Are Two Metrics Better than One?

The Cosmology of **Massive** (Bi)gravity

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Geneva:
Mariele Motta

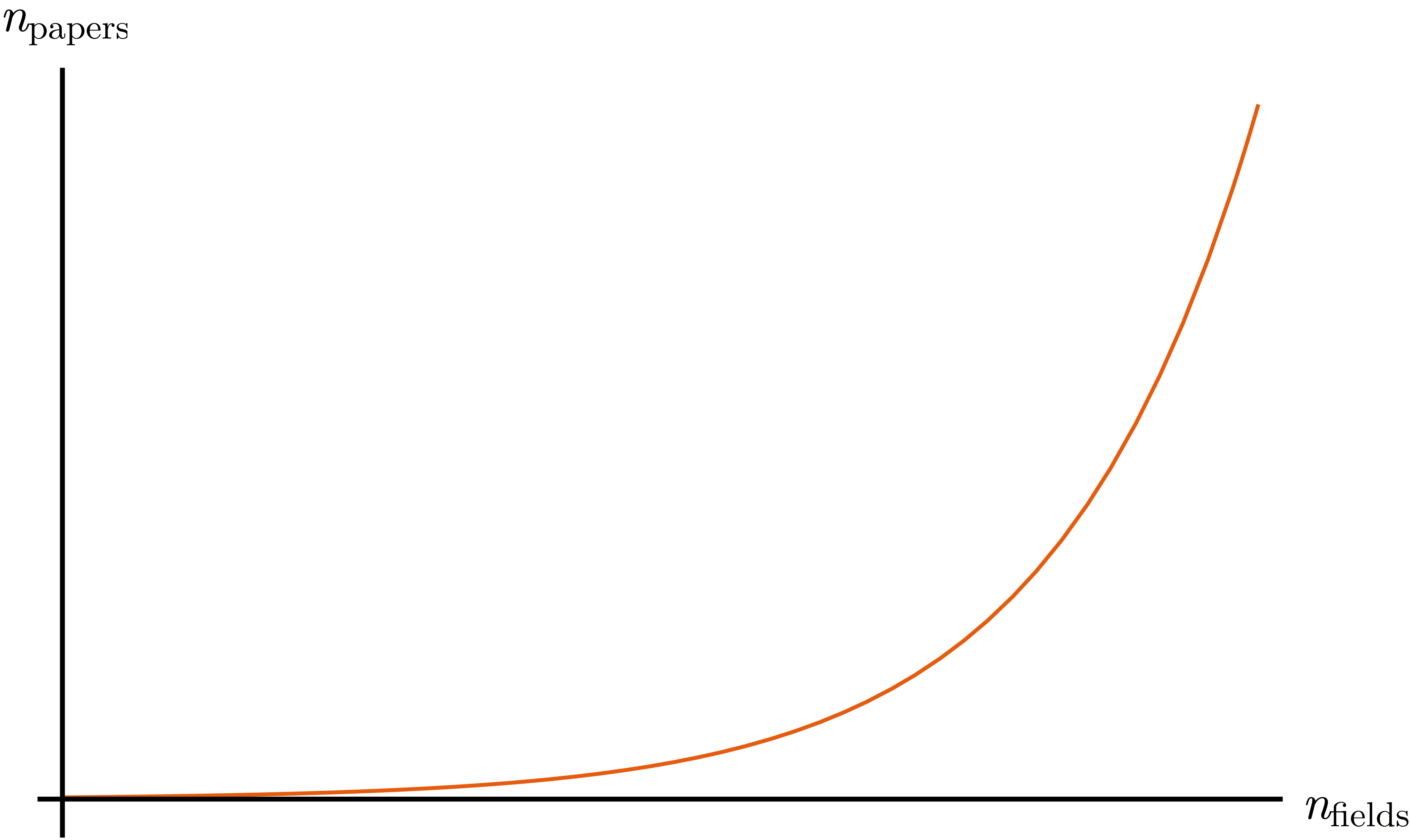
Based on: arXiv:1404.4061
arXiv:1407.4331
arXiv:1409.xxxx
arXiv:1409.xxxx

Outline

- Introduction and motivation
- Background cosmology
- Stability around cosmological backgrounds
- Predictions: subhorizon structure formation
- New frontiers: coupling to both metrics

Why bother with this weird theory with two metrics?

Isn’t one metric enough?


n_{papers}
n_{fields}

Why consider two metrics?

- Field theoretic interest: how do we construct consistent interactions of multiple spin-2 fields?
  - NB "Consistent" crucially includes ghost-free
- My motivation: modified gravity ➔ massive graviton
  - 1) The next decade will see multiple precision tests of GR – we need to understand the alternatives
  - 2) The accelerating universe

Dark energy or modified gravity?

Einstein's equation + the Standard Model + dark matter predict a decelerating universe, but this contradicts observations. The expansion of the Universe is accelerating!

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

What went wrong?? Two possibilities:

- *Dark energy:* Do we need to include new "stuff" on the RHS?
- *Modified gravity:* Are we using the wrong equation to describe gravity at cosmological distances?

Cosmic acceleration has theoretical problems which modified gravity might solve

- **Technically natural self-acceleration**: Certain theories of gravity may have late-time acceleration which does not get destabilized by quantum corrections.
  - This is THE major problem with a simple cosmological constant
- **Degravitation**: Why do we not see a large CC from matter loops? Perhaps an IR modification of gravity makes a CC invisible to gravity
  - This is natural with a massive graviton due to short range

Why consider two metrics?

- Take-home message: Massive bigravity is a natural, exciting, and still largely unexplored new direction in modifying GR.

How can we do gravity beyond GR? Some famous examples

- Brans-Dicke (1961): make Newton's constant dynamical: $G_N = 1/\phi$, gravity couples non-minimally to ϕ

$$S = \int d^4x \frac{1}{16\pi}\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}(\partial\phi)^2\right]$$

- f(R) (2000s): replace Einstein-Hilbert term with a general function f(R) of the Ricci scalar

$$S = \int d^4x \sqrt{-g} f(R)$$

How can we do gravity beyond GR?

