COSMOLOGY WITH WEAK LENSING: THE INFORMATION BEYOND THE POWER SPECTRUM

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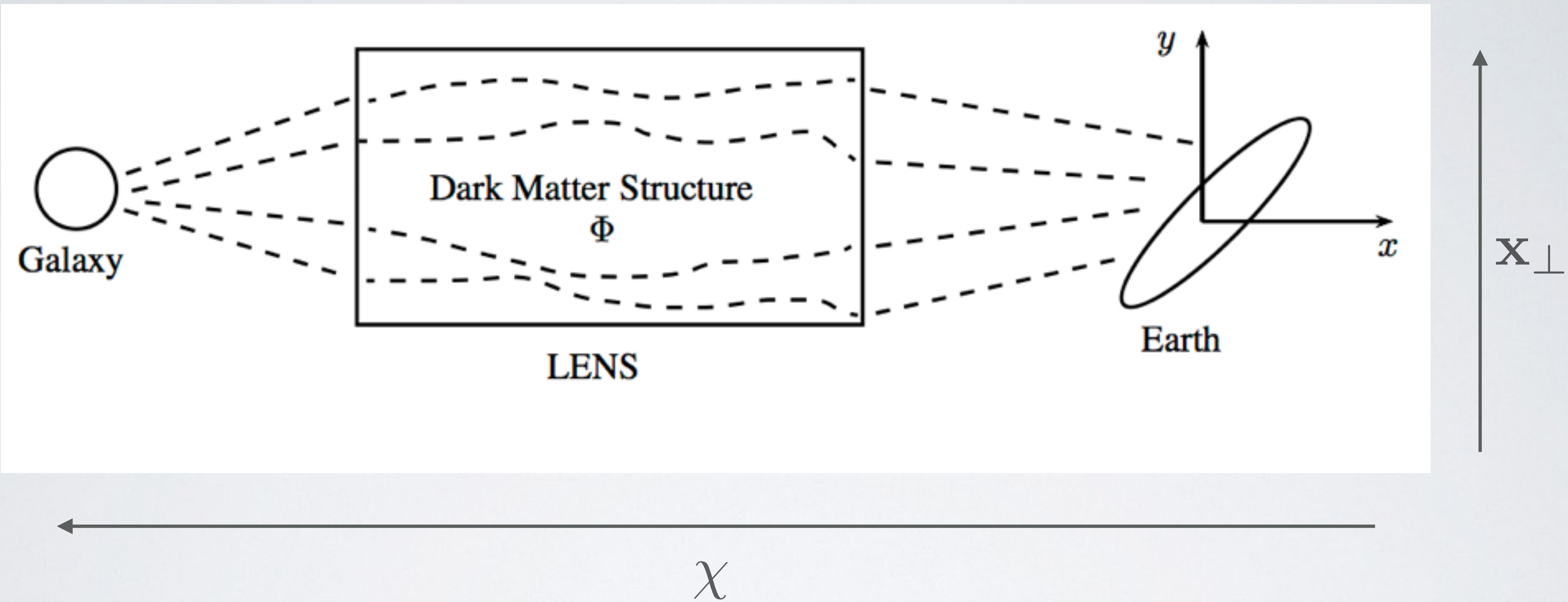
THE ΛCDM MODEL

Explain observations with a few free parameters:

$H_0(100h \text{ km/Mpc}), h(0.7)$	Supernovae IA, CMB,…
$\Omega_m h^2(0.12), \Omega_b h^2(0.022)$	CMB, BAO,…
$\Omega_\Lambda(0.73)$	Supernovae IA,…
$n_s(0.96)$	CMB, BAO,…
$\sigma_8(0.8)$	CMB, Weak Lensing,…
$w(-1???)$	Weak Lensing???
…	…

WEAK LENSING FUNDAMENTALS

THE WEAK LENSING EFFECT



$$\frac{d^2\mathbf{x}_\perp(\chi)}{d\chi^2} = -\frac{2\nabla_\perp\Phi(\mathbf{x}_\perp(\chi),\chi)}{c^2}$$

WL OBSERVABLES

- Observations measure angles $\boldsymbol{\beta} = \mathbf{x}_\perp/\chi$
- Light ray observed at $\boldsymbol{\theta}_s$ was originated at $\boldsymbol{\beta}_s$

$$\mathbf{A}_s = \frac{\partial \boldsymbol{\beta}_s}{\partial \boldsymbol{\theta}_s} = \begin{pmatrix} 1 - \kappa_s - \gamma_s^1 & -\gamma_s^2 \\ \gamma_s^2 & 1 - \kappa_s + \gamma_s^2 \end{pmatrix}$$

- κ_s is the apparent source magnification
- $\boldsymbol{\gamma}_s$ has to do with the apparent source ellipticity
- Possible sources: galaxies, CMB

SOLUTION TO THE WL EQUATIONS

$$\mathbf{x}_{\perp,s} = \mathbf{x}_{\perp,0} - \frac{2}{c^2}\int_0^{\chi_s} d\chi(\chi_s - \chi)\nabla_\perp\Phi(\mathbf{x}_\perp(\chi),\chi)$$

$$\boldsymbol{\beta}(\boldsymbol{\theta},\chi_s) = \boldsymbol{\theta} - \frac{2}{c^2}\int_0^{\chi_s} d\chi\left(\frac{\chi_s - \chi}{\chi_s}\right)\nabla_\perp\Phi(\chi\boldsymbol{\beta}(\boldsymbol{\theta},\chi),\chi)$$

Quadratic algorithm

$$\mathbf{A}(\boldsymbol{\theta},\chi_s) = \mathbb{1} - \frac{2}{c^2}\int_0^{\chi_s} d\chi\chi\left(\frac{\chi_s - \chi}{\chi_s}\right)\mathbf{A}(\boldsymbol{\theta},\chi)\nabla_\perp\nabla_\perp^T\Phi(\chi\boldsymbol{\beta}(\boldsymbol{\theta},\chi),\chi)$$

THE BORN APPROXIMATION

Lensing potential Φ is small, express $\boldsymbol{\beta}, \mathbf{A}$ at $O(\Phi)$

$$\mathbf{A}^{(1)}(\boldsymbol{\theta}, \chi_s) = \mathbb{1} - \frac{2}{c^2} \int_0^{\chi_s} d\chi \chi \left(\frac{\chi_s - \chi}{\chi_s} \right) \nabla_\perp \nabla_\perp^T \Phi(\chi \boldsymbol{\theta}, \chi)$$

$$\kappa^{(1)}(\boldsymbol{\theta}, \chi_s) = \int_0^{\chi_s} d\chi \left(1 - \frac{\chi}{\chi_s} \right) \delta(\chi \boldsymbol{\theta}, \chi)$$

Convergence is the integrated density contrast along the line of sight!

E AND B MODES

$$\tilde{\gamma}_E(\boldsymbol{\ell}) = \frac{(\ell_x^2 - \ell_y^2)\tilde{\gamma}_1(\boldsymbol{\ell}) + 2\ell_x\ell_y\tilde{\gamma}_2(\boldsymbol{\ell})}{\ell_x^2 + \ell_y^2}$$

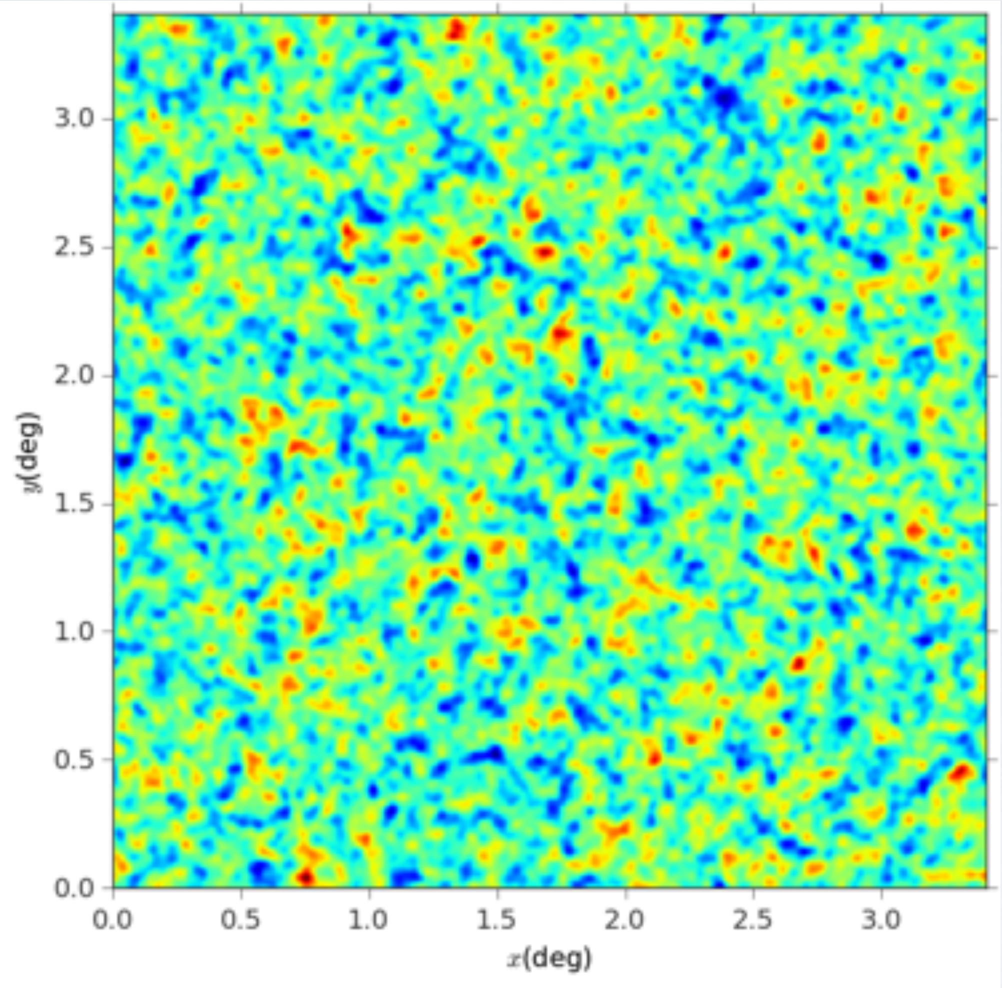
$$\tilde{\gamma}_B(\boldsymbol{\ell}) = \frac{-2\ell_x\ell_y\tilde{\gamma}_1(\boldsymbol{\ell}) + (\ell_x^2 - \ell_y^2)\tilde{\gamma}_2(\boldsymbol{\ell})}{\ell_x^2 + \ell_y^2}$$

$$\gamma_E^{(1)} = \kappa^{(1)} \;\; ; \;\; \gamma_B^{(1)} = 0$$

WHEN IS POST-BORN IMPORTANT?

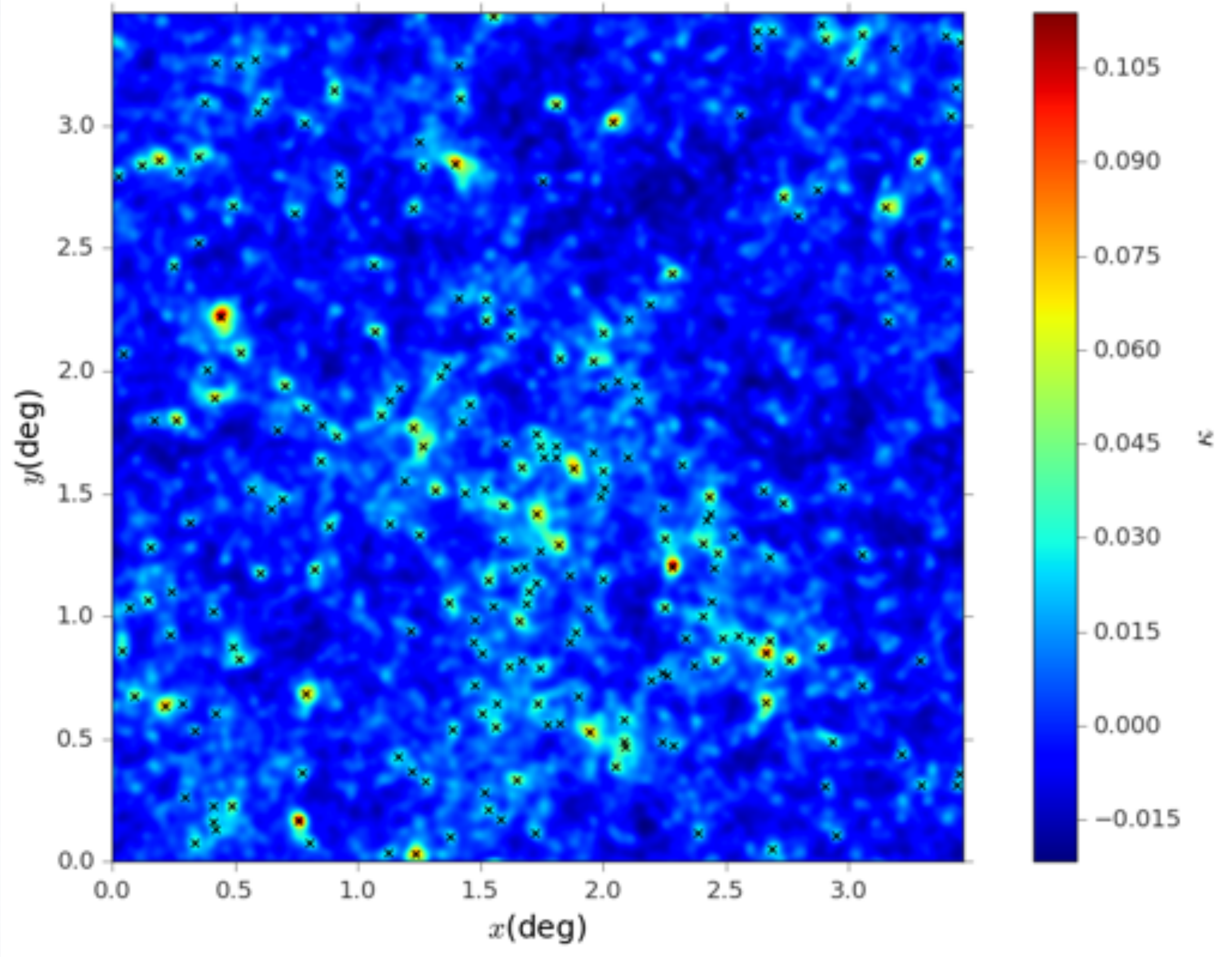
- WL observables are dominated by O(δ)
- CMB Lensing bi-spectrum (Pratten, Lewis 2016)
- Galaxy/CMB lensing: where does convergence NG come from? δ NG, post-Born, work in progress…

