Astrophysical Tests of Gravity

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JS arXiv: 1309.0495
Jain, Vikram, JS arXiv: 1204.6044
Davis, Lim, JS, Shaw arXiv: 1102.5278
see also Chang & Hui 1011.4107

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Outline

Why do People Study Modified Gravity?

Einstein's General Relativity describes gravity perfectly in many situations so why do people study modifications?

Dark Energy:

The universe is accelerating and GR requires ∼69% (Planck) of the universe to be unknown exotic matter in order to account for this.

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Any modification of GR necessarily introduces a new degree of freedom Weinberg 1965.

Why do People Study Modified Gravity?

Beyond the Standard Model Physics:

Many theories of *fundamental physics* predict low-energy effective actions where new scalar degrees of freedom couple to matter e.g. string theory

$$\mathcal{L}_{\text{4-D}} \supset e^{-\phi} R.$$

These are a long-range modification of GR.

Why do People Study Modified Gravity?

Completeness:

So far we have only tested General Relativity in our own galaxy, which corresponds to curvatures and Newtonian potentials of the milky way and selected compact objects e.g. neutron stars.

Gravity could admit interesting features on cosmological scales or galaxies in very different environments whilst still behaving the same in our own galaxy.

Why do we need to Screen the Modifications?

General Relativity has been tested to incredibly high precision over the last 100 years or so! So well that any modification is usually rendered irrelevant once the bounds are imposed e.g. Cassini:

$$\frac{F_5}{F_{\rm N}} < 10^{-5} \quad (10^{-13} \quad \text{for WEP violations})$$

The Get-Out Clause

Is this a dead end?

No

All these experiments have been performed locally, in our own solar system.

There is nothing preventing large-scale modifications of GR provided that we satisfy these local bounds.

Theories with this feature are said to posses *screening mechanisms*.

Screening Fifth-Forces

Scalar Field ϕ coupled to matter.

Expand fluctuations about the background $\phi = \phi_0 + \delta\phi$ to second order:

$$\frac{\mathcal{L}}{\sqrt{-g}} \supset -Z_{\mu\nu}(\phi_0, \partial_\alpha\phi_0, \Box\phi_0, \cdots)\partial^\mu\delta\phi\partial^\nu\delta\phi - m_{\rm eff}^2(\phi_0)\delta\phi^2 - \beta(\phi_0)\frac{\delta\phi}{M_{\rm pl}}T + \cdots$$

The coupling to matter gives rise to new or “fifth” forces.

Screening Fifth-Forces

The fifth-force is screened if:

$$\frac{\mathcal{L}}{\sqrt{-g}} \supset -\underbrace{Z_{\mu\nu}}_{\text{Large}}\partial^\mu\delta\phi\partial^\nu\delta\phi - \underbrace{m_{\text{eff}}^2}_{\text{large}}\delta\phi^2 - \underbrace{\beta(\phi_0)}_{\text{small}}\frac{\delta\phi}{M_{\text{pl}}}T + \cdots$$

Screening Fifth-Forces

$$\frac{\mathcal{L}}{\sqrt{-g}} \supset -\underbrace{Z_{\mu\nu}}_{\text{Vainshtein}}\partial^\mu\delta\phi\partial^\nu\delta\phi - \overbrace{\underbrace{m_{\text{eff}}^2}_{\text{Chameleon}}\delta\phi - \underbrace{\beta(\phi_0)}_{\substack{\text{Symmetron}\\\text{Dilaton}}}\frac{\delta\phi}{M_{\text{pl}}}}^{\text{Conformal Scalar-Tensor Theories}}T + \cdots$$

In this talk we are concerned with conformal scalar-tensor screening only.

Scalar-Tensor Screening

Scalar field is conformally coupled to matter via the "coupling function" $A(\phi)$ in the Einsten Frame:

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2R}{2}-\nabla_\mu\phi\nabla^\mu\phi-V(\phi)\right]+S_{\rm m}\left[\psi_{\rm i};A^2(\phi)g_{\mu\nu}\right]$$

Matter moves on geodesics of the Jordan Frame metric $\tilde{g}_{\mu\nu}=A^2(\phi)g_{\mu\nu}\Rightarrow$ observers in the Einstein frame infer a *fifth-force*

$$\vec{F}_\phi=\frac{\beta(\phi)}{M_{\rm pl}}\vec{\nabla}\phi \quad \beta(\phi)=M_{\rm pl}\frac{\mathrm{d}\ln(A)}{\mathrm{d}\phi}$$

