IR-Resummation in the EFT of LSS

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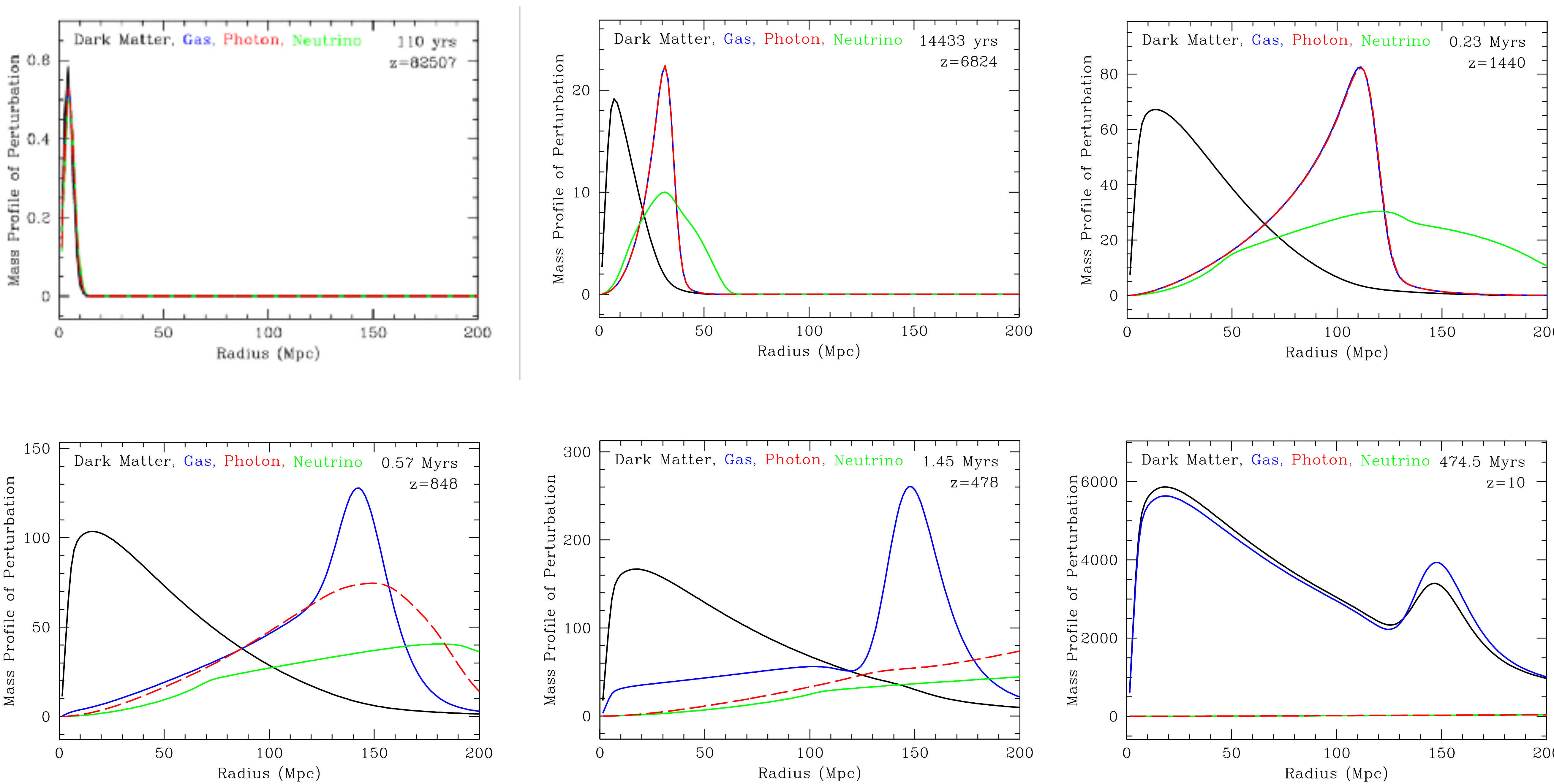
Seminar - Berkeley 2018

Senatore, Trevisan: arXiv 1710.02178, Scoccimarro, Trevisan: arXiv xxxx.xxxx


The Bao
XIAO LONG BAO
NEW!

BAO in pictures

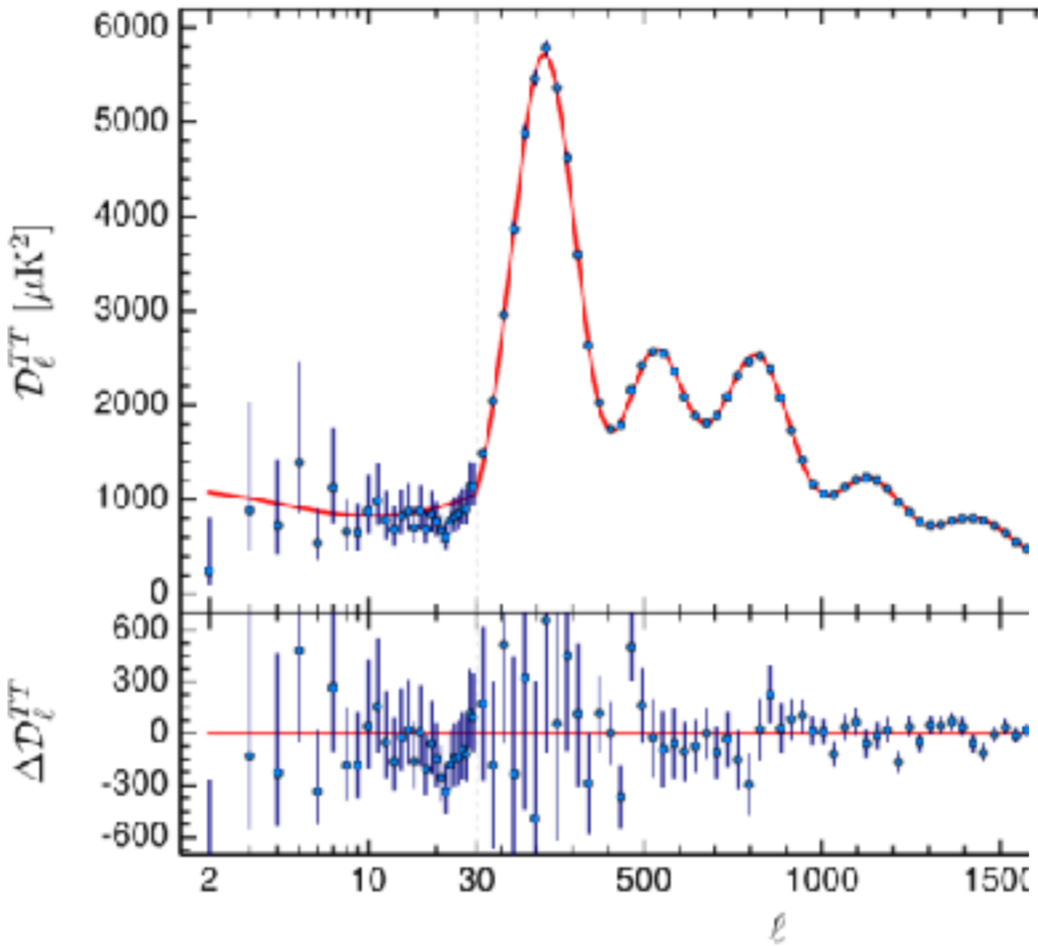


Eisenstein et al. 0604361

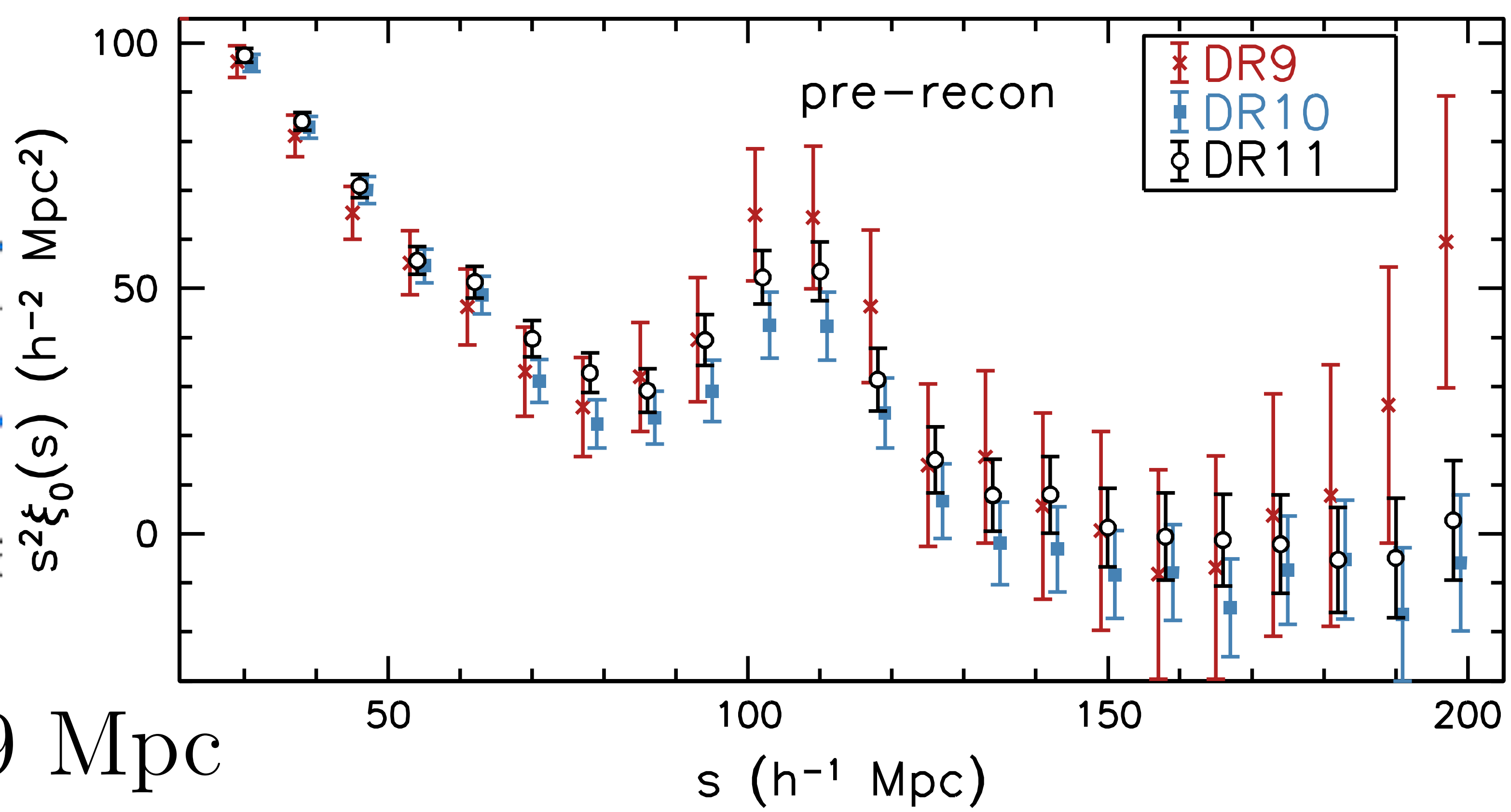

R = 150 Mpc

BAO in the sky

Planck 2015



Anderson et al. 1312.4877



$$r_{drag} = 147.33 \pm 0.49 \text{ Mpc}$$

$$D_V(z = 0.57) = 2056 \pm 20 \text{ Mpc}$$

Outline

- Eulerian PT and IR-resummation
- Resummation of IR mode-coupling for the 2-PF
- IR-resummation for the 3-PF
- A useful numerical approximation

Standard Perturbation Theory (SPT)

