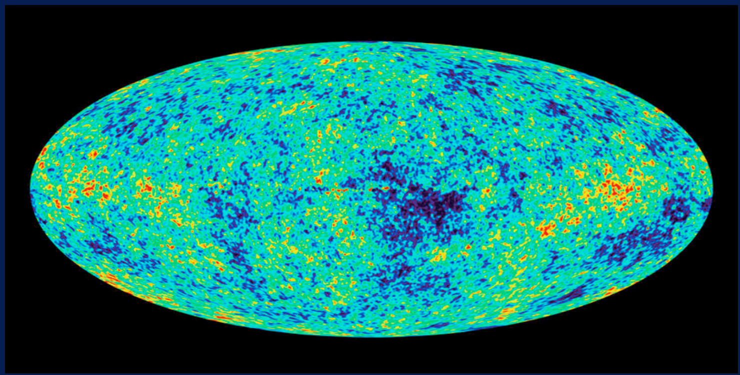
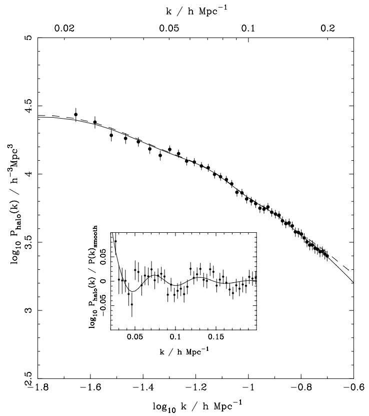
Beyond the standard cosmological model: neutrinos and non-Gaussianity



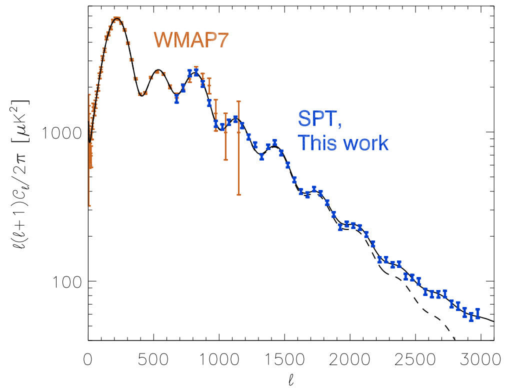
WMAP team

Tristan L. Smith (BCCP, UC Berkeley)

Status of the standard model of cosmology from the CMB



Reid et al. (2010)



Keisler et al. (2011)

Tristan Smith

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

Extensions of the standard cosmological model

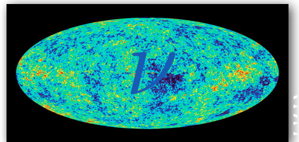
- There are some extensions which are 'expected' at some level:
  - Non-zero gravitational-wave contribution
  - Running of the spectral index
  - Non-zero neutrino mass
  - Time varying dark energy equation of state
- Others would seriously challenge the standard cosmological model:
  - Effective number of neutrino species
  - Non-Gaussianity
  - Parity violating interactions
  - Anisotropic processes

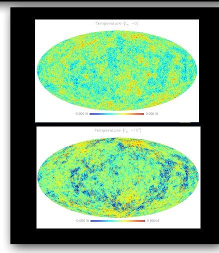
Tristan Smith

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

Outline

- Concentrate on two extensions of the standard cosmological model:
- Constraints on the effective number of neutrino species
- CMB constraints to the primordial non-Gaussian amplitude