Scalar-Tensor Screening

The equations of motion imply an effective potential for ϕ:

$$V_{\rm eff}(\phi) = V(\phi) + \rho A(\phi).$$

If this has a minimum then we can move the field value around as a function of the local density and can screen the fifth-force in dense environments if either:

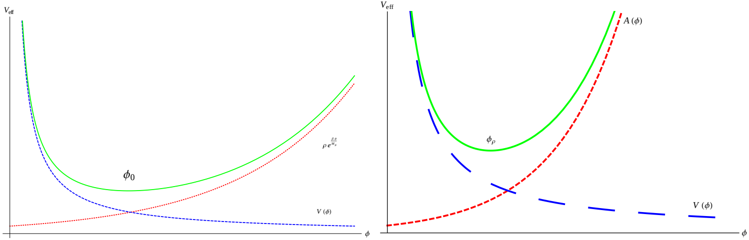
- $m_{\rm eff}(\phi_{\rm min})R \gg 1$ - the force is sub-mm (Chameleon).
- $\beta(\phi) \ll 1$ - the coupling to matter is negligible (Symmetron, Dilaton).

$$m_{\rm eff} = V_{\rm eff}(\phi_{\rm min})_{,\phi\phi} \quad \vec{F}_\phi = \frac{\beta(\phi)}{M_{\rm pl}}\vec{\nabla}\phi$$

Chameleon Screening

Khoury & Weltman 03

$$V(\phi) = \frac{M^{4+n}}{\phi^n} \quad A(\phi) = e^{\beta \frac{\phi}{M_{\rm pl}}}$$



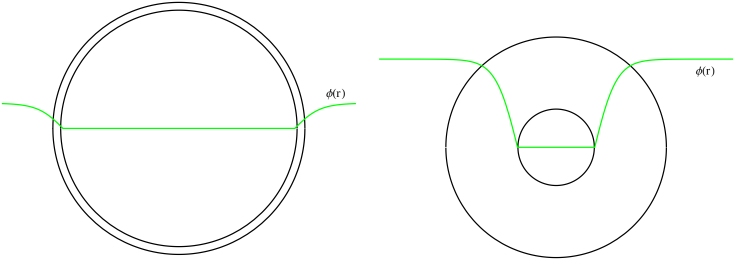
Low ρ High ρ

$$m_{\rm eff}(\phi_\rho) \gg m_{\rm eff}(\phi_0)$$

$f(R)$ gravity is a chameleon under some circumstances.

Spherical Screening

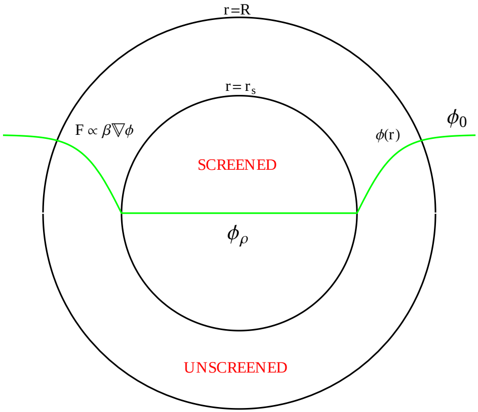
An object will be screened if the field can reach its minimum over most of the object's radius.



screened unscreened

Whether an object is screened or not depends on its Newtonian potential (see later).

Spherical Screening



Screening Parameterisation

There is a model-independent parameterisation perfect for small-scale tests:

$\alpha \equiv 2\beta(\phi_0)^2 \quad G \to G(1+\alpha)$ when the object is fully unscreened.

$f(R)$ gravity has $\alpha = 1/3$.

$$\chi_0 \equiv \frac{\phi_0}{2M_{\rm pl}\beta(\phi_0)} \text{ - the self-screening parameter.}$$

