13th Oc. 2010
informal discussion
@ LBNL, Berkeley

Signature of primordial vector modes on large-scale structure

Atsushi Taruya

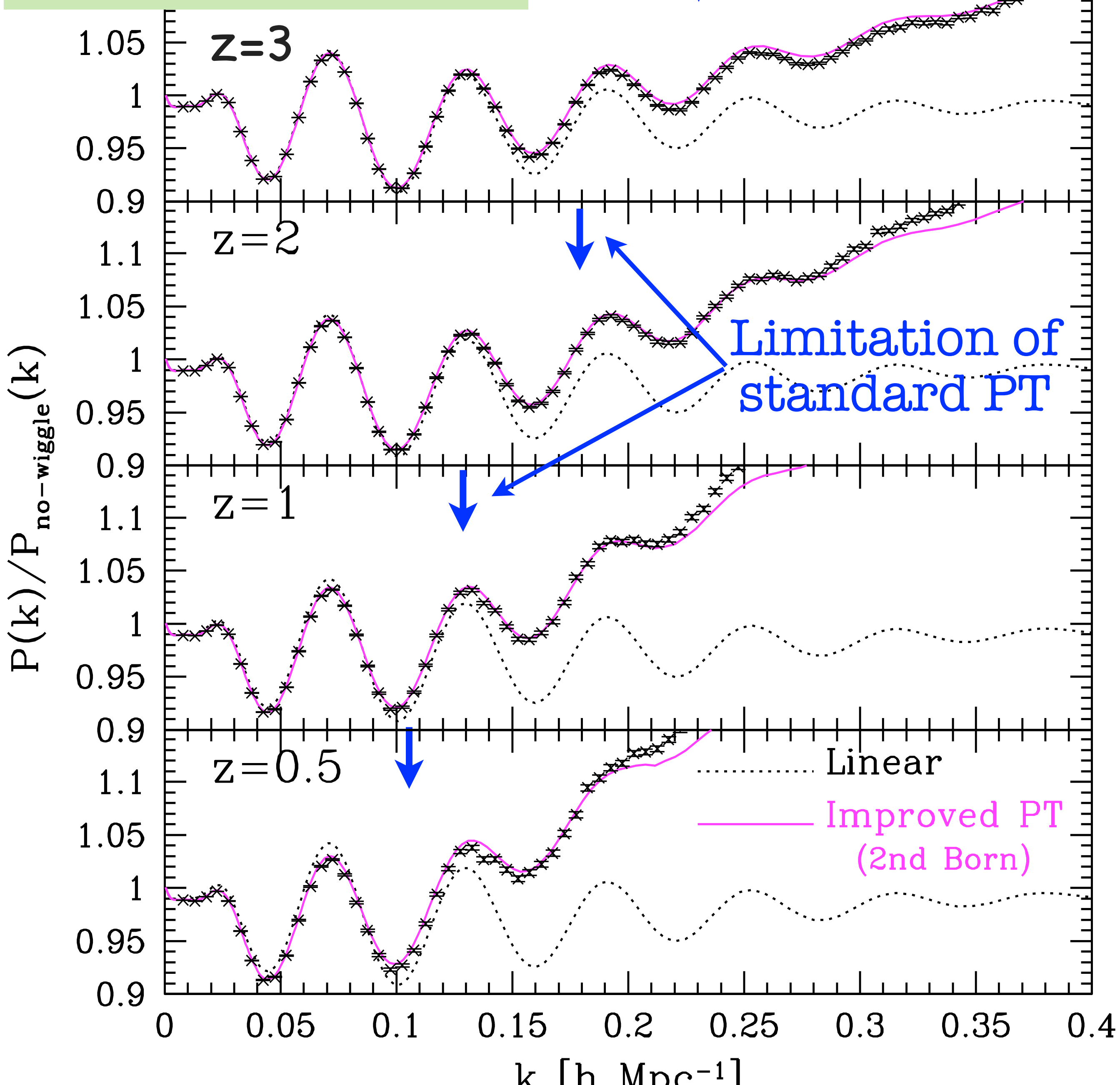
(RESCEU & IPMU, Univ. Tokyo)

