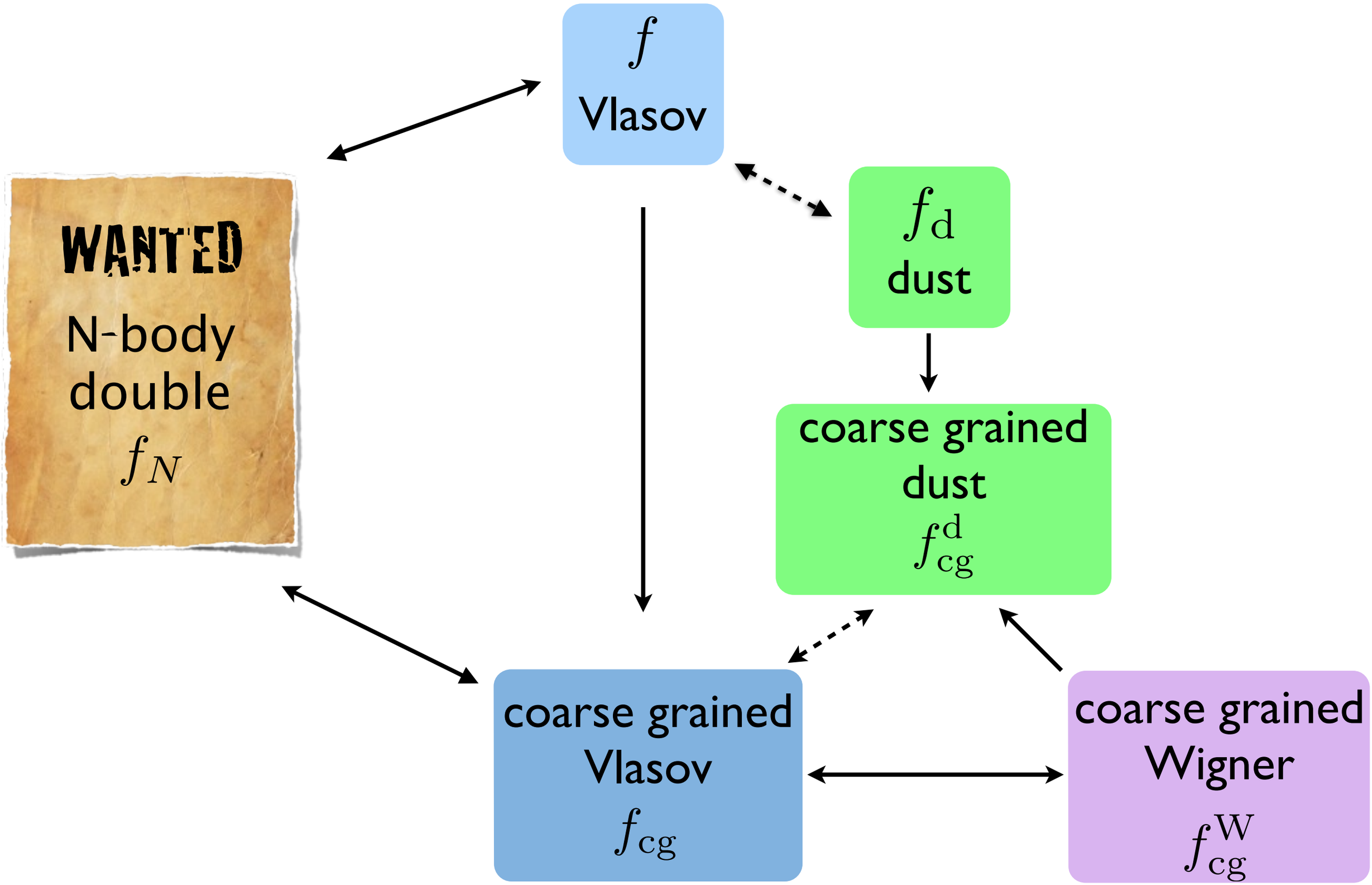
Schrödinger method: cumulants at x = −0.5 Mpc:
all equally important after shell crossing

Schrödinger method at a glance



Dark Matter power spectrum with the Coarse-grained dust model

Application: Perturbation theory



Eulerian Perturbation Theory

Dust model

- express fluid equations in terms of $\delta = n - 1$ and $\theta = \boldsymbol{\nabla} \cdot \boldsymbol{v} \propto \Delta\phi$ (no vorticity)
- perturbative expansion: separation ansatz (fastest growing mode)

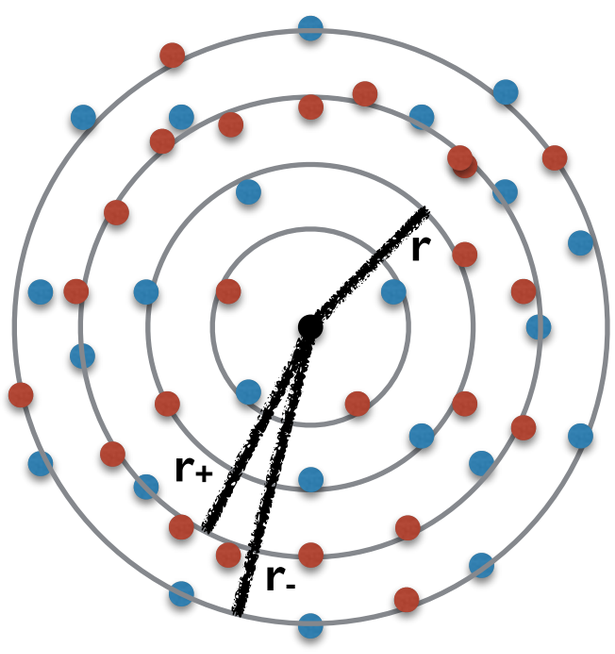
$$\delta(\tau, \boldsymbol{k}) = \sum_{n=1}^{\infty} a^n(\tau)\ \delta_n(\boldsymbol{k}) \qquad \theta(\tau, \boldsymbol{k}) = \mathcal{H}(\tau) \sum_{n=1}^{\infty} a^n(\tau)\ \theta_n(\boldsymbol{k})$$

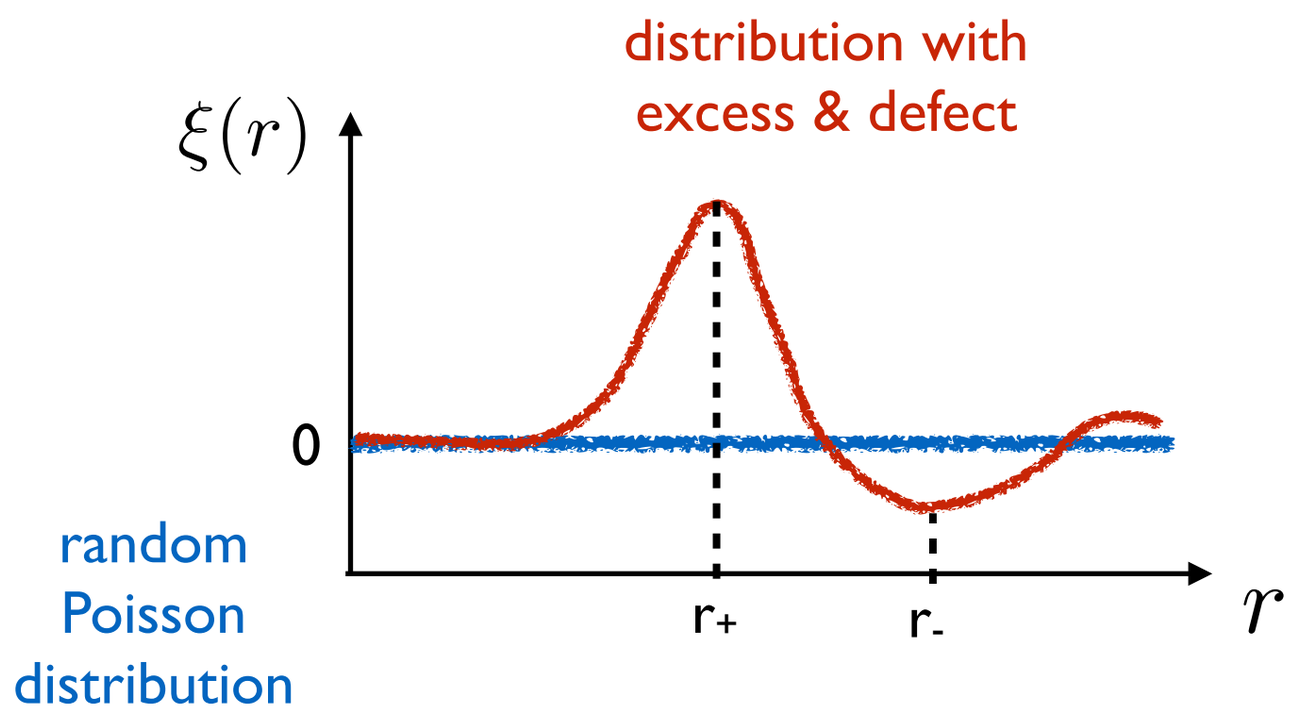
Correlation function

- 2-point correlation: excess probability of finding 2 objects separated by **r**

