Massive Neutrino and Cosmology

~Toward a measurement of the absolute neutrino masses~

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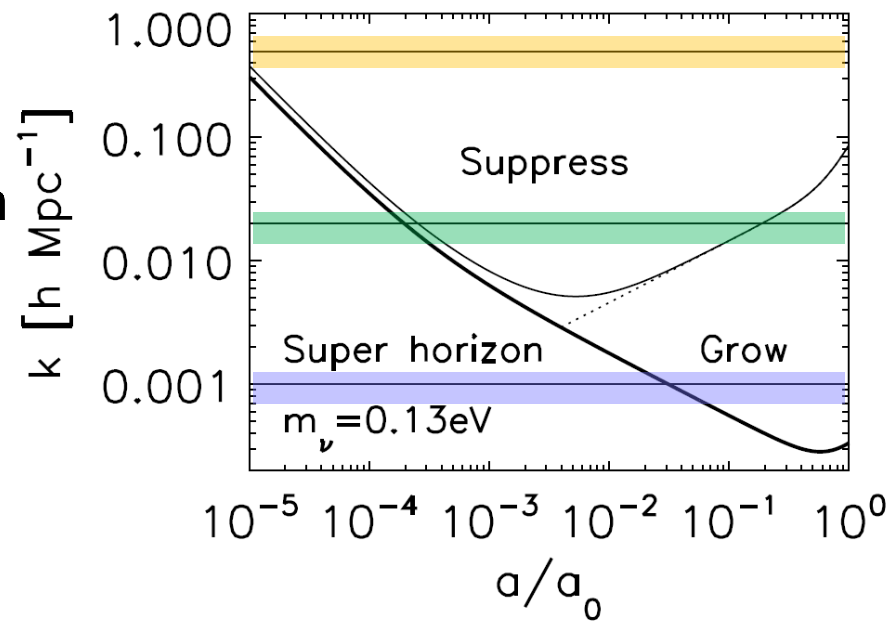
How massive is neutrino?

- Oscillation Experiments (both solar and atmospheric) put ***lower bound***
  - $\Sigma m_{\nu,i}$ >0.056 (0.095) eV
- Cosmology and Astrophysics put ***upper bound***
  - In flat ΛCDM model
    - $\Omega_\nu = \Sigma m_{\nu,i}/94.1h^2$eV < 0.27 → $\Sigma m_{\nu,i} <$ 12 eV
  - Other Constraints from LSS and CMB (i.e., 2dF-gal, SDSS, Ly-α, WMAP, SN-Ia)
    - $\Sigma m_{\nu,i} <$ 0.58 eV (95% CL. from WMAP7-yr+BAO+H_0)
- How do we put a constraint on the mass of neutrino from the power spectrum?

Effect of Neutrino on Structure Formation

~Free streaming scale, $k_{FS}(a)$~

1. Large Scale ($k \ll k_{FS}$)
   - δ_ν grows soon after horizon crossing
   - $\delta_\nu(k,a)=\delta_{cdm}(k,a)$
2. Small Scale ($k \gg k_{FS}$)
   - δ_ν oscillates after horizon crossing (i.e., $\delta_\nu(k,a)\sim 0$)
3. Intermediate Scale
   - δ_ν oscillates first, then grows once $k<k_{FS}(a)$
   - $\delta_\nu(k,a)<\delta_{cdm}(k,a)$



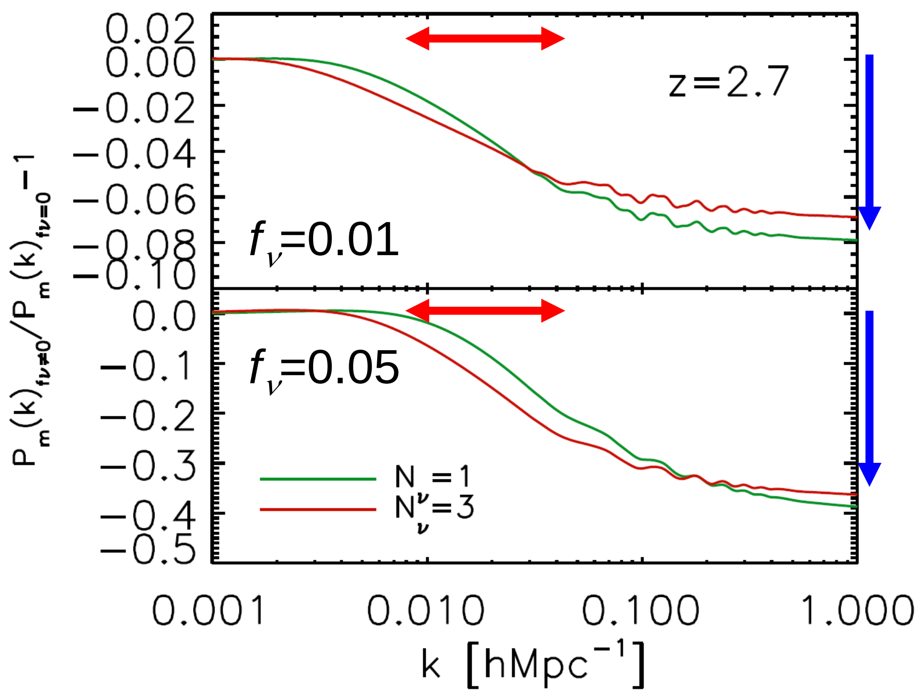
$$\dot{\theta}(\mathbf{k},\tau)+\mathcal{H}(\tau)\theta(\mathbf{k},\tau)+\left[\frac{3}{2}\mathcal{H}^2(\tau)-k^2c_s^2(\tau)\right]\delta(\mathbf{k},\tau)=0$$

$$\boxed{k_{\mathrm{FS,i}}(z)\equiv\sqrt{\frac{3}{2}}\frac{\mathcal{H}(z)}{c_s(z)}}$$

Same definition as Jeans scale

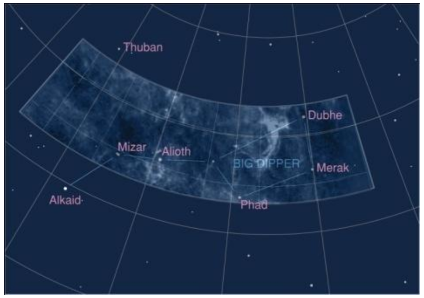
Suppression of Linear Power Spectrum in the presence of Massive Neutrino

- Within the free-streaming scale, $k>k_{FS}$
  - Density contrast of the neutrino is suppressed in a scale dependent way
  - Reduced gravitational potential results in the suppression of the growth rate
- At $k>>k_{FS}$, ***linear*** power spectrum is suppressed by a fixed amount, roughly given by $P_{\Lambda MDM}/P_{\Lambda CDM} \sim 1-8f_\nu = 1-8[\Omega_\nu/\Omega_m]$



- $P_{\Lambda MDM}/P_{\Lambda CDM} \rightarrow \Omega_\nu \rightarrow$ $\boldsymbol{\Sigma m_{\nu,i}}$
- $k_{FS} \rightarrow N_\nu$

A galaxy survey gives power spectrum and puts upper bound on total mass of neutrinos



