Non-Gaussianity in the CMB: primordial and secondary

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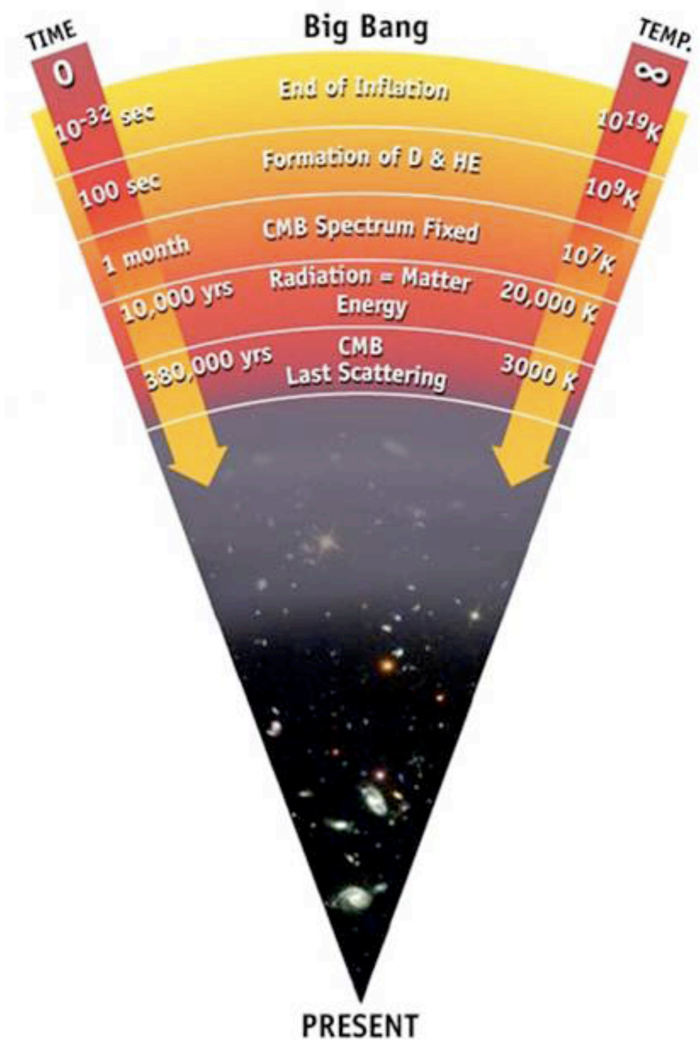
Berkeley, Jan 2009

References:

Smith, Hu and Kaplinghat	astro-ph/0607315
Smith and Zaldarriaga	astro-ph/0612571
Smith, Zahn, Dore and Nolta	arxiv:0705.3980
Smith et al	arxiv:0811.3916
Dvorkin and Smith	arxiv:0812.1566
Smith, Senatore and Zaldarriaga	arxiv:0901.2572
Senatore, Smith and Zaldarriaga	to appear

Cosmic microwave background



Early universe: hot dense proton-electron plasma

(Hydrogen) recombination:

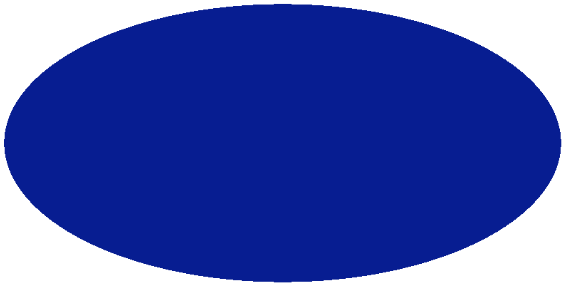
$$p^+e^- \longrightarrow H$$

Universe becomes transparent ($T \approx 3000$ K), photons are no longer coupled to plasma and freestream

These photons are at microwave frequencies today ($T \approx 2.7\ \mathrm{K}, \nu \approx 10^2\ \mathrm{GHz}$) where we observe them as the CMB

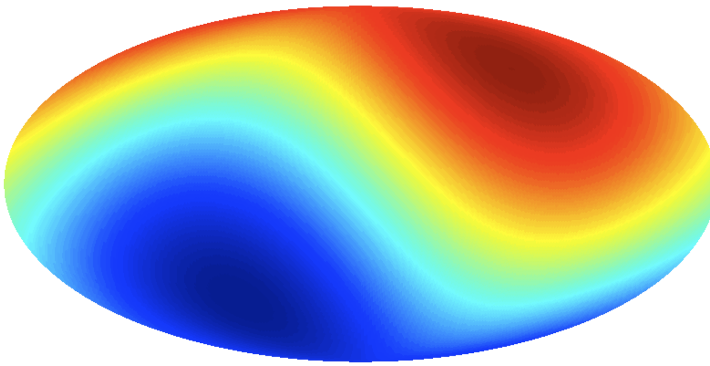
Statistics of the CMB can be both predicted and measured with great accuracy

Microwave sky



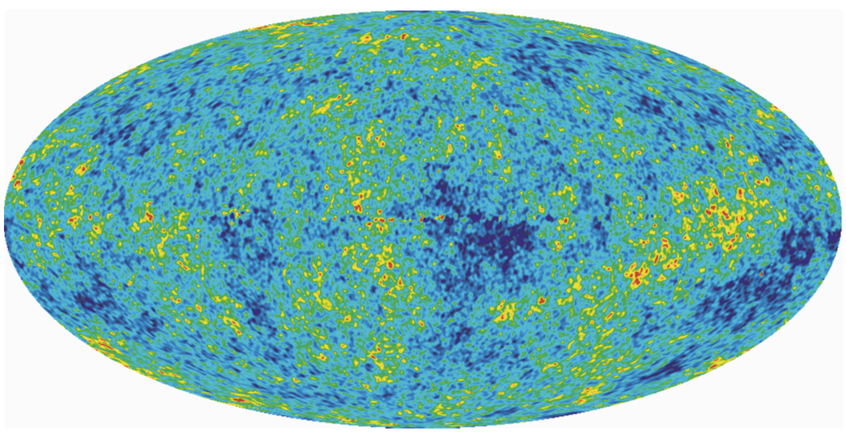
Monopole: perfect blackbody,

$$T_{CMB} = 2.726 \text{ K}$$



Dipole: velocity of earth relative to average velocity of Hubble volume,

$$\Delta T \approx 3 \text{ mK}$$



Anisotropy: Gaussian random fluctuations,

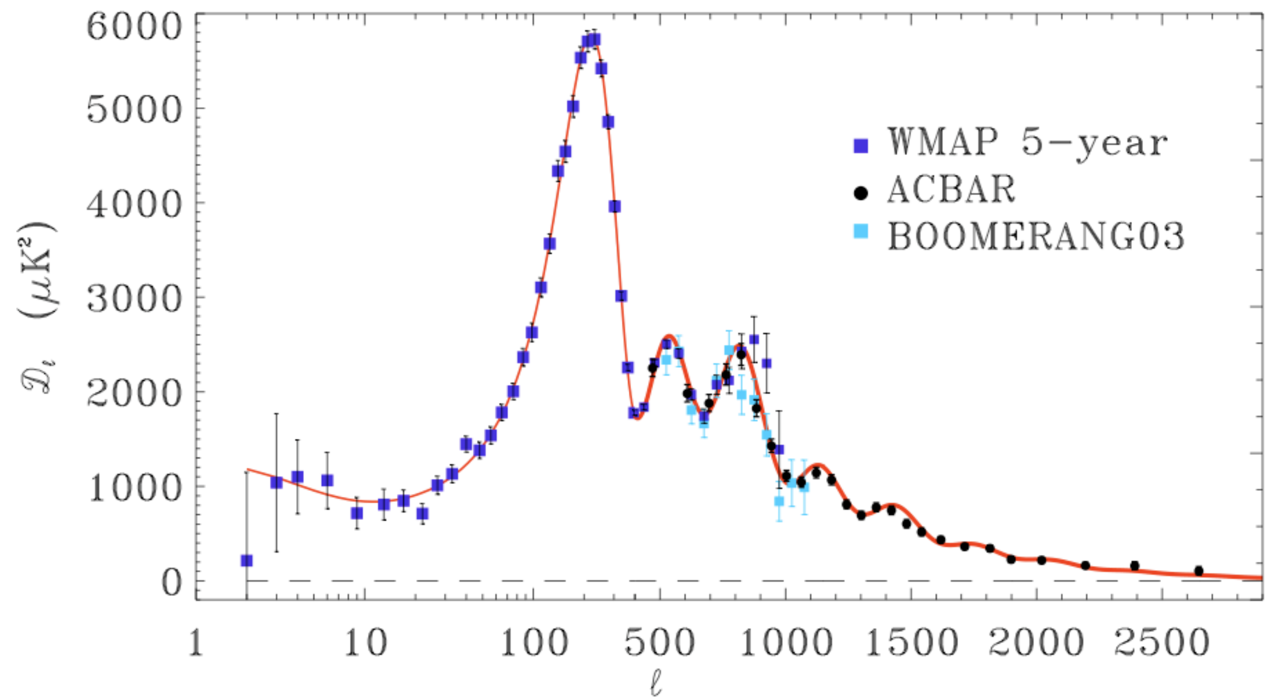
$$\Delta T \approx 100 \text{ uK}$$

"Snapshot of early universe"

CMB power spectrum

Power spectrum: average power in CMB temperature anisotropy as a function of wavenumber ℓ

$$C_\ell = \langle |T(\ell)|^2 \rangle$$



Reichardt et al (ACBAR collaboration), 0801.1491

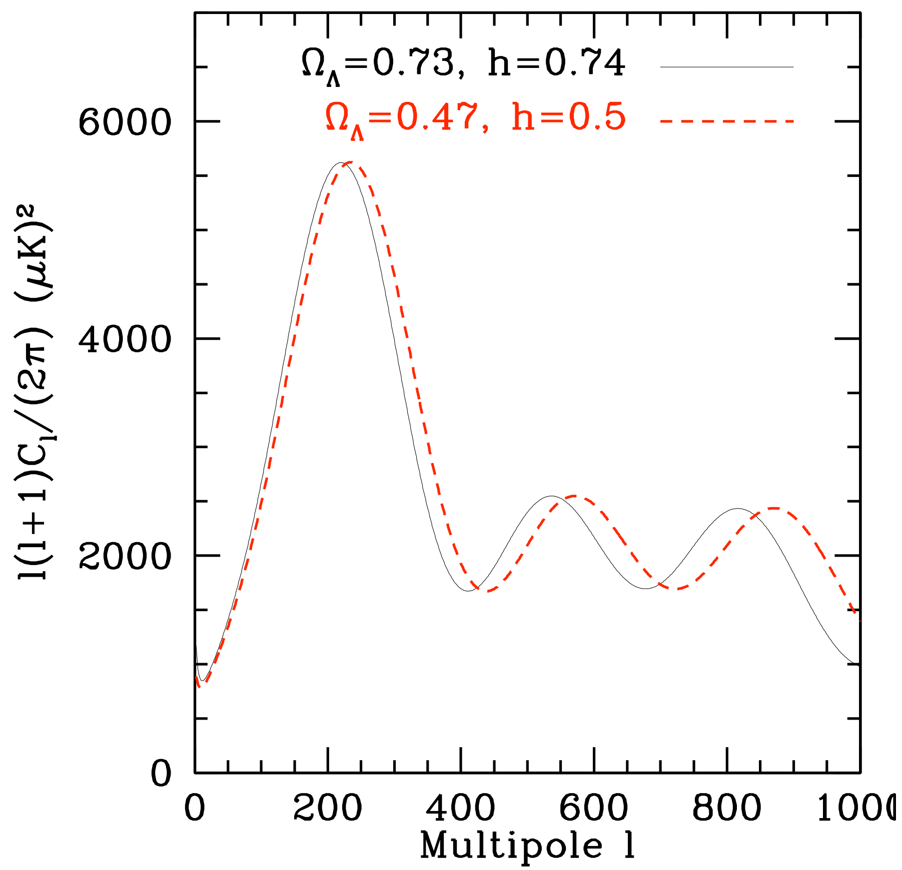
CMB power spectrum: example of phenomenology

Angular scale of acoustic peaks:

$$\ell_a = \pi \frac{D_*}{s_*}$$

D_* ← Angular diameter distance to recombination

s_* ← Sound horizon at recombination



Why is the CMB polarized?

