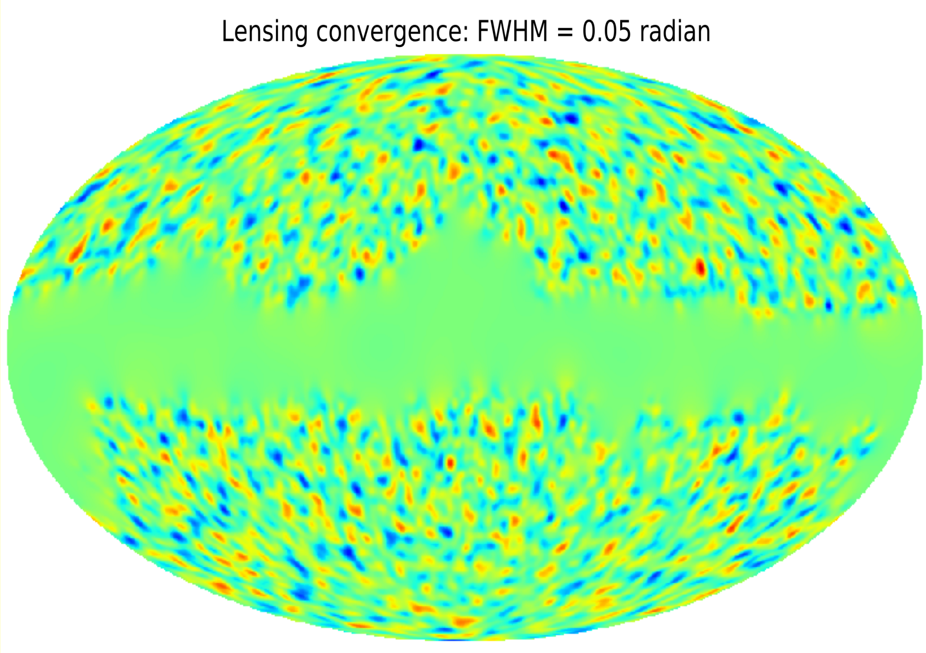
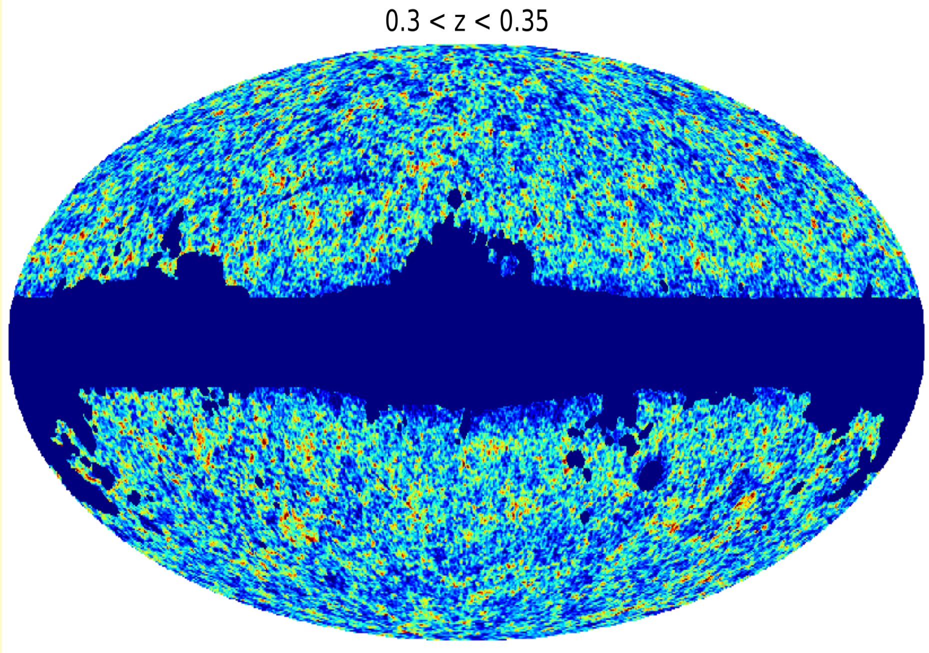
Gravitational lensing of the CMB:

Flux conservation and growth of inhomogeneities

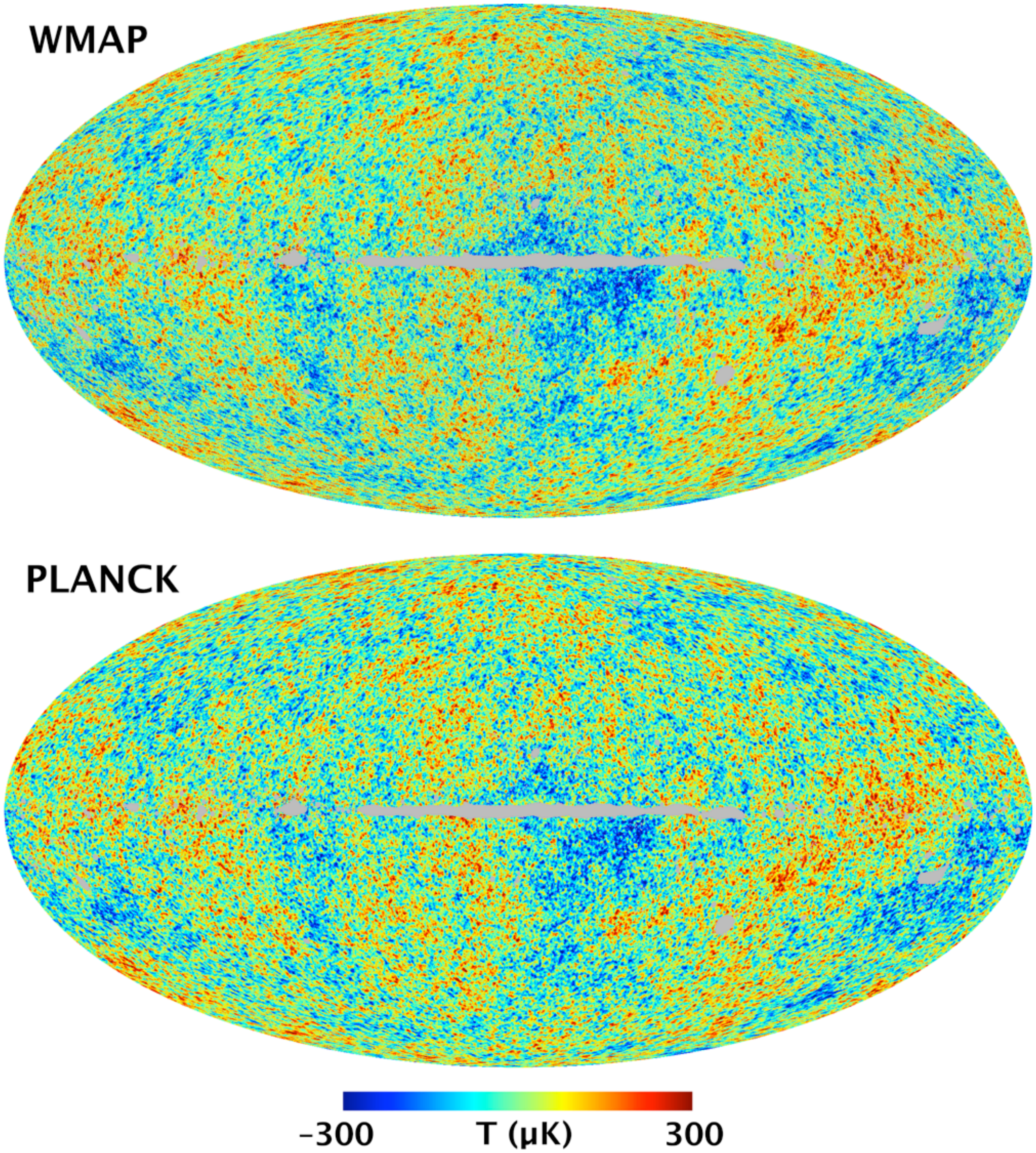




John Peacock

Berkeley

14 April 2015

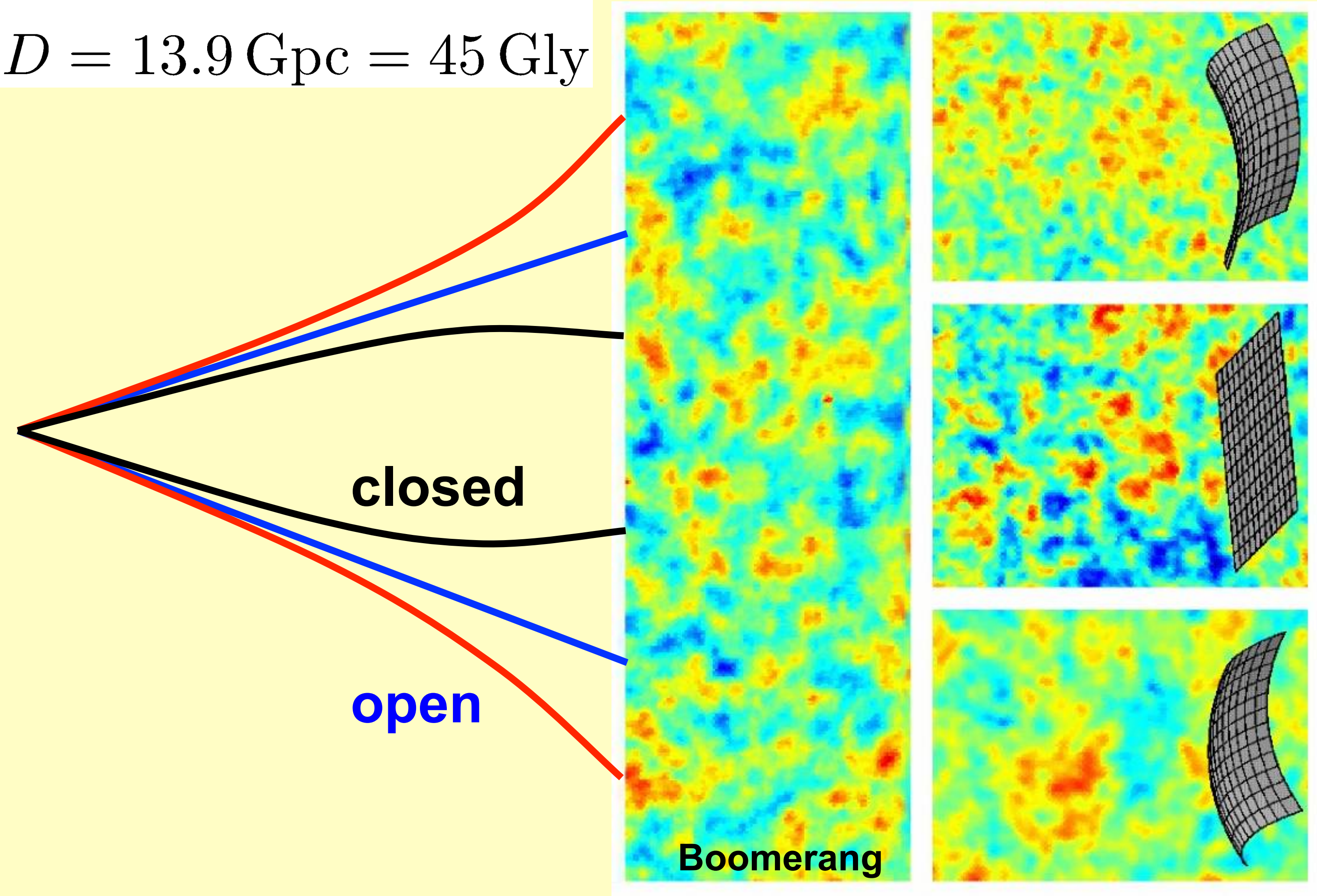


The CMB dominates cosmological knowledge

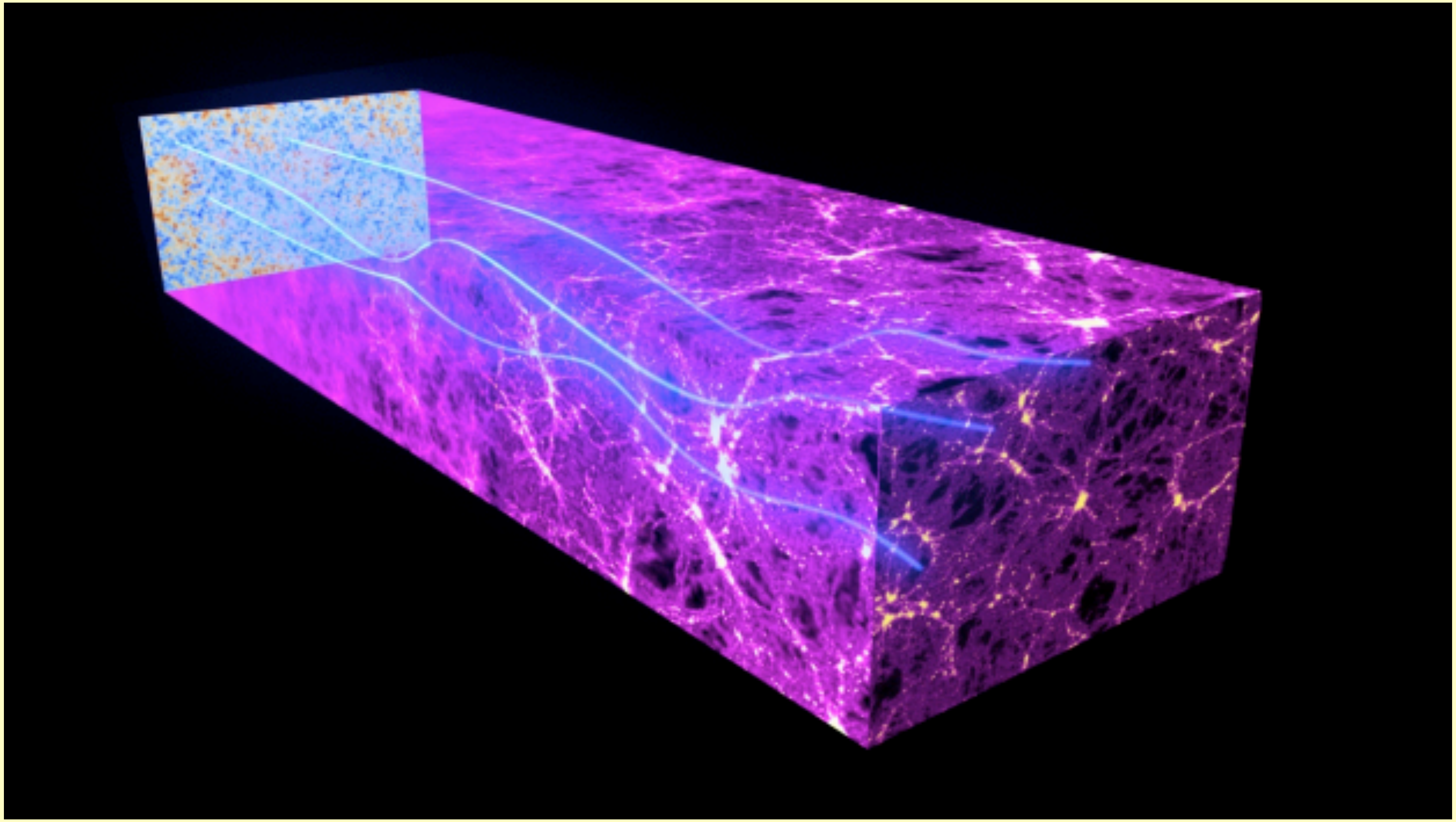
How far away is the CMB?