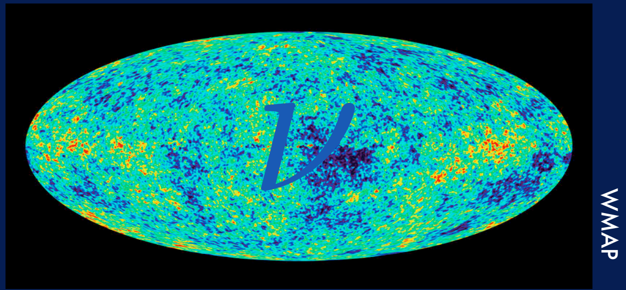




Tristan Smith

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

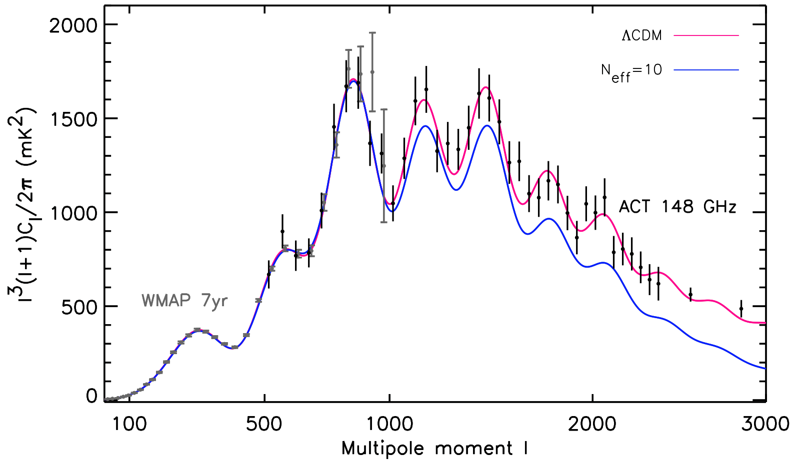
Constraints on neutrino interactions using cosmological observations



Tristan L. Smith
Sudeep Das
Oliver Zahn
(BCCP, UC Berkeley)

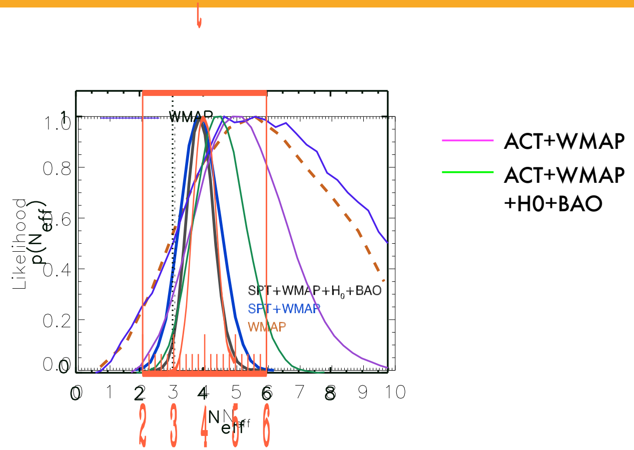
PRD 85, 023001 (2012)

Status of the standard model of cosmology from the CMB



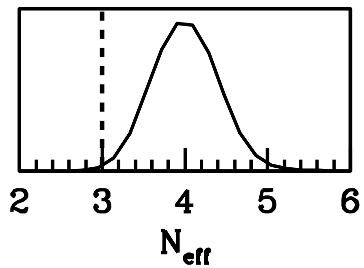
ACT, Dunkley et al. (2011)

Status of the standard model of cosmology from the CMB

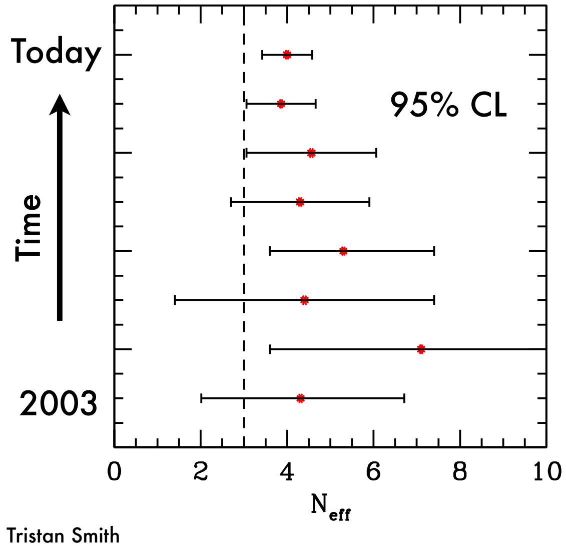


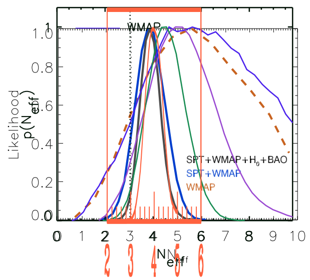
Status of the standard model of cosmology from the CMB

H0+WMAP+ACT+SPT+SDSS



Status of the standard model of cosmology from the CMB





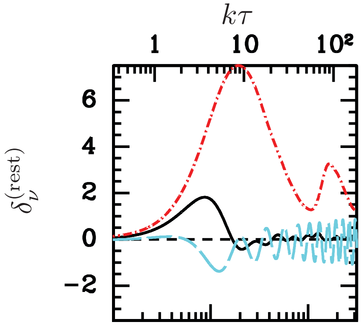
Interpreting extra relativistic energy density

- So far we have a hint that there exists an anomalous radiative background... but no other information
- What can this be? Most explanations (such as sterile neutrinos) suppose that this background will be non-interacting
- We were interested in exploring to what extent the observations can show that this background is non-interacting
- Following Hu (1998) we modify the evolution equations by introducing two new parameters $c_{\rm eff}^2$ and $c_{\rm vis}^2$