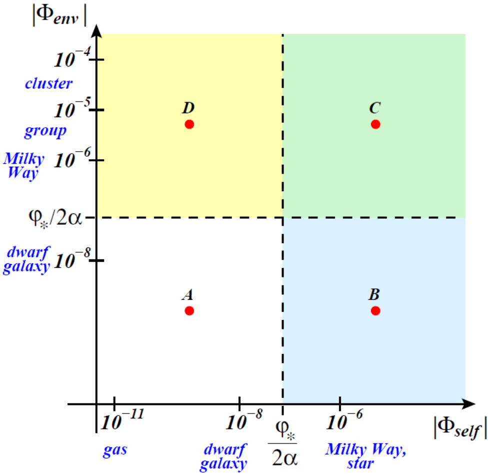
Rule of thumb: an object is partially unscreened if

$$\chi_0 > \Phi_{\rm N}.$$

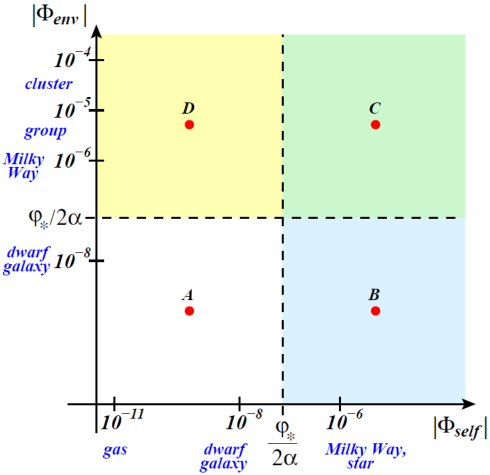
$\Phi_{\rm N}$ is the Newtonian Potential.

Environmental Effects

An object can be self-screened or screened by the potential of its environment.



Environmental Effects



Hui, Nicolis & Stubbs 09

Self-Screening Parameter

$$\chi_0 \sim 10^{-4} - 10^{-5}$$

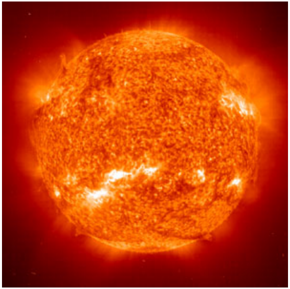
At large χ_0 matter is unscreened over cosmological scales.

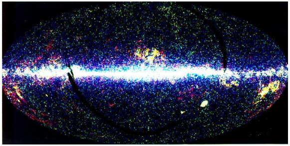
This causes deviations in the halo power spectrum, peculiar velocity power spectrum, mass function...

Current constraint (cluster statistics): $\chi_0 \lesssim 10^{-5}$ Schmidt, Lima, Oyaizu & Hu 09.

Self-Screening Parameter

$$\chi_0 \sim 10^{-6}$$





The Sun and Milky Way have Newtonian potential $\Phi_{\rm N} \sim 10^{-6}$
(Chang & Hui 11, Davis, Lim, JS & Shaw 11)

Does this give a Constraint?

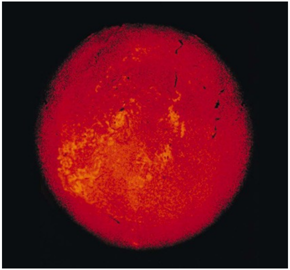
Does this then constrain $\chi_0 \lesssim 10^{-6}$?

Not necessarily: the milky way could be screened by the local group.

This is one of the issues I want to address in this talk.

Self-Screening Parameter

$$\chi_0 \sim 10^{-7}$$



Red giant stars have $\Phi_{\rm N} \sim 10^{-7} - 10^{-8}$ (Jain, Vikram & JS 12).

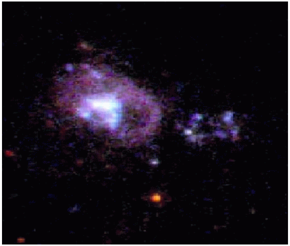
Self-Screening Parameter

Can we use this to probe $\chi_0 < 10^{-6}$?

This is the second issue I want to address.

Self-Screening Parameter

Dwarf galaxies in cosmic voids have $\Phi_{\mathrm{N}} \sim 10^{-8}$.



Current data is not enough for definite constraints (Jain & Vanderplas 11, Vikram, Cabre, Jain & Jake VanderPlas 13).

Dwarf galaxies in voids may host other unscreened astrophysical objects and the data is better for these.

Dwarf Galaxies as Laboratories

Dwarf galaxies in voids have $\Phi_{\rm N} \sim 10^{-8}$: they are the perfect testing ground for scalar-tensor theories.

Any object where $\Phi_{\rm N} < \chi_0$ will be unscreened in a dwarf galaxy located in a cosmic void!