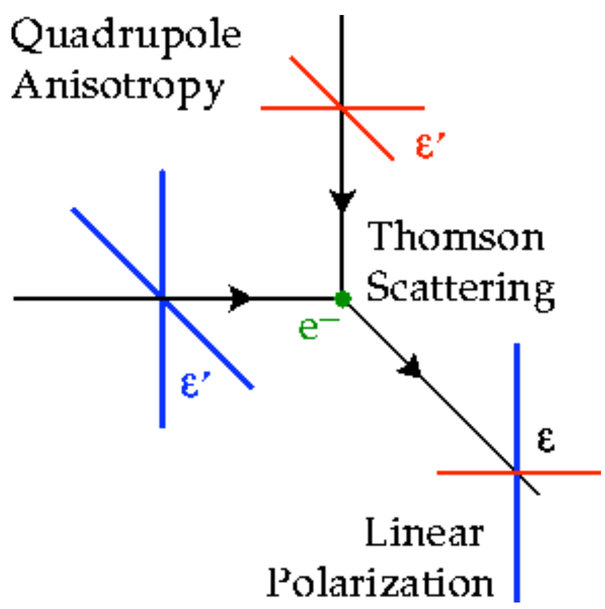
Polarization is generated by Thomson scattering

$$e^{-}\gamma \rightarrow e^{-}\gamma$$

Local temperature quadrupole
=> linear polarization

Only generated in transition between tightly coupled and freestreaming regimes

Polarization is a weaker signal than temperature (6 uK vs 100 uK)

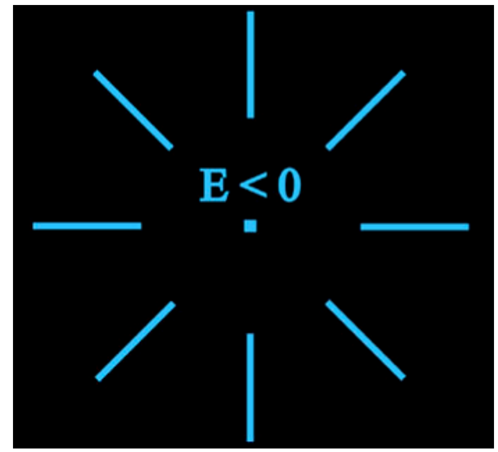


Hu & White

CMB polarization: E-B decomposition

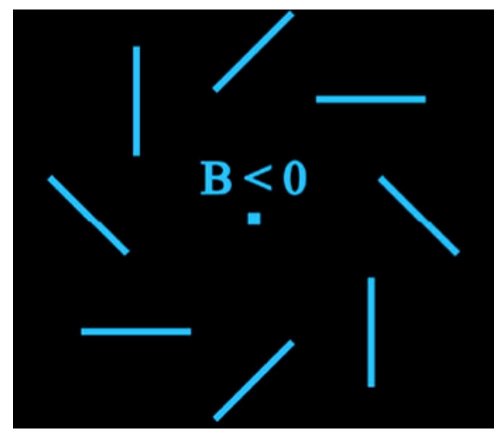
Represent CMB polarization on sky by traceless symmetric tensor:

$$\Pi_{ab} = \langle E_a^* E_b \rangle - \frac{1}{2}\langle |E|^2 \rangle g_{ab}$$



E-modes: “gradient-like” field

$$\Pi_{ab} = \left(\nabla_a \nabla_b - \frac{1}{2} g_{ab} \nabla^2 \right) \phi$$



B-modes: “curl-like” field

$$\Pi_{ab} = \left(\frac{1}{2}\epsilon_{ac} \nabla^c \nabla_b + \frac{1}{2}\epsilon_{bc} \nabla^c \nabla_a \right) \phi$$

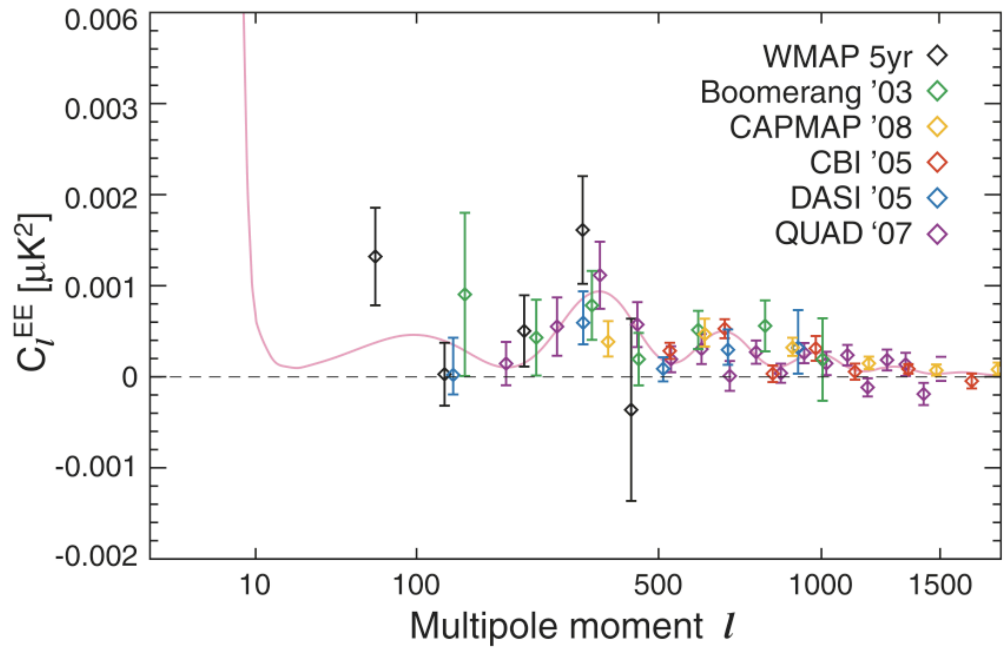
This is the analog of the gradient/curl decomposition for a vector field

CMB polarization: power spectra

E-mode power spectrum

Acoustic peaks sensitive to same qualitative information as temperature

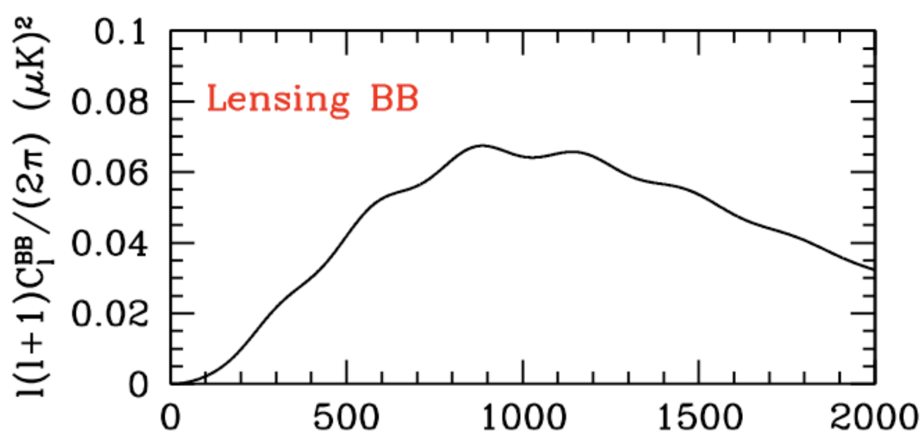
Large signal at low l generated after reionization



Nolta et al (WMAP collaboration), 0803.0593

B-mode power spectrum

Not generated by linear scalar perturbations



“Beyond the power spectrum”

So far, we have only discussed the power spectrum (or equivalently, the 2-point correlation function) of the CMB

For a Gaussian field, the 2-point function contains all the information

To a good approximation, the CMB is a Gaussian field,
but there are weak non-Gaussian signals that can be “excavated”.

1. What non-Gaussian signals should we look for?

2. What higher-point statistics should we use to look for each signal?

Outline

1. Secondary non-Gaussianity: gravitational lensing
   how are the statistics of the CMB affected?
   what cosmological information is added?

2. Secondary non-Gaussianity: reionization
   a new statistic which isolates the epoch of patchy reionization

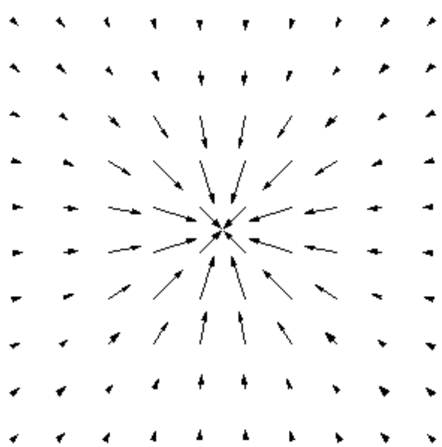
3. Primordial non-Gaussianity
   what signals can one look for?
   new results from WMAP (first optimal analysis)

Part 1: CMB lensing

CMB photons are deflected by gravitational potentials between last scattering and observer. This remaps the CMB while preserving surface brightness:

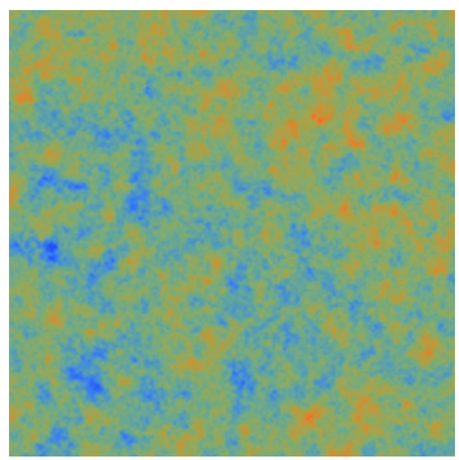
$$\Delta T(\widehat{\mathbf{n}}) \rightarrow \Delta T(\widehat{\mathbf{n}} + \mathbf{d}(\widehat{\mathbf{n}}))$$

where $\mathbf{d}(\widehat{\mathbf{n}})$ is a vector field giving the deflection angle along line of sight