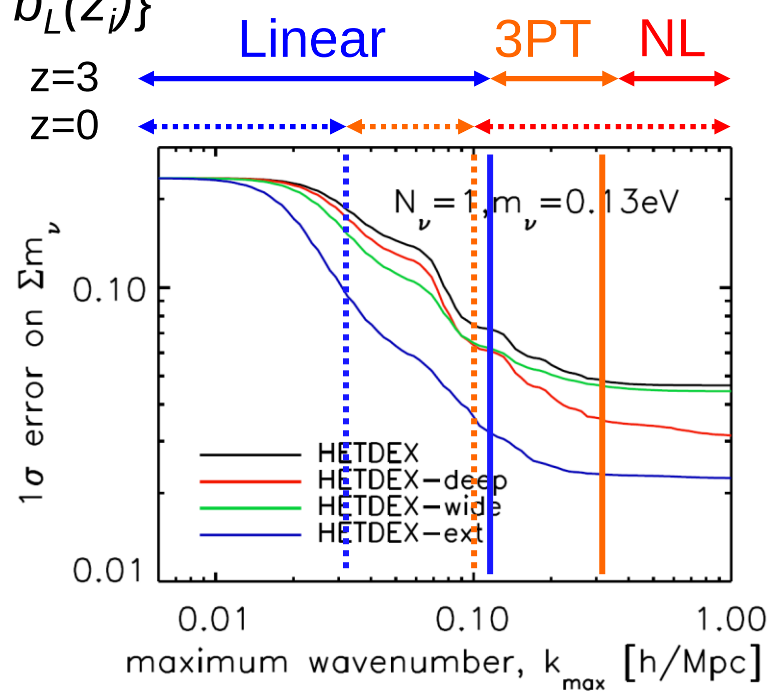
HETDEX

- Hobby-Eberly Telescope Dark Energy Experiment (HETDEX) is a spectroscopic survey that measures…
  - three-dimensional distribution of Lyman-α galaxies in *(RA, Dec, z)*
  - 0.7 million Ly-α galaxies
  - 420 sq. deg. of sky at $\boldsymbol{1.9 < z < 3.5}$ (less contaminated by non-linearity)
  - V~3 $h^{-3}Gpc^3$, n_{gal}~$0.0003h^3Mpc^{-3}$
- Measure both D_A and H with ~1 % accuracy
- *Galaxy Power Spectrum* (GPS) can be used to decipher the cosmological information encoded in the galaxy distribution
  - **Baryon Acoustic Oscillations → Robust (insensitive to NL)**
  - **2D power spectrum (AP-test) → Better (>2x) constraints than BAO only** (Yamamoto et al.,2005: Rassat et al., 2008: MS et al., 2009)
- ***Put tight constraints on the total mass of neutrinos from the 2D power spectrum (BAO cannot measure total mass of neutrino!)***

Marginalized 1-σ error on $m_{\nu,tot}$

$p=\{\Omega_m, \Omega_m h^2, \Omega_b h^2, f_\nu, n_s, \alpha_s, \delta_R, \tau, w_0, b_L(z_i)\}$

- Baseline HETDEX is shot-noise limited at $k>0.2hMpc^{-1}$
  → no gain from small scale information
- Linear theory gives competitive upper limit on $m_{\nu,tot}$
- Further improvement from mildly non-linear regime
- High-z survey has a leverage on the constraining power on $m_{\nu,tot}$
- Need to understand non-linear effects to gain information at mildly-nonlinear regime
  - NL structure growth (CDM)
  - NL structure growth (ν+CDM)
  - NL bias (Jeong&Komatsu 2009)
  - NL redshift space distortion



Carlson, White and Padmanabhan (2009)
Jeong and Komatsu (2006)

$k_{\max}$ [hMpc^{-1}]	0.1	0.2	0.3
HETDEX	0.075 (0.088)	0.055 (0.069)	0.049 (0.063)
HETDEX-Deep	0.064 (0.077)	0.043 (0.056)	0.036 (0.047)
HETDEX-Wide	0.065 (0.079)	0.050 (0.066)	0.046 (0.062)
HETDEX-EX	0.036 (0.048)	0.025 (0.036)	0.023 (0.033)

1-σ errors of $m_{\nu,tot}$ in eV with N_ν = 1 (3)

NL structure growth (ν+CDM)

(MS & Komatsu 2009)

NEW

3PT with Non-Linear Pressure ~Introduction~

- 3PT (1-loop SPT) had been constructed only for CDM
  - Recently applied to real data (Saito et al., 2010)
  - Planned and on-going galaxy surveys at high-z requires understanding at mildly non-linear regime
- First attempt to study multi-fluid system perturbatively in mildly non-linear regime
- Possible application to the baryon physics includes
  - Ly-α forest
  - 21-cm background
- Extension to the CDM+neutrino NL power spectrum
  - Free-streaming scale and mildly non-linear regime roughly coincides
  - Need NL theory to exploit information on a power spectrum

3PT with Non-Linear Pressure ~Flow Chart~

$$\dot{\delta}_c + \nabla \cdot [(1+\delta_c)v_c] = 0$$
$$\dot{\delta}_b + \nabla \cdot [(1+\delta_b)v_b] = 0$$
$$\dot{v}_c + (v_c \cdot \nabla)v_c = -\frac{\dot{a}}{a}v_c - \boxed{\nabla\phi}$$
$$\dot{v}_b + (v_b \cdot \nabla)v_b = -\frac{\dot{a}}{a}v_b - \boxed{\nabla\phi} - c_s^2\frac{\nabla\delta_b}{1+\delta_b}$$
$$\nabla^2\phi = 4\pi G a^2(\bar{\rho}_c\delta_c + \bar{\rho}_b\delta_b)$$

$$g_1(\mathbf{k},\tau) \equiv \frac{\delta_{1,b}(\mathbf{k},\tau)}{\delta_{1,c}(\mathbf{k},\tau)}$$

Baryon and CDM are gravitationally coupled

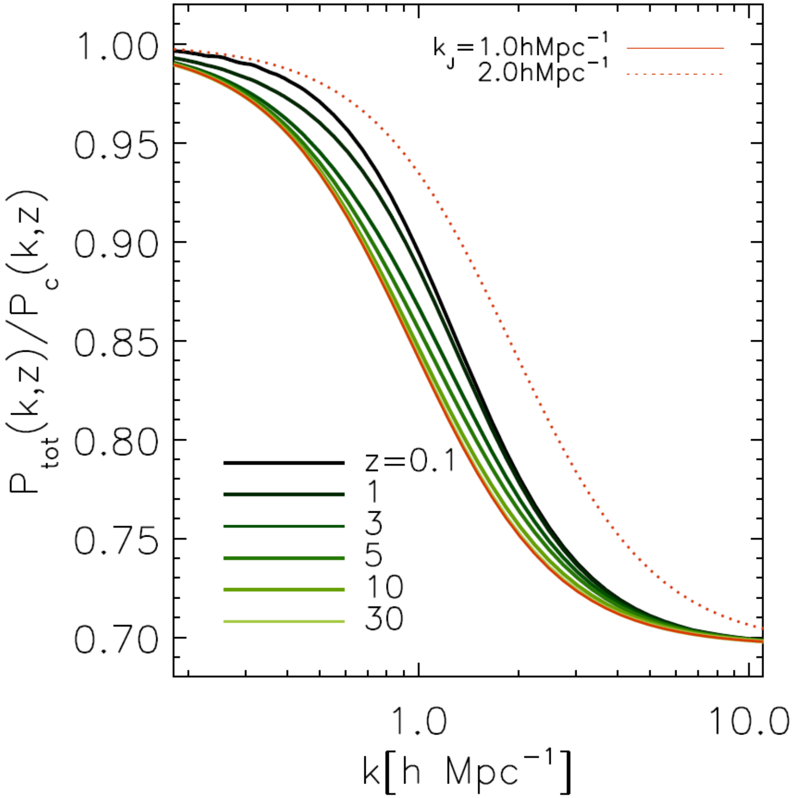
$$\ddot{g}_1^{(0)}(\mathbf{k},\tau) + \frac{6}{\tau}\dot{g}_1^{(0)}(\mathbf{k},\tau) + \frac{6}{\tau^2}\left(1+\frac{k^2}{k_J^2}\right)g_1^{(0)}(\mathbf{k},\tau) = \frac{6}{\tau^2}.$$

$$g_1^{(0)}(k,\tau) = \frac{1}{1+\frac{k^2}{k_J^2}} + \mathcal{O}\left(\tau^{m(k)}\right)$$

