Non-linear Structure Formation in Modified Gravity

Fabian Schmidt
Caltech - TAPIR

with **Wayne Hu, Marcos Lima, Alexey Vikhlinin, Hiroaki Oyaizu**

Berkeley TAC seminar, 11/30/09

The Universe is Accelerating

- **3 ingredients of standard cosmology:**

Homogeneity & Isotropy
- FRW metric

General Relativity (GR)
- Friedmann Equation

Decelerating Universe

Stress-Energy Content
- Matter & Radiation

What is wrong ?
Or missing ?

Physics behind Acceleration

- **Modify any of the ingredients:**

~~Homogeneity & Isotropy~~
~~- FRW metric~~

General Relativity (GR)
- Friedmann Equation

Backreaction? Averaging?

Accelerating Universe

Stress-Energy Content
- Matter & Radiation

Physics behind Acceleration

- **Modify any of the ingredients:**

Homogeneity & Isotropy
- FRW metric

General Relativity (GR)
- Friedmann Equation

Accelerating Universe

Stress-Energy Content
- Matter & Radiation

and
Dark Energy ?

Physics behind Acceleration

- **Modify any of the ingredients:**

Homogeneity & Isotropy
- FRW metric

~~General Relativity (GR)~~
~~- Friedmann Equation~~

Modified Gravity ?

Accelerating Universe

(focus of this talk)

Stress-Energy Content
- Matter & Radiation

Physics behind Acceleration

- **Minimal solution: cosmological constant Λ**

Homogeneity & Isotropy
- FRW metric

General Relativity (GR)
- Friedmann Equation

Accelerating Universe

Λ

Stress-Energy Content
- Matter & Radiation

Modified Gravity: Challenges

Theoretical Challenge:

- **Gravity constrained on wide range of scales:**
  - Early Universe: BBN, CMB
  - Growth of structure
  - Solar System

- **Idea: reduce to GR in high-curvature regime**
  - Applies to Early Universe as well as high-density regions today

Modified Gravity: Challenges

Observational Challenge:

- **How can we distinguish Modified Gravity from GR + Dark Energy ?**
  - (Almost) any expansion possible with Dark Energy

- **Beyond background: growth of structure**
  - Predictions straightforward in *linear regime*
  - *Non-linear regime* less so...
  - Compare modified gravity with Dark Energy model with *identical expansion history*

Probing gravity: linear vs nonlinear regime

Linear regime: CMB, SN, ISW, *BAO*

- Parametrizing gravity possible --> model-independent constraints
- Limited statistical/constraining power (e.g. *f(R)*)

Non-linear regime: galaxy clustering, weak lensing, cluster abundance

- No general parametrization: non-linear mechanism of gravity model important
- Specialized N-body simulations necessary
- Wealth of observables available
- Lots of statistics and S/N

Modified Gravity Models

Two known and fully worked models achieving acceleration:

- **DGP braneworld model**
  - Gravity “leaks” into large extra dimension
- **f(R) model**
  - Phenomenological extension of GR
  - Equivalent to scalar-tensor theory

- **Both use *non-linear mechanism* to restore GR locally**

f(R) Gravity

- **Simplest workable modified gravity model**
- **Generalize Lagrangian of General Relativity:**

$$\mathcal{L}_g = \frac{1}{16\pi G}(R - 2\Lambda) \longrightarrow \frac{1}{16\pi G}(R + f(R))$$

- **Choose function which (in ΛCDM limit) becomes:**

$$f(R) \approx -2\Lambda - f_{R0}\frac{R_0^2}{R},$$

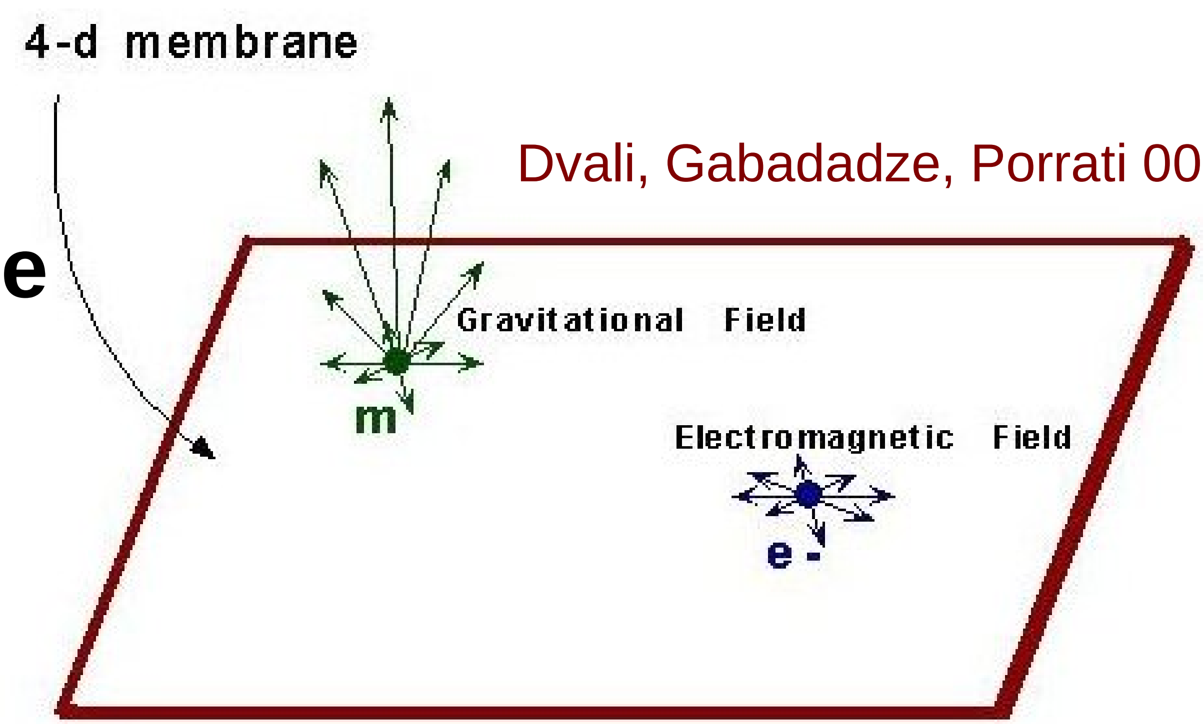
Hu & Sawicki, PRD 07

f(R) Gravity

- **f(R) model produces ΛCDM expansion history without true Λ**
  - Difference in H(z) of order $f_{R0} \ll 1$
- **Equivalent to scalar-tensor theory**
  - Scalar field $f_R \equiv \frac{df}{dR}$ --> 5th force
  - Grav. force enhanced by 4/3 within $\lambda_C = \sqrt{3 f_{RR}}$
- ***Chameleon effect:* recover GR locally**
  - Scalar field decouples in high-density regions

DGP Braneworld cosmology

- **Dvali-Gabadadze-Porrati model:**
  - Matter / radiation confined to 4D brane in 5D Minkowski space
  - Action constructed to reduce to GR on small scales



Dvali, Gabadadze, Porrati 00

- **Cross-over scale**

$$r_c \equiv \frac{G^{(5)}}{2G^{(4)}}$$

DGP Braneworld cosmology

- **Grav. force scales as** $$F \sim \begin{cases} r^{-2}, & r \lesssim r_c \\ r^{-3}, & r \gtrsim r_c \end{cases}$$

- **Cosmology: modified Friedmann eqn:**

$$\boxed{H^2 + \varepsilon \frac{H}{r_c} = \frac{8\pi G}{3}\rho}$$

Deffayet 01

  - $\varepsilon = 1$: ***Normal branch, decelerating***
  - $\varepsilon = -1$: ***Self-accelerating branch***
  - GR limit: $H \gg \dfrac{1}{r_c}$

DGP Branches

- **Self-accelerating branch**
  - Accelerating today if $r_c \sim H_0^{-1} \sim 3000\,\mathrm{Mpc/h}$
  - w_{eff} ~ -0.5 ... -0.8
  - *~4σ conflict* with CMB+Supernovae

Fang et al. 08

- **Normal branch**
  - Have to add Λ or *dark energy* on brane

Lombriser et al. 09,
FS 09b

Growth of structure in DGP

Koyama & Maartens 06,
Nicolis & Rattazzi 04

- **Large scales $\gtrsim r_c,\ H^{-1}$:**
  - 5D treatment of perturbations necessary

- **Sub-horizon scales: effective scalar-tensor theory**
  - Massless field φ - *brane-bending mode*
  - φ contributes to dynamical potential:

$$\boxed{\Psi = \Psi_N + \frac{1}{2}\varphi}$$

Newtonian pot.

  - *Normal branch:* φ attractive
  - *Self-acc. branch:* φ *repulsive*

Brane-bending mode

- **On linear scales :** $\varphi = \dfrac{2}{3\beta}\Psi_N, \quad \beta(a) \propto H\, r_c$
  - Effective grav. constant

$$G_N \to G_{\rm eff} = G_N \left(1 + \frac{1}{3\beta(a)}\right)$$

Brane-bending mode

- **On linear scales :** $\varphi = \dfrac{2}{3\beta}\Psi_N, \;\; \beta(a) \propto H\, r_c$

- **When** $\delta\rho/\bar{\rho} \gtrsim 1$**, non-linear interactions of** φ **important:**

$$\boxed{\nabla^2\varphi + \frac{r_c^2}{3\beta\, a^2}\left[(\nabla^2\varphi)^2 - (\nabla_i\nabla_j\varphi)(\nabla^i\nabla^j\varphi)\right] = \frac{8\pi\, G\, a^2}{3\beta}\delta\rho}$$

  - Non-linear interactions **generic** to braneworld models *(Gauss-Codazzi eq.)*

Time derivatives neglected: sub-horizon scales

Non-linear interactions

$$\nabla^2\varphi + \frac{r_c^2}{3\beta\, a^2}\left[(\nabla^2\varphi)^2 - (\nabla_i\nabla_j\varphi)(\nabla^i\nabla^j\varphi)\right] = \frac{8\pi\, G\, a^2}{3\beta}\delta\rho$$

- **Hard: non-linear in derivatives of φ**
  - No superposition principle
- **Only numerical solution in general**
  - As part of N-body simulation

Non-linear interactions

$$\nabla^2\varphi + \frac{r_c^2}{3\beta\, a^2}\left[(\nabla^2\varphi)^2 - (\nabla_i\nabla_j\varphi)(\nabla^i\nabla^j\varphi)\right] = \frac{8\pi\, G\, a^2}{3\beta}\delta\rho$$

- **Two analytically solvable cases:**

1. Plane wave: $\varphi \sim e^{i\mathbf{k}\cdot\mathbf{x}} \Rightarrow -k^2\varphi = \frac{8\pi G a^2}{3\beta}\delta\rho$

Non-linearity cancels !

Non-linear interactions

$$\nabla^2\varphi + \frac{r_c^2}{3\beta\, a^2}\left[(\nabla^2\varphi)^2 - (\nabla_i\nabla_j\varphi)(\nabla^i\nabla^j\varphi)\right] = \frac{8\pi\, G\, a^2}{3\beta}\delta\rho$$

- **Two analytically solvable cases:**

2. Spherically symmetric mass

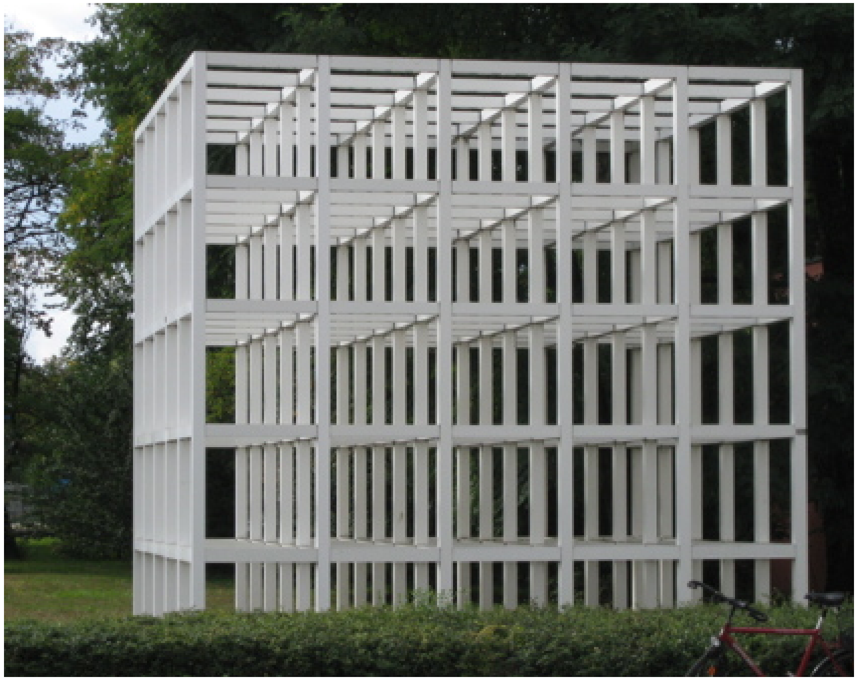
- φ *saturates within Vainshtein radius* $R_* = \left(\frac{8 r_s r_c^2}{9\beta^2}\right)^{1/3}$
- *See later.*

$$R_{*,\odot} \sim 100\,\mathrm{pc} \quad \mathrm{for} \quad r_c \sim \mathrm{Gpc}$$

Simulating DGP

- **Need self-consistent solution of nonlinear φ field and dark matter**

- ***Particle-mesh code:***
  - Density and potential are evaluated on cubic grid
  - Given modified potential, propagation of particles unchanged



Main task: solve for potential

FS 09a, Oyaizu 08

- **Newtonian potential Ψ_N:**
  - Obtained via Fourier transform of density
- **Brane-bending mode φ:**
  - Non-linear relaxation scheme (Newton-Raphson)
    - Parallelized with multi-grid acceleration
- **Finally:** $\Psi = \Psi_N + \frac{1}{2}\varphi$
- **Non-linear relaxation *time-consuming:***
  - CPU time ~20x that of ordinary GR simulations

Simulated Models

sDGP: Best-fit flat self-accelerating DGP model

Fang et al. 08

- No Λ or dark energy

nDGP: normal-branch with dark energy

- Exact ΛCDM expansion history: r_c unconstrained

FS 09b

- Contrived model... but fully understood
- Effective model for generalized braneworlds

