Using the post-Newtonian formalism to understand theories of gravity in cosmology

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T. Clifton and VAAS
Based on **arXiv:** *1803.01157 (today!)*
VAAS and T. Clifton
Based on **arXiv:** *1610.08039, Class. Quan. Grav. 34 (2017) 065003*
VAAS, P. Fleury and T. Clifton
Based on **arXiv:** *1705.02328, JCAP 07 (2017) 028* .

In memory of Jonathon Gleason

1. Parameterizing theories of gravity in cosmology
   - Post-Newtonian formalism
   - Parameterized post-Newtonian Formalism
   - Slip and gravitational constant parameter
   - Parameterized post-Newtonian Cosmology
   - Results
2. Inhomogeneities in cosmology
   - Building a post-Newtonian cosmology
   - Ray-tracing and Hubble diagrams in Post-Newtonian Cosmology
   - Results
3. Summary and Future Work

Post-Newtonian formalism

- In the limit of slow motions ($v \ll c$) and weak gravitational fields ($\Phi \ll 1$), we use an expansion parameter

$$\epsilon \equiv \frac{|\boldsymbol{v}|}{c} \ll 1,$$

where $\boldsymbol{v} = v^{\alpha}$ is the 3-velocity associated with the matter fields.

- The orders of smallness of other quantities are

$$v^2 \sim \frac{\rho}{p} \sim U \sim \Pi \sim \epsilon^2$$

(Will, 1993)

Parameterized post-Newtonian (PPN) Formalism

At leading order the PPN metric is given by

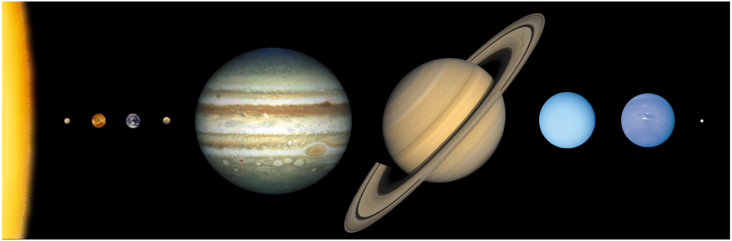
$$ds^2 \equiv (-1 + 2\alpha U)dt^2 + (1 + 2\gamma U)\delta_{\mu\nu}dx^\mu dx^\nu \ ,$$

where

- $U \sim \epsilon^2$ is the Newtonian gravitational potential.
- $\nabla^2 U \equiv -4\pi G \rho_M$.
- We assume U has asymptotically flat solutions.
- α and γ are constant PPN parameters.

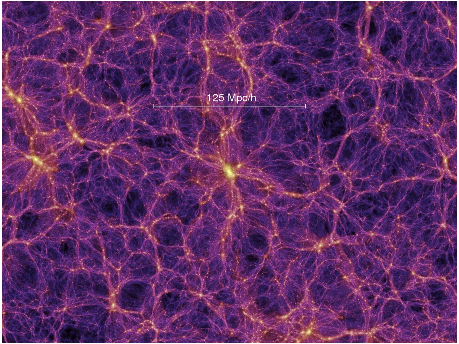
(Will, 1993)

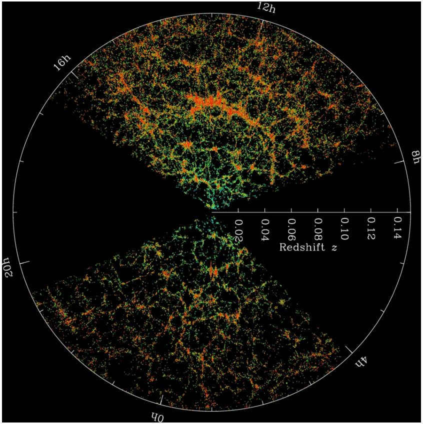
Applicable scales of PPN



Credit: Adapted from NASA / JPL images

Cosmological scales





Credit: Adapted from SDSS / Millenium Simulation images

Slip ζ and gravitational constant parameter μ

For a perturbed FLRW background

$$ds^2 = a^2 \left[-(1-2\Phi)d\tau^2 + (1+2\Psi)\frac{(dx^2+dy^2+dz^2)}{\left(1+\frac{1}{4}\kappa r^2\right)^2} \right]$$