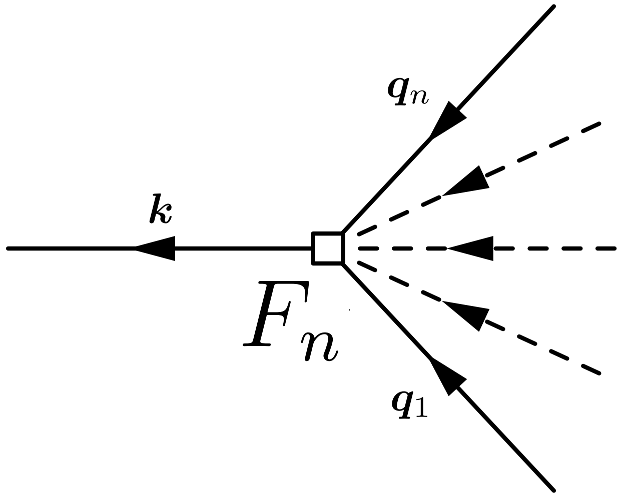
Fluid Equations:

$$\partial_\tau \delta + \vec{\nabla} \cdot [(1+\delta)\vec{v}] = 0,$$

$$\partial_\tau \vec{v} + \mathcal{H}\,\vec{v} + \vec{v} \cdot \vec{\nabla}\vec{v} + \vec{\nabla}\phi = 0$$

$$\triangle \phi = \frac{3}{2}\mathcal{H}^2 \Omega_{\mathrm{m}} \delta$$

Perturbative solution: $\delta_n(\vec{k})$



$\delta^n_{\mathrm{linear}}(\vec{q})$

Effective Field Theory

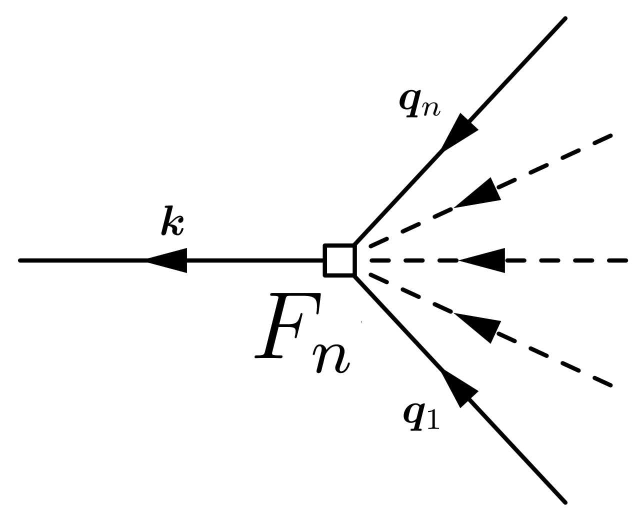
Fluid Equations:

$$\partial_\tau \delta + \vec{\nabla} \cdot [(1+\delta)\vec{v}] = 0,$$

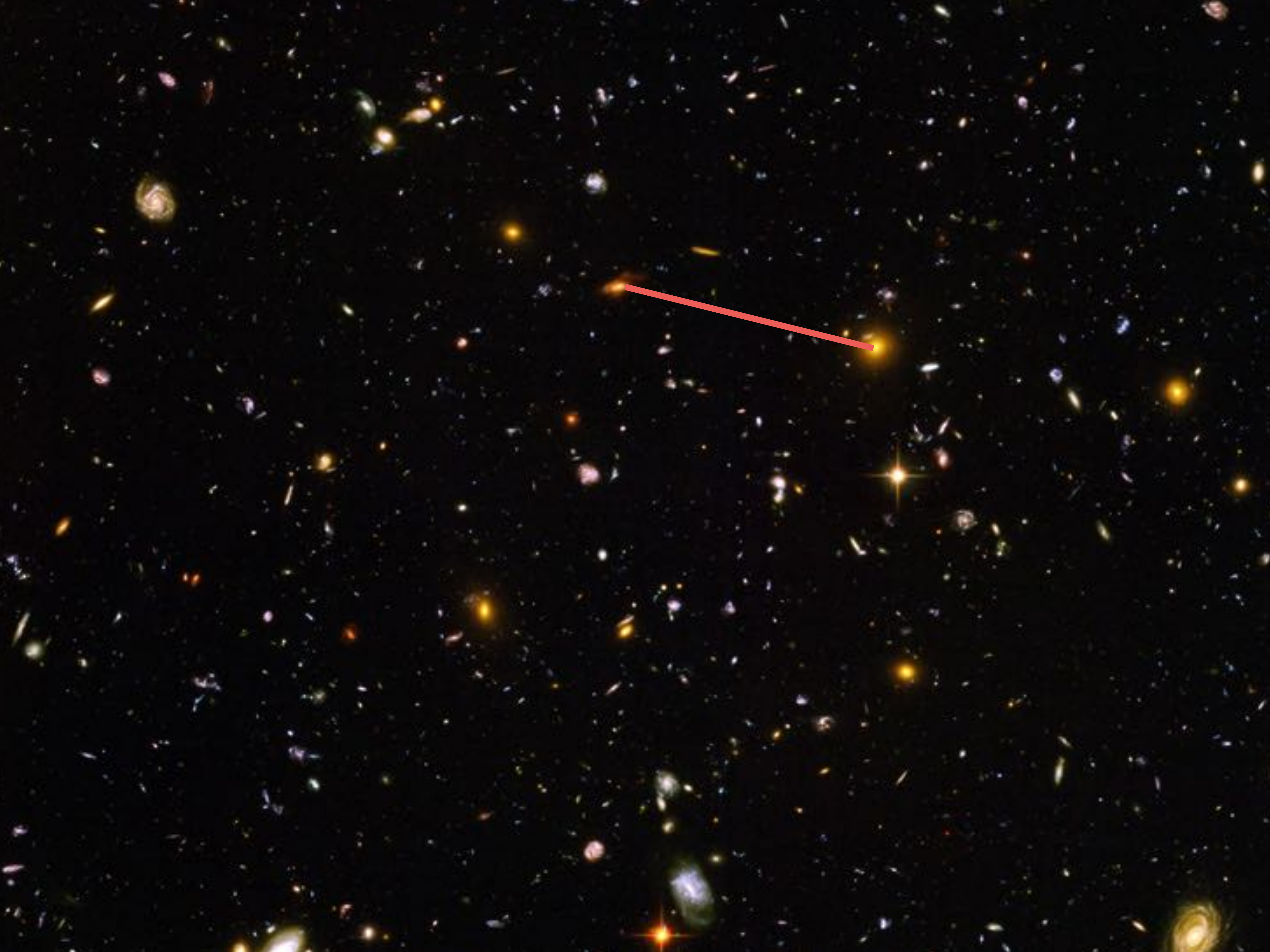
$$\partial_\tau \vec{v} + \mathcal{H}\,\vec{v} + \vec{v} \cdot \vec{\nabla}\vec{v} + \vec{\nabla}\phi = - \frac{\vec{\nabla} \cdot \tau}{a\rho}$$

$$\triangle \phi = \frac{3}{2}\mathcal{H}^2 \Omega_{\mathrm{m}} \delta$$

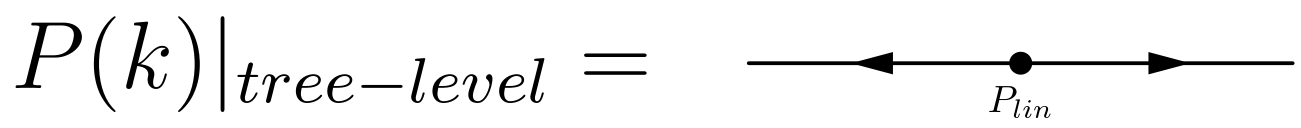
Perturbative solution: $\delta_n(\vec{k})$

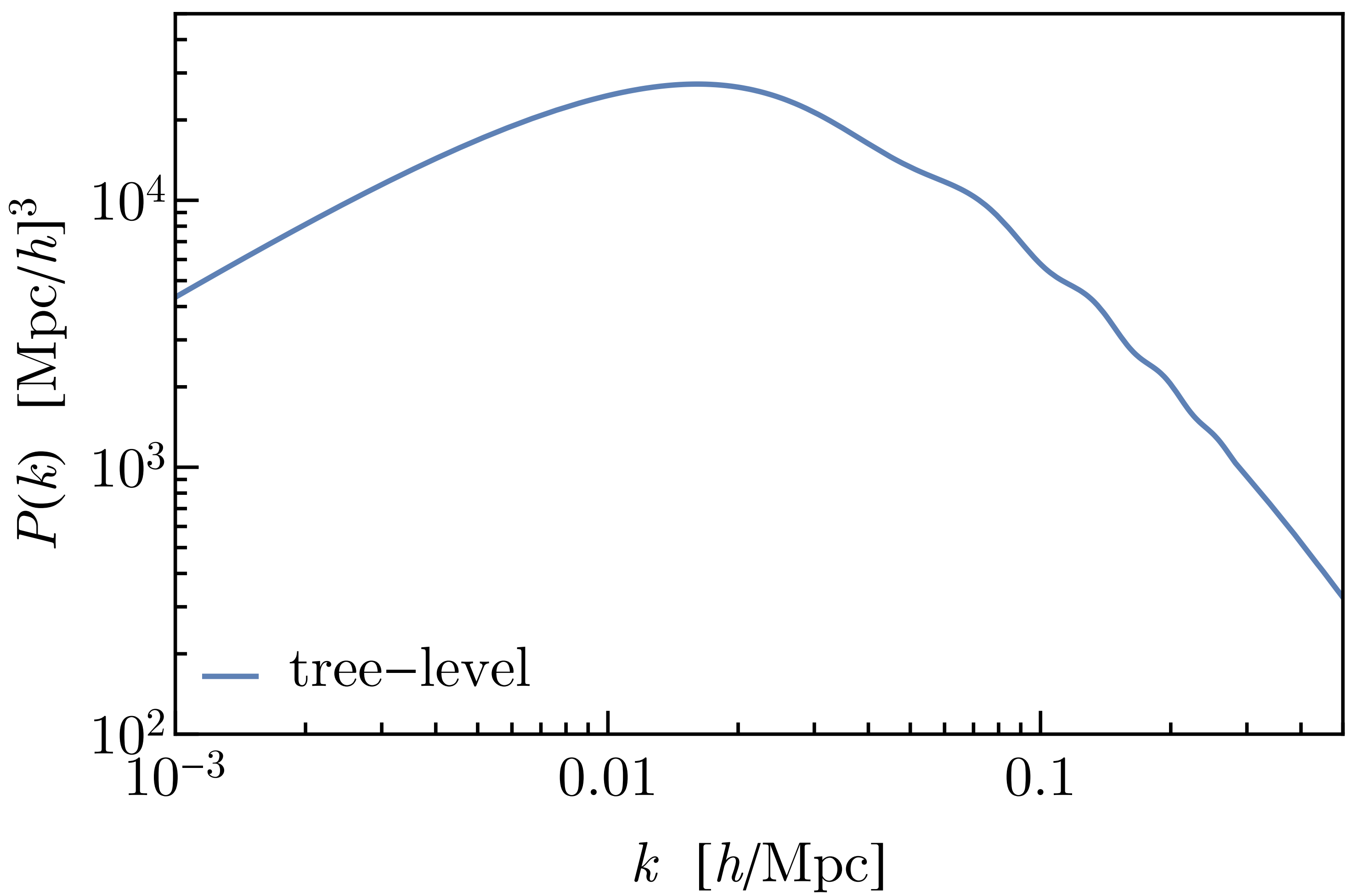


$\delta^n_{\mathrm{linear}}(\vec{q})$



SPT 2-Point Function (2-PF)



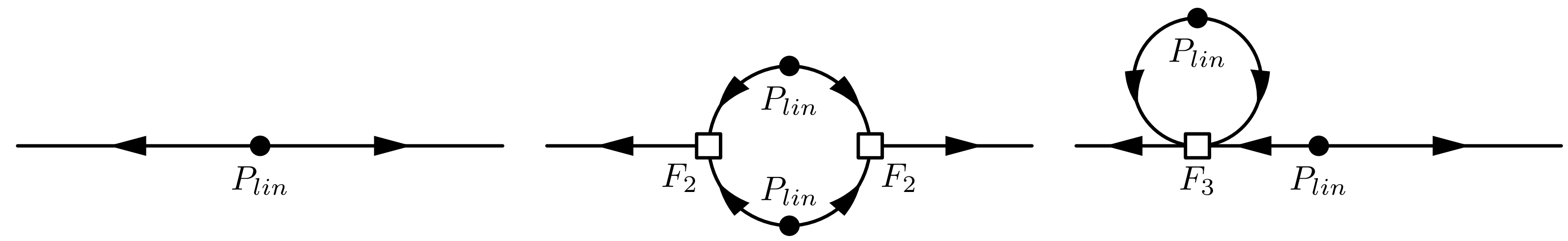


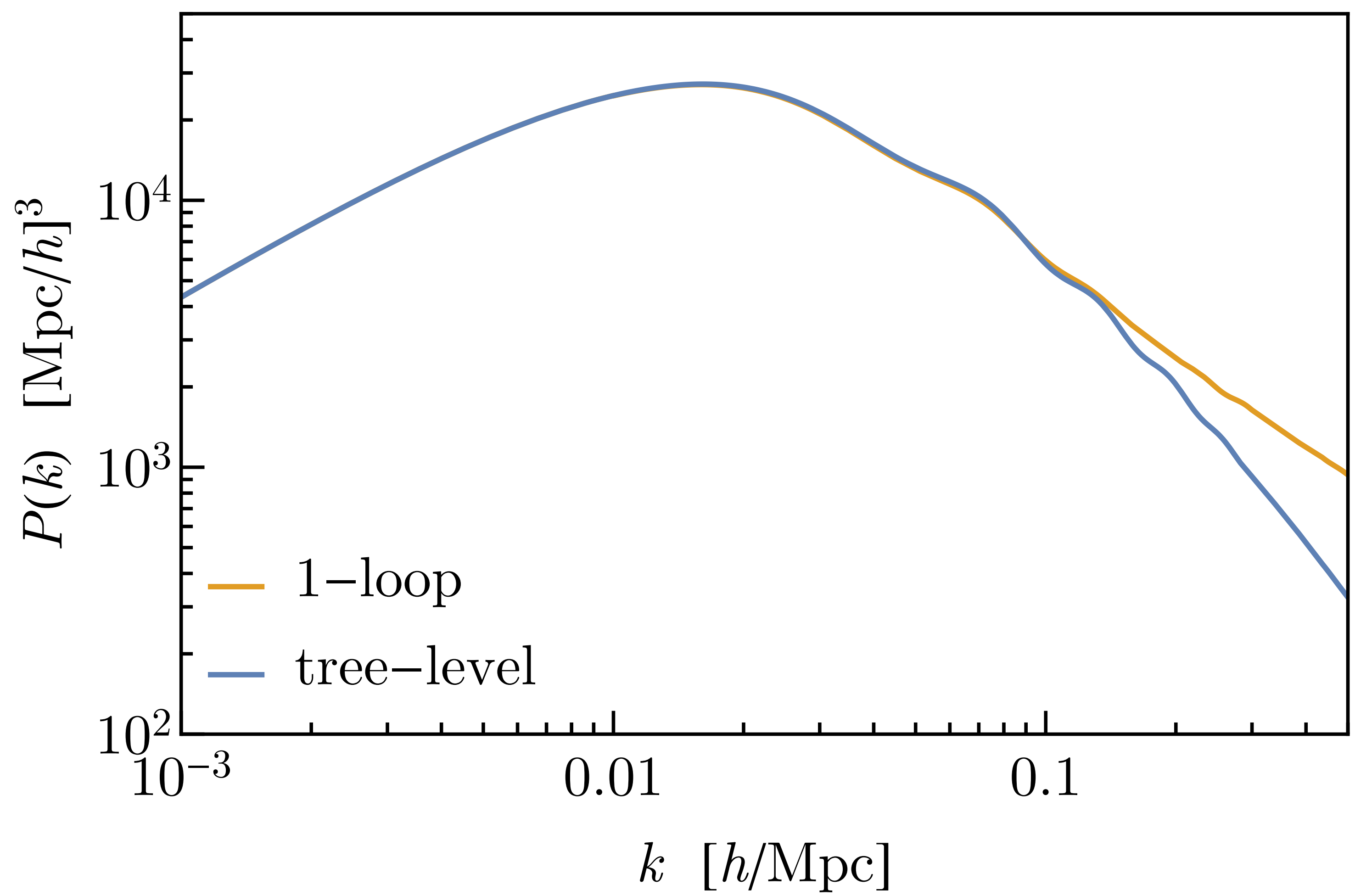
Where is the BAO in Fourier space?