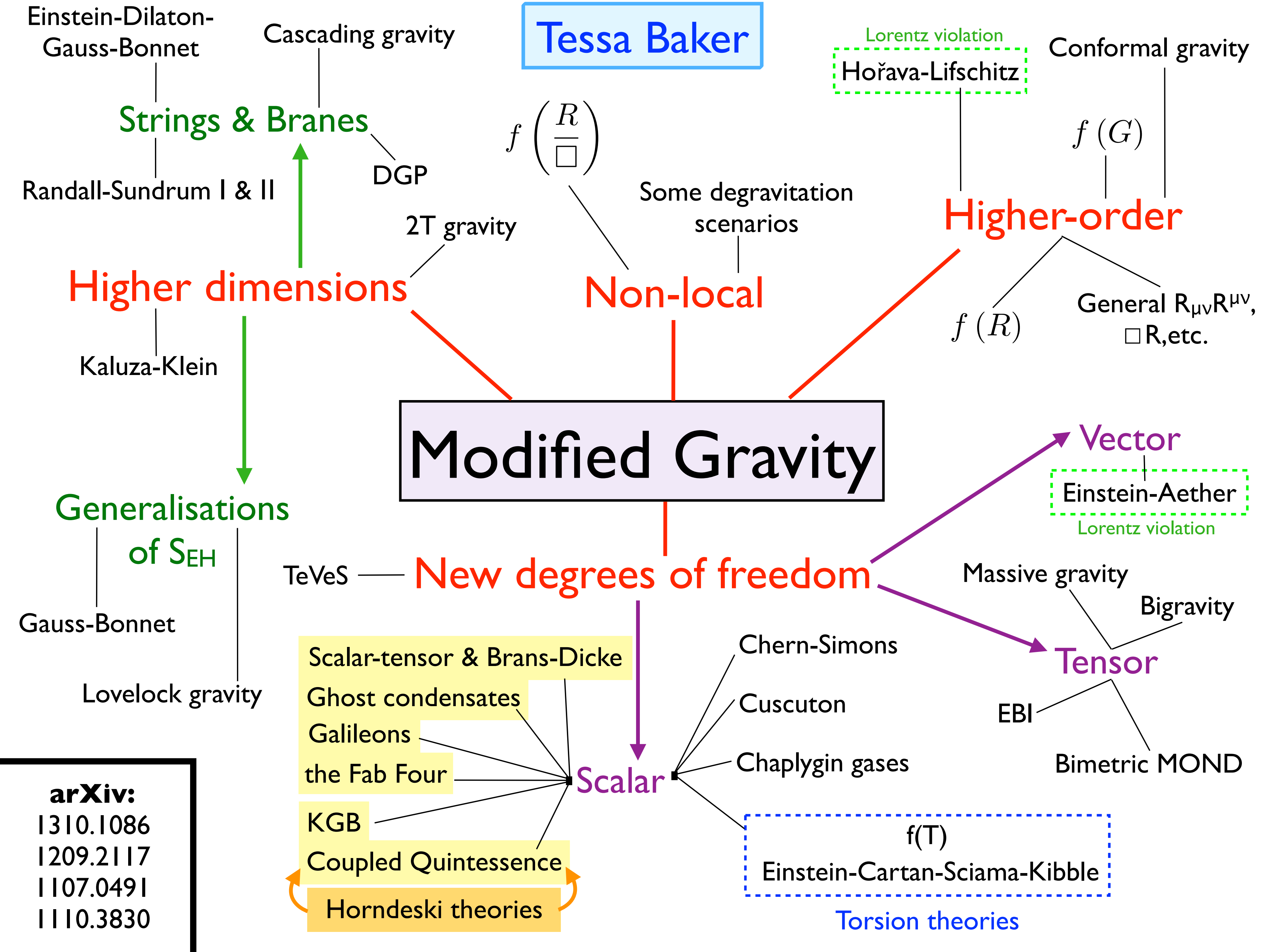
- These theories are generally *not simple*
  - Even f(R) looks elegant in the action, but from a *degrees of freedom* standpoint it is a theory of a scalar field non-minimally coupled to the metric, just like Brans-Dicke, Galileons, Horndeski, etc.

How can we do gravity beyond GR?

- Most attempts at modifying GR are guided by *Lovelock's theorem* (Lovelock, 1971):

> GR is the **unique** theory of gravity which
>
> - Only involves a rank-2 tensor
> - Has second-order equations of motion
> - Is in 4D
> - Is local and Lorentz invariant

- The game of modifying gravity is played by breaking one or more of these assumptions



Tessa Baker
Modified Gravity
Higher dimensions
Strings & Branes
Einstein-Dilaton-Gauss-Bonnet
Cascading gravity
DGP
Randall-Sundrum I & II
2T gravity
Kaluza-Klein
Generalisations of S_{EH}
Gauss-Bonnet
Lovelock gravity
Non-local
$f\left(\frac{R}{\Box}\right)$
Some degravitation scenarios
Higher-order
Lorentz violation
Hořava-Lifschitz
Conformal gravity
$f(G)$
$f(R)$
General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$, etc.
New degrees of freedom
TeVeS
Vector
Einstein-Aether
Lorentz violation
Tensor
Massive gravity
Bigravity
EBI
Bimetric MOND
Scalar
Scalar-tensor & Brans-Dicke
Ghost condensates
Galileons
the Fab Four
KGB
Coupled Quintessence
Horndeski theories
Chern-Simons
Cuscuton
Chaplygin gases
f(T)
Einstein-Cartan-Sciama-Kibble
Torsion theories
arXiv:
1310.1086
1209.2117
1107.0491
1110.3830

Another path: degrees of freedom (*or*, Lovelock or Weinberg?)

- GR is unique.
- But instead of thinking about that uniqueness through Lovelock's theorem, we can also remember that (Weinberg, others, 1960s)…

GR = massless spin-2

- A natural way to modify GR: give the graviton mass!

Non-linear massive gravity is a very recent development

- At the linear level, the correct theory of a massive graviton has been known since 1939 (Fierz, Pauli)
- But in the 1970s, several issues – most notably a dangerous ghost instability (mode with wrong-sign kinetic term) – were discovered

Non-linear massive gravity is a very recent development

- Only in 2010 were these issues overcome when de Rham, Gabadadze, and Tolley (dRGT) wrote down *the* ghost-free, non-linear theory of massive gravity
- See the reviews by
  de Rham arXiv:1401.4173, and
  Hinterbichler arXiv:1105.3735

dRGT Massive Gravity in a Nutshell

- The unique non-linear action for a single massive spin-2 graviton is

$$S = -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R$$

$$+ m^2 M_g^2 \int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$

where $f_{\mu\nu}$ is an arbitrary reference metric which must be chosen at the start

- β_n are the free parameters; the graviton mass is $\sim m^2\beta_n$
- The e_n are elementary symmetric polynomials given by…

$$S = -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R + m^2M_g^2\int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$

For a matrix X, the elementary symmetric polynomials are ([] = trace)

$$e_0\left(X\right) \equiv 1,$$

$$e_1\left(X\right) \equiv \left[X\right],$$

$$e_2\left(X\right) \equiv \frac{1}{2}\left(\left[X\right]^2 - \left[X^2\right]\right),$$

$$e_3\left(X\right) \equiv \frac{1}{6}\left(\left[X\right]^3 - 3\left[X\right]\left[X^2\right] + 2\left[X^3\right]\right),$$

$$e_4\left(X\right) \equiv \det\left(X\right)$$

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Much ado about a reference metric?

- There is a simple (heuristic) reason that massive gravity needs a second metric: you can't construct a non-trivial interaction term from one metric alone:

$$g^{\mu\alpha}g_{\nu\alpha} = \delta^\mu_\nu, \quad (g_{\mu\nu})^2 = 4, \quad \ldots$$

- We need to introduce a second metric to construct interaction terms.
- ➔ There are many dRGT massive gravity theories
- What should this metric be?

From massive gravity to massive bigravity

- Simple idea (Hassan and Rosen, 2011): make the reference metric dynamical

$$S = -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R(g) - \frac{M_f^2}{2}\int d^4x\sqrt{-\det f}R(f)$$
$$+ m^2 M_g^2\int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$

- Resulting theory: one massless graviton and one massive – massive bigravity

From massive gravity to massive bigravity

- By moving from dRGT to bimetric massive gravity, we avoid the issue of choosing a reference metric (Minkowski? (A)dS? Other?)
  - Trading a constant matrix ($f_{\mu\nu}$) for a constant scalar (M_f) – simplification!
  - Better yet, M_f is redundant
- Allows for stable, flat FRW cosmological solutions (do not exist in dRGT)
- Bigravity is a very sensible theory to consider

Massive bigravity has self-accelerating cosmologies

- Homogeneous and isotropic solution:

$$ds_g^2 = a^2\left(-d\tau^2 + d\vec{x}^2\right),$$

$$ds_f^2 = -X^2 d\tau^2 + Y^2 d\vec{x}^2$$

the background dynamics are determined by

$$3\mathcal{H}^2 = \frac{a^2\rho}{M_g^2} + m^2a^2\left(\beta_0 + 3\beta_1 y + 3\beta_2 y^2 + \beta_3 y^3\right)$$

$$y \equiv \frac{Y}{a}$$

$$\beta_3 y^4 + \left(3\beta_2 - \beta_4\right) y^3 + 3\left(\beta_1 - \beta_3\right) y^2 + \left(\frac{\rho}{M_g^2 m^2} + \beta_0 - 3\beta_2\right) y - \beta_1 = 0$$