WL IMAGE ANALYSIS



GAUSSIAN

REAL

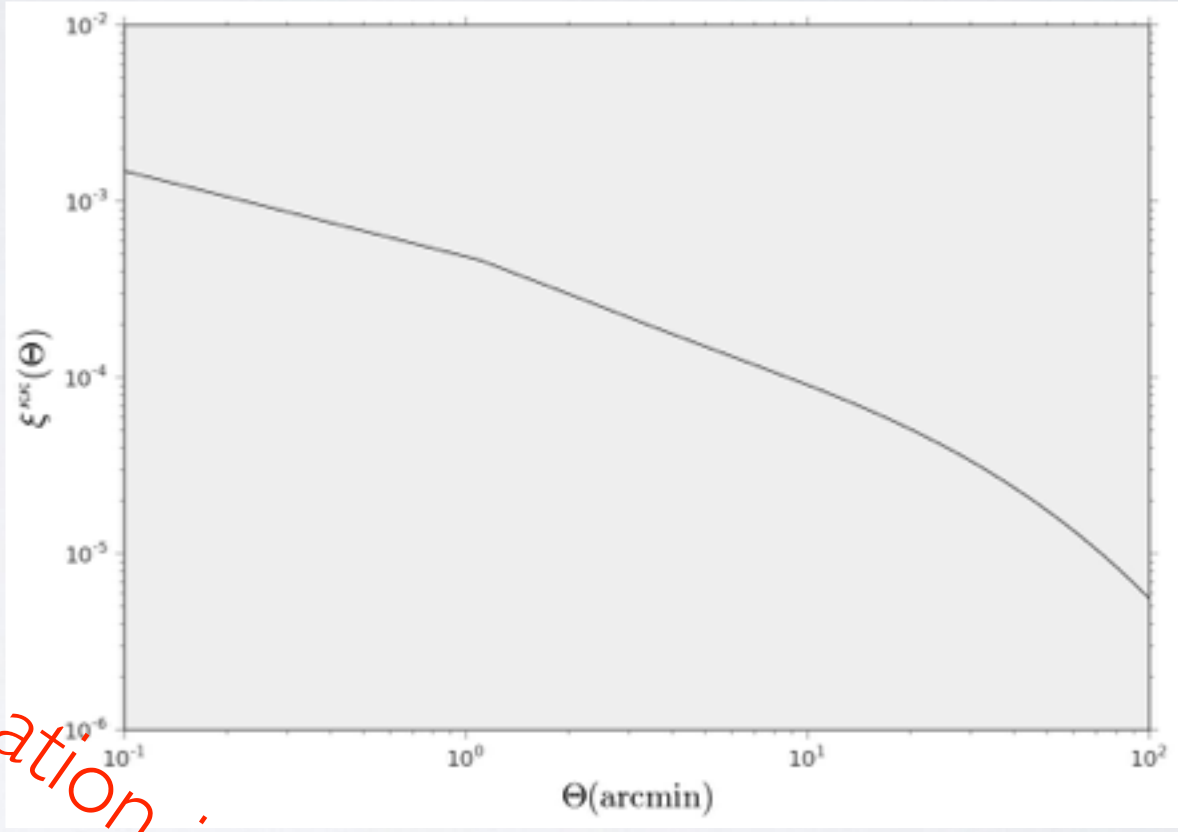


Convergence field is Non-Gaussian!

QUADRATIC STATISTICS

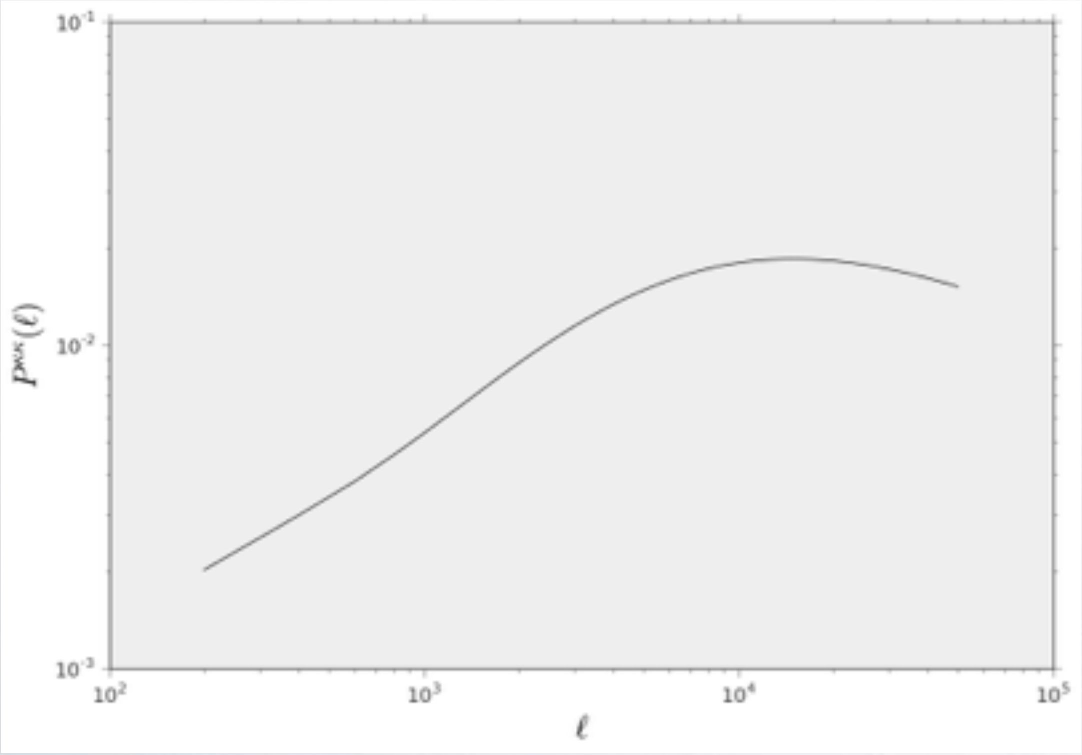
Two-Point correlation function

$$\xi^{\kappa\kappa}(\boldsymbol{\Theta}) = \langle \kappa(\boldsymbol{\theta})\kappa(\boldsymbol{\theta}+\boldsymbol{\Theta})\rangle$$



NG: some information is missing!!

Angular Power Spectrum

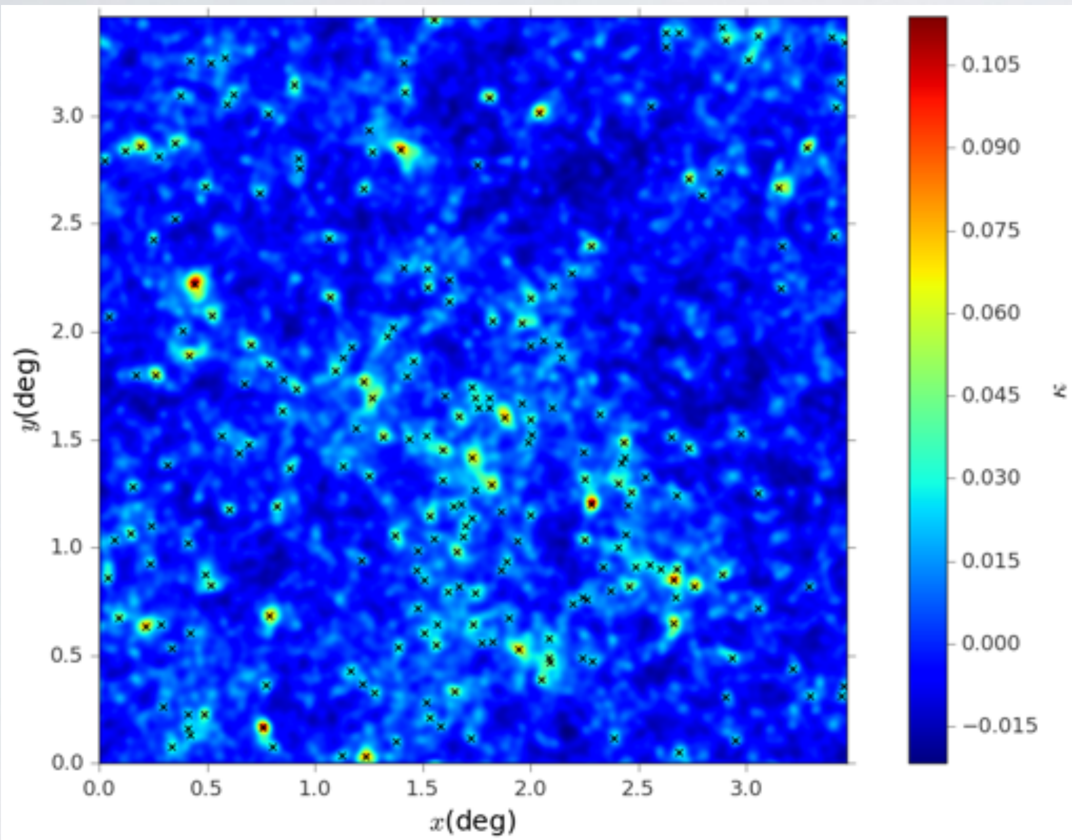


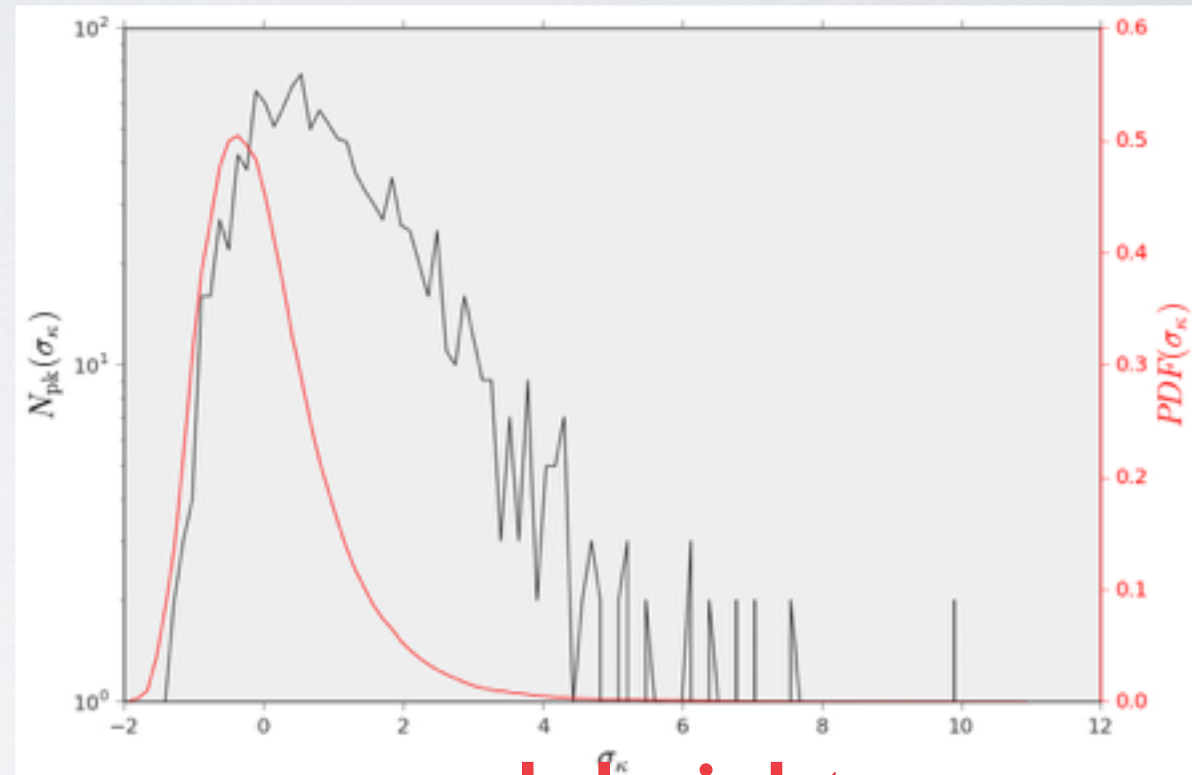
$$\langle \tilde{\kappa}(\boldsymbol{\ell})\tilde{\kappa}(\boldsymbol{\ell}')\rangle = (2\pi)^2\delta_D(\boldsymbol{\ell}+\boldsymbol{\ell}')P^{\kappa\kappa}(\ell)$$