A Screening Map

Screening maps (Cabre et al. 12) for $\chi_0 = 10^{-5}$, 10^{-6} and 10^{-7} have been made using SDSS data and are publicly available:

$$\Phi_{\rm N}^{\rm self} > \chi_0 \Rightarrow \text{ self-screened}$$

$$\Phi_{\rm N}^{\rm ext} > \chi_0 \Rightarrow \text{ environmentally-screened}$$

The maps have been calibrated and tested against N-body code predictions.

This Talk

In the rest of this talk I want to use the effect of scalar-tensor gravity on stars to place new constraints:

1. Does the local group remove the constraint $\chi_0 \lesssim 10^{-6}$?
2. Place new constraints using stellar effects.

These constraints are currently the strongest in the literature.

Stars in Modified Gravity

Main Idea:

- Stars are balls of hydrogen (or other elements) gas which support themselves against gravitational collapse by burning fuel in their centre in order to provide an outward pressure.
- An unscreened star feels a stronger gravitational force in its outer layers.
- This means it needs to burn more fuel per unit time in order to stave off collapse.
- This releases more energy per unit time so the star is brighter.

Equations of Stellar Structure

$$\frac{\mathrm{d}\overbrace{P}^{\text{Pressure}}}{\mathrm{d}r} = -\frac{GM(r)\overbrace{\rho(r)}^{\text{Density}}}{r^2} \quad \text{Hydrostatic Equilibrium}$$

$+$ mass conservation, radiative transfer, energy generation.

Equations of Stellar Structure

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM(r)\rho(r)}{r^2},$$

Modifying gravity only changes the gravitational physics (G). Therefore the rest of the stellar processes are unaffected.

Scaling Relations

The equations of stellar structure do not close, we need to specify the pressure:

$$P_{\rm gas} = \frac{\rho k_{\rm B} T}{\mu m_{\rm H}} \qquad P_{\rm rad} = \frac{1}{3} a T^4$$

Equations are self-similar:

$$P \rightarrow P_{\rm c} x_P$$

Rescaling gives gross properties e.g.

$$P_{\rm c} \propto \frac{M^2}{R^4}$$

Scaling Relations

Rescale $G \to G(1+\alpha)$ ($\alpha = 1/3$ in $f(R)$).

$$\text{Gas Pressure: } L \propto G^4 M^3 \Rightarrow \frac{L_{\text{MG}}}{L_{\text{GR}}} = (1+\alpha)^4$$

$$\text{Radiation Pressure: } L \propto GM \Rightarrow \frac{L_{\text{MG}}}{L_{\text{GR}}} = (1+\alpha)$$

Gas(Radiation) pressure is dominant in low(high) mass stars $\Rightarrow$ expect a turnoff in high-mass stars.

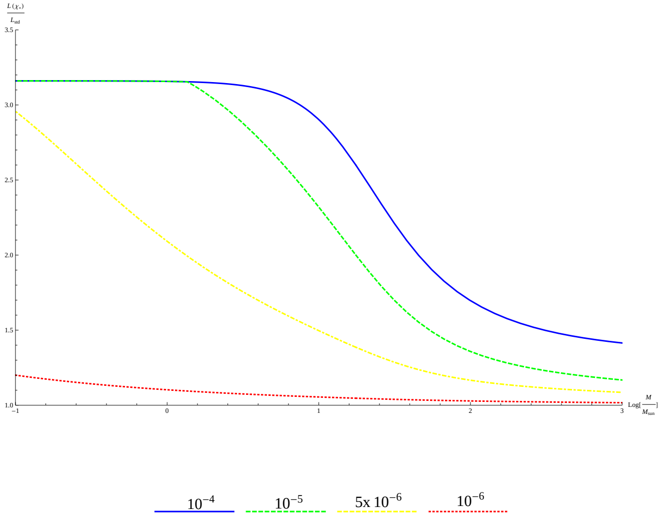
Lane-Emden Models

Lane-Model:

- Stars with no physics - balls of gas which collapse under their own gravity.
- Gas and radiation pressure.
- Equation of state: $P = K\rho^{\frac{4}{3}}$.
- $G(r > r_{\rm s}) = G(1+\alpha)$ but we correct for surface effects.

We take $\alpha = 1/3$ corresponding to $f(R)$ gravity.

Luminosity Enhancement



Implementation into MESA

Need to go beyond the simple polytropic approximation to compare to actual data.