General parameterization

$$\frac{1}{3}\nabla^2\Psi - \mathcal{H}^2\Phi - \mathcal{H}\dot{\Psi} + \kappa\Psi = -\frac{4\pi G}{3}\,\mu\,\delta\rho\,a^2$$

and

$$\frac{1}{3}\nabla^2\Phi + 2\dot{\mathcal{H}}\Phi + \mathcal{H}\dot{\Phi} + \ddot{\Psi} + \mathcal{H}\dot{\Psi} = -\frac{4\pi G}{3}\,\mu(1-\zeta)\,\delta\rho\,a^2\,,$$

Small-scale quasi static regime (Amendola et al. 2008)

$$\mu = -\frac{\nabla^2\Psi}{4\pi G\,\delta\rho\,a^2} \qquad \text{and} \qquad \zeta = 1 - \frac{\nabla^2\Phi}{\nabla^2\Psi}\,.$$

Weak Fields to Cosmology

arXiv: 1803.01157

- Coordinate Transformations: perturbed FLRW $\rightarrow$ perturbed Minkowski

$$\Phi = \frac{1}{2}h_{00} - \frac{1}{2}\dot{\mathcal{H}}r^2$$
$$\Psi = \frac{1}{6}\delta^{\mu\nu}h_{\mu\nu} + \frac{1}{4}\left(\mathcal{H}^2 + \kappa\right)r^2\,,$$

  where the post-Newtonian metric is given by $g_{ab} = \eta_{ab} + h_{ab}$.

- Post-Newtonian potentials

$$h_{00} = 2\alpha U + \frac{1}{3}\alpha_c\,\tilde{r}^2 \quad \text{and} \quad \delta^{\mu\nu}h_{\mu\nu} = 6\gamma U + \gamma_c\,\tilde{r}^2\,,$$

  where U is the Newtonian gravitational potential.

- **Note:** Now α, α_C, γ and γ_C are functions of time only.

ΛCDM

arXiv: 1610.08039

The PPNC parameters are given by

$$\alpha = \gamma = 1,$$

$$\alpha_C = \Lambda,$$

$$\gamma_C = -\frac{\Lambda}{2} \, .$$

Scalar-tensor theories of gravity

arXiv: 1610.08039

The Lagrangian of such a theory is given by

$$L = \frac{1}{16\pi G}\left[\phi R - \frac{\omega(\phi)}{\phi} g^{ab}\phi_{;a}\phi_{;b} - 2\phi\Lambda(\phi)\right] + L_m(\psi, g_{ab}) \ ,$$

The PPNC parameters are given by

$$\alpha = \left(\frac{2\omega+4}{2\omega+3}\right)\frac{1}{\bar{\phi}} \ , \quad \gamma = \left(\frac{2\omega+2}{2\omega+3}\right)\frac{1}{\bar{\phi}}$$

$$\alpha_C = -\left(\frac{2\omega+4}{2\omega+3}\right)\sum_I \frac{4\pi G\rho_I}{\bar{\phi}} + \left(\frac{2\omega+2}{2\omega+3}\right)\left(-\sum_I \frac{12\pi G p_I}{\bar{\phi}} + \Lambda(\bar{\phi})\right) - \frac{\omega(\bar{\phi})}{\bar{\phi}^2}\dot{\bar{\phi}}^2 - \frac{\ddot{\bar{\phi}}}{\bar{\phi}}$$
$$+ \frac{1}{2\omega+3}\left(\frac{\omega'\dot{\bar{\phi}}^2}{2\bar{\phi}} + \bar{\phi}\Lambda'(\bar{\phi})\right) \ ,$$

$$\gamma_C = -\left(\frac{2\omega+2}{2\omega+3}\right)\sum_I \frac{4\pi G\rho_I}{\bar{\phi}} - \left(\frac{1}{2\omega+3}\right)\left(\sum_I \frac{12\pi G p_I}{\bar{\phi}} + \frac{\omega'}{2\bar{\phi}}\dot{\bar{\phi}}^2 + \bar{\phi}\Lambda'(\bar{\phi})\right)$$
$$- \frac{\omega(\bar{\phi})}{4\bar{\phi}^2}\dot{\bar{\phi}}^2 - \left(\frac{2\omega+1}{4\omega+6}\right)\Lambda(\bar{\phi}) - \frac{\ddot{\bar{\phi}}}{2\bar{\phi}} \ .$$