The BAO signal is ~5% oscillation with freq. 1/150 Mpc

SPT 2-Point Function

$$P(k)\big|_{1-loop} =$$

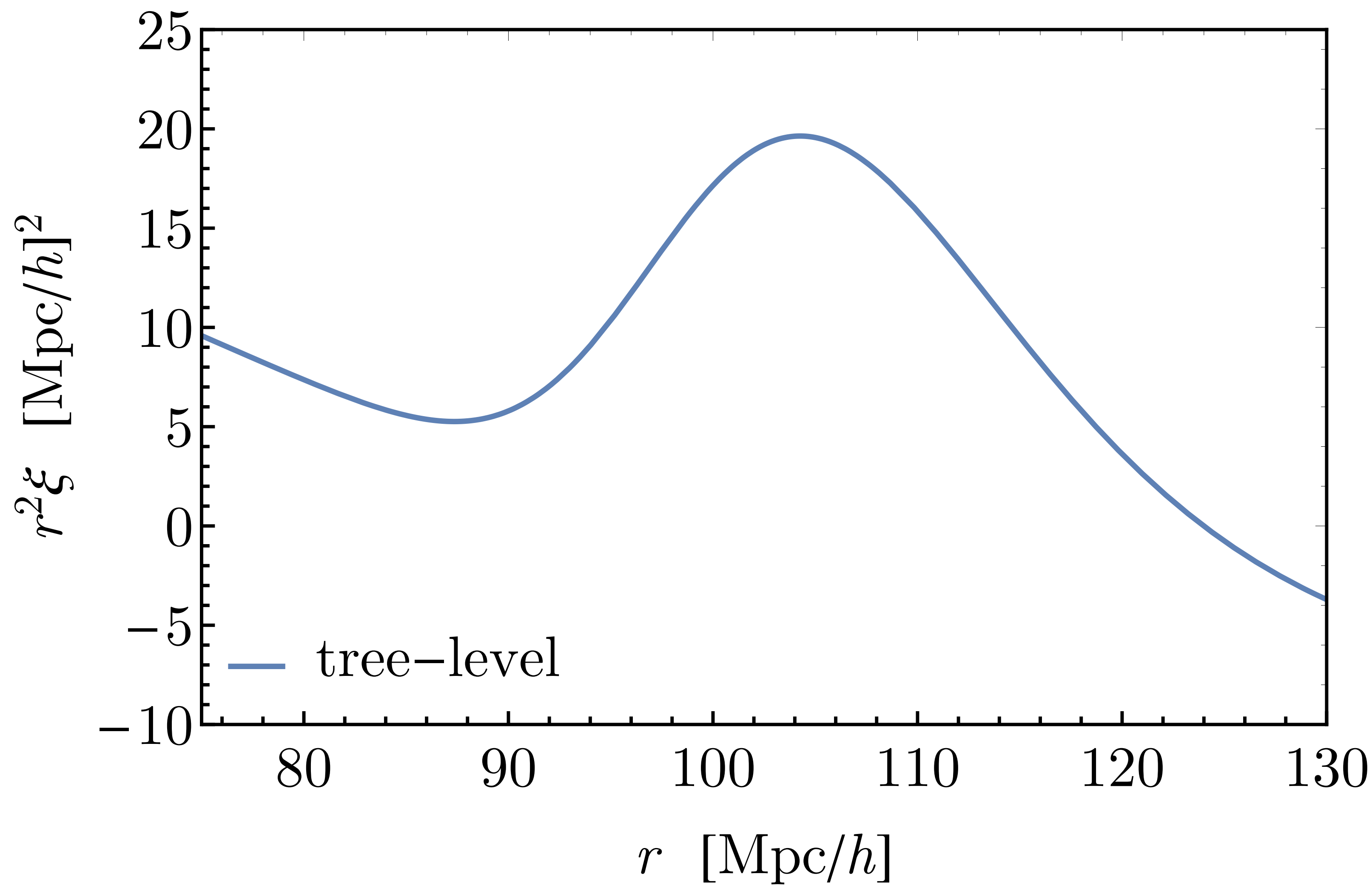




Perturbation Theory should recover the BAO, so the more loops the better, right?

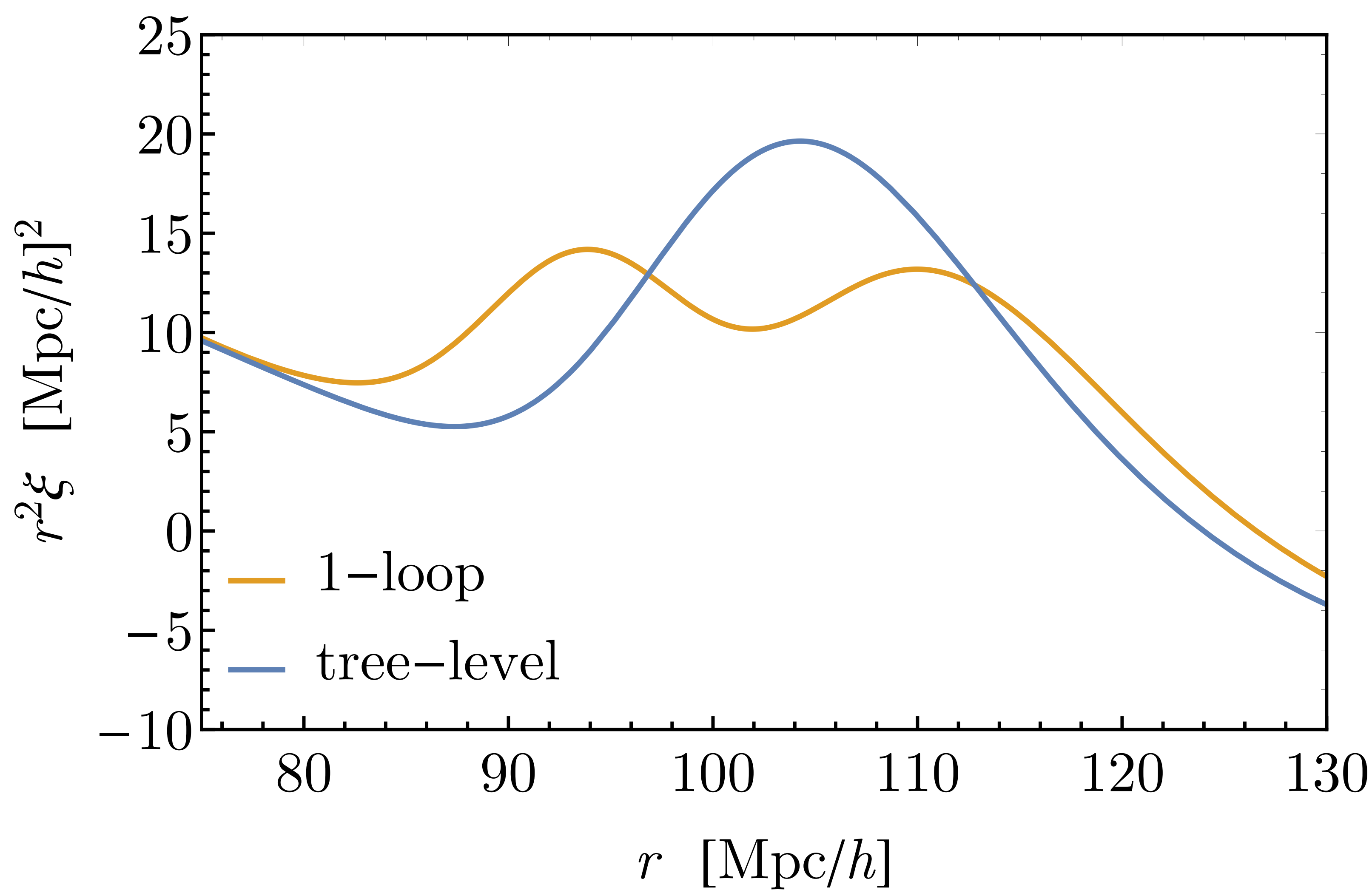
SPT 2-Point Function

SPT completely fails around the BAO scale



SPT 2-Point Function

SPT completely fails around the BAO scale



SPT 2-Point Function

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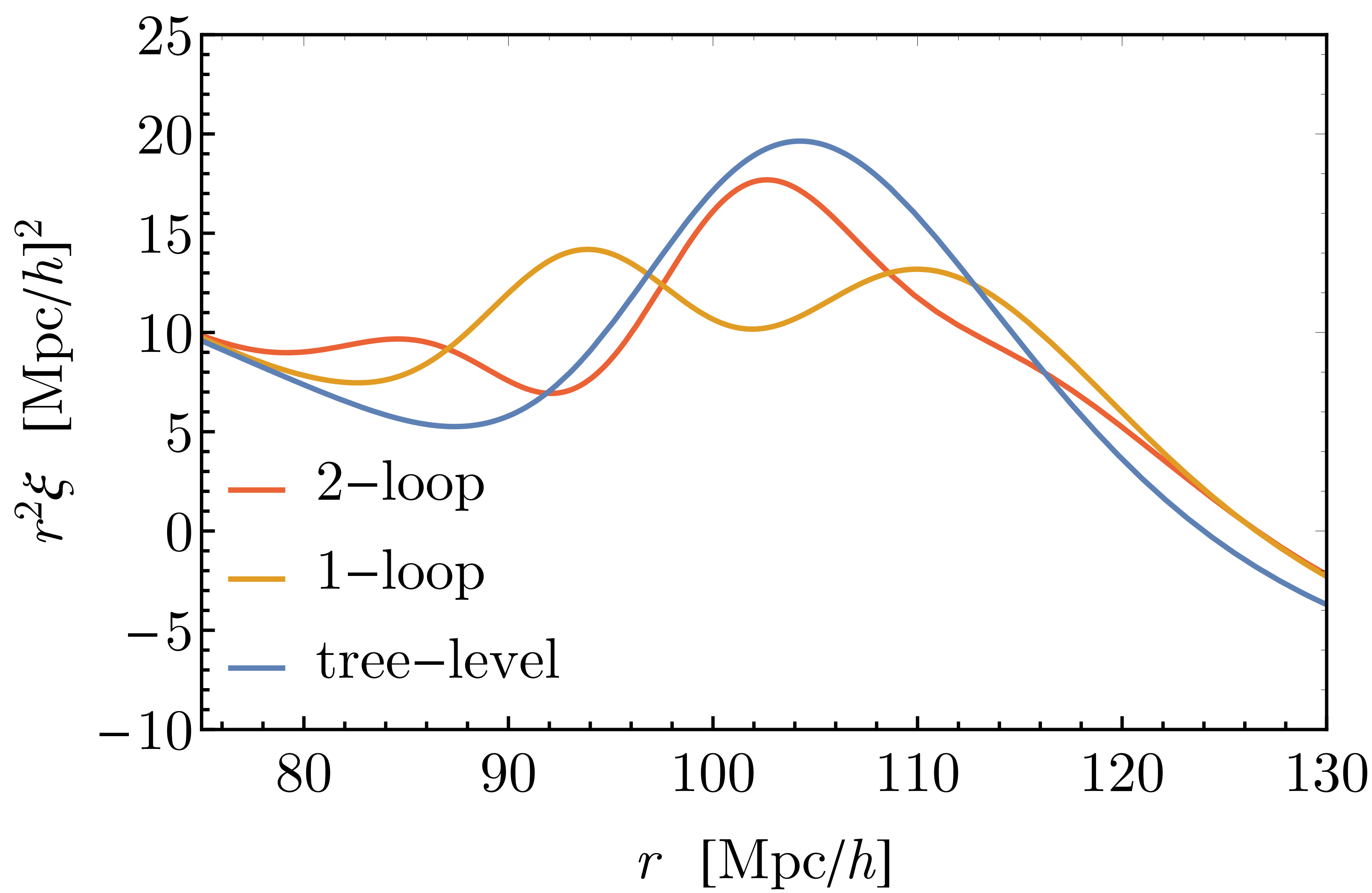


What is going on?

Why SPT fails?