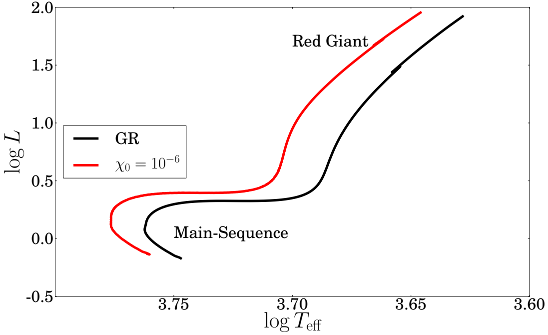
MESA (Paxton et al 11) is a self-consistent stellar evolution code and we have modified it to include modified gravity.

$$G \to G(r) = G\left(1 + \alpha\left[1 - \frac{M(r_s)}{M(r)}\right]\right)$$

Screening radius is found by solving the field equations for a given χ_0.

Implementation into MESA

$$M = 1M_\odot \quad \alpha = 1/3$$



Davis, Lim, JS & Shaw 2011

Distance Indicators as Probes of Modified Gravity

Distance to a galaxy depends on the measurement of variables $\{X\}$ and a conversion using some formula $d=F(\{X\})$.

If this formula is different in MG then application of the GR formula will give the wrong distance e.g. luminosity distance:

$$F=\frac{L}{d^2}.$$

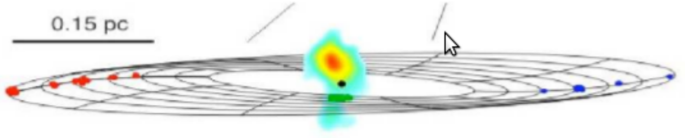
Distance Indicators as Probes of Modified Gravity

Idea: Compare distances found using screened and unscreened distance indicators to the same unscreened galaxies and look for systematic deviations compared to a screened sample.

We use distance indicators in the nearby universe so cosmological effects can be neglected.

Water Masers

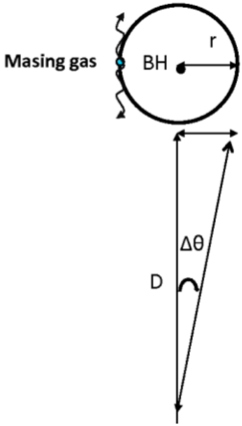
Clouds of H_2O water orbits black holes and mases at 22 GHz.



Kuo 01

Near the black hole $\Phi_N \sim 10^{-5}$ so this is a screened environment for sensible parameter values.

Water Masers

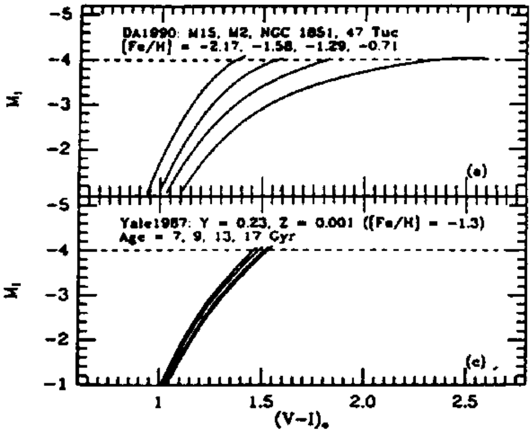


Tip of the Red Giant Branch

$\sim 2M_\odot$ stars are large enough to excite Helium burning at $T \sim 10^8$K. They climb the red giant branch until their core temperature is high enough to ignite the central Helium.

This depends only on nuclear physics so the ignition tends to happen at fixed luminosity and temperature.

Tip of the Red Giant Branch



Are TRGB Distances Screened?

The helium flash is set by nuclear physics.

$\chi_0 \lesssim 10^{-6} \Rightarrow$ core screened $\Rightarrow$ no change in core properties $\Rightarrow$ TRGB is screened.

$\chi_0 \gtrsim 10^{-6} \Rightarrow$ core unscreened $\Rightarrow$ core temperature increases at a faster rate $\Rightarrow$ helium flash happens earlier $\Rightarrow$ peak luminosity goes down.

Masers Vs. TRGB

If $\chi_0 \gtrsim 10^{-6}$ comparing maser and TRGB distances probes MG.