$$D = \int \frac{c}{H(z)}\, dz$$

$$z = 1080 \Rightarrow D = 13.9\,\mathrm{Gpc} = 45\,\mathrm{Gly}$$



Gravitational lensing of the CMB

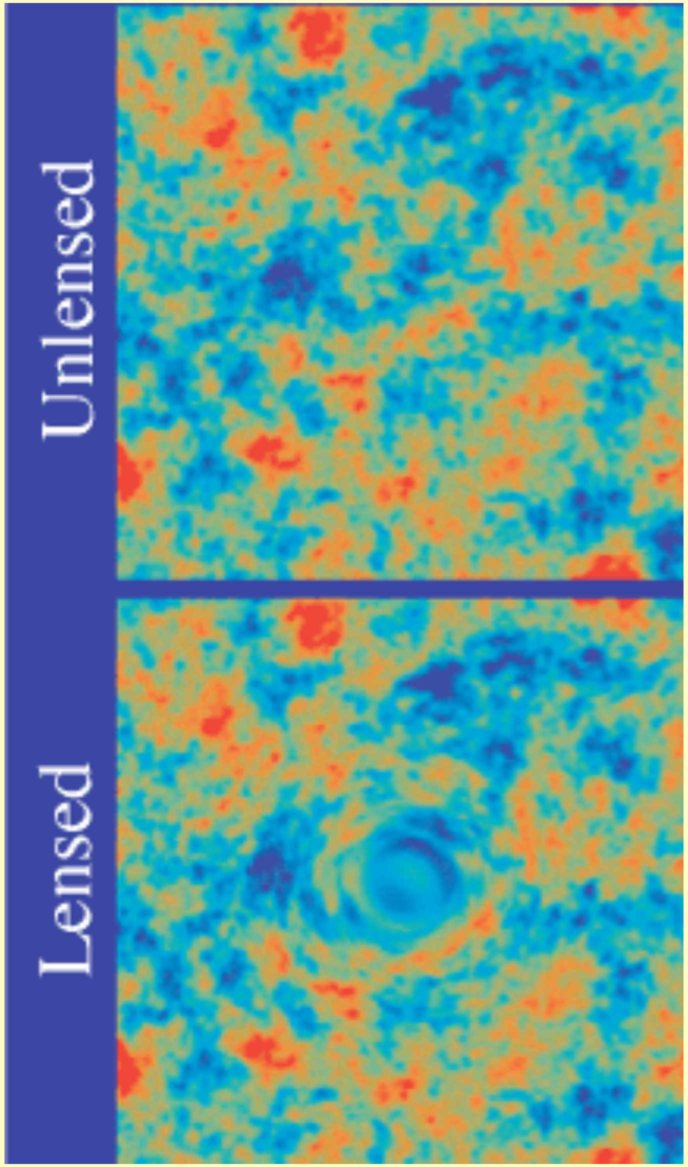


Foreground matter fluctuations deflect light and distort apparent CMB sky map

Outline

- Could accumulated effects of lensing alter mean distances?
  - Clarkson et al. 2014: yes, by ~ 1%
  - Non-Newtonian ‘backreaction’
- Quick overview of gravitational lensing
- Kaiser & JP 2015:
  - Maybe
  - But not backreaction
- Observing CMB lensing
  - Tomographic cross-correlation with WISE+SuperCOSMOS

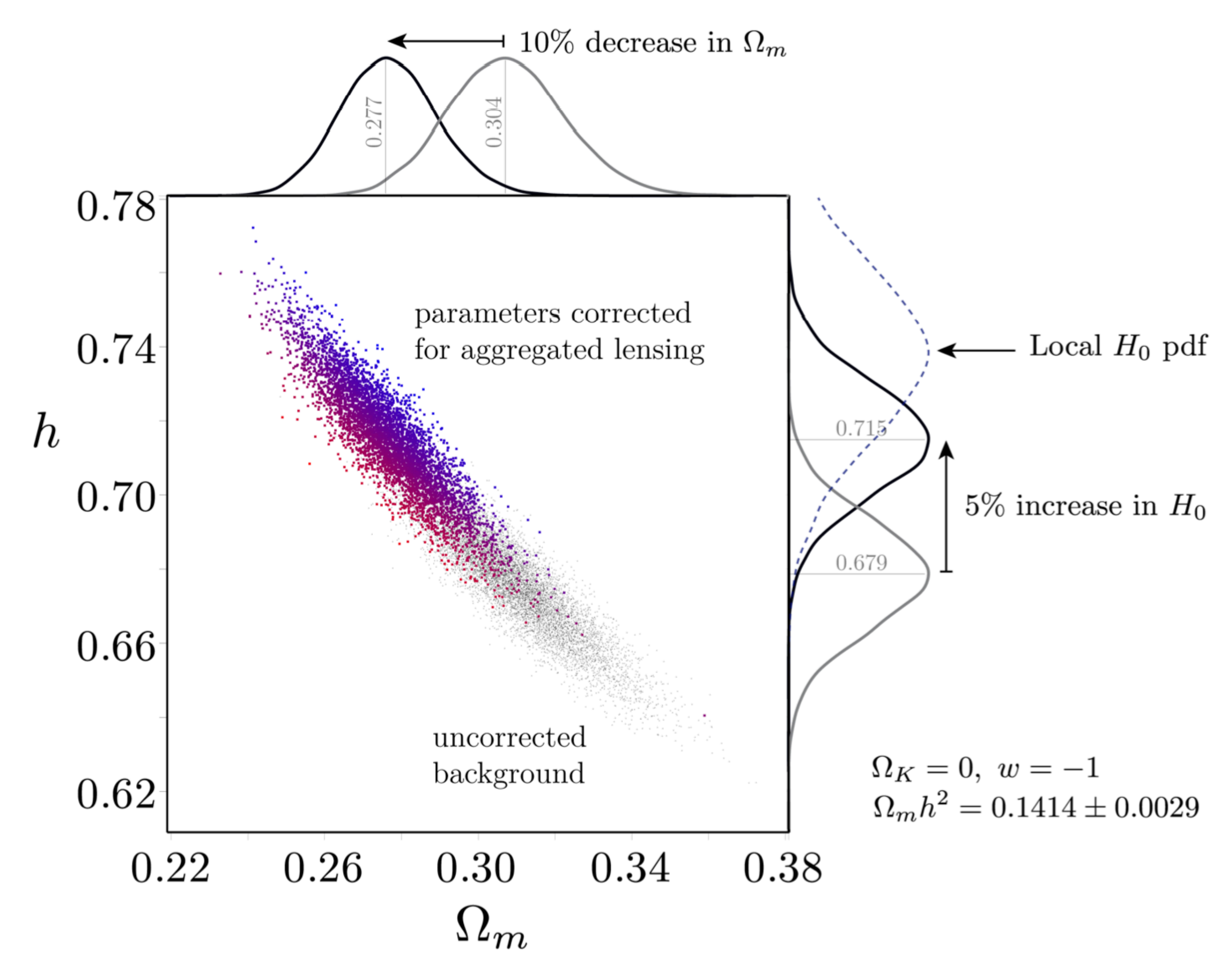
Clarkson et al.'s claim



Clarkson et al. arXiv:1405.786 claim that lensing changes average distance to CMB, with big impact on inferred cosmological parameters

V1: D increased by 7%
V2: D increased by 1% (5% change in inferred H_0)

Clarkson et al.



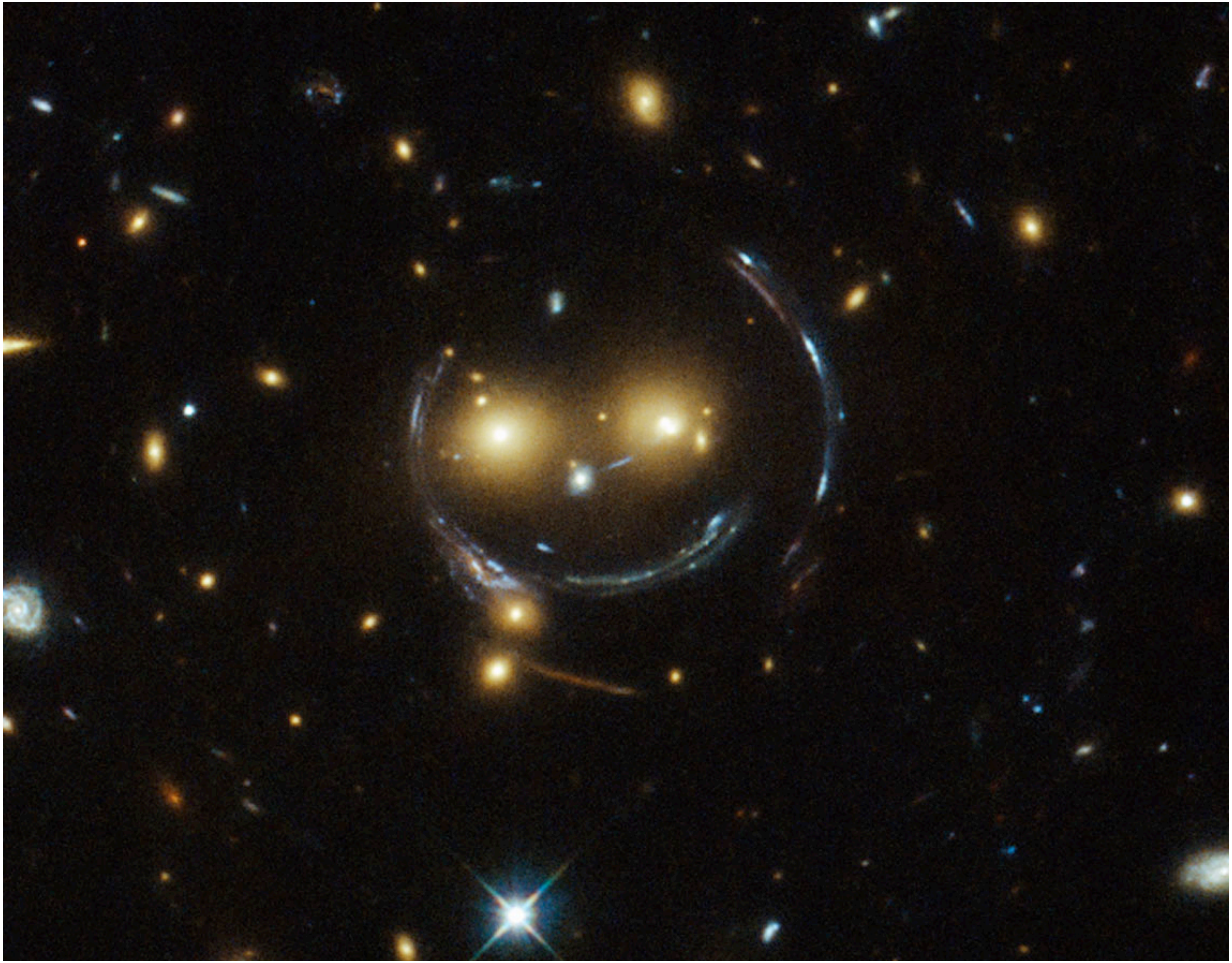
A brief history of gravitational lensing

- Theory-led
  - Einstein (1912): deflection without factor 2
  - Zwicky (1937): clusters can act as lenses
  - Refsdal (1960s): detailed theory of image distortions

- Multiple quasars
  - 1979: Q0957+561 6-arcsec identical pair
  - Rare: prob ~ 0.001 even at z ~ 1

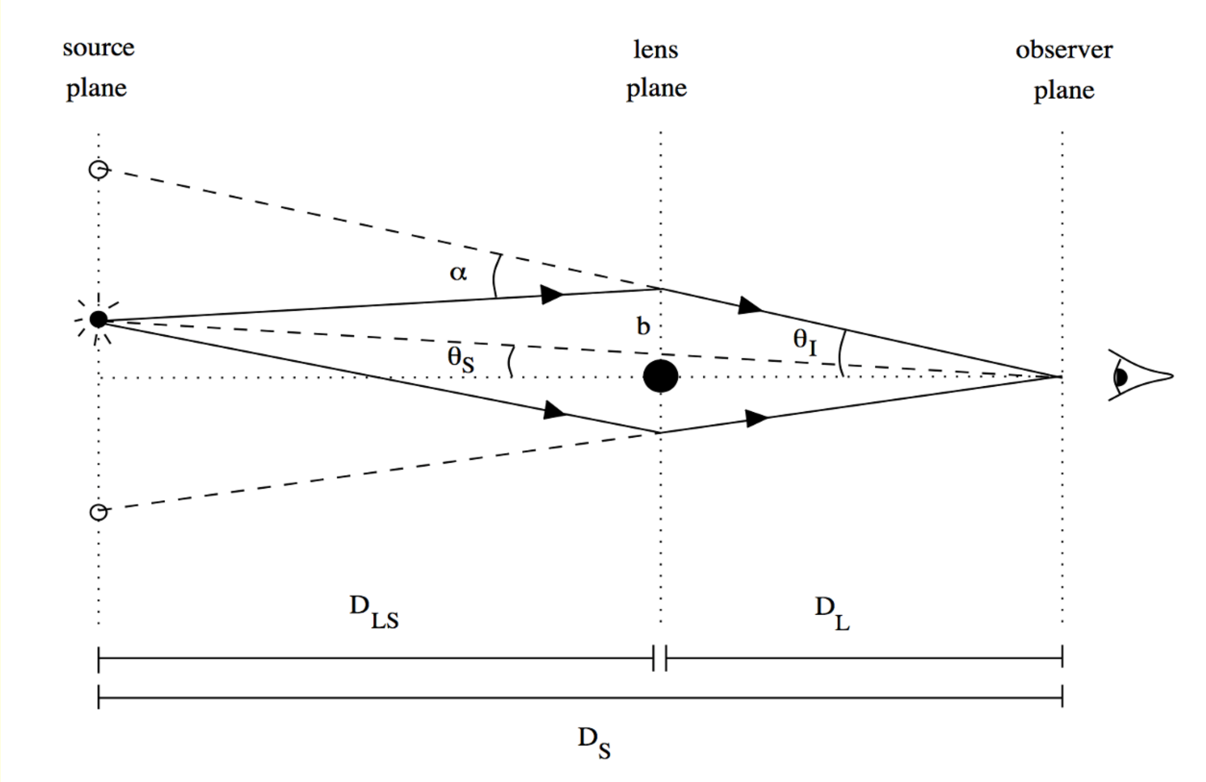
- Cluster arcs and Einstein rings
  - 1988: A370
  - 2015: HST Frontier Fields

- Weak lensing
  - Search for correlated 1% ellipticities
  - Theory (Kaiser 1991)
  - Detections (2000)
  - Dominates future surveys (LSST; Euclid)



Lensing terminology

Sky plane or image plane: where extrapolation of observed rays meets source plane.