Other examples

arXiv: 1610.08039

- Dark energy models like quintessence

$$\rho_I = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_I = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- Vector-tensor theories of gravity

$$L = \frac{1}{16\pi G}\left[R + \omega A_a A^a R + \eta A^a A^b R_{ab} - \epsilon F^{ab} F_{ab} + \tau A_{a;b} A^{a;b}\right] + L_m(\psi, g_{ab})$$

Background Cosmological Expansion

arXiv: 1803.01157

Acceleration equation

$$\dot{\mathcal{H}} = -\frac{4\pi G}{3}\,\alpha\,\langle\rho\rangle\,a^2 + \frac{\alpha_c\,a^2}{3}$$

Constraint equation

$$\mathcal{H}^2 = \frac{8\pi G}{3}\,\gamma\,\langle\rho\rangle\,a^2 - \frac{2\gamma_c\,a^2}{3} - \kappa$$

Additional constraint

$$4\pi G\langle\rho\rangle = \left(\alpha_C + 2\gamma_C + \frac{d\gamma_C}{d\ln a}\right)\Bigg/\left(\alpha - \gamma + \frac{d\gamma}{d\ln a}\right)$$

Small-scale cosmological perturbations

arXiv: 1803.01157

Poisson-like equations in an FLRW background

$$\nabla^2 \Phi = -4\pi G a^2 \alpha \delta\rho \ ,$$
$$\nabla^2 \Psi = -4\pi G a^2 \gamma \delta\rho \ .$$

Horizon-scale perturbations

arXiv: 1803.01157

Parameterizing the large-scale pertubations

$$-\mathcal{H}^2\Phi - \mathcal{H}\dot{\Psi} + \kappa\Psi = -\frac{\delta}{3}\left(\mathcal{H}^2 - \dot{\mathcal{H}} + \kappa\right)$$

and

$$2\dot{\mathcal{H}}\Phi + \mathcal{H}\dot{\Phi} + \ddot{\Psi} + \mathcal{H}\dot{\Psi} = \frac{\delta}{3}\left(\frac{2\dot{\mathcal{H}}\mathcal{H} - \ddot{\mathcal{H}}}{\mathcal{H}}\right)$$

Horizon-scale perturbations

arXiv: 1803.01157

Parameterizing the large-scale pertubations

$$-\mathcal{H}^2\Phi - \mathcal{H}\dot{\Psi} + \kappa\Psi = -\frac{4\pi G}{3}\delta\rho\, a^2\left(\gamma - \frac{1}{3}\gamma' + \frac{1}{12\pi G\langle\rho\rangle}\gamma_c'\right)$$

and

$$2\dot{\mathcal{H}}\Phi + \mathcal{H}\dot{\Phi} + \ddot{\Psi} + \mathcal{H}\dot{\Psi} = -\frac{4\pi G}{3}\delta\rho\, a^2\left(\alpha - \frac{1}{3}\alpha' + \frac{1}{12\pi G\langle\rho\rangle}\alpha'{}_c\right).$$

where primes denote $d/d\ln a$.

Main Results

arXiv: 1803.01157

Slip ζ and gravitational constant parameter μ

Small-scale limit

$$\lim_{k\to\infty} \mu = \gamma$$

$$\lim_{k\to\infty} \zeta = 1 - \frac{\alpha}{\gamma},$$

Large-scale limit

$$\lim_{k\to 0} \mu = \gamma - \frac{1}{3}\gamma' + \frac{1}{12\pi G\langle\rho\rangle}\gamma_c'$$

$$\lim_{k\to 0} \zeta = 1 - \frac{\alpha - \alpha'/3 + \alpha_c'/12\pi G\langle\rho\rangle}{\gamma - \gamma'/3 + \gamma_c'/12\pi G\langle\rho\rangle}.$$