Only have one galaxy with simultaneous measurements (NGC 4258). This is a spiral galaxy so $\Phi_{\rm N} \sim 10^{-6}$:

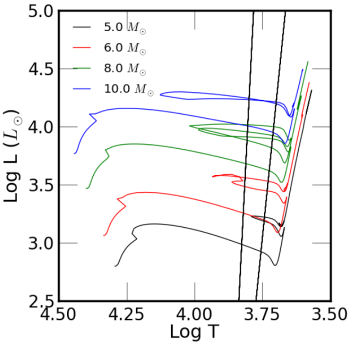
Maser:	$d = 7.2 \pm 0.2$ MPc
Maser:	$d = 7.1 \pm 0.2$ MPc
TRGB:	$d = 7.18 \pm 0.13 \pm 0.4$ MPc

Agreement sets a new constraint $\chi_0 \lesssim 10^{-6}$.

The local group screening argument no-longer holds.

Cepheid Distances

$5 - 10M_{\odot}$ stars have short-lived phases during their red giant tracks called loops. Along part of these loops the stars are unstable to *Cepheid Pulsations*.



Cepheid Distances

There is a well-known period-luminosity relation with $\Pi \propto (G\rho)^{-1/2}$.

$$\underbrace{M_{\rm B}}_{\sim\log(L)} = \mu \underbrace{\log(\Pi)}_{\sim\log(R)} + \nu \underbrace{(B-V)}_{\sim\log(T_{\rm e})} + \gamma$$

$$L = 4\pi R^2 \sigma T^4$$

Cepheid Distances

$$\frac{\Delta d}{d} \approx -0.3 \frac{\Delta G}{G}$$

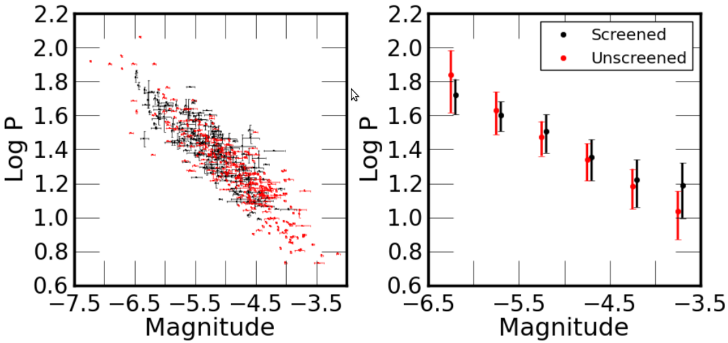
We compute $\langle G \rangle$ using MESA.

When $\chi_0 \lesssim 10^{-6}$ TRGB is screened but Cepheids are not:

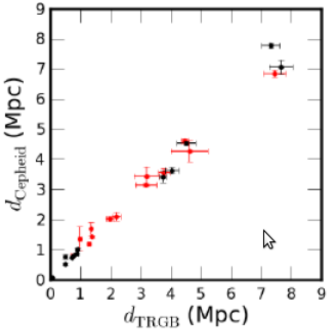
$$\Delta d = d_{\rm Cepheid} - d_{\rm TRGB}$$

By comparing Δd in screened and unscreened galaxies we can probe $\chi_0 < 10^{-6}$.

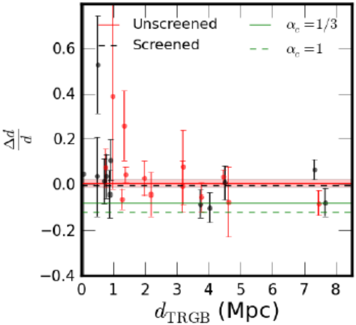
Measurements of Cepheids



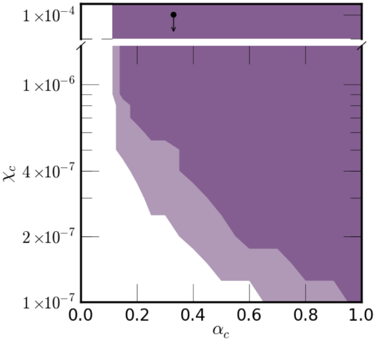
Measurements of Cepheids



Distance Comparisons



Constraints



Jain, Vikram & JS 2012

$$\chi_0 \lesssim \; 4 \times 10^{-7} \quad (f(R) \text{ Gravity})$$

These are currently the strongest constraints in the literature.

Can we do Better?

$$\frac{d_{\rm MG}-d_{\rm GR}}{d_{\rm GR}}\approx -0.3\frac{\Delta G}{G}$$

is an approximation - need full hydrodynamic perturbation theory.