Repeat the procedure for n=2 and 3 to get $g_2(k)$ and $g_3(k)$

- Re-construct the "total" 3PT power spectrum from "CDM" 3PT power spectrum and $g_n(k)$
- Approximations/Assumptions
  - Universe is flat and Matter Dominated at the epoch of interest (EdS)
  - Jeans scale is time independent
  - Sound speed is spatially homogeneous (grad[c_s]=0).

3PT with Non-Linear Pressure ~Results~



- For a given Jeans scale, k_J, effective filtering scale is shifted toward smaller scale due to non-linearity in the density contrast

- The effect is larger for lower redshift and larger k_J

TABLE 1
RATIO OF THE EFFECTIVE AND THE LINEAR FILTERING SCALES, $k_{F,eff}/k_J$

k_J (h Mpc^{-1})	z=0.1	1.0	3.0	5.0	10	30
0.1	1.08	1.04	1.01	1.00	1.00	1.00
0.5	1.37	1.21	1.07	1.03	1.01	1.00
1.0	1.43	1.32	1.14	1.08	1.03	1.00
3.0	1.41	1.38	1.28	1.20	1.08	1.01
5.0	1.40	1.39	1.32	1.24	1.12	1.02
10	1.41	1.40	1.35	1.29	1.16	1.03

NOTE. — This table shows the ratios of the effective ($k_{F,eff}$) and the linear (k_J) filtering scales for different redshifts and k_J. The ratios are closer to unity at higher redshifts because non-linearities are weaker.

*** 3PT is *not* valid for this small scale (i.e., >0.1h/Mpc for z=0)
Point is, at *k~k$_J$*, effect of non-linearity is *non-negligible*

Application to Massive Neutrino

~Linear approximation vs. Full 3PT treatment~

$$P_{tot}(k,\tau) = f_c^2 P_c(k,\tau) + f_c f_b P_{bc}(k,\tau) + f_b^2 P_b(k,\tau)$$

$$P_i(k,\tau) = P_{11,i}(k,\tau) + P_{22,i}(k,\tau) + 2P_{13,i}(k,\tau)$$

MS & Komatsu (2009)

$$P_{tot}^{\rm STT}(k,z) = f_c^2 P_c(k,z) + 2 f_c f_\nu P_{11,\nu c}(k,z) + f_\nu^2 P_{11,\nu}(k,z)$$

$$P_{tot}^{\rm STT}(k,z) = \underline{f_c^2 P_c(k)} + [2 f_c f_\nu g_1(k) + f_\nu^2 g_1^2(k)] P_{11,c}(k,z)$$

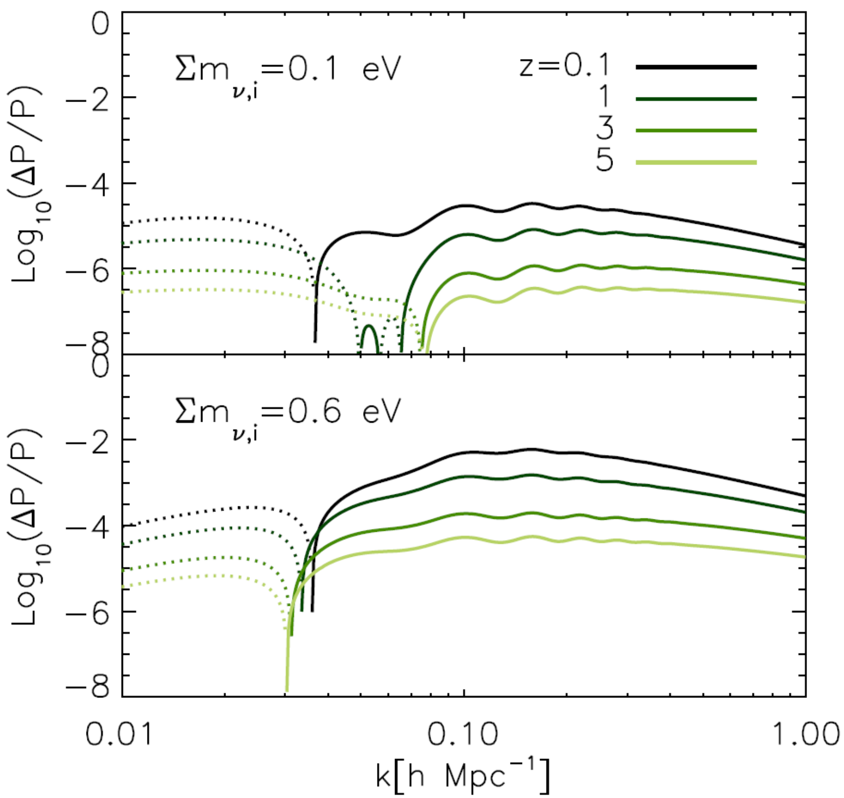
Use 3PT only for $P_c(k)$

Saito et al. (2008)

Non-linear **matter** power spectrum

~Linear approximation vs. Full 3PT treatment~

Matter Power Spectrum frac. diff.

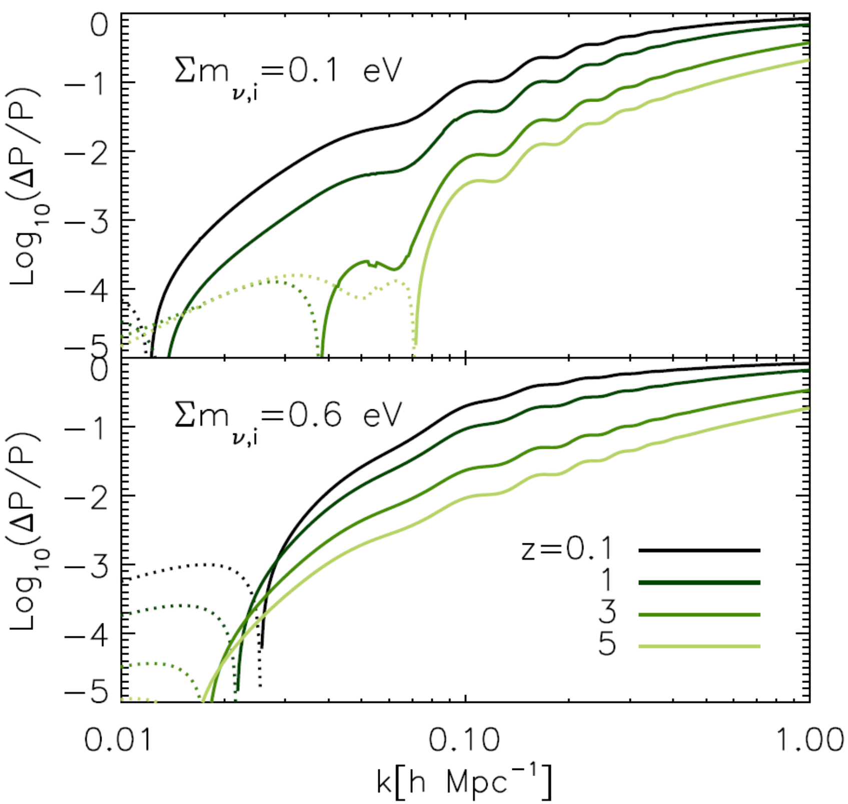


- Linear approximation well approximates **Full 3PT** treatment for small neutrino mass and for larger redshift
- For heavier neutrino mass, non-linear effect becomes non-negligible especially at low redshift
- Current constraint on the neutrino mass ($\Sigma m_{\nu,i}$< 0.58 eV) suggests linear approximation is good for the total **matter** power spectrum