Interpreting extra relativistic energy density

- Changes to the effective sound-speed modifies the pressure support

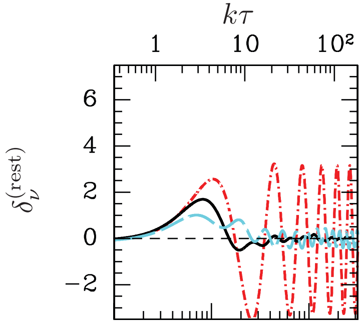
$c_{\rm eff}^2 = 1/3$ —
0.8 – –
0.1 -·-·-



Interpreting extra relativistic energy density

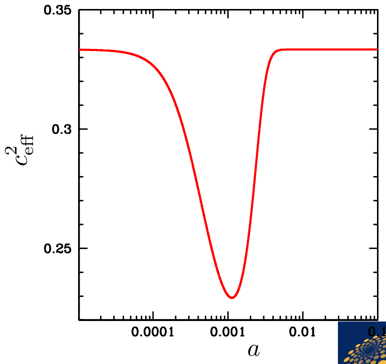
- Changes to the viscosity parameter controls to what extent the fluid is imperfect (i.e., anisotropic stress)

$c_{\rm vis}^2 = 1/3$ —
1 – –
0 -·-·-



What do these new parameters physically mean?

- These parameters give some measure of the interactions this anomalous background may have
- An analogy with the tightly coupled photon-baryon fluid
- Bell et al. (2006) considered a model where some neutrinos are tightly coupled to a scalar field

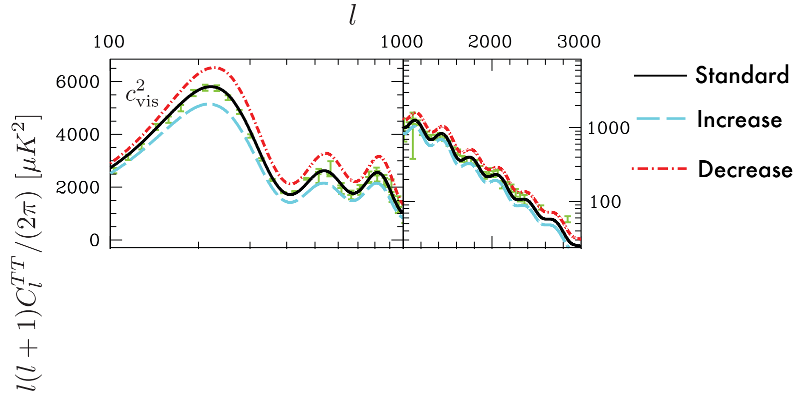


Tristan Smith

BERKELEY CENTER for COSMOLOGICAL PHYSICS

Interpreting extra relativistic energy density

- The effects on the CMB spectrum

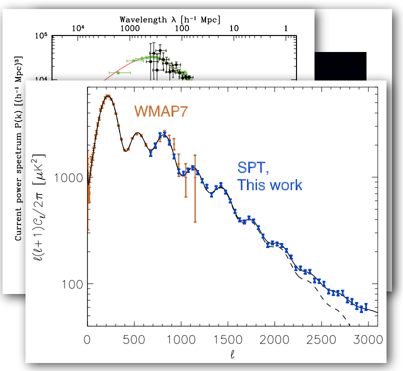


Tristan Smith

BERKELEY CENTER for COSMOLOGICAL PHYSICS

The data

- We used various combinations of CMB and large-scale structure data:



- H_0 from HST
- SDSS matter power spectrum
- Lya forest
- CMB
  - WMAP7
  - ACBAR
  - ACT
  - SPT

Tristan Smith

BERKELEY CENTER for COSMOLOGICAL PHYSICS

The Results

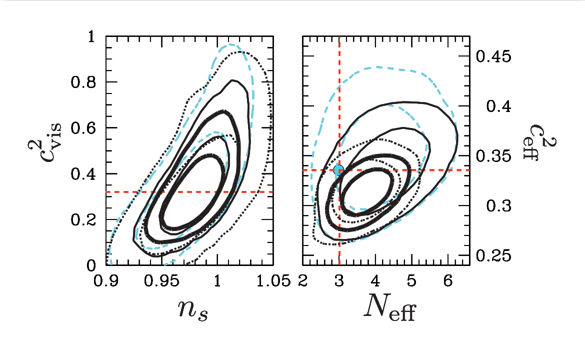
Marginalized 1D constraints

$N_{\rm eff}$	$c^2_{\rm vis}$	$c^2_{\rm eff}$
$4.0^{+0.17+0.58}_{-0.18-0.57}$	$1/3$	$1/3$