Development of new analytic method

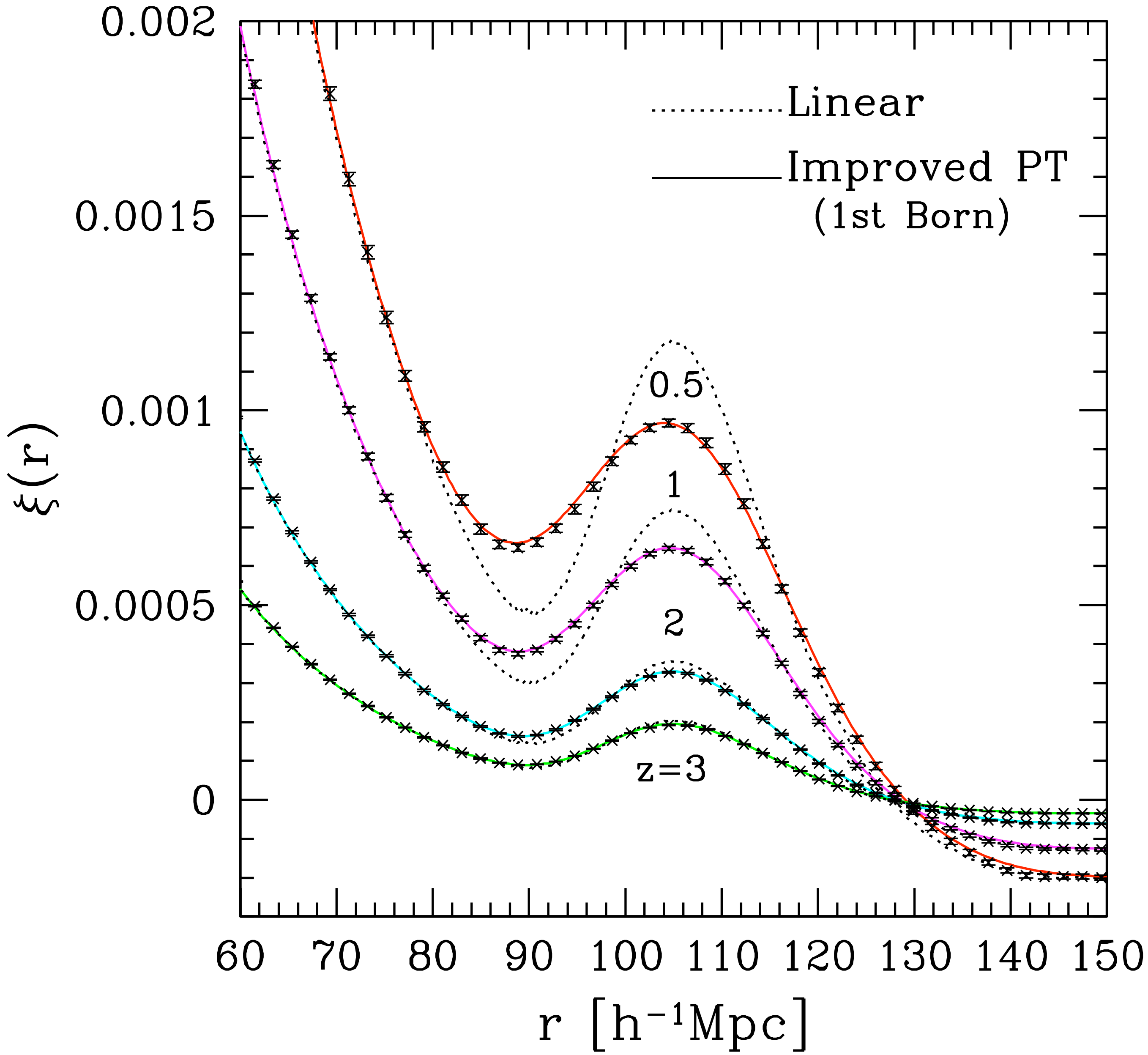
An improved treatment of perturbation theory (PT) to deal with non-linear gravitational evolution

AT & Hiramatsu (2008)
AT, Nishimichi, Saito & Hiramatsu (2009)

Power spectrum



Correlation function



Redshift-space power spectrum

Based on improved treatment of perturbation theory (PT) for gravitational clustering,

AT, Nishimichi & Saito ('10)

$$P^{(\mathrm{S})}(k,\mu) = e^{-(k\mu f\sigma_{\mathrm{v}})^2}\left[P_{\delta\delta}(k) - 2f\,\mu^2 P_{\delta v}(k) + f^2\mu^4 P_{vv}(k) + A(k,\mu) + B(k,\mu)\right]$$

$$A(k,\mu) = -2\,k\,\mu\int\frac{d^3\mathbf{p}}{(2\pi)^3}\,\frac{p_z}{p^2}B_\sigma(\mathbf{p},\mathbf{k}-\mathbf{p},-\mathbf{k})$$

antiphase oscillation

Mode-coupling btw. velocity & density

(low-k approx.)