Non-linear neutrino power spectrum

~Linear approximation vs. Full 3PT treatment~

Neutrino Power Spectrum frac. diff.



- Linear approx. fails to follow the Full 3PT treatment
  - → δ_ν **is indeed non-linear**
- Linear approx. works well for the total **matter** power spectrum because of the small fraction (mass) of neutrino, f_ν, not because of the linearity of the neutrino density contrast, δ_ν

$f_\nu = 1 - f_c \sim 0.01$ for $m_{\nu,tot} \sim 0.1eV$

$$P_{tot}(k) = f_c^2 P_c(k) + 2 f_c (1 - f_c) P_{b,c}(k) + (1 - f_c)^2 P_b(k)$$

3PT with Non-Linear Pressure ~Implications~

- Significant change in the shape of the ***baryon/neutrino*** power spectrum → $\delta_\nu \sim \delta_{\nu,1} + \delta_{\nu,2} + \delta_{\nu,3}$
- Jeans mass can be ~3 times smaller

  →Smaller objects than the linear theory prediction can be formed at a given redshift
- Saito et al. (2008) approximates total ***matter*** power spectrum with a linear order neutrino perturbation
  - $\delta_m \sim f_{CDM}\, \delta_{CDM} + f_\nu\, \delta_\nu$
  - $\delta_{CDM} \sim \delta_{CDM,1} + \delta_{CDM,2} + \delta_{CDM,3}$
  - $\delta_\nu \sim \delta_{\nu,1}$
- Linear approximation is good enough for total ***matter*** power spectrum as long as $\Sigma m_{\nu,i} < 0.6$ eV

Is Massive Neutrino fluid? ***NO***

- Nevertheless, attempts to include massive neutrino into non-linear perturbation theory so far is based on fluid approximation
- Do we need NL-CAMB?
- Is fluid approximation valid for massive neutrino?
- If so, why and how?

Is Fluid Approximation Valid for Massive Neutrino?

or for collision-less particles in general ?

(MS & Komatsu 2010)

Linear Theory

- In our previous work, we approximated the pressure-full component to be ***fluid***, neglecting anisotropic stress and higher order moments in the Boltzmann hierarchy

- 3PT is based on linear theory, and any higher order perturbation theory should converge to the linear theory at large scale and high redshift

  ➔ Check the validity of the fluid approximation in linear theory

- Solve perturbed Boltzmann equations truncating the hierarchy at arbitrary moment, and compare the results in EdS universe (fixed gravitational potential)

Boltzmann Hierarchy

Ma & Bertschinger (1995)

- Energy density of neutrino is given as energy weighted integration of the phase space distribution function
- Its perturbation is given as a energy weighted integration of the **perturbed distribution function**
- Evolution of perturbed distribution function, Ψ, is described by **linearly perturbed Boltzmann equation**

$$f_0 = f_0(\epsilon) = \frac{g_s}{h_P^3} \frac{1}{e^{\epsilon/k_B T_0} \pm 1}$$

$$f(x^i, P_j, \tau) = f_0(q)[1 + \underline{\Psi(x^i, q, n_j, \tau)}]$$

perturbation on distribution function

$$\frac{\partial \Psi}{\partial \tau} + i \frac{q}{\epsilon}(\mathbf{k} \cdot \hat{\mathbf{n}})\Psi + \frac{d \ln f_0}{d \ln q}\left[\dot{\phi} - i \frac{\epsilon}{q}(\mathbf{k} \cdot \hat{\mathbf{n}})\psi\right] = \frac{1}{f_0}\left(\frac{\partial f}{\partial \tau}\right)_C$$

$$\boxed{\Psi(\mathbf{k}, \hat{\mathbf{n}}, q, \tau) = \sum_{l=0}^{\infty} (-i)^l (2l+1) \Psi_l(\mathbf{k}, q, \tau) P_l(\hat{k} \cdot \hat{\mathbf{n}})}$$

(Legendre Expansion)

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon}\Psi_1 - \dot{\phi}\frac{d \ln f_0}{d \ln q},$$

$$\dot{\Psi}_1 = \frac{qk}{3\epsilon}(\Psi_0 - 2\Psi_2) - \frac{\epsilon k}{3q}\psi \frac{d \ln f_0}{d \ln q},$$

$$\dot{\Psi}_l = \frac{qk}{(2l+1)\epsilon}[l\Psi_{l-1} - (l+1)\Psi_{l+1}], \quad l \geq 2.$$

Truncate hierarchy at $l=l_{max}$

$$\bar{\rho}_h = a^{-4} \int q^2 dq \, d\Omega \, \epsilon f_0(q)$$

$$\delta\rho_h = 4\pi a^{-4} \int q^2 dq \, \epsilon f_0(q) \Psi_0$$

Numerical Confirmation of the Fluid Approximation

- When gravity dominates the evolution of Ψ_l (ε>>q), Ψ_0 and Ψ_1 will be independent of the higher order moments
  - depends on m_ν, k and z
- Truncating Ψ_l for l >1 is equivalent to neglect the anisotropic stress
- How high l_{max} should we use for massive neutrino to achieve the desired accuracy?
  - < 1% in density contrast, δ
- Compare Ψ_0 (l_{max}=1,2,3) with ***exact*** solution of Ψ_0

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon}\Psi_1 - \dot{\phi}\frac{d\ln f_0}{d\ln q},$$

$$\dot{\Psi}_1 = \frac{qk}{3\epsilon}(\Psi_0 - \cancel{2\Psi_2}) - \frac{\epsilon k}{3q}\psi\frac{d\ln f_0}{d\ln q},$$

$$\cancel{\dot{\Psi}_l = \frac{qk}{(2l+1)\epsilon}[l\Psi_{l-1} - (l+1)\Psi_{l+1}]}, \quad l \geq 2.$$

$$\Updownarrow$$

$$\dot{\delta} = -(1+w)(\theta - 3\dot{\phi}) - 3\frac{\dot{a}}{a}\left(\frac{\delta P}{\delta \rho} - w\right)\delta,$$

$$\dot{\theta} = -\frac{\dot{a}}{a}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{\delta P/\delta\rho}{1+w}k^2\delta - \cancel{k^2\sigma} + k^2\psi$$

$$\cancel{\dot{\sigma} = \ldots}$$

Exact Solution of Ψ_0 (k,q,η)