$$dP = n[1 + \xi(r)]dV$$

homogeneity & isotropy: $\xi(\boldsymbol{r}) = \xi(r)$





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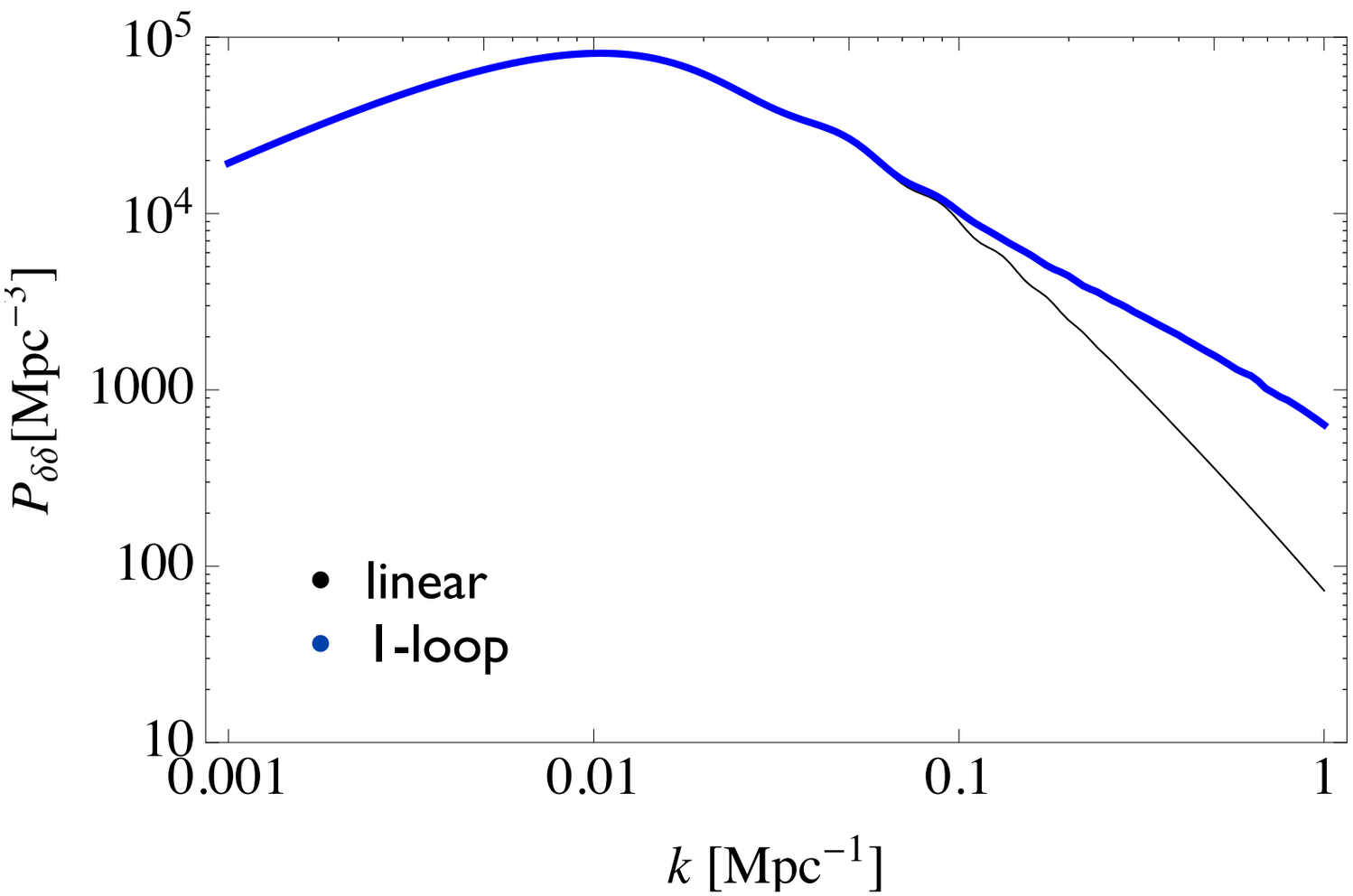
Density power spectrum



correlation function
=
FT of power spectrum

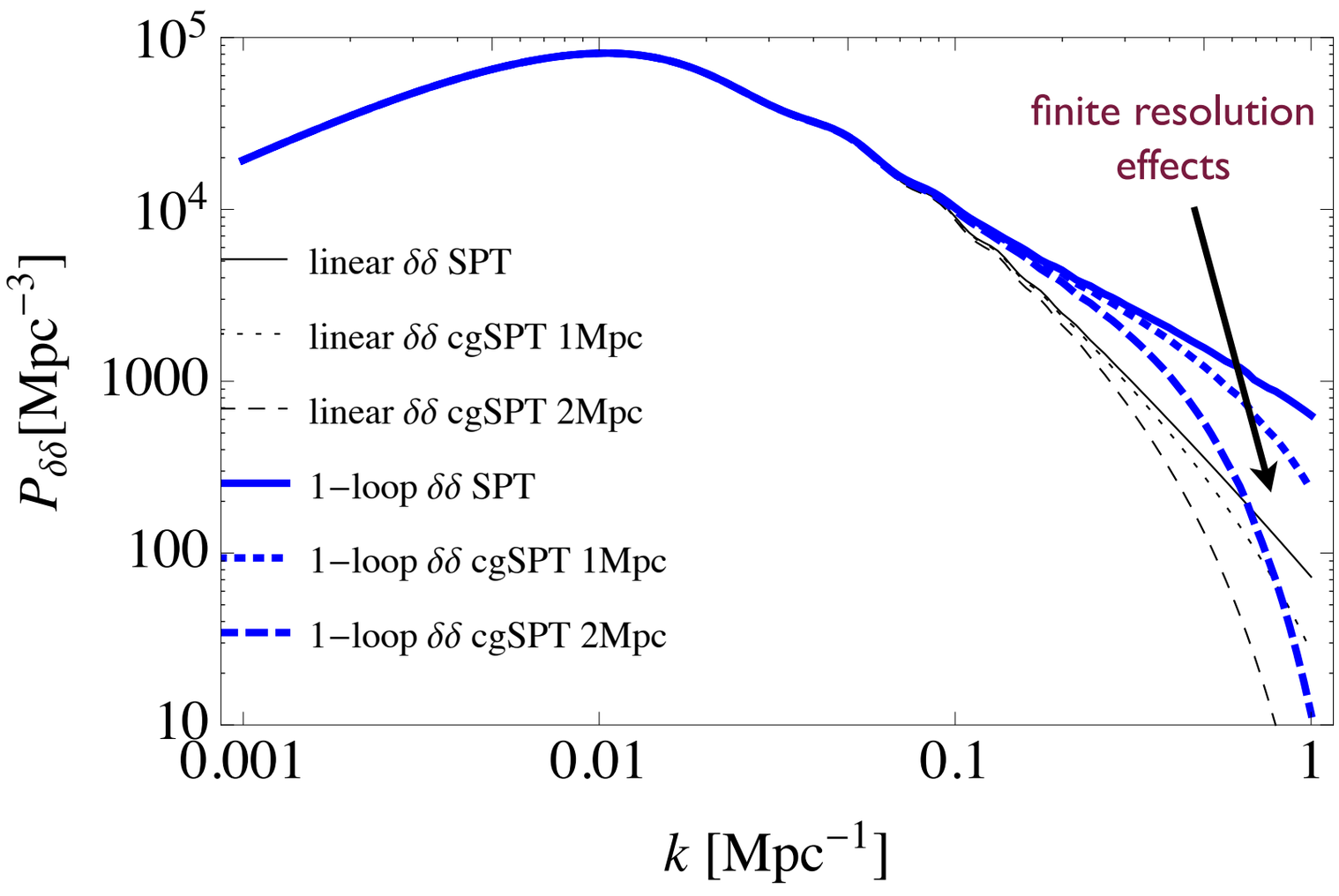
$$\xi(r) = \frac{1}{2\pi^2} \int dk\ k^2 P(k) \frac{\sin(kr)}{kr}$$

Eulerian Perturbation Theory

Coarse grained dust model

- consider only σ_x correction in Schrödinger method
- in 1st order: smoothing of input power spectrum

Density power spectrum



trivial effect:

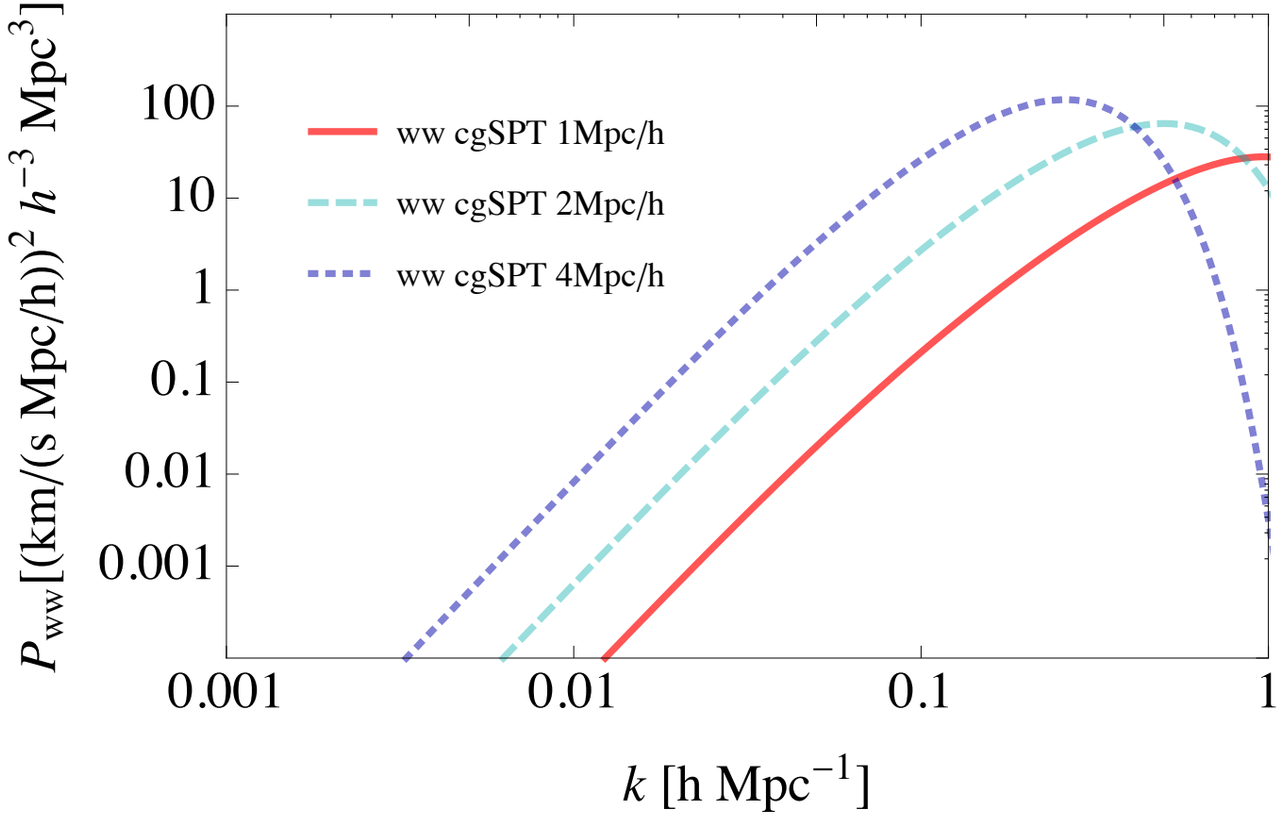
density power spectrum gets smoothed

Eulerian Perturbation Theory

Coarse grained dust model

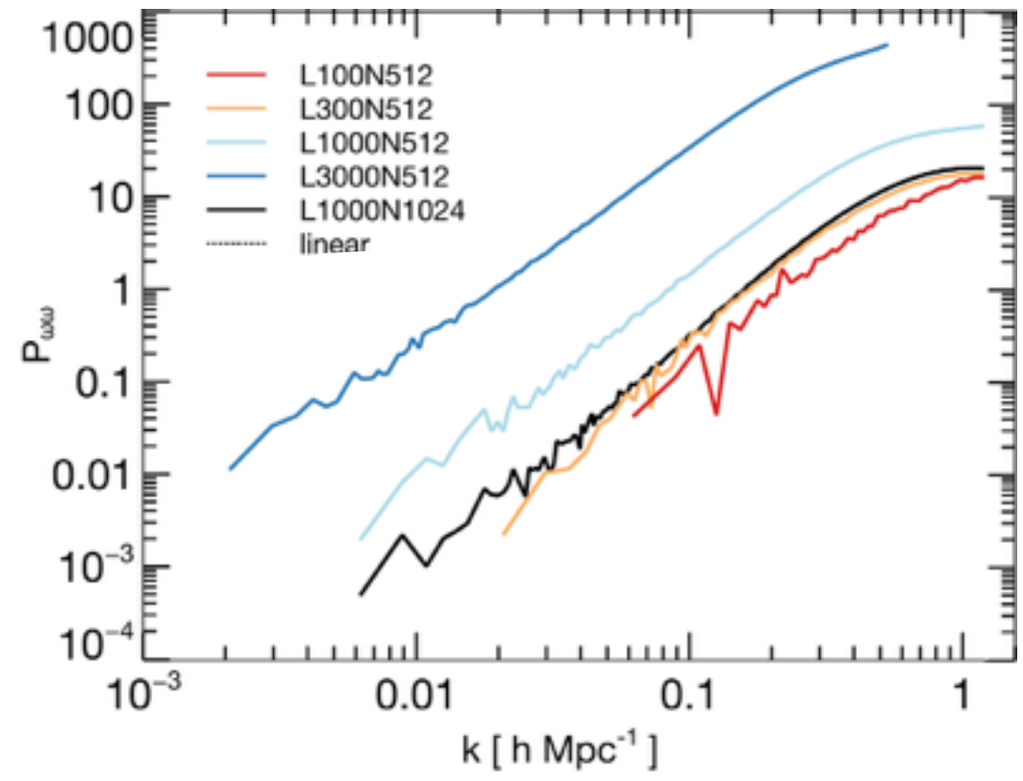
- same procedure, but mass-weighted velocity $\bar{\boldsymbol{v}} := \frac{\overline{n\boldsymbol{v}}}{\bar{n}}$
- large scale vorticity $\bar{\boldsymbol{w}} := \boldsymbol{\nabla} \times \bar{\boldsymbol{v}} \neq 0$ **!**

Vorticity power spectrum $P_{ww}(k)$



corresponding N-body data

Hahn, Angulo & Abel (2014, arXiv:1404.2280)



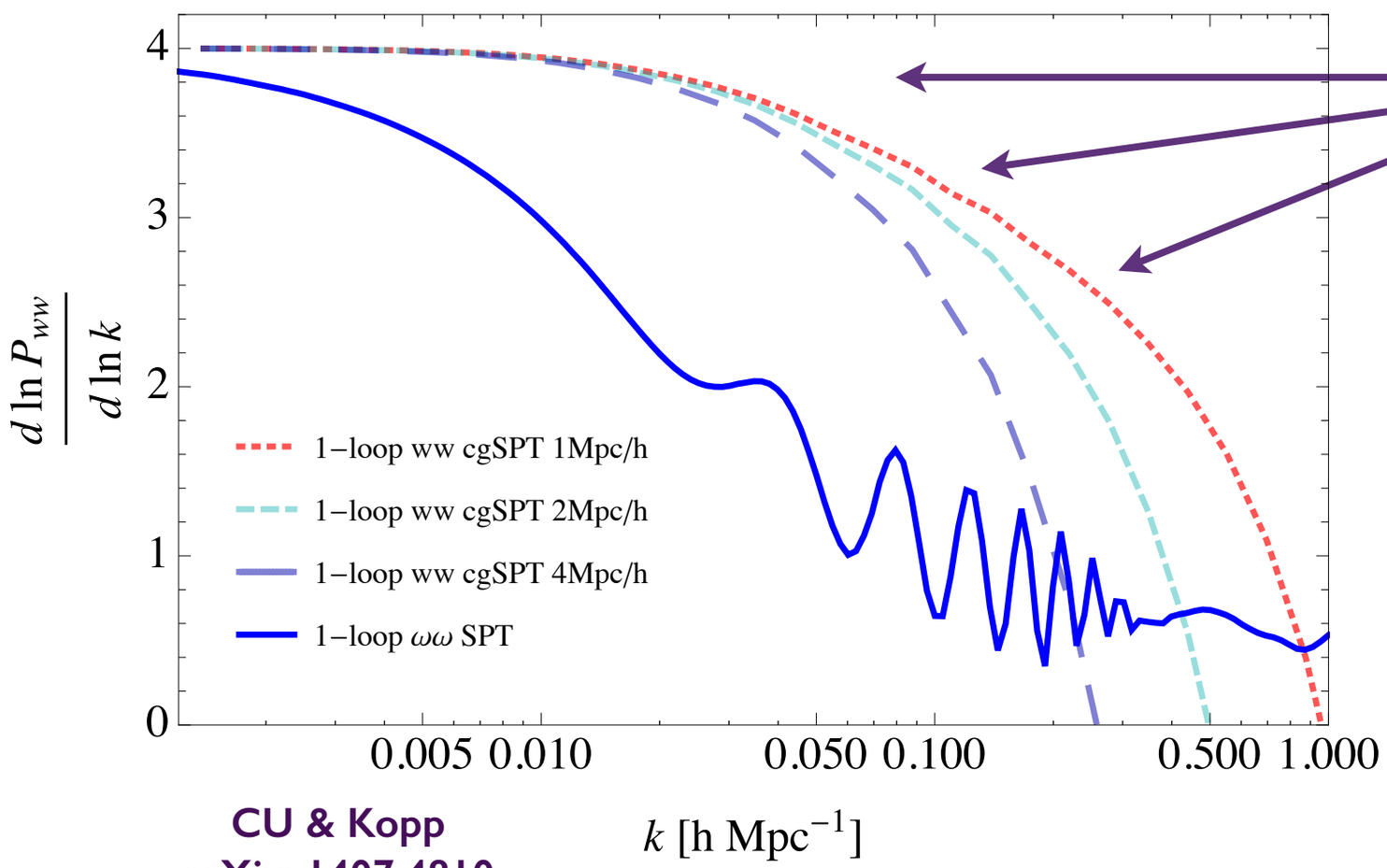
CU & Kopp
arXiv: 1407.4810

Eulerian Perturbation Theory

Coarse grained dust model

- similar to dust, but mass-weighted velocity $\bar{\boldsymbol{v}} := \frac{\overline{n\boldsymbol{v}}}{\bar{n}}$
- large scale vorticity $\bar{\boldsymbol{w}} := \boldsymbol{\nabla} \times \bar{\boldsymbol{v}} \neq 0$ **!**