Tristan Smith

BERKELEY CENTER for COSMOLOGICAL PHYSICS

The Results



WMAP+SDSS

WMAP+SDSS + Lya

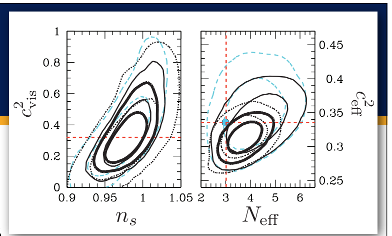
CMB-only

All

Tristan Smith

BERKELEY CENTER for COSMOLOGICAL PHYSICS

Conlusions and future directions



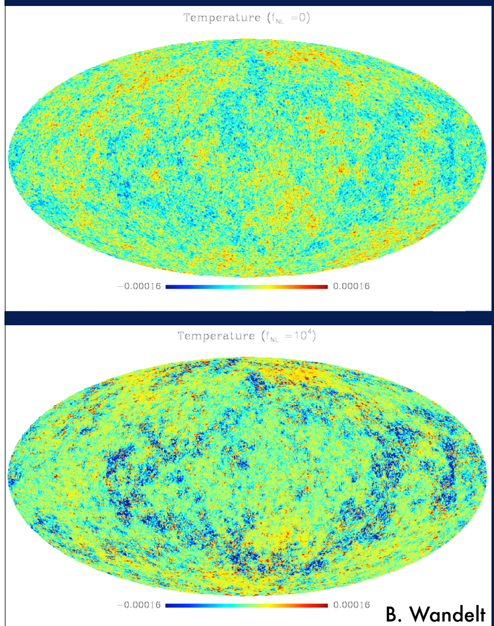
- These constraints provide further evidence that there may be extra non-interacting neutrino-like degrees of freedom
- Planck will be able to constrain:

$$N_{\rm eff} = 3.0 \pm 0.17$$
$$c^2_{\rm eff} = 0.333 \pm 0.004$$
$$c^2_{\rm vis} = 0.333 \pm 0.026$$

- Extend parameterization for neutrino mass
- Explore to what extent the data is able to constrain a time evolving $c_{\rm eff}(a)$ and $c_{\rm vis}(a)$

Tristan Smith

BERKELEY CENTER for COSMOLOGICAL PHYSICS



B. Wandelt

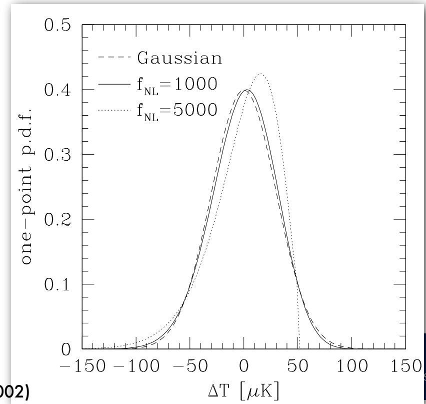
The PDFs of non-Gaussianity estimators

In collaboration with Marc Kamionkowski, Alan Heavens, and Benjamin Wandelt

Phys. Rev. D 83, 023007 (2011)
Phys. Rev. D 84, 063013 (2011)

Non-Gaussian estimation from the CMB

- The standard cosmological model predicts that the primordial fluctuations obey Gaussian statistics
- It is simple to think of a few basic ways to test this prediction
- One way is to look at the PDF of the temperature fluctuations in the CMB
- It turns out that the signal-to-noise using the PDF is sub-optimal



Komatsu (2002)

Tristan Smith

Non-Gaussian estimation from the CMB

* Instead, we want to use the fact that any process which is Gaussian is uniquely determined by its mean, μ , and variance, σ

$$\langle (x-\mu)^1 \rangle = 0$$
$$\langle (x-\mu)^2 \rangle = \sigma^2$$
$$\langle (x-\mu)^3 \rangle = 0$$
$$\langle (x-\mu)^4 \rangle = 3\sigma^4$$

* An obvious test of Gaussianity then asks: is the third moment zero? and is the fourth moment just given by the Gaussian piece?