Lensing deflection: $\theta_{\rm I} - \theta_{\rm S} = -\nabla_\theta \psi$

Lensing potential: $\psi = 2 \int \frac{D_{\rm LS}}{D_{\rm L} D_{\rm S}} \, \Phi \, d\ell$

Lensing convergence: $\nabla^2_\theta \psi = 2\kappa \quad \kappa = 4\pi G \int \frac{D_{\rm L} D_{\rm LS}}{D_{\rm S}} \, \rho \, d\ell$

Lensing screen magnification

Surface brightness conserved: flux amplification = area magnification from coordinate transformation.

$$\mu^{-1} = \det\left(\partial\theta_{\rm S}/\partial\theta_{\rm I}\right) = \det\begin{pmatrix} 1-\kappa-\gamma & 0 \\ 0 & 1-\kappa+\gamma \end{pmatrix}$$

$$\Rightarrow \mu = (1 - 2\kappa + \kappa^2 - \gamma^2)^{-1} \simeq 1 + 2\kappa + 3\kappa^2 + \gamma^2 + \cdots$$

Weak lensing: add shear and convergence from all screens. Apparently we expect magnification on average:

$$\langle\gamma^2\rangle = \langle\kappa^2\rangle \Rightarrow \langle\mu\rangle \simeq 1 + 4\kappa^2$$

The focusing theorem

Seems to be consistent with a result due to Seitz, Schneider & Ehlers (1994), using the optical scalar equations of Sachs (1961). Define D via D^2 = area of light beam

$$\ddot{D}/D = -(R + \Sigma^2)$$

R from Ricci tensor; $\sum$ is ‘shear’ from Weyl tensor. Expressed as deviations from homogeneous universe, <R>=0, so beam is always focused

What is distance?

- Comoving: dr = c dt x (1+z)
- Angular-diameter: D = r / (1+z)
- Luminosity: D = r x (1+z)
- Curvature?

Normally care about converting angles on sky to proper sizes, so define via radio of area on source plane and solid angle:

$$D \equiv \sqrt{\frac{dA}{d\Omega}}$$

Thus lensing does change apparent distance:

$$D = D_0/\mu$$

Clarkson et al.

What is the distance to the CMB? How relativistic corrections remove the tension with local H_0 measurements

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The success of precision cosmology depends not only on accurate observations, but also on the theoretical model – which must be understood to at least the same level of precision. Subtle relativistic effects can lead to biased measurements if they are neglected. One such effect gives a systematic shift in the distance-redshift relation away from its background value, due to the accumulation of all possible lensing events. We estimate the expectation value of this aggregated lensing using second-order perturbations about a concordance background, and show that the distance to last scattering is shifted by several percent. Neglecting this shift leads to significant bias in the background cosmological parameters. We show that this removes the tension between local measurements of H_0 and those measured through the CMB and favours a closed universe.

Clarkson et al.

$$\langle \Delta \rangle \simeq \frac{3}{2} \left\langle \left(\frac{\delta d_A}{\chi_s} \right)^2 \right\rangle = \frac{3}{2} \left\langle \kappa^2 \right\rangle , \tag{1.5}$$

where κ is the usual linear lensing convergence. This is actually the leading contribution to the expected change to large distances. We prove this remarkably simple and important result in a variety of ways in several appendices. It implies that the total area of a sphere of constant redshift will be larger than in the background. Physically this is because a sphere about us in redshift space is not a sphere in real space — lensing implies that this 'sphere' becomes significantly crumpled in real space, and hence has a larger area. When interpreted

4 Conclusions

We have demonstrated an important overall shift in the distance redshift relation when the aggregate of all lensing events is considered, calculated by averaging over an ensemble of universes. This result is a consequence of flux conservation at second-order in perturbation theory. This is a purely relativistic effect with no Newtonian counterpart — and it is the first quantitative prediction for a significant change to the background cosmology when averaging over structure [21]. The extraordinary amplification of aggregated lensing comes mainly from the integrated lensing of structure on scales in the range 1–100 Mpc, although structure down to 10kpc scales contributes significantly. We have estimated the size of the effect using

$$
\begin{aligned}
\hat{D}_A \;=\;& a(\chi_s)\chi_s\Big\{1+\left(1-\frac{1}{\mathcal{H}_s\chi_s}\right)\Phi_o+\frac{1}{2}\Big[\left(1-\frac{1}{\mathcal{H}_s\chi_s}\right)\Phi_o^{(2)}-\left(1-\frac{1}{\mathcal{H}_s\chi_s}\right)\omega_{\parallel o}-\left(\frac{\mathcal{H}'}{\mathcal{H}^2}-7\right)\Phi_o^2\Big]-\left(2-\frac{1}{\mathcal{H}_s\chi_s}\right)\Phi_s \\
&+\frac{1}{2}\Big[-\Psi_s^{(2)}-\left(1-\frac{1}{\mathcal{H}_s\chi_s}\right)\Phi_s+\frac{1}{2}\left(1-\frac{2}{\mathcal{H}_s\chi_s}\right)\omega_{\parallel s}-\frac{1}{2}h_{\parallel s}-\left(\frac{\mathcal{H}'}{\mathcal{H}^2}-\frac{2}{\chi_s\mathcal{H}_s}-7\right)\Phi_s^2-2\left(1-\frac{2}{\chi_s\mathcal{H}_s}\right)\chi_s\Phi_s\Phi_s' \\
&-2\chi_s\Phi_s\nabla_\parallel\Phi_s\Big]+\frac{1}{\mathcal{H}_s\chi_s}\nabla_\parallel v_o+\frac{1}{2}\Big[\frac{1}{\mathcal{H}_s\chi_s}\nabla_\parallel v_o^{(2)}-\frac{1}{\mathcal{H}_s\chi_s}v_{\parallel o}-\left(\frac{\mathcal{H}'}{\mathcal{H}^2}-2\right)\nabla_\parallel v_o\nabla_\parallel v_o+\frac{1}{\chi_s\mathcal{H}_s}\nabla_{\perp i}v_o\nabla_\perp^i v_o\Big] \\
&+\left(1-\frac{1}{\mathcal{H}_s\chi_s}\right)\nabla_\parallel v_s+\frac{1}{2}\Big[\left(1-\frac{1}{\mathcal{H}_s\chi_s}\right)\nabla_\parallel v_s^{(2)}+\left(1-\frac{1}{\mathcal{H}_s\chi_s}\right)v_{\parallel s}+\left(\frac{\mathcal{H}'}{\mathcal{H}^2}-\frac{2}{\chi_s\mathcal{H}_s}-1\right)\nabla_\parallel v_s\nabla_\parallel v_s \\
&+2\chi\left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\nabla_\parallel v_s\left(\nabla_\parallel v_s'-\nabla_\parallel^2 v_s\right)+\left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\nabla_{\perp i}v_s\nabla_\perp^i v_s-2\left(\frac{\mathcal{H}'}{\mathcal{H}^2}-\frac{1}{\chi_s\mathcal{H}_s}+4\right)\Phi_s\Phi_o \\
&+2\left(1-\frac{2}{\chi_s\mathcal{H}_s}\right)\chi_s\Phi_o\Phi_s'-2\chi\Phi_o\nabla_\parallel\Phi_s-2\left(\frac{\mathcal{H}'}{\mathcal{H}^2}-\frac{1}{\chi_s\mathcal{H}_s}\right)\nabla_\parallel v_s\nabla_\parallel v_o-2\chi_s\left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\nabla_\parallel v_o\left(\nabla_\parallel v_s'-\nabla_\parallel^2 v_s\right) \\
&-2\left(\frac{\mathcal{H}'}{\mathcal{H}^2}+\frac{1}{2\chi_s\mathcal{H}_s}+\frac{11}{2}\right)\Phi_o\nabla_\parallel v_o-2\left(\frac{\mathcal{H}'}{\mathcal{H}^2}-\frac{1}{\chi_s\mathcal{H}_s}+3\right)\Phi_s\nabla_\parallel v_s+2\left(\frac{\mathcal{H}'}{\mathcal{H}^2}-1\right)\Phi_o\nabla_\parallel v_s+2\chi_s\left(1-\frac{2}{\chi_s\mathcal{H}_s}\right)\Phi_s'\nabla_\parallel v_s \\
&+2\chi_s\nabla_\parallel\Phi_s\nabla_\parallel v_s-2\chi_s\left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\Phi_s(\nabla_\parallel v_s'-\nabla_\parallel^2 v_s)-2\chi_s\left(1-\frac{2}{\chi_s\mathcal{H}_s}\right)\Phi_s'\nabla_\parallel v_o-2\chi_s\nabla_\parallel\Phi_s\nabla_\parallel v_o \\
&+2\left(\frac{\mathcal{H}'}{\mathcal{H}^2}+2\right)\Phi_s\nabla_\parallel v_o+2\left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\chi_s\Phi_o(\nabla_\parallel v_s'-\nabla_\parallel^2 v_s)\Big]+\frac{2}{\chi_s}\int_\chi\Phi+\frac{1}{2}\Big[\frac{1}{\chi_s}\int_\chi(\Phi^{(2)}+\Psi^{(2)}) \\
&-\frac{1}{\chi_s}\int_\chi\frac{(\chi-\chi_s)}{\chi}\omega_\parallel+\frac{1}{\chi_s}\int_\chi 3\frac{(\chi-\chi_s)}{\chi}h_\parallel\Big]-2\left(1-\frac{1}{\mathcal{H}_s\chi}\right)\int_\chi\Phi'+\frac{1}{2}\Big[-\left(1-\frac{1}{\mathcal{H}_s\chi_s}\right)\left(\int_\chi(\Phi'_{(2)}+\Psi'_{(2)})\right) \\
&-\int_\chi\left(1-\frac{1}{\mathcal{H}_s\chi_s}+\frac{(2\chi-\chi_s)}{2\chi_s}\right)\omega_\parallel^{(2)\prime}+\int_\chi\left(1-\frac{1}{\mathcal{H}_s\chi_s}-\frac{(2\chi-\chi_s)}{\chi_s}\right)h_\parallel{}'\Big]+\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\nabla_\perp^2\Phi \\
&+\frac{1}{2}\Big[\frac{1}{2}\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\nabla_\perp^2(\Phi^{(2)}+\Psi^{(2)})-\frac{1}{2}\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\nabla_\perp^2\omega_\parallel-\frac{1}{2}\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\nabla_\perp^2 h_\parallel\Big]+\frac{1}{2}\Big\{-2\Phi_o\Big[\left(1+\frac{2}{\chi_s\mathcal{H}_s}\right)\frac{2}{\chi}\int_\chi\Phi \\
&+2\left(\frac{\mathcal{H}'}{\mathcal{H}^2}+\frac{3}{\chi_s\mathcal{H}_s}-4\right)\int_\chi\Phi'-\left(2+\frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\chi\nabla_\perp^2\Phi+\left(9+\frac{2}{\chi_s\mathcal{H}_s}\right)\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\nabla_\perp^2\Phi+4\int_\chi\frac{\chi}{\chi_s}\Phi' \\
&+4\int_\chi\frac{(\chi-\chi_s)}{\chi_s}\Phi'-4\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\Phi''\Big]+2\Phi_s\Big[\left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\frac{4}{\chi}\int_\chi\Phi+2\left(\frac{\mathcal{H}'}{\mathcal{H}^2}-\frac{2}{\chi_s\mathcal{H}_s}+2\right)\int_\chi\Phi' \\
&-\left(2+\frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\chi\nabla_\perp^2\Phi+2\left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\nabla_\perp^2\Phi\Big]+2\nabla_\parallel v_o\Big[\frac{2}{\chi}\left(1-\frac{2}{\chi_s\mathcal{H}_s}\right)\int_\chi\Phi-2\int_\chi\Phi' \\
&-\left(2-\frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\chi\nabla_\perp^2\Phi+2\left(2-\frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\nabla_\perp^2\Phi\Big]-2\nabla_\parallel v_s\Big[\frac{2}{\chi_s}\left(5-\frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\Phi-4\left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\int_{\lambda_s}\Phi' \\
&-\left(2-\frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\chi\nabla_\perp^2\Phi+\left(3-\frac{2}{\chi_s\mathcal{H}_s}\right)\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\nabla_\perp^2\Phi\Big]+4\left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\nabla_\perp v_s\int_\chi\nabla_\perp^i\Phi \\
&+4\Big[\frac{2}{\chi_s}\left(3-\frac{2}{\chi_s\mathcal{H}_s}\right)\int_\chi\Phi+\left(\frac{\mathcal{H}'}{\mathcal{H}^2}+\frac{1}{\chi_s\mathcal{H}_s}-2\right)\int_\chi\Phi'-\left(2-\frac{1}{\chi_s\mathcal{H}_s}\right)\int_\chi\chi\nabla_\perp^2\Phi+\left(3-\frac{2}{\chi_s\mathcal{H}_s}\right)\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\nabla_\perp^2\Phi\Big]\int_\chi\Phi' \\
&-4\chi\left(1-\frac{2}{\chi_s\mathcal{H}_s}\right)\chi_s\Phi_s'\int_\chi\Phi'-4\chi_s\nabla_\parallel\Phi_s\int_\chi\Phi'-4\left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\left[\chi\left(\nabla_\parallel v_s'-\nabla_\parallel^2 v_s\right)\right]\int_\chi\Phi'+16\int_\chi\frac{(\chi-\chi_s)}{\chi_s}\Phi\Phi' \\
&+\frac{2}{\chi_s}\int_\lambda\Phi^2-4\int_\chi\frac{1}{\chi\chi_s}\Phi\int_{\tilde\chi}\Phi(\tilde\chi)+8\int_\chi\Phi'\int_{\tilde\chi}\Phi'(\tilde\chi)+8\int_\chi\Phi\Phi'-4\int_\chi\frac{(\chi-\chi_s)}{\chi\chi_s}\left[\frac{1}{\chi}\Phi\int_{\tilde\chi}\Phi(\tilde\chi)+\Phi^2-2\Phi'\int_{\tilde\chi}\Phi(\tilde\chi)\right] \\
&-4\int_\chi\frac{1}{\chi}\Phi\int_{\tilde\chi}\frac{(\tilde\chi-\chi)\tilde\chi}{\chi_s}\nabla_\perp^2\Phi(\tilde\chi)-4\int_\chi\frac{(\chi-\chi_s)}{\chi^2\chi_s}\Phi\int_{\tilde\chi}(\tilde\chi-\chi)\tilde\chi\nabla_\perp^2\Phi(\tilde\chi)+4\int_\chi\frac{(\chi-\chi_s)^2}{\chi_s}\Phi\nabla_\perp^2\Phi \\
&-4\int_\chi\Phi\int_{\tilde\chi}\frac{\tilde\chi}{\chi}\nabla_\perp^2\Phi(\tilde\chi)+14\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\Phi\nabla_\perp^2\Phi+2\int_\chi\frac{(\chi-\chi_s)}{\chi_s}\Phi\int_{\tilde\chi}\tilde\chi\nabla_\perp^2\Phi(\tilde\chi)+4\int_\chi\frac{(\chi-\chi_s)}{\chi_s}\nabla_\perp^2\Phi\int_{\tilde\chi}\Phi(\tilde\chi) \\
&+4\int_\chi\frac{(\chi-\chi_s)}{\chi_s}\Phi'\int_{\tilde\chi}\tilde\chi\nabla_\perp^2\Phi(\tilde\chi)+4\int_\chi\frac{(\chi-\chi_s)}{\chi}\Phi\int_{\tilde\chi}\frac{(\tilde\chi-\chi)\tilde\chi}{\chi}\nabla_\perp^2\Phi(\tilde\chi)-8\int_\chi\frac{(\chi-\chi_s)}{\chi_s}\nabla_{\perp i}\Phi\int_{\tilde\chi}\frac{\tilde\chi}{\chi}\nabla_\perp^i\Phi(\tilde\chi) \\
&-8\int_\chi\frac{\chi}{\chi_s}\nabla_{\perp i}\Phi\int_{\tilde\chi}\nabla_\perp^i\Phi(\tilde\chi)-\frac{8}{\chi_s}\int_\chi\int_{\tilde\chi}\nabla_{\perp i}\Phi(\tilde\chi)\int_{\tilde\chi}\nabla_\perp^i\Phi(\tilde\chi)\,,+12\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\nabla_{\perp i}\Phi\nabla_\perp^i\Phi \\
&-8\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\nabla_{\perp i}\Phi'\int_{\tilde\chi}\nabla_\perp^i\Phi(\tilde\chi)-8\left(1-\frac{1}{\chi_s\mathcal{H}_s}\right)\int_{\tilde\chi}\frac{\tilde\chi}{\chi}\nabla_\perp^i\Phi'(\tilde\chi)\int_{\tilde\chi}\nabla_\perp^i\Phi(\tilde\chi) \\
&-4\int_\chi\frac{(\chi-\chi_s)\chi}{\chi_s}\int_{\tilde\chi}\nabla_{\perp\langle i}\nabla_{\perp j\rangle}\Phi(\tilde\chi)\int_{\tilde\chi}\nabla_\perp^{\langle i}\nabla_\perp^{j\rangle}\Phi(\tilde\chi)-2\int_\chi\frac{(\chi-\chi_s)}{\chi_s}\nabla_\perp^2\Phi\int_{\tilde\chi}(\tilde\chi-\chi)\tilde\chi\nabla_\perp^2\Phi(\tilde\chi) \\
&+8\int_\chi\frac{(\chi-\chi_s)}{\chi_s}\int_{\tilde\chi}\nabla_{\perp i}\Phi(\tilde\chi)\int_{\tilde\chi}\frac{\tilde\chi^2}{\chi}\nabla_\perp^i\nabla^2\Phi(\tilde\chi)-4\int_\chi\frac{(\chi-\chi_s)}{\chi_s}\nabla_{\perp i}\Phi\int_{\tilde\chi}\frac{(\tilde\chi-\chi)\tilde\chi^2}{\chi}\nabla_\perp^i\nabla_\perp^2\Phi(\tilde\chi)\Big\}\Big\}.
\end{aligned}
\tag{B1}
$$