As ρ -> 0, y -> constant, so the mass term approaches a (positive) constant ➔ late-time acceleration

- NB: We are choosing (for now) to only couple matter to one metric, $g_{\mu\nu}$

Massive bigravity effectively competes with ΛCDM

- A comprehensive comparison to background data was undertaken by **Akrami, Koivisto, & Sandstad [arXiv:1209.0457]**
- Data sets:
  - Luminosity distances from Type Ia supernovae (Union 2.1)
  - Position of the first CMB peak – angular scale of sound horizon at recombination (WMAP7)
  - Baryon-acoustic oscillations (2dFGRS, 6dFGS, SDSS and WiggleZ)

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- A comprehensive comparison to background data was undertaken by **Akrami, Koivisto, & Sandstad (2012), arXiv:1209.0457**
- Take-home points:
  - No exact ΛCDM without explicit cosmological constant (vacuum energy)
    - Dynamical dark energy
    - Phantom behavior ($w < -1$) is common
  - ✓ Viable alternative to ΛCDM

Massive bigravity effectively competes with ΛCDM

Y. Akrami, T. Koivisto, and M. Sandstad [arXiv:1209.0457]
See also F. Könnig, A. Patil, and L. Amendola [arXiv:1312.3208];
ARS, Y. Akrami, and T. Koivisto [arXiv:1404.4061]

$$B_i \equiv \frac{m^2}{H_0^2}\beta_i$$

Model	B_0	B_1	B_2	B_3	B_4	Ω_m	χ^2_{min}	p-value	log-evidence
ΛCDM	free	0	0	0	0	free	546.54	0.8709	-278.50
(B_1, Ω_m^0)	0	free	0	0	0	free	551.60	0.8355	-281.73
(B_2, Ω_m^0)	0	0	free	0	0	free	894.00	< 0.0001	-450.25
(B_3, Ω_m^0)	0	0	0	free	0	free	1700.50	< 0.0001	-850.26
(B_1, B_2, Ω_m^0)	0	free	free	0	0	free	546.52	0.8646	-279.77
(B_1, B_3, Ω_m^0)	0	free	0	free	0	free	542.82	0.8878	-280.10
(B_2, B_3, Ω_m^0)	0	0	free	free	0	free	548.04	0.8543	-280.91
(B_1, B_4, Ω_m^0)	0	free	0	0	free	free	548.86	0.8485	-281.42
(B_2, B_4, Ω_m^0)	0	0	free	0	free	free	806.82	< 0.0001	-420.87
(B_3, B_4, Ω_m^0)	0	0	0	free	free	free	685.30	0.0023	-351.14
$(B_1, B_2, B_3, \Omega_m^0)$	0	free	free	free	0	free	546.50	0.8582	-279.61
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full bigravity model	free	free	free	free	free	free	546.50	0.8445	-279.82

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$\sim B_1 \, \mathrm{Tr}[\sqrt{g^{-1} f}]$

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Scalar perturbations in massive bigravity

- Extensive analysis of perturbations undertaken by ARS, Akrami, and Koivisto, arXiv:1404.4061
  Könnig, Akrami, Amendola, Motta, and ARS, arXiv:1407.4331
  - See also Könnig and Amendola, arXiv:1402.1988
- Linearize metrics around FRW backgrounds, restrict to scalar perturbations {$E_{g,f}$, $A_{g,f}$, $F_{g,f}$, and $B_{g,f}$}:

$$ds_g^2 = a^2 \left\{-(1+E_g)d\tau^2 + 2\partial_i F_g d\tau dx^i + \left[(1+A_g)\delta_{ij} + \partial_i\partial_j B_g\right] dx^i dx^j\right\}$$

$$ds_f^2 = -X^2(1+E_f)d\tau^2 + 2XY\partial_i F_f d\tau dx^i + Y^2\left[(1+A_f)\delta_{ij} + \partial_i\partial_j B_f\right] dx^i dx^j$$

- Full linearized Einstein equations (in cosmic or conformal time) can be found in ARS, Akrami, and Koivisto, arXiv: 1404.4061

Scalar fluctuations can suffer from instabilities

- Usual story: solve perturbed Einstein equations in quasistatic limit: $k^2\Phi \gg H^2\Phi \sim H\dot{\Phi} \sim \ddot{\Phi}$
- This is valid only if perturbations vary on Hubble timescales
- Cannot trust quasistatic limit if perturbations are unstable
- Check for instability by solving full system of perturbation equations

Scalar fluctuations can suffer from instabilities

- Degree of freedom count: ten total variables
  - Four $g_{\mu\nu}$ perturbations: E_g, A_g, B_g, F_g
  - Four $f_{\mu\nu}$ perturbations: E_f, A_f, B_f, F_f
  - Two perfect fluid perturbations: δ and θ
- Eight are redundant:
  - Four of these are nondynamical/auxiliary (E_g, F_g, E_f, F_f)
  - Two can be gauged away
  - After integrating out auxiliary variables, one of the dynamical variables becomes auxiliary
- End result: only two independent degrees of freedom

Scalar fluctuations can suffer from instabilities

- Choose g-metric Bardeen variables:

$$\Phi \equiv A_g - H\left(F_g + B_g'\right)$$

$$\Psi \equiv E_g - H\left(F_g + B_g'\right) - F_g' - B_g''$$

- Then *entire* system of 10 perturbed Einstein/fluid equations can be reduced to two coupled equations:

$$X_i'' + F_{ij}X_j' + S_{ij}X_j = 0$$

where

$$X_i = \{\Phi, \Psi\}$$

Scalar fluctuations can suffer from instabilities

- Ten perturbed Einstein/fluid equations can be reduced to two coupled equations:

$$X_i'' + F_{ij} X_j' + S_{ij} X_j = 0$$

where

$$X_i = \{\Phi, \Psi\}$$

- Under assumption (WKB) that F_{ij}, S_{ij} vary slowly, this is solved by

$$X_i = X_i^0 e^{i\omega N}$$

with N = ln a

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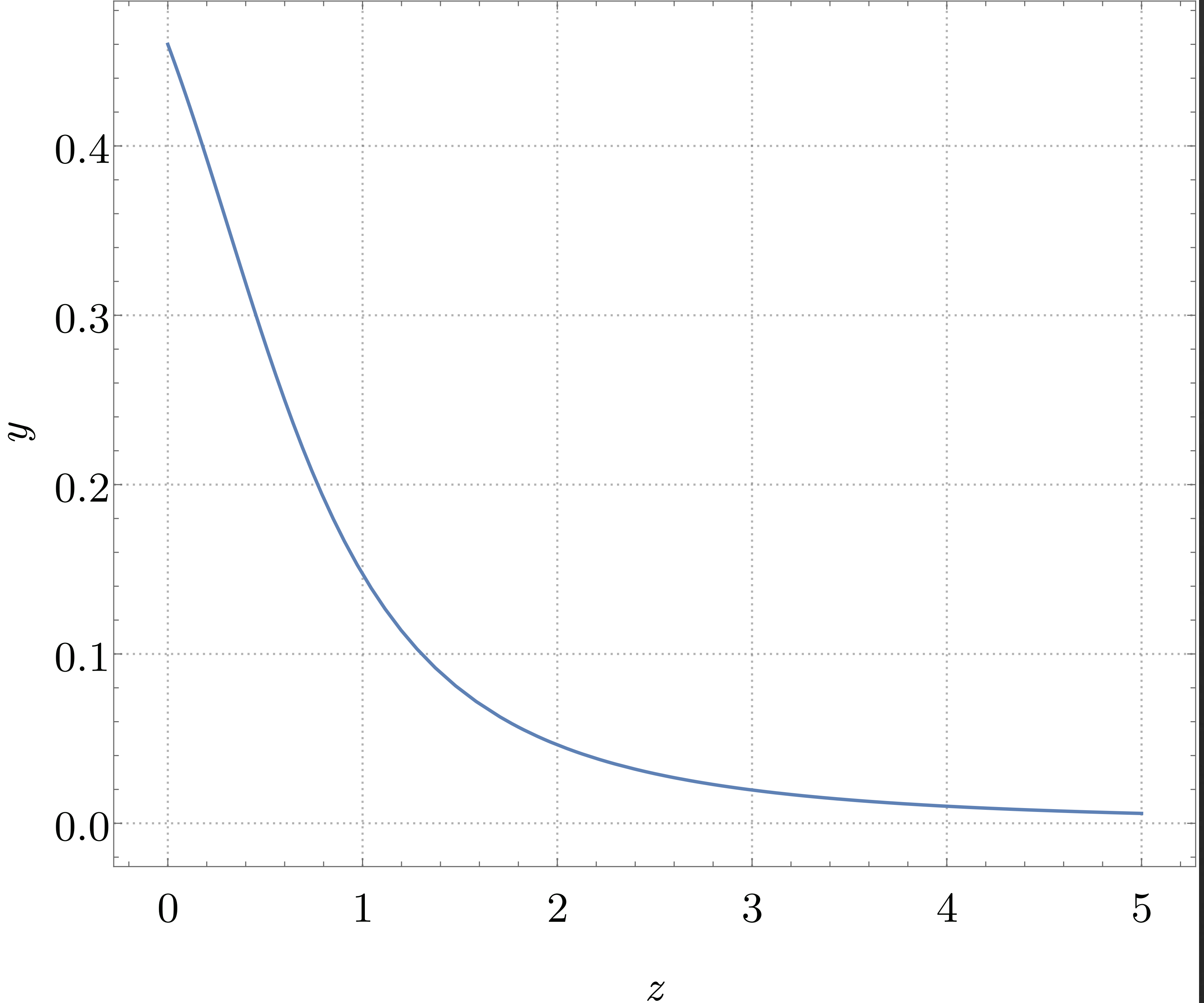
Scalar fluctuations can suffer from instabilities

- B_1-only model – simplest allowed by background

$$\omega_{B_1} = \pm \frac{k}{H} \frac{\sqrt{-1 + 12y^2 + 9y^4}}{1 + 3y^2}$$

- Unstable for small y (early times)


0.4
0.3
0.2
0.1
0.0
y
0
1
2
3
4
5
z

Scalar fluctuations can suffer from instabilities

- B_1-only model – simplest allowed by background

$$\omega_{B_1} = \pm \frac{k}{H} \frac{\sqrt{-1 + 12y^2 + 9y^4}}{1 + 3y^2}$$

- Unstable for small y (early times)
- For realistic parameters, model is only (linearly) stable for z <~ 0.5

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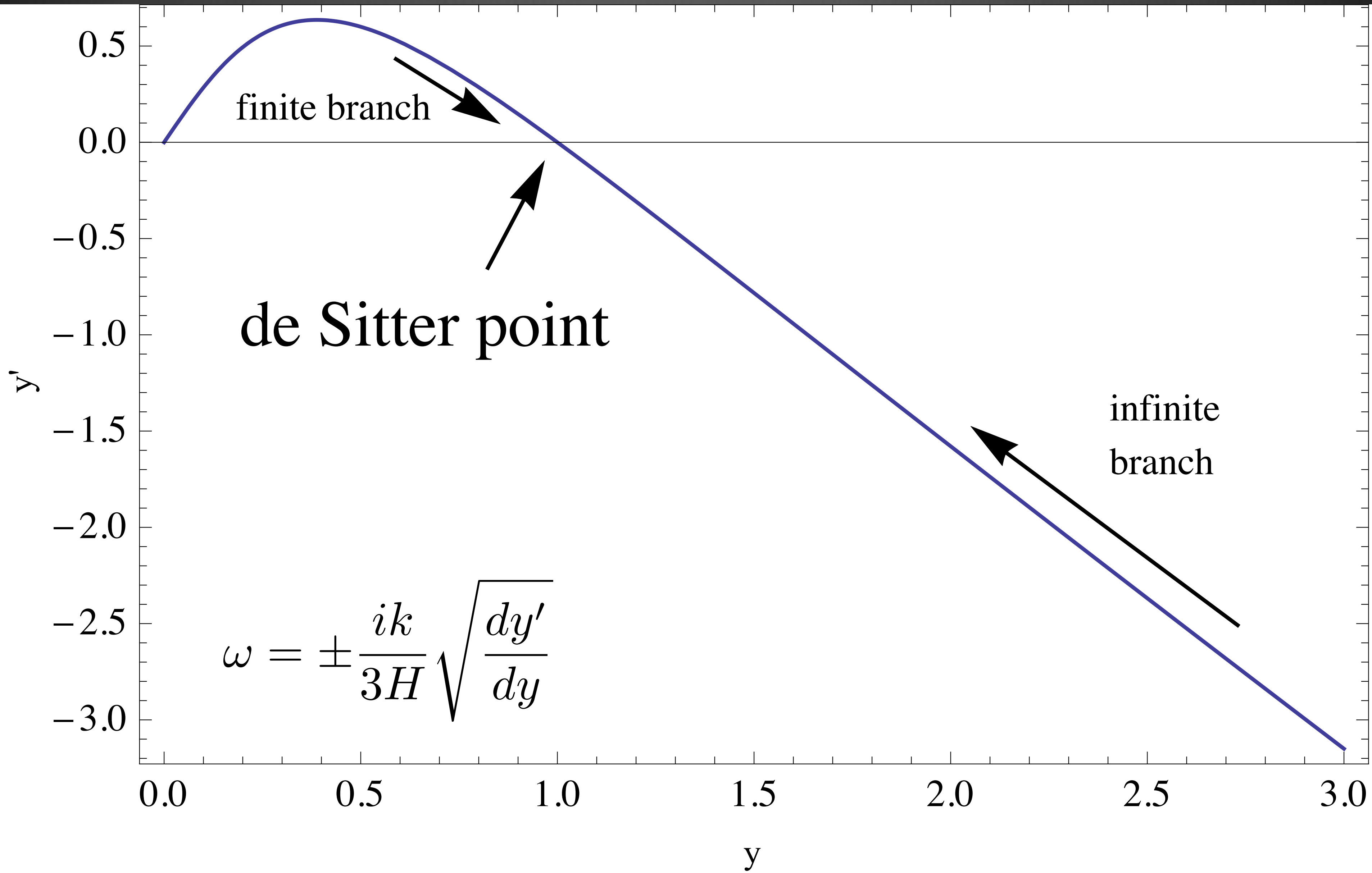
Scalar fluctuations can suffer from instabilities

- The instability is avoided by infinite-branch solutions, where y starts off at infinity at early times
- Background viability requires $B_1 > 0$
- Existence of infinite branch requires $0 < B_4 < 2B_1$ – i.e., turn on the f-metric cosmological constant

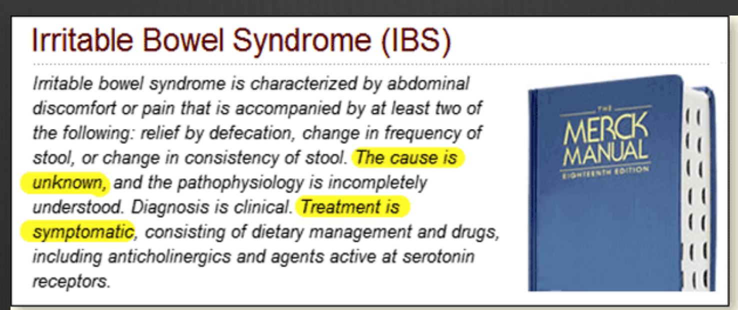
B_1-B_4 model: background dynamics



$$\omega = \pm \frac{ik}{3H}\sqrt{\frac{dy'}{dy}}$$

Scalar fluctuations can suffer from instabilities

- The instability is avoided by infinite-branch models, where y starts off at infinity at early times
- Background viability: $B_1 > 0$
- Infinite branch: $0 < B_4 < 2B_1$ – i.e., turn on the f-metric cosmological constant
- Catchy name: infinite-branch bigravity (IBB)
  - (Earlier proposal, infinite-branch solution (IBS), did not catch on)



Irritable Bowel Syndrome (IBS)

Irritable bowel syndrome is characterized by abdominal discomfort or pain that is accompanied by at least two of the following: relief by defecation, change in frequency of stool, or change in consistency of stool. The cause is unknown, and the pathophysiology is incompletely understood. Diagnosis is clinical. Treatment is symptomatic, consisting of dietary management and drugs, including anticholinergics and agents active at serotonin receptors.