NEW

- Instead of expanding the Boltzmann equation ($d\Psi/d\tau=0$), we first find a formal solution, and expand the solution.

$$|l-l'| \le l'' \le l+l'$$

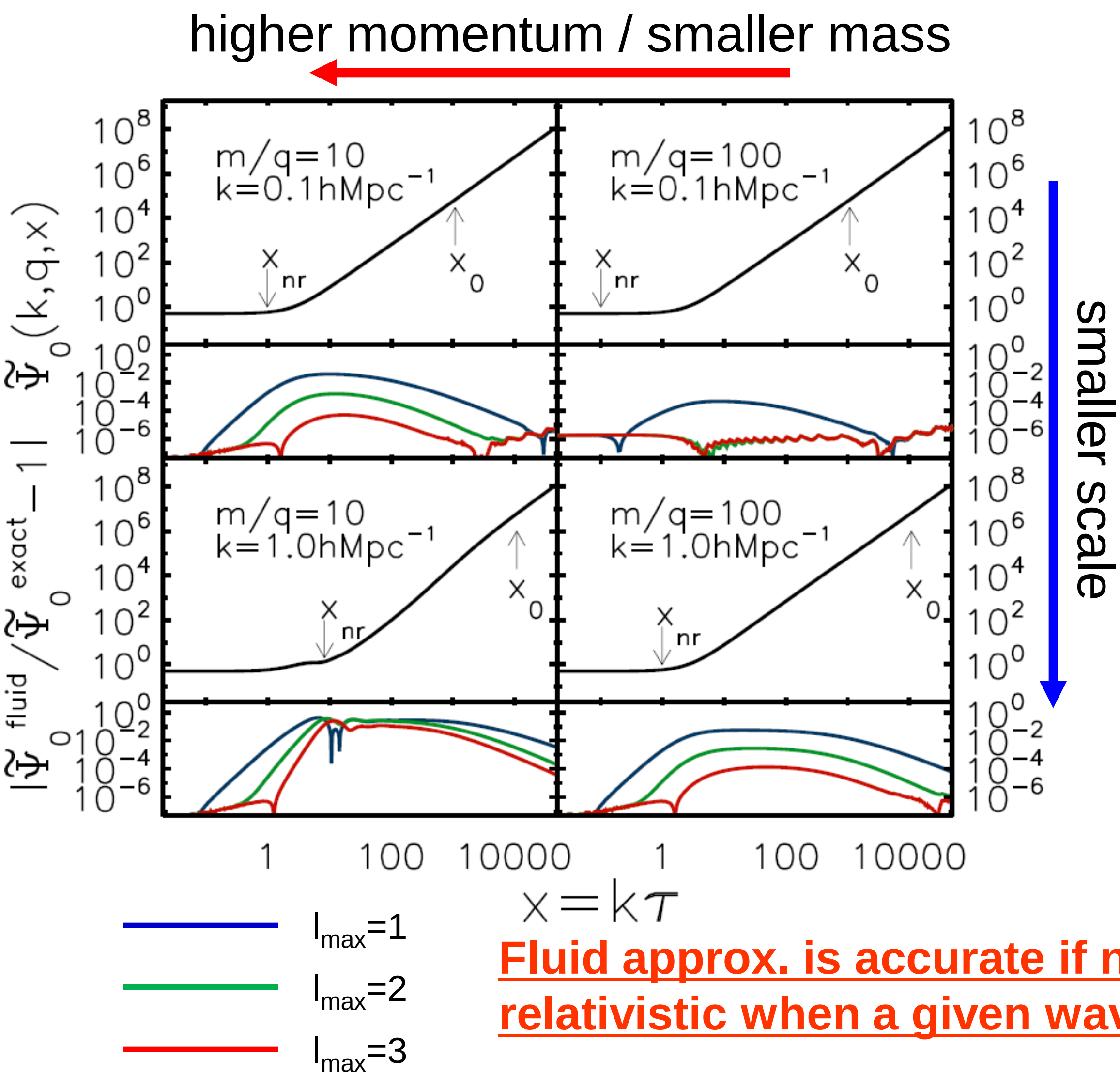
$$\tilde{\Psi}_l(k,q,x) = \sum_{l'}\sum_{l''}(-i)^{l'+l''-l}(2l'+1)(2l''+1)\tilde{\Psi}_{l'}(k,q,x_i)j_{l''}(z-z_i)\begin{pmatrix} l & l' & l'' \\ 0 & 0 & 0 \end{pmatrix}^2$$
$$- \psi(k)\int_{x_i}^{x} dx' \frac{\epsilon(q,x')}{q}\left[\frac{l}{2l+1}j_{l-1}(z-z') - \frac{l+1}{2l+1}j_{l+1}(z-z')\right].$$

Infinite sum

- The solution for Ψ_l above is equivalent to solving infinite order of Boltzmann hierarchy.
- Since initial values (super horizon) of Ψ_l is suppressed for higher l, as $\Psi_l \sim x^l$, we truncate the initial values of $\Psi_{l'}$ at $l'>2$.

$$\tilde{\Psi}_0(k,q,x) = \tilde{\Psi}_0(k,q,x_i)j_0(z-z_i) - 3\tilde{\Psi}_1(x_i)j_1(z-z_i) + 5\tilde{\Psi}_2(x_i)j_2(z-z_i)$$
$$+ \psi(k)\int_{x_i}^{x} dx' \frac{\epsilon(q,x')}{q} j_1(z-z'), \qquad z(x) \equiv \int_C^x \frac{q}{\epsilon(q,x')}dx'$$

Fluid Approx. vs. Exact Solution: $\Psi_0\ (k,q,\eta)$



- Neutrino with small m/q become non-relativistic after horizon-crossing
  → large l_{max} is required
- At small scale, relative importance of high l increases as k^l
  → large l_{max} is required

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon}\Psi_1 - \dot{\phi}\frac{d\ln f_0}{d\ln q},$$

$$\dot{\Psi}_1 = \frac{qk}{3\epsilon}(\Psi_0 - 2\Psi_2) - \frac{\epsilon k}{3q}\psi\frac{d\ln f_0}{d\ln q},$$

$$\dot{\Psi}_l = \frac{qk}{(2l+1)\epsilon}[l\Psi_{l-1} - (l+1)\Psi_{l+1}], \quad l \geq 2.$$

Fluid approx. is accurate if neutrinos were already non-relativistic when a given wavenumber entered the horizon

Fluid Approx. vs. Exact Solution : δ_ν(k,a)