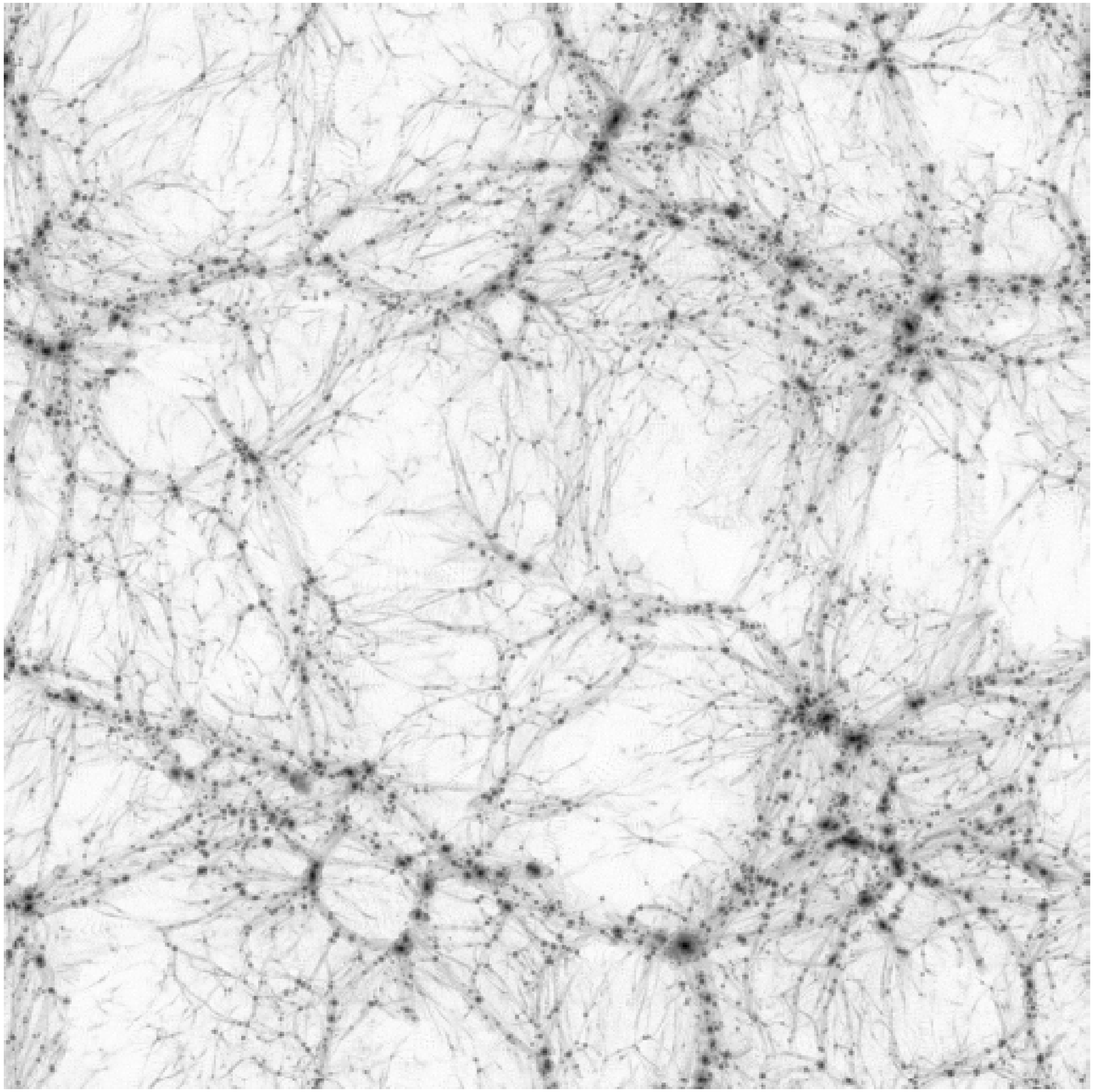
Cosmology: parameters

	QCDM	sDGP	ΛCDM	nDGP–1	nDGP–2
Ω_m	0.258	0.258	0.259	0.259	0.259
Ω_Λ (eff.)	0	0	0.741	0.741	0.741
r_c [Mpc]	∞	6118	∞	500	3000

- **Simulate three models in each case:**
  - GR with identical expansion history: "QCDM" / ΛCDM
  - Full DGP
  - Linearized DGP

Box sizes: 400, 256, 128, 64 Mpc/h
3-6 runs each

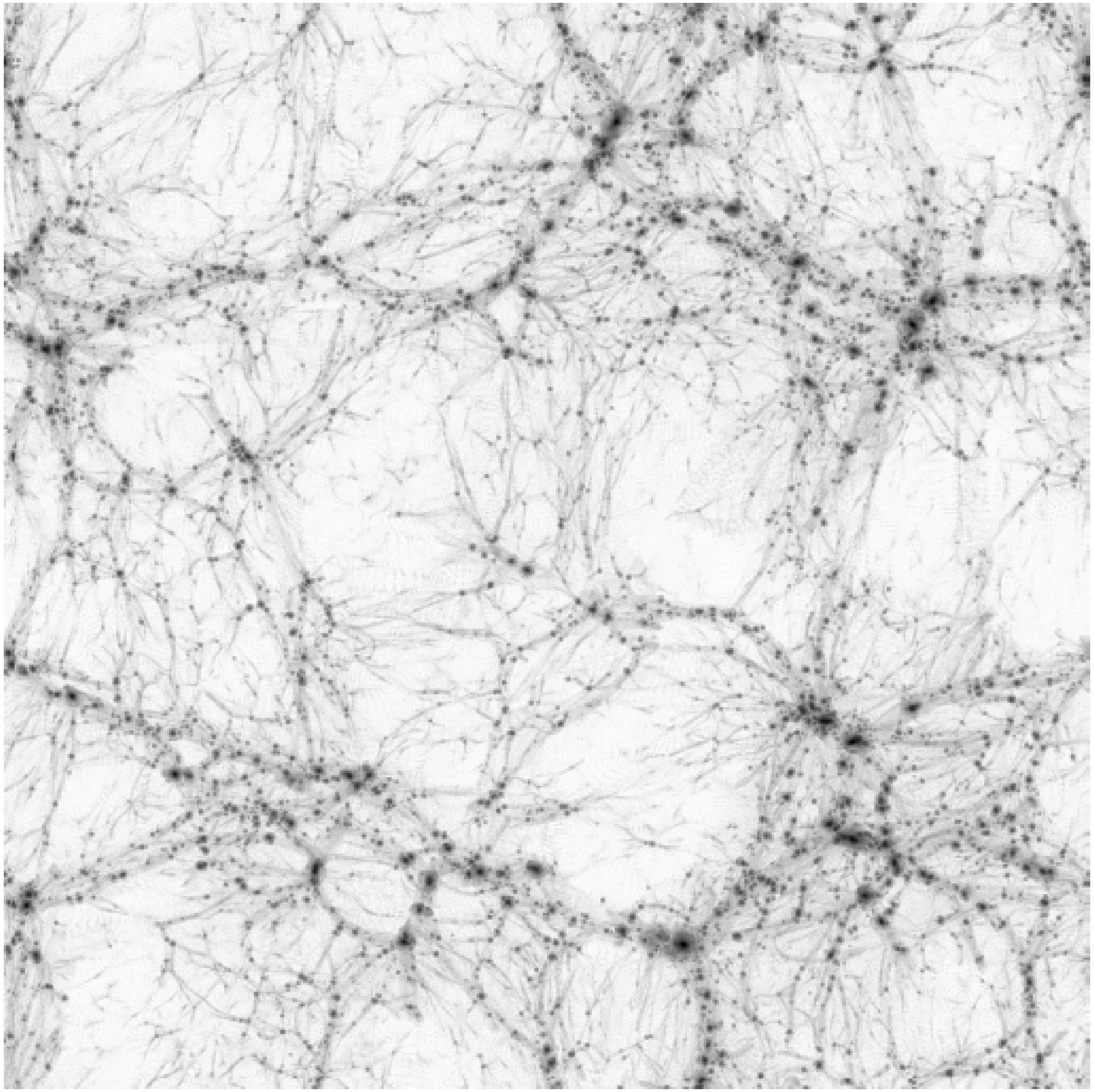
Results: Structure Formation



Density field
Slice through simulation at z=0, size: 64 Mpc/h

GR – ΛCDM

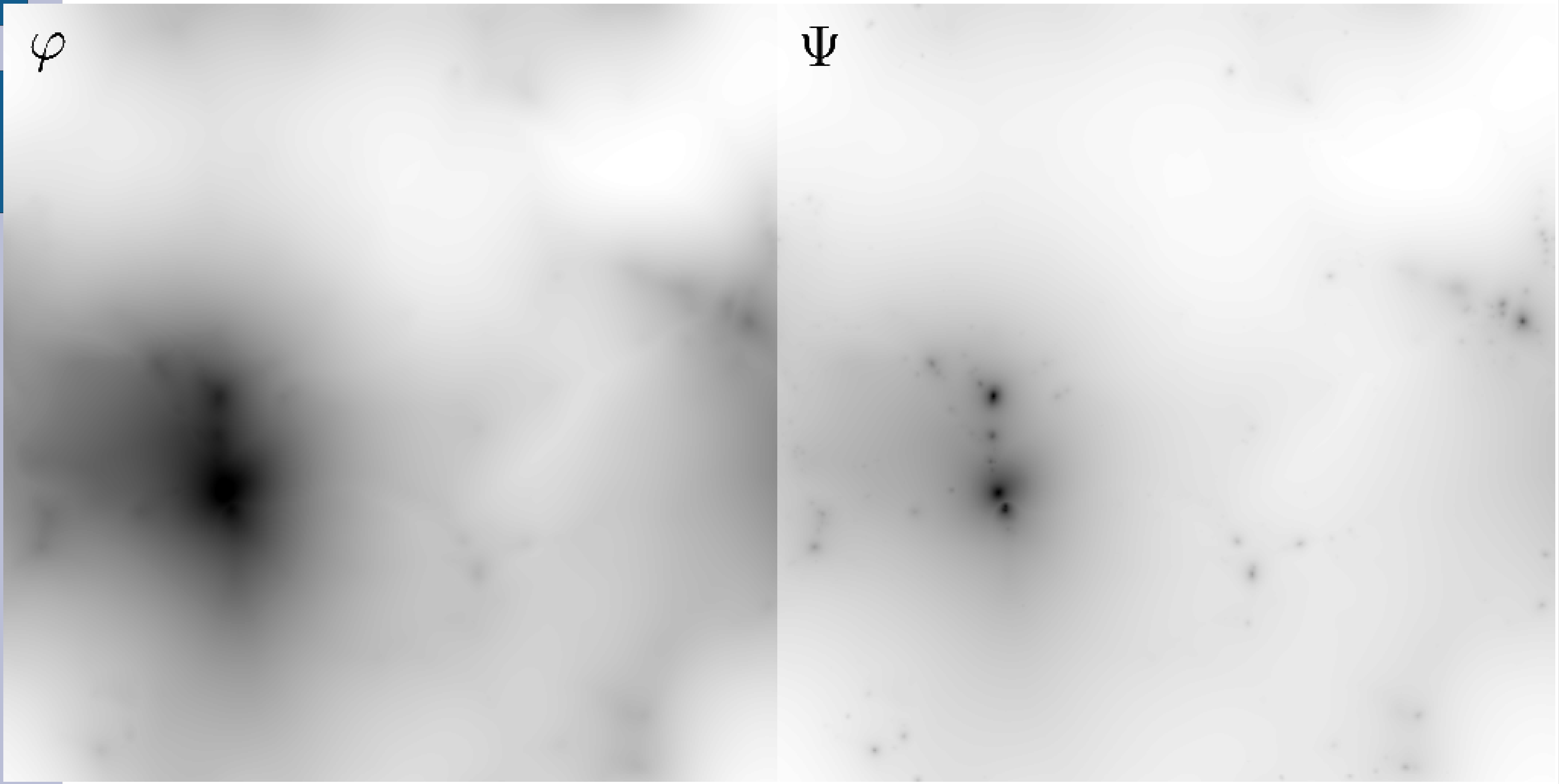
Results: Structure Formation



Density field
Slice through simulation at z=0, size: 64 Mpc/h

DGP normal branch + DE
r_c = 3000 Mpc

Brane-bending mode & Potential



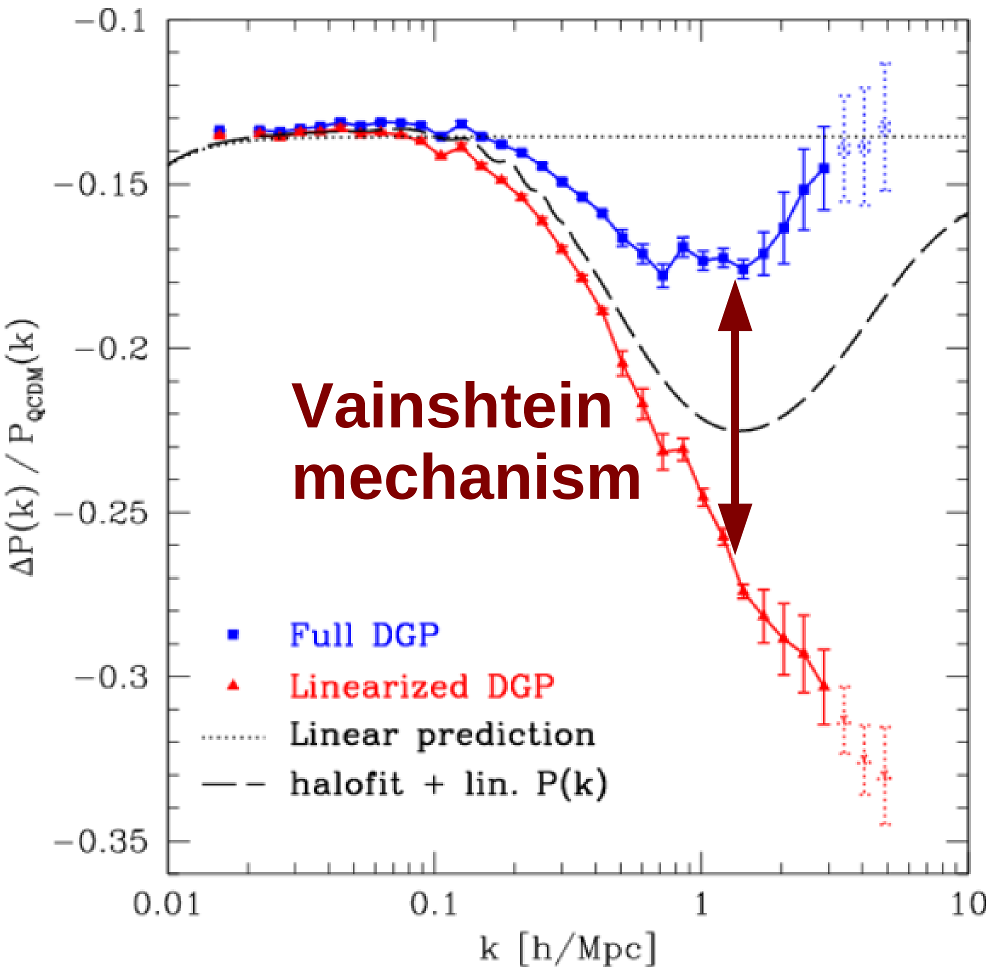
sDGP simulation, 64 Mpc/h box, z=0

Results: Matter Power Spectrum

FS 09a,b

- **Full and linearized DGP vs GR** (z=0)

sDGP



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FS 09a,b

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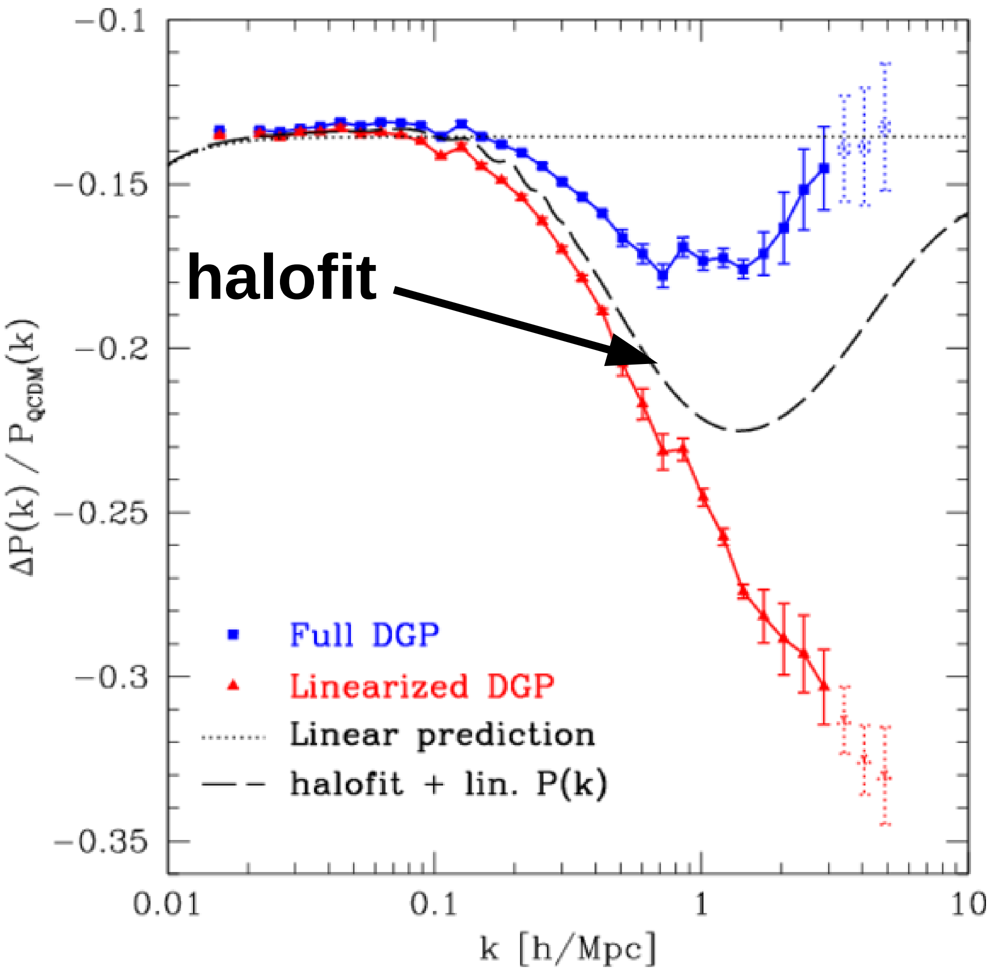