Let's focus on the 1-loop correction for a mode ***k***

$$P_{1-\mathrm{loop}}(k) \sim \frac{1}{2}\int_{p\ll\Lambda} \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{(\boldsymbol{p}\cdot\boldsymbol{k})^2}{p^4}\left[P_{\mathrm{lin}}(|\boldsymbol{k}-\boldsymbol{p}|)+P_{\mathrm{lin}}(|\boldsymbol{k}+\boldsymbol{p}|)-2P_{\mathrm{lin}}(|\boldsymbol{k}|)\right]P_{\mathrm{lin}}(p).$$

$$P_{\mathrm{lin}} \propto k^n$$

For a smooth component $\sim P_{\mathrm{lin}}(k)\dfrac{p^2}{k^2},$

Very long modes ($p \ll k$) contribute little to the loop

Why SPT fails?

The smooth component receives small contributions from loop integrals

$$P_{1-\mathrm{loop}}(k) \sim P_{\mathrm{lin}}(k) \int_{p \ll k} \frac{\mathrm{d}^3 p}{(2\pi)^3} P_{\mathrm{lin}}(p)$$
$$= P_{\mathrm{lin}}(k) \epsilon_{\delta_<}$$

and $\quad \epsilon_{\delta_<} \ll 1$

which is good for PT

Why SPT fails?

Let's go back to the 1-loop expression

$$P_{1-\text{loop}}(k) \sim \frac{1}{2}\int_{p\ll\Lambda} \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{(\boldsymbol{p}\cdot\boldsymbol{k})^2}{p^4}\left[P_{\text{lin}}(|\boldsymbol{k}-\boldsymbol{p}|)+P_{\text{lin}}(|\boldsymbol{k}+\boldsymbol{p}|)-2P_{\text{lin}}(|\boldsymbol{k}|)\right]P_{\text{lin}}(p).$$

$$P_{\text{lin}} \propto \sin(k/k_{osc})$$

For the BAO $\sim P^w_{\text{lin}}(k)\left(\cos\left(p\ell_{\text{BAO}}\right)-1\right)$

There is an infrared contribution from $\ell^{-1}_{\text{BAO}} \lesssim p \lesssim k$

Why SPT fails?

The wiggly component (BAO) receives large infrared (IR-enhanced) contribution from loop integrals

$$P^{w}_{1-\text{loop}}(k) \sim k^2 P^{w}_{\text{lin}}(k) \int_{\ell_{\text{BAO}}^{-1} \lesssim p \lesssim k} \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{P_{\text{lin}}(p)}{p^2}$$
$$= P^{w}_{\text{lin}}(k)\epsilon_{s_<}.$$

and $\quad \epsilon_{s_<} \approx 1$

Bad for doing PT! Better not to expand

A brief excursus on Lagrangian PT (LPT)

Instead of using comoving coordinates, use fluid coordinates

$$\vec{x}(\vec{q},t) = \vec{q} + \vec{s}(\vec{q},t)$$

and sum over all initial positions

$$1 + \delta(\vec{x},t) = \int \mathrm{d}^3 q\, \delta_D^3(\vec{x} - \vec{q} - \vec{s}(\vec{q},t))$$

to obtain the 2-PF

$$P(k) = \int \mathrm{d}^3 q_{12}\, e^{-i\vec{k}\cdot\vec{q}_{12}} \left\langle e^{-i\vec{k}\cdot(\vec{s}(q_1)-\vec{s}(q_2))} \right\rangle$$

A brief excursus on Lagrangian PT (LPT)

The linear order solution for the displacement field is

$$\vec{s}(p) \simeq \vec{s}_1(p) = i\frac{\vec{p}}{p^2}\delta_{\rm lin}(p),$$

and leads to the Zel'dovich approximation:

$$P(k) = \int \mathrm{d}^3 q_{12}\, e^{-i\vec{k}\cdot\vec{q}_{12}} e^{-\frac{1}{2}k_i k_j \langle s_i s_j\rangle(q_{12})}$$

where

$$\langle s_i s_j \rangle \sim \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\frac{P_{\rm lin}(p)}{p^2}$$

To fix just resum…

- IR-resummation (à la EPT) Senatore, Zaldarriaga '15

  numerically more demanding

- Consistency relations (using EP) Baldauf et. al '15

  split P_{lin} into smooth and wiggly components

  NLO corrections?

- Lagrangian PT (à la Zeldovich) many people

  short modes should not be resummed

  calculations are more cumbersome, especially EFT

- Time-Sliced PT (à la QFT) Blas et al. '15

  need to split P_{lin} into smooth and wiggly components

  manually check the IR-enhanced contribution

  resummation relies on separation of scales

  calculations are more cumbersome, especially NLO and UV

IR- Resummation and NLO Corrections

IR-resummation

LPT calculations involve the average of an exponential

$$P(k) = \int \mathrm{d}^3 q_{12}\, e^{-i\vec{k}\cdot\vec{q}_{12}} \left\langle e^{-i\vec{k}\cdot(\vec{s}(q_1)-\vec{s}(q_2))} \right\rangle$$

which can be done as

$$\left\langle e^{-i\boldsymbol{k}\cdot\boldsymbol{\Delta}(\boldsymbol{q})} \right\rangle = \exp\left[\sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \left\langle (\boldsymbol{k}\cdot\boldsymbol{\Delta}(\boldsymbol{q}))^n \right\rangle_c \right] = K(\boldsymbol{k},\boldsymbol{q}),$$

Once expanded to some order in P_{lin}

Lagrangian PT = Standard PT

IR-resummation

The idea is to resum IR modes ($\sim \epsilon_{s<}$)

$K|_N$: IR-resummed, up to order *N*

$K||_N$: not IR-resummed, up to order *N*

K_0 : contains only IR-displacements

$$K|_N = K_0 \cdot \frac{K}{K_0}\Big|\Big|_N = \sum_{j=0}^{N} R||_{N-j} K_j$$

to get*

$$\xi(r)|_N = \sum_{j=1}^{N} \int \mathrm{d}^3 q\, \xi_j^E(q) \tilde{R}||_{N-j}(\boldsymbol{r} - \boldsymbol{q}, \boldsymbol{r})$$

*after an approximation which is parametrically justified

IR-resummation with NLO terms

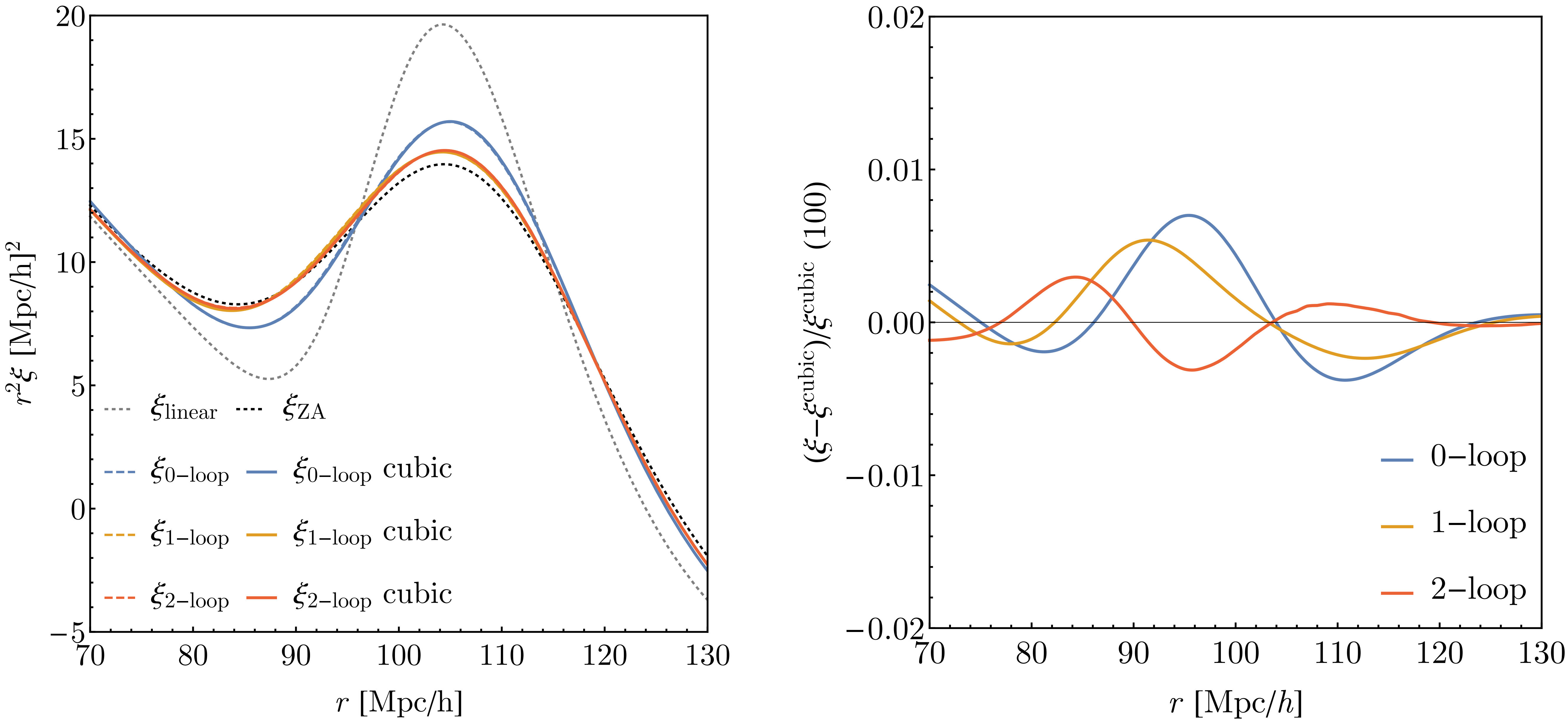
$$K_0(\boldsymbol{k}, \boldsymbol{q}) \equiv \exp\left[-\frac{1}{2}k_i k_j A^{\mathrm{IR}}_{ij}(\boldsymbol{q}) \qquad\qquad\qquad \right]$$

What is the NLO IR-enhanced term?

$$s_n \sim s_1 \delta_1^{n-1} \quad \nearrow \quad \langle ss \rangle\big|_{1-loop} \sim \langle ss\delta\delta \rangle$$

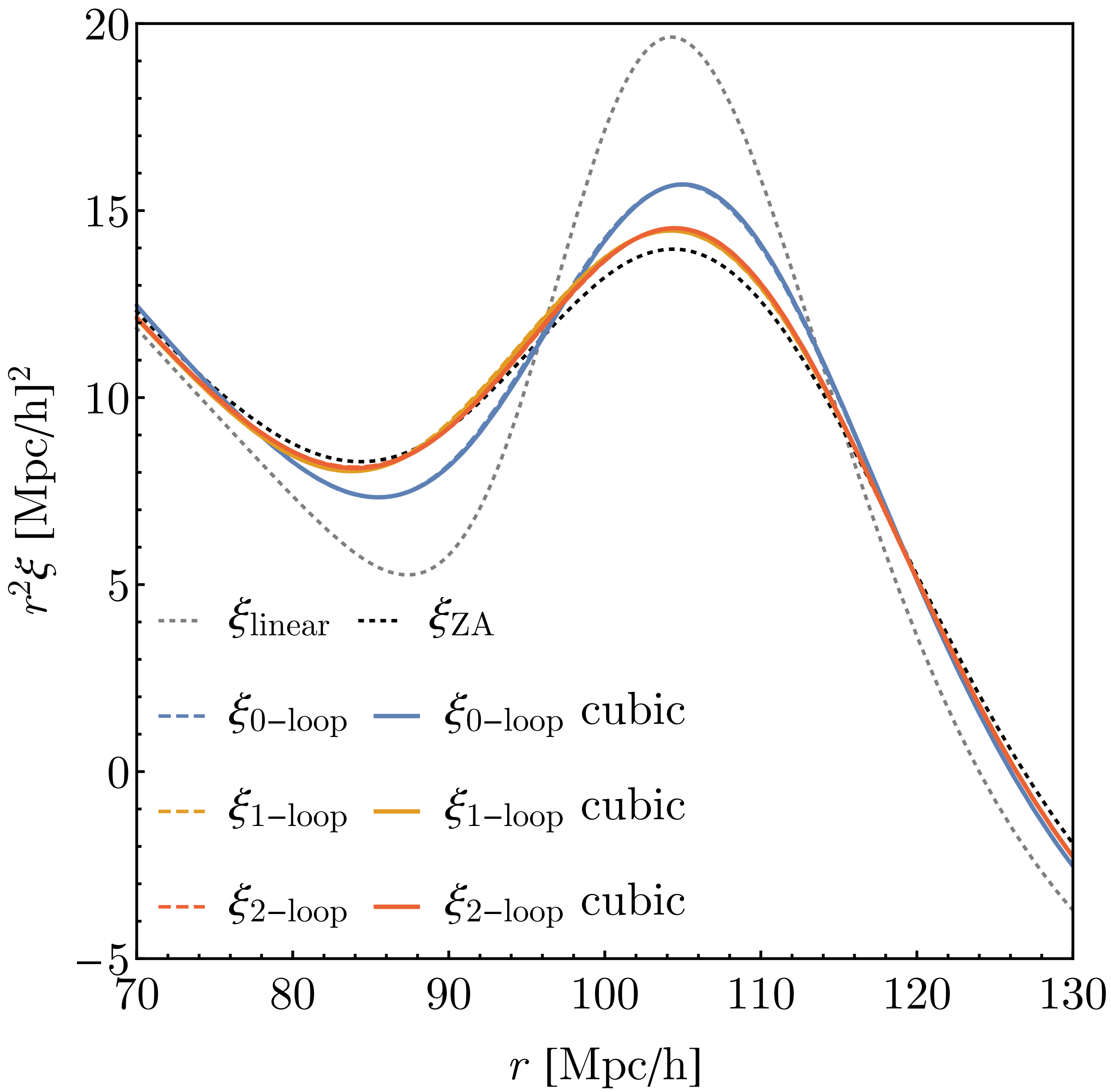
$$\searrow \quad \langle sss \rangle\big|_{tree} \sim \langle sss\delta \rangle$$