- Error on Ψ_0 (k,q,η) is large for large *k*, small m_ν/q and low *z*

$$\delta\rho_h = 4\pi a^{-4}\int q^2 dq\, \epsilon f_0(q)\Psi_0$$

- Integrant is ***exponentially suppressed for small m_ν/q***

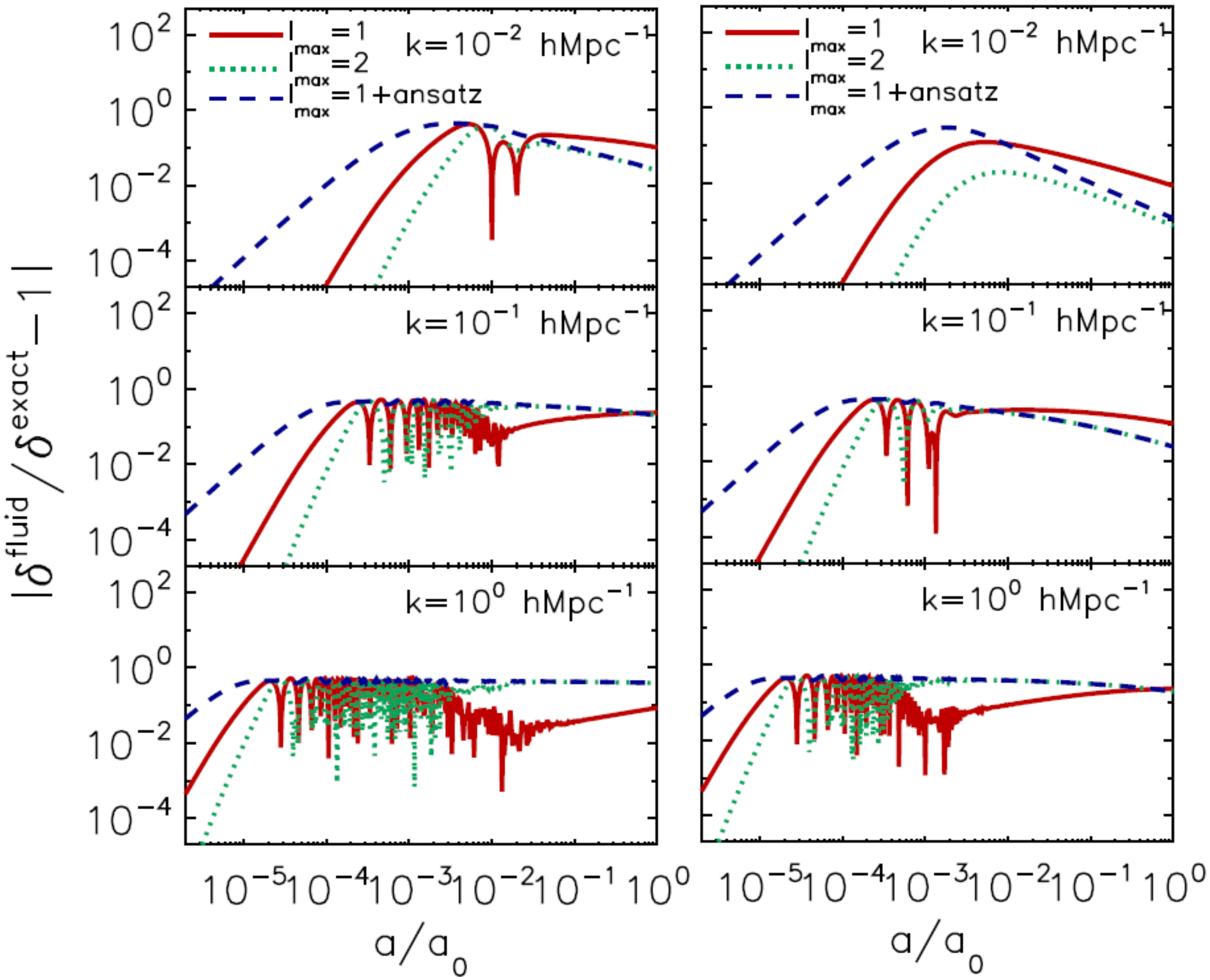
$$f_0(q) \sim \frac{1}{e^{q/T}+1} = \frac{1}{e^{(q/m_\nu)(m_\nu/T_{\nu,0})(a_{eq}/a_0)}+1}$$

- For fixed $m_\nu/T_{\nu,0}$>>10^4 (m_ν>>1eV), contribution to δ_ν from high momentum neutrino with m_ν/q<<3 will be greatly suppressed
  - → Error from relativistic neutrino does not count
  - → Fluid Approximation is ***valid***

- For sufficiently light neutrino (small $m_\nu/T_{\nu,0}$), large error will be propagated from neutrinos with high *q* in the perturbed distribution function, Ψ_l, to δ_ν
  - → Fluid Approximation is ***NOT valid***

Fluid Approx. vs. Exact Solution : $\delta_\nu(k,a)$

- For small mass neutrino (m_ν=0.05 eV), fluid approx. is ***limited*** to large scale, and late time
- For large mass neutrino (m_ν=0.5 eV), fluid approx. is still ***limited*** to few~20% accuracy



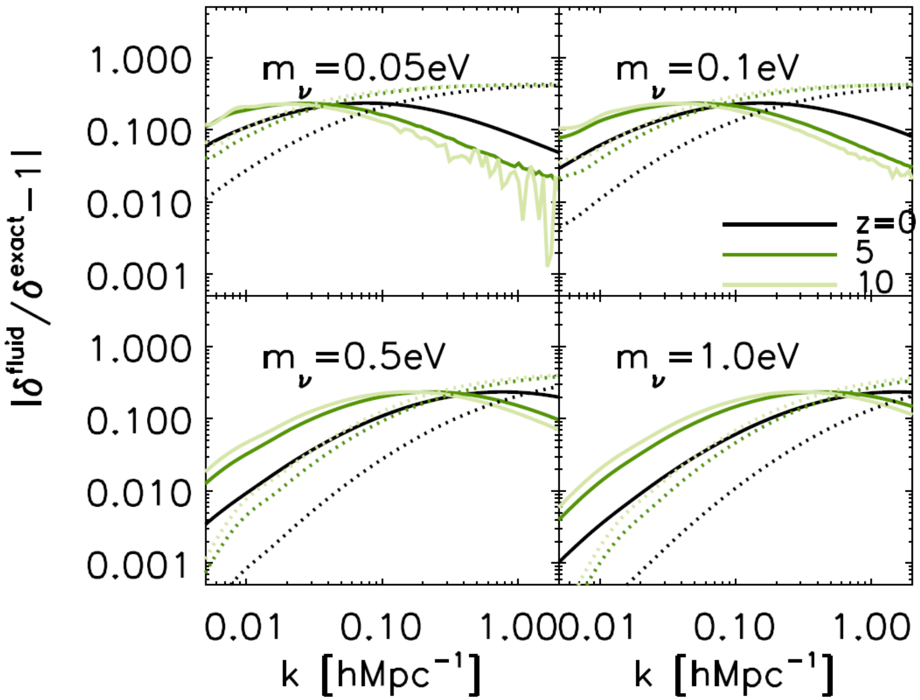
Fluid Approx. vs. Exact Solution : $\delta_\nu(k,a)$

- At least, one of the neutrino species has a mass of ~ 0.05eV
- Structure formation is mostly affected by the most massive species
- Fluid approximation accuracy is limited to ~25% over the wavenumber, where 3PT is applied (~ 0.3 *h* Mpc^{-1} for z~3)

$$P_m \sim \delta^2 = f_{cdm}^2\delta_{cdm}^2 + f_{cdm} f_\nu \delta_{cdm}\delta_\nu + f_\nu^2 \delta_\nu^2$$

$\boxed{<1\%}$

$$\boxed{\begin{array}{l} 0.004 < f_\nu < 0.04 \\ 0.05 < m_\nu < 0.5eV \end{array}}$$



Including anisotropic stress term (l_{max}=2) improves the accuracy

Anisotropic Stress (l_{max}=2) ?