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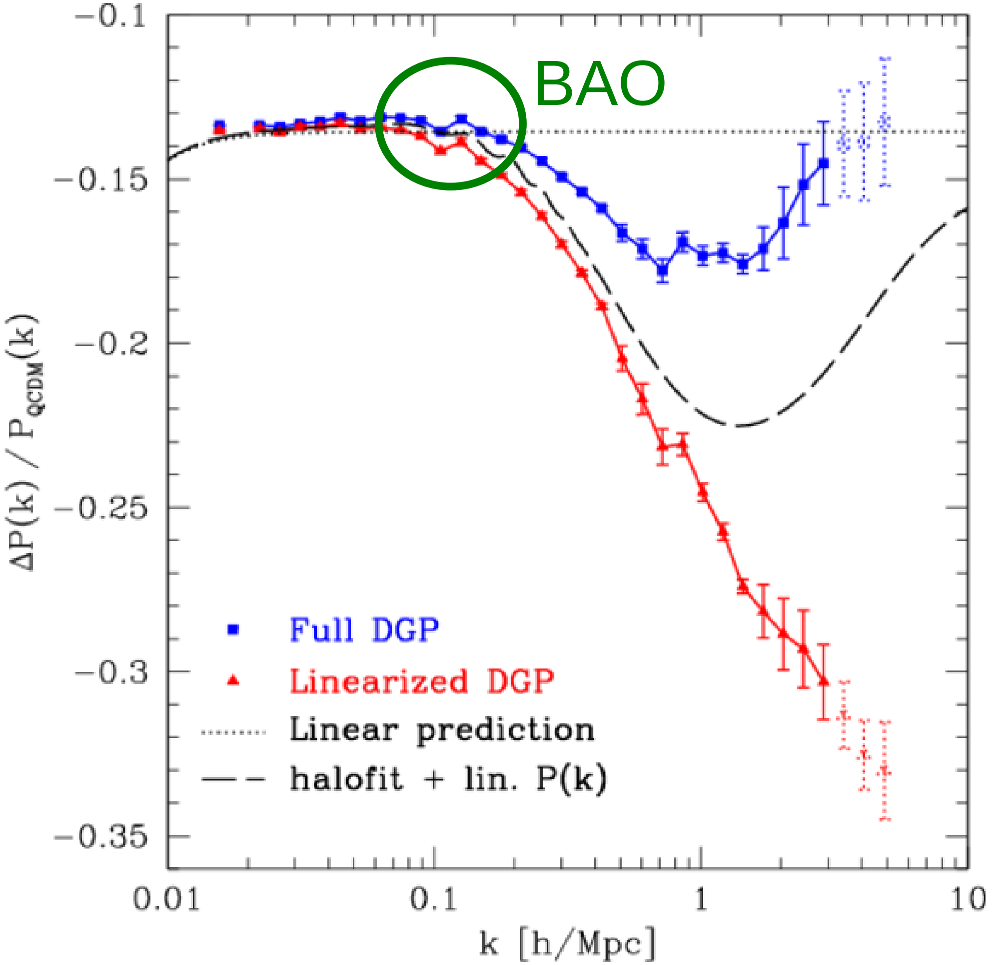
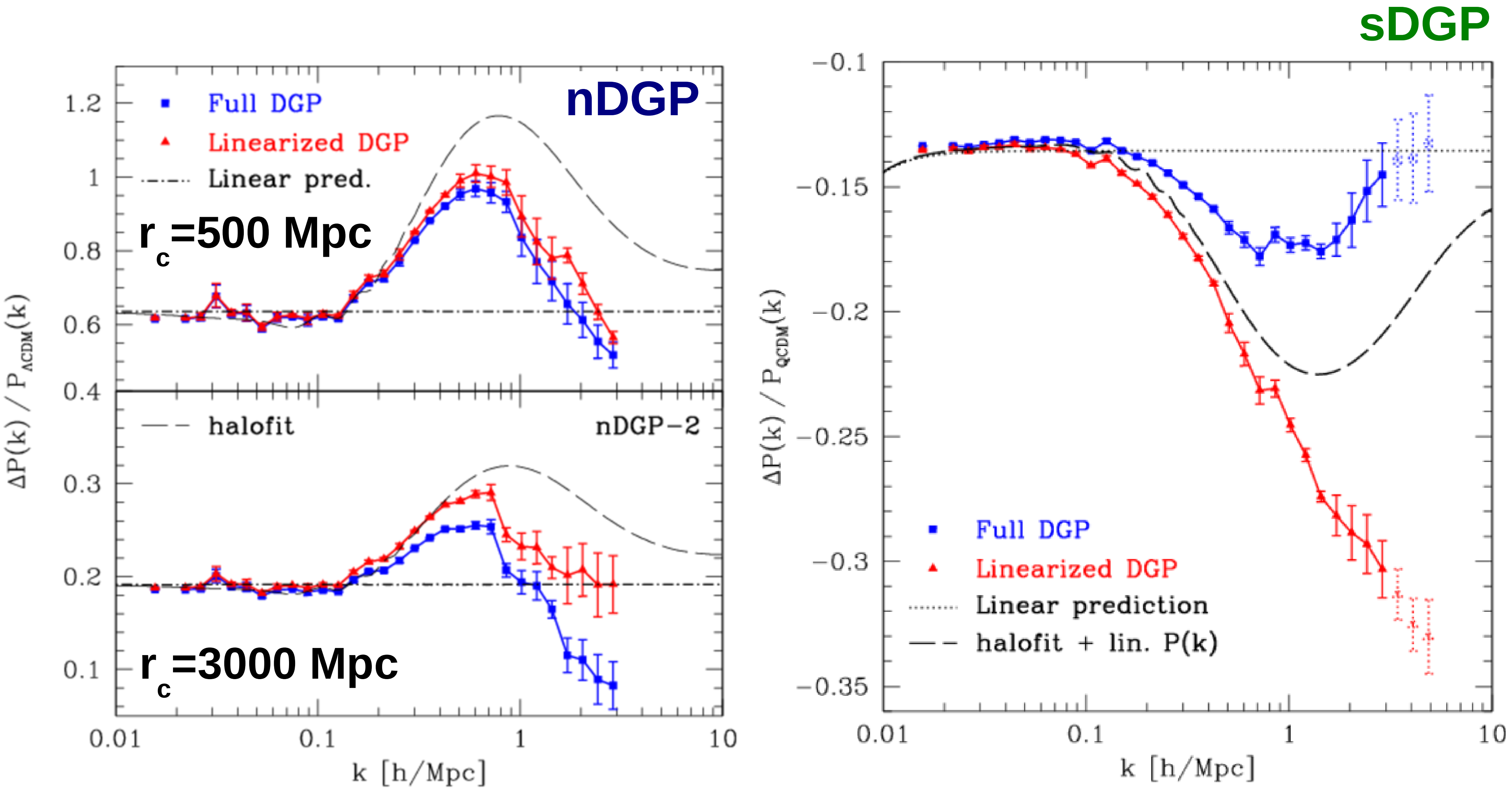


Results: Matter Power Spectrum

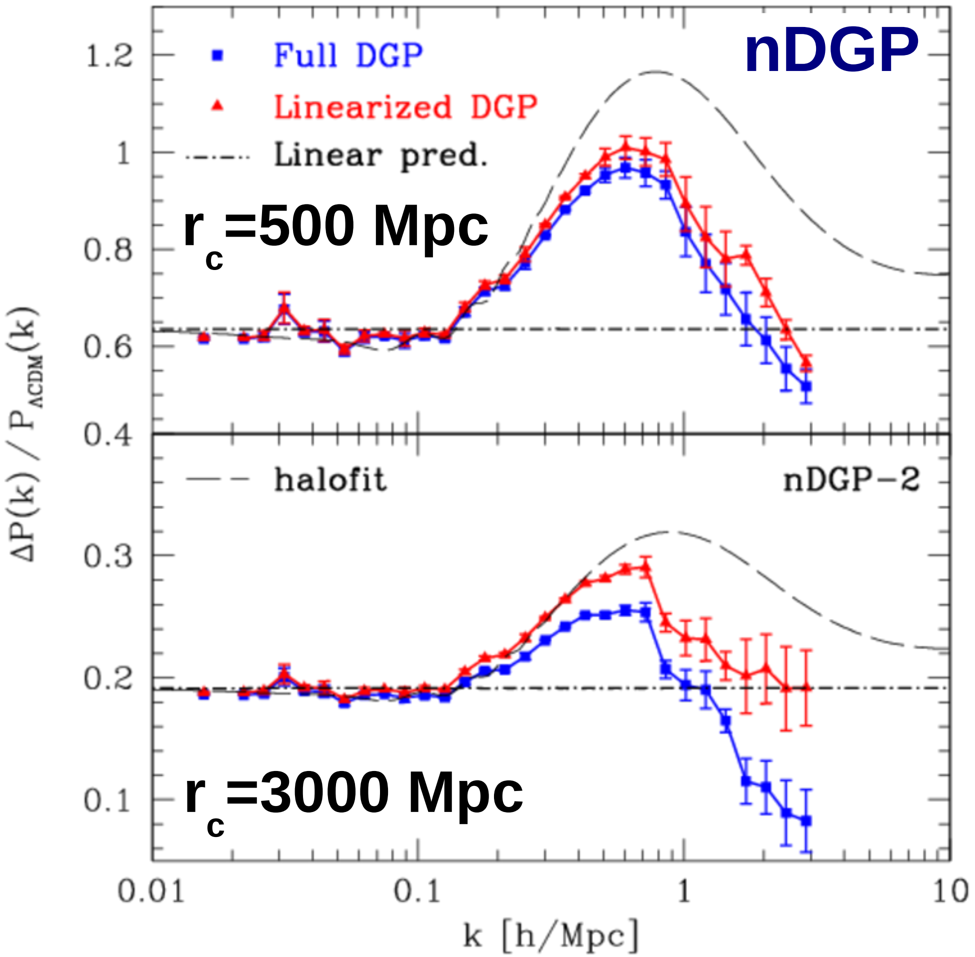
FS 09a,b

- **Full** and **linearized** **DGP vs GR** (z=0)



Results: Matter Power Spectrum

- **Can we model DGP effects without running 300 hr simulations ?**



  - Extend predictions to different cosmological parameter sets
  - Understand physics behind DGP effects

Halo Model of Large-Scale Structure

Goal: map *linear initial density* field to *non-linear large-scale structure* today

Ansatz: all matter in bound ***dark matter halos***

Basic halo properties:

1. Mass function: abundance

2. Halo bias: clustering

3. Halo density profiles: interior matter distribution

Halo Model of Large-Scale Structure

Goal: map *linear initial density* field to *non-linear large-scale structure* today

Ansatz: all matter in bound ***dark matter halos***

Basic halo properties:

1. Mass function

2. Halo bias

Spherical Collapse & Press-Schechter Theory (Sheth-Tormen)

3. Halo density profiles

NFW profile + concentration relation

Press-Schechter Ansatz

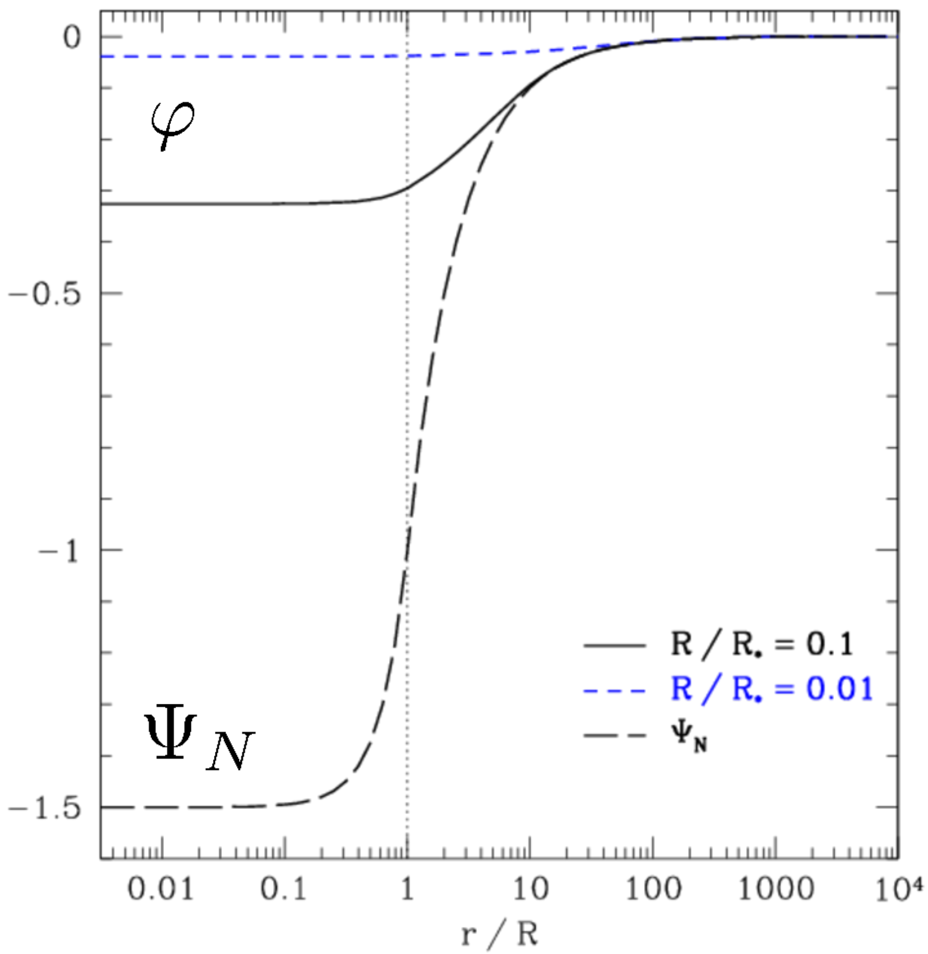
- **Regions in *initial density field* with** $\delta(R) > \delta_c$ **are in collapsed *halos today***
  - $\delta(R)$: density field smoothed at scale R
    $\rightarrow M = 4\pi/3\bar{\rho}R^3$
  - $\mathrm{Var}(\delta(R)) = \sigma^2(M)$ <--- linear P(k)
- δ_c **calculated from *collapse of spherical tophat* perturbation**
- **"Size" of halo set by *virial radius***