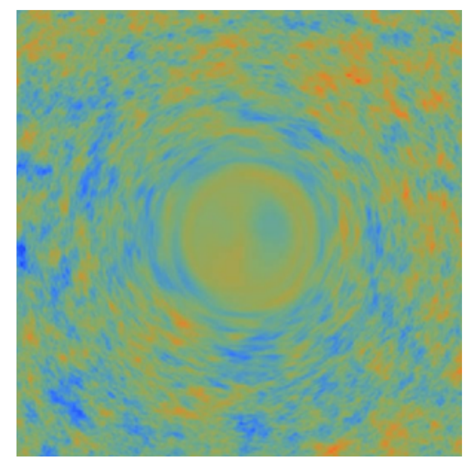
Deflection angles

+



Unlensed CMB

$\longrightarrow$



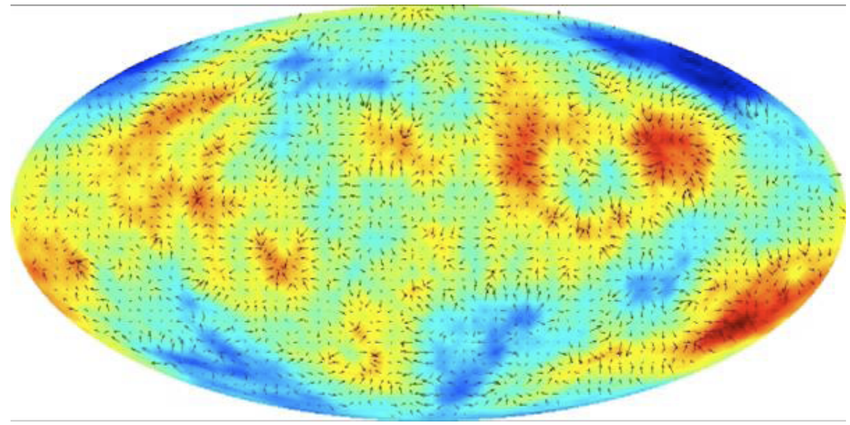
Lensed CMB

Wayne Hu

CMB lensing: theory

To first order, the deflection field $\mathbf{d}(\widehat{\mathbf{n}})$ is a pure gradient:

$$\mathbf{d}_a(\widehat{\mathbf{n}}) = \nabla_a \phi(\widehat{\mathbf{n}})$$



Antony Lewis

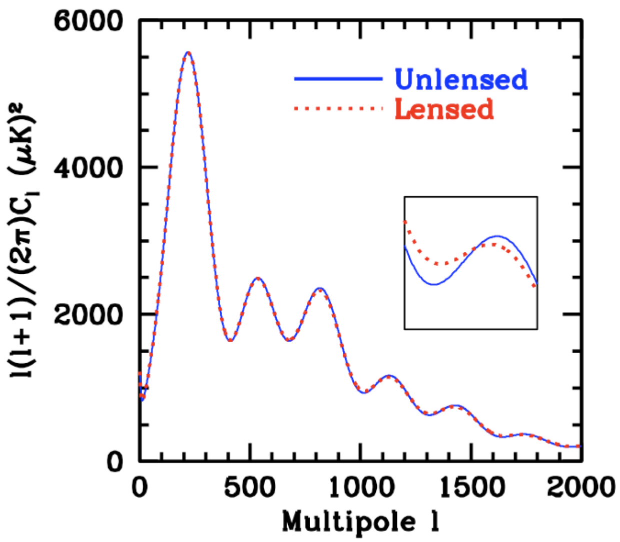
where the lensing potential is given by the line-of-sight integral

$$\phi(\widehat{\mathbf{n}}) = -2 \int_0^{\chi_*} d\chi \left(\frac{\chi_* - \chi}{\chi \chi_*} \right) \Psi(\chi \widehat{\mathbf{n}}, \eta_0 - \chi)$$

Redshift kernel: broad peak at z ~ 2
RMS deflection: ~2.5 arcmin, coherent on degree scales (l ~ 100)

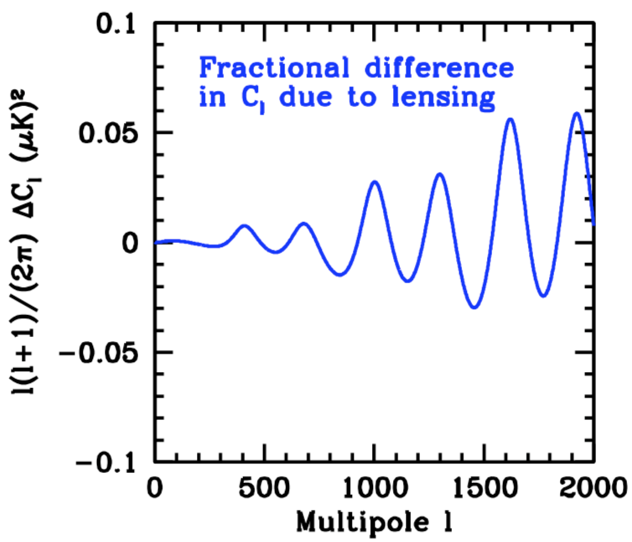
CMB acquires sensitivity to new parameters (e.g. neutrino mass)

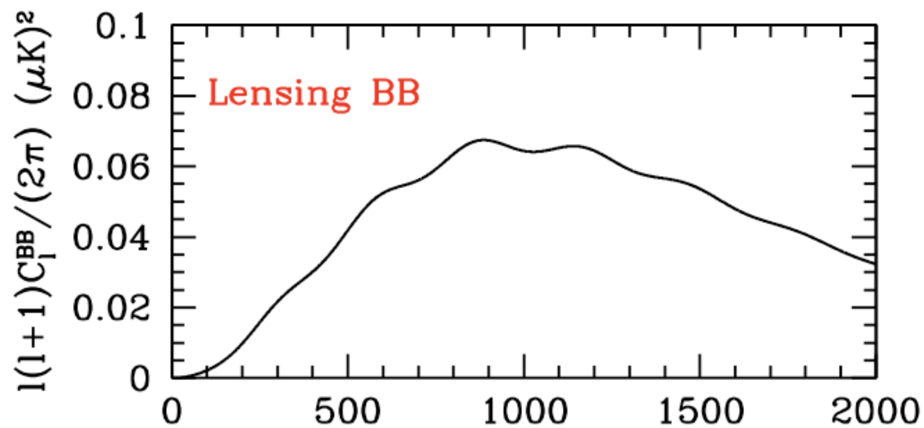
CMB lensing: power spectrum



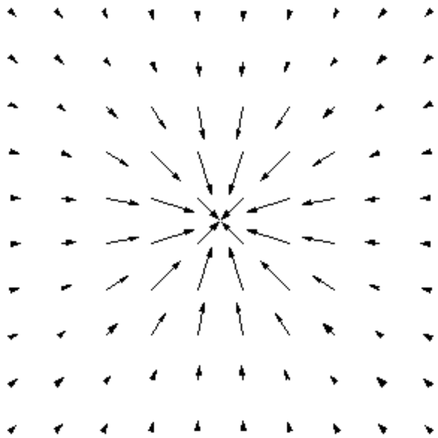
Gravitational lensing appears in temperature and polarization, but polarization is more sensitive

B-mode polarization is generated



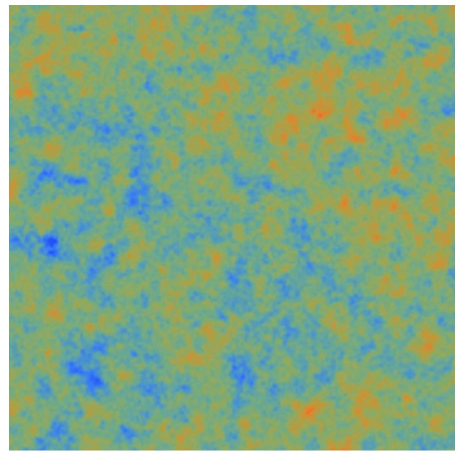


CMB lens reconstruction: idea



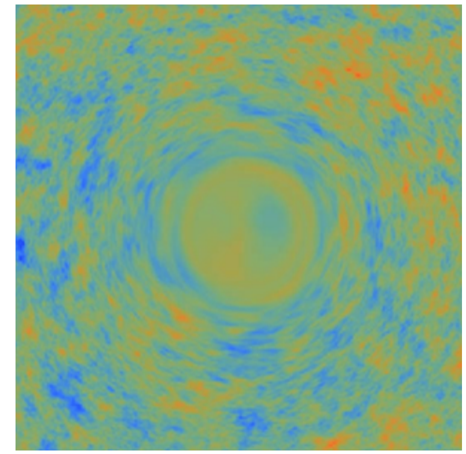
Deflection angles

+



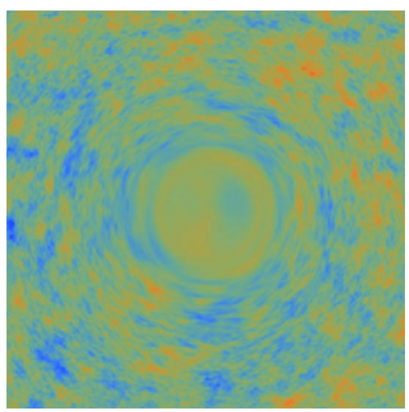
Unlensed CMB

$\longrightarrow$



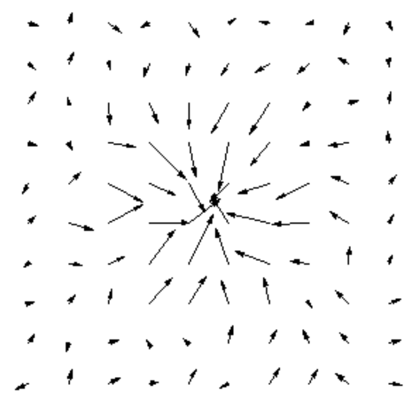
Lensed CMB

Idea: from observed CMB, reconstruct deflection angles (Hu 2001)



Lensed CMB

$\longrightarrow$



Reconstruction + noise

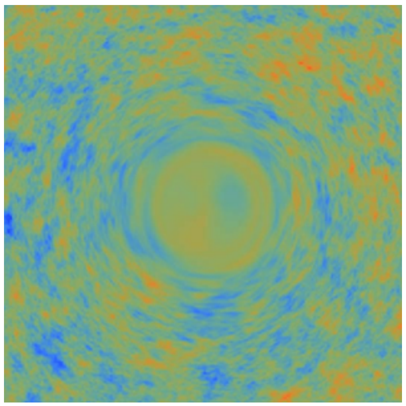
CMB lens reconstruction: quadratic estimator

Lensing potential weakly correlates Fourier modes with $\mathbf{l} \neq \mathbf{l}'$

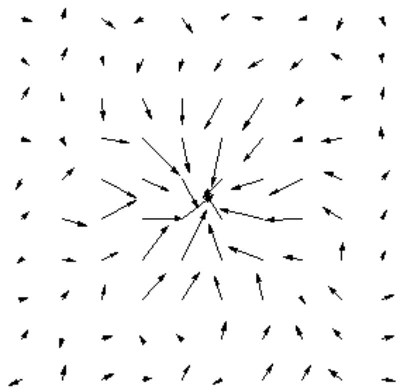
$$\langle T(\mathbf{l})\, T(\mathbf{l}')^* \rangle \propto [-\mathbf{l}' \cdot (\mathbf{l} - \mathbf{l}')]\, \varphi(\mathbf{l} - \mathbf{l}')$$

Formally: can define estimator $\widehat{\varphi}(\mathbf{l})$ which is **quadratic** in CMB temperature:

$$\widehat{\varphi}(\mathbf{l}) \propto \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} [(\mathbf{l}\cdot\mathbf{l}_1) C_{l_1} + (\mathbf{l}\cdot\mathbf{l}_2) C_{l_2}] \frac{T(\mathbf{l}_1) T(\mathbf{l}_2)}{C_{l_1}^{\rm tot} C_{l_2}^{\rm tot}} \qquad (\mathbf{l}_2 = \mathbf{l} - \mathbf{l}_1)$$



$\longrightarrow$



Lensed CMB

Reconstruction + noise

Intuitively: use hot and cold spots of CMB as local probes of lensing potential (analagous to cosmic shear: galaxy ellipticities are used as probes)