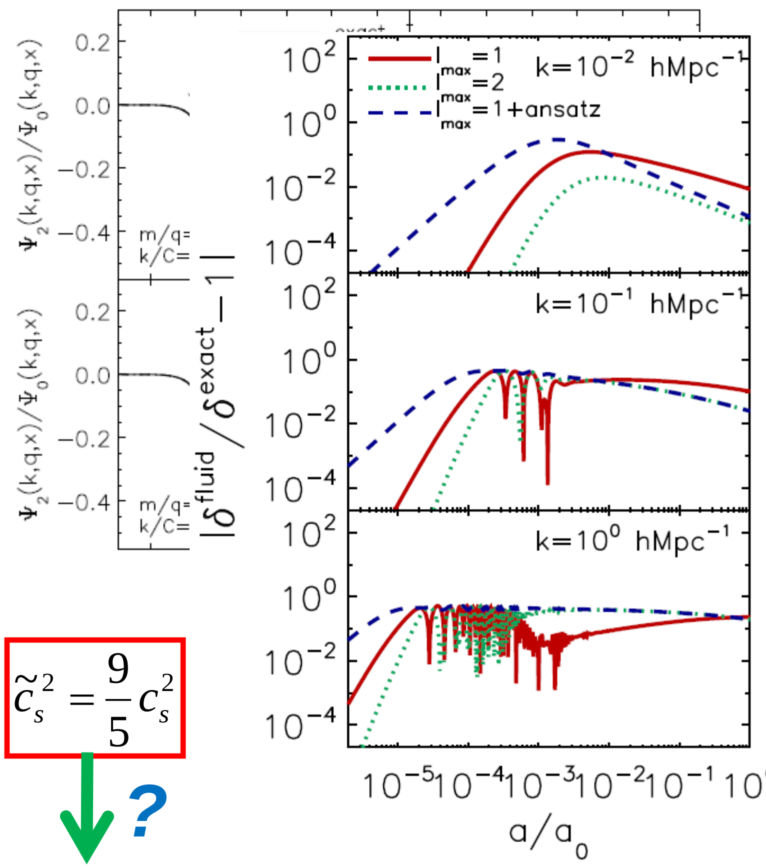
- For l_{max}=2, we have a useful relation between Ψ_0 and Ψ_2

$$\frac{\partial}{\partial x}\left[\tilde{\Psi}_2(k,q,x)+\frac{2}{5}\tilde{\Psi}_0(k,q,x)+\frac{2}{5}\phi(k,x)\right]=0$$

- Neglecting evolution of ϕ, Ψ_2 is proportional to Ψ_0, and we have

$$k^2\sigma(k,\tau)\simeq-\frac{4}{5}\frac{\delta P(k,\tau)/\delta\rho(k,\tau)}{1+w(\tau)}k^2\delta(k,\tau)$$

- This is equivalent to increasing pressure by 9/5
- Similarly, including an ansatz for diffusion term in the Euler equation can improve accuracy



$$\boxed{\tilde{c}_s^{\,2}=\frac{9}{5}c_s^2}$$

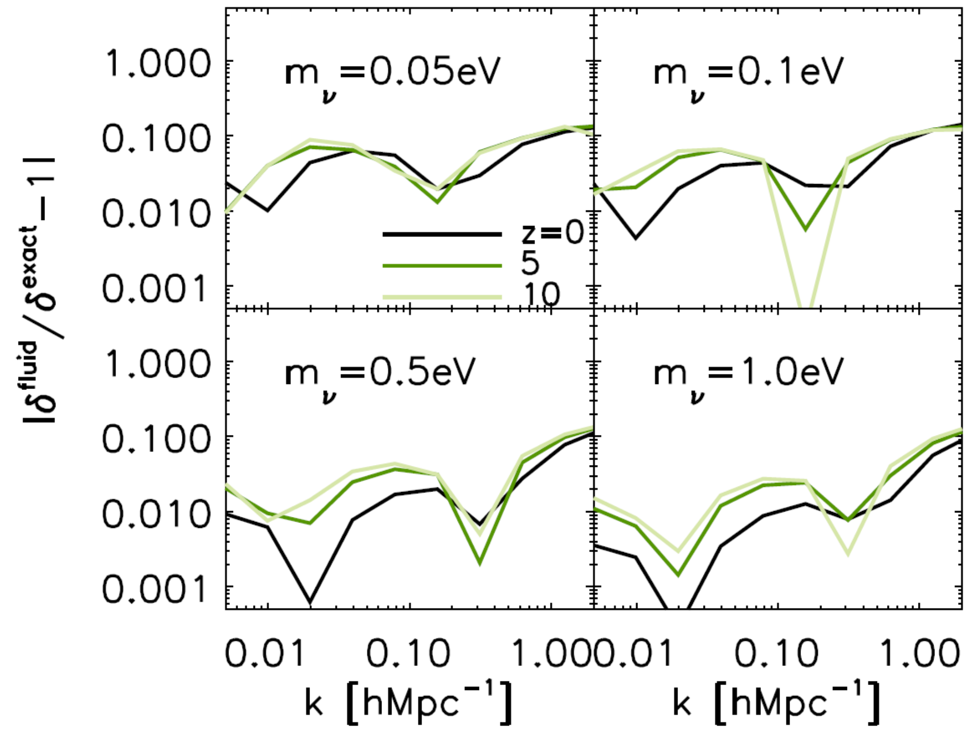
?

$$\boxed{\tilde{c}_s^{\,2}=F(k,m_\nu)c_s^2}$$

Preliminary Result

Find Appropriate Ansatz

- What about adding extra diffusion term to Euler equation?
- Applying appropriate ansatz will improve the accuracy of fluid approximation (2~10 times better!)



$$\dot{\delta}_\nu(k,\tau) = -[1+w(\tau)][\theta_\nu(k,\tau) - 3\dot{\phi}(k,\tau)] - 3\frac{\dot{a}(\tau)}{a(\tau)}\left[c_s^2(k,\tau) - w(\tau)\right]\delta_\nu(k,\tau),$$

$$\dot{\theta}_\nu(k,\tau) = -\frac{\dot{a}(\tau)}{a(\tau)}[1 - 3c_s^2(k,\tau) + \beta(m_\nu)]\theta_\nu(k,\tau) + [1 - \alpha(m_\nu)]\frac{k^2 c_s^2(k,\tau)\delta_\nu(k,\tau)}{1+w(\tau)} - k^2\sigma_\nu(k,\tau) + k^2\psi(k,\tau)$$

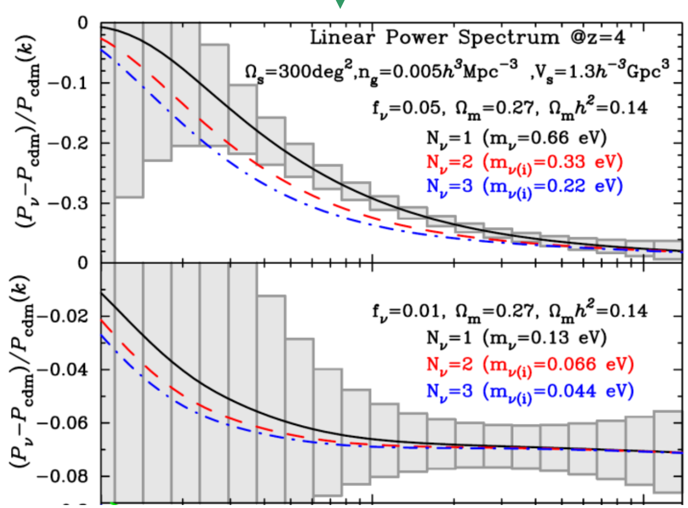
Conclusions

- Future and on-going LSS surveys combined with Planck can put a significant constraint on the total mass of the neutrinos ($\Delta m_{\nu,tot}$<0.1 *eV*)
- To exploit the information in a given power spectrum, we need to understand various non-linearities including massive neutrinos
- 3PT has been constructed for a mixed fluid of CDM and pressureful component (NEW) ➔ possible extension to massive neutrino
- We developed exact solution for perturbed distribution function, Ψ_l (NEW)
- Fluid approximation accuracy is limited to *<25%* for massive neutrino with 0.05<m_ν<0.5 eV for a range of wavenumber, where perturbation theory concerns (NEW)
  ➔ *<1%* accuracy in matter power spectrum
- If necessary, accuracy of fluid approximation can be further improved by introducing appropriate ansatz (*<10%* so far)