Spherical Collapse in DGP

FS, Hu, Lima 09

- **Spherical mass: φ eq. solvable analytically**
  - φ *saturates* within Vainshtein radius

- **Collapse not self-similar in general**

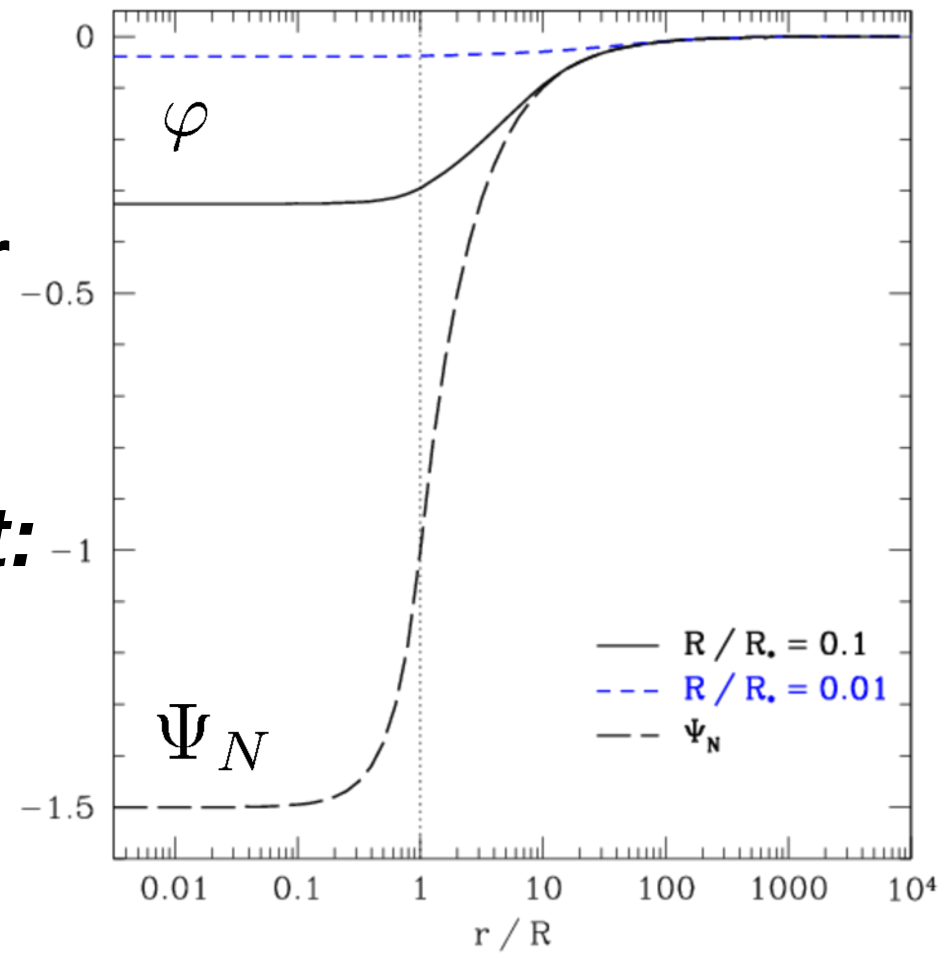


Spherical Collapse in DGP

FS, Hu, Lima 09

- **Spherical mass: φ eq. solvable analytically**
  - φ *saturates* within Vainshtein radius
- **Collapse not self-similar in general**
- **In case of *perfect tophat:***
  - Collapse self-similar
  - Modified force described by

$$G_N \longrightarrow G_{\rm DGP}(\delta, a)$$



Spherical Collapse in DGP

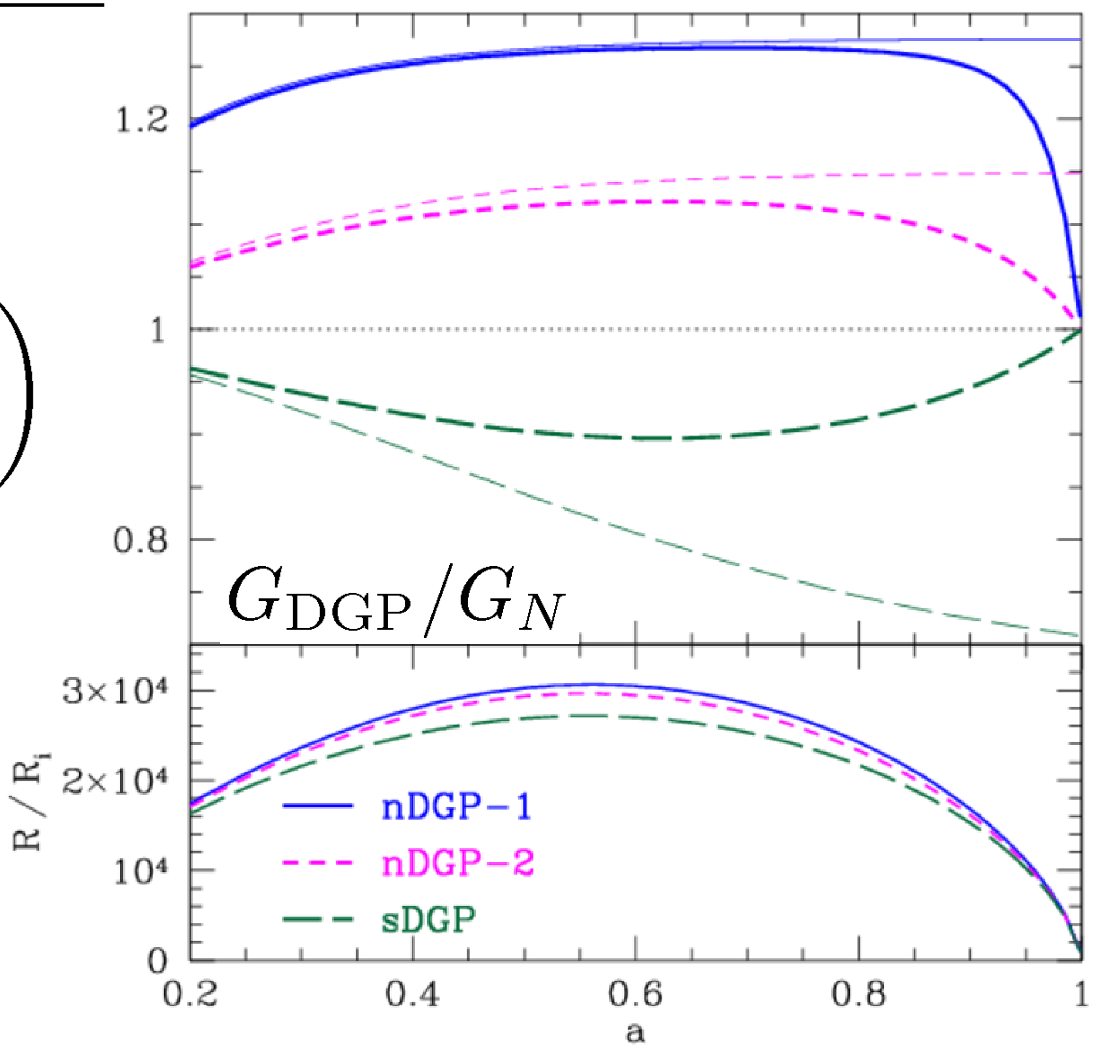
FS, Hu, Lima 09

$$\frac{G_{\rm DGP}}{G_N} = 1 + \frac{2}{3\beta}\,\frac{\sqrt{1+\delta/\delta_*}-1}{\delta/\delta_*}$$

- **Linear limit:** $\delta \ll \delta_*$

$$G_{\rm DGP} \to G_N \left(1 + \frac{1}{3\beta}\right)$$

$$\delta_* \sim (H_0\, r_c)^{-2}$$



Spherical Collapse in DGP

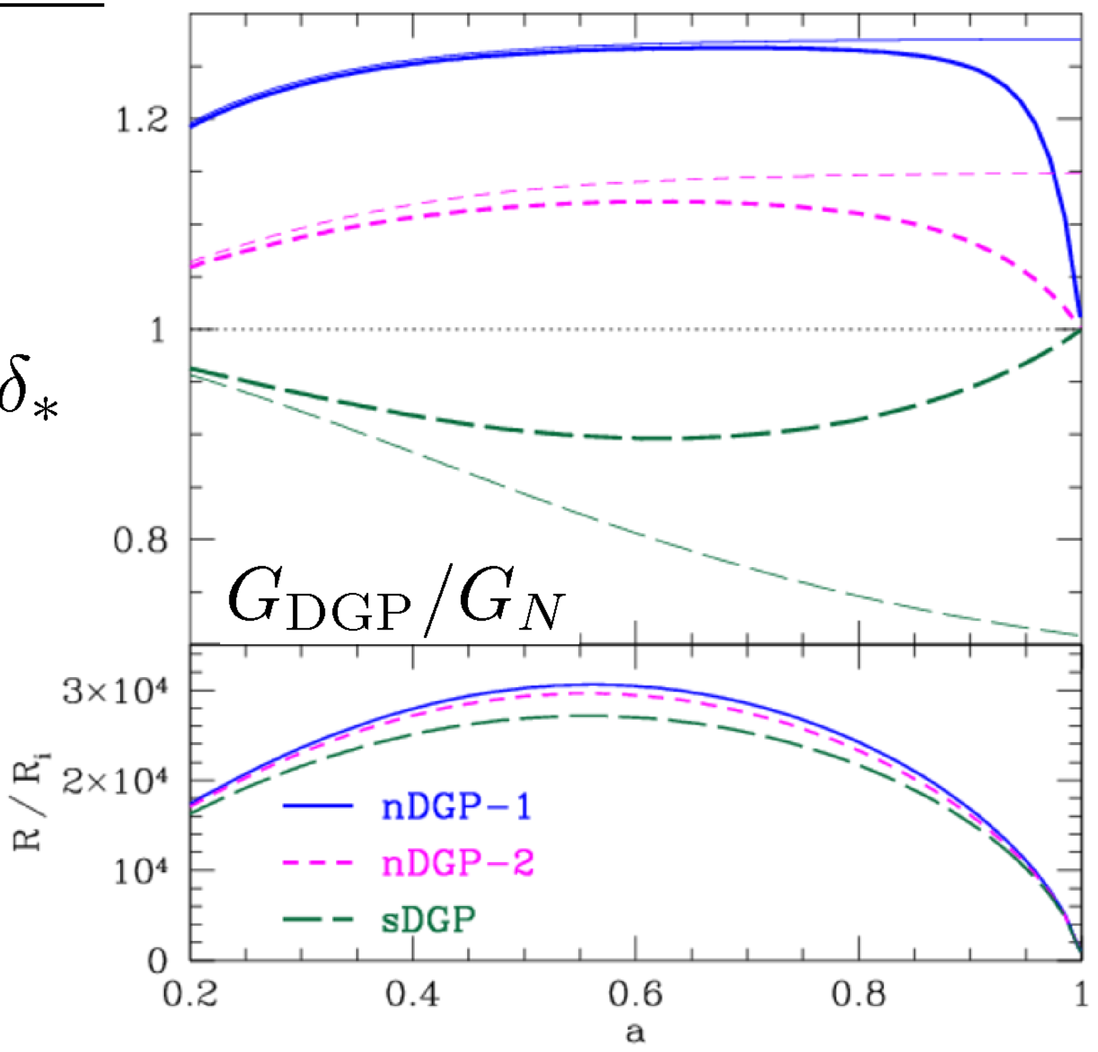
FS, Hu, Lima 09

$$\frac{G_{\rm DGP}}{G_N} = 1 + \frac{2}{3\beta}\frac{\sqrt{1+\delta/\delta_*}-1}{\delta/\delta_*}$$

- **Vainshtein limit:**

$$G_{\rm DGP} \to G_N \quad \text{for} \quad \delta \gg \delta_*$$

$$\delta_* \sim (H_0\, r_c)^{-2}$$

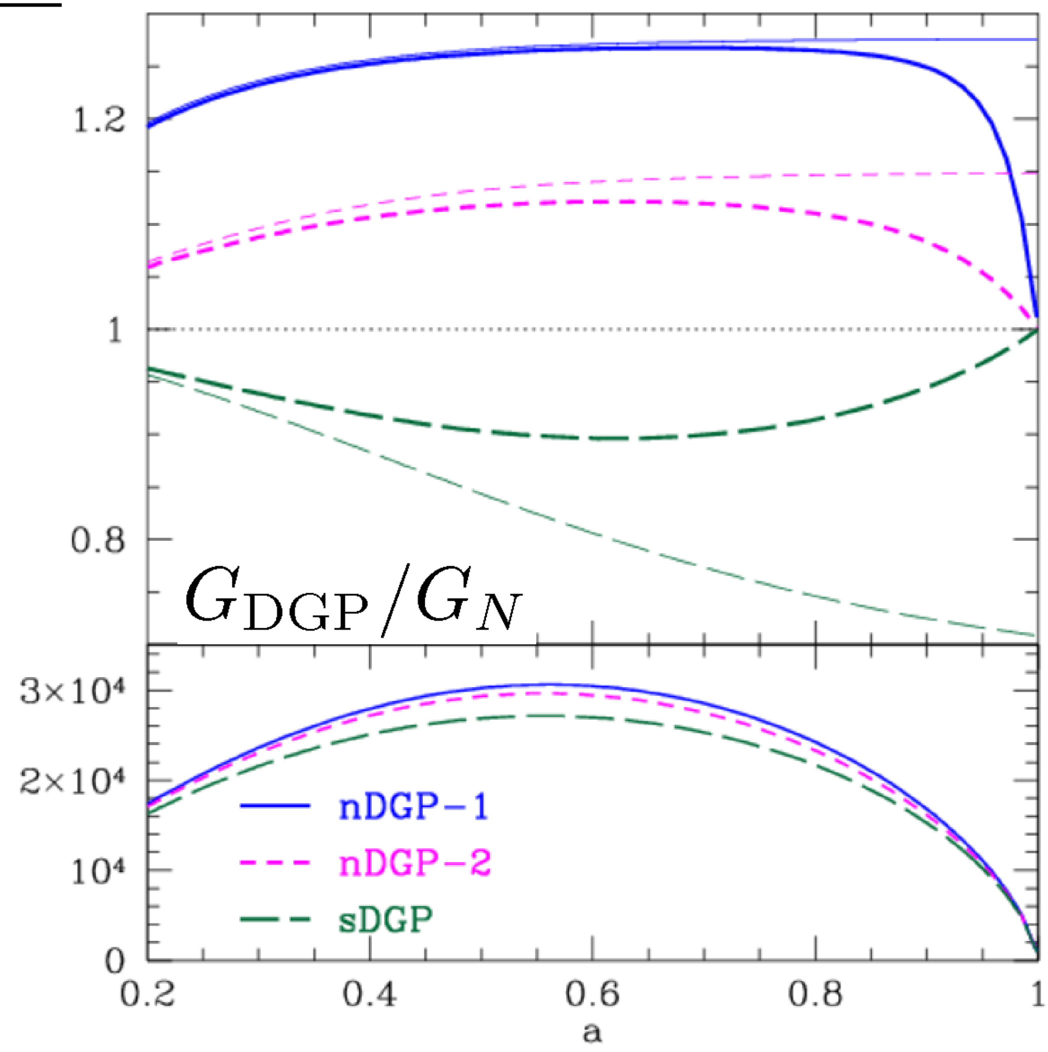


Spherical Collapse in DGP

FS, Hu, Lima 09

$$\frac{G_{\rm DGP}}{G_N} = 1 + \frac{2}{3\beta}\frac{\sqrt{1+\delta/\delta_*}-1}{\delta/\delta_*}$$