Spectral index of vorticity power spectrum



CU & Kopp
arXiv: 1407.4810

corresponding results EFT of LSS

Carrasco, Foreman, Green, Senatore
(2013, arXiv: 1310.0464)

$$n_{\boldsymbol{w}} = \begin{cases} 4 & \text{for} \quad k \lesssim 0.1 \\ 3.6 & \text{for} \quad 0.1 \lesssim k \lesssim 0.3 \\ 2.8 & \text{for} \quad 0.3 \lesssim k \lesssim 0.6 \end{cases}$$

usual estimate for vorticity
arising from mass-weighted velocity

$$\boldsymbol{\omega} \sim \frac{\boldsymbol{\nabla} \times [(1+\delta)\boldsymbol{v}]}{1+\delta}$$

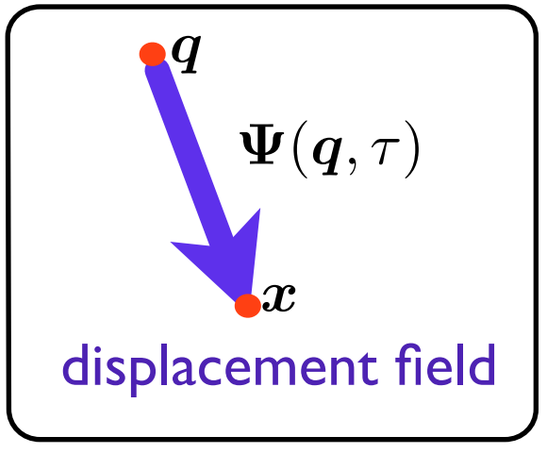
Halo correlation in redshift space with the Coarse-grained dust model

Lagrangian Perturbation Theory

Lagrangian perturbation theory

- central quantity: $\Psi(q,\tau)$, perturbative expansion
- relation to density: mass conservation $[1+\delta(\boldsymbol{x})]d^3x = d^3q$

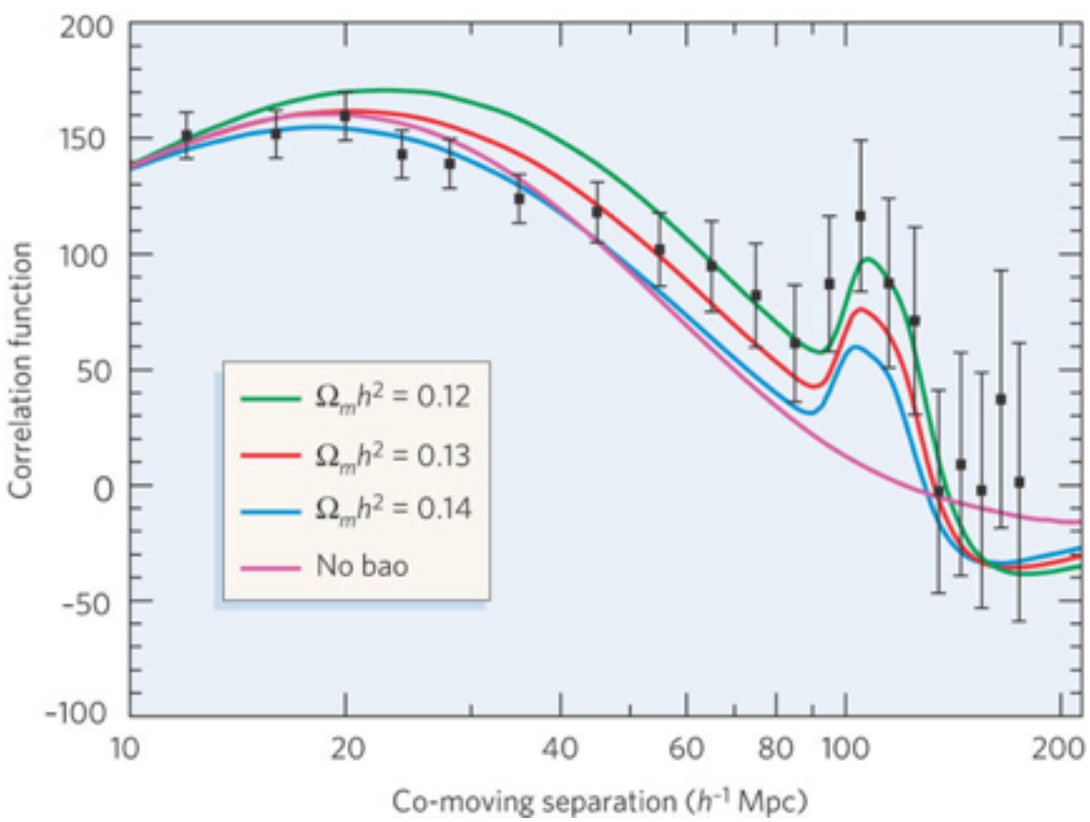
Rampf & Buchert (2012, JCAP 6)



Correlation function

- 2-point correlation function for biased tracers: halos or galaxies

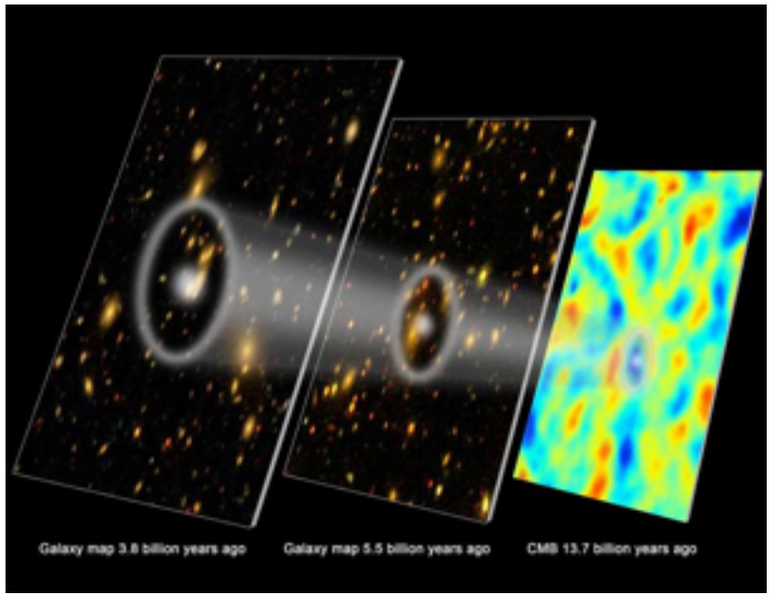
$$dP = n[1+\xi(r)]dV$$ powerful probe for cosmology



Eisenstein et al.

2005 ApJ, 633

baryon acoustic oscillations
„standard ruler“

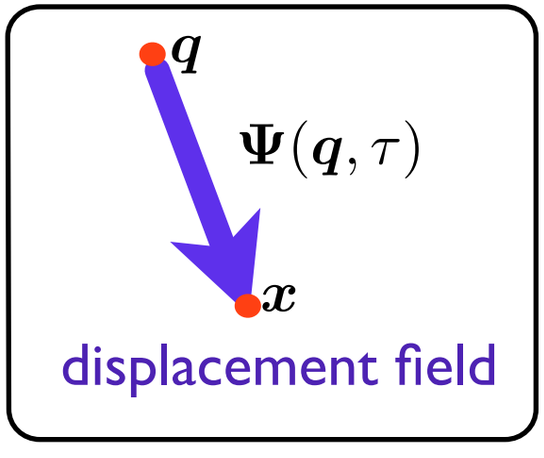


Zel'dovich approximation

(Post-)Zel'dovich approximation

- **Zel'dovich**
  - 1st order Lagrangian PT
  - relation fully non-perturbatively: physically motivated resummation

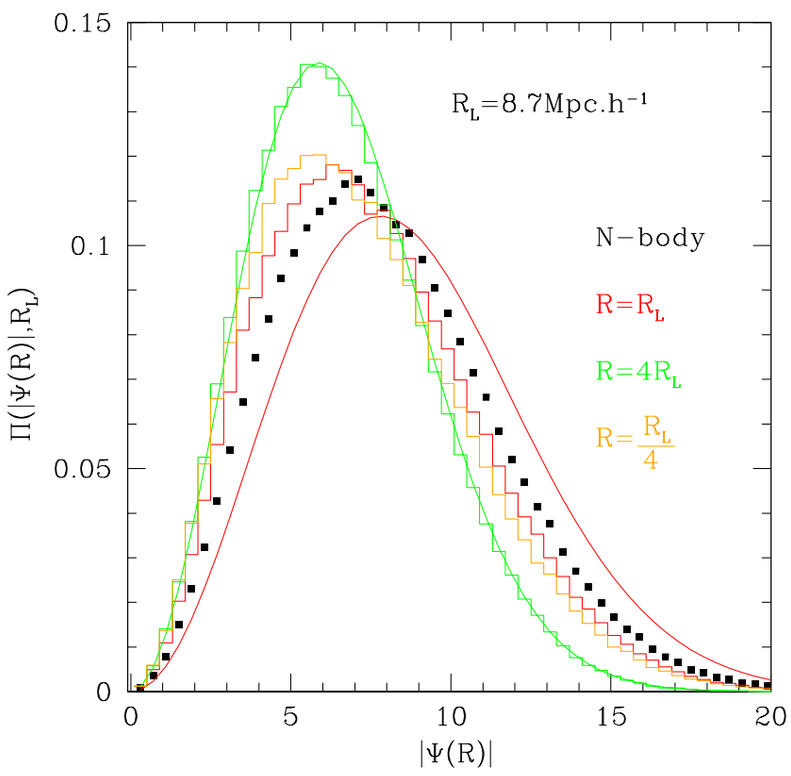
Zel'dovich (1970, A&A 5, 84)