$$\left\langle\theta(\mathbf{k}_1)\left\{\delta(\mathbf{k}_2)-\mu_2^2\,\theta(\mathbf{k}_2)\right\}\left\{\delta(\mathbf{k}_3)-\mu_3^2\,\theta(\mathbf{k}_3)\right\}\right\rangle = (2\pi)^3\delta_D(\mathbf{k}_{123})\,B_\sigma(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

$$B(k,\mu) = (k\,\mu)^2\int\frac{d^3\mathbf{p}}{(2\pi)^3}F(\mathbf{p})F(\mathbf{k}-\mathbf{p})$$

small in amplitude (<1-2%)

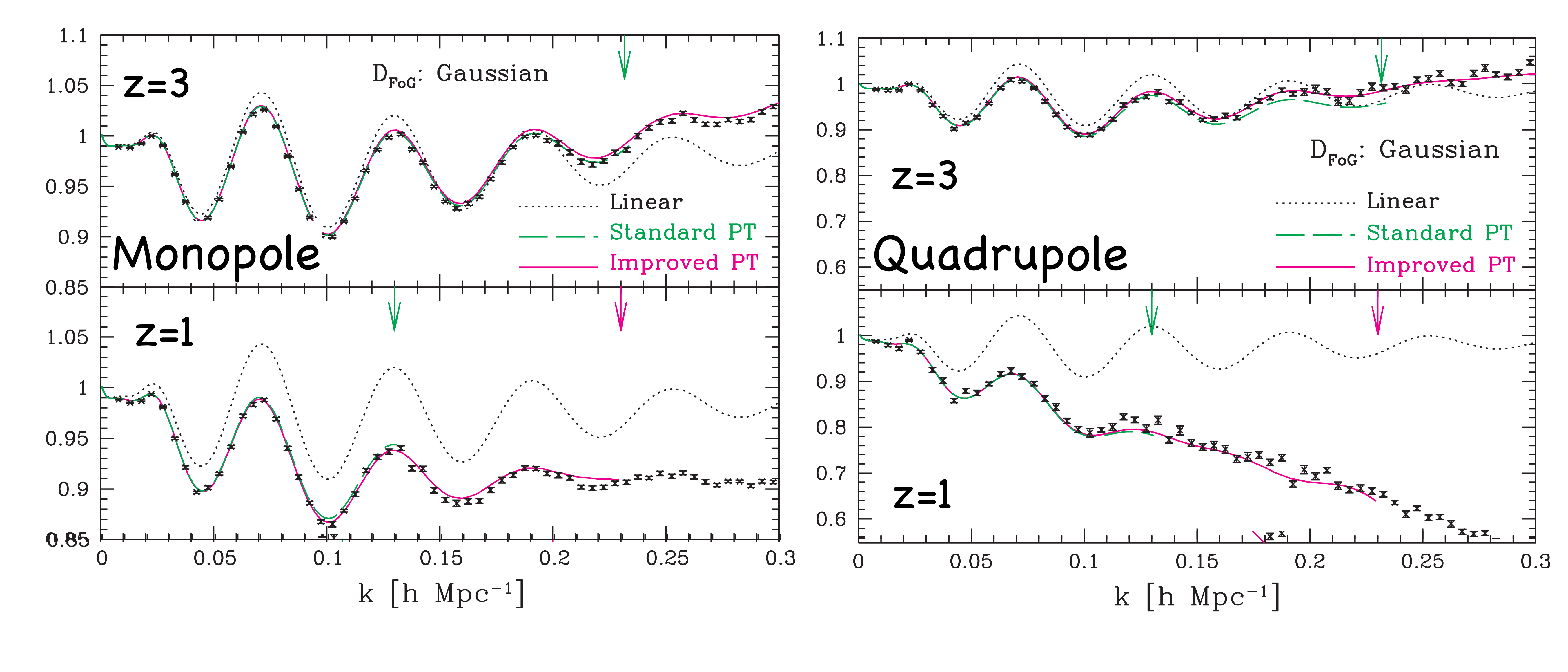
$$F(\mathbf{p}) \equiv \frac{p_z}{p^2}\left\{P_{\delta v}(p) - \frac{p_z^2}{p^2}P_{vv}(p)\right\}$$

These also depend on 'f'