HIGHER ORDER STATISTICS

- N-point correlation functions
- Peak (local maxima) counts
- Minkowski Functionals (topology)

PEAK COUNTS

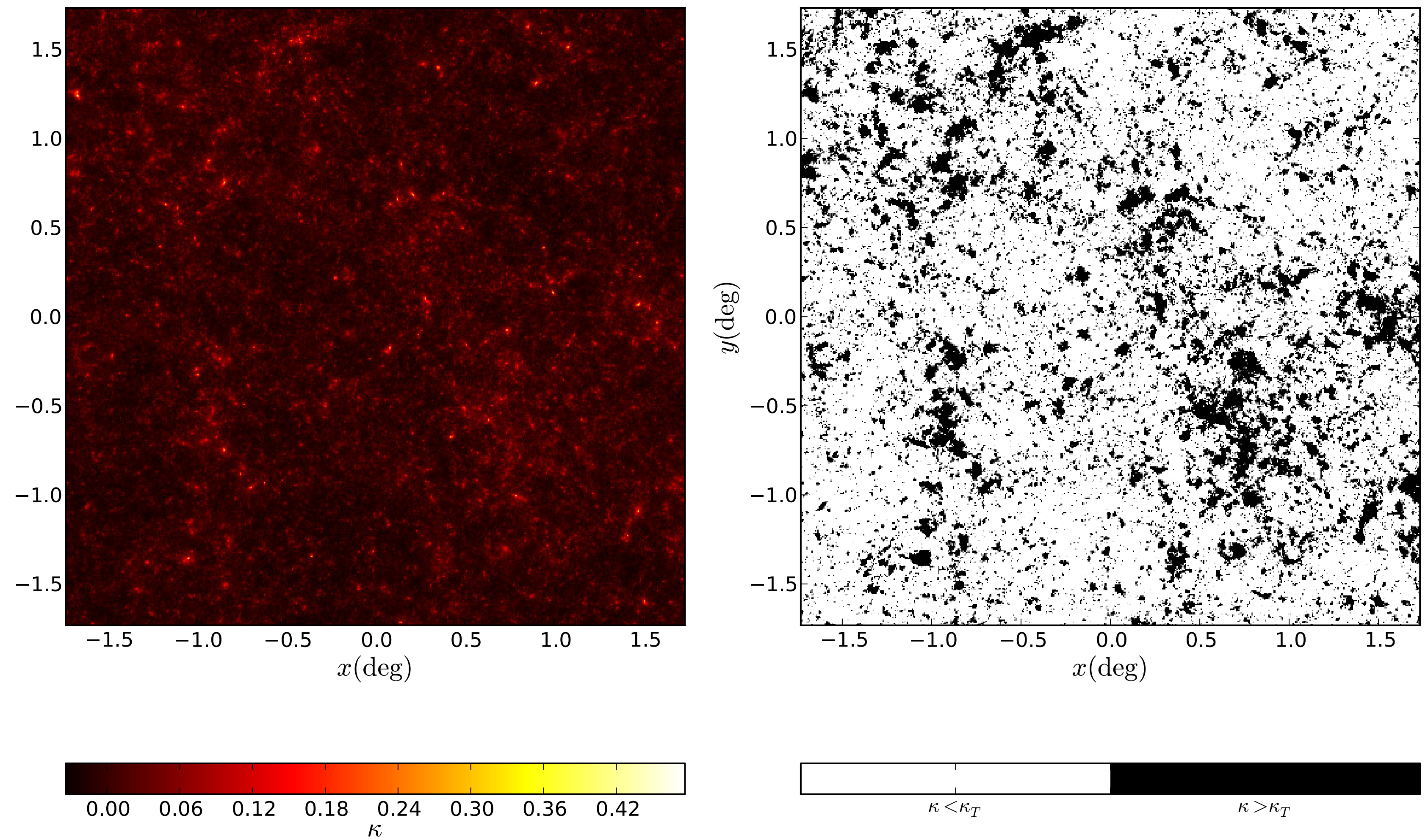




In the Gaussian case the peak histograms are entirely determined by quadratic statistics

$$\sigma_0^2 = \langle \kappa^2 \rangle \;\; ; \;\; \sigma_1^2 = \langle |\nabla \kappa|^2 \rangle \;\; ; \;\; \sigma_2^2 = \langle \kappa \nabla^2 \kappa \rangle$$

MINKOWSKI FUNCTIONALS



THRESHOLD KAPPA MAP

$$V_0(\kappa_T) \rightarrow \text{Area statistic}$$
$$V_1(\kappa_T) \rightarrow \text{Perimeter statistic}$$
$$V_2(\kappa_T) \rightarrow \text{Genus statistic}$$

$$V_k(\nu) = A_k \left(\frac{\sigma_0}{\sigma_1}\right)^k e^{-\nu^2/2}[H^{(0)}_{k-1}(\nu) + \sigma_0 H^{(1)}_{k-1}(\nu) + \sigma_0^2 H^{(2)}_{k-1}(\nu) + ...]$$

NG contributions

Munshi et. al. 2012

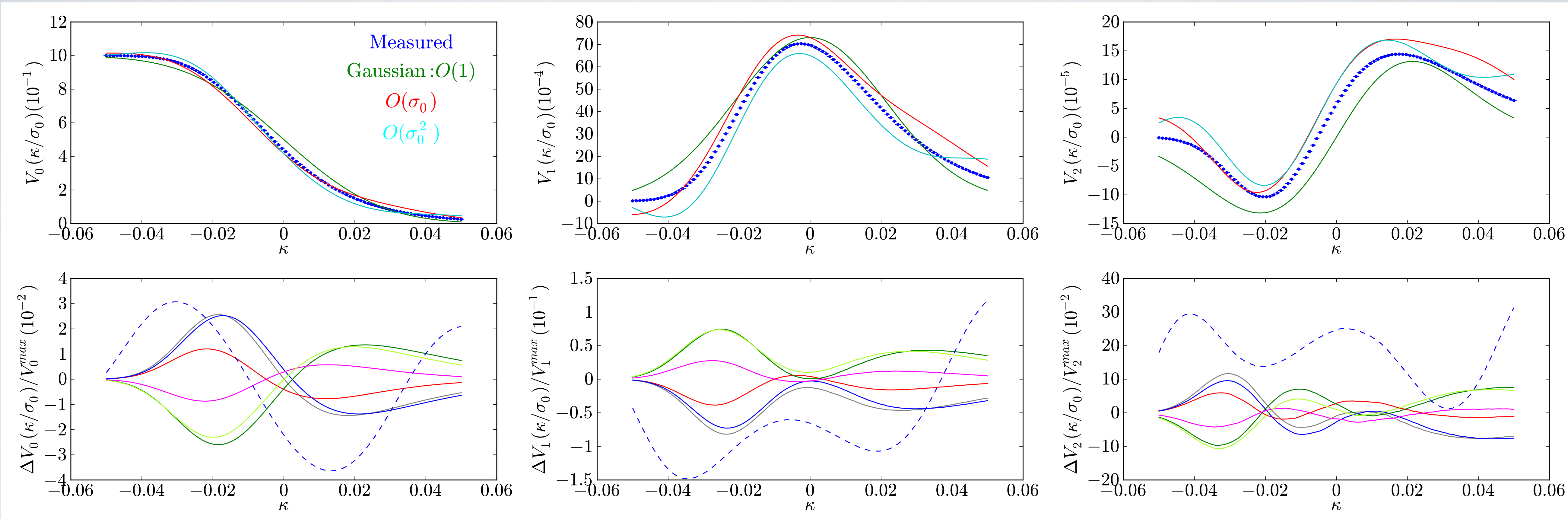
$$S_0 = \frac{\langle \kappa^3 \rangle}{\sigma_0^4}, \quad S_1 = \frac{\langle \kappa^2 \nabla^2 \kappa \rangle}{\sigma_0^2 \sigma_1^2}, \quad S_2 = \frac{2\langle |\nabla \kappa|^2 \nabla^2 \kappa \rangle}{\sigma_1^4}$$

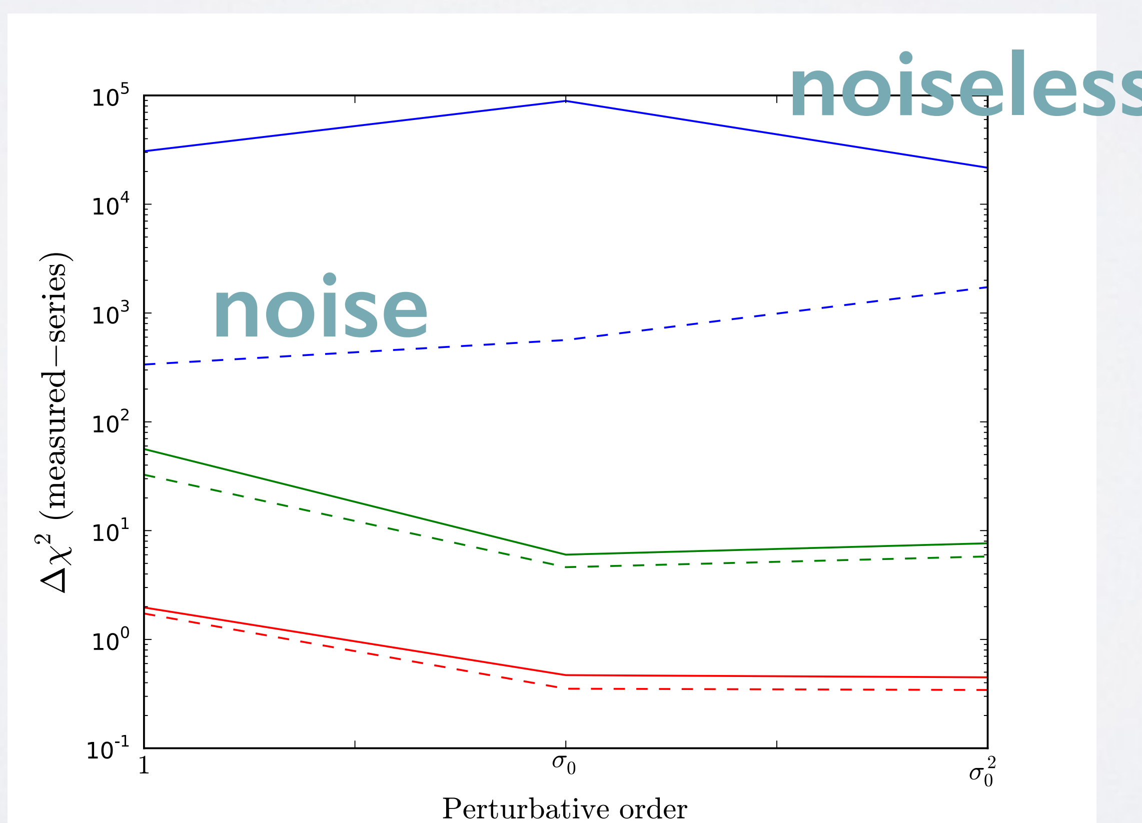
Higher order correlations!

$$K_0 = \frac{\langle \kappa^4 \rangle_c}{\sigma_0^6}, \quad K_1 = \frac{\langle \kappa^3 \nabla^2 \kappa \rangle_c}{\sigma_0^4 \sigma_1^2},$$

$$K_2 = \frac{\langle \kappa |\nabla \kappa|^2 \nabla^2 \kappa \rangle_c}{\sigma_0^2 \sigma_1^4}, \quad K_3 = \frac{\langle |\nabla \kappa|^4 \rangle_c}{\sigma_0^2 \sigma_1^4}.$$