- **truncated Zel'dovich**
  - Zel'dovich with smoothed input power spectrum
  - improves agreement with N-body

Coles, Melott, Shandarin (1993, MNRAS 260)

theoretical implementation
coarse-grained dust model



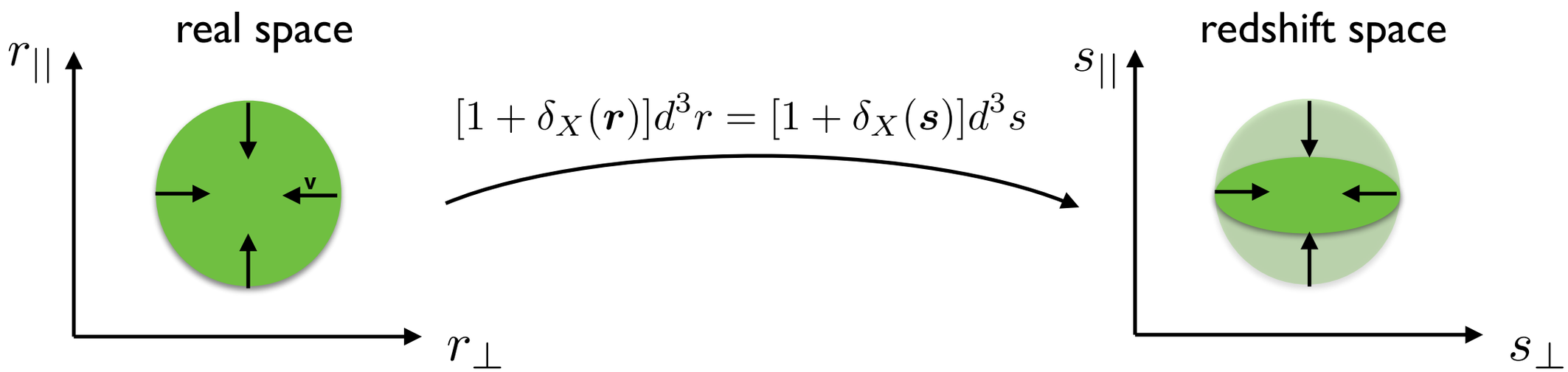
- **Post Zel'dovich**
  - higher order Lagrangian PT
  - relation partially non-perturbatively: Convolution LPT

Carlson et al. (2012, MNRAS 429)

Redshift space distortions

Redshift space distortions

- observations in redshift space affected by velocities $s_{||} = r_{||} + v/\mathcal{H} \quad s_\perp = r_\perp$



Gaussian streaming model

- joint probability distribution: density & velocity

Redshift space correlation function

real space correlation

Gaussian velocity distribution

$$1+\xi_X(s_{||}, s_\perp, t) = \int_{-\infty}^{\infty} \frac{dr_{||}}{\sqrt{2\pi}\sigma_{12}(r, r_{||}, t)} (1+\xi_X(r,t)) \exp\left[-\frac{\left(s_{||} - r_{||} - v_{12}(r,t) r_{||}/r\right)^2}{2\sigma_{12}^2(r, r_{||}, t)}\right]$$

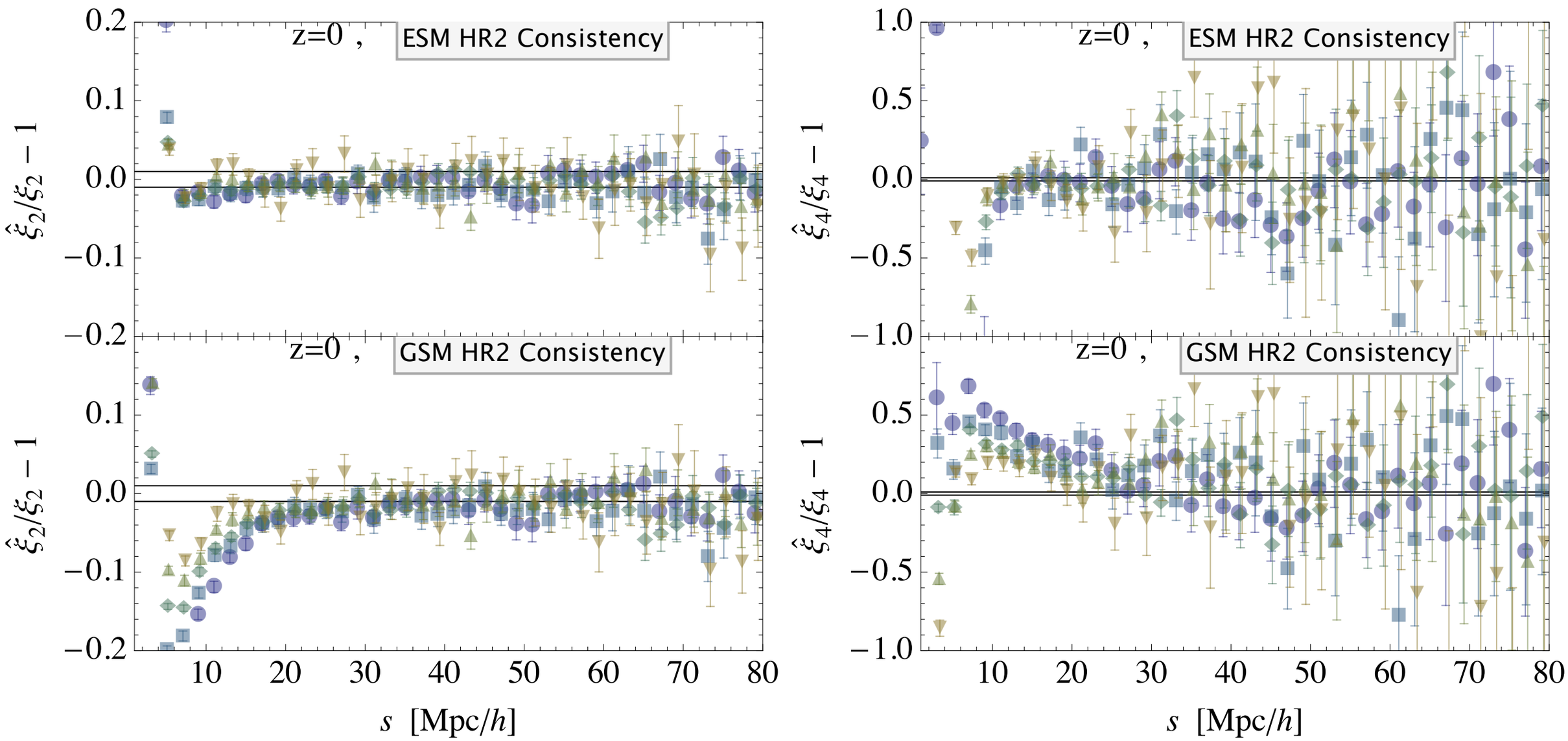
pairwise velocity: mean & variance

Reid & White (2011, MNRAS 417)

Redshift space distortions

Edgeworth streaming model

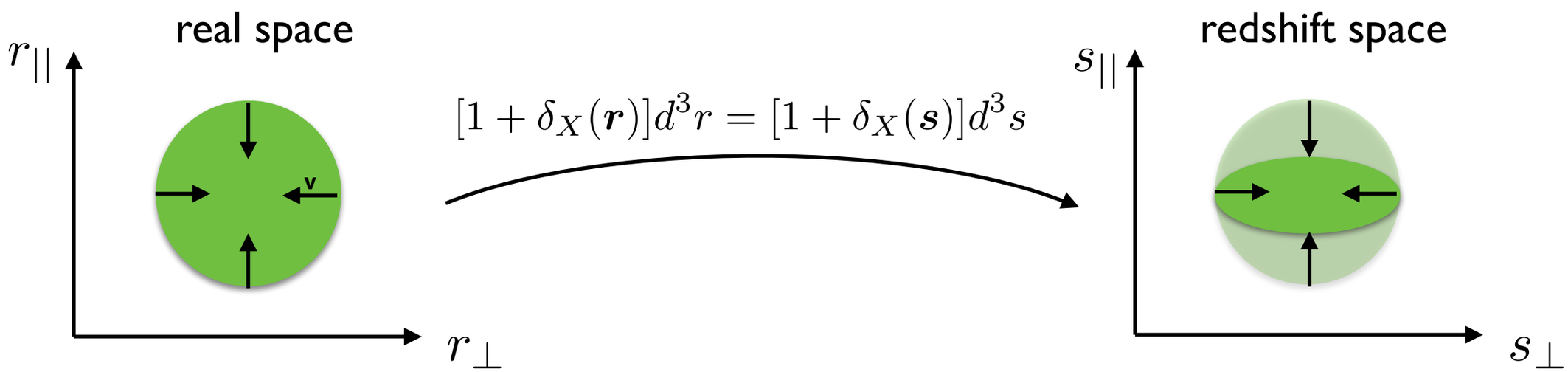
- generalization of Gaussian streaming model
  - non-Gaussian corrections improve on small scales: 1% down to 10 Mpc
  - general class of distribution functions