- **Calculate δ_c using**
  - $G_{\rm DGP}$ and
  - $G_N\left(1+\dfrac{1}{3\beta}\right)$
  - Limiting cases of non-linear interactions



Virial Theorem

- $\boxed{2T(R_{\rm vir}) + W(R_{\rm vir}) = 0}$ with: $T = \frac{3}{10} M \dot{R}^2$

  Virial radius

  $$W = -\int d^3x \rho\, \boldsymbol{x} \cdot \boldsymbol{\nabla} \Psi$$

- ***Standard* approach:** $E_{\rm tot} = T + U = {\rm const.}$
- **However,** $E_{\rm tot}$ **not conserved if:**
  - Gravitational forces evolve
  - (Effective) dark energy density evolves

- **Applies to DE with** $w \neq -1$ **as well**

Virial Theorem

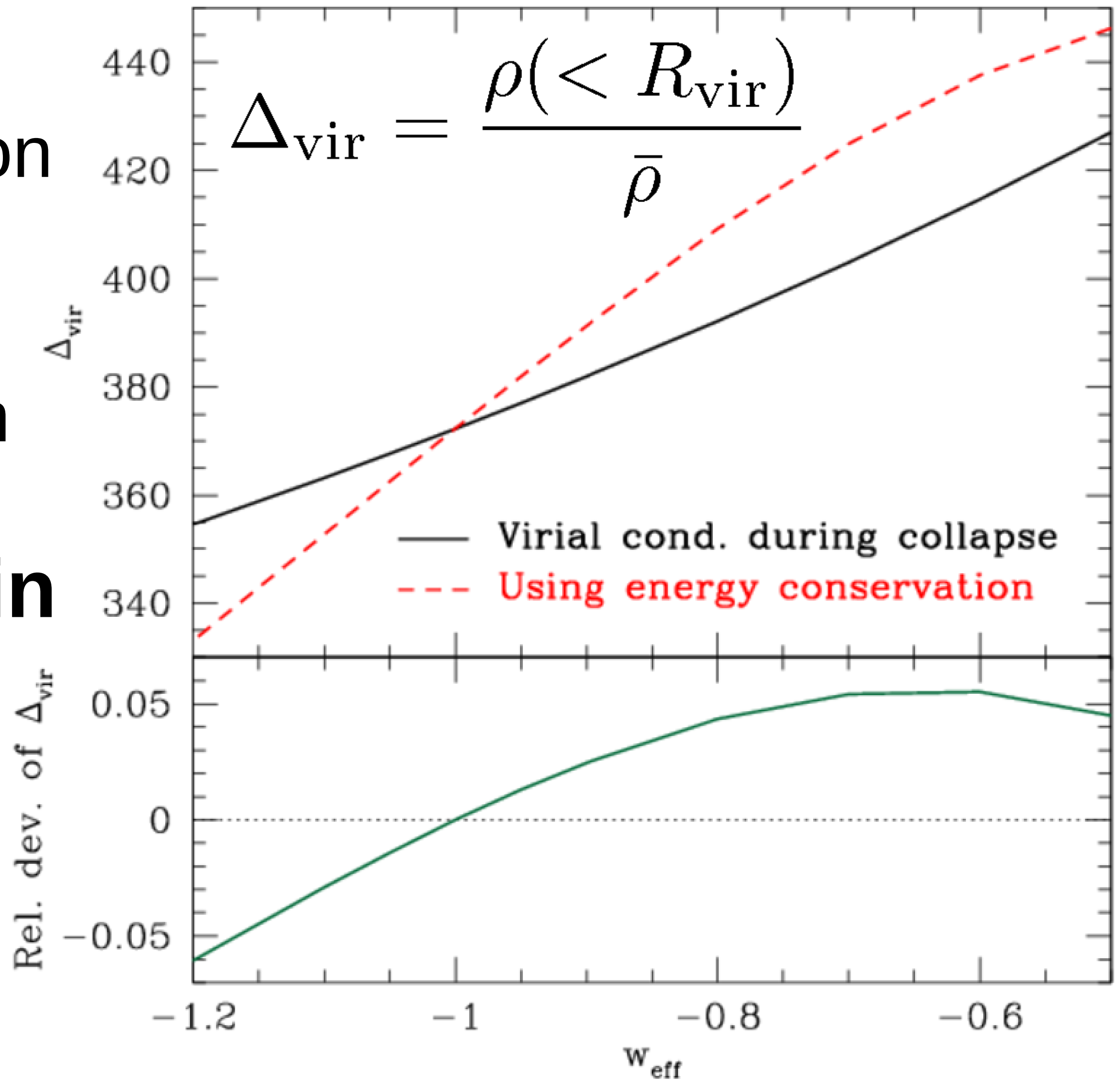
FS, Hu, Lima 09

- **Our approach:**
  - evaluate virial condition *during collapse*
  - does not assume energy conservation

Virial Theorem

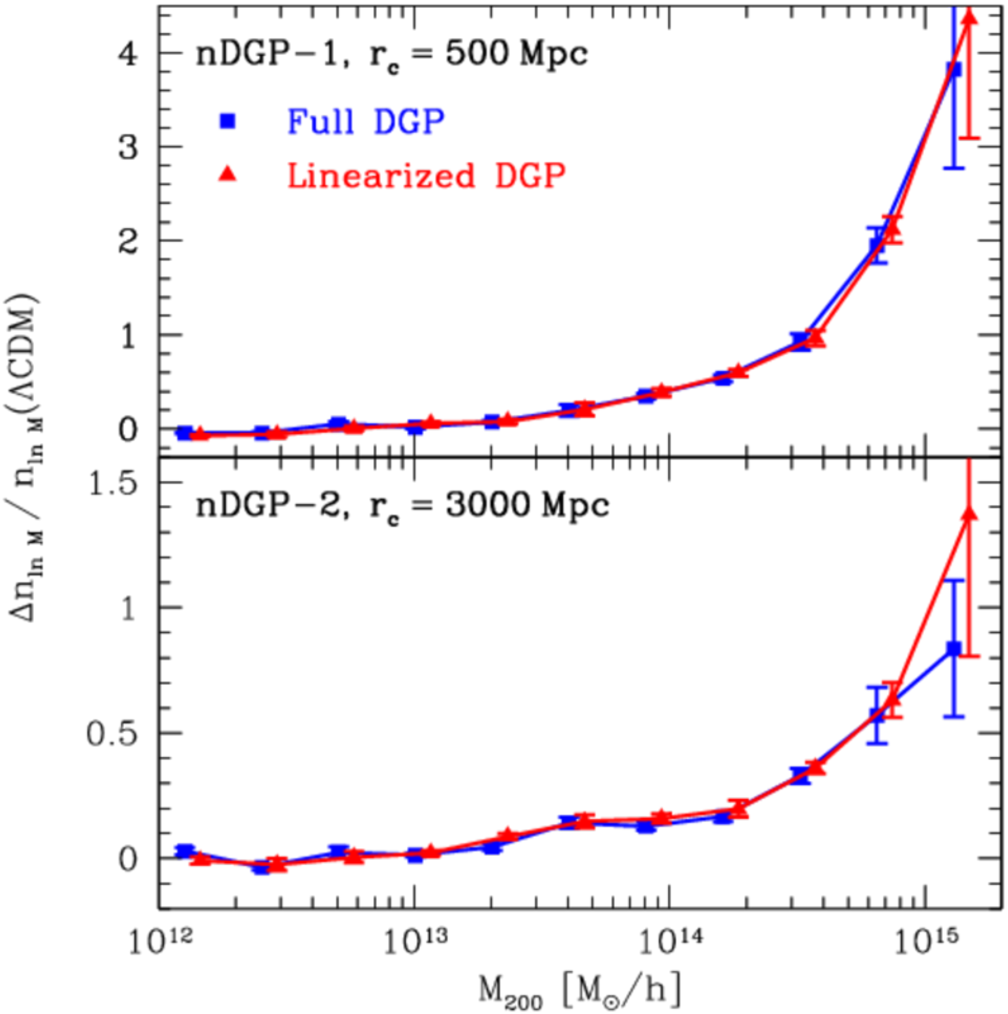
FS, Hu, Lima 09

- **Our approach:**
  - evaluate virial condition *during collapse*
  - does not assume energy conservation
- **Some differences in virial overdensity in quintessence models**



Halo mass function in DGP

Sensitive probe of growth of structure

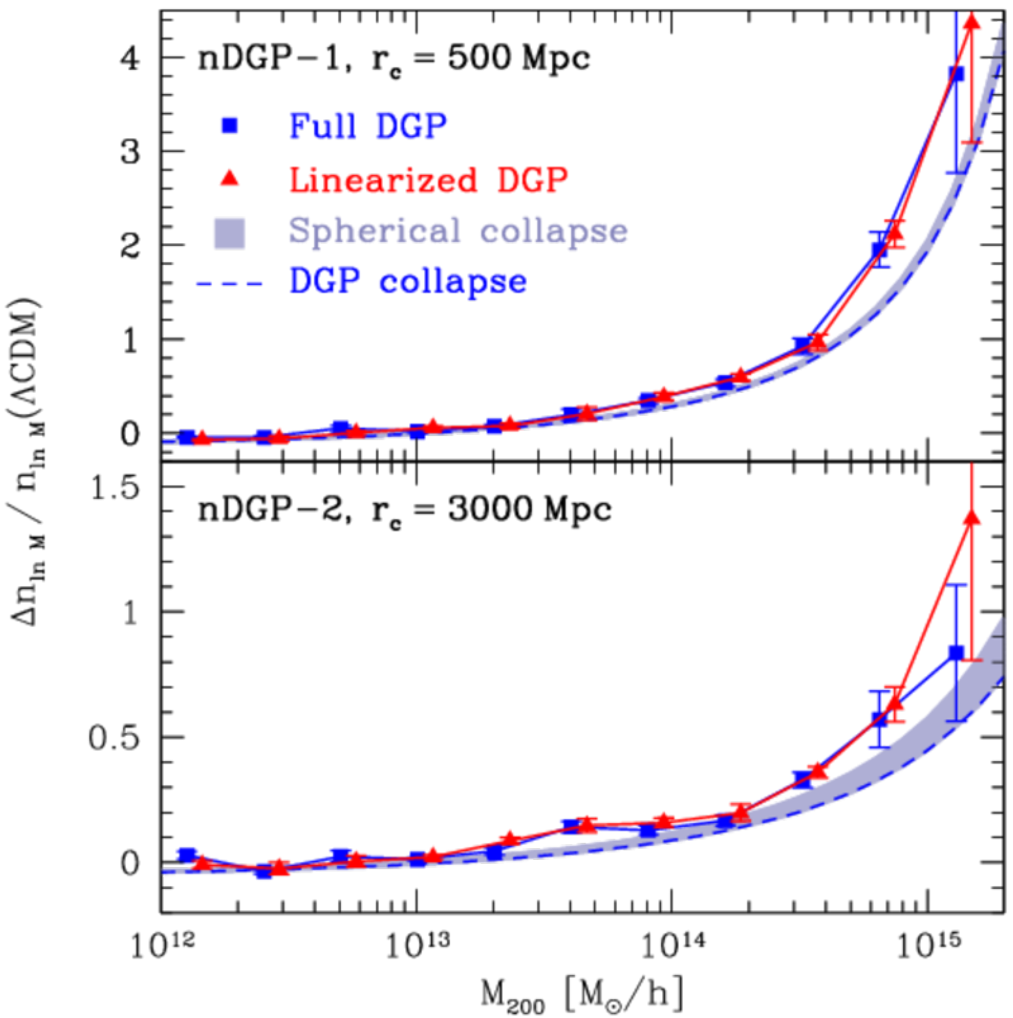


nDGP: relative deviation of dn/dln M from ΛCDM

Order unity enhancement at cluster masses

Halo mass function in DGP

Spherical collapse + Sheth-Tormen mass fct.



nDGP: relative deviation of dn/dln M from ΛCDM

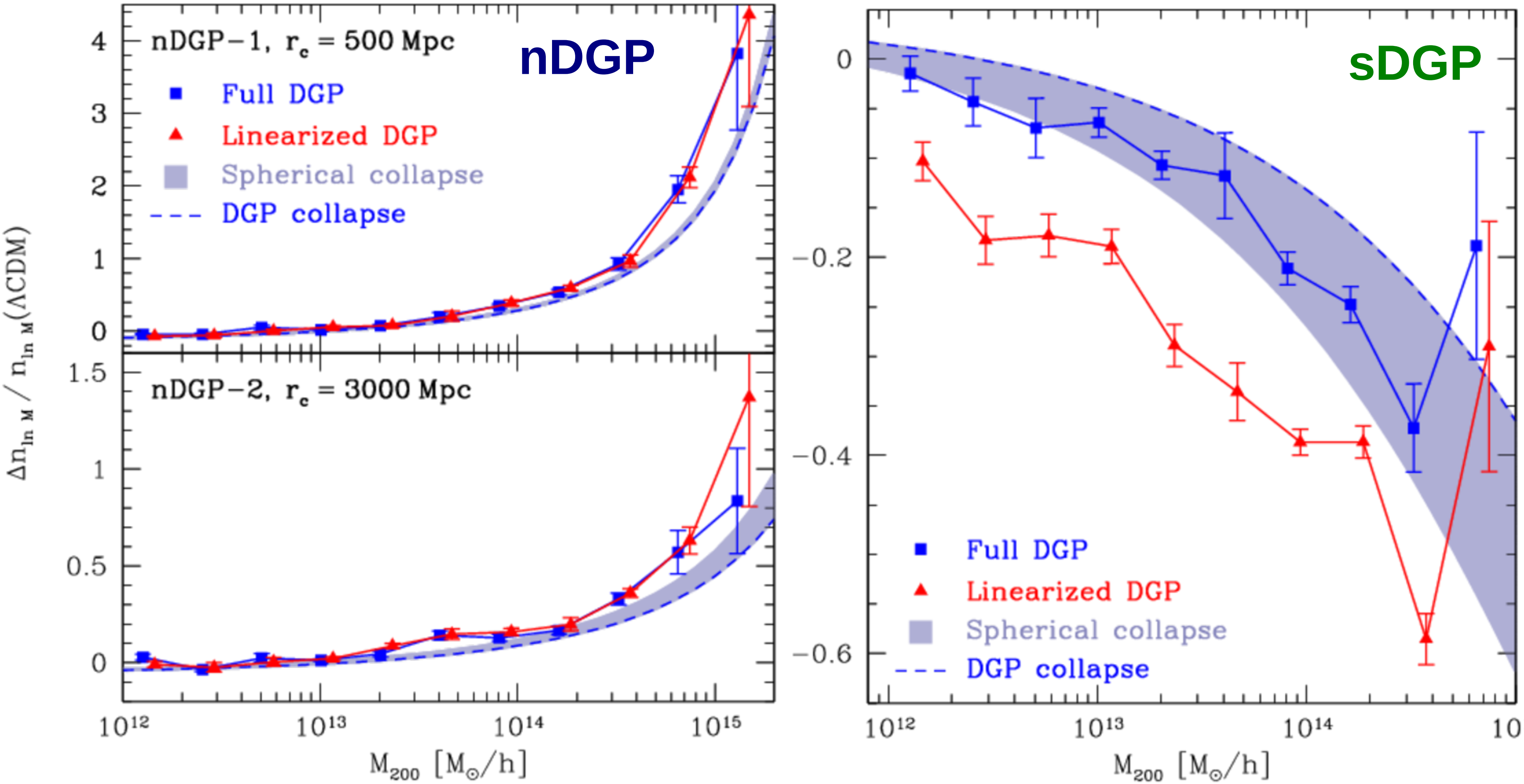
Order unity enhancement at cluster masses ~ 10^{14} M_o