AP et. al. , arXiv:1309.4460



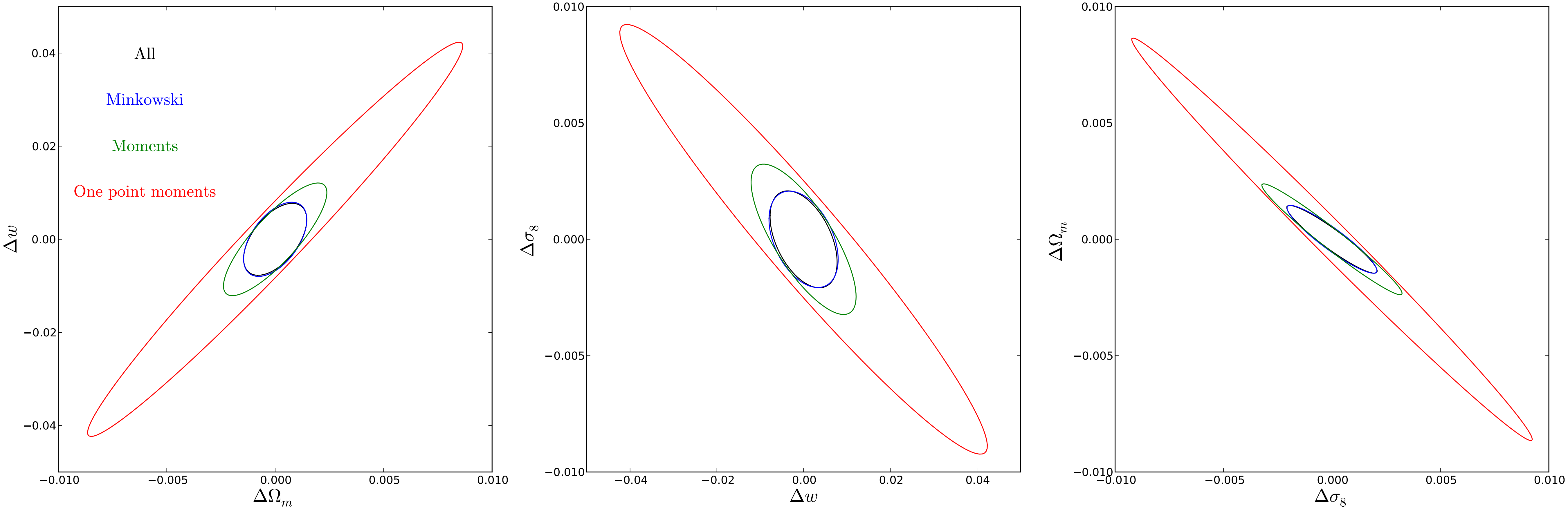


1 arcmin smoothing

5 arcmin smoothing

15 arcmin smoothing

LSST FORECAST



ARE NG STATISTICS USEFUL?

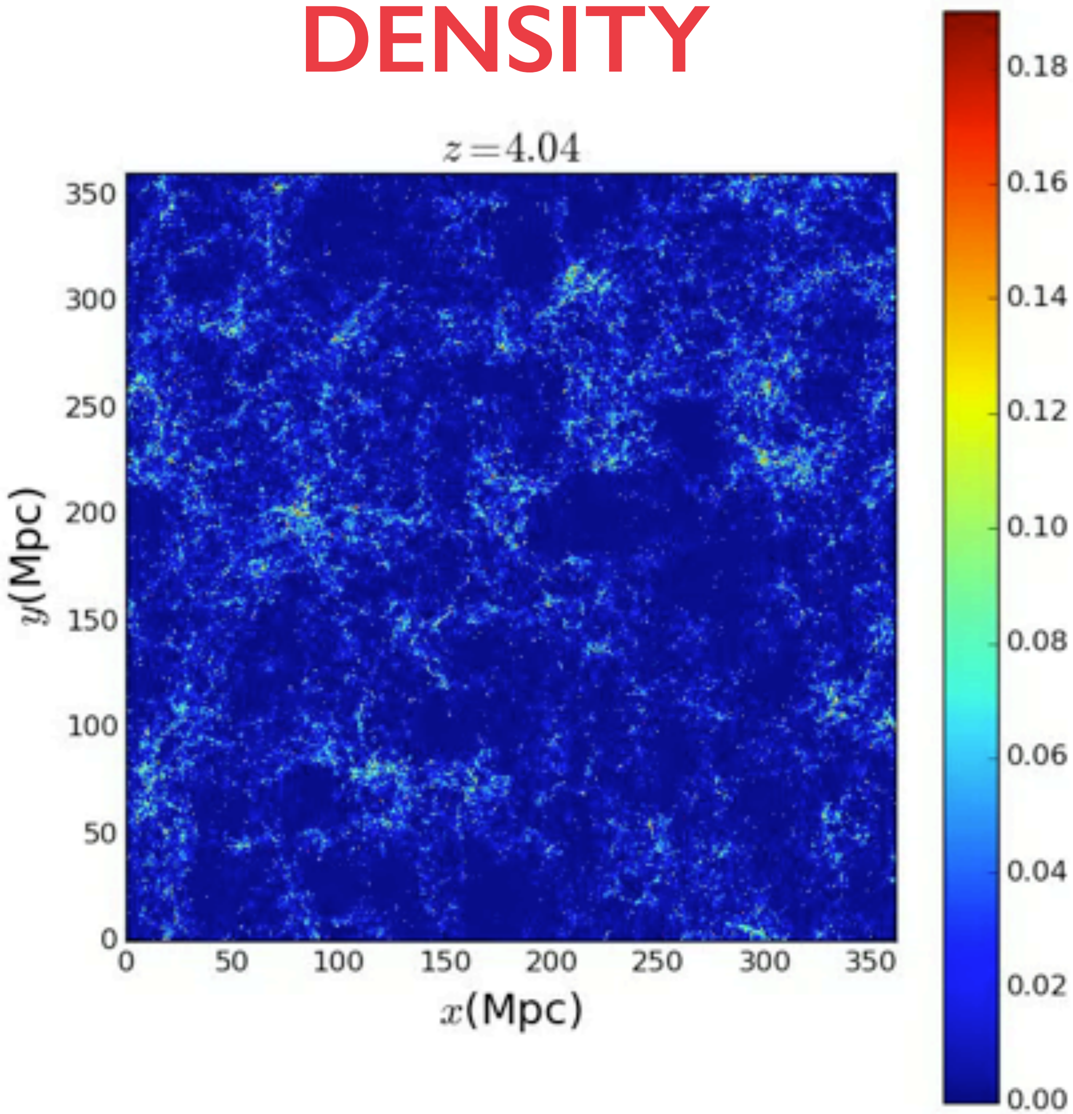
- Peaks, MFs = Contributions from higher-than-quadratic correlation functions
- No available analytical tools for studying wCDM constraining power with NG statistics
- Pixel-level numerical simulations needed!

WL NUMERICAL SIMULATIONS

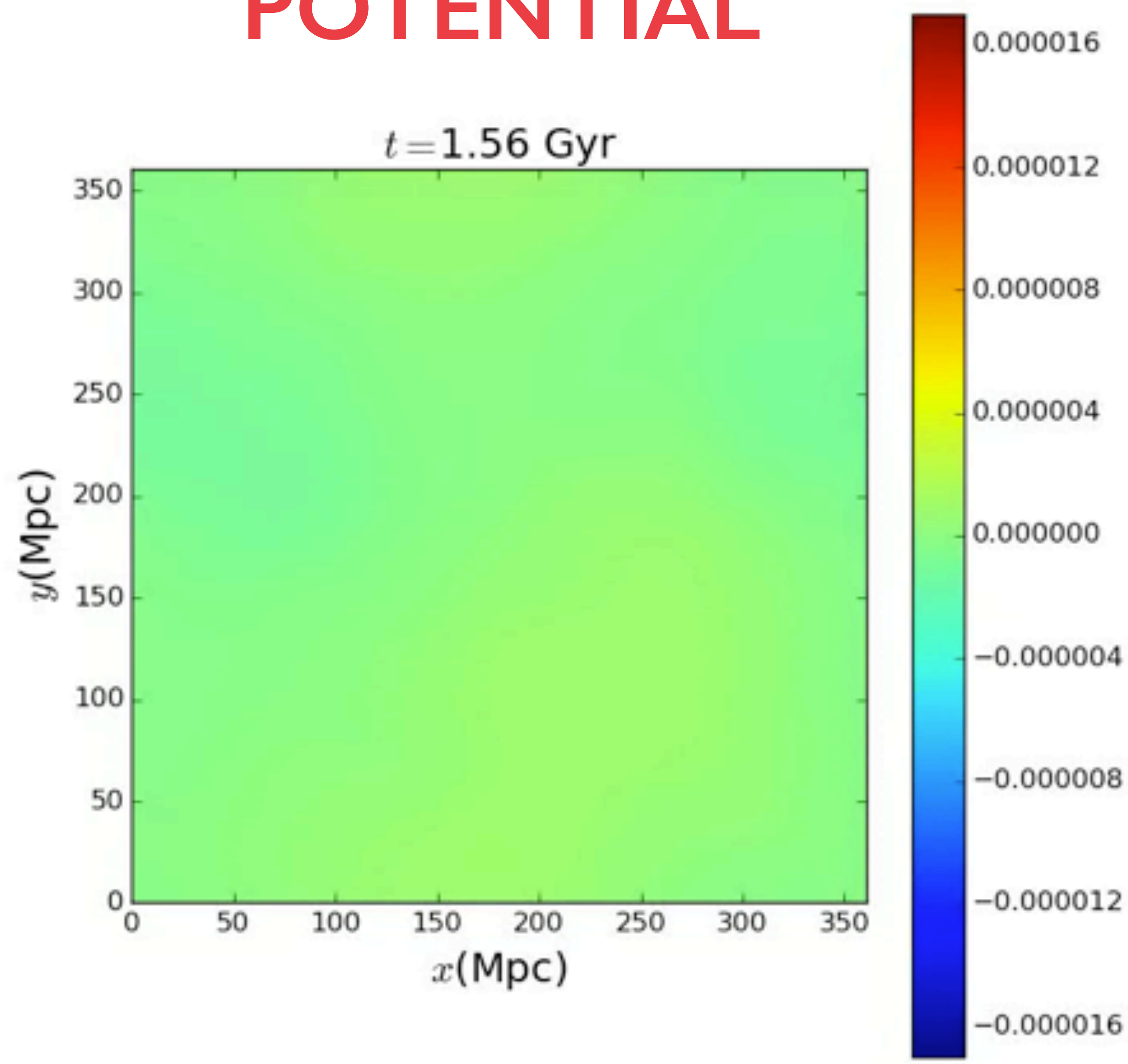
LENSING POTENTIAL SIMULATIONS

- N-body simulations using public code Gadget-2 (Springel 2005)
- Galaxy lensing: $(N_p, L_b) = (512^3, 240\mathrm{Mpc}/h)$
- CMB lensing: $(N_p, L_b) = (1024^3, 500\mathrm{Mpc}/h)$
- Measure density contrast with gridding (4096^2surface pixels)
- Solve Poisson equation $\nabla^2\Phi = \delta$ via FFT
- Explore wCDM parameter space to build emulators