Redshift space distortions

Redshift space distortions

- observations in redshift space affected by velocities $s_{||} = r_{||} + v/\mathcal{H} \quad s_\perp = r_\perp$



Gaussian streaming model

- joint probability distribution: density & velocity

Wang, Reid & White (2014, MNRAS 437)

Lagrangian Perturbation Theory

Redshift space correlation function

real space correlation

Gaussian velocity distribution

$$1+\xi_X(s_{||}, s_\perp, t) = \int_{-\infty}^{\infty} \frac{dr_{||}}{\sqrt{2\pi}\sigma_{12}(r, r_{||}, t)} (1+\xi_X(r,t)) \exp\left[-\frac{\left(s_{||} - r_{||} - v_{12}(r,t) r_{||}/r\right)^2}{2\sigma_{12}^2(r, r_{||}, t)}\right]$$

pairwise velocity: mean & variance

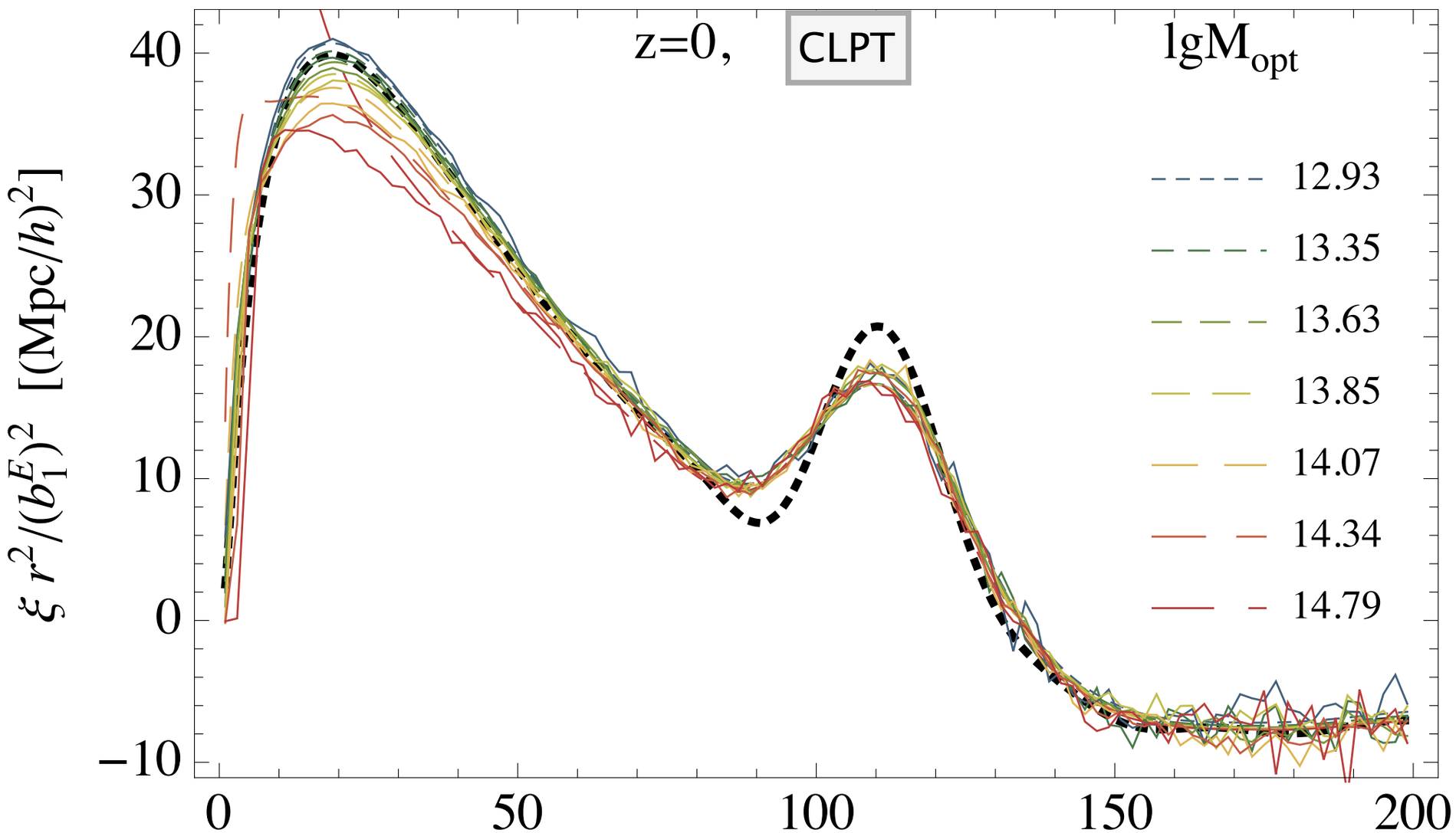
Reid & White (2011, MNRAS 417)

Redshift space distortions

Streaming parameters: truncated CLPT

Real space correlation function ξ(r)

- best agreement for 1 Mpc/h
- smoothing in R(M) worse - need peak bias Baldauf, Desjacques & Seljak (arXiv: 1405.5885)

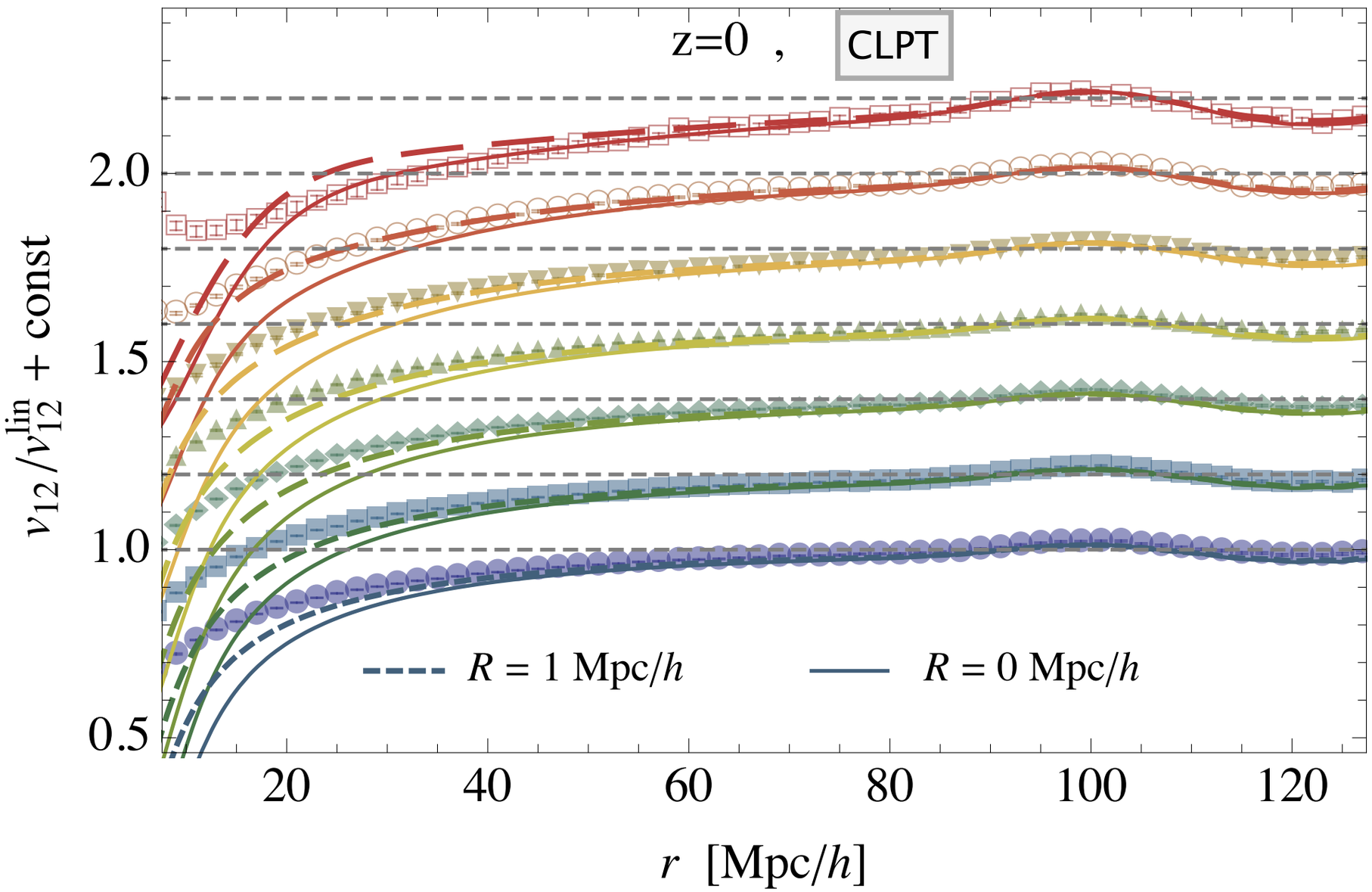


Redshift space distortions

Streaming parameters: truncated CLPT

Pairwise velocity $v_{12}(r)$

- best agreement for 1 Mpc/h, small scales: acceleration important
- smoothing in R(M) worse - velocity bias? Baldauf, Desjacques & Seljak (arXiv: 1405.5885)