Spot the backreaction…

Gravity = glass

Clarkson et al. analysis deals with “subtle relativistic effects”

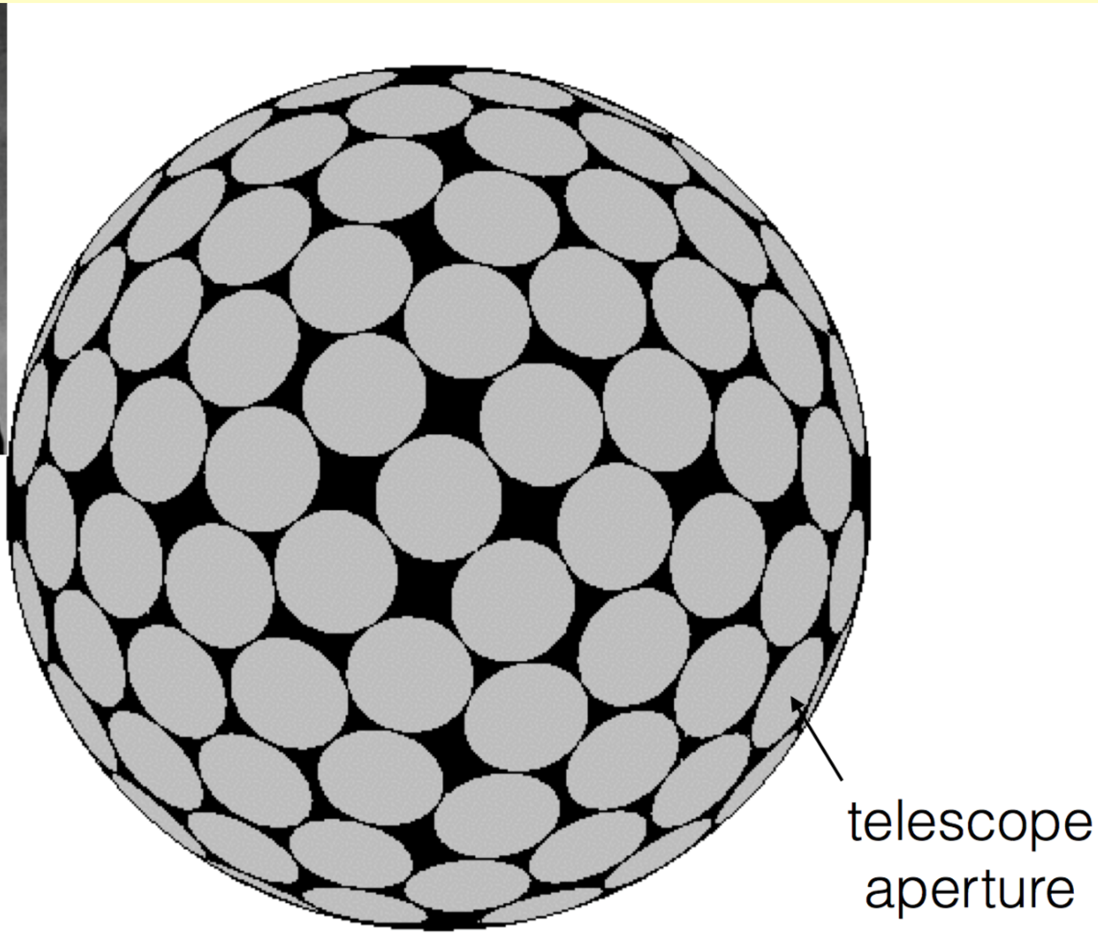
$$d\tau^2 = (1 + 2\Phi)\, dt^2 - (1 - 2\Phi)\, [dx^2 + dy^2 + dz^2]$$

So coordinate speed of light responds to a refractive index

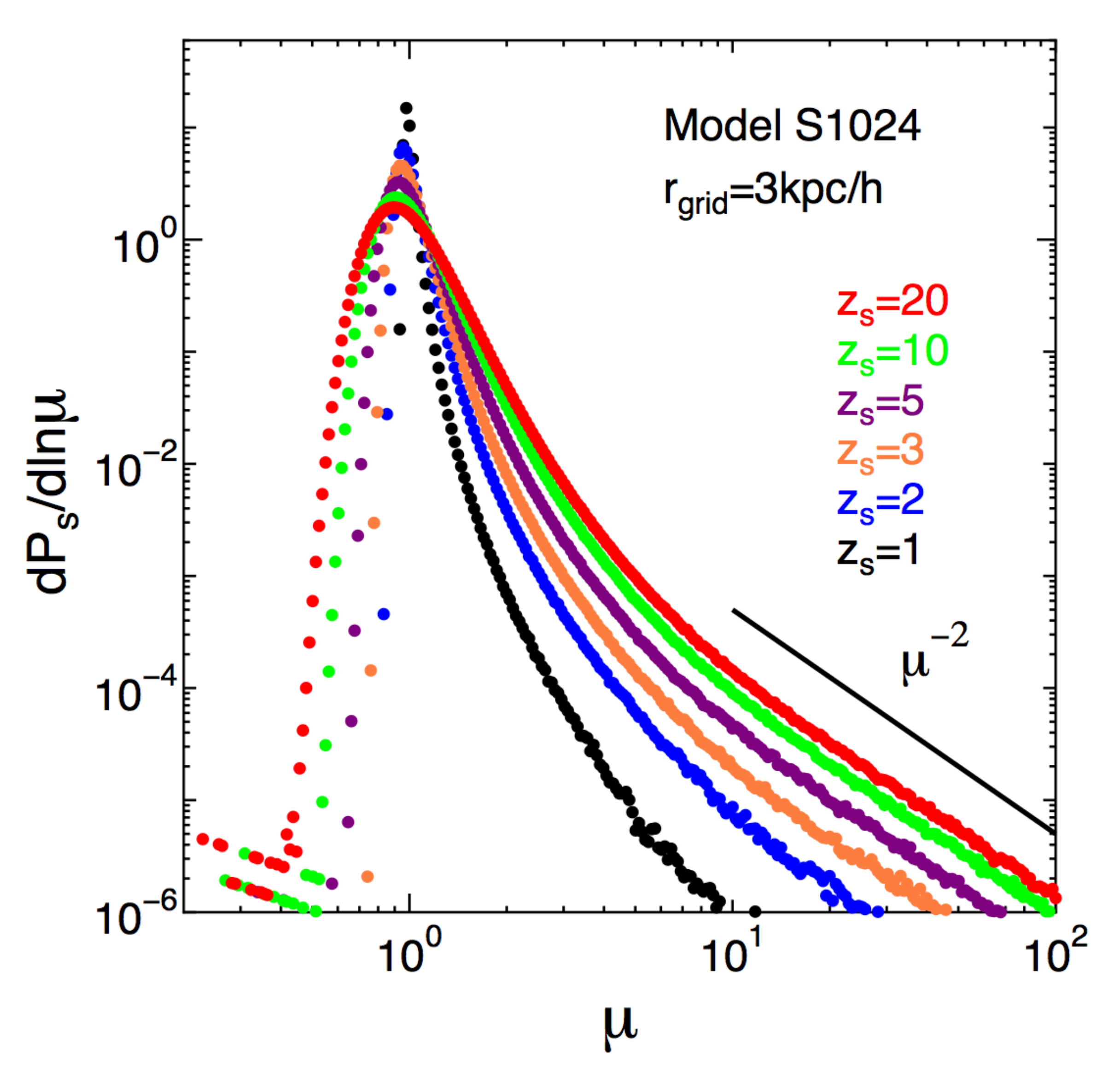
$n = 1 - 2\Phi$

− Hence GR factor 2 in light deflection. Should be able to understand average effects of gravitational optics via ‘lumpy glass’ analogy

Flux conservation: Weinberg 1976



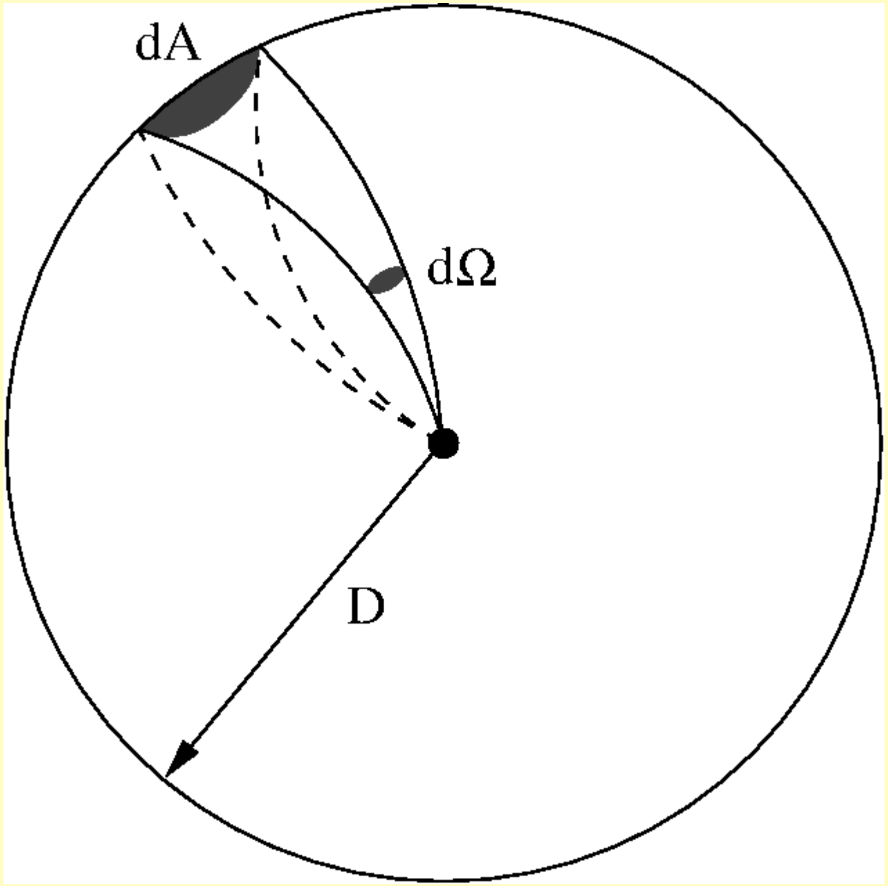
Centre sphere on source. All photons emerge through one telescope or another, so mean magnification = 1



Magnification PDF from ray-tracing simulations

(Takahashi et al. 2011)

Two conservation laws



Consider emission from fixed sphere. Lensing magnification is ratio of solid angles:

$\mu = d\Omega / (dA/D^2)$

$\langle \mu \rangle = 1$ (ave over dA) (Weinberg 1976)