Non-Gaussian estimation from the CMB

* To make progress we use a parameterization for the level of non-Gaussianity in the CMB maps

$$\Phi(\vec{x}) = \phi(\vec{x}) + f_{\rm nl}\left[\phi(\vec{x})^2 - \langle \phi(\vec{x})^2 \rangle\right]$$

where ϕ is a Gaussian random field and Φ is the primordial curvature potential

* We can see that, for instance, the three-point function is now non-zero:

$$\langle \Phi(\vec{x}_1)\Phi(\vec{x}_2)\Phi(\vec{x}_3)\rangle \sim f_{\rm nl}\langle \phi(\vec{x}_1)\phi(\vec{x}_3)\rangle\langle \phi(\vec{x}_2)\phi(\vec{x}_3)\rangle$$

Non-Gaussian estimation from the CMB

* The harmonic coefficients of the temperature field on the sky are related to the primordial curvature potential

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3\mathbf{k}}{(2\pi)^3}\Phi(\mathbf{k})g_{Tl}(k)Y^*_{lm}(\hat{\mathbf{k}})$$

so that correlations in harmonic space are also non-Gaussian

$$\langle a_{l_1m_1}a_{l_2m_2}\rangle = C_{l_1}\delta_{l_1,l_2}\delta_{m_1m_2}$$
$$\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle \sim f_{\rm nl}B_{l_1,l_2,l_3}$$
$$\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}a_{l_4m_4}\rangle_c \sim \tau_{\rm nl}T_{l_1,l_2,l_3,l_4}$$

Why do we want to do this?

* Any constraint to primordial non-Gaussianity probes the physics of the very early universe
* In particular, assuming that inflation was driven by a single field one can show [Creminelli and Zaldarriaga (2004)]

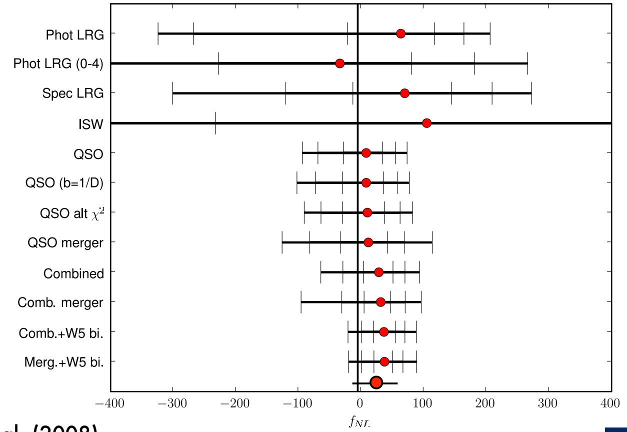
$$f_{\rm nl} \approx \frac{5}{12}(1-n_s) \longrightarrow f_{\rm nl} \approx 0.02$$

* So that if we find $f_{\rm nl} > 0.02$ then *all single field inflationary models will be ruled out*

* Measurement of the amplitude of the trispectrum, $\tau_{\rm nl}$, would give us additional constraints on the early-universe physics which produces non-Gaussianities

Non-Gaussian estimation

* Constraints to non-Gaussianity



Slosar et al. (2008)

Tristan Smith

Non-Gaussian estimation

* Constraints to non-Gaussianity
* WMAP constraint on bispectrum:

$$f_{\rm nl} = 32 \pm 21 \ (68\%)$$

Komatsu et al. (2010)

* WMAP constraint on trispectrum:

$$\tau_{\rm nl} = (0.96 \pm 0.68) \times 10^4$$

Smidt et al. (2010)

Tristan Smith

Non-Gaussian estimation

* For the rest of this talk, we will work in a simplified limit: flat-sky, Sachs-Wolfe limit

$$T(\hat{n}) = \int \frac{d^3\vec{k}}{(2\pi)^3}\Phi(\vec{k}) \int_0^{\tau_0} d\tau e^{i\vec{k}\cdot\hat{n}(\tau_0-\tau)} S(k,\tau)$$

$$a_{lm} = \int T(\hat{n}) Y_{lm}(\hat{n}) d^2\hat{n}$$

flat-sky: $a(\vec{l}) = \int T(\hat{n}) e^{i\vec{l}\cdot\hat{n}} d^2\hat{n}$

Sachs-Wolfe: $S_{SW}(k,\tau) = \frac{1}{3}\delta(\tau - \tau_D)$

Tristan Smith

Estimators for $f_{\rm nl}$ and $\tau_{\rm nl}$

* Expectation values of the harmonic coefficients are given by

$$\langle a(\vec{l}_1) a(\vec{l}_2) a(\vec{l}_3)\rangle = f_{\rm nl} B(l_1, l_2, l_3)\delta_{\vec{l}_1+\vec{l}_2+\vec{l}_3}$$

$$\langle a(\vec{l}_1) a(\vec{l}_2) a(\vec{l}_3) a(\vec{l}_4)\rangle = \tau_{\rm nl} T(\vec{l}_1, \vec{l}_2, \vec{l}_3, \vec{l}_4)\delta_{\vec{l}_1+\vec{l}_2+\vec{l}_3+\vec{l}_4}$$