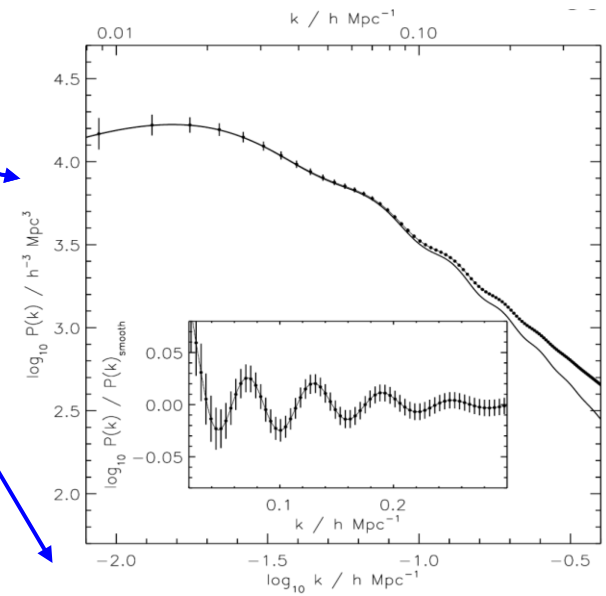
Thank you!

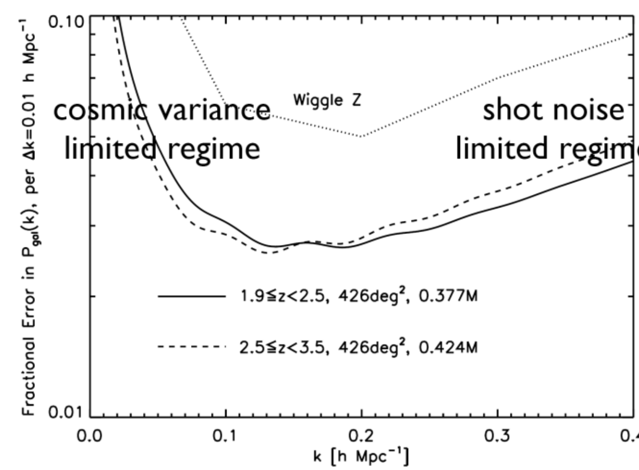
HETDEX/m_ν/N_ν

- HETDEX is shot-noise limited at $k > 0.2 h\,Mpc^{-1}$
- Power spectrum is sensitive to the total mass of neutrinos, $m_{\nu,tot}$, <u>not the number of species, N_ν</u>



Takada etal. 2006





application/caveats/discussions

- EdS+massive neutrino → ϕ and ψ are not constant
  - Contribution of neutrinos to the gravitational potential is small ($0.01<f_\nu<0.05$)
  - This small contribution is important to understand the amplitude of $P(k)$, but does not change k_{max} significantly (k_{max} will be decreased slightly)
- EdS+massive neutrino → ϕ and ψ are not constant
  - Once fluid approximation becomes valid at some $z>1$, ψ is already large enough.

    → unless ψ decreases faster than a^{-2}, fluid approximation stays valid ($\varepsilon \sim a$)

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon}\Psi_1 - \dot{\phi}\frac{d\ln f_0}{d\ln q}\,,$$

$$\dot{\Psi}_1 = \frac{qk}{3\epsilon}(\Psi_0 - 2\Psi_2) - \frac{\epsilon k}{3q}\psi\frac{d\ln f_0}{d\ln q}\,,$$

$$\dot{\Psi}_l = \frac{qk}{(2l+1)\epsilon}[l\Psi_{l-1} - (l+1)\Psi_{l+1}]\,, \quad l \geq 2\,.$$

As long as gravitational term remains dominant, fluid approximation is valid

application/caveats/discussions

- Exact solution is also available for Ψ_l with time dependent potentials ϕ and ψ
  - Now, ϕ and ψ are also subject to integration over time

$$\tilde{\Psi}_l(k,q,x) = \sum_{l'}\sum_{l''}(-i)^{l'+l''-l}(2l'+1)(2l''+1)\tilde{\Psi}_{l'}(k,q,x_i)j_{l''}(z-z_i)\begin{pmatrix} l & l' & l'' \\ 0 & 0 & 0 \end{pmatrix}^2$$
$$-\int_{x_i}^{x} dx' \left[\frac{\epsilon(q,x')}{q}\psi(k,x') - \frac{q}{\epsilon(q,x')}\phi(k,x')\right]\left[\frac{l}{2l+1}j_{l-1}(z-z') - \frac{l+1}{2l+1}j_{l+1}(z-z')\right]$$
$$+\phi(k,x_i)j_l(z-z_i) - \phi(k,x)\delta_{l0},$$

application/caveats/discussions

- Fluid Approximation (l_{max}=1) is equivalent to continuity and Euler equations with $\sigma = 0$
- w and $\delta P/\delta\rho$ are time dependent
  - We need to calculate Ψ_0 from Boltzmann equation with l_{max}=1
- $\delta P/\delta\rho$ cannot be replaced with velocity dispersion as in Takada et al. (2006)

$$\dot\delta(k,\tau) = -[1+w(\tau)][\theta(k,\tau) - 3\dot\phi(k,\tau)] - 3\frac{\dot a(\tau)}{a(\tau)}\left[\frac{\delta P(k,\tau)}{\delta\rho(k,\tau)} - w(\tau)\right]\delta(k,\tau)$$

$$\dot\theta(k,\tau) = -\frac{\dot a(\tau)}{a(\tau)}[1-3w(\tau)]\theta(k,\tau) - \frac{\dot w(\tau)}{1+w(\tau)}\theta(k,\tau) + \frac{\delta P(k,\tau)/\delta\rho(k,\tau)}{1+w(\tau)}k^2\delta(k,\tau) - k^2\sigma(k,\tau) + k^2\psi$$

$$c_s^2(k,\tau) \equiv \frac{\delta P(k,\tau)}{\delta\rho(k,\tau)} = \frac{1}{3}\frac{\int q^2 dq \frac{q^2}{\epsilon(q,\tau)} f_0(q)\Psi_0(k,q,\tau)}{\int q^2 dq\,\epsilon(q,\tau) f_0(q)\Psi_0(k,q,\tau)}$$

In non-relativistic limit, we have

$$c_s^2(\tau) \to \frac{1}{3}\sigma_\nu^2(\tau) + \frac{2}{9}\frac{\sigma_\nu^2(\tau)}{1+\frac{1}{3}\sigma_\nu^2(\tau)} \simeq \frac{5}{9}\sigma_\nu^2(\tau)$$