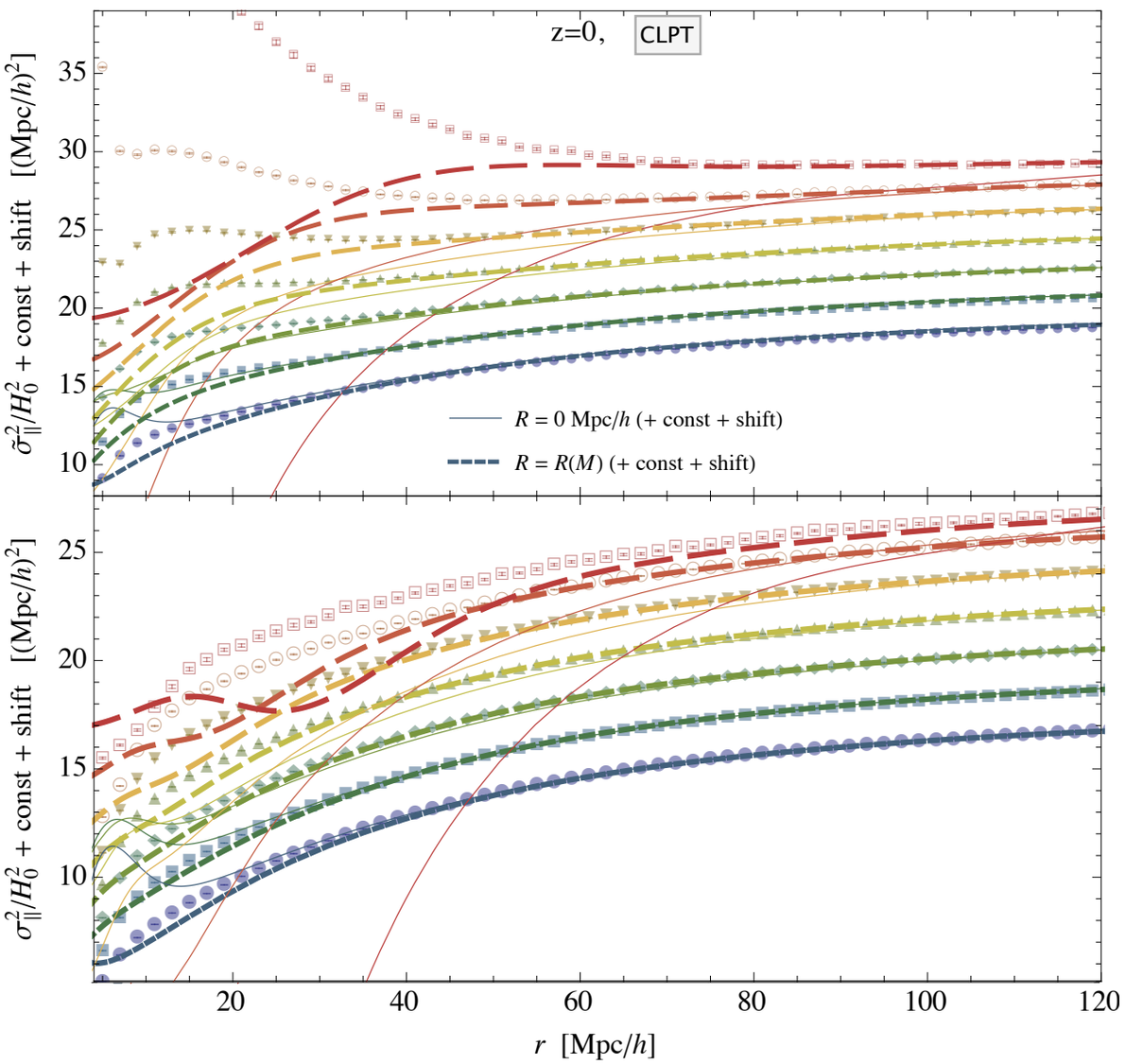


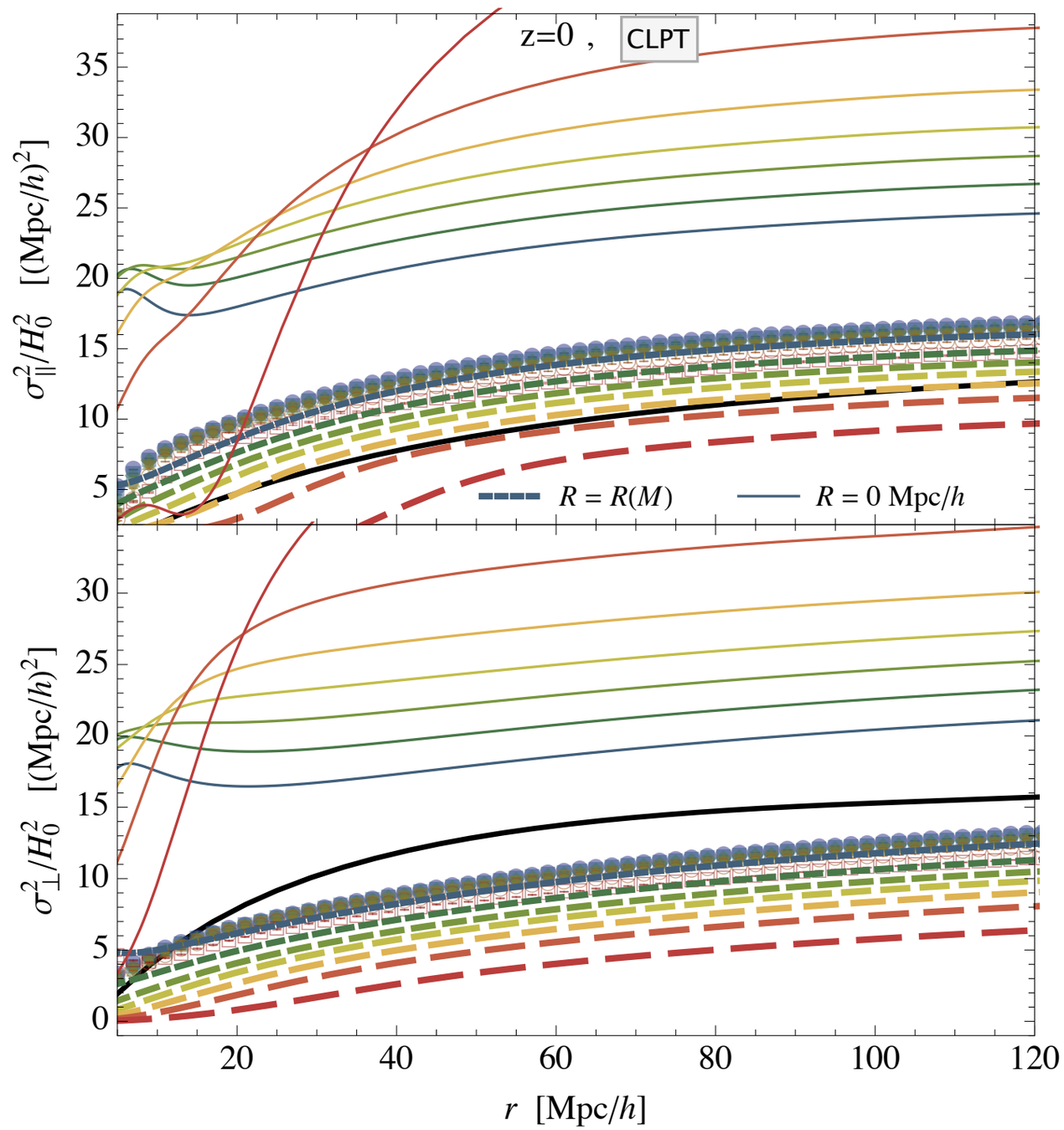
Redshift space distortions

Streaming parameters: truncated CLPT

Pairwise velocity dispersion $\sigma_{12}(r)$

- best agreement for R(M)
- consider cumulant not moment



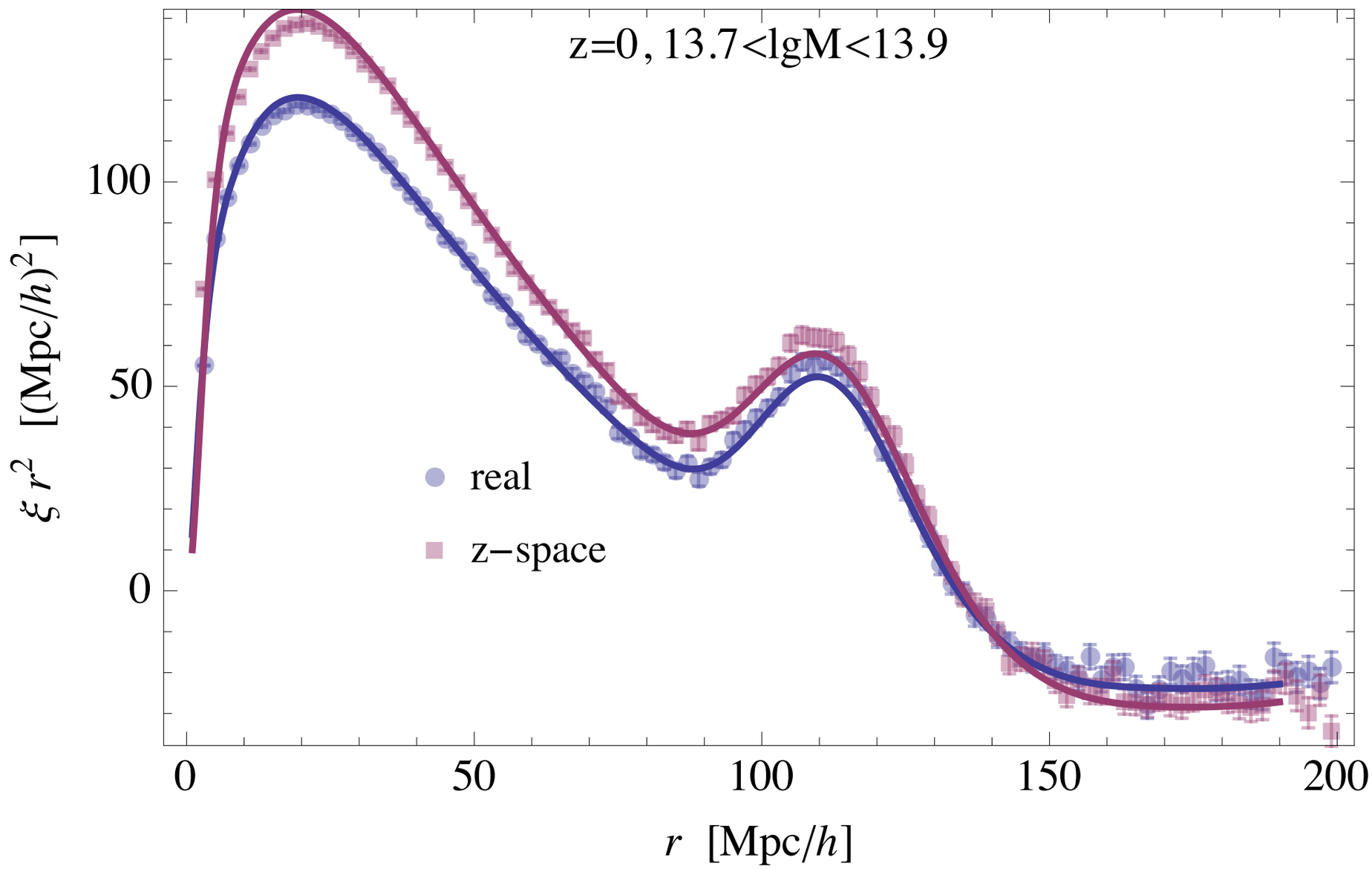


Redshift space distortions

Gaussian streaming model: truncated CLPT

- real space $\xi(r)$ & redshift space monopole $\xi_0(s)$
- well described by Convolution LPT

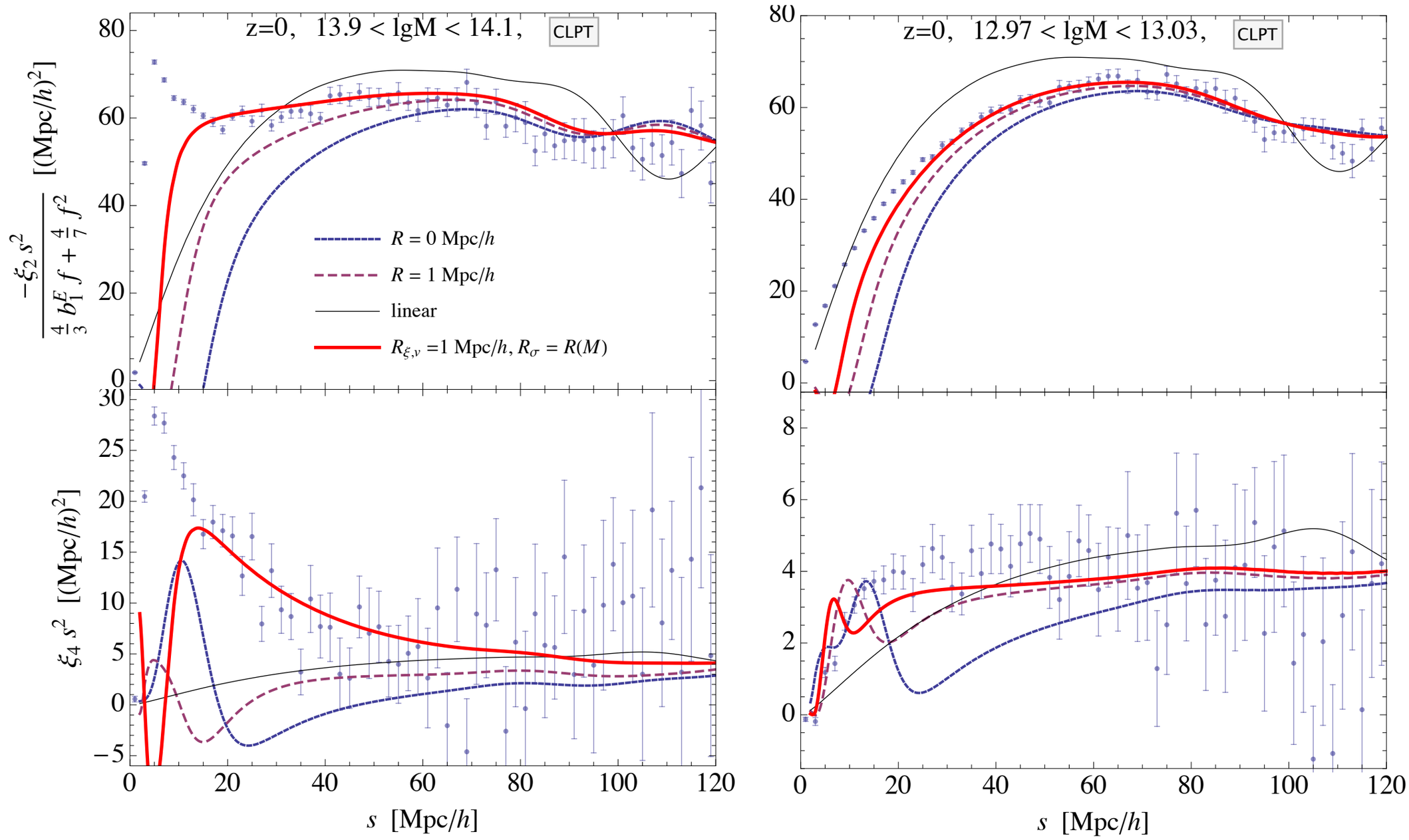
Carlson et al.
(2012, MNRAS 429)



Redshift space distortions

Gaussian streaming model: truncated CLPT

- redshift space quadrupole $\xi_2(s)$ & hexadecapole $\xi_4(s)$
- truncated CLPT with hybrid smoothing outperforms CLPT