IR-resummation with NLO terms



Senatore, Trevisan: arXiv 1710.02178

IR-resummation with NLO terms



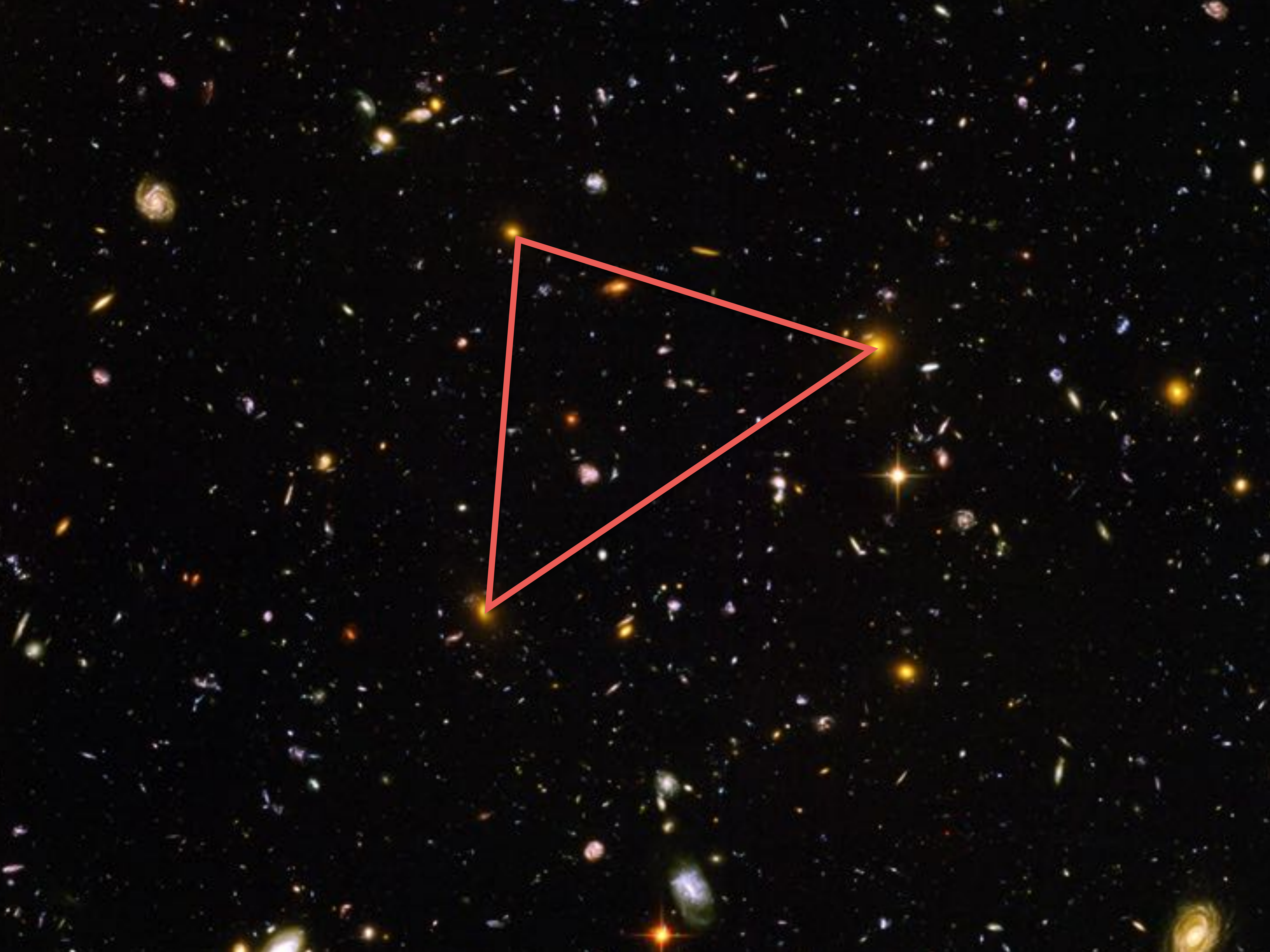
$$\frac{\text{NLO}}{\text{loop}} \sim \left(\frac{k_{long}}{k_{short}}\right)^{n+2} \epsilon_{s_<}$$