$\langle 1/\mu \rangle = 1$ (ave over $d\Omega$) (Kibble & Lieu 2005; if no multiple lensing)

Critical distinction between source-plane averaging and sky-plane averaging makes sense of some paradoxes

Statistical bias in weak lensing

Recall expression for magnification:

$$\mu = (1 - 2\kappa + \kappa^2 - \gamma^2)^{-1}$$
$$\simeq 1 + 2\kappa + 3\kappa^2 + \gamma^2 + \cdots$$

$$\langle 1/\mu \rangle_\Omega = 1 \text{ (via } \langle \kappa \rangle_\Omega = 0 \text{ and } \langle \gamma^2 \rangle = \langle \kappa^2 \rangle)$$

$$\langle \mu \rangle_\Omega \simeq 1 + 4\langle \kappa^2 \rangle$$

$$\langle \mu \rangle_A = 1 \Rightarrow \langle \kappa \rangle_A = -2\langle \kappa^2 \rangle$$

Origin of Clarkson et al. claim of net mean magnification

Note sources are typically seen via underdense sightlines

Malmquist-esque effects

Recall $(D/D_0)^2 = 1/\mu$

- So D^2 is unchanged in sky average
- And $1/D^2$ is unchanged in source average

Other nonlinear combinations have a bias

$$\langle D/D_0 \rangle_A \simeq 1 + \tfrac{3}{2}\langle \kappa^2 \rangle$$
$$\langle (D/D_0)^2 \rangle_A \simeq 1 + 4\langle \kappa^2 \rangle$$

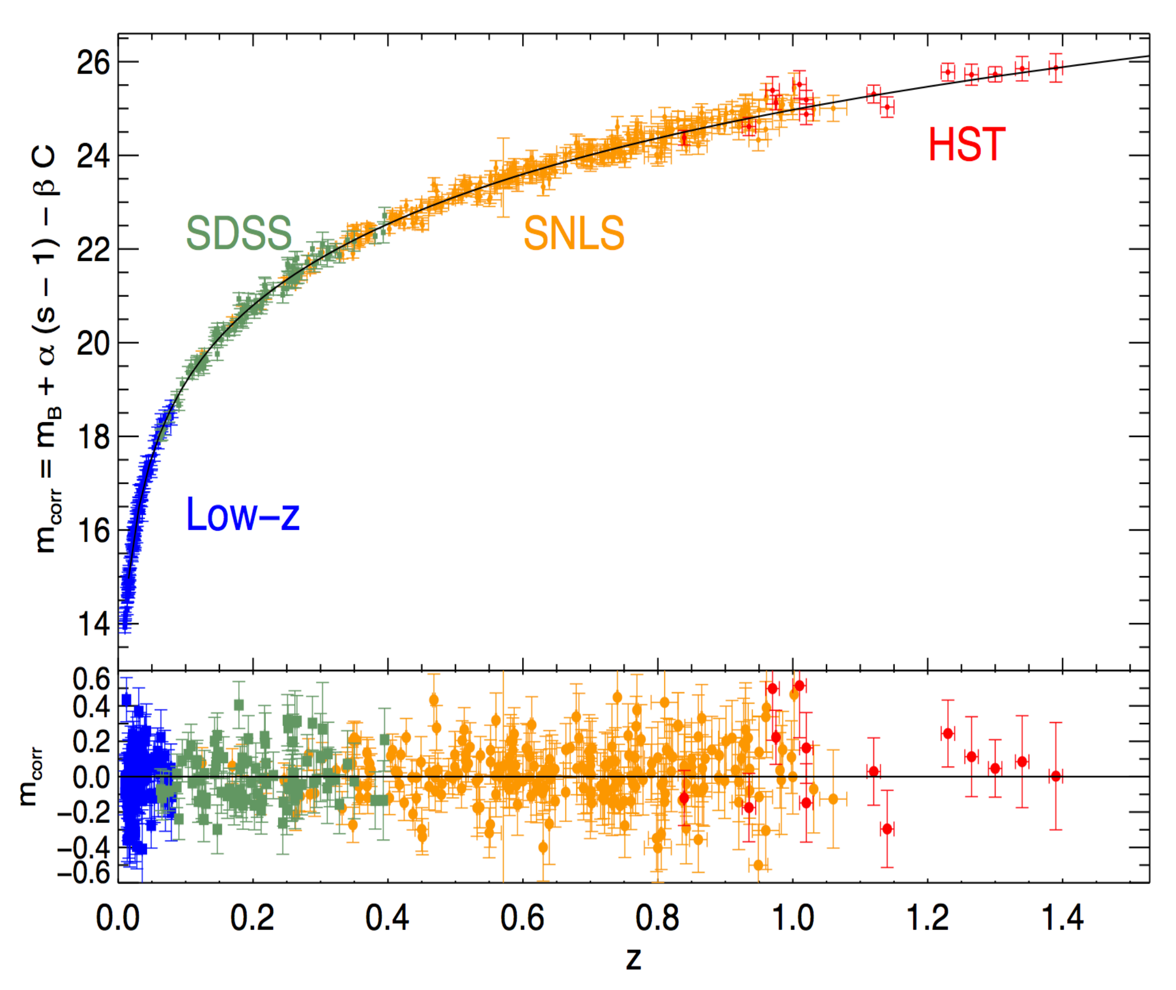
Distance and area bias claimed by Clarkson et al.

Makes sense of focusing equation: this is for D, not D^2

CMB could be biased:

$$\ell_{\rm peak} \propto 1/\mu^{1/2}$$
$$\langle 1/\mu^{1/2} \rangle_\Omega \simeq 1 - \tfrac{1}{2}\langle \kappa^2 \rangle$$

Supernova cosmology



$$\langle \ln \mu \rangle_A \simeq -2\langle \kappa^2 \rangle$$
$$\simeq -(1/2)\sigma_m^2$$

$$\sigma_m \simeq 0.06z$$

Regressing on m wrong in principle. Error 0.01 in w for data to z=2

The loophole

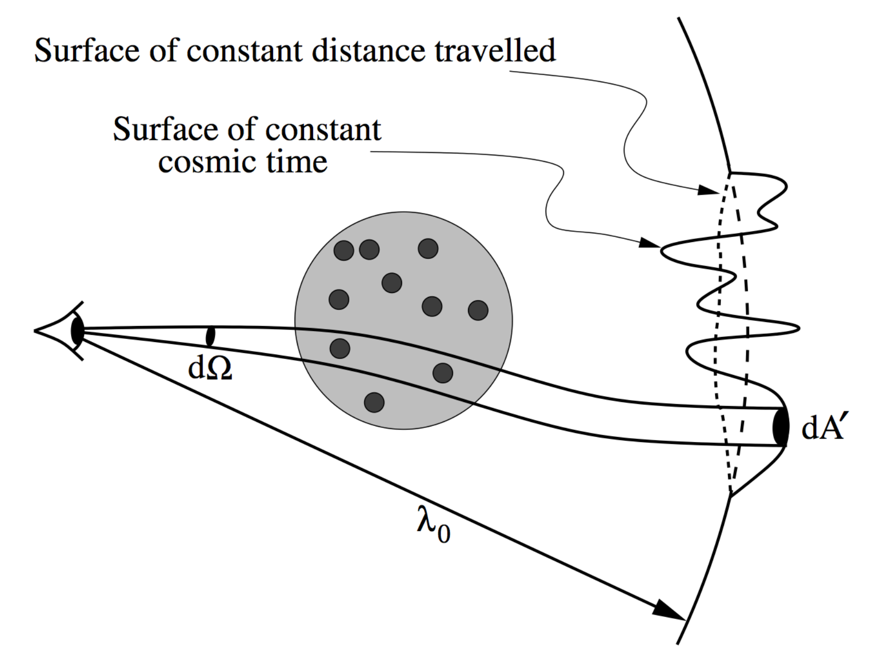
All arguments so far assumed that area of source sphere is unchanged

− But Clarkson et al. claim it is changed, by amount of order convergence^2

− If true, would destroy conservation theorems that seem to explain other Clarkson et al. results ???

Since we are interested in a surface of fixed redshift (= fixed cosmic time), lensing may shift the area

Two changes to A of source sphere



(1) Wiggly paths are longer: reduces A

(2) Potential time delay zero on average, but crinkles surface: increases A

NK + JP: both effects 2^{nd} order in potential

$$\left\langle \frac{\Delta A}{A} \right\rangle = \frac{1}{\lambda_s^2} \int d\lambda (16\lambda(\lambda_s - \lambda) + 8\lambda^2) J(\lambda)$$

$$J = 2\pi \int d\ln k \; k \; \Delta_\Phi^2$$

If deflection is clumps of size L & depth Φ, total deflection is ~ Φ $(D/L)^{1/2}$. ΔA/A ~ $\Phi^2 D/L$ ~ θ^2 ~ $(\mathrm{arcmin})^2$ ~ 10^{-7}

Real CMB lensing

- Accounted for correctly at 2nd order

Lensing re-maps the temperature according to

$$\tilde{\Theta}(\mathbf{x}) = \Theta(\mathbf{x}') = \Theta(\mathbf{x} + \boldsymbol{\nabla}\psi)$$
$$\approx \Theta(\mathbf{x}) + \nabla^a\psi(\mathbf{x})\nabla_a\Theta(\mathbf{x}) + \frac{1}{2}\nabla^a\psi(\mathbf{x})\nabla^b\psi(\mathbf{x})\nabla_a\nabla_b\Theta(\mathbf{x}) + \ldots \quad (4.7)$$

As discussed, the series expansion is not a good approximation on all scales. However it can be used to get qualitatively correct results for the lensed C_l, and is useful for giving a simple derivation to aid understanding. Introducing the Fourier transform of the lensing potential, $\psi(\mathbf{l})$, we have

$$\boldsymbol{\nabla}\psi(\mathbf{x}) = i\int \frac{\mathrm{d}^2\mathbf{l}}{2\pi}\,\mathbf{l}\psi(\mathbf{l})e^{i\mathbf{l}\cdot\mathbf{x}}, \qquad \boldsymbol{\nabla}\Theta(\mathbf{x}) = i\int \frac{\mathrm{d}^2\mathbf{l}}{2\pi}\,\mathbf{l}\Theta(\mathbf{l})e^{i\mathbf{l}\cdot\mathbf{x}}. \quad (4.8)$$

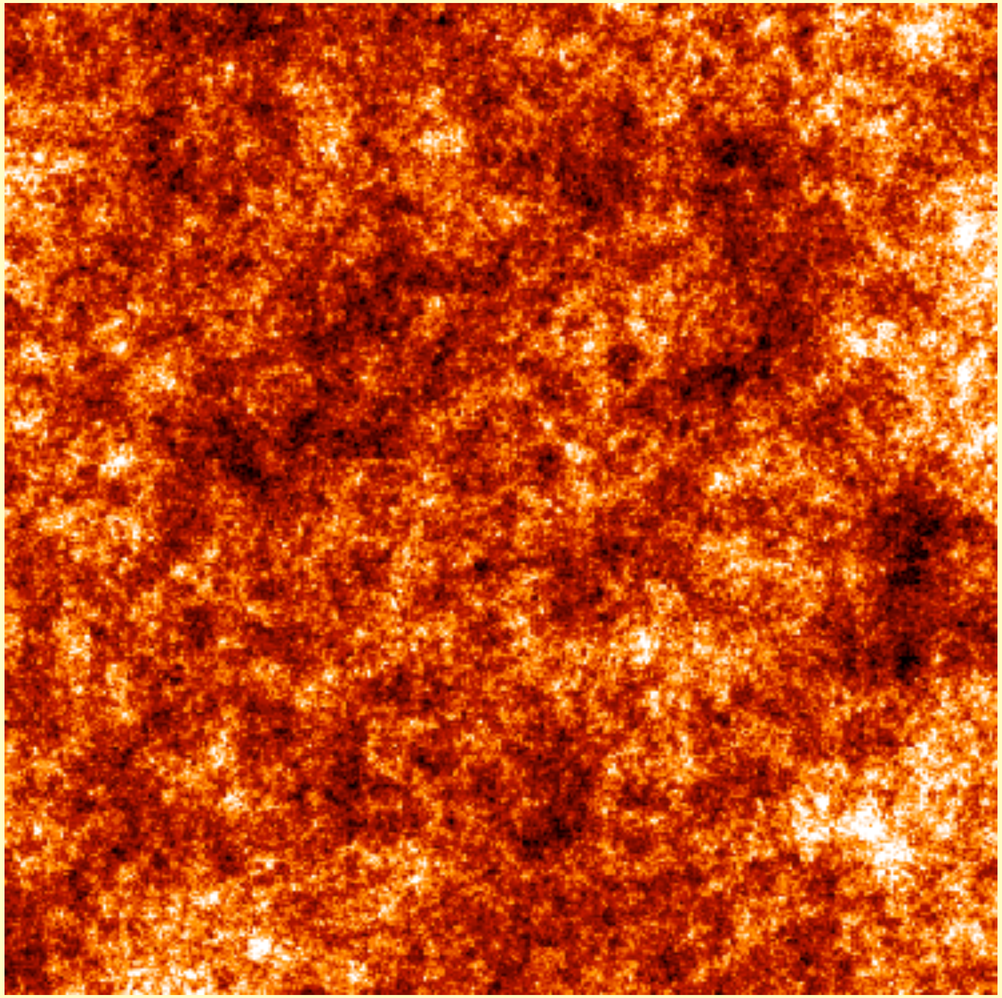
Taking the Fourier transform of $\tilde{\Theta}(\mathbf{x})$ and substituting we get the Fourier components to second order in ψ

$$\tilde{\Theta}(\mathbf{l}) \approx \Theta(\mathbf{l}) - \int \frac{\mathrm{d}^2\mathbf{l}'}{2\pi}\,\mathbf{l}'\cdot(\mathbf{l}-\mathbf{l}')\psi(\mathbf{l}-\mathbf{l}')\Theta(\mathbf{l}')$$
$$-\frac{1}{2}\int\frac{\mathrm{d}^2\mathbf{l}_1}{2\pi}\int\frac{\mathrm{d}^2\mathbf{l}_2}{2\pi}\,\mathbf{l}_1\cdot[\mathbf{l}_1+\mathbf{l}_2-\mathbf{l}]\,\mathbf{l}_1\cdot\mathbf{l}_2\Theta(\mathbf{l}_1)\psi(\mathbf{l}_2)\psi^*(\mathbf{l}_1+\mathbf{l}_2-\mathbf{l}). \quad (4.9)$$