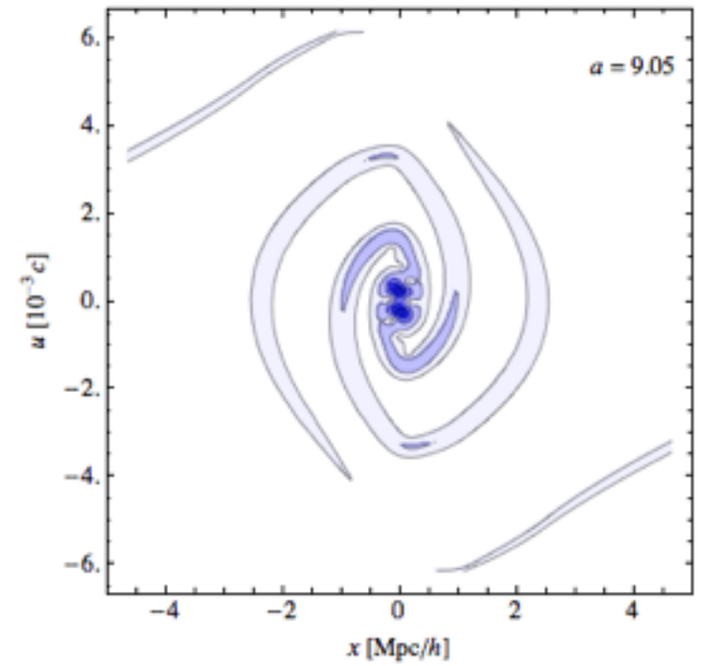


Conclusion & Prospects

Schrödinger method

- models CDM using self-gravitating scalar field
- analytical tool for structure formation
  - multi-streaming & virialization

CU, Kopp & Haugg
(2014, PRD 90, 023517)



Coarse-grained dust model

- mass-weighted velocity
  - vorticity compatible with N-body

CU & Kopp
(arXiv: 1407.4810)

Halo correlation in redshift space

- generalization of Gaussian streaming model
- truncated Post-Zel'dovich approximation

Kopp, CU, Haugg & Achitouv
(in preparation)

Prospects

- disentangle limitations of dust & perturbation theory
- understand universal halo density profiles (NFW)
- DM models: wavelike (axion), warm & (non-)relativistic neutrinos