There are two new features in modified gravity hydrodynamics:

1. The modified periods are even smaller than GR.
2. The stars are more stable.

Modified Gravity Hydrodynamics

$$\ddot{\vec{r}} = -\frac{1}{\rho}\nabla P + \underbrace{\vec{f}}_{\text{force per unit mass}}$$

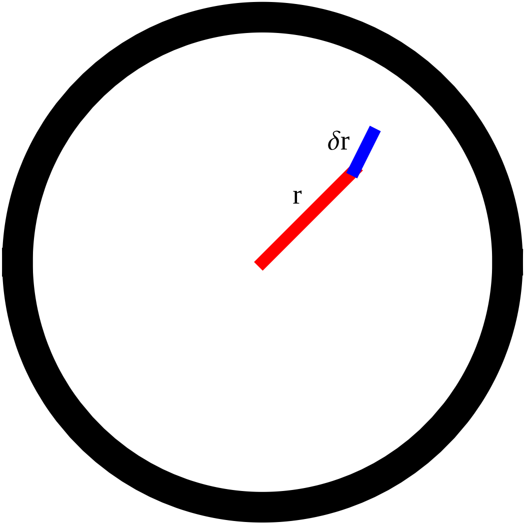
$$\vec{f} = -\frac{GM(r)}{r^2} - \frac{\beta(\phi)}{M_{\rm pl}}\nabla\phi$$

Look at radial perturbations only:

$$r = r_0 + \delta r$$

The rest of the hydrodynamic equations do not depend on gravity.

Modified Gravity Hydrodynamics



Linear Adiabatic Wave Equation

$$\frac{\delta r}{r} = \xi(r)e^{i\omega t} \quad \Gamma_{1,0} = \frac{\partial \ln P_0}{\partial \ln \rho_0}$$

General Relativity:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^4\Gamma_{1,0}P_0\frac{\mathrm{d}\xi}{\mathrm{d}r}\right) + r^3\frac{\mathrm{d}}{\mathrm{d}r}\left[\left(3\Gamma_{1,0}-4\right)P_0\right]\xi + r^4\rho_0\omega^2\xi = 0$$

This is a Sturm-Liouville problem.

The eigenfrequencies give the periods of oscillation.

Modified Linear Adiabatic Wave Equation

$$\frac{\delta r}{r} = \xi(r)e^{i\omega t} \quad \Gamma_{1,0} = \frac{\partial \ln P_0}{\partial \ln \rho_0}$$

Modified Gravity:

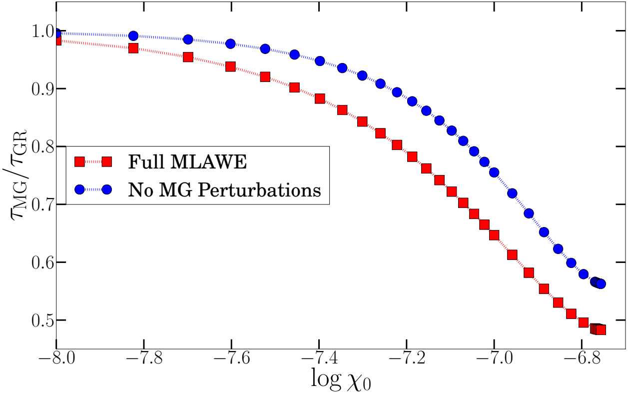
$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^4\Gamma_{1,0}P_0\frac{\mathrm{d}\xi}{\mathrm{d}r}\right) + r^3\frac{\mathrm{d}}{\mathrm{d}r}\left[\left(3\Gamma_{1,0}-4\right)P_0\right]\xi - 4\pi\alpha G r^4\rho_0^2\xi + r^4\rho_0\omega^2\xi = 0$$

This is a Sturm-Liouville problem.

The eigenfrequencies give the modified periods of oscillation.

Example: Lane-Emden Models

Lane-Emden model with $P = K\rho^{\frac{5}{3}}$:



$$\frac{GM}{R} = 10^{-7}$$

Cepheid Distances

α	χ_0	$\Delta d/d$ (approx)	$\Delta d/d$ (LAWE)	$\Delta d/d$ (MLAWE)
$1/3$	4×10^{-7}	-0.03	-0.04	-0.12
$1/2$	4×10^{-7}	-0.05	-0.06	-0.16
1	2×10^{-7}	-0.06	-0.07	-0.19

Full hydrodynamic prediction is 3 times larger than the equilibrium prediction!