CMB lens reconstruction: higher-point statistics

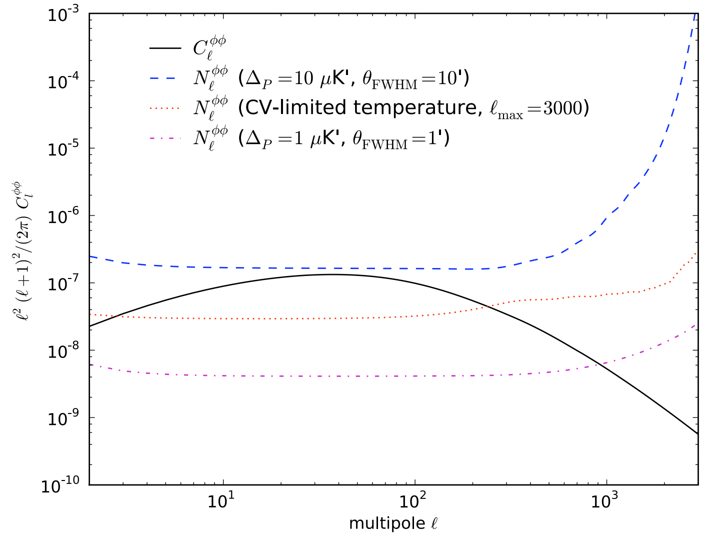
Lens reconstruction naturally leads to higher-point statistics

e.g. take CMB temperature $T(\mathbf{n})$

$\longrightarrow \widehat{\varphi}(\mathbf{l})$ (apply quadratic estimator)

$\longrightarrow \widehat{C}_\ell^{\varphi\varphi}$ (take power spectrum)

defines 4-point estimator in the CMB



Smith et al, 0811.3916

Or: take CMB temperature $T(\mathbf{n})$, galaxy counts $g(\mathbf{n})$

$\longrightarrow \widehat{\varphi}(\mathbf{l})$ (apply quadratic estimator)

$\longrightarrow \widehat{C}_\ell^{\varphi g}$ (take cross power spectrum)

Defines (2+1)-point estimator in the (CMB,galaxy) fields

Can think of the lensing signal formally as a contribution to the 3-point or 4-point function, but lens reconstruction is more intuitive

CMB lensing: lens reconstruction vs power spectrum

Consider two methods for extracting CMB lensing signal:

1. measure CMB power spectra $C_\ell^{TT}, C_\ell^{TE}, C_\ell^{EE}, C_\ell^{BB}$
   lensing makes some contribution, unlensed spectra act as "noise"
2. measure power spectrum $C_\ell^{\phi\phi}$ of lens potential (4-point in CMB)
   within reconstruction noise (power spectrum $N_\ell^{\phi\phi}$)

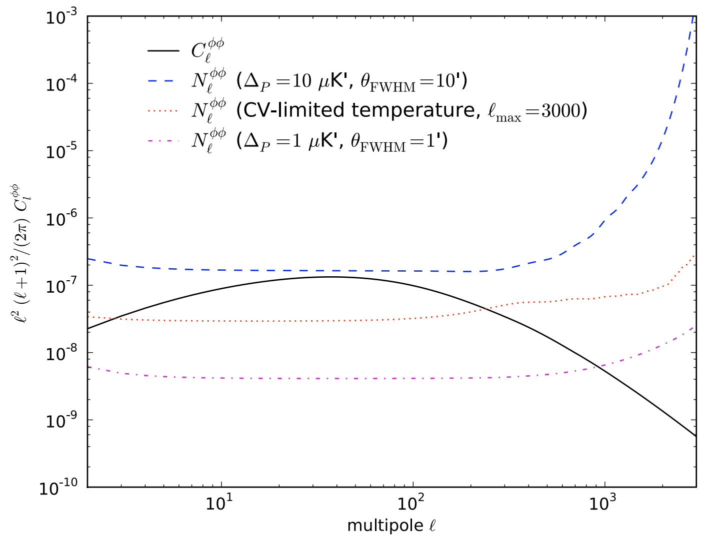
Second method has better signal-to-noise and is more flexible

e.g. CMB lensing detection in WMAP3:

C_ℓ^{TT} : lensing is "detected" at 0.3 sigma

$C_\ell^{\phi\phi}$: 1 sigma

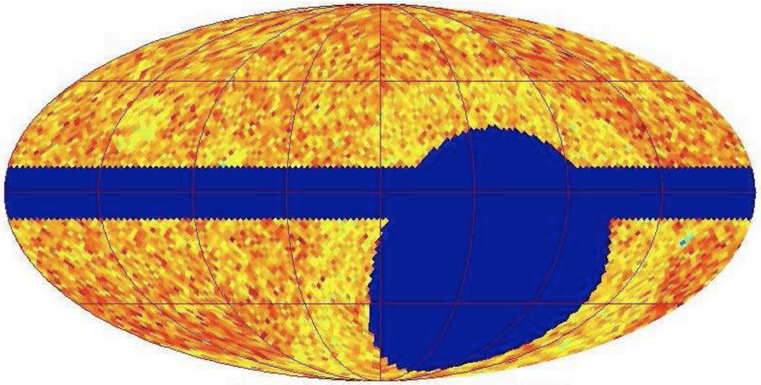
$C_\ell^{\phi g}$: 3.5 sigma (g = NVSS galaxy counts)



Smith et al, 0811.3916

NVSS: NRAO VLA Sky Survey

1.4 GHz source catalog, 50% complete at 2.5 mJy



Mostly extragalactic sources:

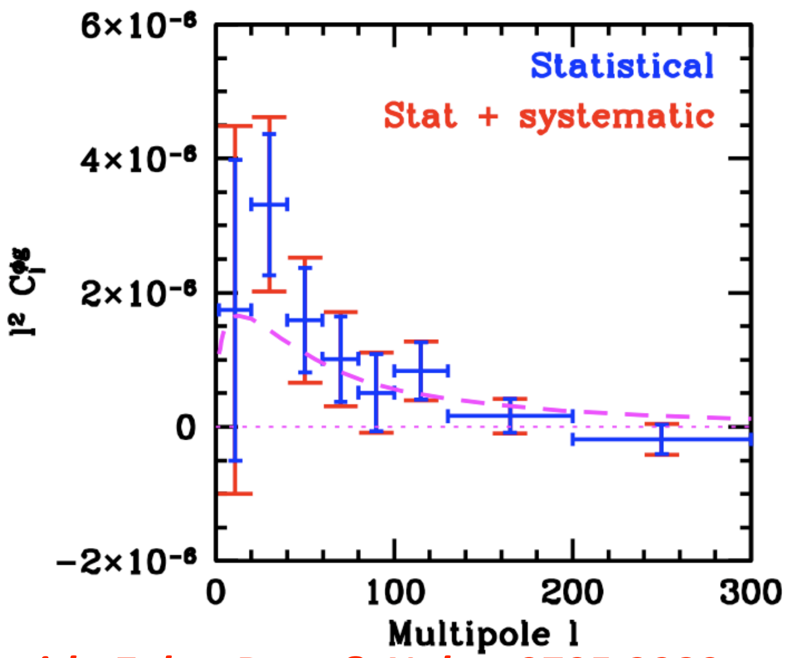
AGN-powered radio galaxies
Quasars
Star-forming galaxies

Well-suited for cross-correlating to WMAP lensing potential:

Nearly full sky coverage $(f_{\rm sky} = 0.8)$
Low shot noise $(N_{\rm gal} = 1.8 \times 10^6)$
High median redshift $(z_{\rm median} = 0.9)$

WMAP3-NVSS result with systematic errors

		Beam			Galactic			Point source + SZ			
$(\ell_{\min}, \ell_{\max})$	Statistical	Asymmetry	Uncertainty	Total	Dust	Free-free	Total	Unresolved	Resolved	Total	Stat + systematic
(2, 20)	17.4 ± 22.4	± 0.9	± 0.3	± 1.2	± 0.4	± 1.4	± 3.6	± 10.9	± 0.5	± 11.4	17.4 ± 27.4
(20, 40)	33.2 ± 10.5	± 0.2	± 0.1	± 0.3	± 0.2	± 0.5	± 1.4	± 4.9	± 1.0	± 5.9	33.2 ± 13.0
(40, 60)	15.9 ± 7.8	± 0.1	± 0.1	± 0.2	± 0.2	± 0.3	± 1.0	± 2.8	± 1.5	± 4.3	15.9 ± 9.3
(60, 80)	10.1 ± 6.3	± 0.1	± 0.1	± 0.2	± 0.1	± 0.3	± 0.8	± 2.0	± 0.3	± 2.3	10.1 ± 7.0
(80, 100)	5.1 ± 5.8	± 0.1	± 0.1	± 0.2	± 0.1	± 0.3	± 0.8	± 1.1	± 0.2	± 1.3	5.1 ± 6.0
(100, 130)	8.3 ± 4.3	± 0.1	< 0.1	± 0.2	± 0.1	± 0.2	± 0.6	± 0.6	± 0.2	± 0.8	8.3 ± 4.4
(130, 200)	1.6 ± 2.5	< 0.1	< 0.1	± 0.1	± 0.1	± 0.1	± 0.4	± 0.3	± 0.1	± 0.4	1.6 ± 2.6
(200, 300)	-1.9 ± 2.2	< 0.1	< 0.1	± 0.1	± 0.1	± 0.1	± 0.4	± 0.3	± 0.1	± 0.4	-1.9 ± 2.3



Smith, Zahn, Dore & Nolta, 0705.3980

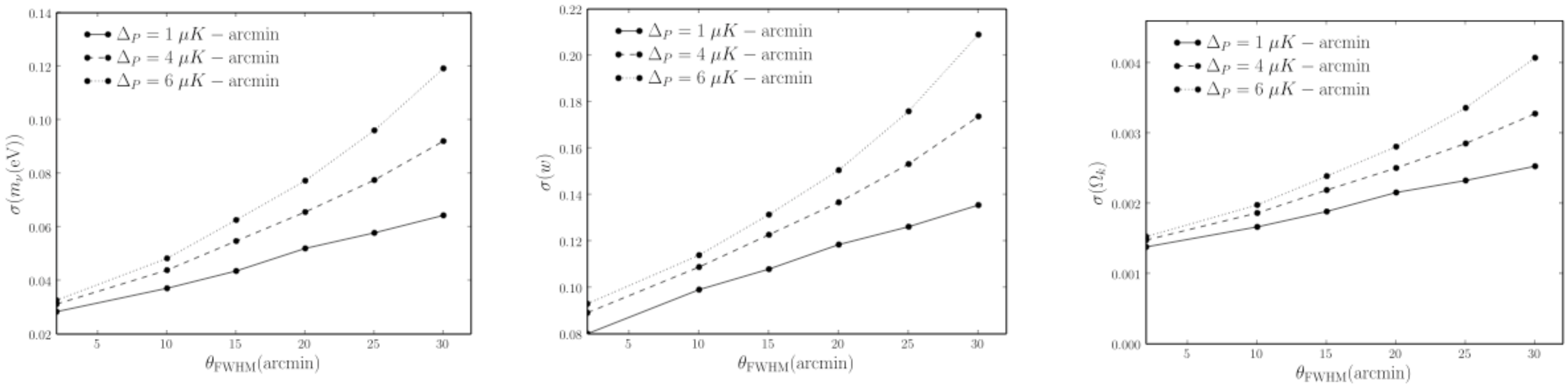
Detection significance: fit in one large bandpower, in multiple of fiducial model

Result: 1.15 +/- 0.34, i.e. a 3.4 sigma detection, in agreement with the expected level

CMB lens reconstruction: future prospects

$C_\ell^{\phi\phi}$ Is sensitive to late-universe observables such as:

neutrino mass dark energy equation of state curvature



Smith et al, 0811.3916

(forecasts for an all-sky polarization satellite)

Need high-resolution CMB measurements: many experiments on horizon (SPT, ACT, Planck, Polarbear…)

Probes large angular scales at high redshift (e.g. early dark energy)

Outline

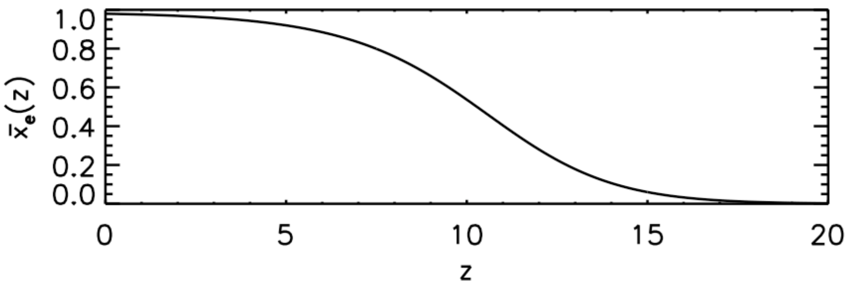
1. Secondary non-Gaussianity: gravitational lensing
   how are the statistics of the CMB affected?
   what cosmological information is added?