Role of corrections

Including corrections

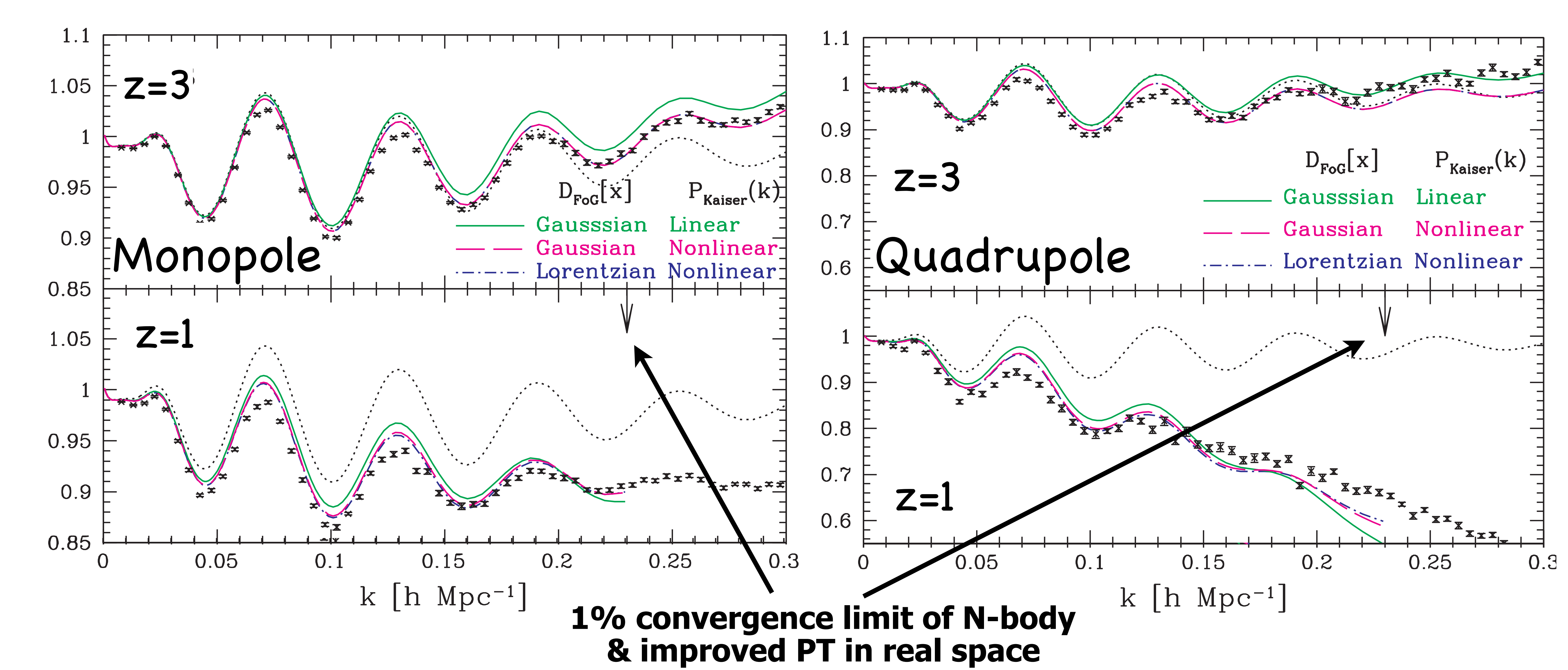
$$P^{(\mathrm{S})}(k,\mu) = \sum_{\ell=\mathrm{even}} P_\ell^{(\mathrm{S})}(k)\,\mathcal{P}_\ell(\mu)$$



Role of corrections

Neglecting corrections

$$P^{(\mathrm{S})}(k,\mu) = \sum_{\ell=\mathrm{even}} P_\ell^{(\mathrm{S})}(k)\,\mathcal{P}_\ell(\mu)$$



Even within the 1% convergence limit, the agreement is not so good (few % in monopole, >5% in quadrupole)

Today’s Topic

Cosmic Antique

Cosmic Twister

Endangered species

Vector modes

Vector modes in cosmology

Classifying the spatial inhomogeneities as

scalar・vector・tensor

$S(\vec{x},t),\ V_i(\vec{x},t),\ T_{ij}(\vec{x},t)$

Intuitively,

vector modes are analogous to shear and/or vorticity in fluid mechanics (divergence-free)

Roles in cosmology

- **Scalar**：cosmic expansion・gravitational clustering
- **Vector**・**tensor**：remnants, sub-dominant components

→ minor, but helpful to probe early-universe physics

Vector modes in cosmology

$$\delta g_{i0} = a^2 \sigma_i \qquad \frac{1}{\mathcal{H}}(\nabla \times \boldsymbol{v})_i = \omega_i$$

Evolution equations

$$\dot{\sigma}_i + 2\mathcal{H}\,\sigma_i = 8\pi\,G\,a^2 P\,\Pi_i^{(\mathrm{V})} \quad + \quad \text{(New sources)}$$

$$\dot{q}_i + (1 - 3\,c_s^2)\mathcal{H}\,q_i = \frac{P}{P+\rho}\Pi_i^{(\mathrm{V})} \quad ; \quad q_i \equiv \mathcal{H}\,\omega_i$$

Anisotropic stress (matter)

Causal seeds / Cosmic string

2nd-order perterbation

Modification of gravity in vector sector (gravity)