Illustration

arXiv: 1803.01157

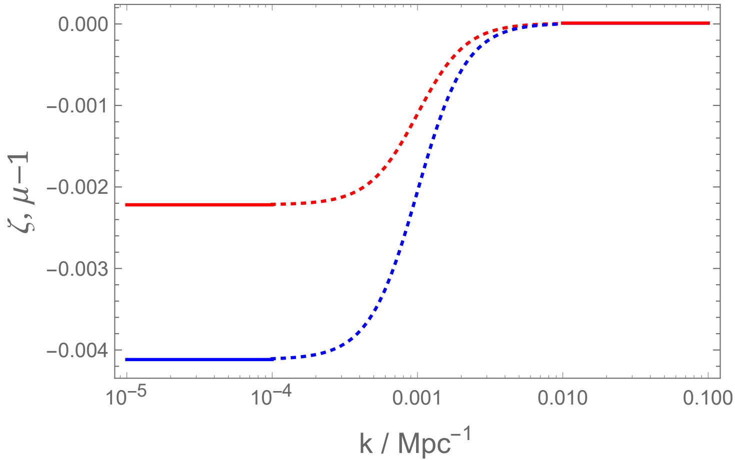


Figure: The small-scale ($k \gtrsim 0.01\,\mathrm{Mpc}^{-1}$) and large-scale ($k \lesssim 10^{-4}\,\mathrm{Mpc}^{-1}$) limits of the ζ (red) and μ (blue) parameters at the present time, connected by an interpolating tanh function (dotted).

Compare PPNC with other approaches

Effective Field Theory (EFT) approaches (Bellini et al. 2014)

- Parameterize the action directly - valid on linear scales.
- In general contains 5 independent functions of time α_M, α_B, α_T, α_K, α_H for perturbations.

Parameterized post-Friedmann (PPF) approaches (Baker et al. 2011)

- Parameterize the field equations directly
- Contains a large number of space-time dependent functions

Building a post-Newtonian cosmology

Arxiv:1503.08747

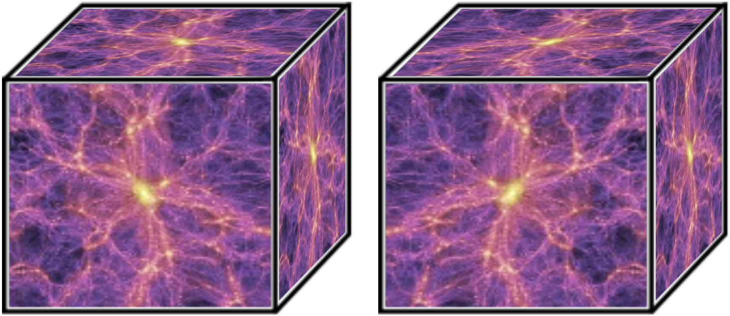
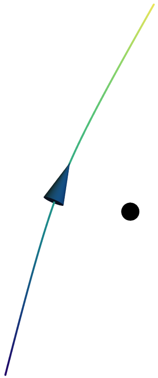
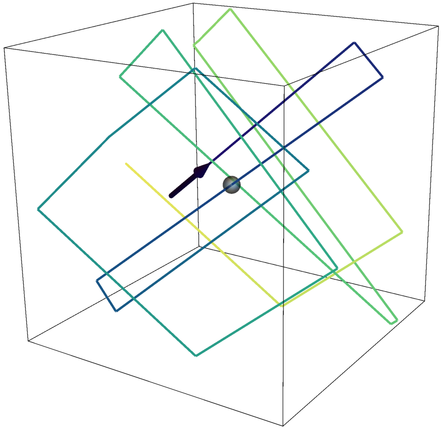