and for the BAO

$$\epsilon_{s_<} \approx 1$$

$$-1 < n < -2$$

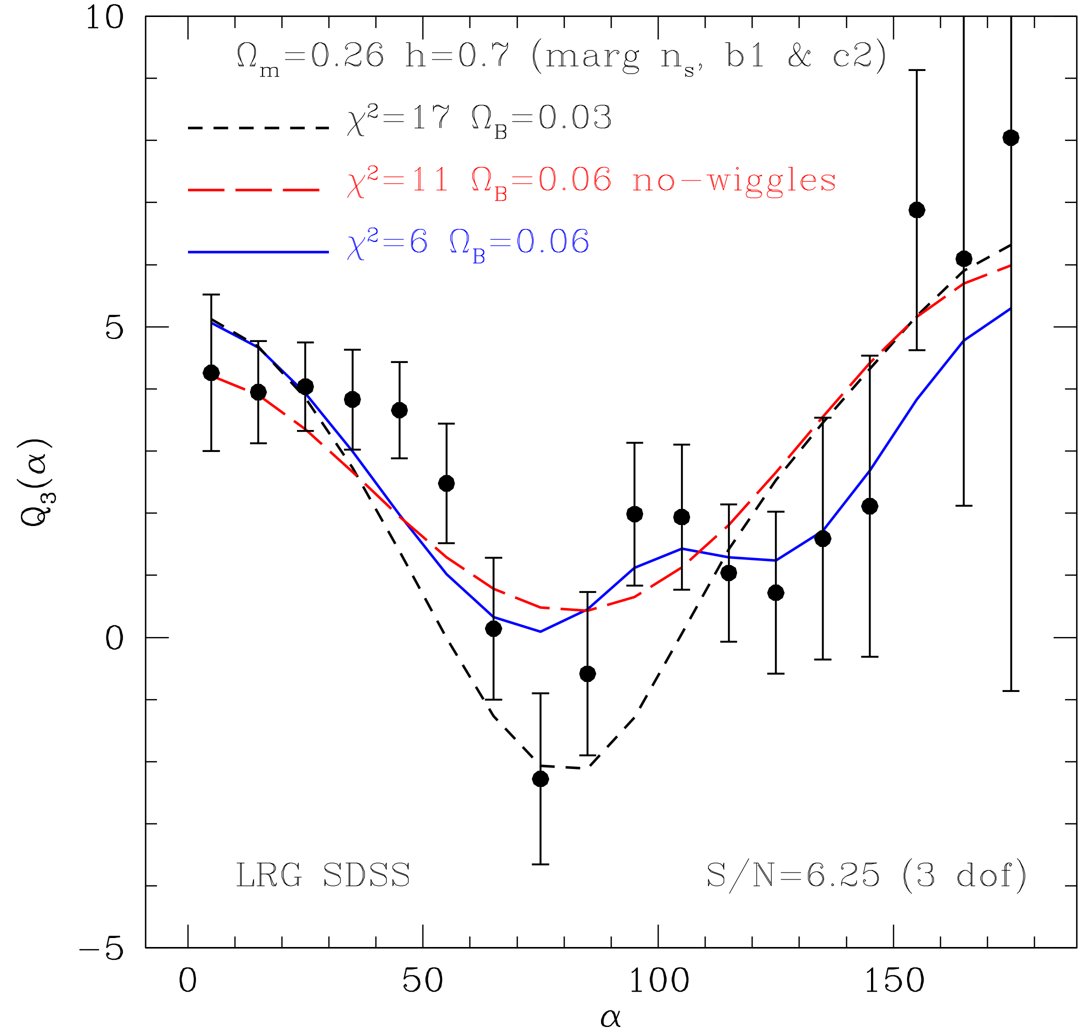
Senatore, Trevisan: arXiv 1710.02178

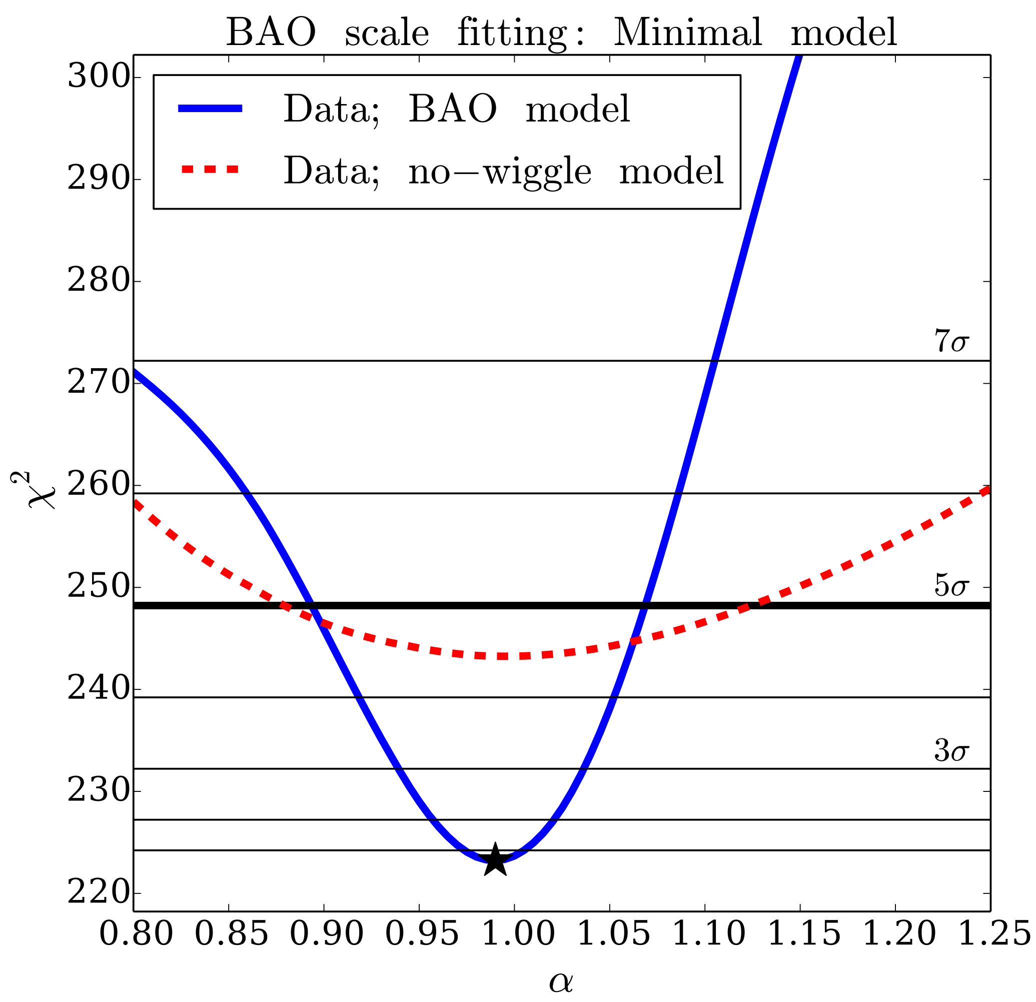


IR-resummation for the 3-PF

Scoccimarro, Trevisan: arXiv xxxx.xxxx

Detection of the BAO in the 3-PF

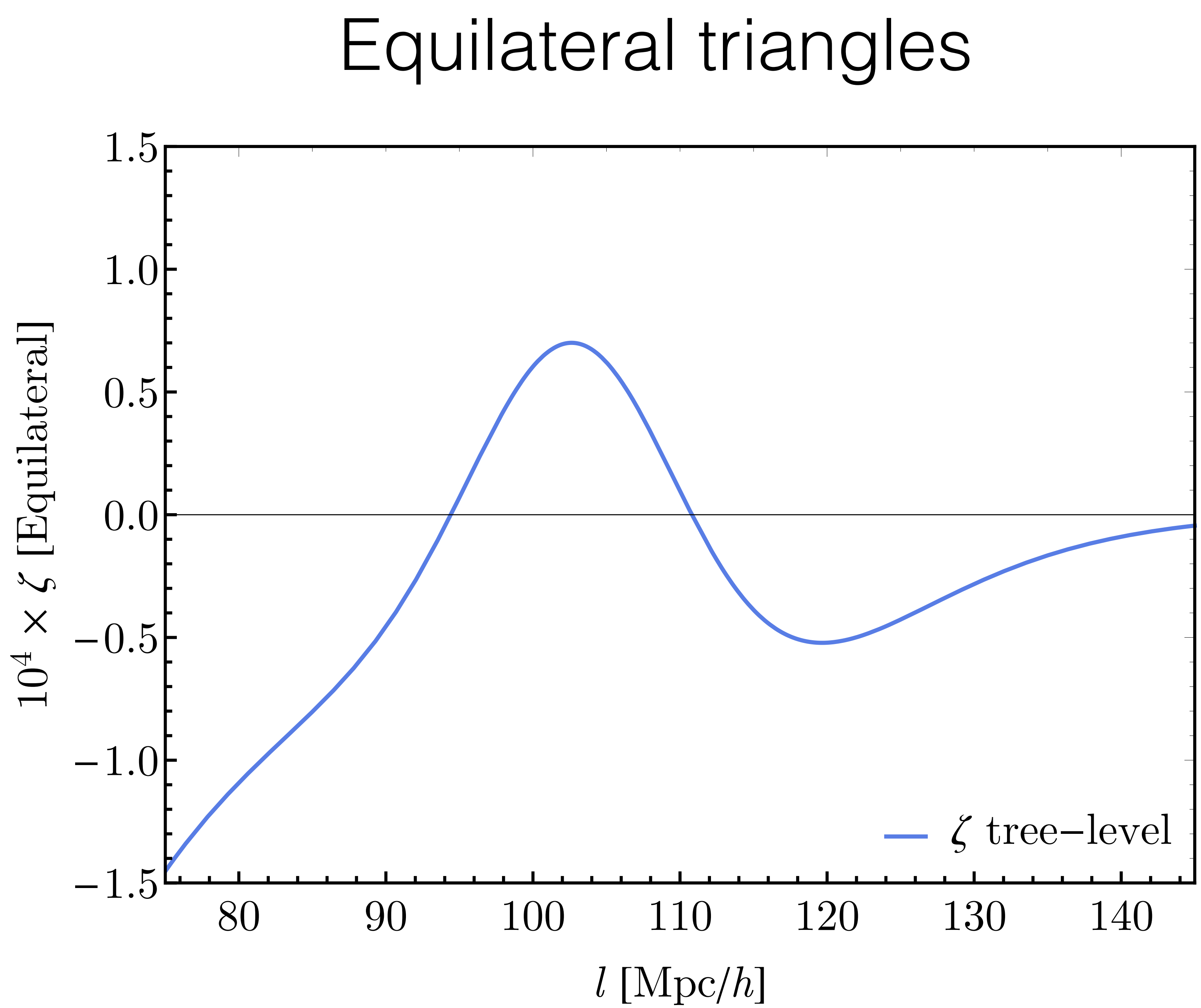




Analyses use $P_{\rm phys}(k) = [P(k) - P_{\rm nw}(k)] \exp\left[-k^2\Sigma_{\rm nl}^2/2\right] + P_{\rm nw}(k),$

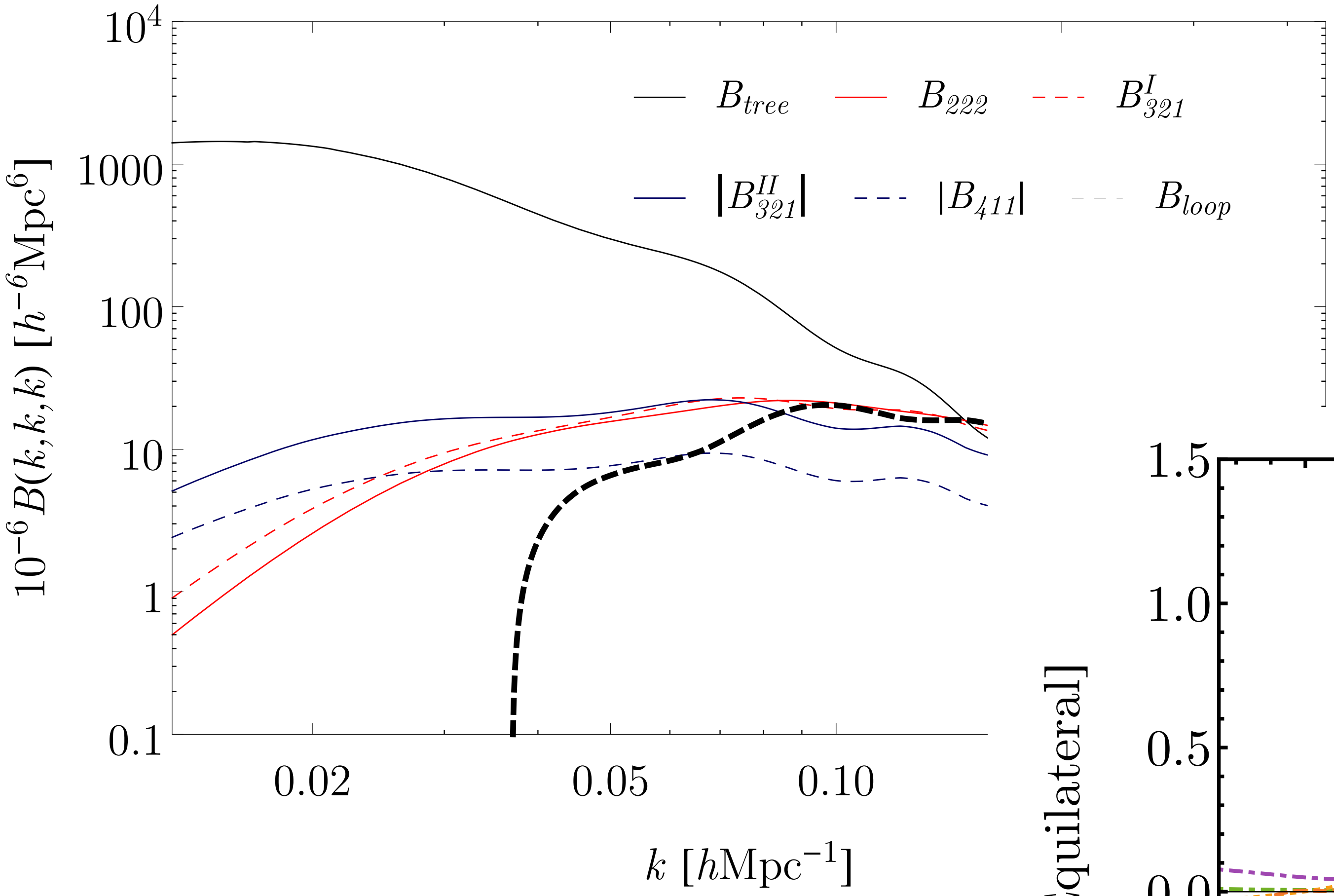
plugged into the tree-level SPT 3-PF

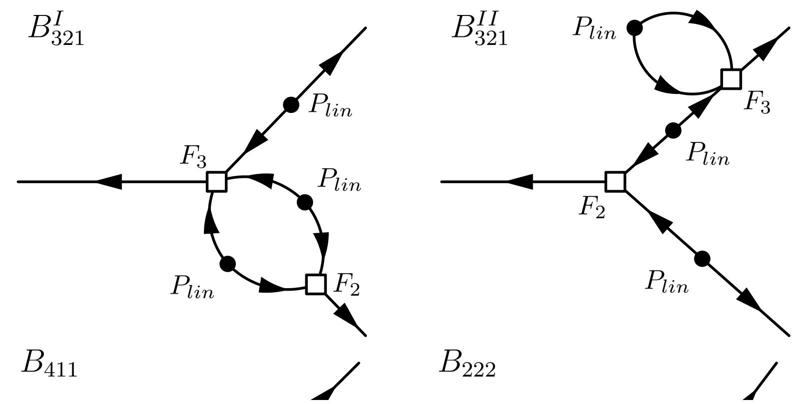
SPT predictions for the 3-PF

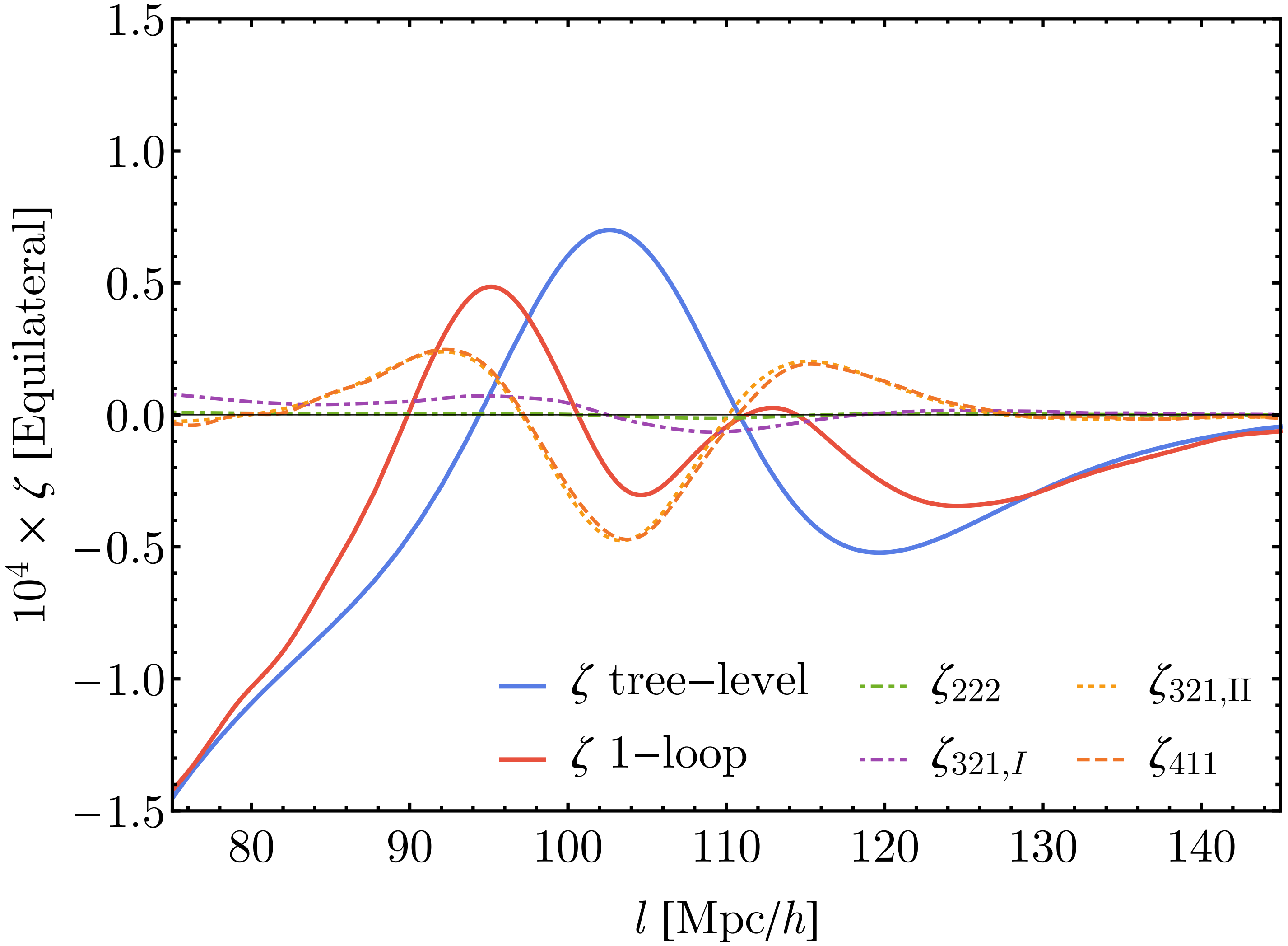


SPT predictions for the 3-PF

Baldauf et.al 1406.4135







IR-resummation for the 3-PF

Start from

$$\langle (1+\delta(\vec{x}_1))(1+\delta(\vec{x}_2))(1+\delta(\vec{x}_3))\rangle = \int đ^6k\, \mathrm{d}^6q\, e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{q})}\, K(\boldsymbol{k},\boldsymbol{q}),$$

where

$$K(\boldsymbol{k},\boldsymbol{q}) = \left\langle e^{-i\boldsymbol{k}\cdot\boldsymbol{\Delta}(\boldsymbol{q})}\right\rangle \qquad \boldsymbol{\Delta}(\boldsymbol{q}) = (\vec{s}(\vec{q}_1)-\vec{s}(\vec{q}_3), \vec{s}(\vec{q}_2)-\vec{s}(\vec{q}_3))$$

to obtain the IR-resummed 3-PF

$$\zeta^{\mathrm{IR+}}(\vec{x}_1,\vec{x}_2,\vec{x}_3)|_N = \int \mathrm{d}^6r \sum_{j=2}^{N} \zeta_j(\boldsymbol{r}) \left[K_0^{-1}(-i\nabla_{\boldsymbol{y}},\boldsymbol{x})\Big|\Big|_{N-j} G(\boldsymbol{x},\boldsymbol{y})\right]_{\boldsymbol{y}=\boldsymbol{x}-\boldsymbol{r}}$$