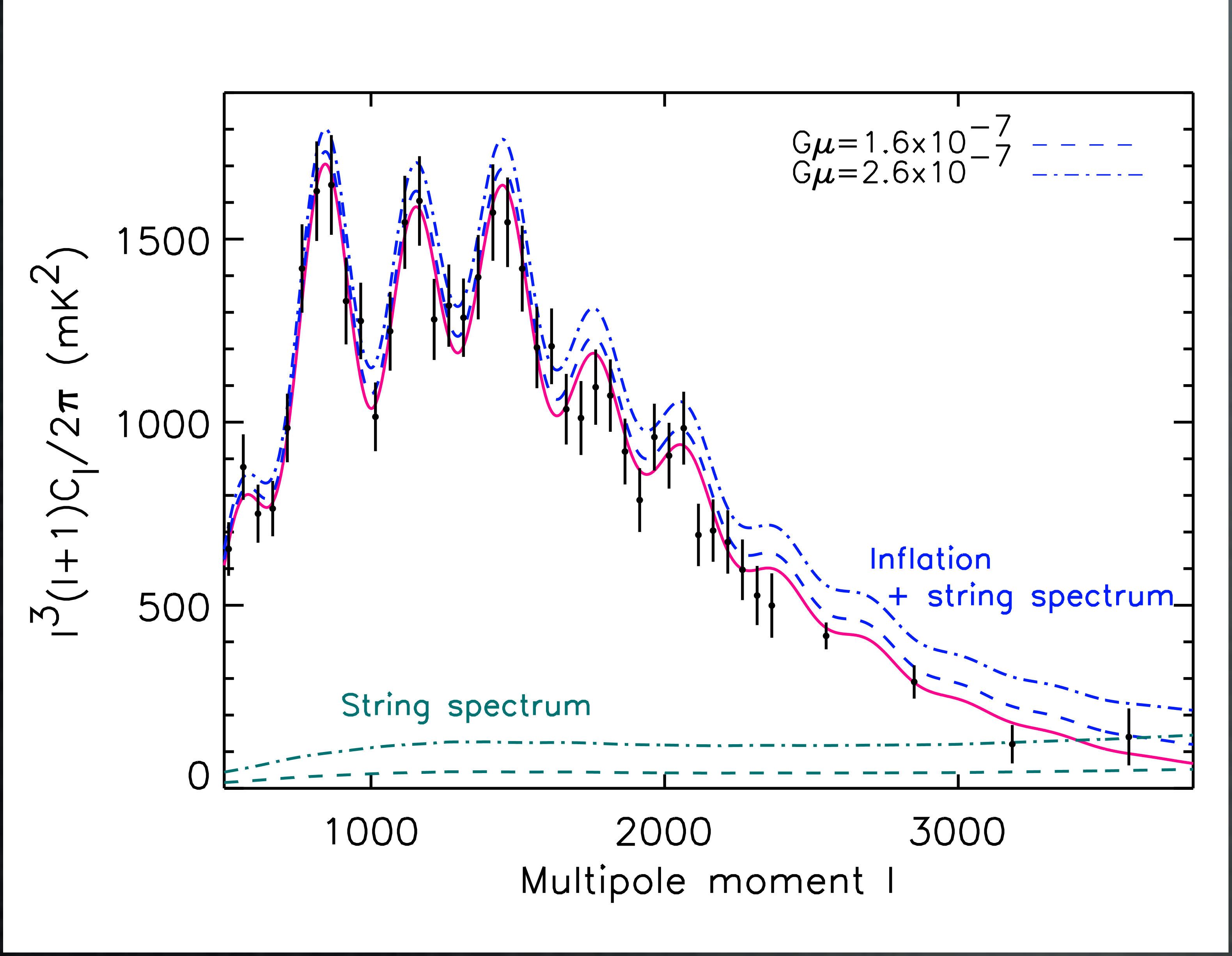
Cosmological vector fields

(Einstein-Aether / Extended Horava-Lifshitz)

In addition, anisotropic inflation may also produce vector modes

CMB constraints on vector modes

Cosmic strings as a source producing vector modes



Dunkley et al. ('10)

For Nambu string,

$$G\mu < 1.6 \times 10^{-7} \ (95\%\mathrm{CL})$$

(WMAP+ACT)

Future feasibility

- B-mode polarizations

  Seljak & Slosar ('06)

- Lensing effect on CMB

  Yamauchi et al. in prep.