DENSITY



POTENTIAL



RAY-TRACING

Multi-lens-plane algorithm (Jain, Seljak, White 1999), (Hilbert et. al. 2009)

$$\boldsymbol{\beta}_k = \boldsymbol{\theta} + \sum_{i=1}^{k} \delta\boldsymbol{\beta}_i$$

$$\delta\boldsymbol{\beta}_k = (B_k - 1)\delta\boldsymbol{\beta}_{k-1} + C_k \boldsymbol{\alpha}_k \ ; \ \delta\boldsymbol{\beta}_0 = 0$$

$$\mathbf{A}_k = \mathbb{1}_{2\times 2} + \sum_{i=1}^{k} \delta\mathbf{A}_i$$

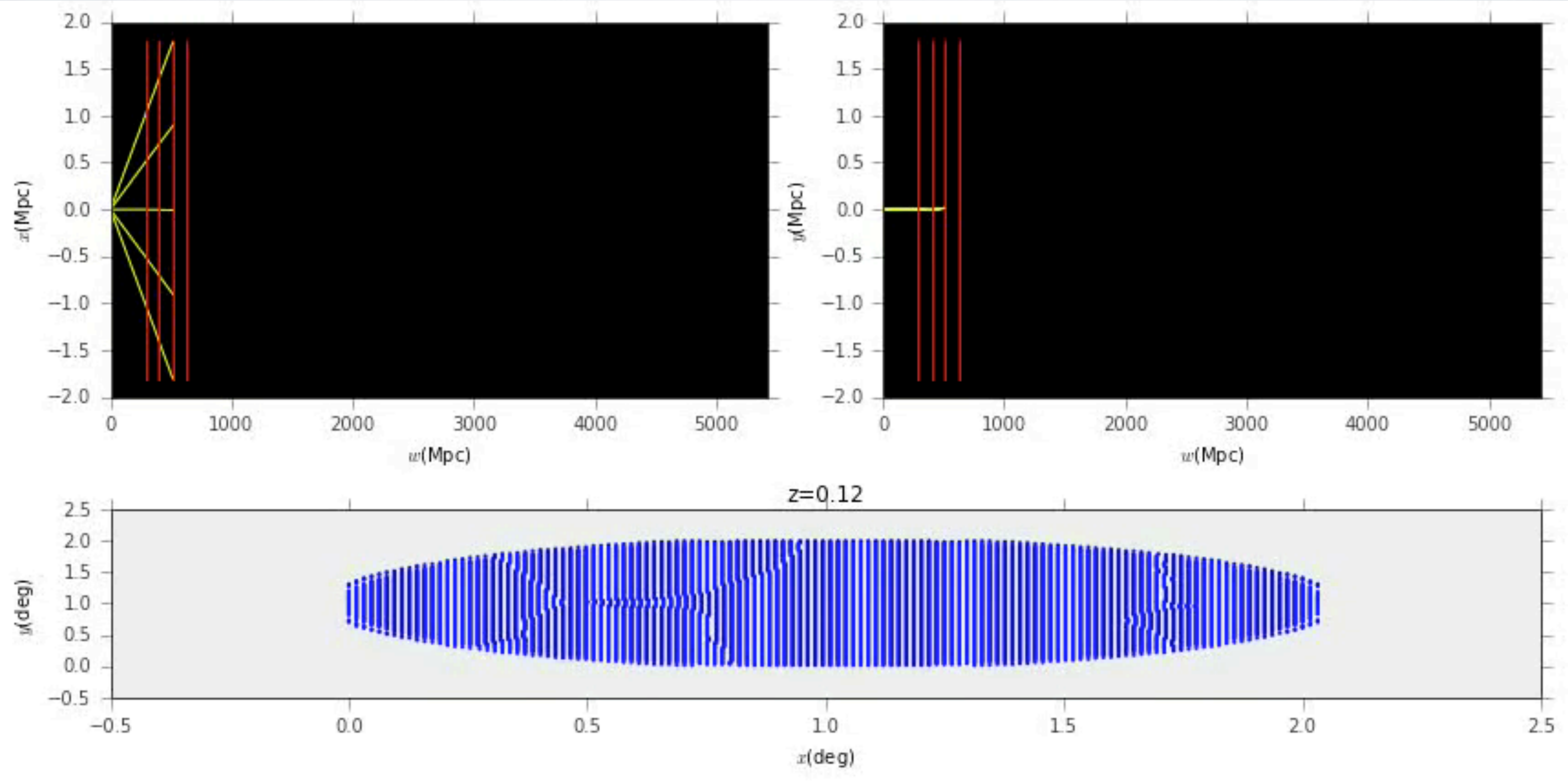
$$\delta\mathbf{A}_k = (B_k - 1)\delta\mathbf{A}_{k-1} + C_k \mathbf{T}_k \mathbf{A}_k \ ; \ \delta\mathbf{A}_0 = 0$$

Linear algorithm

$$\boldsymbol{\alpha}_k = \nabla_\beta \Phi_k(\chi_k \boldsymbol{\beta}_k) \ ; \ \mathbf{T}_k = \nabla_\beta \nabla_\beta^T \Phi_k(\chi_k \boldsymbol{\beta}_k)$$

(AP, arXiv:1606.01903)

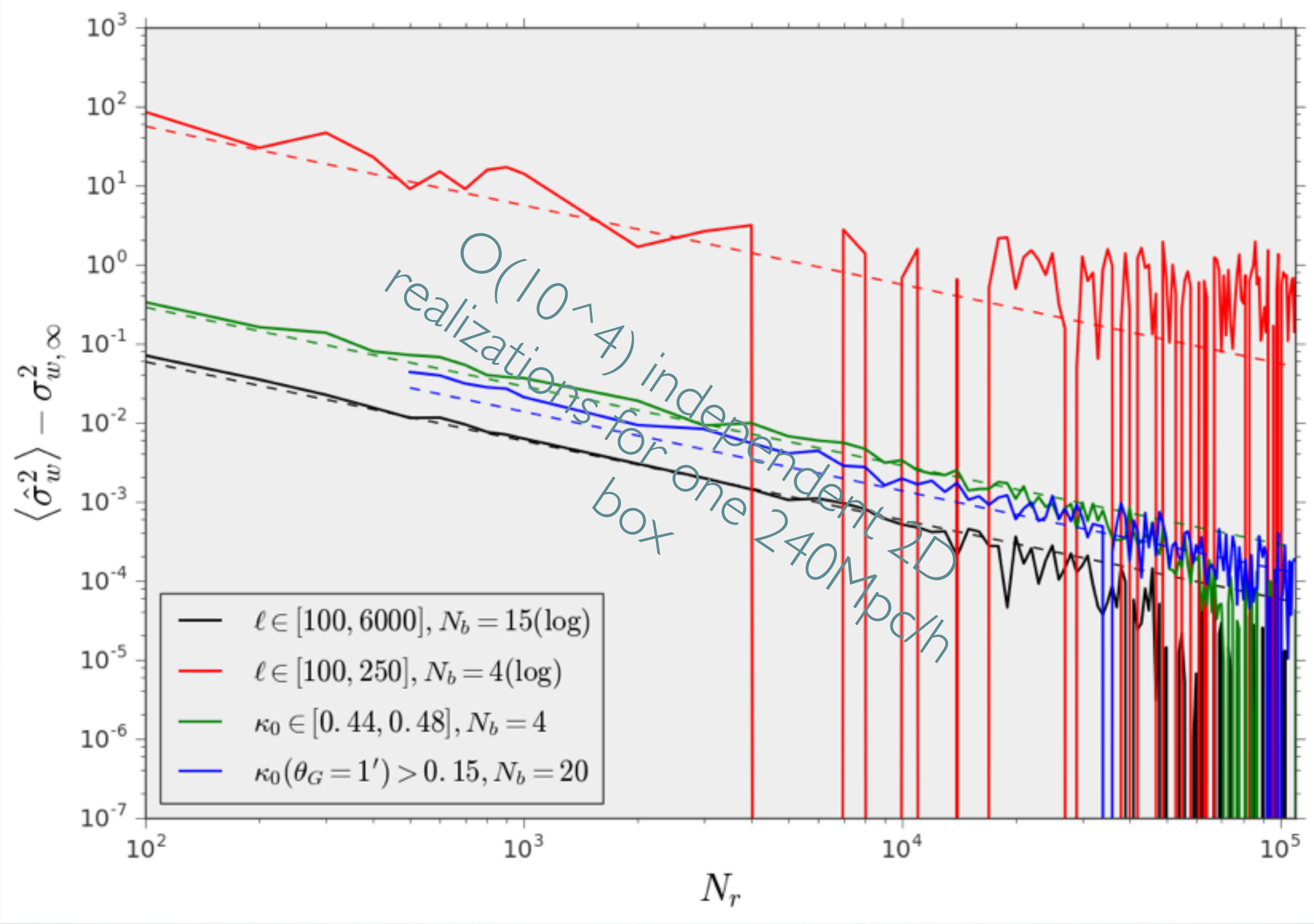
RAY TRACING EXAMPLE



RECYCLING OF 3D BOXES

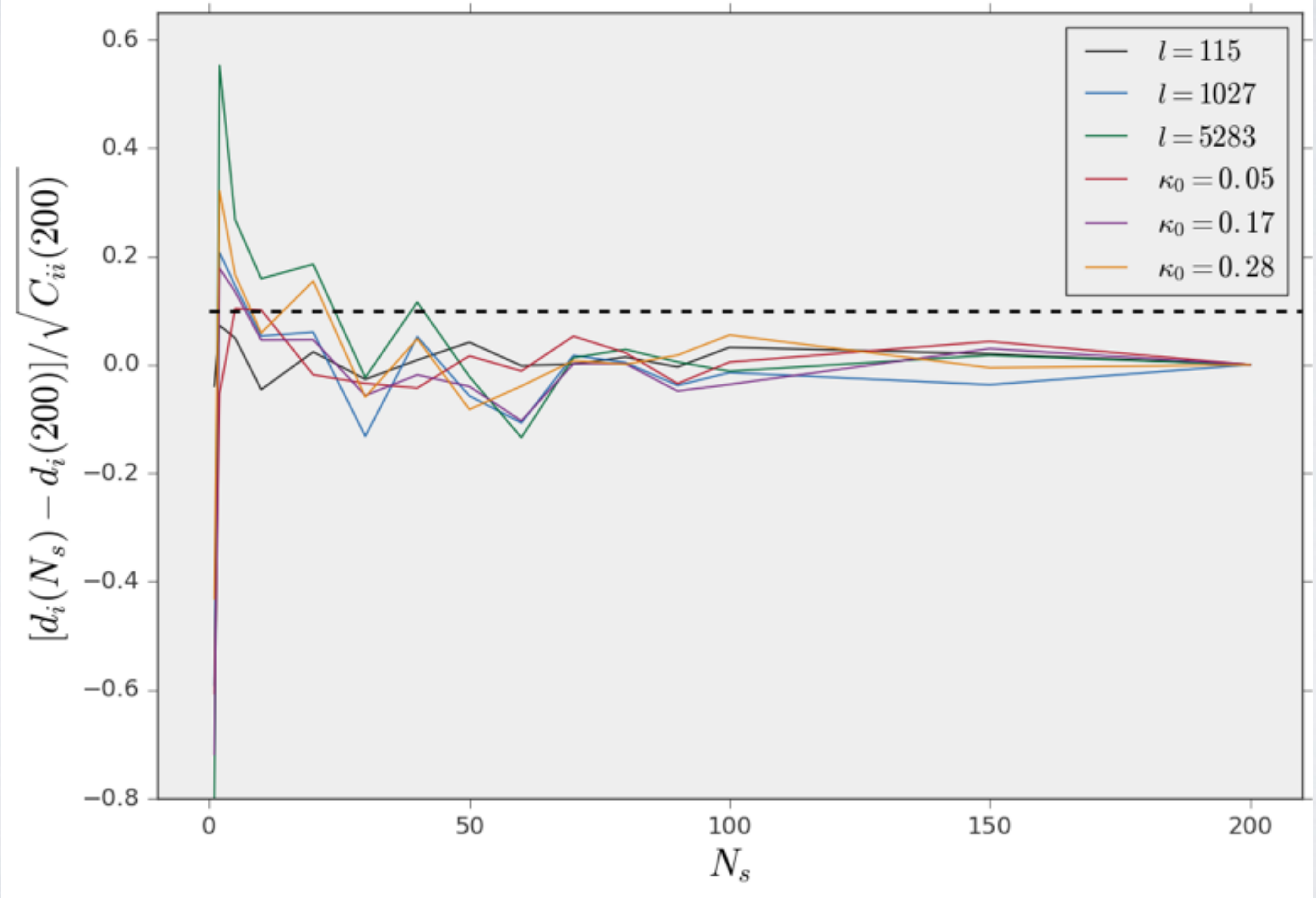
- Construct pseudo-independent realizations of 2D lensing potential
- Slice each 3D snapshot in 9 planes (3 slices per axis x,y,z)
- For each lens k, choose a random slice
- Periodically shift the slice along the axes (randomly)
- Place the slice on the line of sight and compute deflections/Jacobians
- Repeat for the remaining lenses
- Allows to generate multiple pseudo-random realizations of the same FOV using a single 3D box!