2. Secondary non-Gaussianity: reionization
   a new statistic which isolates the epoch of patchy reionization

3. Primordial non-Gaussianity
   what signals can one look for?
   new results from WMAP (first optimal analysis)

Part 2: reionization and the CMB

How does reionization affect the CMB?
Consider homogenous reionization first
(i.e., x_e = function of z only)



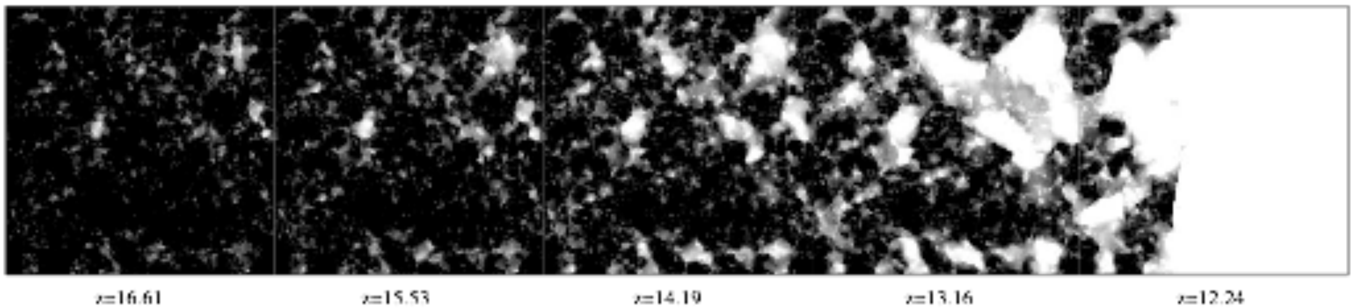
Dvorkin & Smith, 0812.1566

1. Screening: overall amplitude of CMB power spectrum = $A_s e^{-2\tau}$
   Creates degeneracy: harder to measure A (tau acts as "nuisance parameter")
   Conversely, if degeneracy can be broken, can constrain reionization
   (e.g. WMAP5: $\tau = 0.087 \pm 0.017$)

2. New polarization from Thomson scattering after reionization:
   Generates E-mode peak at $\ell \sim 8$ (horizon size) that can break degeneracy

3. Doppler effect: contributes to large-scale temperature anisotropy
   Mostly cancels along line-of-sight since v = "pure gradient" to lowest order

Reionization and the CMB: patchy reionization



Zahn et al (2005)

Consider a reionization bubble with optical depth $\tau_{\rm bub}$ and radial velocity v_r
What CMB anisotropy does the bubble generate?

1. Screening:

$$\begin{aligned} \Delta T(\mathbf{n}) &= -\tau_{\rm bub} T(\mathbf{n}) \\ \Delta[Q \pm iU](\mathbf{n}) &= -\tau_{\rm bub} (Q \pm iU)(\mathbf{n}) \end{aligned}$$

2. "Thomson":

$$\Delta(Q \pm iU)(\mathbf{n}) = (\tau_{\rm bub}) \frac{\sqrt{6}}{10} \sum_{m=-2}^{2} {}_{\pm 2}Y_{2m}(\mathbf{n}) T_{2m}(z, \mathbf{n})$$

3. kSZ:

$$\Delta T(\mathbf{n}) = -\tau_{\rm bub} (v_r / c)$$

Patchy reconstruction: quadratic estimator?

Idea: if reionization is patchy, the optical depth is a field $\tau(\mathbf{n})$, not a constant

Can we write down a quadratic estimator $\widehat{\tau}(\mathbf{l})$ (“tau reconstruction”)
analogous to the estimator $\widehat{\varphi}(\mathbf{l})$? (“lens reconstruction”)

Lens reconstruction “works” because lensing generates anisotropy which looks (heuristically) like (φ) x (unlensed CMB):

$$\begin{aligned} T_{\rm lensed} &= T_{\rm unlensed} + (\nabla\varphi)\cdot(\nabla T_{\rm unlensed}) + \cdots \\ (Q \pm iU)_{\rm lensed} &= (Q \pm iU)_{\rm unlensed} + (\nabla\varphi)\cdot\nabla(Q \pm iU)_{\rm unlensed} + \cdots \end{aligned}$$

Question: Does patchy reionization generate anisotropy of the same form, with φ replaced by τ?

Patchy reconstruction: quadratic estimator?

1. Screening effect

Heuristically: amplitude of recombination peaks is modulated by tau
(region of sky with larger tau => lower observed acoustic peaks)

$$\begin{aligned} \Delta T(\mathbf{n}) &= -\tau(\mathbf{n}) T(\mathbf{n}) \\ \Delta[Q \pm iU](\mathbf{n}) &= -\tau(\mathbf{n})(Q \pm iU)(\mathbf{n}) \end{aligned}$$

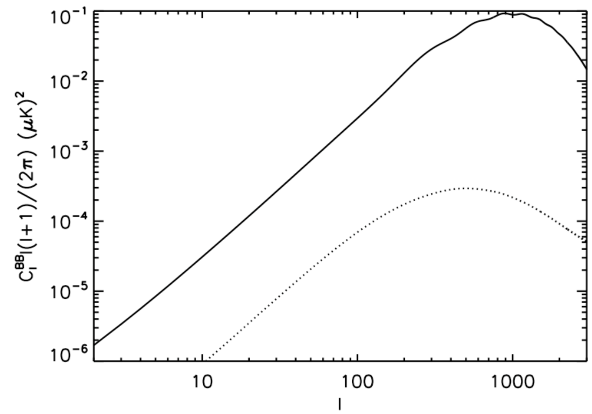
Anisotropy generated by the screening effect has form (τ) x (unlensed CMB)
=> screening effect is "captured" by quadratic estimator formalism

Patchy reconstruction: quadratic estimator?

2. Bubble scattering (polarization only)

Heuristically: large-scale E-mode from reionization is modulated by tau
(region of sky with larger tau => larger reionization E-mode)

Generates B-modes, but much smaller than lensing at power spectrum level



Dvorkin & Smith, 0812.1566

$$\Delta(Q \pm iU)(\mathbf{n}) = \frac{\sqrt{6}}{10} \int \frac{d\tau(\mathbf{n})}{dz} \sum_{m=-2}^{2} {}_{\pm 2}Y_{2m}(\mathbf{n}) T_{2m}(z, \mathbf{n})$$

$$\approx \frac{\tau(\mathbf{n})}{\tau_0} (Q \pm iU)_{\mathrm{reion}}(\mathbf{n})$$

Anisotropy generated by Thomson scattering has form (τ) x (unlensed CMB)
=> Thomson scattering is "captured" by quadratic estimator formalism

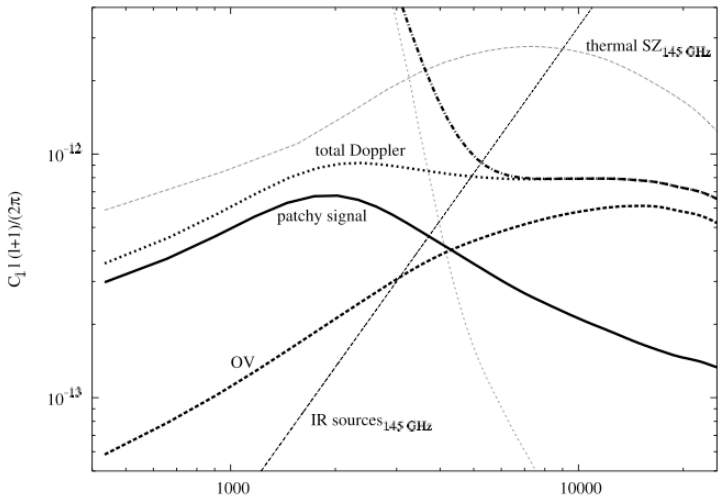
Patchy reconstruction: quadratic estimator?

3. kSZ (temperature only)

The kSZ anisotropy is a line-of-sight integral:

$$\Delta T(\mathbf{n}) = -\frac{1}{c}\int dz\,\frac{d\tau}{dz}v_r(z,\mathbf{n})$$

kSZ from reionization contributes to the small-scale temperature power spectrum



Zahn et al (2005)

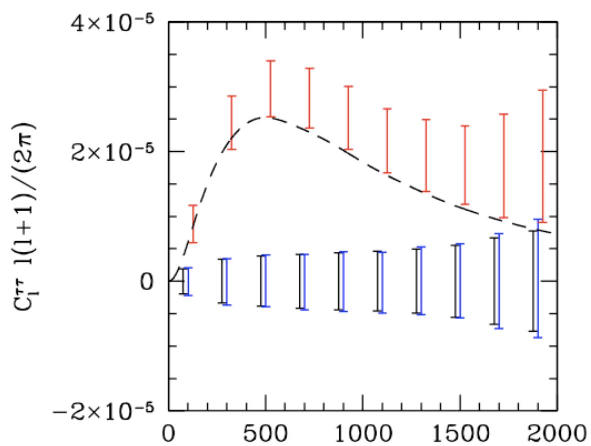
Anisotropy generated by kSZ does not have form (τ) x (unlensed CMB)
=> kSZ is not "captured" by quadratic estimator formalism

Patchy reionization: Quadratic estimator

Conclusion: screening and polarized Thomson effects can be used to construct a quadratic estimator for $\tau(\mathbf{n})$ (math is the same as lens reconstruction, but with different l-weighting)

From here, one can construct estimators as in the lensing case:

$$\begin{aligned}\widehat{\tau}(\mathbf{l}) &= \text{(2-point in CMB fields T,E,B)} \\ \widehat{C}_\ell^{\tau\tau} &= \text{(4-point in CMB fields)} \\ \widehat{C}_\ell^{\tau X} &= \text{((2+1)-point in CMB + cross-correlation field X)}\end{aligned}$$