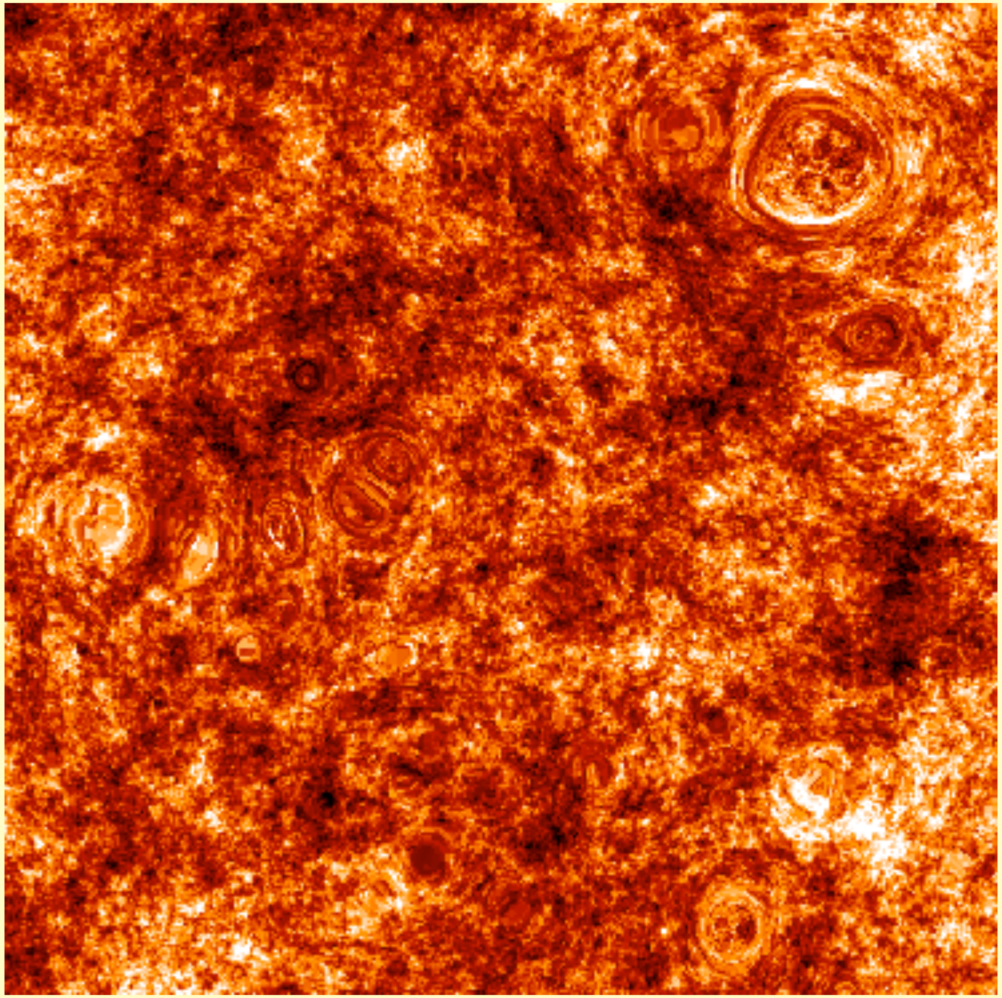
Lewis & Challinor (2006)

- Map of foreground lensing possible
  - Local magnification would shift peak in power spectrum
  - In practice use induced non-Gaussian signature

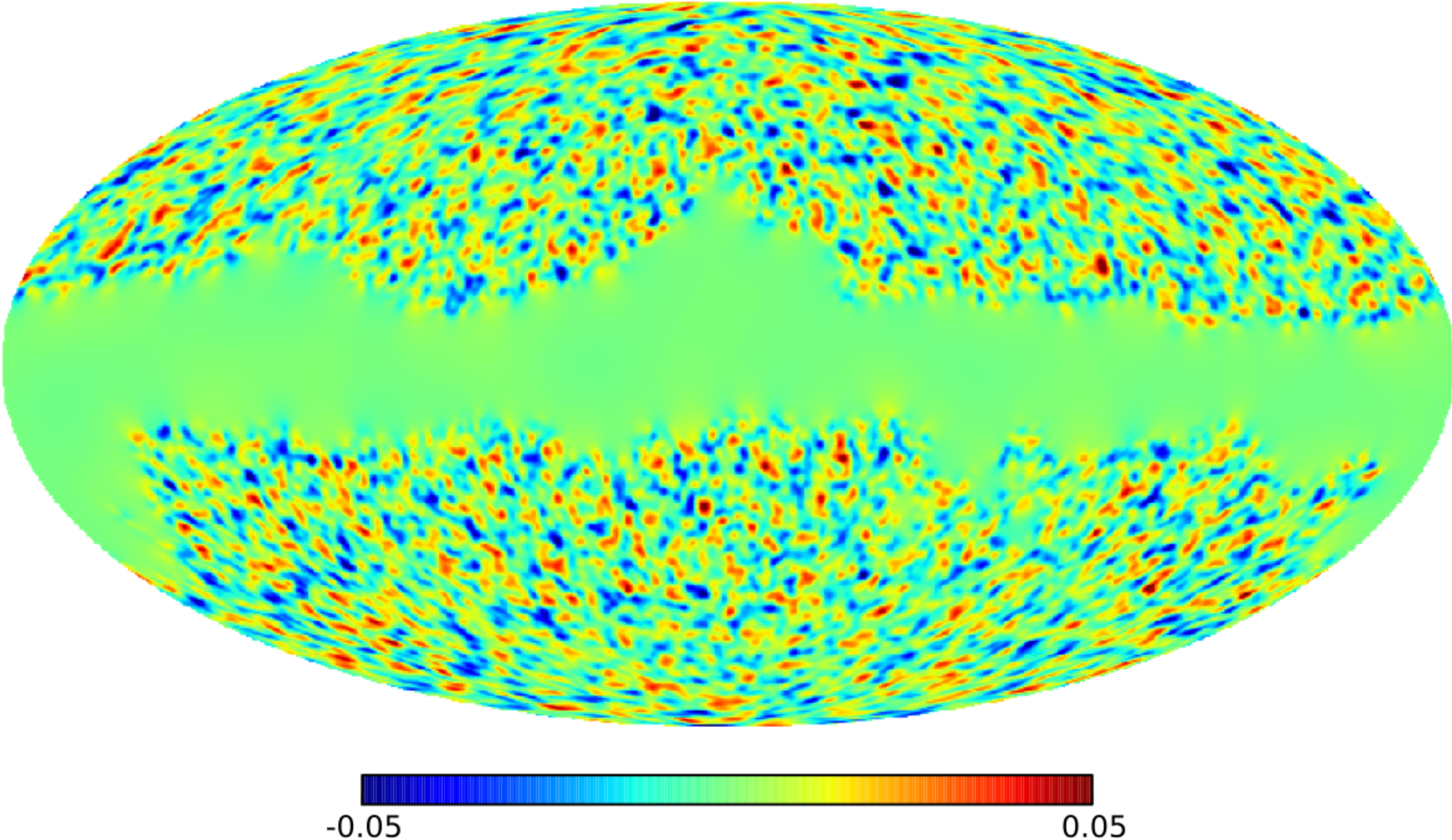
Unlensed CMB: 6 arcmin image (MPIA)



Lensed CMB: 6 arcmin image (MPIA)