BOX RECYCLING



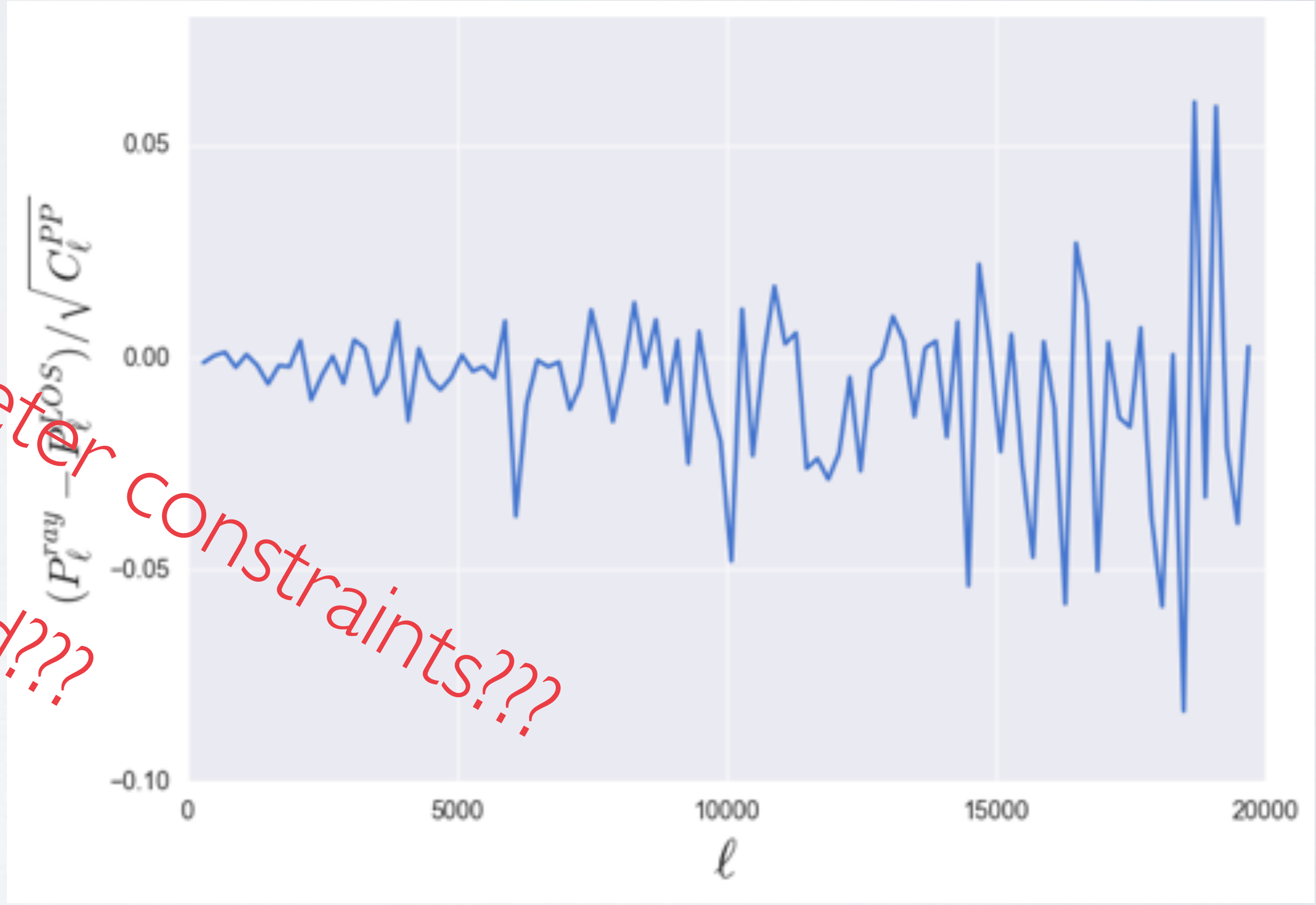
AP, Haiman, May, arXiv:1601.06792

FORWARD MODELING



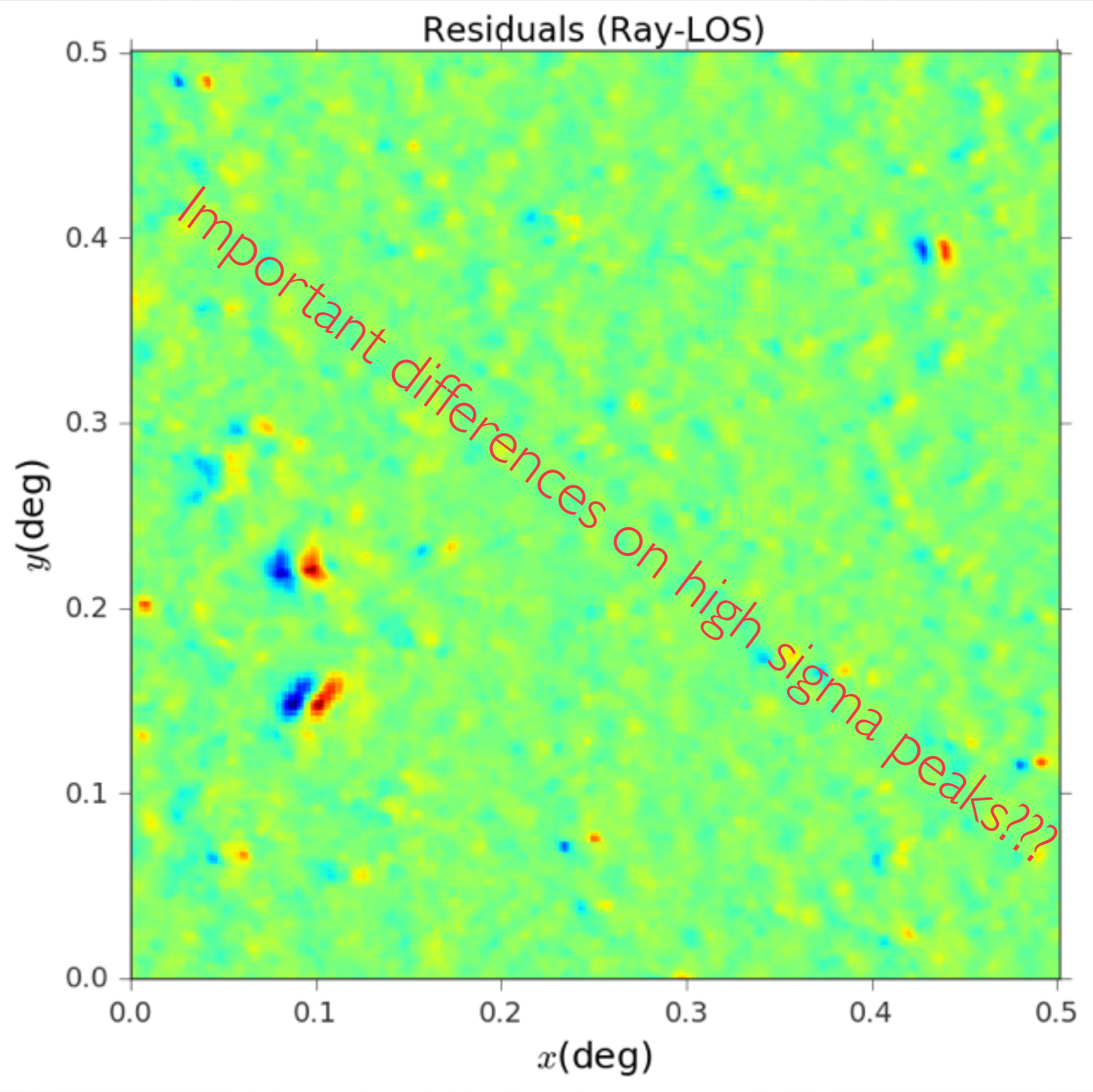
BORN APPROXIMATION VALIDITY

Work in progress…



Effect on parameter constraints???

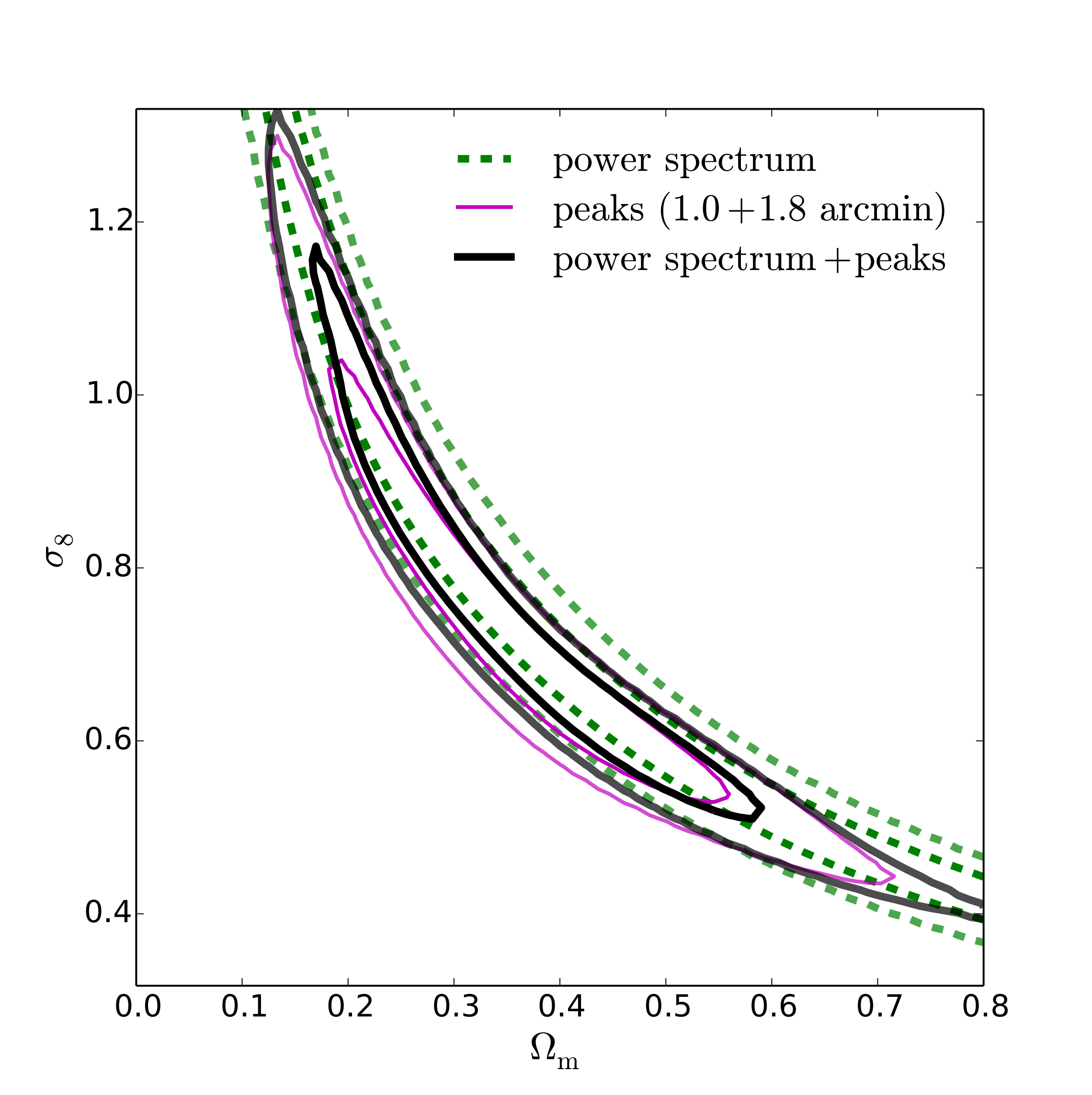
LSST, Euclid???

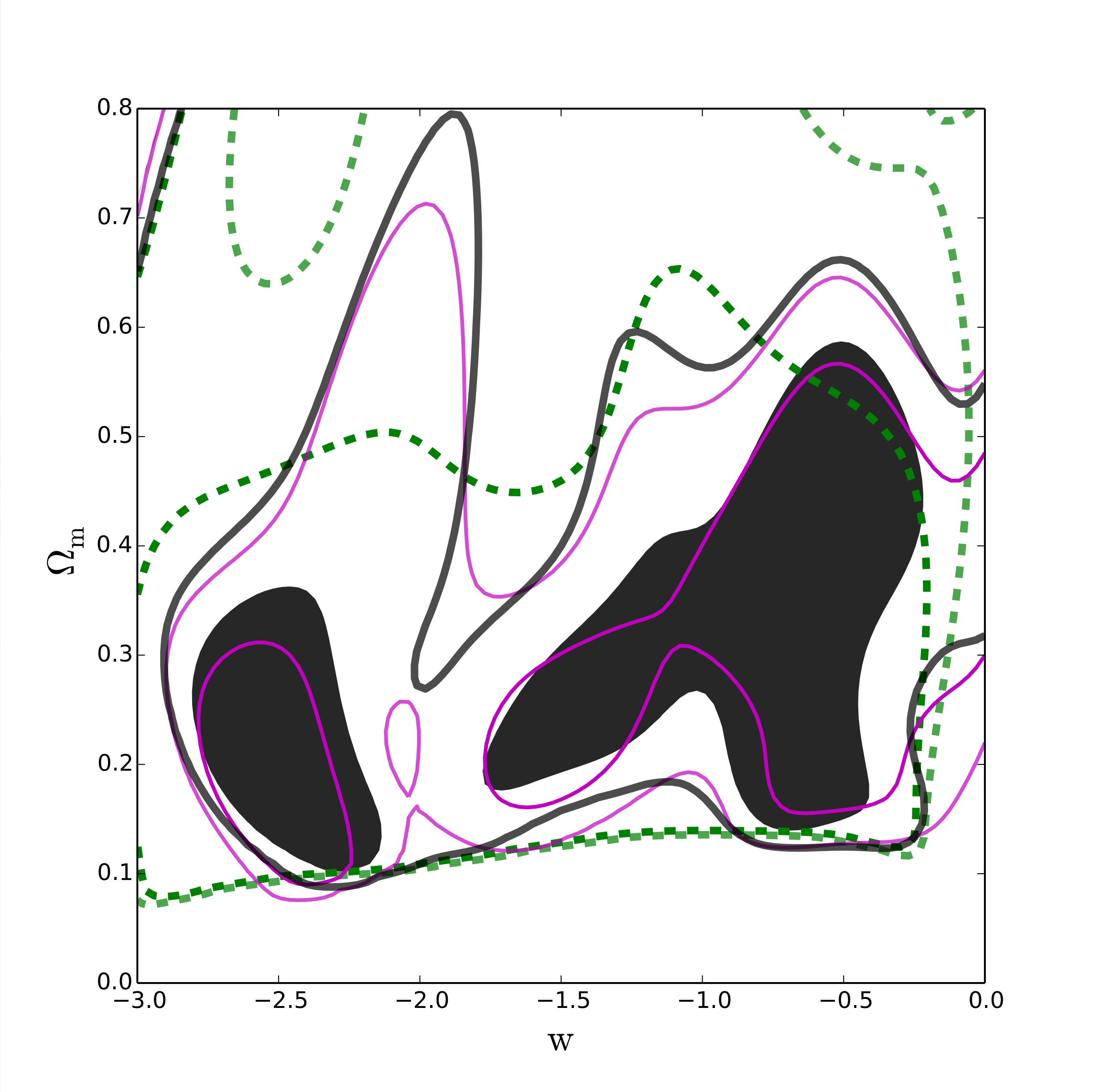

Residuals (Ray-LOS)
Important differences on high sigma peaks???
y(deg)
x(deg)
0.0
0.1
0.2
0.3
0.4
0.5