Dvorkin & Smith, 0812.1566

Quadratic estimator: exaggerated example

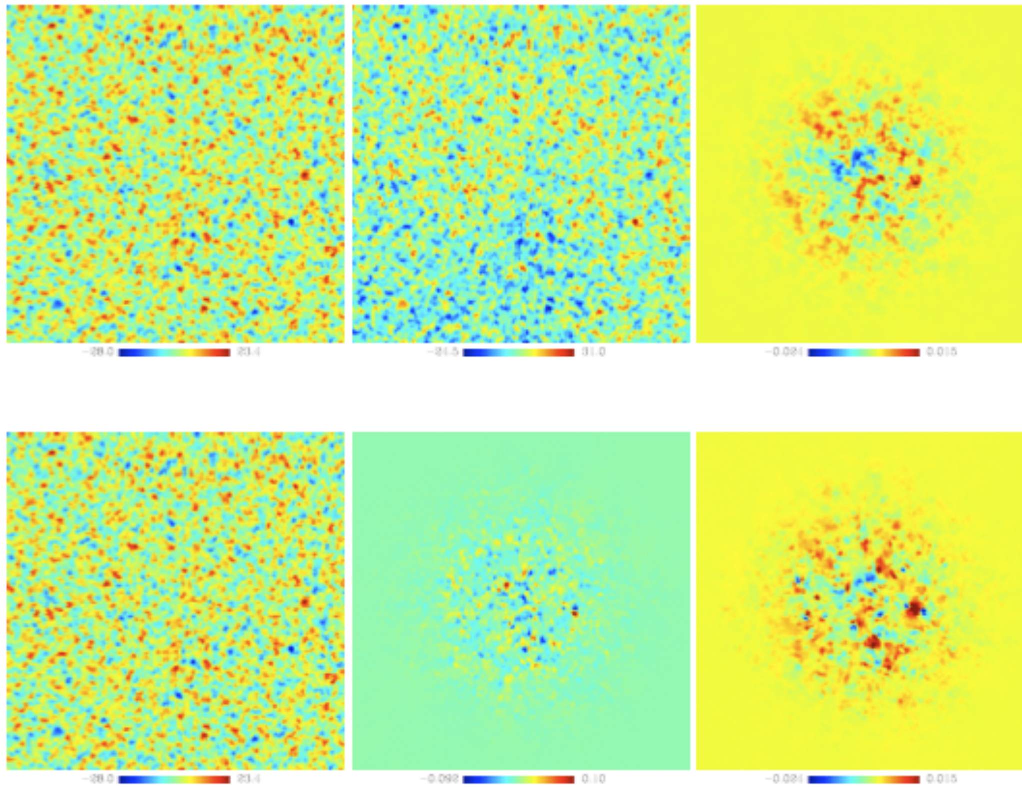
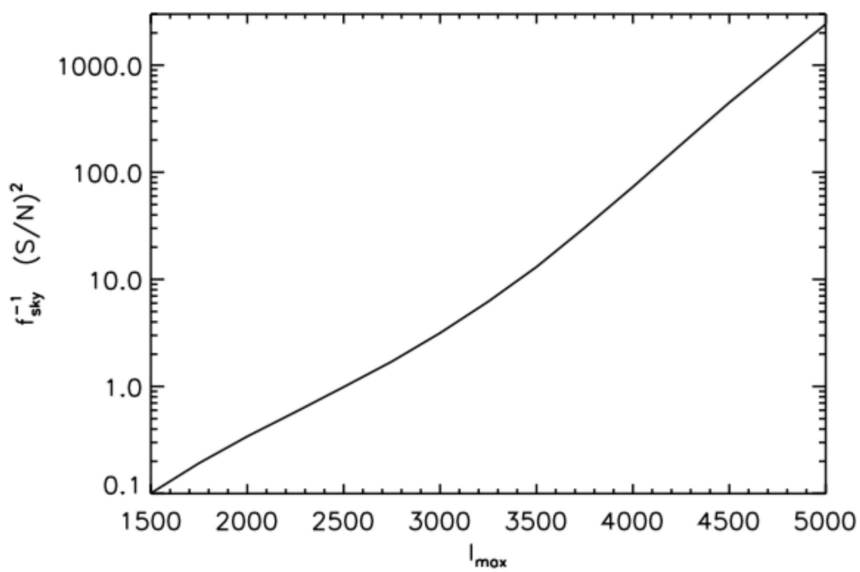


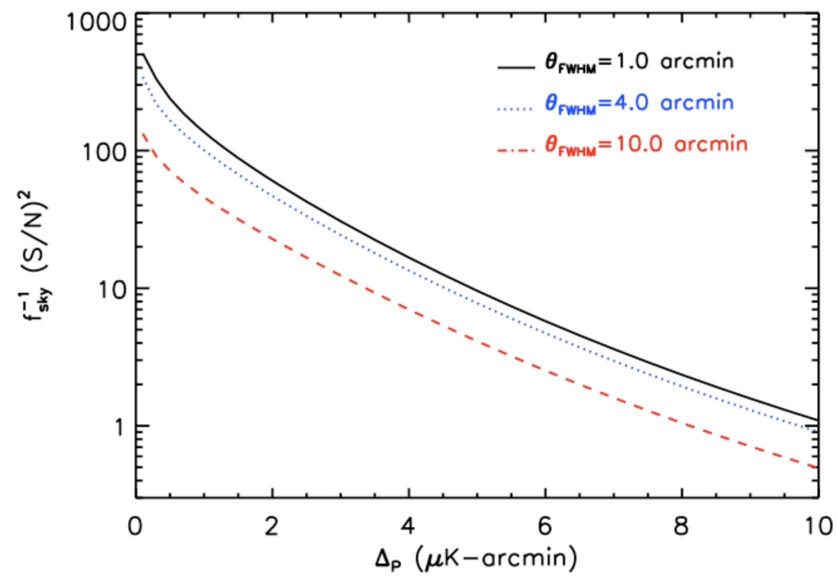
FIG. 5: An exaggerated example of the quadratic estimator $\widehat{\tau}_{\ell m}$ defined in §IV assuming the constant quadrupole approximation. *Top row* (left to right): the primary E-mode E_0, the response field E_1 and the $\Delta\tau$-field, on a 100 deg^2 patch of sky. For visual purposes, the $\Delta\tau$-field has been multiplied by a Gaussian window function, and we have artificially increased the signal-to-noise of the reconstruction by omitting the lensed B-mode and assuming cosmic variance limited measurements to $\ell_{\max} = 2000$. Note that $E_1 \approx -E_0$ on small scales, while the E_1-field has power added at large scales. *Bottom row* (left to right): the E-mode and B-mode components of the total observed polarization (Eq. (31)), and the quadratic reconstruction $\widehat{\tau}_{\ell m}$. Note that B-modes appear where there are τ fluctuations. In this figure, the units of the E-mode and B-mode polarization fields are in μK and $\Delta\tau$ is dimensionless.

Dvorkin & Smith, 0812.1566

Quadratic estimator: signal-to-noise



Temperature



Polarization

Dvorkin & Smith, 0812.1566

Ideas for improving the signal-to-noise:

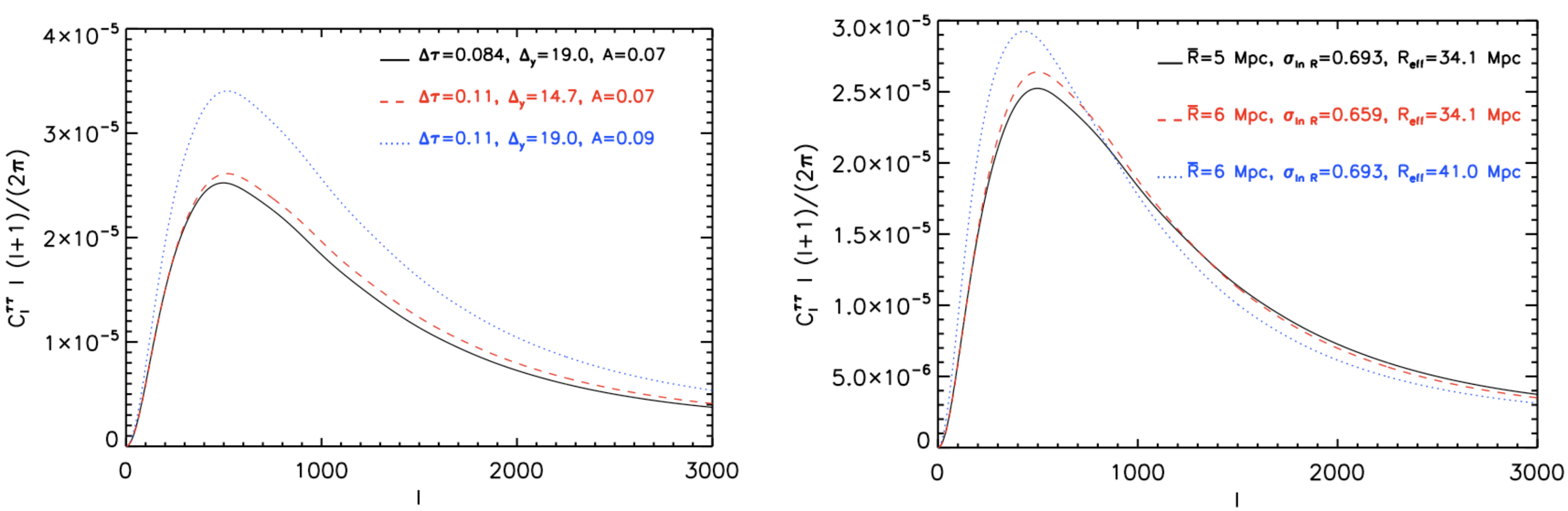
1. cross-correlate with other tracers of reionization
2. combine with lens reconstruction to jointly solve for tau and phi

Parameter forecasts in a toy reionization model

Two observables: Amplitude of the power spectrum is determined by

$$A = \int \frac{dz}{H(z)} \frac{(1+z)^4}{\chi(z)^2} x_e(z)[1 - x_e(z)]$$

Location of peak is determined by the typical bubble radius R



Dvorkin & Smith, 0812.1566

A high-sensitivity satellite experiment can constrain both at the ~10% level

Reionization and the CMB: conclusion

Two methods for extracting reionization information from the CMB:

1. kSZ from small-scale power spectrum
2. quadratic estimator $\widehat{\tau}$

Complementary strengths and weaknesses:

1. "sees" kSZ, but not screening or bubble scattering
   experimental sensitivity already exists (SPT/ACT)
   probes reionization on small scales $(2000 \lesssim \ell \lesssim 4000)$
   power is mixed with other secondaries, modeling uncertainty

2. "sees" screening and bubble scattering, but not kSZ
   more futuristic, requires low-noise polarization
   probes tau fluctuations on large scales $(\ell = D_{\rm reionization}/R_{\rm bubble} \approx 400)$
   isolates patchy signal, map can be cross-correlated, or used for "de-ionizing"

Unsolved problem: find statistic to isolate the kSZ signal in temperature!

Outline

1. Secondary non-Gaussianity: gravitational lensing
   how are the statistics of the CMB affected?
   what cosmological information is added?

2. Secondary non-Gaussianity: reionization
   a new statistic which isolates the epoch of patchy reionization

3. Primordial non-Gaussianity
   what signals can one look for?
   new results from WMAP (first optimal analysis)

Part 3: Primordial non-Gaussianity

In "vanilla" models of inflation, the initial fluctuations are Gaussian
The 3-point correlation function function is zero:

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = 0$$

However, more exotic models can predict nonzero three-point functions
"Local" shape: e.g. curvaton model

$$\zeta(x) = \zeta_G(x) + f_{NL}^{\mathrm{local}} \zeta_G(x)^2$$

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle \sim f_{NL}^{\mathrm{local}} \left(\frac{1}{k_1^3 k_2^3} + \mathrm{symm.} \right) \delta^3 \left(\sum_i k_i \right)$$

"Equilateral" shape: higher-derivative interactions, DBI inflation

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle \sim f_{NL}^{\mathrm{equil.}} \left(\prod_{i=1}^{3} \frac{k_1 + k_2 + k_3 - 2k_i}{k_i^3} \right) \delta^3 (\sum_i k_i)$$

Are these the only shapes to look for?