Instability does not rule models out

- Instability -> breakdown of linear perturbation theory
  - Nothing more
  - Nothing less
- Cannot take quasistatic limit for unstable models
- Need nonlinear techniques to make structure formation predictions

Scalar perturbations in the quasistatic limit

ARS, Y. Akrami, and T. Koivisto, arXiv:1404.4061 (gory details)

- We can take the quasistatic limit for infinite-branch bigravity
- Specializing to this limit, and assuming only dust (P=0)…
  - Five perturbations ($E_{g,f}$, $A_{g,f}$, and B_f - B_g) are determined algebraically in terms of the density perturbation δ
  - Meanwhile, δ is determined by the same evolution equation as in GR:

$$\delta'' + \mathcal{H}\delta' + \frac{1}{2}k^2 E_g(\delta) = 0$$

$$\delta'' + \mathcal{H}\delta' + \frac{1}{2}k^2 E_g(\delta) = 0$$

(GR *and* massive bigravity)

- In GR, there is no anisotropic stress so E_g (time-time perturbation) is related to δ through Poisson's equation,

$$k^2 E_g = -(a^2 \bar{\rho}/M_g^2)\delta$$

- In bigravity, the relation beteen E_g and δ is significantly more complicated
  ➔ modified structure growth

The "observables": Modified gravity parameters

$$ds_g^2 = a^2 \left[-(1+E_g)d\tau^2 + (1+A_g)\delta_{ij}dx^i dx^j \right]$$

We calculate three parameters which are commonly used to distinguish modified gravity from GR:

- **Growth rate/index** (f/γ): measures growth of structures

$$f(a,k) \equiv \frac{d\log\delta}{d\log a} \approx \Omega_m^\gamma$$

- Modification of Newton's constant in Poisson eq. (Q):

$$\frac{k^2}{a^2}A_g \equiv \frac{Q(a,k)\bar{\rho}}{M_g^2}\delta$$

- Anisotropic stress (η):

$$\eta(a,k) \equiv -\frac{A_g}{E_g}$$

GR:

$$\gamma \approx 0.545$$

$$Q = \eta = 1$$

Adam Solomon

The "observables": Modified gravity parameters

- We have analytic solutions (messy) for A_g and E_g as (stuff) x δ, so
  - Can immediately read off analytic expressions for Q and η:

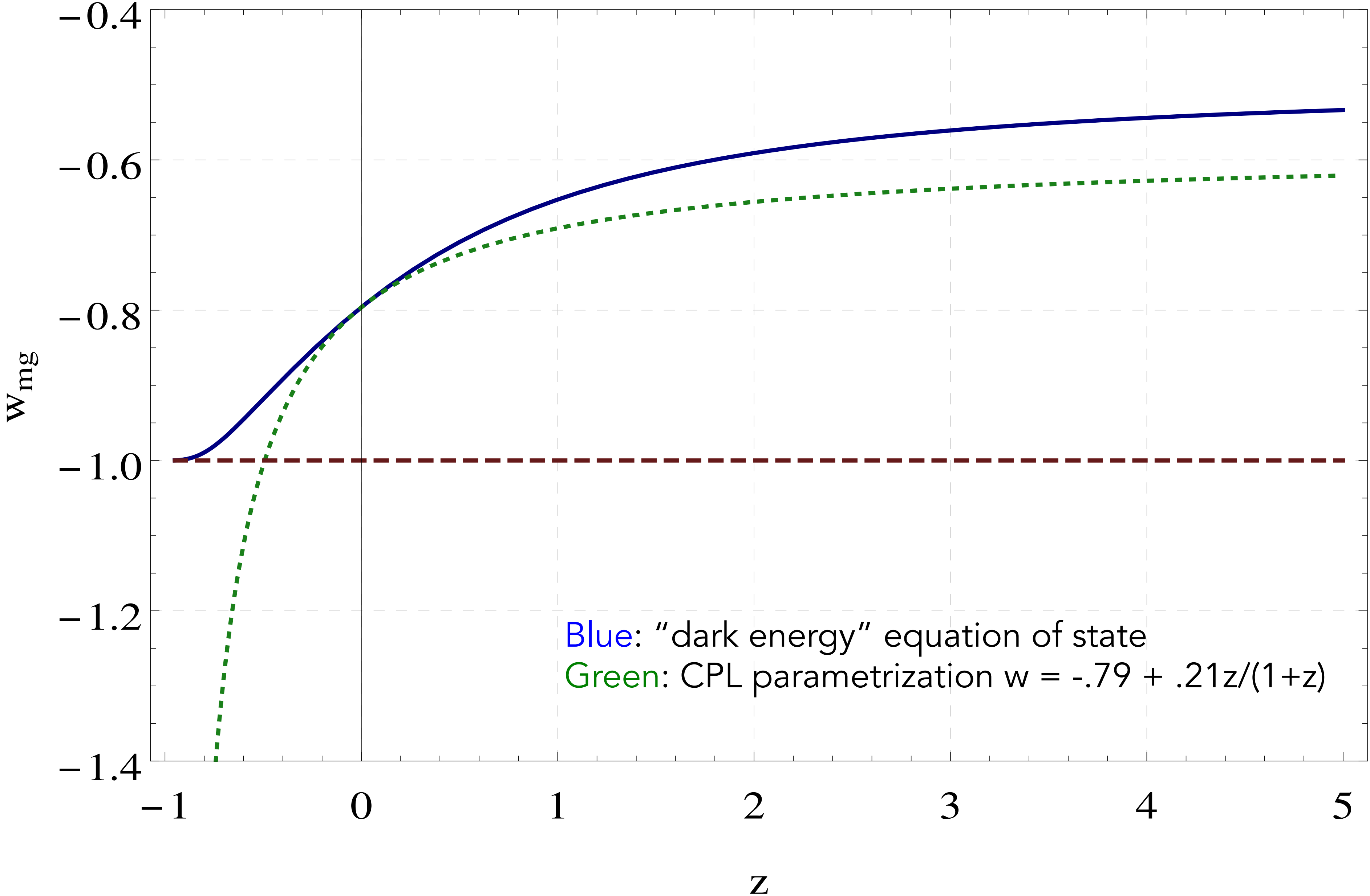
$$Q = h_1 \left(\frac{1 + k^2 h_4}{1 + k^2 h_3} \right), \qquad \eta = h_2 \left(\frac{1 + k^2 h_4}{1 + k^2 h_5} \right)$$

    (h_i are non-trivial functions of time; see ARS, Akrami, and Koivisto arXiv:1404.4061, App. B)
  - Can solve numerically for δ using Q and η:

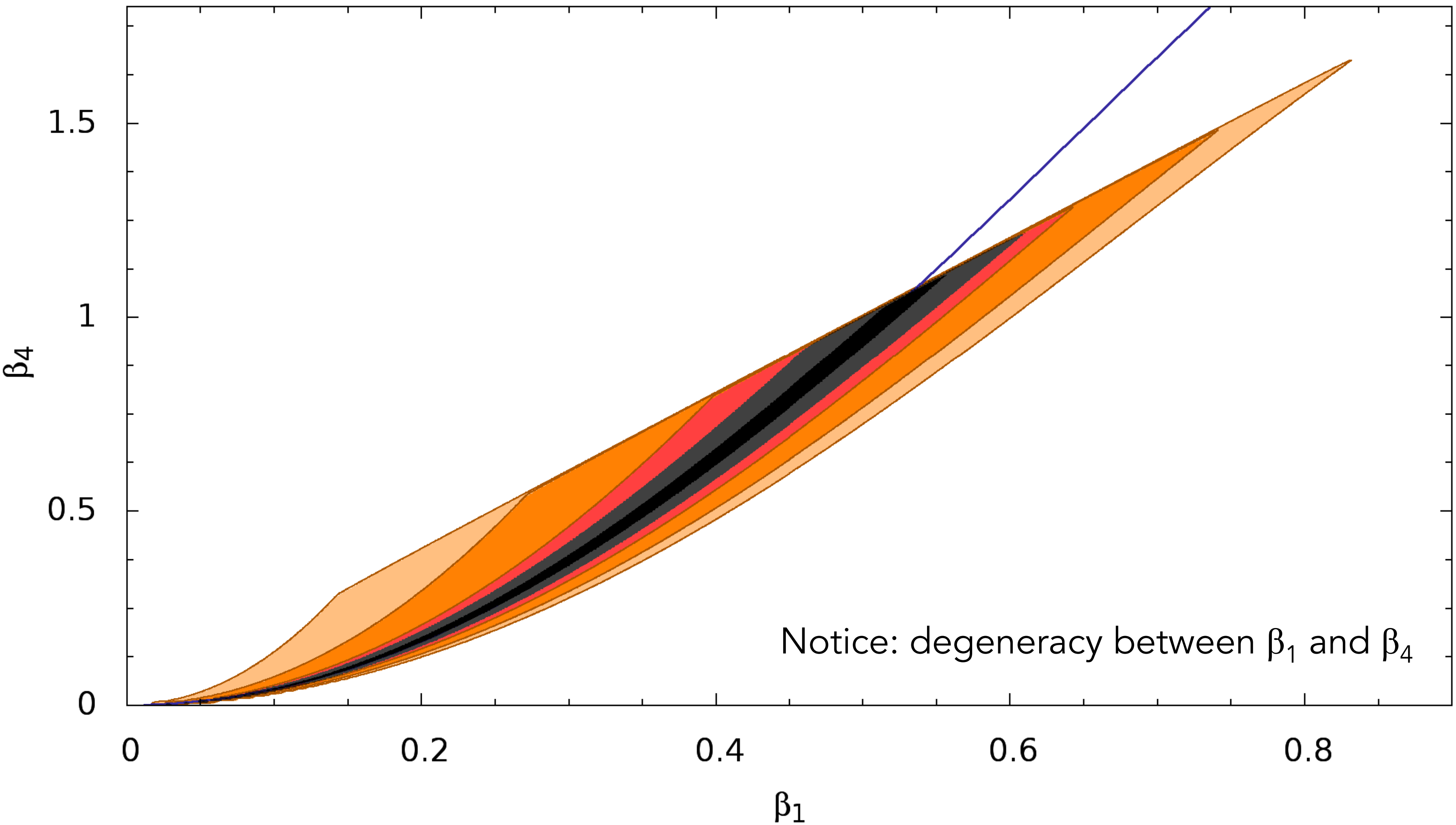
$$\delta'' + \mathcal{H}\delta' - \frac{1}{2}\frac{Q}{\eta}\frac{a^2\bar{\rho}}{M_g^2}\delta = 0$$

Infinite-branch bigravity: Expansion history

Adam Solomon

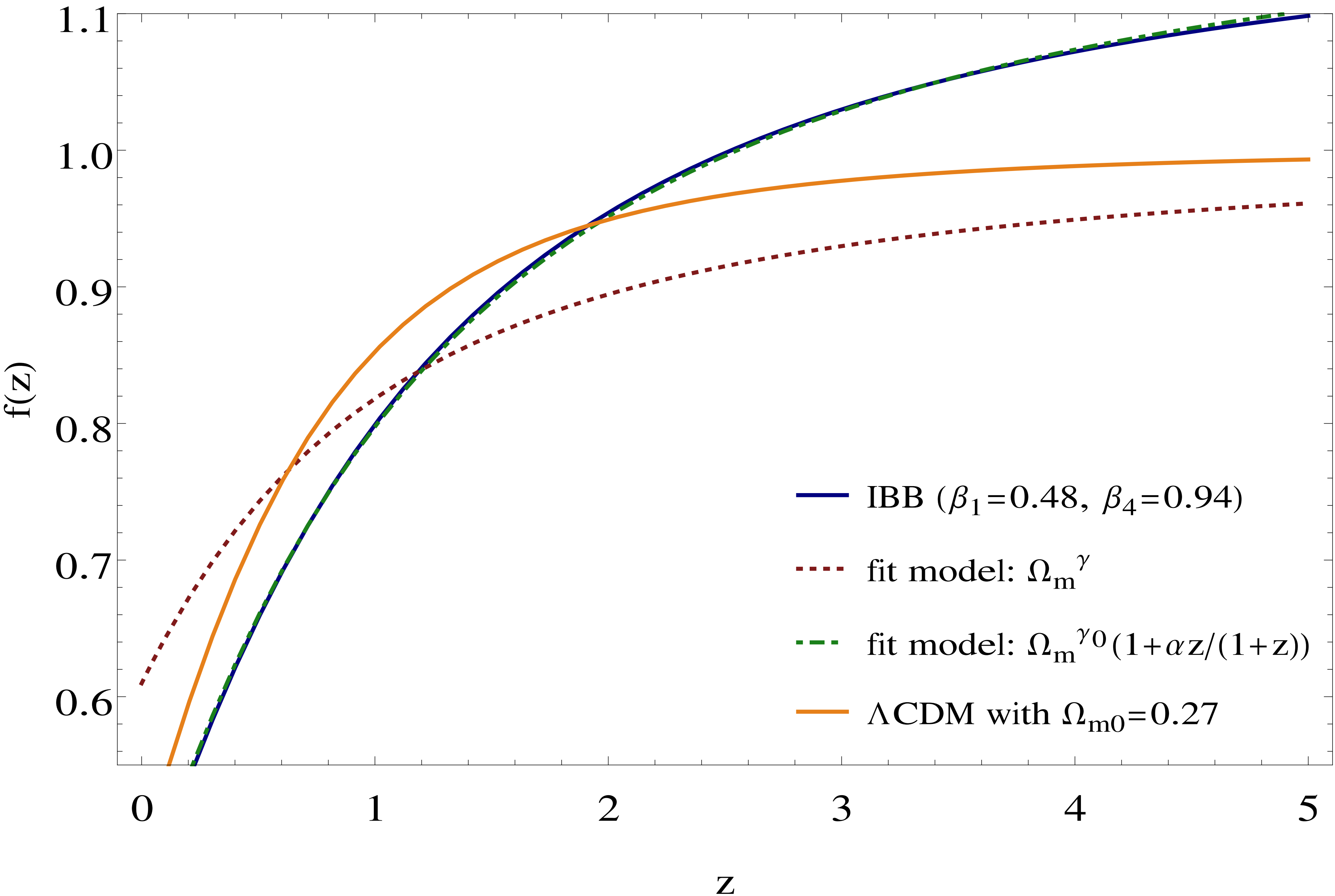

−0.4
−0.6
−0.8
−1.0
−1.2
−1.4
w_mg
−1
0
1
2
3
4
5
z
Blue: "dark energy" equation of state
Green: CPL parametrization w = -.79 + .21z/(1+z)

Constraints from SNe Ia (Union 2.1)