Large-scale structure probes

Galaxy spectroscopic surveys

Redshift distortion ω_i

Galaxy imaging surveys

Cosmic shear σ_i

Vector modes can be separately detected

Redshift distortion

Redshift of galaxies via spectroscopic measurement is inherently affected by the Doppler shift due to peculiar velocity of galaxies

Redshift space   real space

$$\vec{\mathbf{s}} = \vec{\mathbf{r}} + \frac{(\vec{\mathbf{v}}\cdot\hat{\mathbf{z}})}{a\,H(z)}\,\hat{\mathbf{z}}\;;\quad \begin{cases}\mathbf{v} & : \text{Peculiar velocity of galaxies}\\ \hat{\mathbf{z}} & : \text{Observer's line-of-sight direction}\end{cases}$$

Galaxy clustering pattern is apparently distorted

Anisotropic power spectrum:

$$P(k) \longrightarrow P^{(\mathrm{S})}(k,\,\mu)\;;\quad \mu\equiv(\vec{\mathbf{k}}\cdot\hat{\mathbf{z}})/|\vec{\mathbf{k}}|$$

Kaiser Formula

Linear regime (galaxy bias, ignored)

matter P(k) in real space

standard formula $$P^{(\mathrm{S})}(k,\mu) = (1+f\,\mu^2)^2\,P_\delta(k)$$ Kaiser ('87)

growth-rate parameter $f(z) \equiv \dfrac{d\ln D_+}{d\ln a}$ $\left[D_+(z) : \right.$ Linear growth factor $\left.\right]$

Vector modes induces new term

modified formula $$P^{(\mathrm{S})}(k,\mu) = (1+f\,\mu^2)^2 P_\delta(k) + \frac{1}{2}\mu^2(1-\mu^2)\,P_\omega(k)$$

Power spectrum of vorticity

Multipole expansion

Combining P0,P2 & P4, we can separately detect vorticity.

$$P^{(\mathrm{S})}(k,\mu) = \sum_{\ell=0} P_\ell^{(\mathrm{S})}(k)\,\mathcal{P}_\ell(\mu)$$