GALAXY LENSING CONSTRAINTS

CFHT LENS: PS + PEAKS

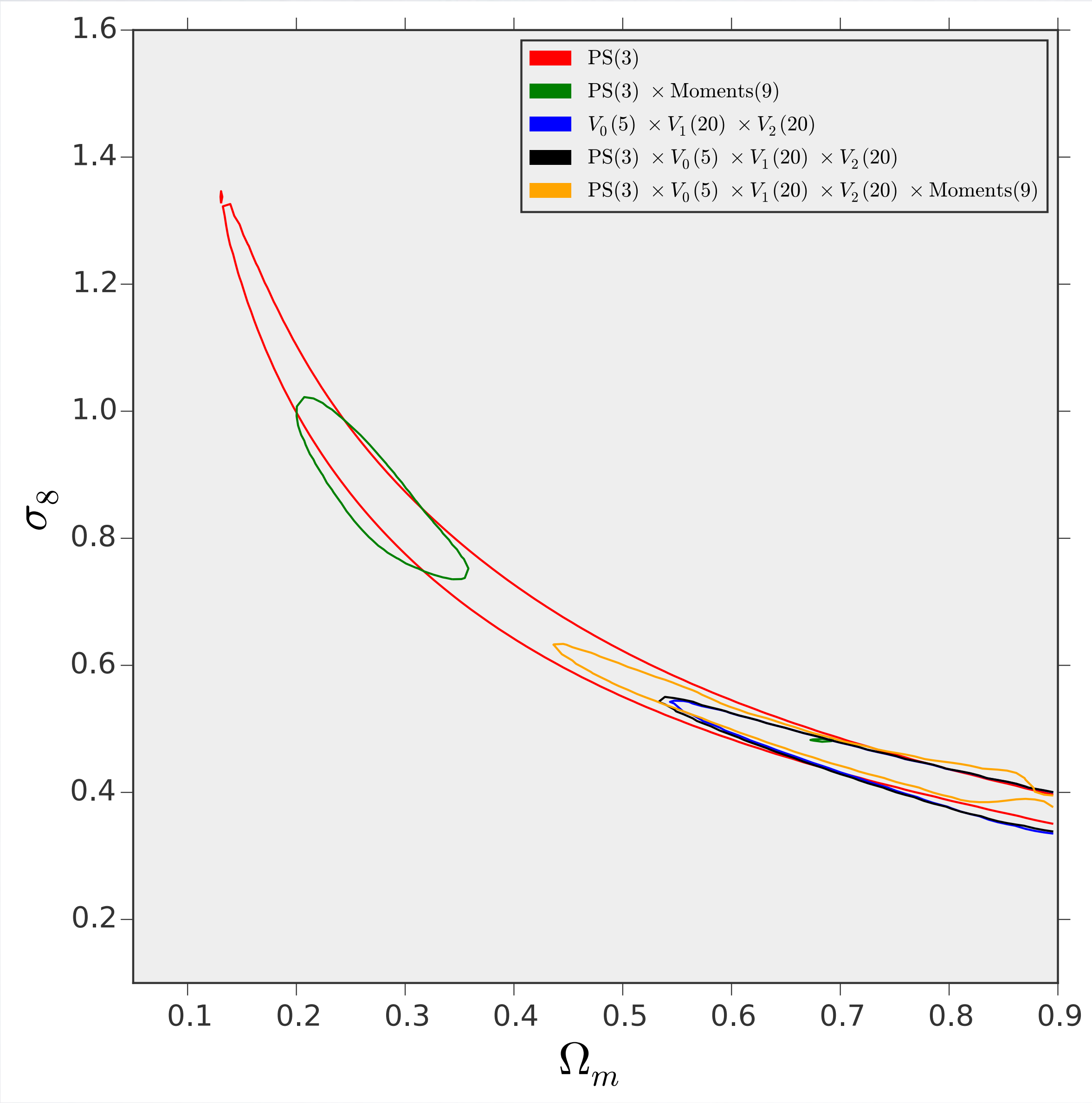
J. Liu, AP, et. al., arXiv:1412.0757v3

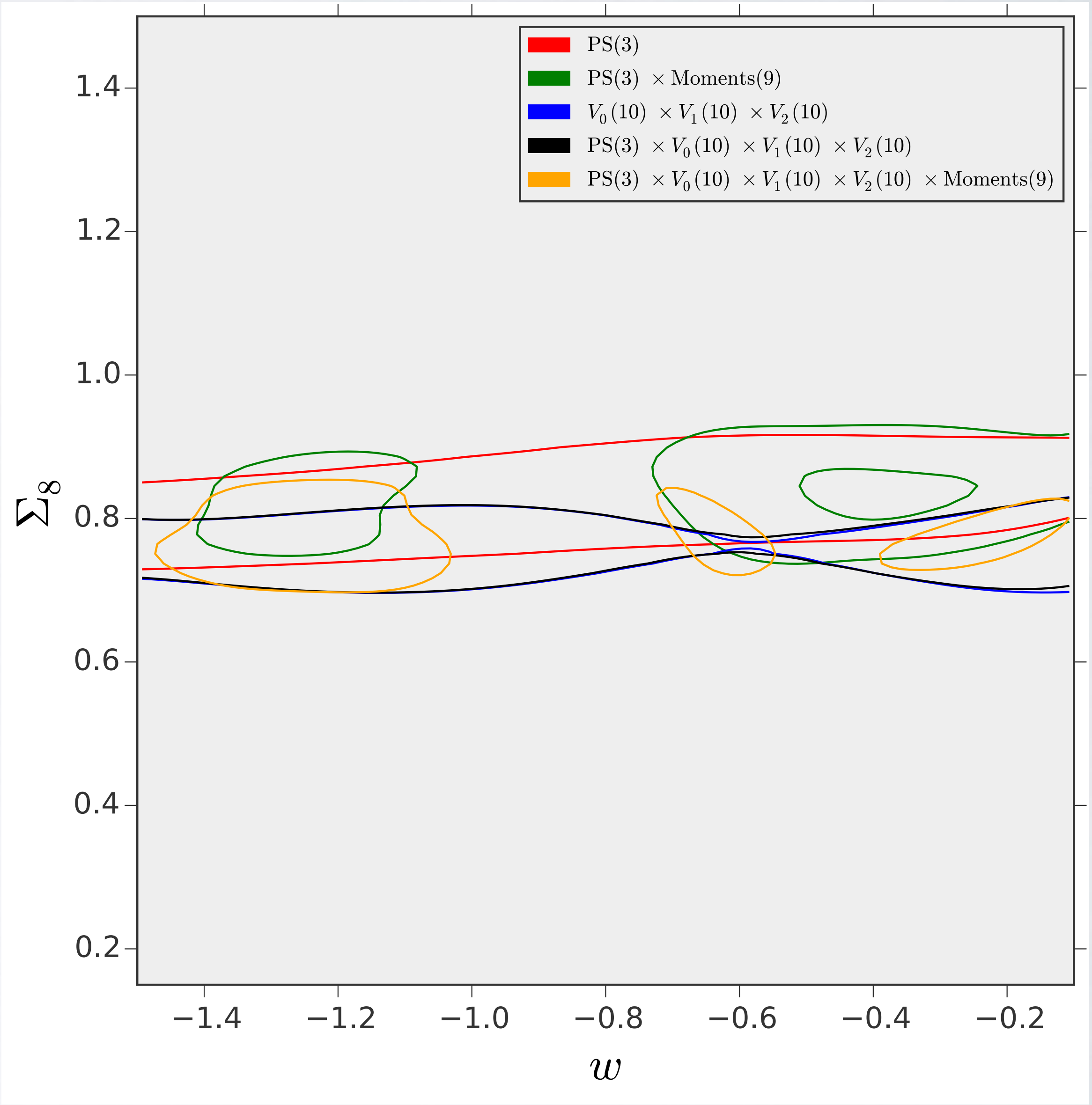




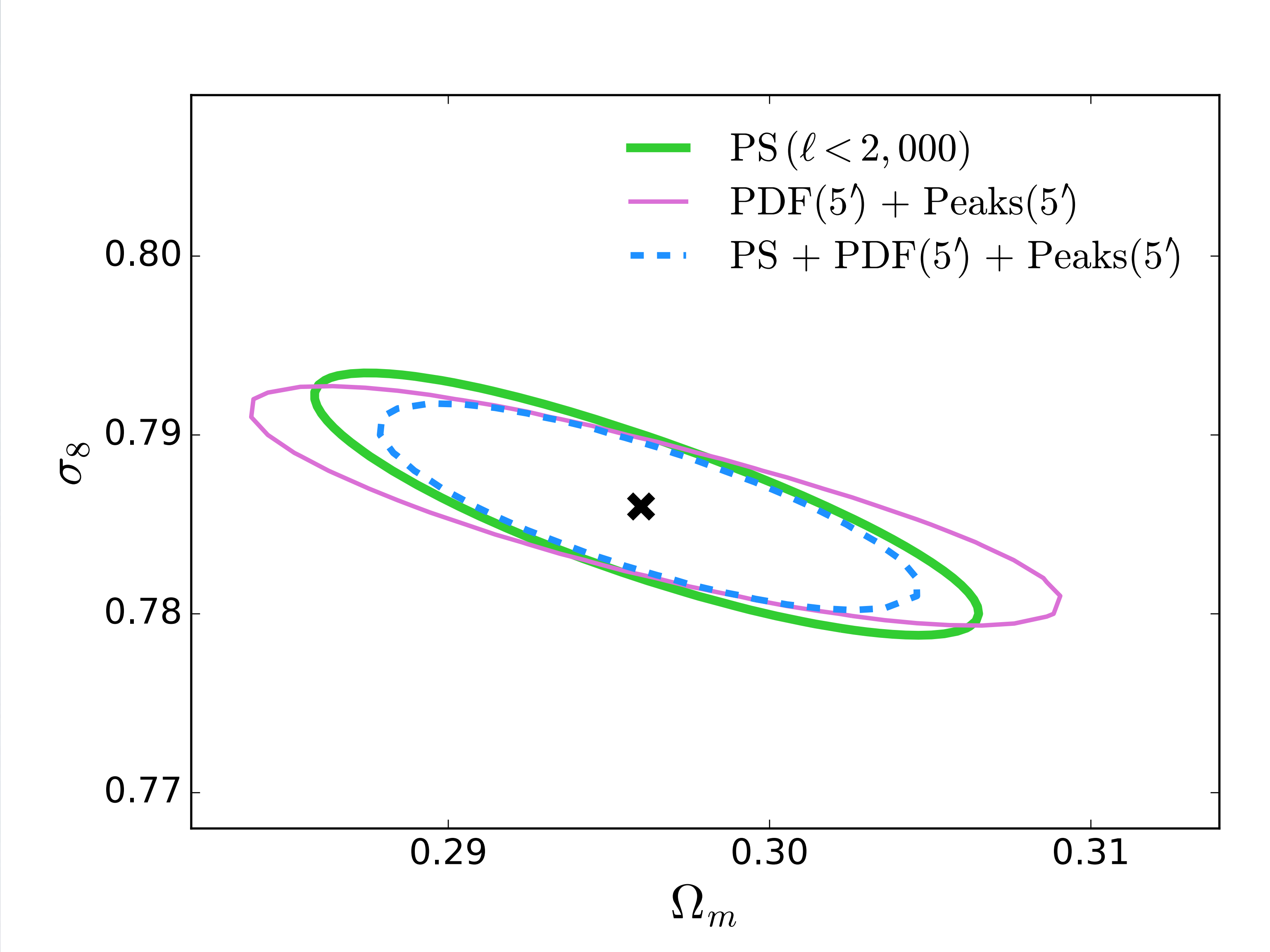
CFHT LENS: PS + MF + MOMENTS

AP, J.Liu, et. al., arXiv:1503.06214





CMB LENSING CONSTRAINTS

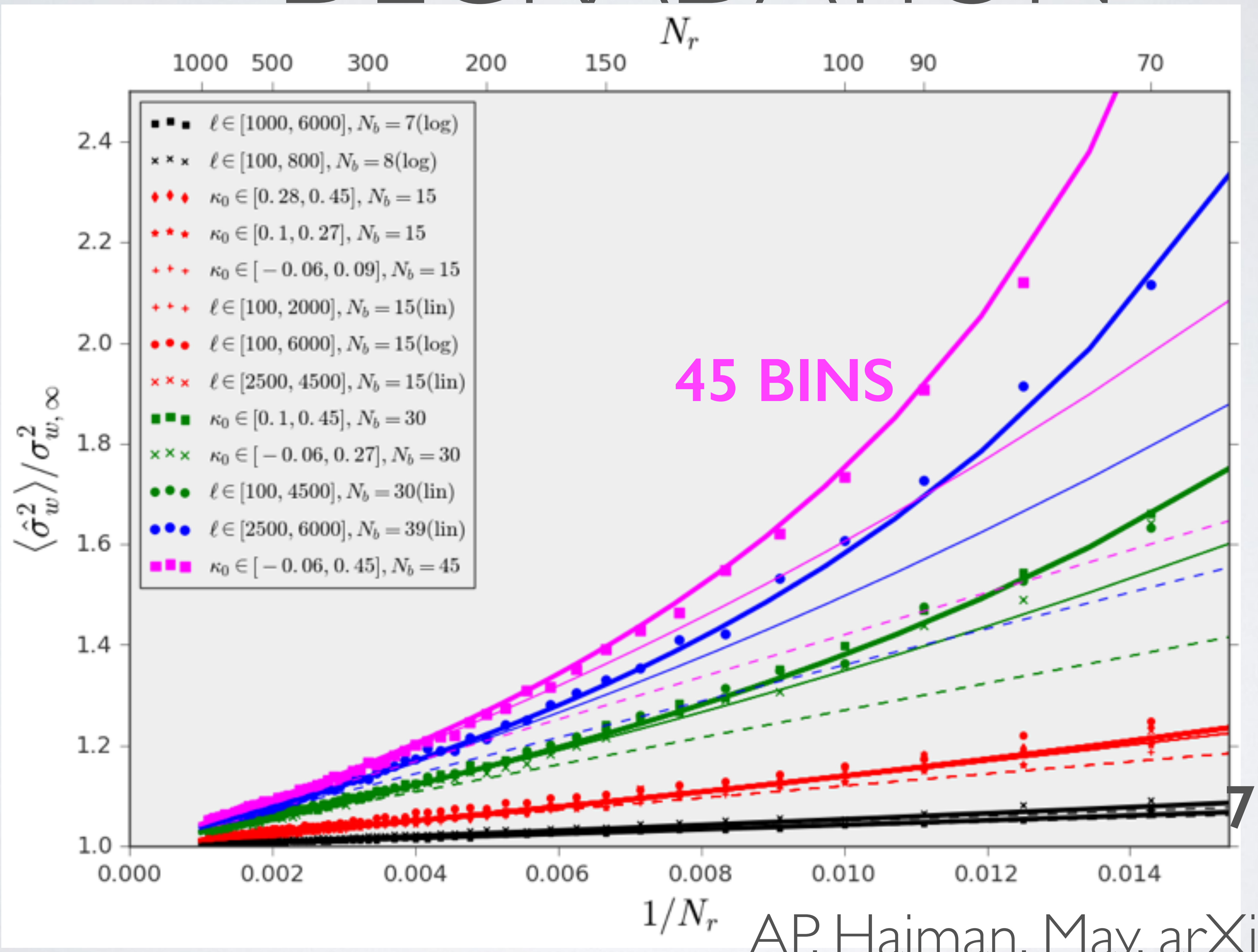


J.Liu, J.C.Hill, B.Sherwin, AP, V.Bohm, Z.Haiman arXiv: 1608.03169

- 30 sigma (PDF), 15 sigma (peak counts) contributions from non-linear LSS to CMB lensing signal
- PS more constraining due to shape information
- NG statistics helpful in breaking degeneracies, when combined with PS
- Moments + MF: work in progress…

WL REQUIREMENTS FOR FUTURE SURVEYS

PARAMETER CONSTRAINT DEGRADATION



AP, Haiman, May, arXiv:1601.06792

LESSONS LEARNED

- Trade-off: high dimensional features=more information, but estimates scatter increases!
- Keep dimensionality low while retaining cosmological information (dimensionality reduction problem)
- Relevant for future surveys (LSST) which use tomographic redshift information!
- Possible solution: linear PCA (investigated), non-linear techniques (LLE, work in progress…)
- Possible solution: shrinkage covariance estimation (Pope, Szapudi, 2007)

SUMMARY

- WL observables are non-Gaussian distributed (δ NG + Post-Born)

- NG statistics (peaks, moments, MF) can complement the PS when analyzing WL data (tighter constraints + break degeneracies)

- Numerical simulations and efficient algorithms needed for NG forward modeling (emulators)

- Dimensionality reduction is a very relevant problem nowadays!

FUTURE PROSPECTS

- Study importance of post-Born effects to NG statistics (trade off simulation efficiency/accuracy)

- Develop new, physics-oriented dimensionality reduction techniques to get the most out of data

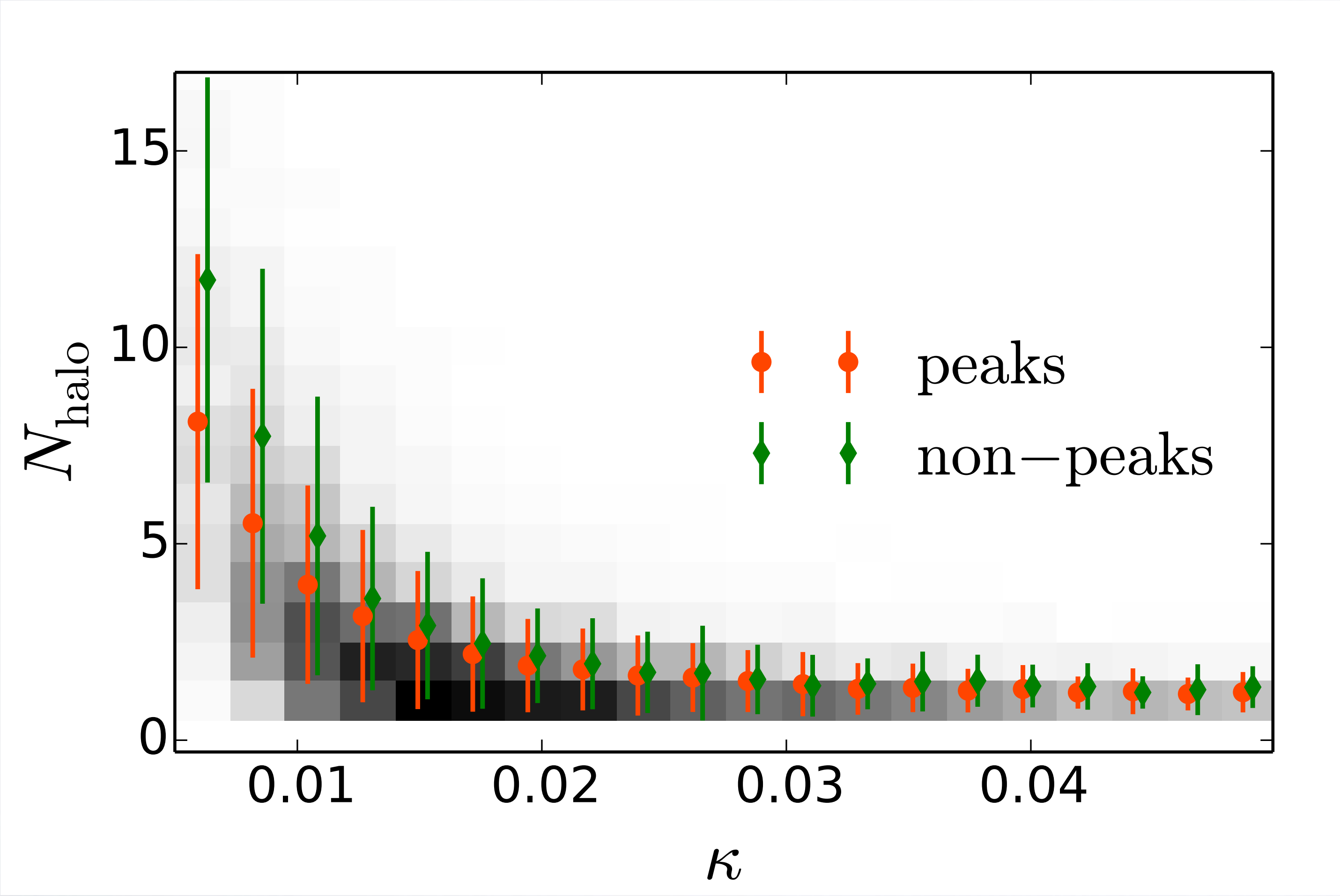
- Address WL systematics (photo-z, intrinsic galaxy alignments, baryon effects)

Thank you for your attention!