* We can construct an estimator as a weighted sum, i.e.

$$\widehat{f_{\rm nl}} = N^{-1} \sum_{\vec{l}_1,\vec{l}_2,\vec{l}_3} a(\vec{l}_1) a(\vec{l}_2) a(\vec{l}_3) W(\vec{l}_1, \vec{l}_2, \vec{l}_3)$$

* We optimize this estimator by requiring the signal-to-noise to be maximized

Tristan Smith

Estimators for $f_{\rm nl}$ and $\tau_{\rm nl}$

* Maximizing the S/N gives an inverse-variance weighted sum:

$$\widehat{f_{\rm nl}} = \frac{1}{N}\sum_{\vec{l}_1+\vec{l}_2+\vec{l}_3=0}\frac{a(\vec{l}_1)a(\vec{l}_2)a(\vec{l}_3)B(l_1,l_2,l_3)}{C_{l_1}C_{l_2}C_{l_3}}$$

$$\widehat{\tau_{\rm nl}} = \frac{1}{N}\sum_{\vec{l}_1+\vec{l}_2+\vec{l}_3+\vec{l}_4=0}\frac{a(\vec{l}_1)a(\vec{l}_2)a(\vec{l}_3)a(\vec{l}_4)T(\vec{l}_1,\vec{l}_2,\vec{l}_3,\vec{l}_4)}{C_{l_1}C_{l_2}C_{l_3}C_{l_4}}$$

* What are the statistics of these estimators? Usually we assume the central limit theorem applies...

Tristan Smith

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

Statistics of the estimators

$$\widehat{f_{\rm nl}} = \frac{1}{N}\sum_{\vec{l}_1+\vec{l}_2+\vec{l}_3=0}\frac{a(\vec{l}_1)a(\vec{l}_2)a(\vec{l}_3)B(l_1,l_2,l_3)}{C_{l_1}C_{l_2}C_{l_3}}$$

$$\widehat{\tau_{\rm nl}} = \frac{1}{N}\sum_{\vec{l}_1+\vec{l}_2+\vec{l}_3+\vec{l}_4=0}\frac{a(\vec{l}_1)a(\vec{l}_2)a(\vec{l}_3)a(\vec{l}_4)T \ldots \vec{l}_3,\vec{l}_4)}{C \ldots}$$

* An observation gives ... measurements
* For $\widehat{f_{\rm nl}}$ we ... gles ($\vec{l}_1$, $\vec{l}_2$, $\vec{l}_3$), $N_{\rm pix}^2$ of these
* For $\widehat{\tau_{\rm nl}}$ w... over all quadrilaterals ($\vec{l}_1$, $\vec{l}_2$, $\vec{l}_3$, $\vec{l}_4$), $N_{\rm pix}^3$ of these

Central limit theorem does not apply!

Tristan Smith

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

Examples of non-Gaussian PDFs

* At its core, these estimators are a weighted sum of the product of Gaussian random variables:

$$\widehat{A} = \sum_{i,j,k} W_{i,j,k}a_i a_j a_k$$

* The simplest case is $W_{i,j,k} = 1$

$$\widehat{A} = \sum_{i,j,k} a_i a_j a_k = \left(\sum_i a_i\right)^3$$

Tristan Smith

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

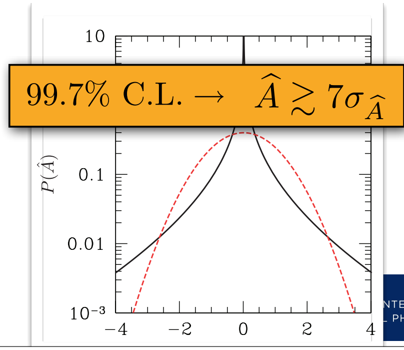
Examples of non-Gaussian PDFs

* At its core, the bispectrum $f_{\rm nl}$ estimator is a weighted sum of the product of three Gaussian random variables:

$$\widehat{A} = \sum_{i,j,k} W_{i,j,k}a_i a_j a_k$$

* The simplest case is $W_{i,j,k} = 1$

$$\widehat{A} = \sum_{i,j,k} a_i a_j a_k = \left(\sum_i a_i\right)^3$$

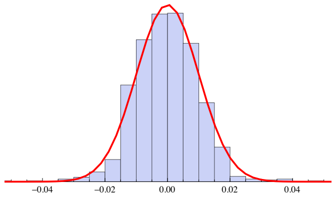


Tristan Smith

...NTER *for* ... PHYSICS

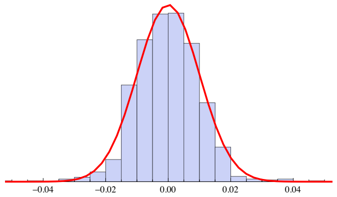
Computationally intensive

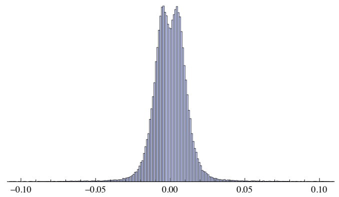
- 1000 realizations takes at least 1000 CPU-hours!
- This is good enough to determine the variance of the estimator, but not to determine the shape of the PDF
- For a non-Gaussian process, 1000 realizations gives the following histogram:



Computationally intensive

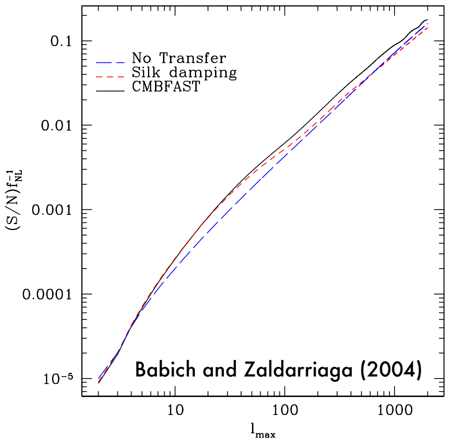
- 1000 realizations takes at least 500 CPU-hours!
- This is good enough to determine the variance of the estimator, but not to determine the shape of the PDF
- For a non-Gaussian process, 1000 realizations gives the following histogram; for 10^5 realizations we find this (!!):





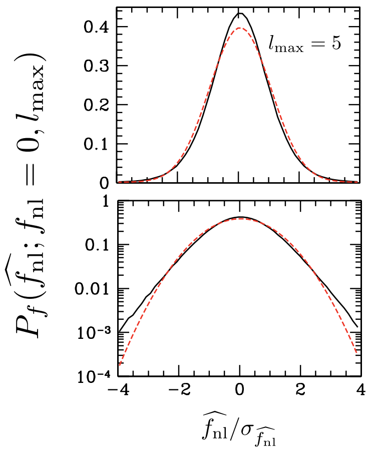
Non-Gaussian estimation

- Flat sky/ Sachs Wolfe will reproduce the correct scalings and give order of magnitude estimates:



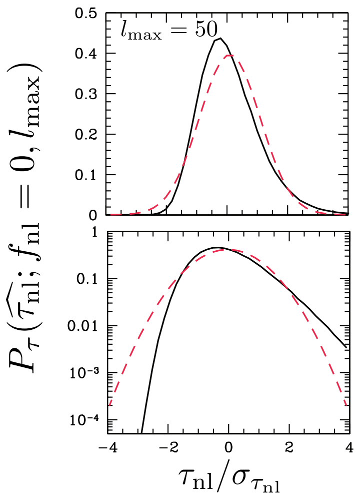
Full shape of the PDF

- First investigate the shape of the PDF for $P_f(\widehat{f_{\rm nl}}; f_{\rm nl} = 0, l_{\rm max})$



Full shape of the PDF

* Doing the same for $P_\tau(\widehat{\tau_{\rm nl}}; f_{\rm nl}=0, l_{\rm max})$, we find it is *highly non-Gaussian*:



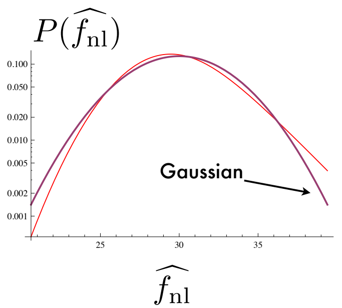
Tristan Smith

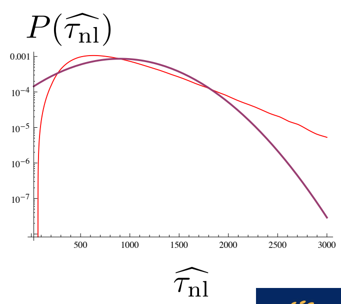
BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

Full shape of the PDF

* For $f_{\rm nl} \neq 0$ the non-Gaussianity in the map imparts *additional* non-Gaussianity to the bispectrum PDF and trispectrum PDF