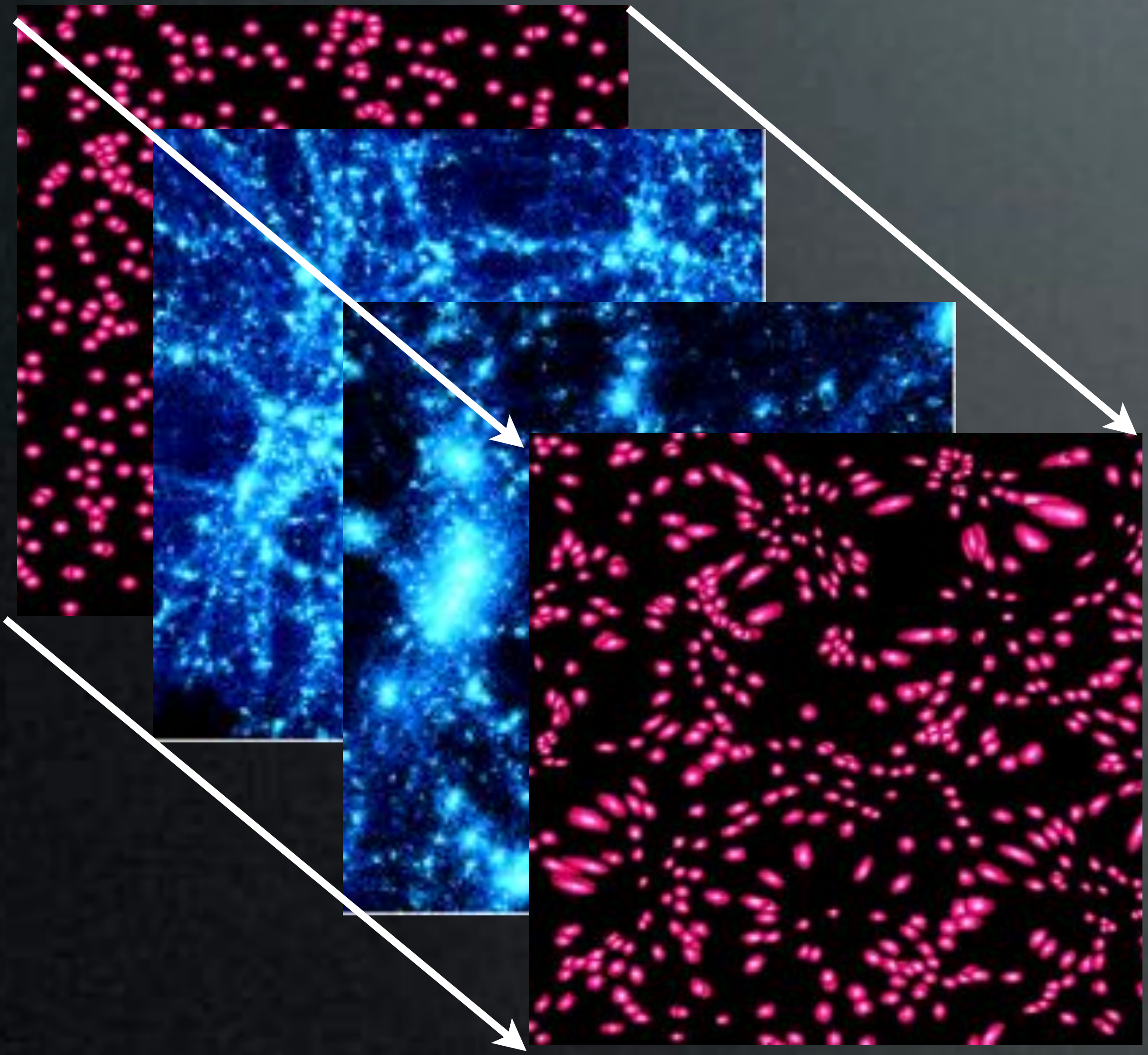
Legendre polynomials

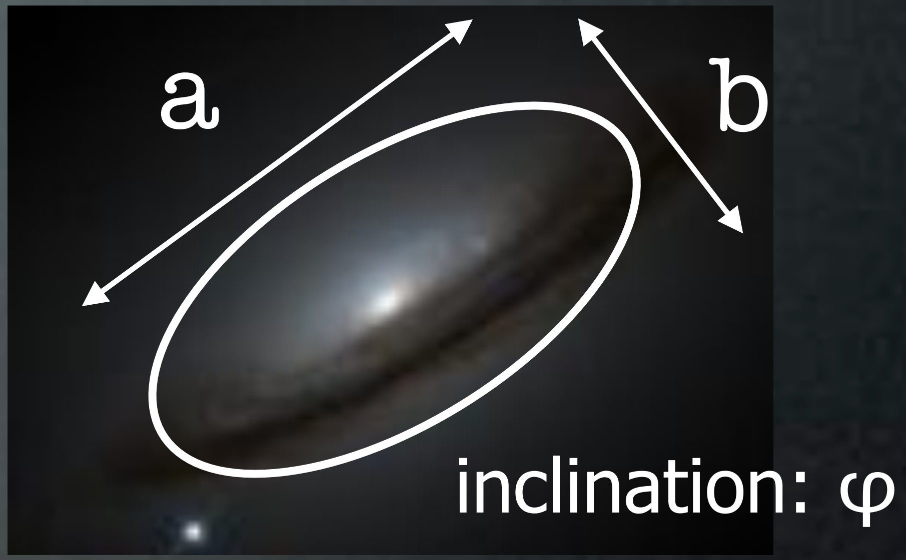
$$\begin{cases} P_0^{(\mathrm{S})}(k) = \left(1+\frac{2}{3}f+\frac{1}{5}f^2\right)P_\delta(k)+\frac{1}{15}P_\omega(k) \\ P_2^{(\mathrm{S})}(k) = \left(\frac{4}{3}f+\frac{4}{7}f^2\right)P_\delta(k)+\frac{1}{21}P_\omega(k) \\ P_4^{(\mathrm{S})}(k) = \frac{8}{35}f^2\,P_\delta(k)-\frac{4}{35}P_\omega(k) \end{cases}$$

Simple Fisher analysis indicates that vorticity component with vector/scalar ratio of 5~10% in amplitude would be detected.

Cosmic Shear

Distortion of distant-galaxy images due to weak gravitational lensing by large-scale structure





Complex ellipticity

$$\chi = \left(\frac{a^2 - b^2}{a^2 + b^2} \right) e^{i\,2\phi} \longrightarrow 2\gamma$$