Effective field theory of inflation (Cheung et al 2007):

$$S_\pi = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\rm Pl}^2 R - M_{\rm Pl}^2 \dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 + \ldots \right] . (28)$$

The two cubic operators do not generate 3-point functions which are precisely equal:

$$\frac{\Delta(k_1,k_2,k_3)}{k_1^3 k_2^3 k_3^3 k_t^3} \left[k_t^3 - k_t \left(\sum_{i<j} k_i k_j \right) - 3 k_1 k_2 k_3 \right] + \frac{12 - 8\alpha}{k_1 k_2 k_3 k_t^3}$$

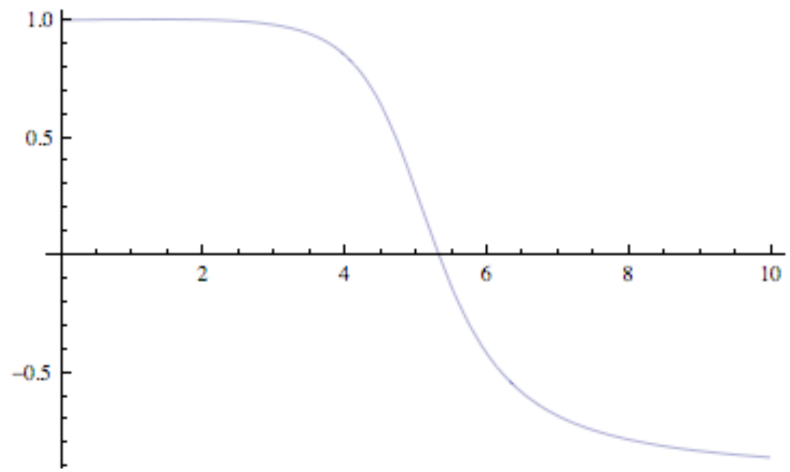
$$k_t = k_1 + k_2 + k_3$$

$$\Delta(k_1,k_2,k_3) = (k_t - 2k_1)(k_t - 2k_2)(k_t - 2k_3)$$

Can orthogonalize the family of bispectra parameterized by α

A new “folded” shape

Consider correlation coefficient with equilateral shape, as function of α:



Senatore, Smith and Zaldarriaga (to appear)

There is a “window” at $\alpha \approx 5.1$ where a new shape appears

Local shape: “squeezed” triangles ($\ell_1 \ll \ell_2, \ell_3$) contain the signal-to-noise
Equilateral shape: equilateral triangles ($\ell_1 \approx \ell_2 \approx \ell_3$)
Folded shape: “folded triangles” $(\ell_1 + \ell_2) \approx \ell_3$

Analysis in WMAP5 forthcoming

More general shape analysis?

Power spectrum analogue: one could either

1. estimate C_l's in l-bins
2. estimate P(k) in k-bands
3. estimate inflationary parameters (n_s, running, ….)

Bispectrum: analogously, one could either:

1. estimate b_{l1,l2,l3} in l bins
2. estimate F(k1,k2,k3) in k bins
3. estimate fnl parameters

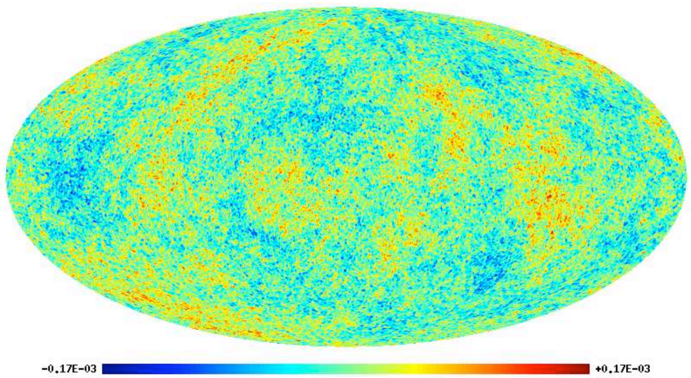
So far, only #3 has been performed on WMAP data,
but #1+2 would be possible using our machinery

…. Is there a good theoretical motivation for doing this?

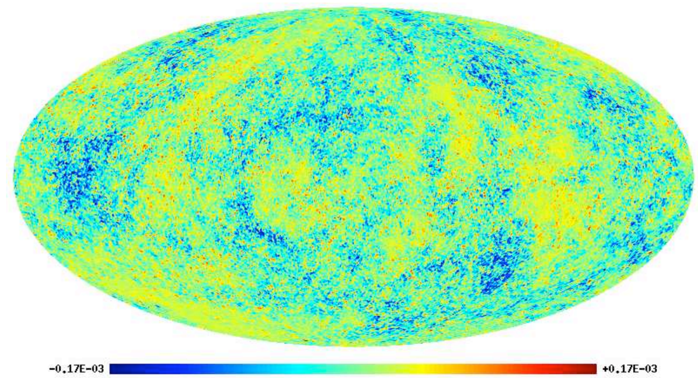
Three-point function in the observed CMB

Non-gaussian initial conditions from inflation
+ linear transfer functions = non-Gaussian CMB

$$f_{NL}^{\rm local} = 0$$



$$f_{NL}^{\rm local} = 3000$$



Michele Liguori

Optimal estimator: sum over triples (l1,l2,l3) with inverse signal-to-noise weighting

$$\langle T(l_1)T(l_2)T(l_3)\rangle \propto f_{NL}\delta^2\left(\sum_i l_i\right)$$

WMAP analysis: background

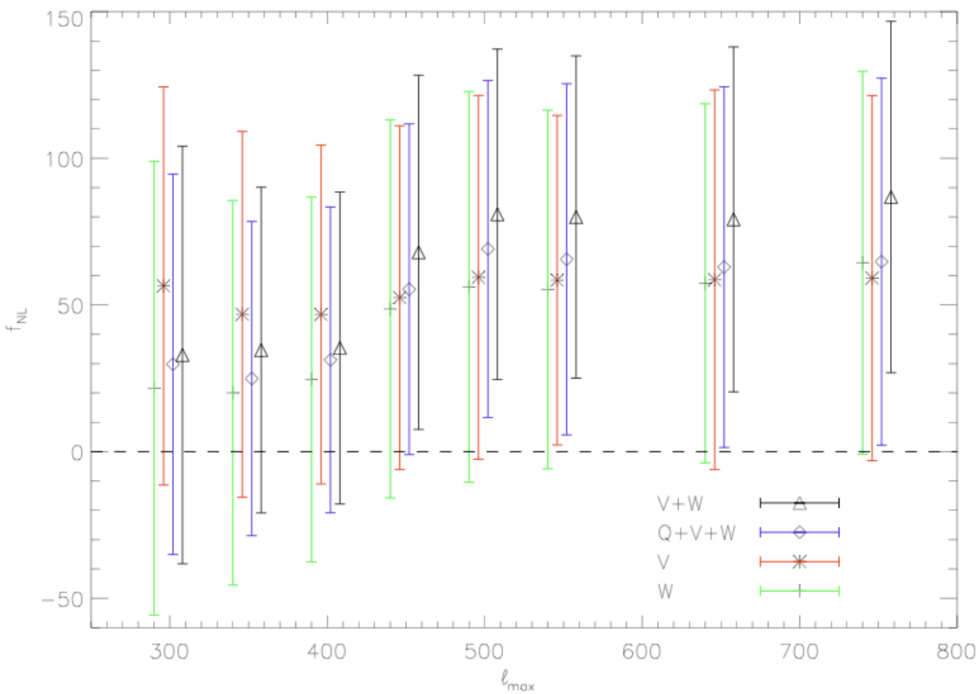
$$f_{NL}^{\mathrm{local}} = 32 \pm 34$$ Creminelli et al (WMAP3)

$$f_{NL}^{\mathrm{local}} = 87 \pm 30$$ Yadav & Wandelt (WMAP3!!)

$$f_{NL}^{\mathrm{local}} = 55 \pm 30$$ Komatsu et al (WMAP5)

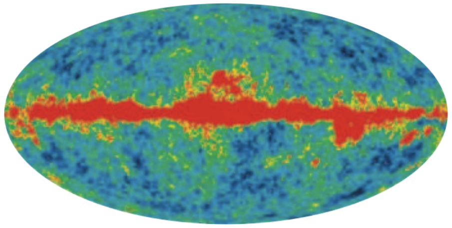
A robust detection would rule out most models of inflation! (e.g. slow-roll)
Which analysis should be believed?

Reason for the discrepancy: “step” at l=450

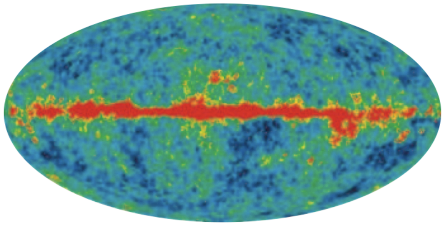


Yadav & Wandelt (2007)

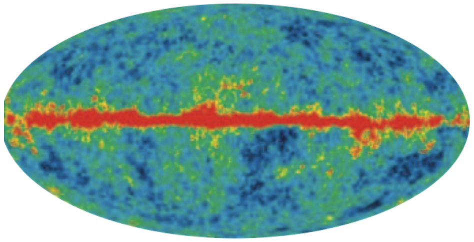
Q-band
(40 GHz)



V-band
(60 GHz)



W-band
(90 GHz)



Must be careful to avoid making a posteriori choices…. !

Use of V+W is motivated a priori
Use of lmax=750 is not

Problem: have estimates of fnl with different values but comparable statistical errors

General principle: in this situation, either

1. have evidence for systematics
2. estimator is significantly suboptimal

e.g. $f_{NL}^{\rm local} = 32 \pm 34$ (Creminelli et al, WMAP3, lmax=350)

$f_{NL}^{\rm local} = 87 \pm 30$ (Yadav & Wandelt, WMAP3, lmax=750)

Is this shift between $\ell_{\rm max} = 350$ and $\ell_{\rm max} = 750$ consistent with statistics?

If so, one should be able to reweight the estimator so that adding the small scales improves $\sigma(f_{NL}^{\rm local})$ by more than 4

Questions:

1. Are the large and small scales consistent? (i.e. do they give a consistent $f_{NL}^{\rm local}$?)
2. Are WMAP3 and WMAP5 consistent?

Optimal estimator

$$d \rightarrow (S+N)^{-1} d$$

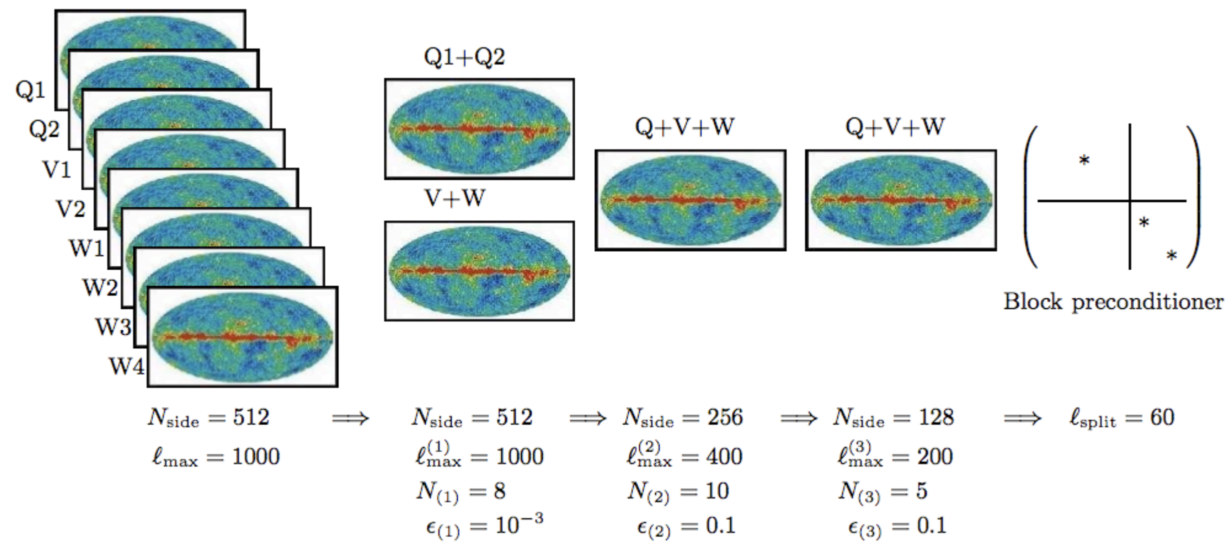
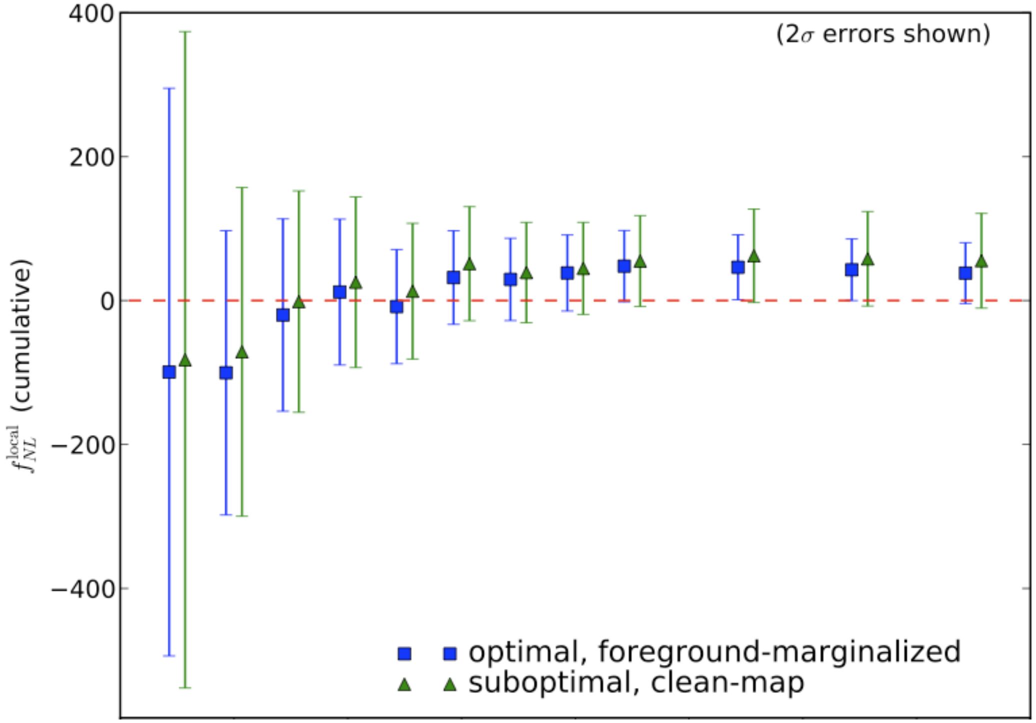