Spherical collapse
- range between "full" and no Vainshtein mechanism

FS, et al. 08,
FS, Hu, Lima 09

Halo mass function in DGP

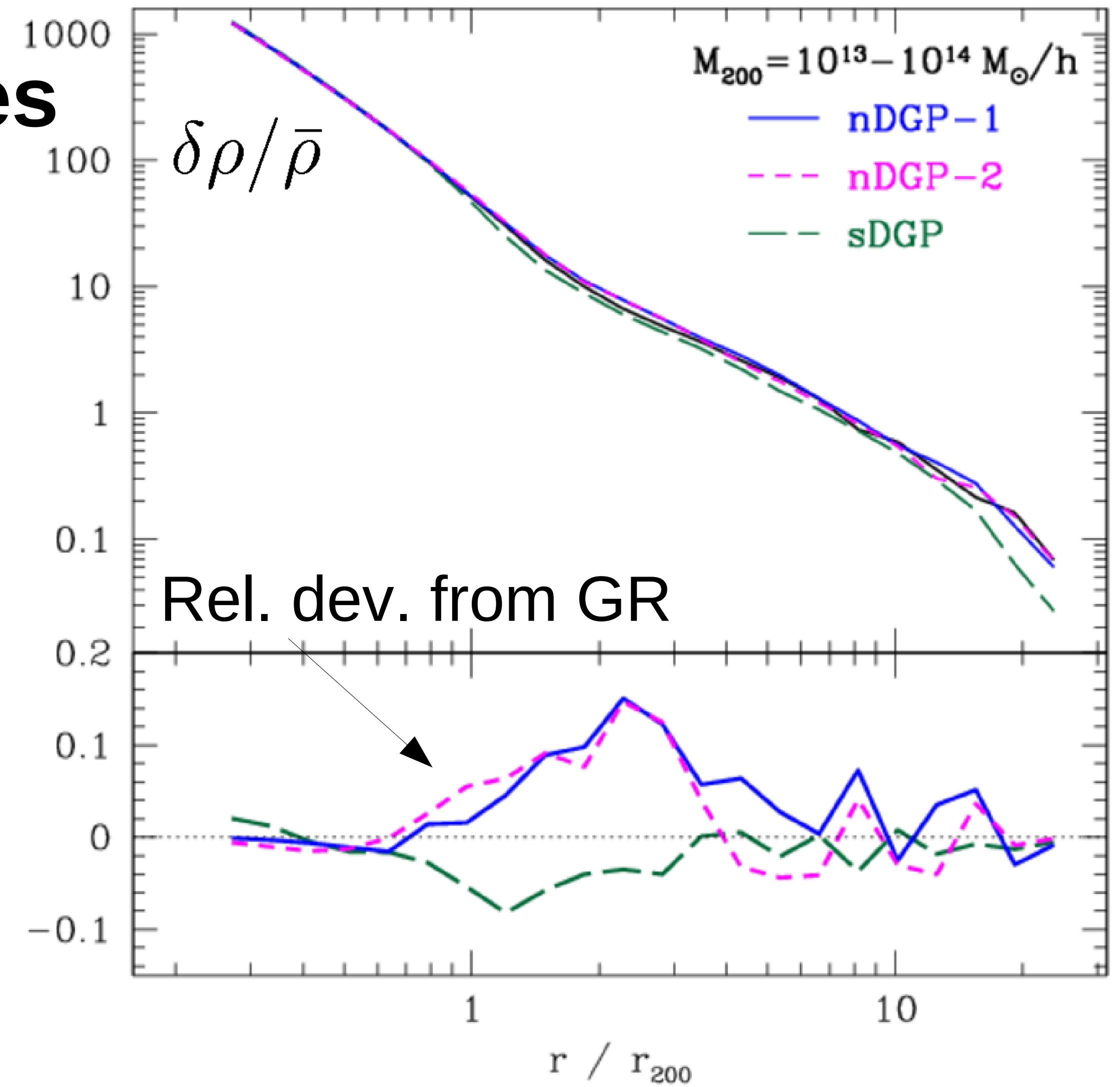
Spherical collapse + Sheth-Tormen mass fct.



Halo Density Profiles

- **No strong DGP effects in inner cores**
  - Scale radius r_s unchanged
  - Cores formed early

- **Some effects in infall region**
  - at few R_{200}



Halo model power spectrum

Halo mass function, bias, + profiles --> P(k)

- $\mathbf{P(\mathit{k}) = P^{2h}(\mathit{k}) + P^{1h}(\mathit{k})}$

2-halo, large scales

1-halo, small scales

- Assume unmodified halo profiles (cf simulations)

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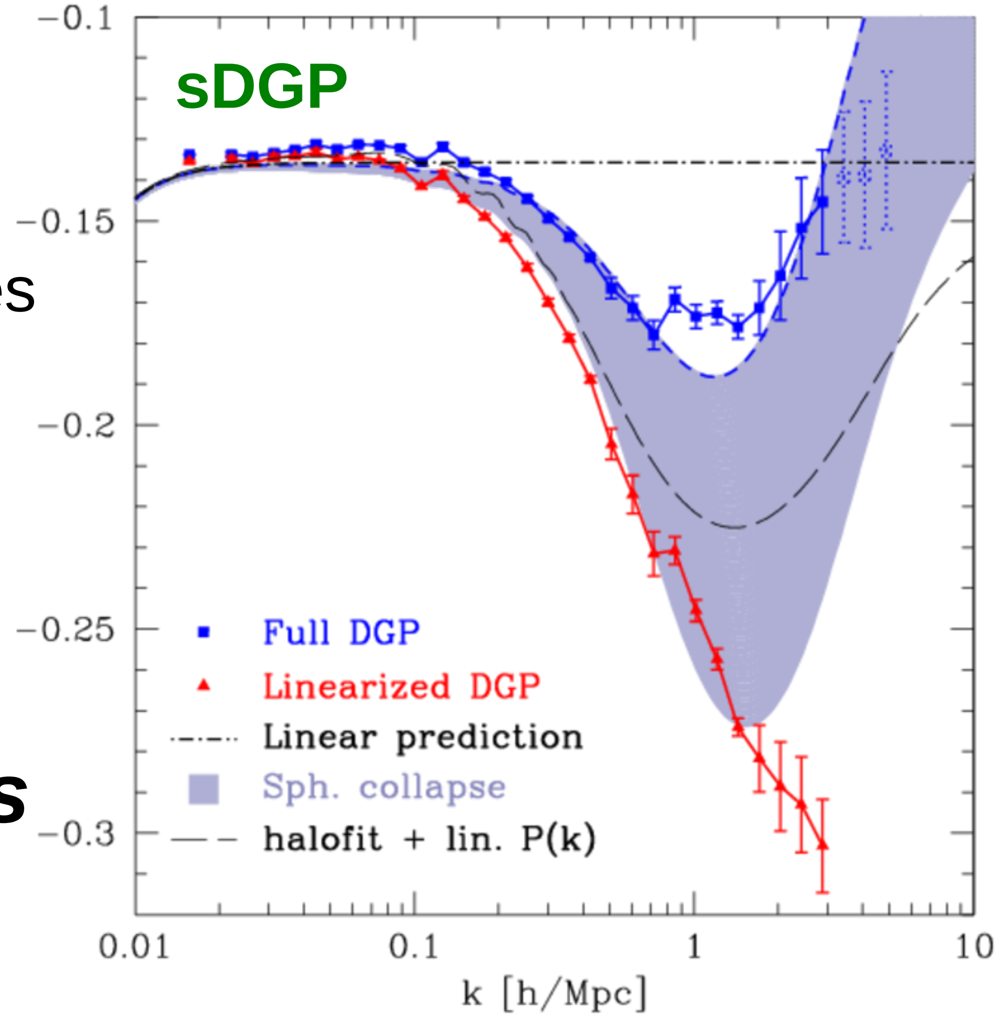
- ***Excellent match to** **full sDGP simulations***



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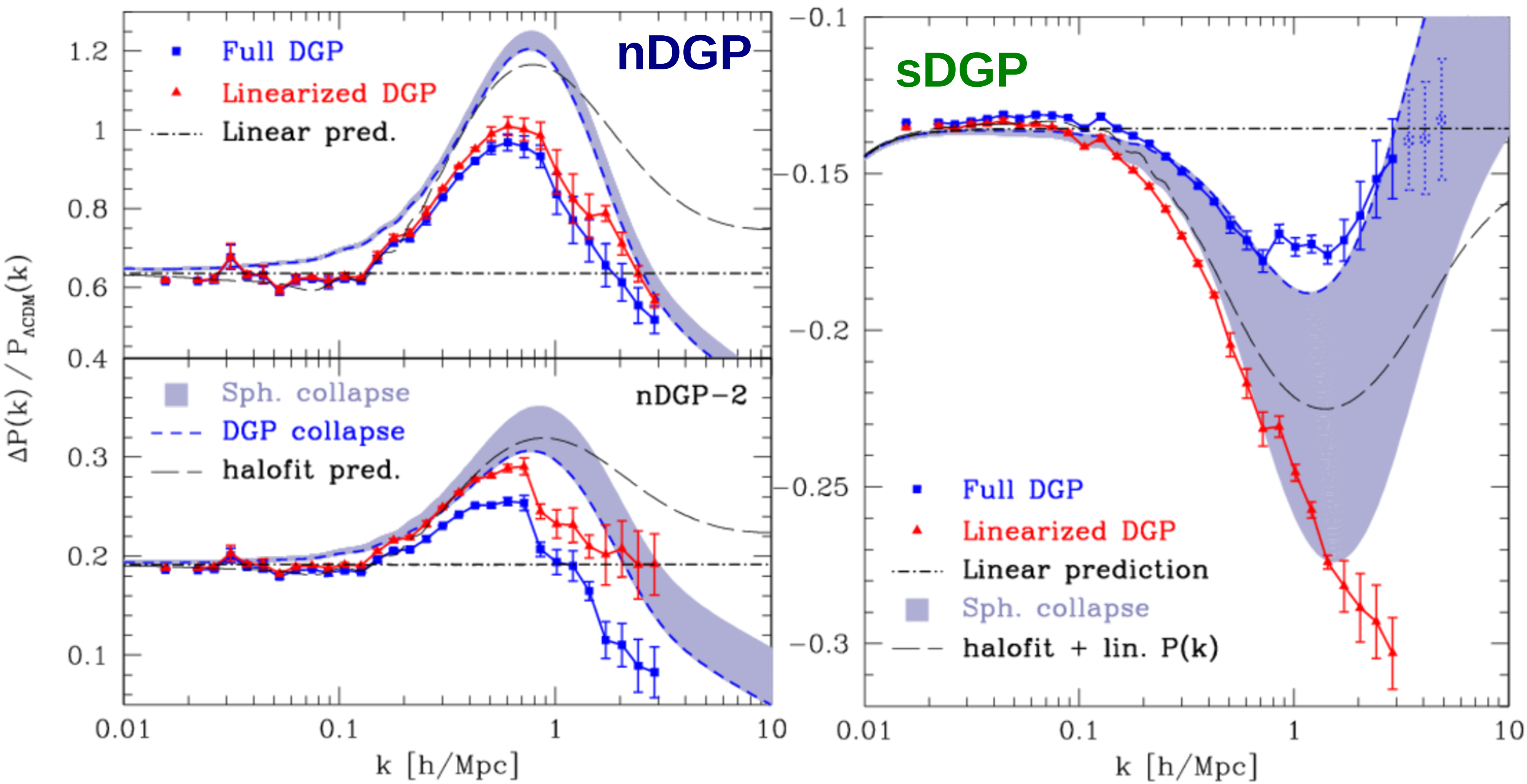
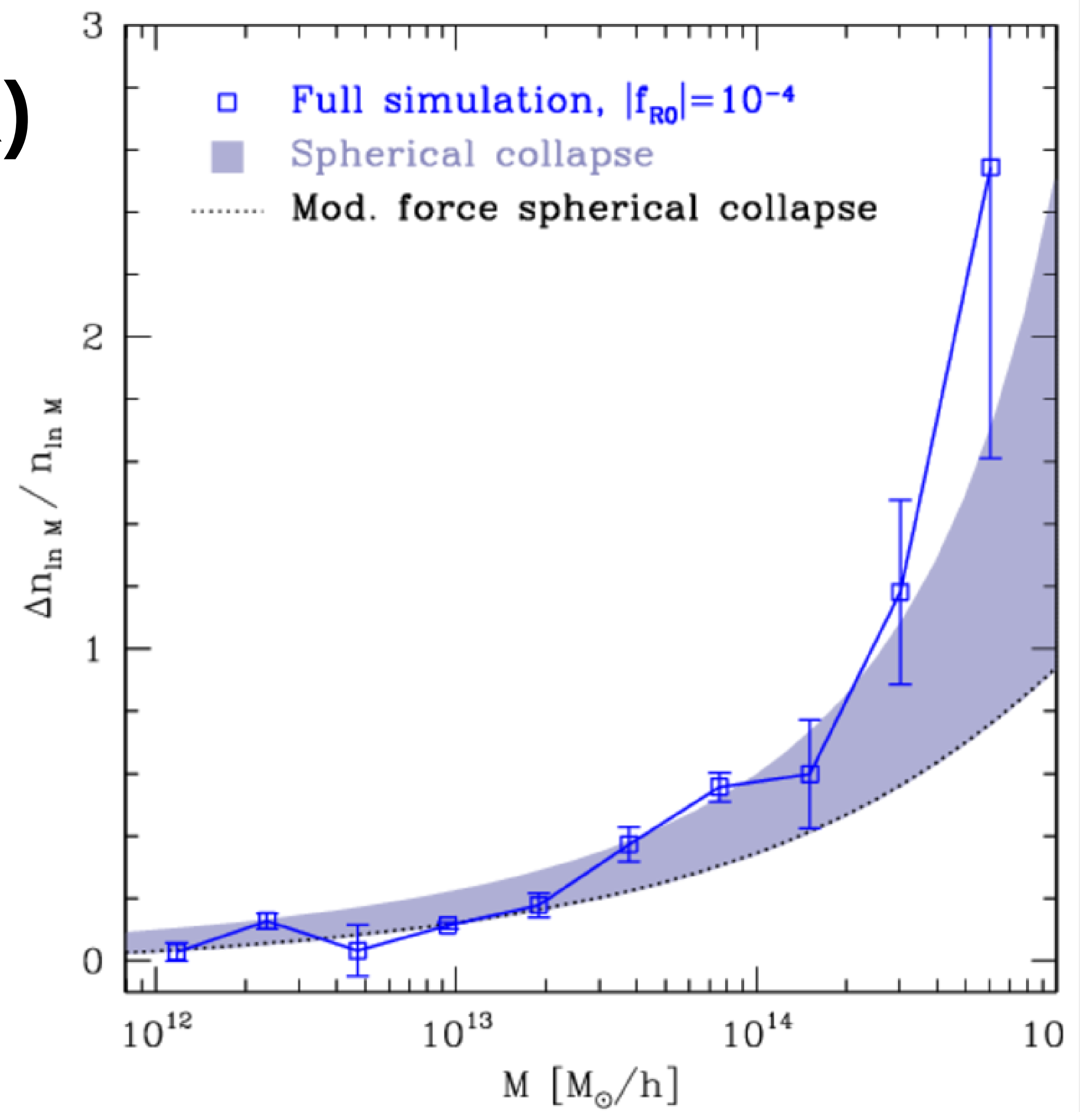


Application: constraining f(R) with Cluster Abundance

- **Mass function enhancement in f(R)**
  - $f_{R0} = 10^{-4}$
- **Spherical collapse predictions**
  - *Conservative:* Lower bound on f(R) effects