shear field

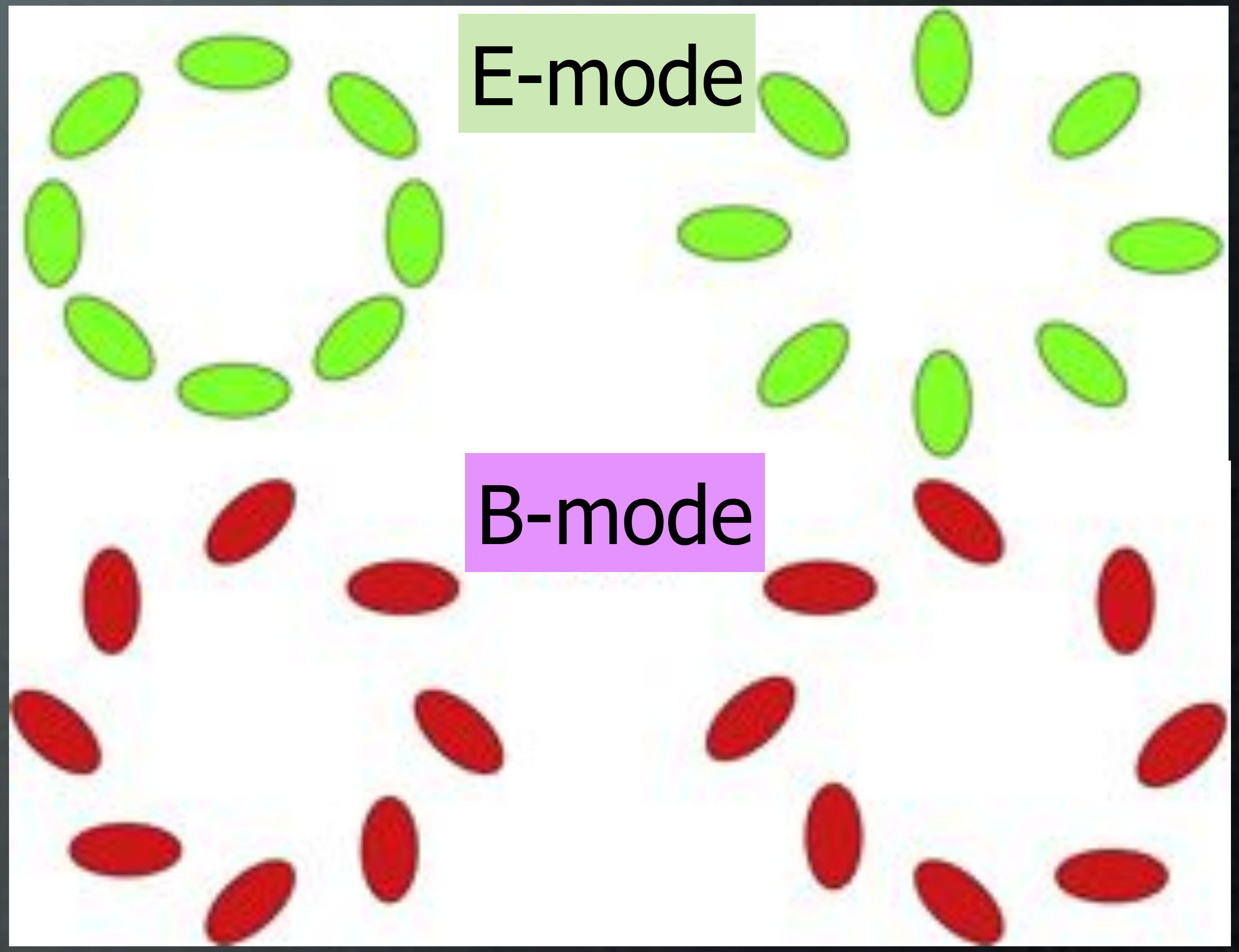
Gravitational Lensing induces spatial correlation in the shear field.

E-/B-mode decomposition

In analogy to CMB polarization,
spatial pattern of shear field is decomposed to E-/B-modes

Notice !

Scalar-type perturbations only generate E-mode pattern.



Vector-type perturbations can produce not only E-mode, but also B-mode cosmic shear (clue to detect vector modes)

Vector cosmic shear: formula

Shear field from vector modes

Flat universe

$$\gamma = -\int_0^{r_s} dr\, \frac{r_s - r}{2r\, r_s}\, r^2\, e_+^i e_+^j \left[e_r^k \partial_i \partial_j \sigma_k - \frac{d}{dr}\partial_j \sigma_i\right]$$

r : comoving radial distance

r_s : comoving radial distance for distant galaxies

$\boldsymbol{e}_r,\ \boldsymbol{e}_+ \equiv \boldsymbol{e}_\theta + i\,\boldsymbol{e}_\varphi$: projection vector

metric fluc. (vector mode)

B-mode angular power spectrum

$$C_\ell^{\rm BB} = \frac{2}{\pi}\int_0^\infty dk\, k^2 \int_0^{r_s} dr \int_0^{r_s} dr' \times \left(\frac{3r_s - 4r}{2r\, r_s}\right)\left(\frac{3r_s - 4r'}{2r'\, r_s}\right) j_\ell(k\,r)\, j_\ell(k\,r')\, P_\sigma(k; r, r')$$

Summary

Detecting vector modes from large-scale structure

- Redshift distortion — via spectroscopic survey
- B-mode cosmic shear — via imaging surveys

➡ Derive basic formulas for power spectra: $P^{(\mathrm{S})}(k,\mu),\ C_\ell^{\mathrm{BB}}$

Feasibility of future observations

Redshift distortion : would detect vorticity component
if vector/scalar ratio of 5~10% @ k<0.1h/Mpc

Lensing B-mode : would detect cosmic strings of $G\mu \lesssim 10^{-7}$

(Thomas et al. '09)

※ Synergy with dark energy survey may be fruitful