(Questions)

CFHT LENS: ORIGIN OF PEAKS

Use CHFT catalog stellar mass info to infer halo masses (Liu, Haiman, arXiv: 1606.01318)



- High peaks: single halo, 10^15 Msun
- Low peaks: 8-10 small halos 10^13 Msun, offset from the line of sight

COVARIANCE MATRIX TECHNOLOGY

If the distribution of feature realizations is Normal:

$$\hat{C}^{ff} = \frac{1}{N_r - 1} \sum_{r=1}^{N_r} (\hat{f}_r - \bar{f})(\hat{f}_r - \bar{f})^T \sim \mathcal{W}(C^{ff}, N_r)$$

$$\langle \hat{C}^{ff} \rangle = C^{ff} \qquad \langle \hat{\Psi} \rangle = \left(\frac{N_r - 1}{N_r - N_b - 2} \right) \Psi$$

PARAMETER CONSTRAINTS: REVIEW

- N_b image features $\hat{f} \rightarrow N_p$ parameter estimates $\hat{p}$
- Forward model $f(p)$ (simulations, analytical, emulators)
- $N_b \times N_b$ feature covariance matrix $\hat{C}^{ff}$ (simulations!)
- Normal data likelihood, flat parameter priors

$$\mathcal{L}(p|f,\hat{f},\hat{C}^{ff}) \propto \exp\left(-\frac{1}{2}\{\hat{f}-f(p)\}^T\{\hat{C}^{ff}\}^{-1}\{\hat{f}-f(p)\}\right)$$

Linearized ML estimator: $\hat{p} = p_0 + (M\hat{\Psi}M^T)^{-1}M\hat{\Psi}(\hat{f}-f_0)$

$f_0 = f(p_0)$; $\hat{\Psi} = (\hat{C}^{ff})^{-1}$; $M_{ij} = \left(\frac{\partial f_j}{p_i}\right)_{\partial p_0}$

ML estimate is unbiased, but how big is the scatter??

$$\hat{\Sigma}_1 = (M\hat{\Psi}M^T)^{-1}$$

$$\hat{\Sigma}_2 = \langle(\hat{p}-p_0)(\hat{p}-p_0)^T\rangle_{\mathrm{observations}} = (M\hat{\Psi}M^T)^{-1}M(\hat{\Psi}C^{ff}\hat{\Psi})M^T(M\hat{\Psi}M^T)^{-1}$$

PARAMETER ERROR DEGRADATION

Error from simulations:

$$\langle \hat{\Sigma}_1 \rangle = \left(\frac{N_r - N_b + N_p - 1}{N_r - N_b - 2} \right) \Sigma = \left[1 + O\left(\frac{1 + N_p}{N_r} \right) \right] \Sigma$$

Real scatter of estimates:

$$\langle \hat{\Sigma}_2 \rangle = \left(\frac{N_r - 2}{N_r - N_b + N_p - 2} \right) \Sigma = \left[1 + O\left(\frac{N_b - N_p}{N_r} \right) \right] \Sigma$$

Grows with feature dimensionality!!!

Taylor, Joachimi, arXiv:1402.6983