FS et al. 08
FS, Vikhlinin, Hu 09

Application: constraining f(R) with Cluster Abundance

- **X-ray clusters**
  - ROSAT survey + Chandra followup
- **Observable: $N(>M_0)$**
- **Treat f(R) effect as effective *σ_8 enhancement***
  - No expensive recomputation of cluster likelihood
  - Neglect information in *shape* of f(R) enhancement
- **CMB constrains primordial normalization**
  - SN, H_0, BAO break parameter degeneracies

FS, Vikhlinin, Hu 09

Application: constraining f(R) with Cluster Abundance

- **Marginalized constraints (95% CL):**

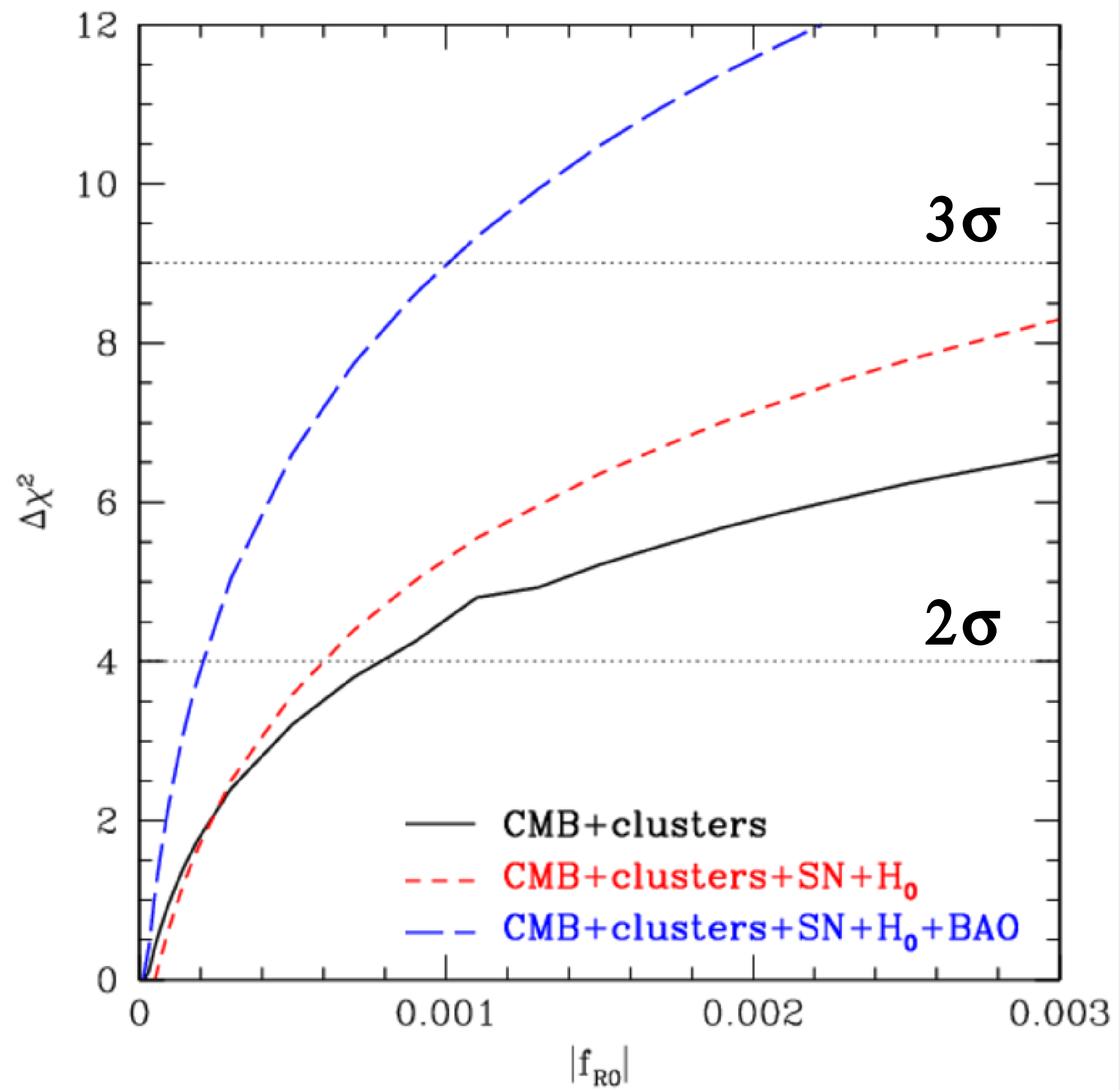
  $$|f_{R0}| < 1.3 \times 10^{-4}$$

  – cf. CMB, SN, g-ISW:

  $$|f_{R0}| < O(0.1)$$

- **Reach of 5th force:**

  $$\lambda_C \lesssim 40\ \mathrm{Mpc}$$



FS, Vikhlinin, Hu 09

Room for Improvement

- **Dominant observational systematic: cluster mass scale**
  - σ(M) = +-9% --> $|f_{R0}| < \mathbf{3 \times 10^{-4}}$ incl. syst.

- **Uses only low-z sample, ~30 clusters**
- **Simple model...**
  - Ignore f(R) effects on dynamical mass (up to 30%)
  - Using "less conservative" collapse parameters:

$$|f_{R0}| \lesssim \mathbf{4 \times 10^{-5}}$$

Conclusions

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- **Simulations of modified gravity are opening the door to probing gravity on Mpc scales**

Conclusions

- **Modified Gravity:** fundamental alternative to *Dark Energy*
- **Understanding structure formation in non-linear regime crucial**
  - prove viability of models and place constraints
- **Simulations of modified gravity are opening the door to probing gravity on Mpc scales**
  - Full simulations of DGP and f(R) now done
  - In progress: understanding non-linear mechanisms generic in modified gravity

Conclusions

- **First self-consistent simulations of DGP model:**
  - Simple *spherical collapse model* seems to capture main effects

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- **First self-consistent simulations of DGP model:**
  - Simple *spherical collapse model* seems to capture main effects
- **Physical *model* + calibration with *simulations* --> observational constraints**
- **Using X-ray clusters, linear regime constraints on f(R) improved by ~1000**

Spherical Collapse in DGP

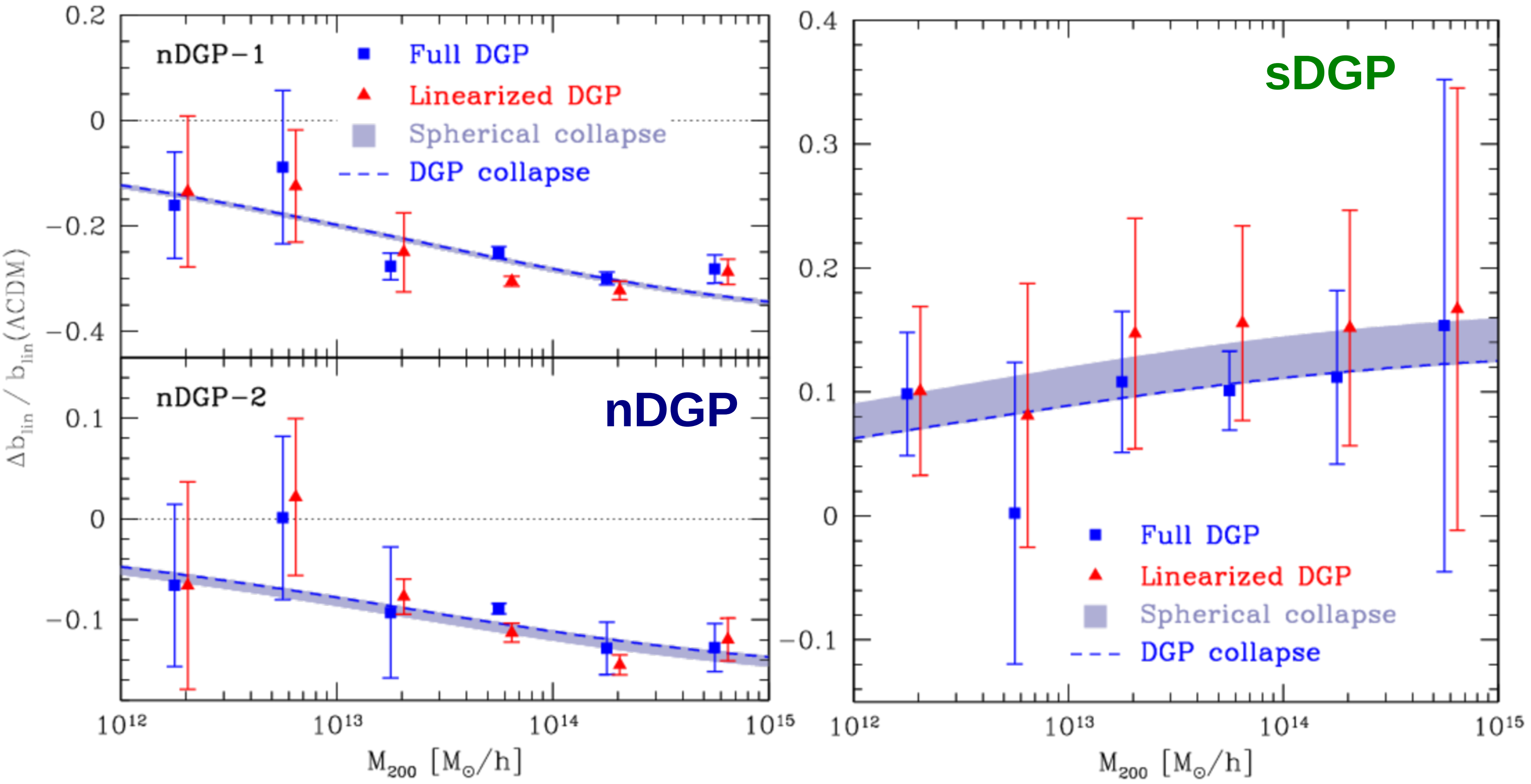
- **Linear collapse threshold δ_c**

	Collapse type/Model:	sDGP	nDGP-1	nDGP-2
δ_c	GR	1.662	1.674	1.674
	DGP	1.627	1.687	1.688
	DGP lin.	1.676	1.678	1.672
Δ_{vir}	GR	399.9	372.3	372.3
	DGP	467.1	300.4	322.8
	DGP lin.	436.4	311.7	339.1

- **Virial overdensity Δ_{vir}**

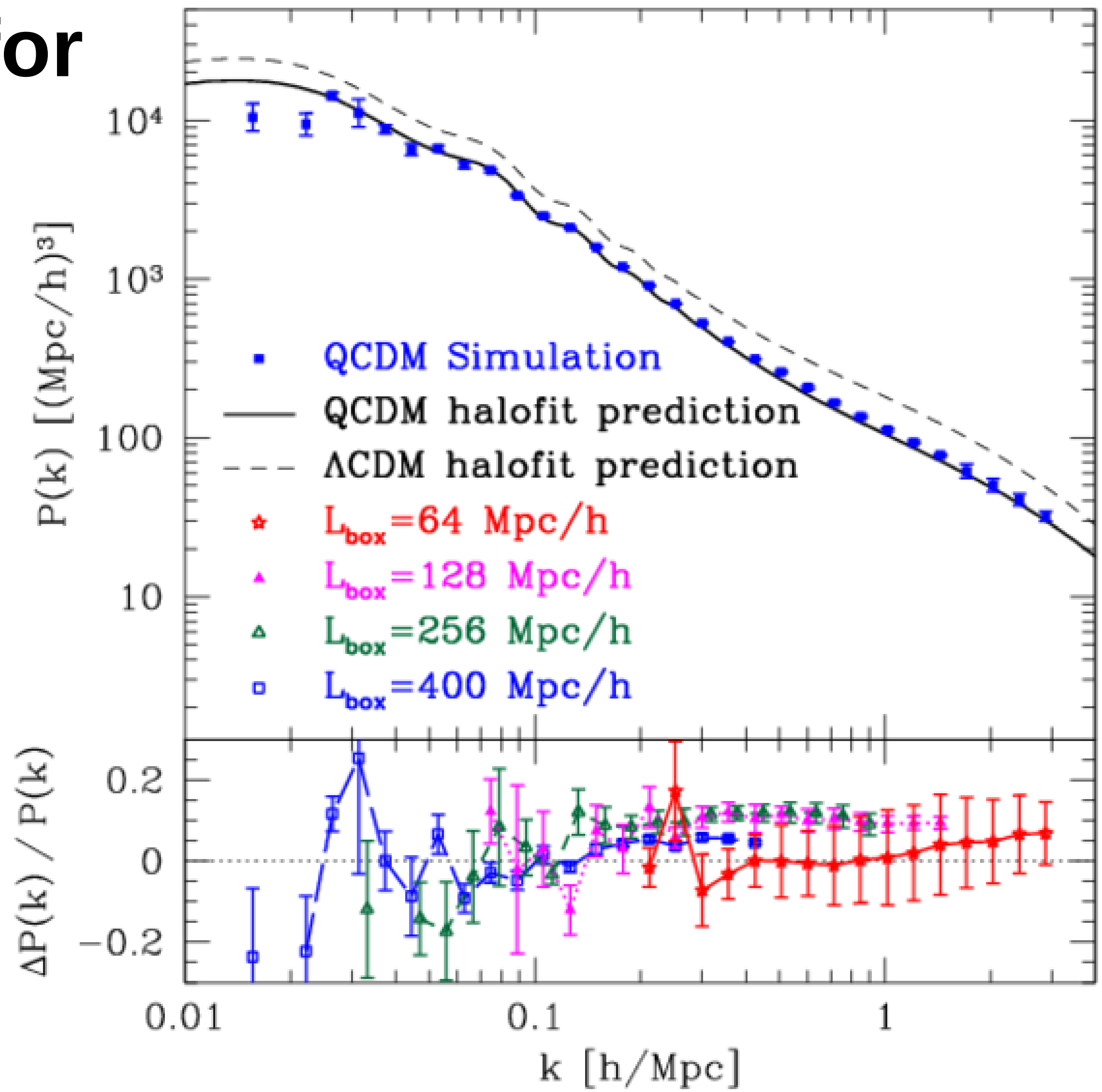
Spherical Collapse + Sheth-Tormen prescription

Halo bias



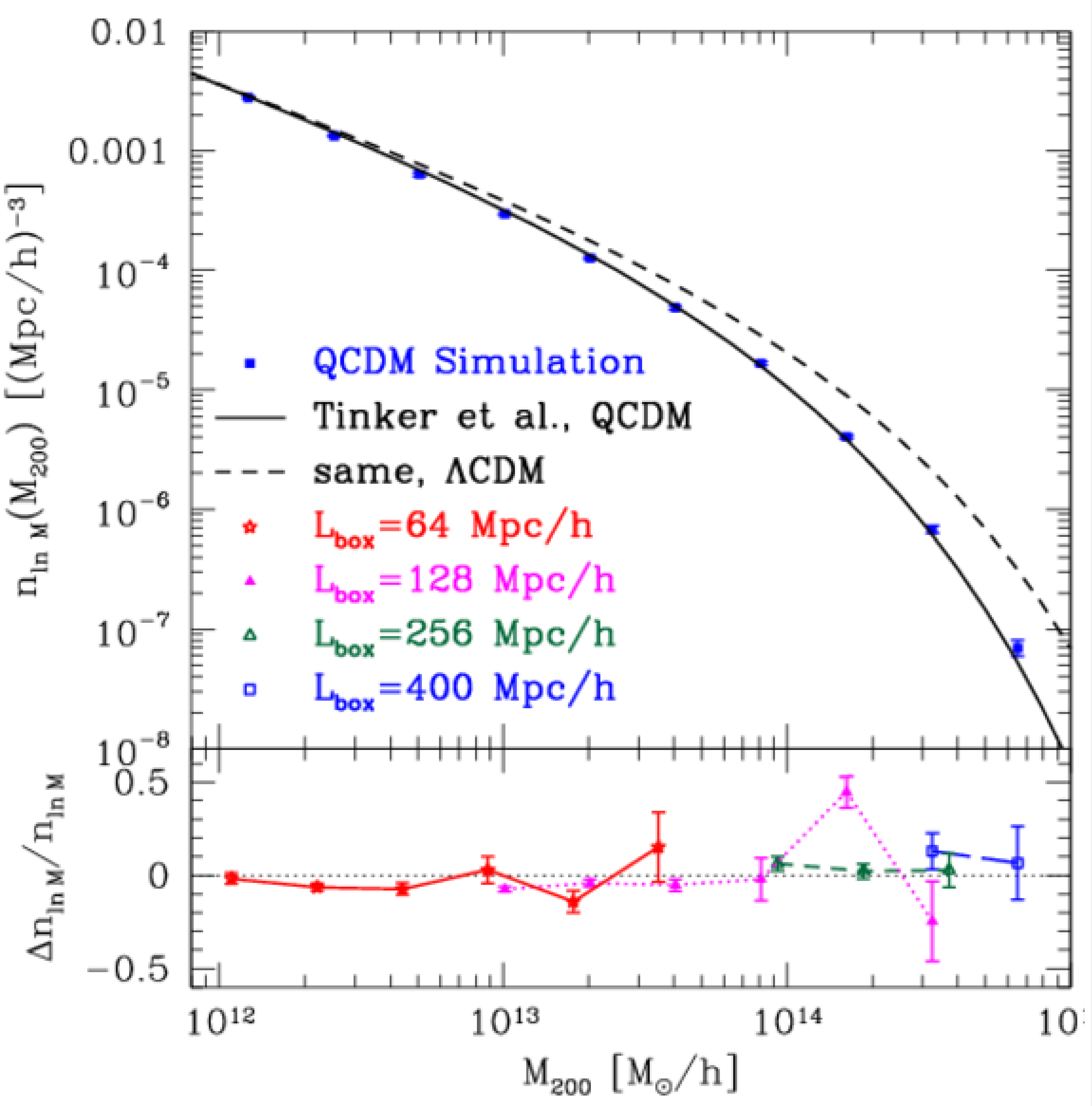
Matter power spectrum

- **Power spectrum for GR (QCDM)**
- **Cross-check with standard fitting formulas**
  - Smith et al.