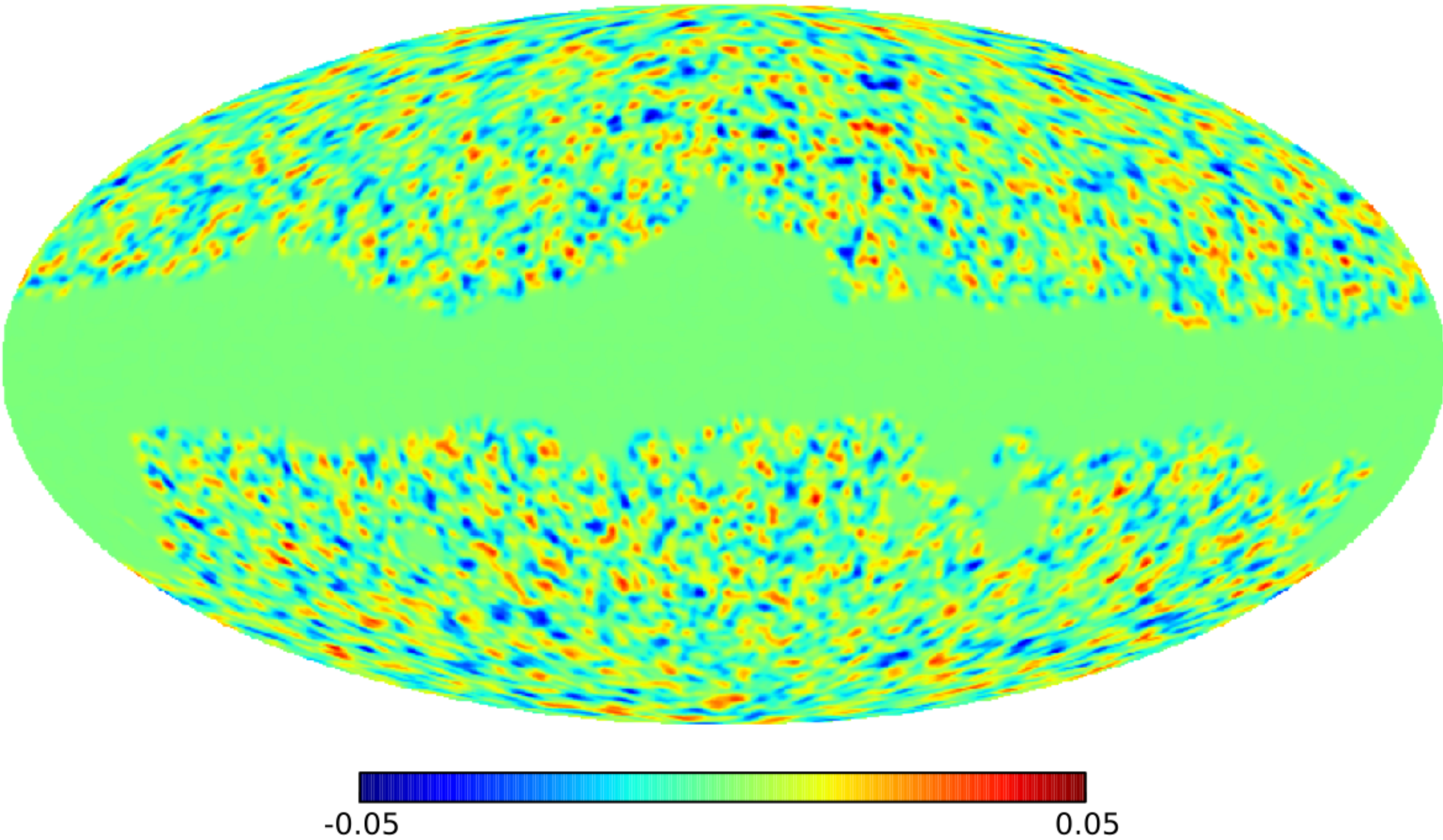


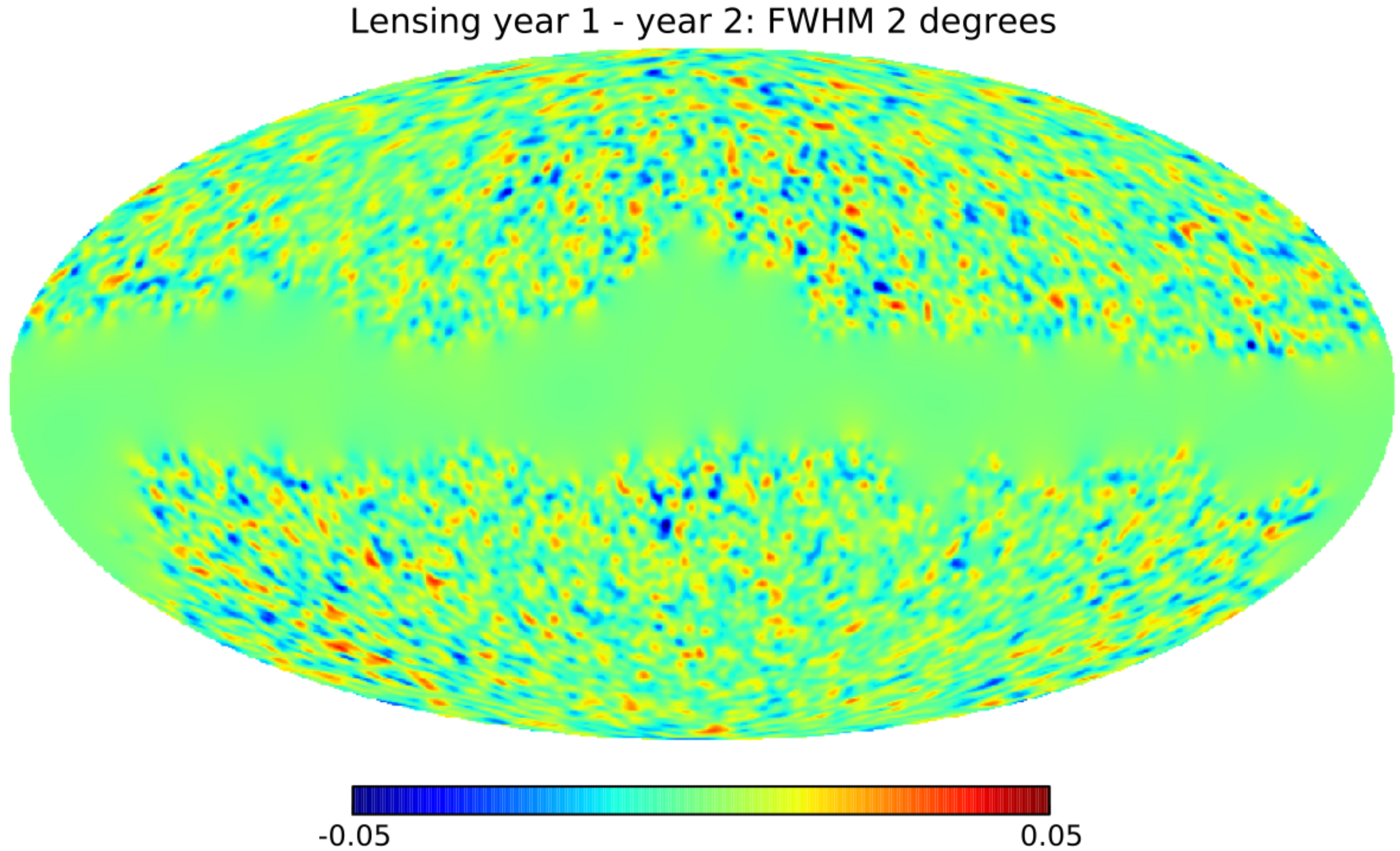
Lensing year 1: FWHM 2 degrees



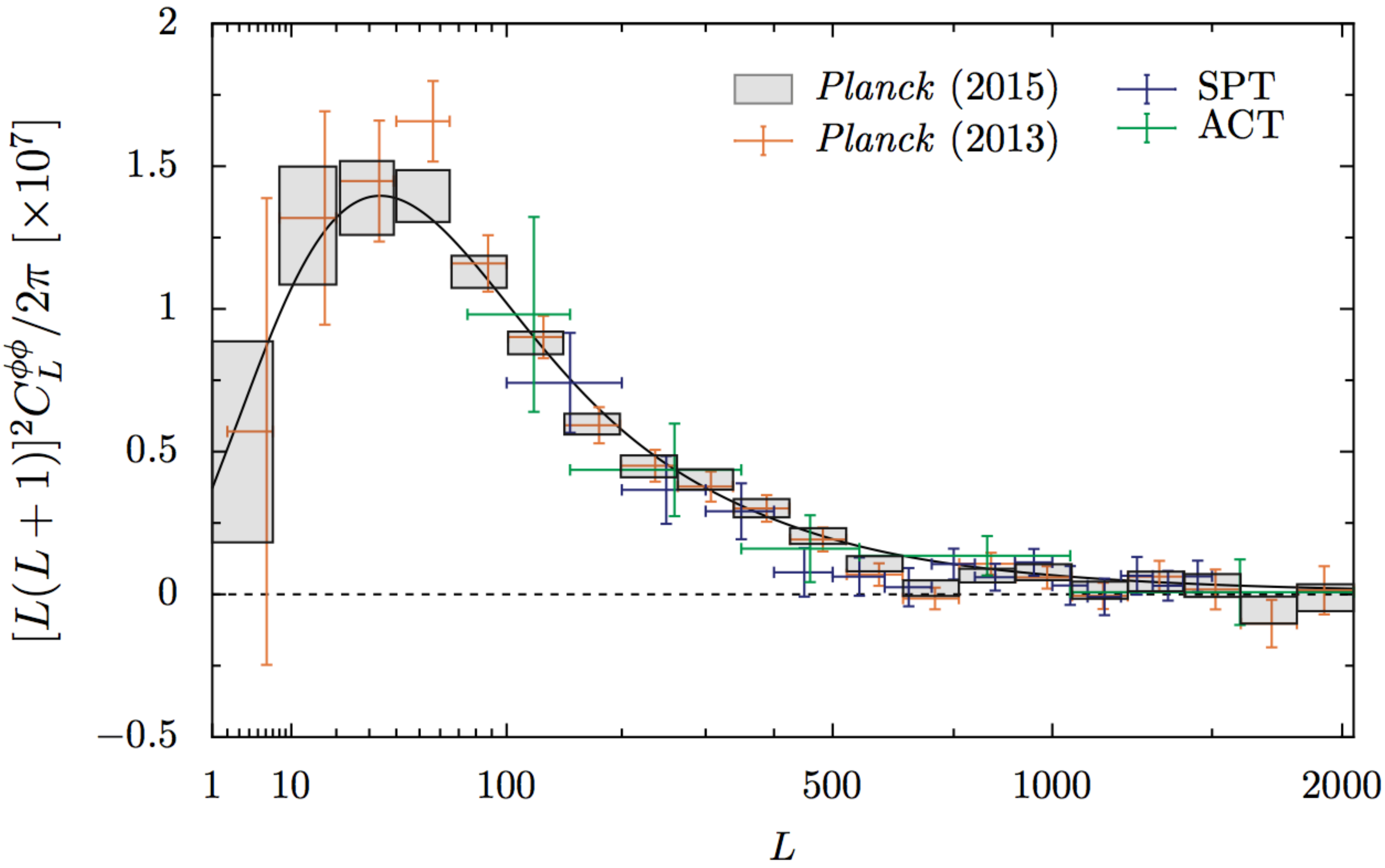
Lensing convergence: projected mass distribution back to z=1100

Lensing year 2: FWHM 2 degrees




Lensing year 1 - year 2: FWHM 2 degrees
-0.05
0.05

Planck lensing power spectrum



Corrected for noise: S/N = 1 at peak

WISE

Wide-field Infrared Survey Explorer

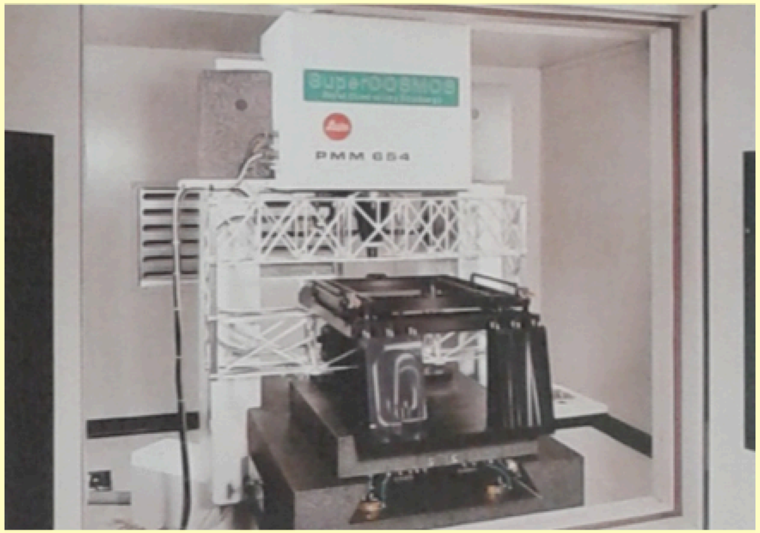
Dec 2009 − Feb 2011

40cm telescope; FWHM > 6’’

All-sky surveys
3.3, 4.7, 12, 23 microns (W1-W4)

~ 500M sources with W1<17
(roughly 50:50 stars & galaxies)

SuperCOSMOS



All-sky optical catalogue from scans of 1980s UKST & POSS2 Schmidt surveys

Depth B<21, R<19.5

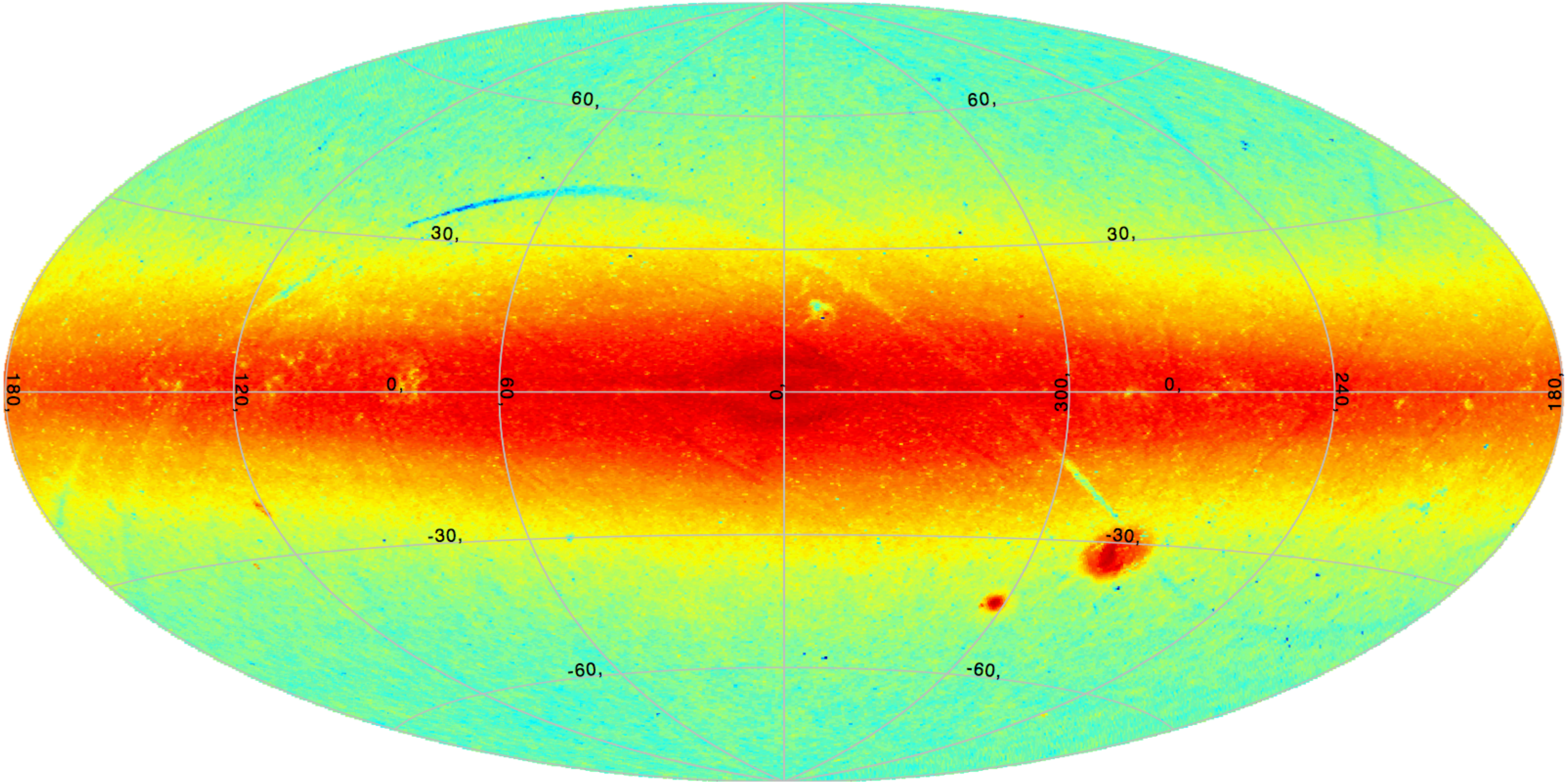
Calibrated for 2dFGRS

~ 200M galaxies; ~ 1B stars

Curated by WFAU

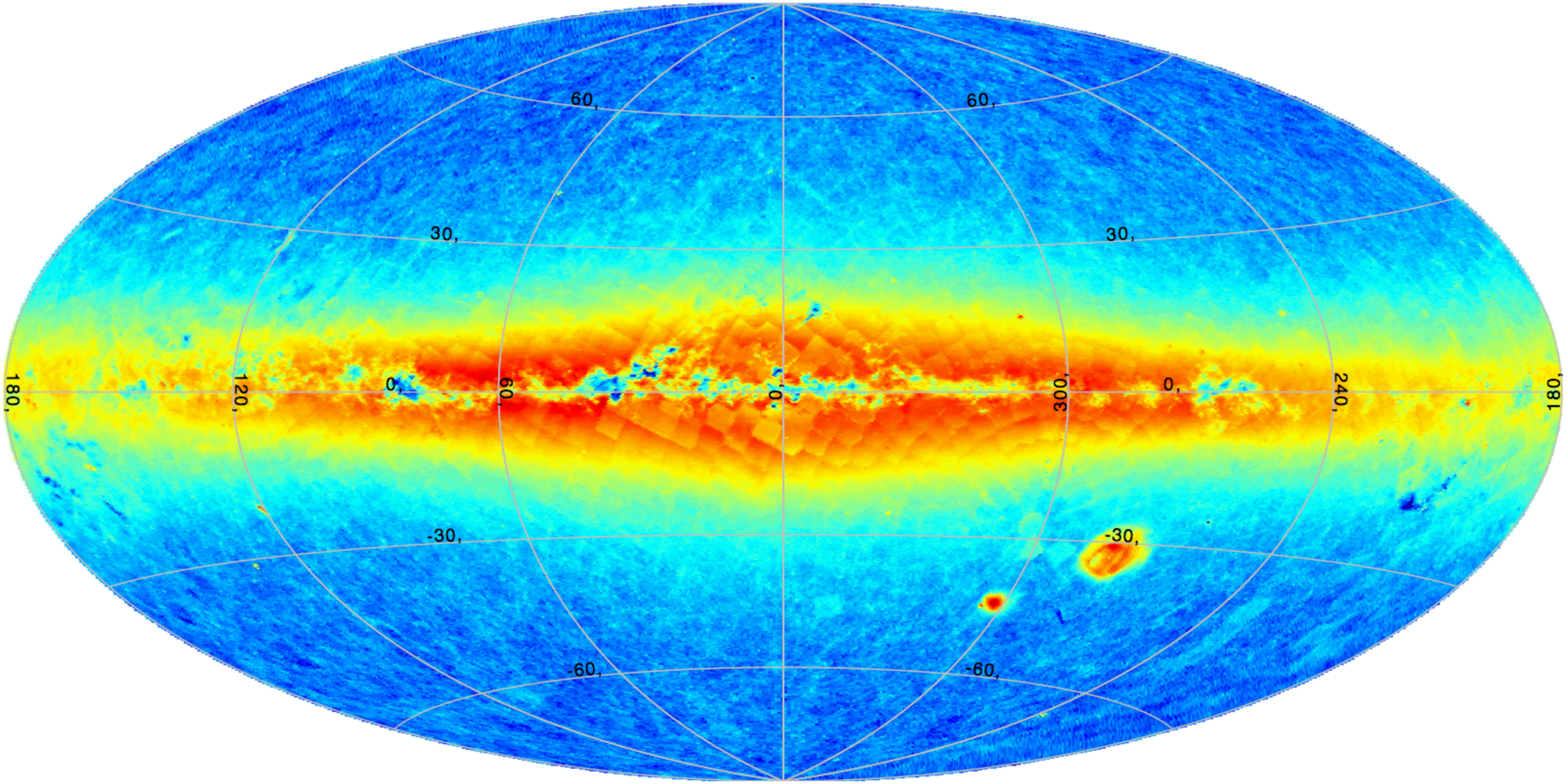
Vincent Reddish (1926-2015)