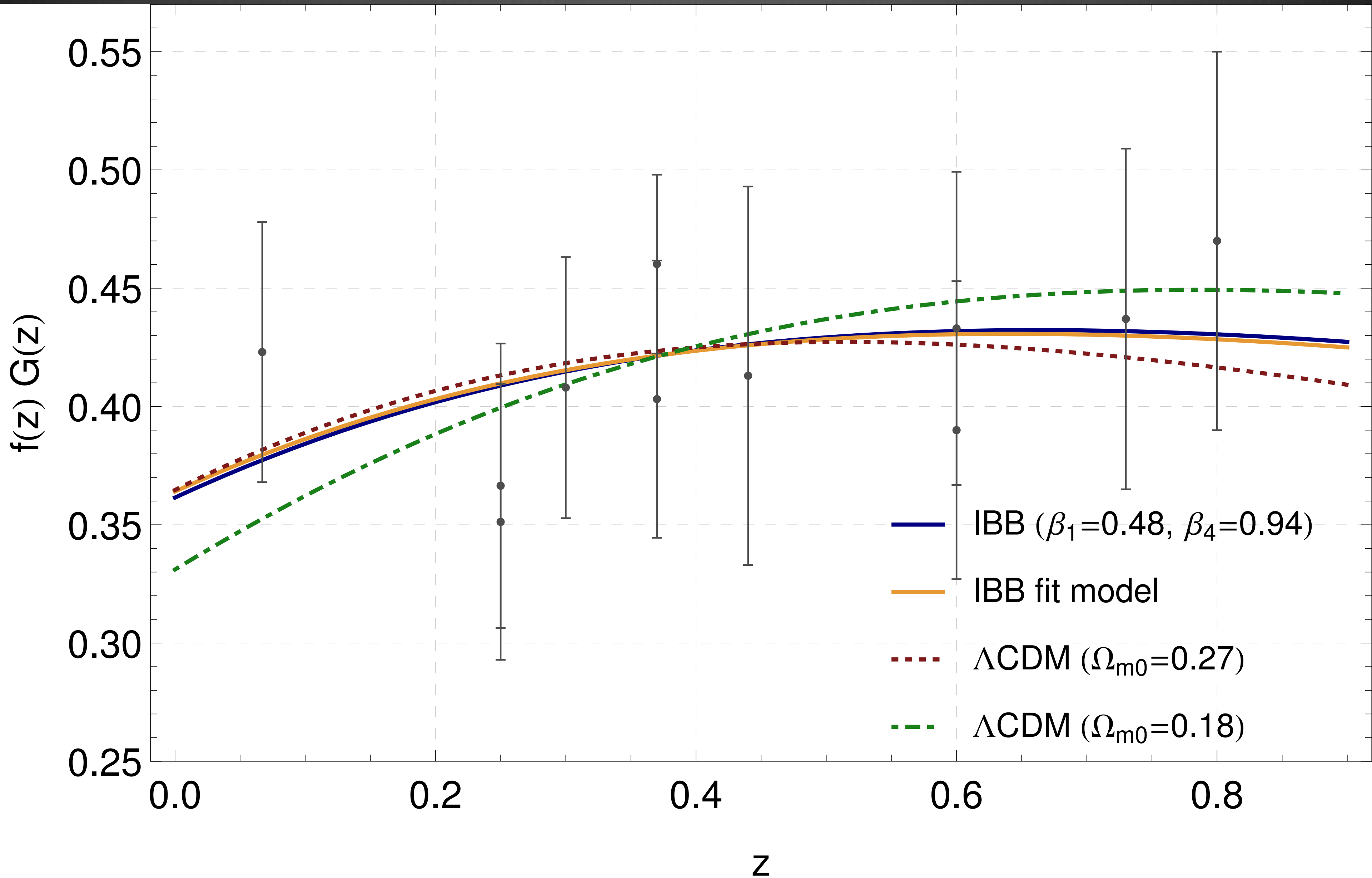


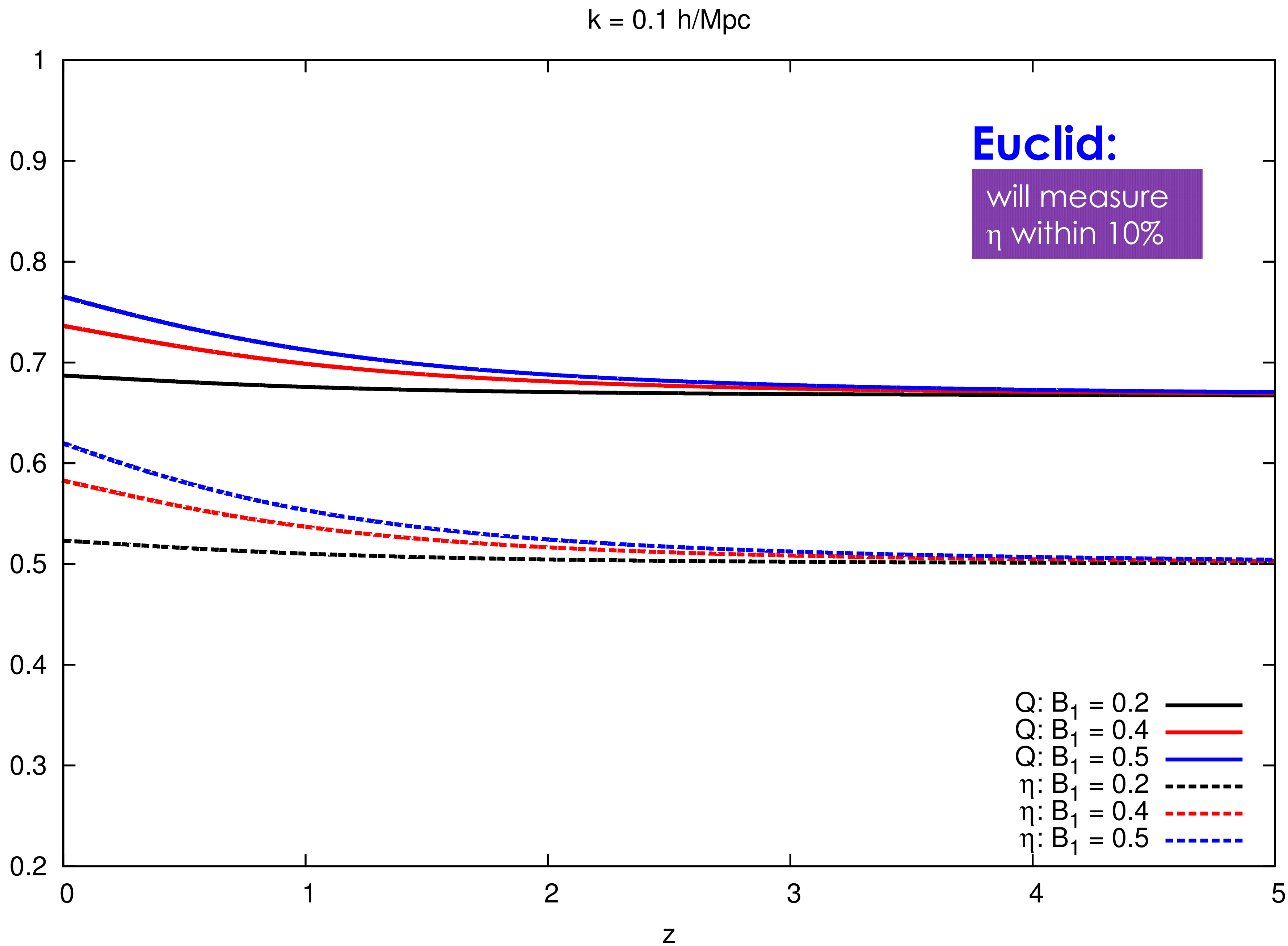
Infinite-branch bigravity: Structure formation

Adam Solomon


1.1
1.0
0.9
0.8
0.7
0.6
f(z)
0
1
2
3
4
5
z
IBB (β_1=0.48, β_4=0.94)
fit model: $\Omega_m{}^{\gamma}$
fit model: $\Omega_m{}^{\gamma_0}(1+\alpha z/(1+z))$
ΛCDM with Ω_{m0}=0.27

Growth constraints: 6dFGS, LRG200, LRG60, BOSS, WiggleZ, and VIPERS (compiled by Macaulay, Wehus, & Eriksen, arXiv:1303.6583)





k = 0.1 h/Mpc
Euclid:
will measure
η within 10%
Q: B1 = 0.2
Q: B1 = 0.4
Q: B1 = 0.5
η: B1 = 0.2
η: B1 = 0.4
η: B1 = 0.5
z
0
1
2
3
4
5
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Euclid and SKA forecasts for infinite-branch bigravity in prep.