Halo mass function

- **Mass function for GR (QCDM)**
- **Cross-check with standard fitting formulas**
  - Tinker et al.



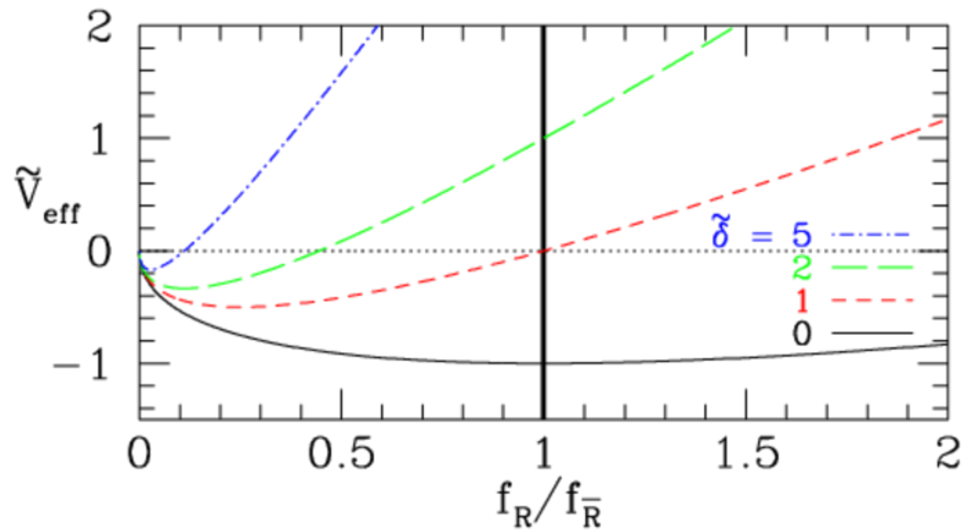
Evading Solar System Tests: Chameleon Mechanism in f(R)

- **Scalar field f_R with density-dependent potential:**

$$\nabla^2 f_R = \frac{\partial \tilde{V}_{\rm eff}(f_R, \rho_m)}{\partial f_R}$$

Khoury & Weltman, PRD, 2004

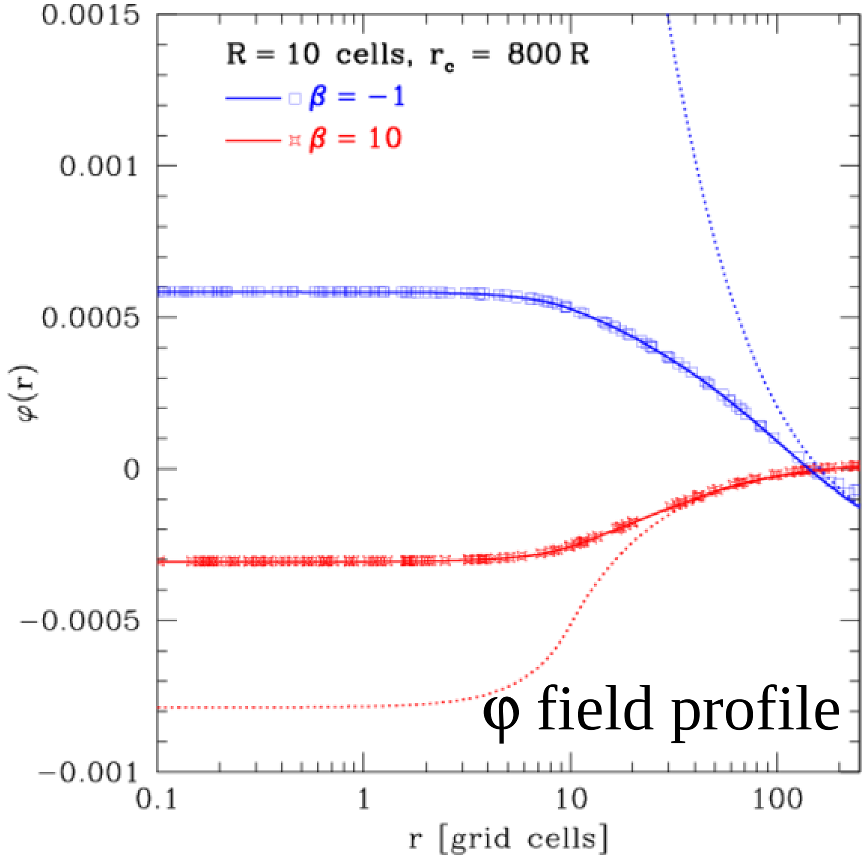
Hu & Sawicki, arXiv:0705.1158

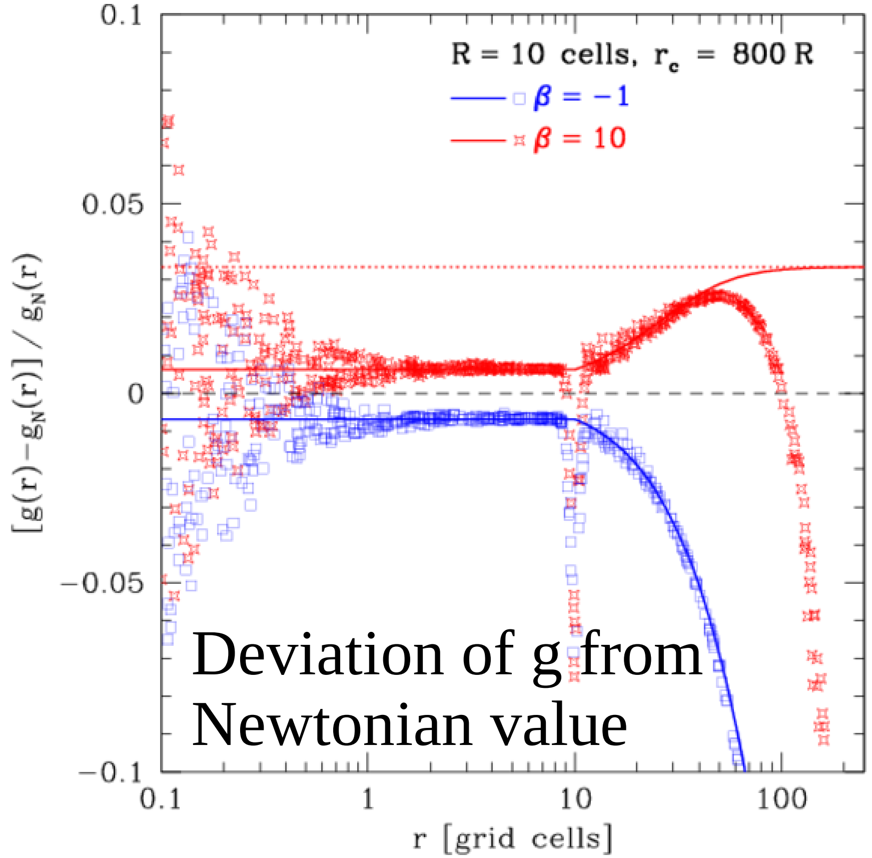


- **GR restored in high-density environments**
  - Chameleon operates when background field small enough: $\overline{f_R} \sim \Psi \lesssim 10^{-5}$

Code Tests

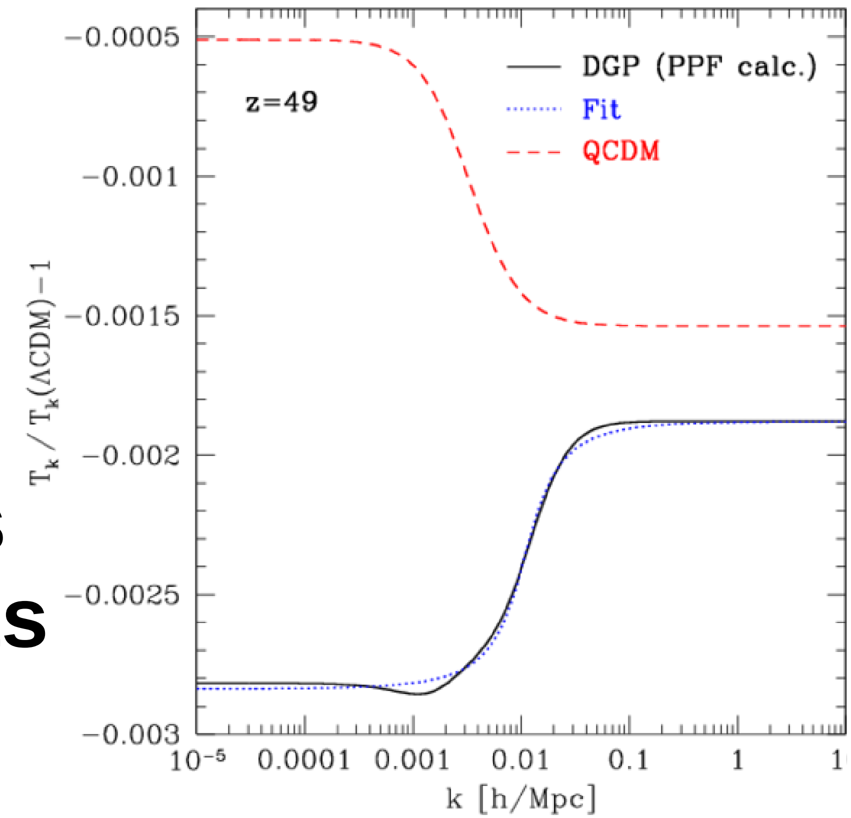
- **Spherical mass (top-hat profile):**
  - Compare with analytical solution





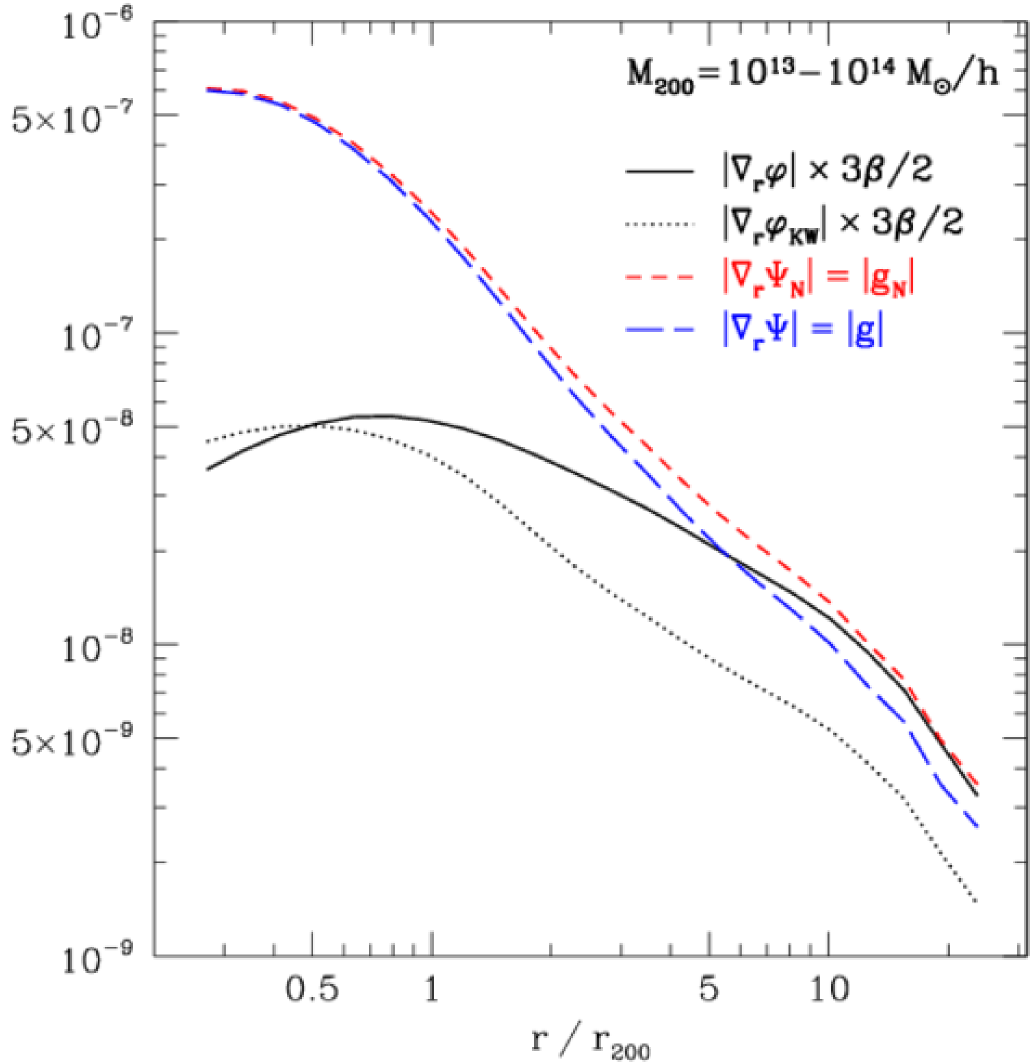
Cosmology: initial conditions

- **At initial redshift (z = 49):**
  - Density: Gaussian random field according to linear power spectrum
  - Generate particle positions, velocities using Zel'dovich approximation



- **Correct initial conditions for early-time DGP effects**

Results: Nonlinear suppression of brane-bending mode



Average halo profiles:

- *brane-bending mode*
- *acceleration* in **DGP** and **GR**

GR is restored inside halos

- Simplified ansatz of Khoury&Wyman only works in inner regions

Constraining f(R) from cluster abundance

- **Constraints marginalized over h, A_s**
  - Degeneracy between Ω_m, f_{R0}