IR-resummation for the 3-PF

Similar to the resummation of the 2PF

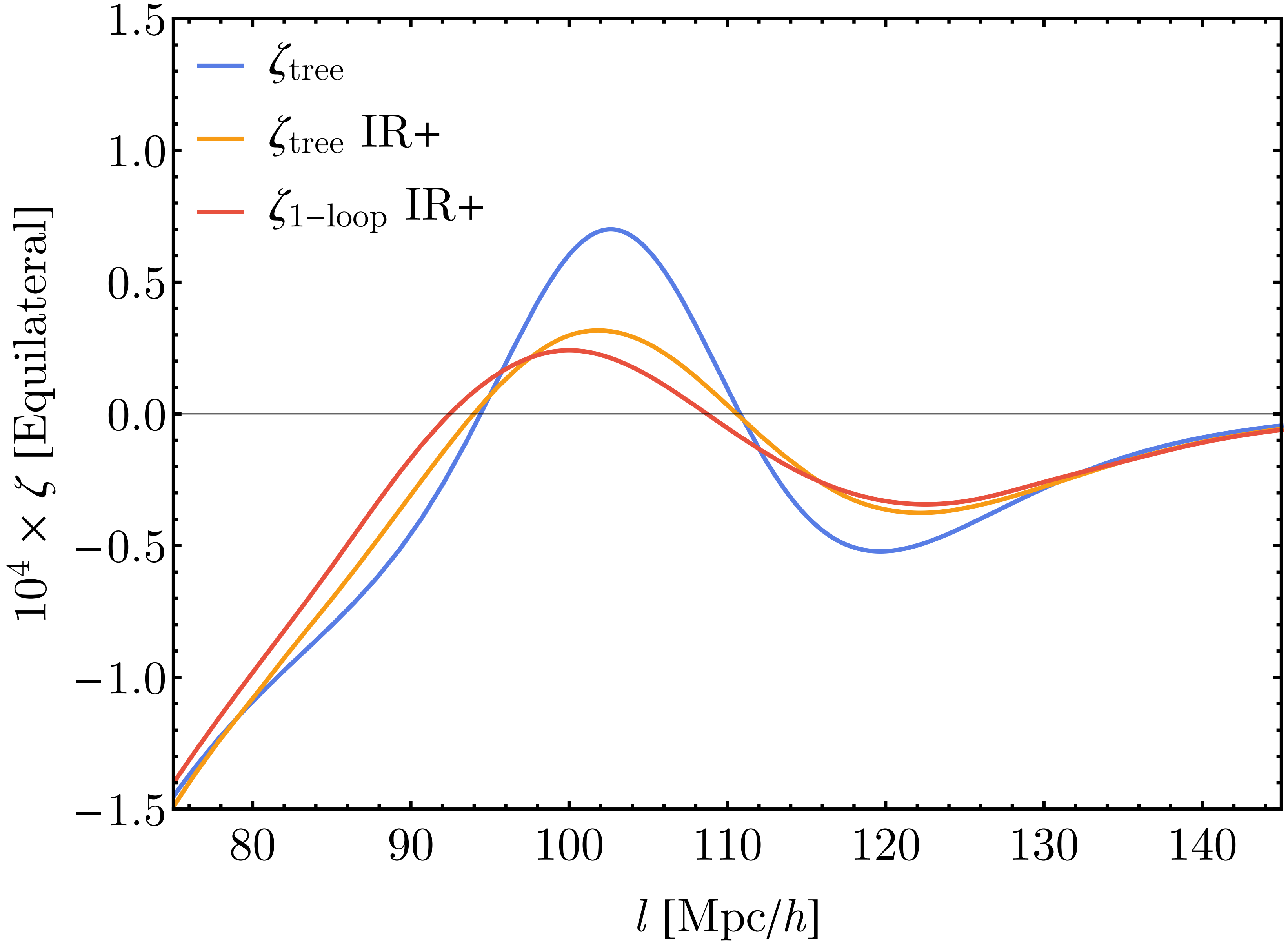
For example at tree-level

$$\zeta^{\mathrm{IR+}}_{tree}(\vec{x}_1,\vec{x}_2,\vec{x}_3) = \int \mathrm{d}^6 r \, \zeta^{\mathrm{E}}_{tree}(\boldsymbol{r}) \underline{G(\boldsymbol{x},\boldsymbol{x}-\boldsymbol{r})}$$

Gaussian-like kernel induced by long displacement modes

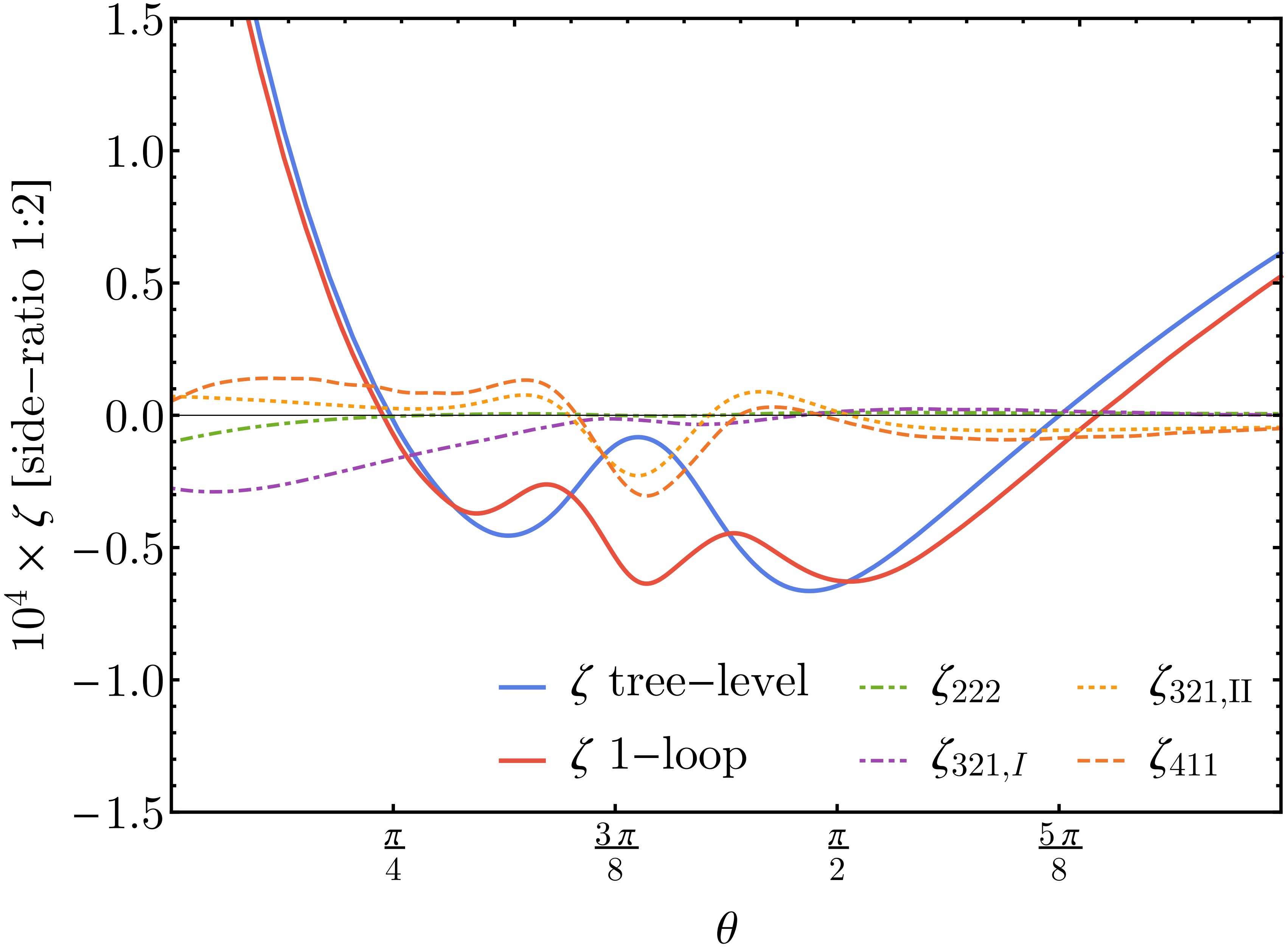
Scoccimarro, Trevisan: in preparation

IR-resummation for the 3-PF



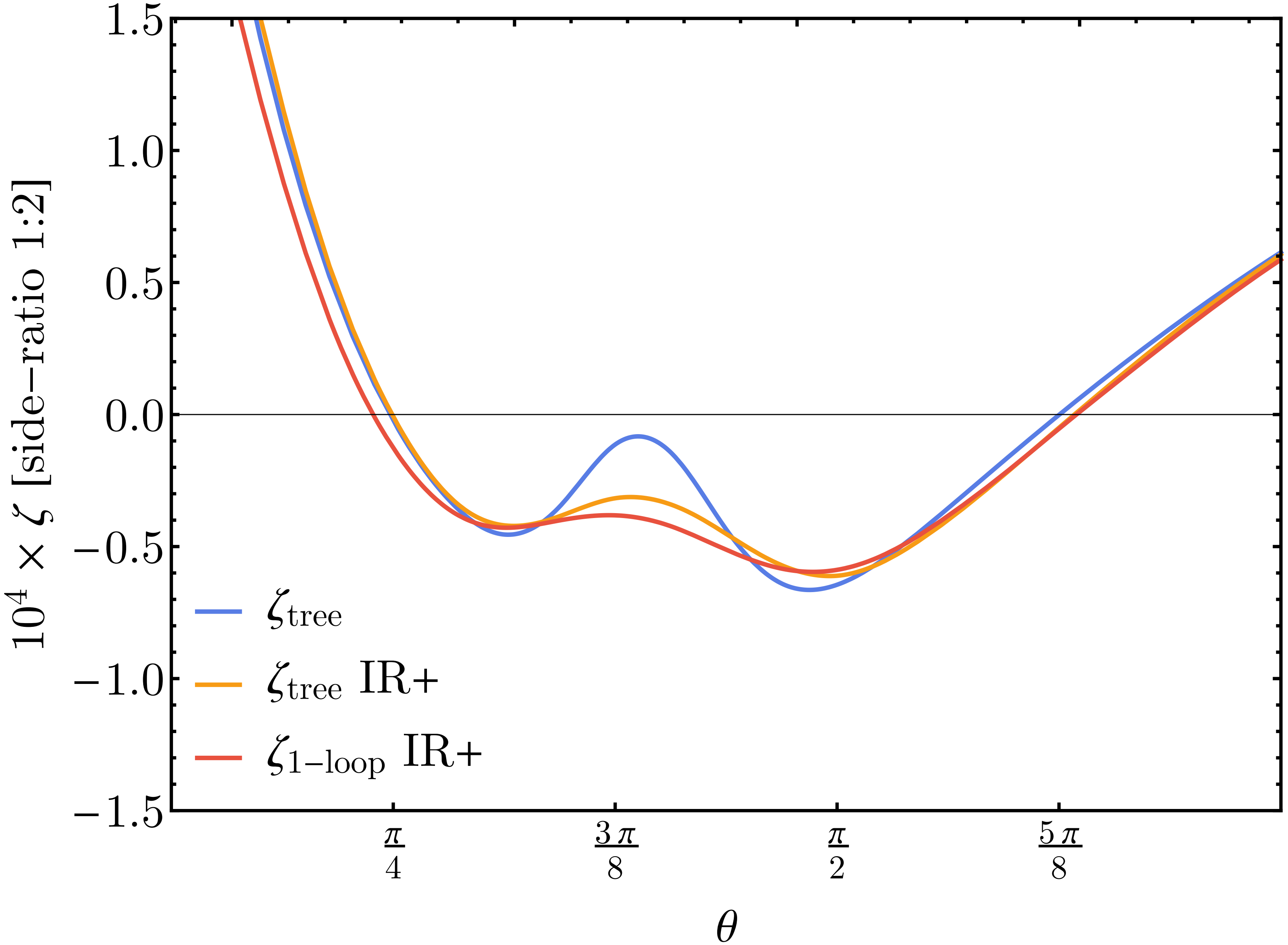
Scoccimarro, Trevisan: in preparation

IR-resummation for the 3-PF



Scoccimarro, Trevisan: in preparation

IR-resummation for the 3-PF

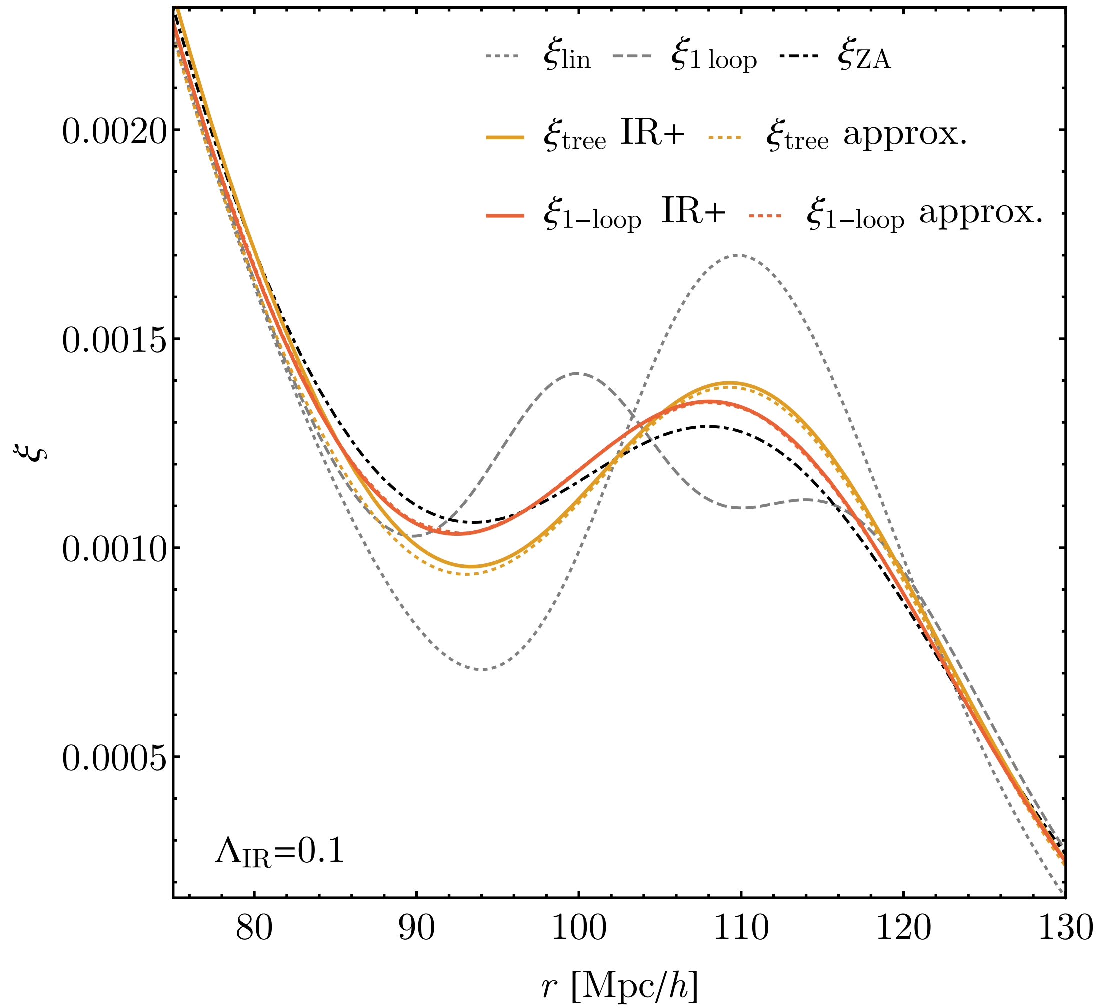


Scoccimarro, Trevisan: in preparation

A numerical approximation

$$P_{\rm lin} \to P^{\rm nw}_{\rm lin}(k) + e^{-k^2\Sigma^2_\Lambda} P^{\rm w}_{\rm lin}(k)$$

$$\Sigma^2_\Lambda = \frac{1}{3}\int^\Lambda đ^3p \frac{P_{\rm lin}(p)}{p^2}(1 - j_0(p\ell_{\rm BAO}) + 2j_0(p\ell_{\rm BAO}))$$