[work with Yashar Akrami (Oslo), Phil Bull (Oslo), Tomi Koivisto (Nordita), and Domenico Sapone (Madrid)]

Bimetric Cosmology: Summary

- Some bimetric models do not give sensible backgrounds; others have instability
  - NB – instability does not necessarily rule a model out
- One viable and stable model – infinite-branch bigravity (IBB)
- IBB deviates from ΛCDM at background level and in structure formation. Euclid (2020s) should settle the issue.
- Extensive analysis of perturbations undertaken by ARS, Akrami, & Koivisto in arXiv:1404.4061; stability by Könnig, Akrami, Amendola, Motta, & ARS in arXiv:1407.4331
  - See also Könnig and Amendola, arXiv:1402.1988
- In prep: Euclid forecasts, ISW

Generalization: Doubly-coupled bigravity

- Question: Does the dRGT/Hassan-Rosen bigravity action privilege either metric?
- No: The vacuum action (kinetic and potential terms) is symmetric under exchange of the two metrics:

$$S = -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R(g) - \frac{M_f^2}{2}\int d^4x\sqrt{-\det f}R(f) + m^2 M_g^2\int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$

Symmetry: $g_{\mu\nu} \Leftrightarrow f_{\mu\nu}, \quad M_g \Leftrightarrow M_f, \quad \beta_n \Leftrightarrow \beta_{4-n}$

Generalization: Doubly-coupled bigravity

- Most bimetric matter couplings reintroduce the ghost
  - Recent development: arXiv:1408.0487, arXiv:1408.1678
- Candidate ghost-free double coupling (1408.1678): matter couples to an effective (Jordan-frame) metric:

$$g^{\text{eff}}_{\mu\nu} = \alpha^2 g_{\mu\nu} + 2\alpha\beta g_{\mu\alpha}\left(\sqrt{g^{-1}f}\right)^{\alpha}_{\ \nu} + \beta^2 f_{\mu\nu}$$

- Rationale (see 1408.1678, 1408.5131): $\sqrt{(-\det g_{\text{eff}})}$ is of the same form as the massive gravity/bigravity interaction terms!
  - Matter loops will generate ghost-free interactions between g and f

Doubly-coupled cosmology

Enander, ARS, Akrami, and Mörtsell [arXiv:1409.xxxx – early next week]

- Novel features (compared to singly-coupled):
  - Can have conformally-related solutions, $f_{\mu\nu} = (\beta/\alpha)^2 g_{\mu\nu}$
  - These solutions can mimic exact ΛCDM (no dynamical DE)
    - Only for special parameter choices
  - Models with only $\beta_2 \neq 0$ or $\beta_3 \neq 0$ are now viable at background level

Doubly-coupled cosmology

- Candidate partially massless theory has non-trivial dynamics
  - $\beta_0 = \beta_4 = 3\beta_2$, $\beta_1 = \beta_3 = 0$: has partially-massless symmetry around maximally symmetric (dS) solutions (arXiv:1208.1797)
  - New gauge symmetry which eliminates the helicity-0 mode (no fifth force, no vDVZ discontinuity)
    - Fixes and protects the value of the CC/vacuum energy
    - Attractive solution to the CC problems!
  - However the singly-coupled version does not have non-trivial cosmologies
  - ✓ This doubly-coupled bimetric theory results in a natural candidate PM gravity with viable cosmology
  - Remains to be seen: is this really partially massless?
    - All backgrounds? Fully non-linear symmetry?

Avoids instabilities?

- At early times, on finite branch, y -> β/α rather than 0
- Instability in singly-coupled theory occurred at small y
- ➢ Can double coupling exorcise the instability?

Are massive cosmologies viable?

- A single massive graviton (dRGT massive gravity) lacks flat FRW solutions (and open solutions are unstable)
- 1408.1678: double coupling can cure this!
- ARS, Enander, Akrami, Koivisto, Könnig, and Mörtsell [arXiv:1409.xxxx]:
  - That conclusion relies on existence of a scalar rolling down a nontrivial potential.
  - Cosmologies dominated by dust and other w=const. fluids still do not exist
  - Are these ruled out? Either way, very strange cosmologies!

Summary

- Sensible theory exists of massive gravitons and interacting spin-2 fields
- Late-time acceleration can be addressed (self-acceleration)
- Dynamical dark energy – serious competitor to ΛCDM!
- Clear non-GR signatures in large-scale structure: Euclid
- Can couple both metrics to matter: truly bimetric gravity
- Exciting cosmological implications: exact ΛCDM, partial masslessness, etc.
- Can we do cosmology with a single massive graviton?

Bicollaborators:

Oslo:
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Madrid:
Domenico Sapone

Stockholm/Nordita:
Jonas Enander
Tomi Koivisto
Edvard Mörtsell

Geneva:
Mariele Motta

Based on: arXiv:1404.4061
arXiv:1407.4331
arXiv:1409.xxxx
arXiv:1409.xxxx

What’s next?

- Singly-coupled bigravity:
  - Forecasts for Euclid
  - Superhorizon scales: CMB (Boltzmann + ISW), inflation, tensor modes
  - Nonlinear regime (N-body simulations)
  - Inflation from bigravity
- Doubly-coupled bigravity:
  - Cosmological constraints (subhorizon, superhorizon, nonlinear)
  - Statistical analysis against background data (SNe, CMB, BAO)
  - Linear stability
  - Local constraints
- Doubly-coupled massive gravity:
  - Is the theory sensible?

Subhorizon evolution equations
g metric

- Energy constraint (0-0 Einstein equation):

$$\left(\frac{k}{a}\right)^2\left(A_g+\frac{m^2}{2}yPa^2(B_f-B_g)\right)+\frac{3}{2}m^2yP\left(A_g-A_f\right)=\frac{\bar{\rho}}{M_g^2}\delta$$

- Trace i-j Einstein equation:

$$\left(\mathcal{H}'-\mathcal{H}^2+\frac{a^2\bar{\rho}}{2M_g^2}\right)E_g+m^2a^2\left[\frac{1}{2}xP\left(E_f-E_g\right)+yQ\left(A_f-A_g\right)\right]=0$$

- Off-diagonal (traceless) i-j Einstein equation:

$$A_g+E_g+m^2a^2yQ(B_f-B_g)=0$$

Subhorizon evolution equations
f metric

- Energy constraint (0-0 Einstein equation):

$$\left(\frac{k}{a}\right)^2\left(A_f-\frac{m^2}{2}\frac{Pa^2}{y}(B_f-B_g)\right)+\frac{3m^2}{2}\frac{P}{y}(A_f-A_g)=0$$

- Trace i-j Einstein equation:

$$\left[-K'+\left(H+\frac{x'}{x}\right)K\right]E_f+m^2\frac{a^2x}{y^2}\left[\frac{1}{2}P\left(E_f-E_g\right)+Q\left(A_f-A_g\right)\right]=0$$

- Off-diagonal (traceless) i-j Einstein equation:

$$A_f+E_f-m^2\frac{Qa^2}{x}(B_f-B_g)=0$$