FIG. 20: Preconditioner chain for multigrid $(S+N)^{-1}$ filtering, using noise maps from the three-year WMAP dataset. From left to right, each set of maps represents one conjugate gradient inversion problem, which is preconditioned by the "faster and cruder" approximation which appears next in the chain, obtained by either reducing resolution or the number of distinct beams retained in the problem.

Motivation for optimal estimator:

1. smaller error bars ($\sigma(f_{NL}^{\rm local}) = 21$ for WMAP5)
2. eliminate arbitrary choices: different implementations should agree precisely
3. cannot get multiple estimates with comparable errors;
   get single "bottom line" estimate for a given dataset

Results: WMAP5 optimal analysis



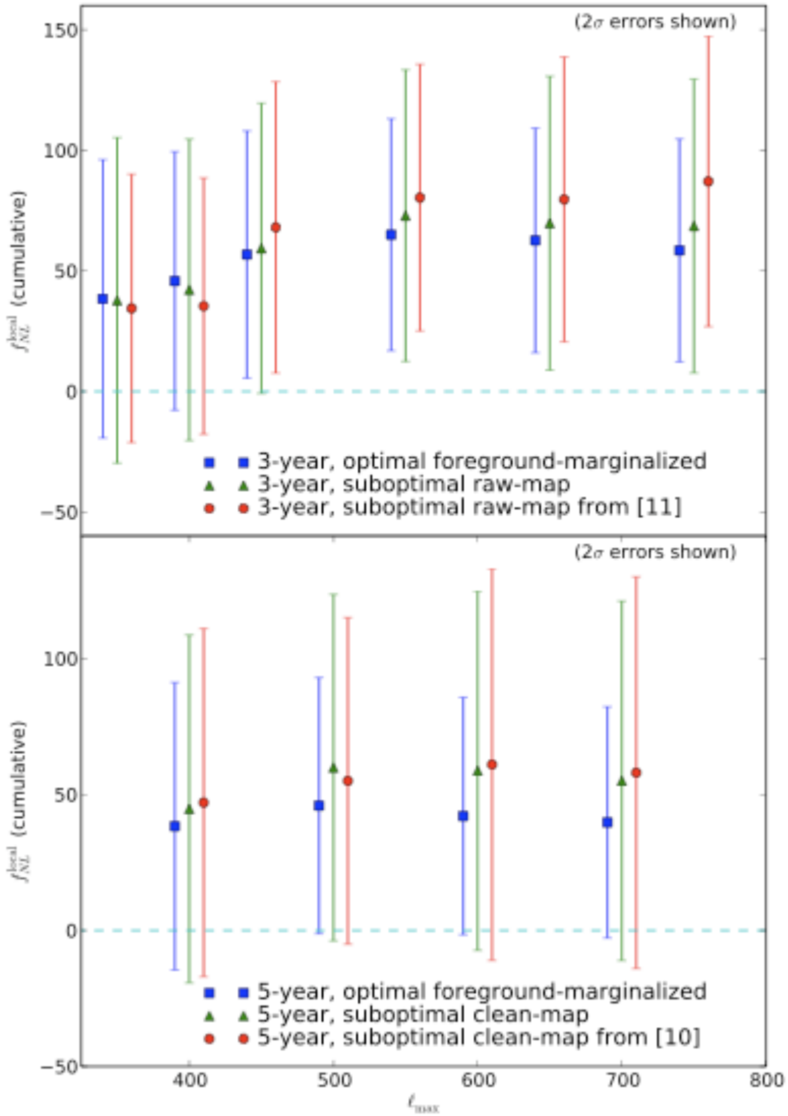
Smith, Senatore and Zaldarriaga (to appear)

WMAP5 suboptimal: $-11 < f_{NL}^{\rm local} < 121$ (95% CL)

WMAP5 optimal: $-4 < f_{NL}^{\rm local} < 80$ (95% CL)

+SDSS (Slosar et al 2008): $-1 < f_{NL}^{\rm local} < 61$ (95% CL)

Results: are the large and small scales consistent?



Smith, Senatore and Zaldarriaga (to appear)

Quantify this by defining:

$$\Delta f_{NL} = \widehat{f}_{NL}(\ell_{\max} = 750) - \widehat{f}_{NL}(\ell_{\max} = 350)$$

In simulation: $\Delta f_{NL} = 19 \quad (\text{at } 1\sigma)$

In WMAP3, we find:

$\Delta f_{NL} = 20$ (optimal estimator)

$\Delta f_{NL} = 32$ (suboptimal)

some disagreement w/Yadav-Wandelt (2007):

$\Delta f_{NL} = 52$

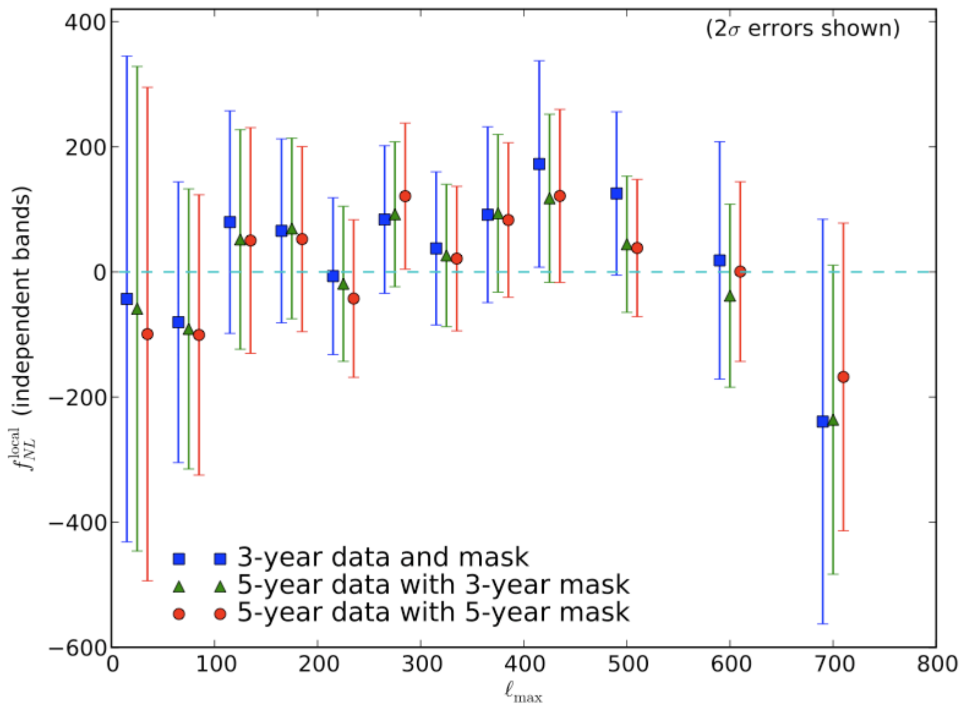
In WMAP5, we find:

$\Delta f_{NL} = 9$ (optimal estimator)

$\Delta f_{NL} = 16$ (suboptimal)

with good agreement with Komatsu (2008)

Results: are WMAP3 and WMAP5 consistent?



Smith, Senatore and Zaldarriaga (to appear)

Define: $\Delta f_{NL} = \widehat{f}_{NL}(\mathrm{WMAP5}) - \widehat{f}_{NL}(\mathrm{WMAP3})$

We find: $\Delta f_{NL} = -20$ (optimal estimator)

$\Delta f_{NL} = -25$ (suboptimal)

In simulation: $\Delta f_{NL} = 16 \quad (\text{at } 1\sigma)$, so WMAP3 and WMAP5 are consistent

Speculation: what about the high significance in WMAP3?

Yadav & Wandelt (2007): 2.9 sigma, $\widehat{f}_{NL}^{\rm local} = 87$
Our pipeline: 2.3 sigma, $\widehat{f}_{NL}^{\rm local} = 69$ (suboptimal estimator)
2.5 sigma, $\widehat{f}_{NL}^{\rm local} = 56$ (optimal)

Probability of a 2.5 sigma result by chance: 1.25%
Can we really "dismiss" the high significance in seen WMAP3?

How many equally "interesting" parameters could we have looked for?

$$f_{NL}^{\rm local}, f_{NL}^{\rm equilateral}, r, w_0, \Omega_K, (dn_s/d\ln k), \alpha_0, \alpha_{-1}$$

In addition, might have found something in WMAP1-only, or in WMAP5-only….

Now consider: what happens when we add more data?
We have $\widehat{f}_{NL}^{\rm local} = 56$ in WMAP3
Suppose $f_{NL}^{\rm local}$ is really 56; then we expect to get $f_{NL}^{\rm local} \approx 56$ in WMAP5
Suppose WMAP3 is statistical fluke; then we expect a shift toward zero….

Primordial non-Gaussianity: systematic tests

- large-scale foregrounds: $f_{NL}^{\rm local} \approx \rm few$ (conservative galactic mask, after cleaning)
- small-scale foregrounds: negligible
- point sources: negligible
- three-point function from secondaries: $f_{NL}^{\rm local} \approx \rm few$ (largest: ISW-lensing)

Primordial non-Gaussianity: conclusions

- Systematics seem completely under control!
- First implementation of optimal estimator: reduces $\sigma(f_{NL}^{\rm local})$ from 30 to 21
- Eliminates arbitrary choices which complicate interpretation of results
- We currently find $f_{NL}^{\rm local}$ just under 2 sigma

$$-4 < f_{NL}^{\rm local} < 81 \ \rm (WMAP5)$$
$$-1 < f_{NL}^{\rm local} < 61 \ \rm (WMAP5{+}Sloan)$$