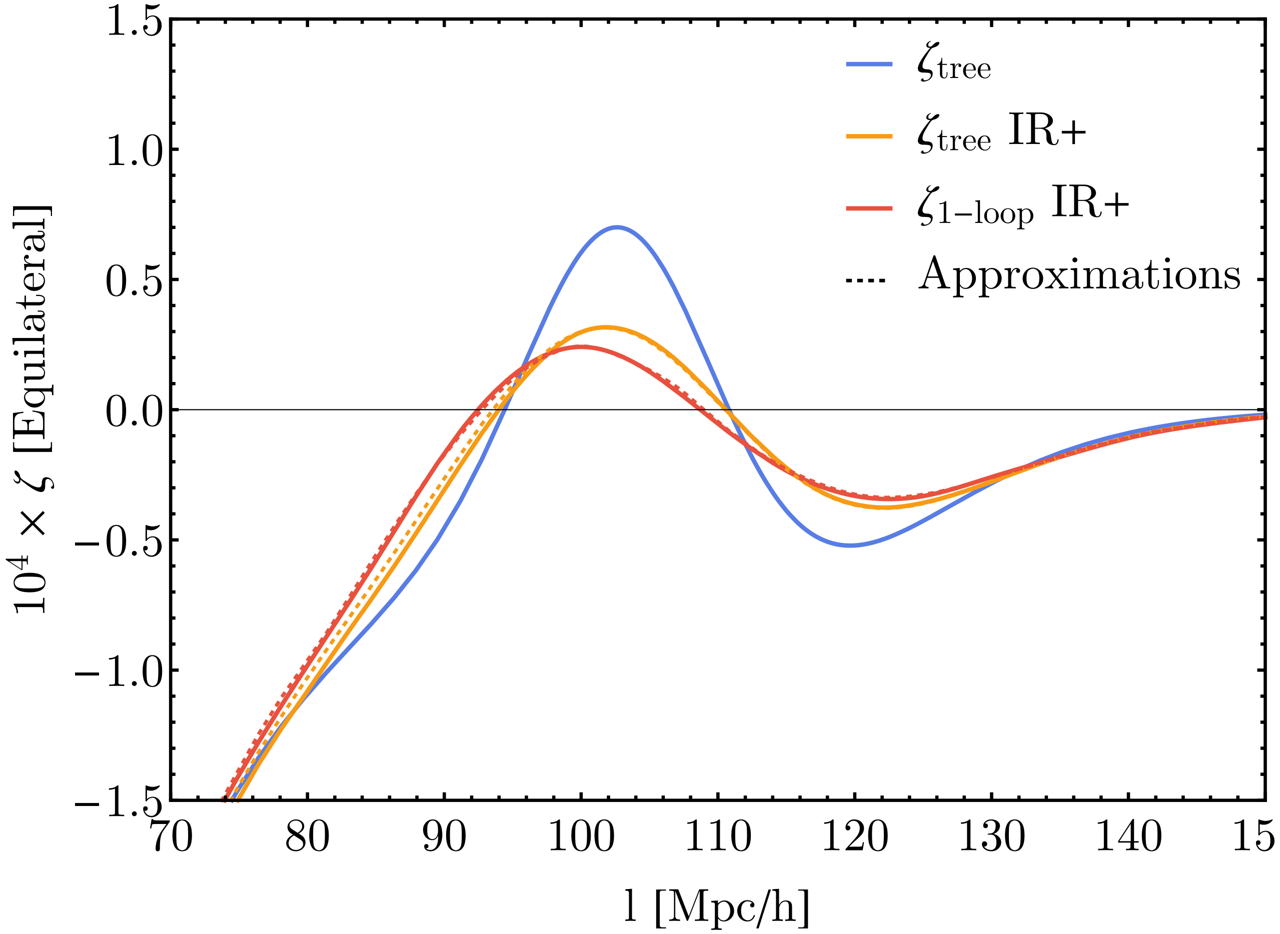


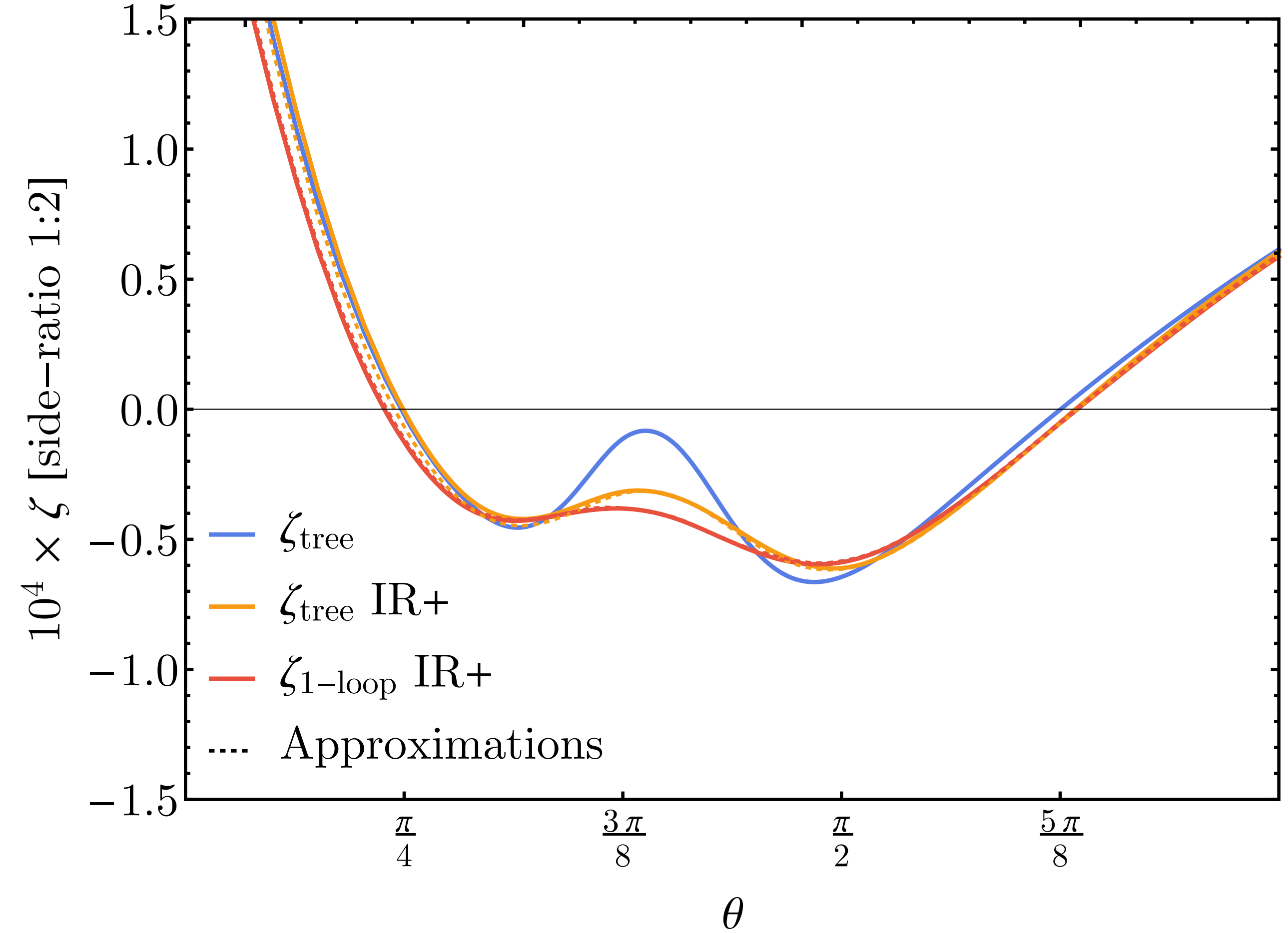
Scoccimarro, Trevisan: in preparation

A numerical approximation

$$P_{\rm lin} \to P_{\rm lin}^{\rm nw}(k) + e^{-k^2\Sigma_\Lambda^2} P_{\rm lin}^{\rm w}(k)$$

$$\Sigma_\Lambda^2 = \frac{1}{3}\int^\Lambda đ^3p \frac{P_{\rm lin}(p)}{p^2}\left(1 - j_0(p\ell_{\rm BAO}) + 2j_0(p\ell_{\rm BAO})\right)$$

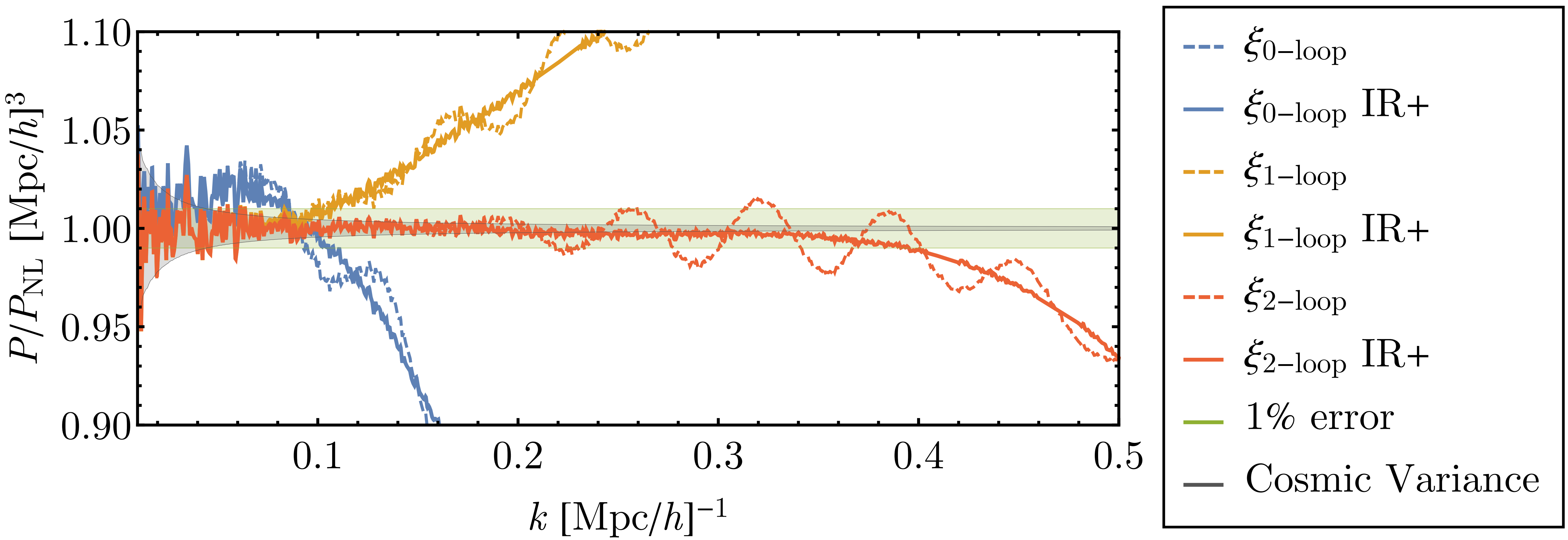




Scoccimarro, Trevisan: in preparation

Conclusions

- Although IR-enhanced, IR mode-coupling leads to corrections <1% in the 2-PF
- IR-resummation fixes also the 3-PF
- Tree-level and 1-loop 3PF agree quite well
- Num. approx. very close already at 1-loop
- 3-PF contains additional information: how well can we constraint?? reconstruction??



1.10
1.05
1.00
0.95
0.90
P/P_{NL} [Mpc/h]^3
0.1
0.2
0.3
0.4
0.5
k [Mpc/h]$^{-1}$
$\xi_{0-\mathrm{loop}}$
$\xi_{0-\mathrm{loop}}$ IR+
$\xi_{1-\mathrm{loop}}$
$\xi_{1-\mathrm{loop}}$ IR+
$\xi_{2-\mathrm{loop}}$
$\xi_{2-\mathrm{loop}}$ IR+
1% error
Cosmic Variance