* For $l_{\rm max} \simeq 3000$, $f_{\rm nl}=30$, and $\tau_{\rm nl}=900$





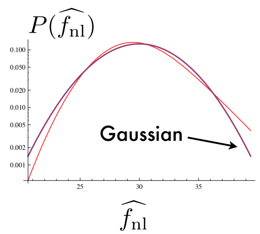
Tristan Smith

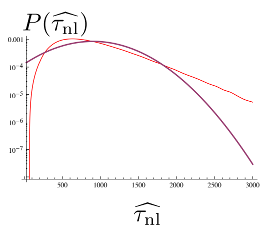
BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

Full shape of the PDF

* For $f_{\rm nl} \neq 0$ the non-Gaussianity in the map imparts *additional* non-Gaussianity to the bispectrum PDF and trispectrum PDF

* For $l_{\rm max} \simeq 3000$, $f_{\rm nl}=30$, and $\tau_{\rm nl}=900$





* In addition, the variance of these estimators depends on the value of $f_{\rm nl}$ and $\tau_{\rm nl}$...

Tristan Smith

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

Evolution of the variance

* The fact that the variance depends on $f_{\rm nl}$ and $\tau_{\rm nl}$ is easy to see:

$$\widehat{f_{\rm nl}} = \frac{1}{N}\sum_{\vec{l}_1+\vec{l}_2+\vec{l}_3=0} \frac{a(\vec{l}_1)a(\vec{l}_2)a(\vec{l}_3)B(l_1,l_2,l_3)}{C_{l_1}C_{l_2}C_{l_3}}$$

$$a(\vec{l}) = \bar{a}(\vec{l}) + f_{\rm nl}\delta a^2(\vec{l})$$

$$\widehat{f_{\rm nl}} = F_0 + f_{\rm nl}F_1 + f_{\rm nl}^2 F_2 + f_{\rm nl}^3 F_3$$

$$\langle \widehat{f_{\rm nl}}^2 \rangle = \sum_{i,j}\langle F_i F_j\rangle (f_{\rm nl})^{i+j}$$

Tristan Smith

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

Evolution of the variance

* Our calculations show that the variances of these estimators scale with $l_{\max}$ as:

$$\sigma^2_{f_{\rm nl}} = \frac{1}{72 A l_{\max}^2 \ln(l_{\max})} + \frac{f_{\rm nl}^2}{2\ln^3(l_{\max})}$$

$$\sigma^2_{\tau_{\rm nl}} = \frac{1.74\times 10^{-2}}{A^2 l_{\max}^4} + \frac{0.028\tau_{\rm nl}}{A l_{\max}^2} + 0.23\tau_{\rm nl}^2$$

* Now we have everything we need to evaluate the significance of a hypothetical detection...

Tristan Smith

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

What could have gone wrong?

* Our full calculations give

$$f_{\rm nl} = 30^{+7.25}_{-5.5}\ (95\%\ \text{C.L.})$$
$$\tau_{\rm nl} < 250\ (95\%\ \text{C.L.})$$

* If we did not take into account the non-Gaussian shape of the PDF then we would have concluded

$$f_{\rm nl} = 30^{+7.5}_{-5.4}\ (95\%\ \text{C.L.})$$
$$\tau_{\rm nl} < 1000\ (95\%\ \text{C.L.})$$

* If, in addition, we did not take into account how the variance depends on the amplitudes we would have concluded

$$f_{\rm nl} = 30 \pm 2.8\ (95\%\ \text{C.L.})$$
$$\tau_{\rm nl} < 90\ (95\%\ \text{C.L.})$$

Tristan Smith

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

Dispelling a claim

* In 2006 Kogo and Komatsu claimed that for large enough $l_{\max}$ the trispectrum estimator has a larger S/N than the bispectrum estimator

$$\sigma^2_{f_{\rm nl}} = \frac{1}{72 A l_{\max}^2 \ln(l_{\max})}$$

$$\sigma^2_{\tau_{\rm nl}} = \frac{1.74\times 10^{-2}}{A^2 l_{\max}^4}$$

* With the correct scaling, we can see that the bispectrum estimator will always have a higher S/N

Tristan Smith

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS

Conclusions

* Small-scale CMB observations show an anomalously large value for $N_{\rm eff}$
* Explored how observations probe the interaction/clustering properties of this anomalous radiative energy density
* We found that with an expanded parameterization the data is still at odds with the standard neutrino sector at > 95 % CL *and* consistent with a non-interacting fluid
* The central limit theorem does not apply to non-Gaussian estimators– PDFs of these estimators may, themselves, be non-Gaussian
* The effect on the bispectrum estimator is small; the effect on the trispectrum estimator is large– must be included when stating the significance of a measurement

Tristan Smith

BERKELEY CENTER *for* COSMOLOGICAL PHYSICS