Can improve the constraints using the same data sets.

Future Constraints

Vary χ_0 until $\Delta d/d$ matches the approximation at fixed α:

α	χ_0
$1/3$	9×10^{-8}
$1/2$	7×10^{-8}
1	3×10^{-8}

Table: JS, 2013

The constraints can be greatly improved. This is work in progress

Stellar Stability

$$\frac{\delta r}{r} = \xi(r)e^{i\omega t} \quad \Gamma_{1,0} = \frac{\partial \ln P_0}{\partial \ln \rho_0}$$

We can use the variational method to find an upper bound on the fundamental frequency.

$$\omega_0^2 \leq F[P_0, \rho_0]$$

The star is unstable if this is imaginary.

General Relativity:

$$\Gamma_{1,0}^{\text{critical}} = \frac{4}{3}$$

Stellar Stability

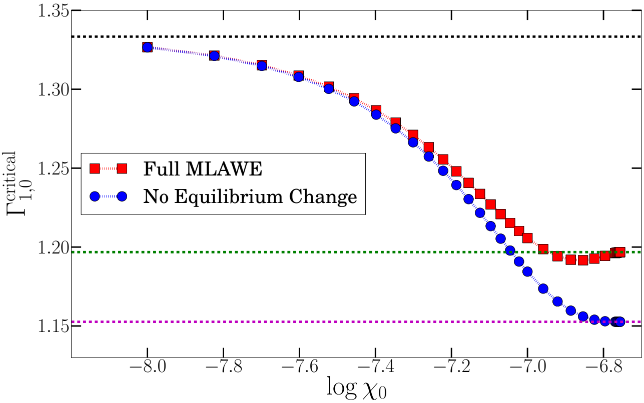
Modified Gravity:

$$\Gamma_{1,0}^{\text{critical}} = \frac{4}{3} - \alpha g(\chi_0)$$

$g(\chi_0)$ is an $\mathcal{O}(1)$ composition dependent factor which depends on how unscreened the star is.

$$g(\chi_0) = \frac{\int \mathrm{d}r 4\pi r^4 \rho_0(r)^2}{\int \mathrm{d}r 3r^2 P_0(r)}$$

Stability of Lane-Emden Models

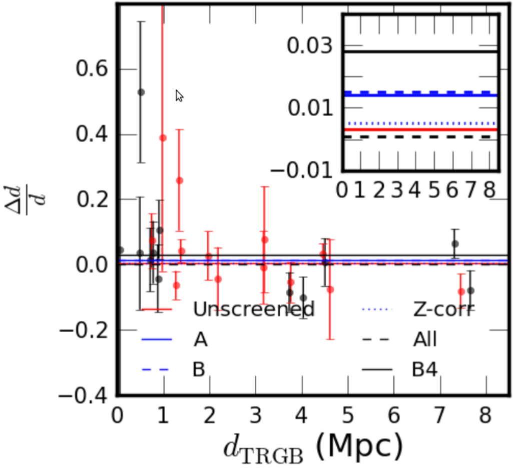


The dip is present because the modified equilibrium structure competes with the perturbations.

Summary

- Stars in unscreened dwarf galaxies may themselves be unscreened and exhibit new and novel features.
- By exploiting these new features we can probe MG to new levels:
  1. By comparing water maser and TRGB distances we constrain $\chi_0 \lesssim 10^{-6}$.
  2. By comparing Cepheid and TRGB distances we constrain $\chi_0 < 4 \times 10^{-7}$ for $f(R)$.
- These are currently the strongest constraints in the literature.
- Solving the full hydrodynamic problem shows that these can be greatly improved.
- There is lots of interesting things going on and lots of work to be done....

Systematics



Code Comparisons

We compare our GR dimensionless frequencies

$$\tilde{\omega}^2 = \frac{(n+1)\omega^2}{4\pi G\rho_c}$$

to those found by Hurley, Roberts & Wright 1966:

n	Hurley, Roberts & Wright	Me
0.5	0.37071	0.370714029
1.0	0.38331	0.38331184243
1.5	0.37640	0.376399032288
2.0	0.35087	0.350866992807
2.5	0.30389	0.303893585012
3.0	0.22774	0.227742000109
3.25	0.17731	0.177307835186
3.5	0.12404	0.124042556661
4.0	0.04056	0.0405613874985