Figure: This figure was produced using an image from D. J. Croton *et al.*, 2005

Illustrations



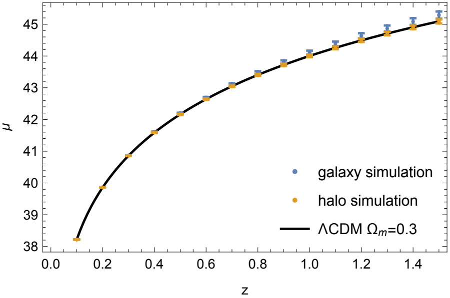
(a) Point Mass

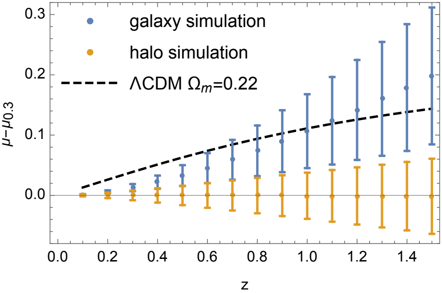


(b) Homogeneous Halo

Hubble Diagrams

Arxiv: 1705.02328

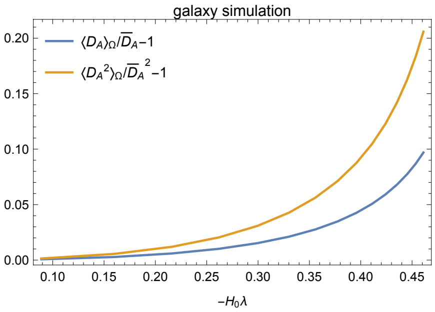


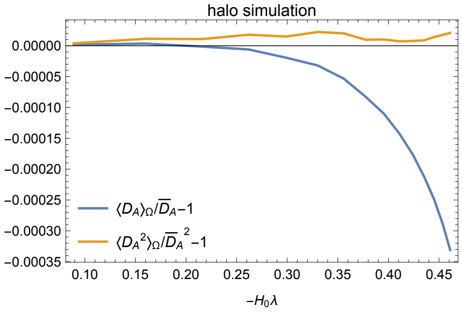


Comparison with Weinberg-Kibble-Lieu Theorem

Arxiv: 1705.02328

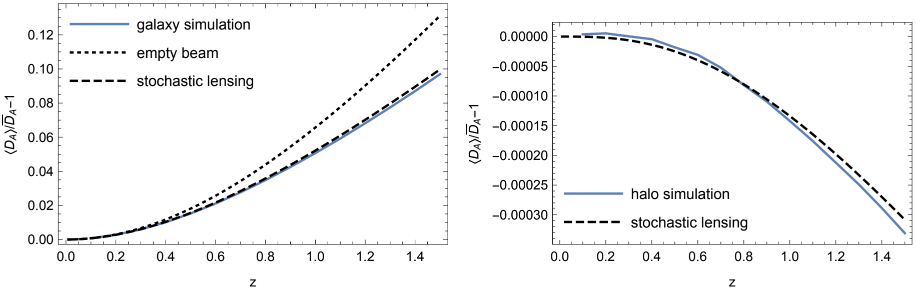
$$\langle D_A^2(\lambda)\rangle_\Omega \equiv \frac{1}{4\pi}\int_{\lambda=\mathrm{cst}} D_A^2\, d\Omega = \bar{D}_A^2(\lambda)\,.$$





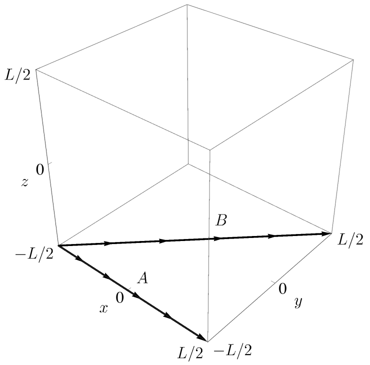
Comparison with Stochastic Lensing Formalism

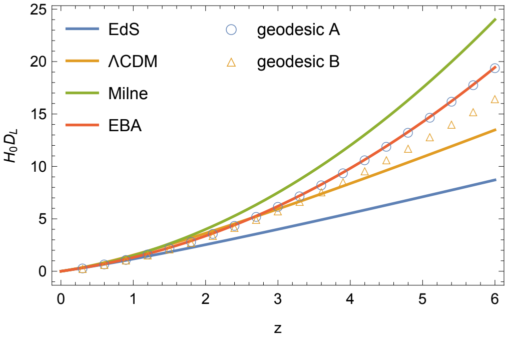
Arxiv: 1705.02328



Comparison with Numerical Relativity

Arxiv: 1705.02328





Summary and Future Work

- We have constructed a parametrization that requires only 4 functions of time to parameterize a large class of metric theories of gravity in cosmology.
- In GR, we have calculated the effect of small-scale inhomogeneities on observables in these type of models.
- Apply it to more general distributions of matter
- Constrain the parameters using observations

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