WISE



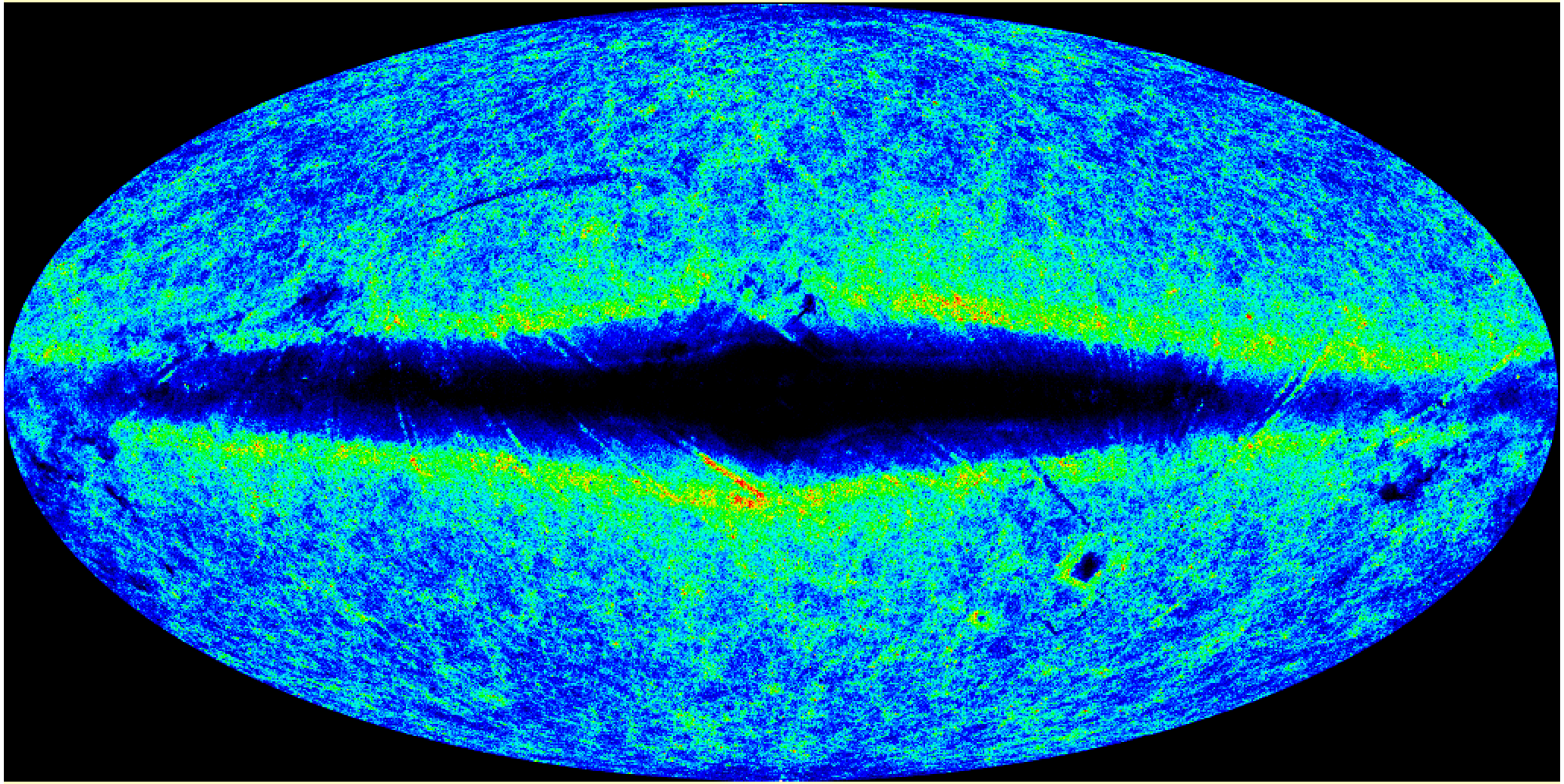
W1<17: 488M

Super-COSMOS extended



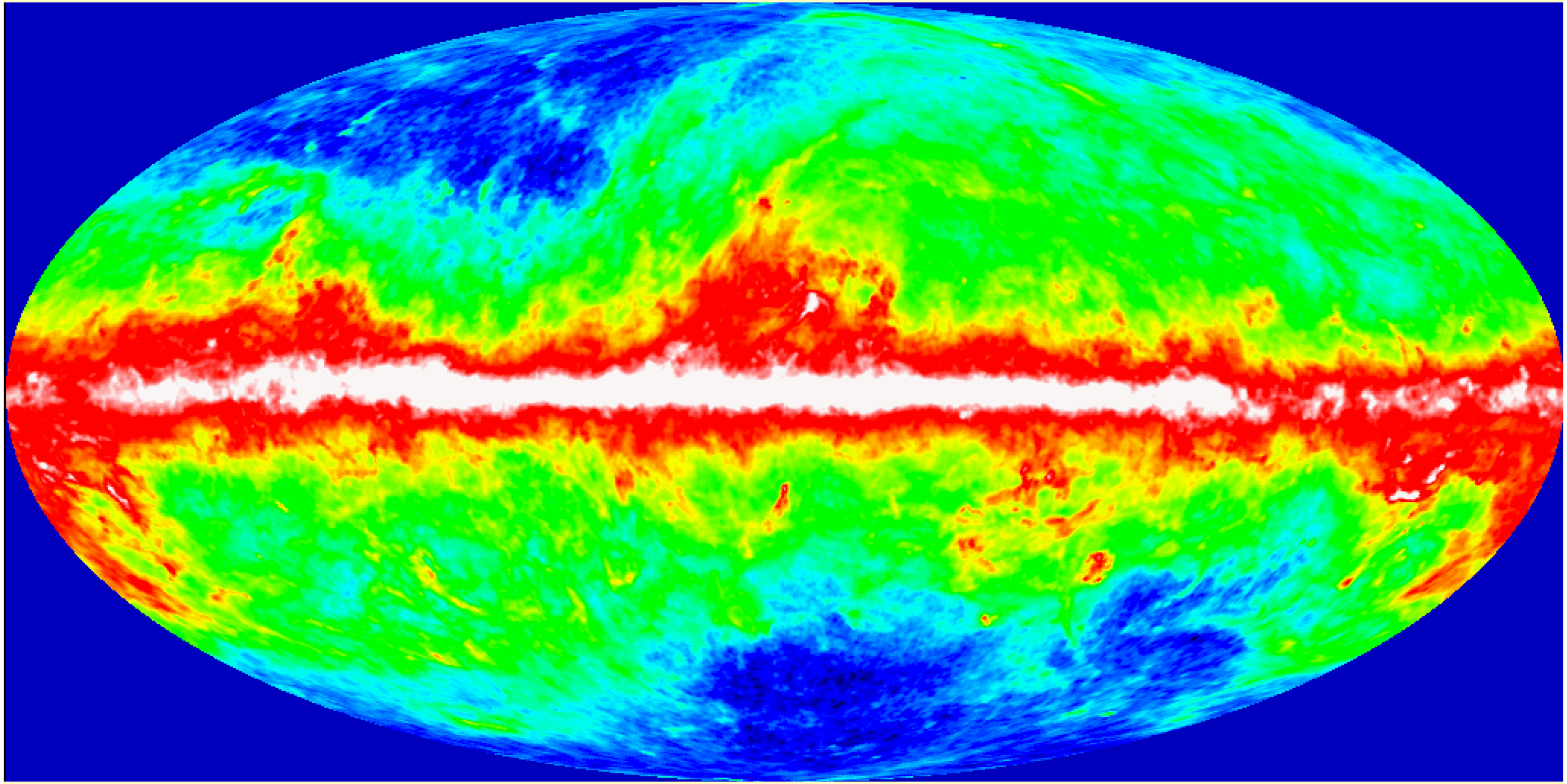
B<21, R<19.5: 204M

WISE: W1-W2 > 0

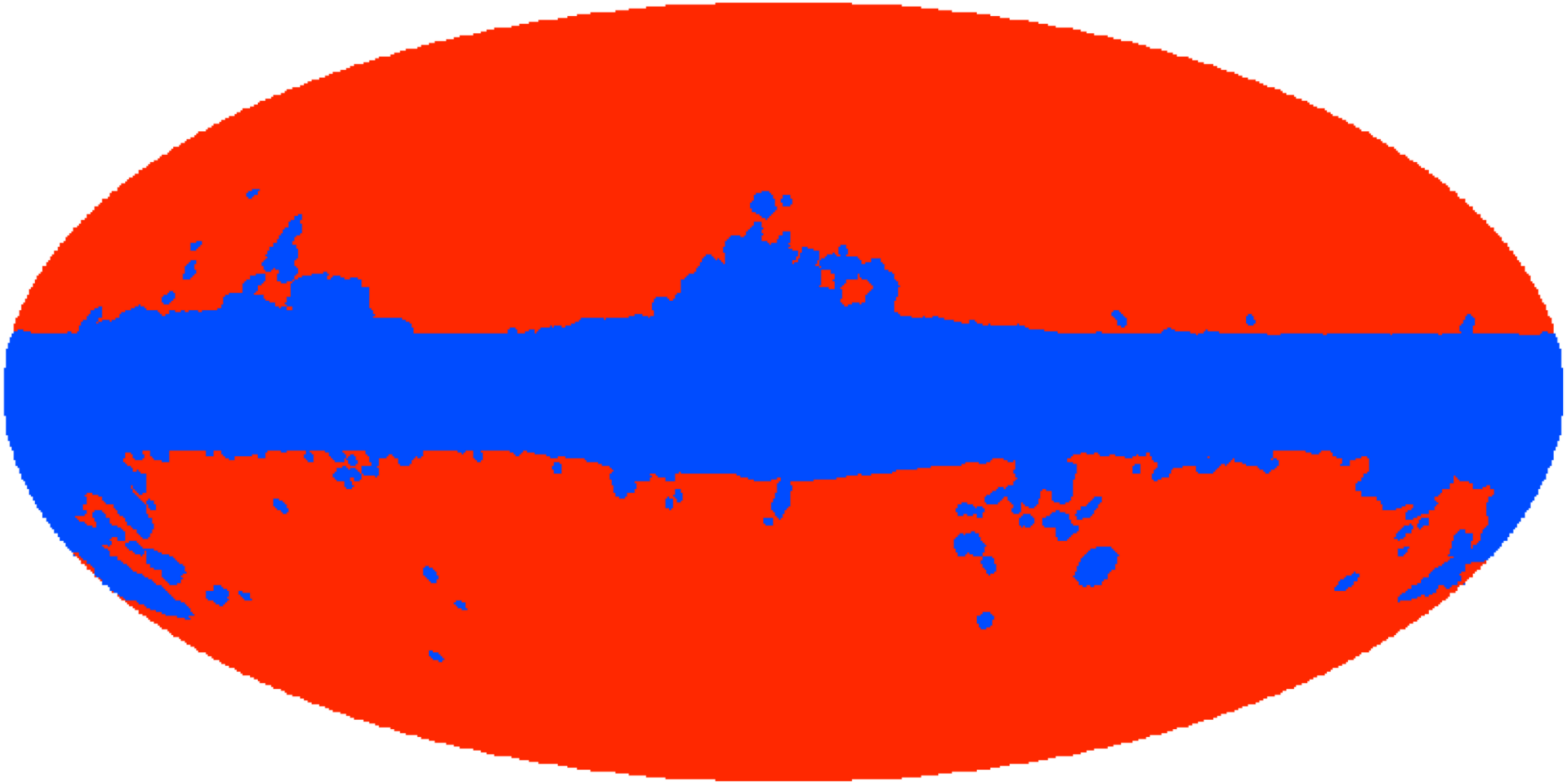


20M after masking

E(B-V) is part of the mask



Bayesian mask



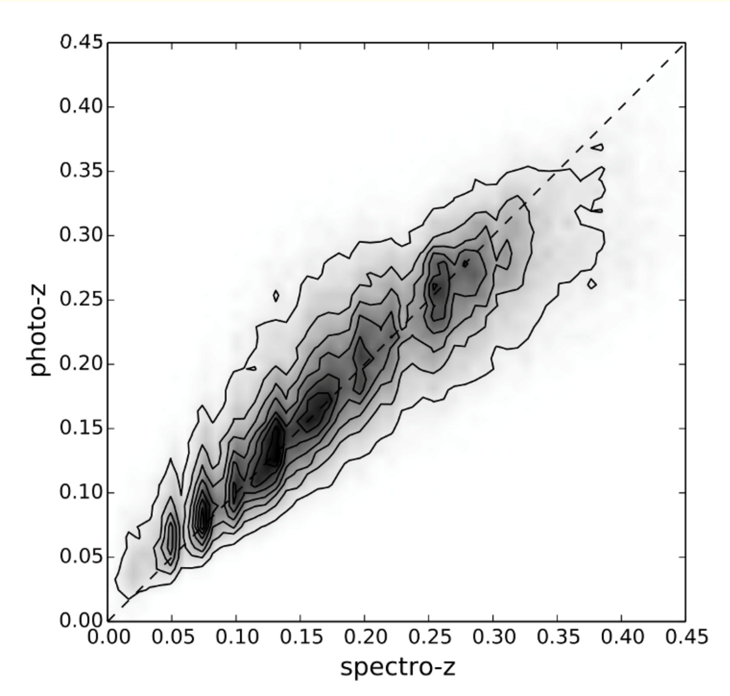
Clip outliers to educate E(B-V) prior, then iterate clipping

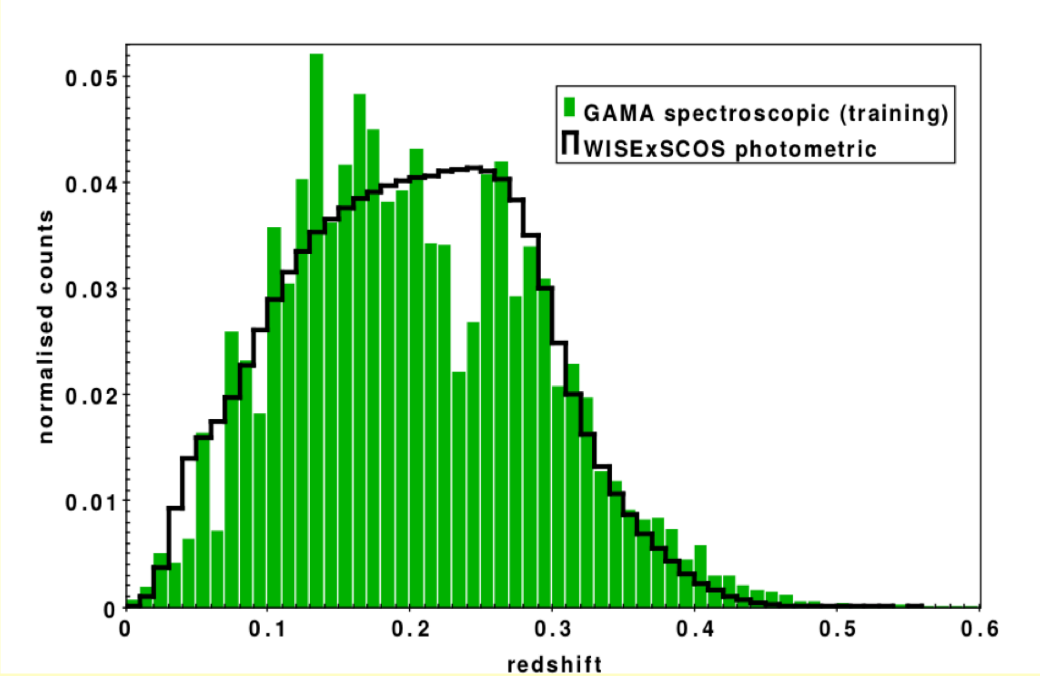
Photometric redshifts

ANNz Using (B,R,W1,W2) and GAMA spectroscopy

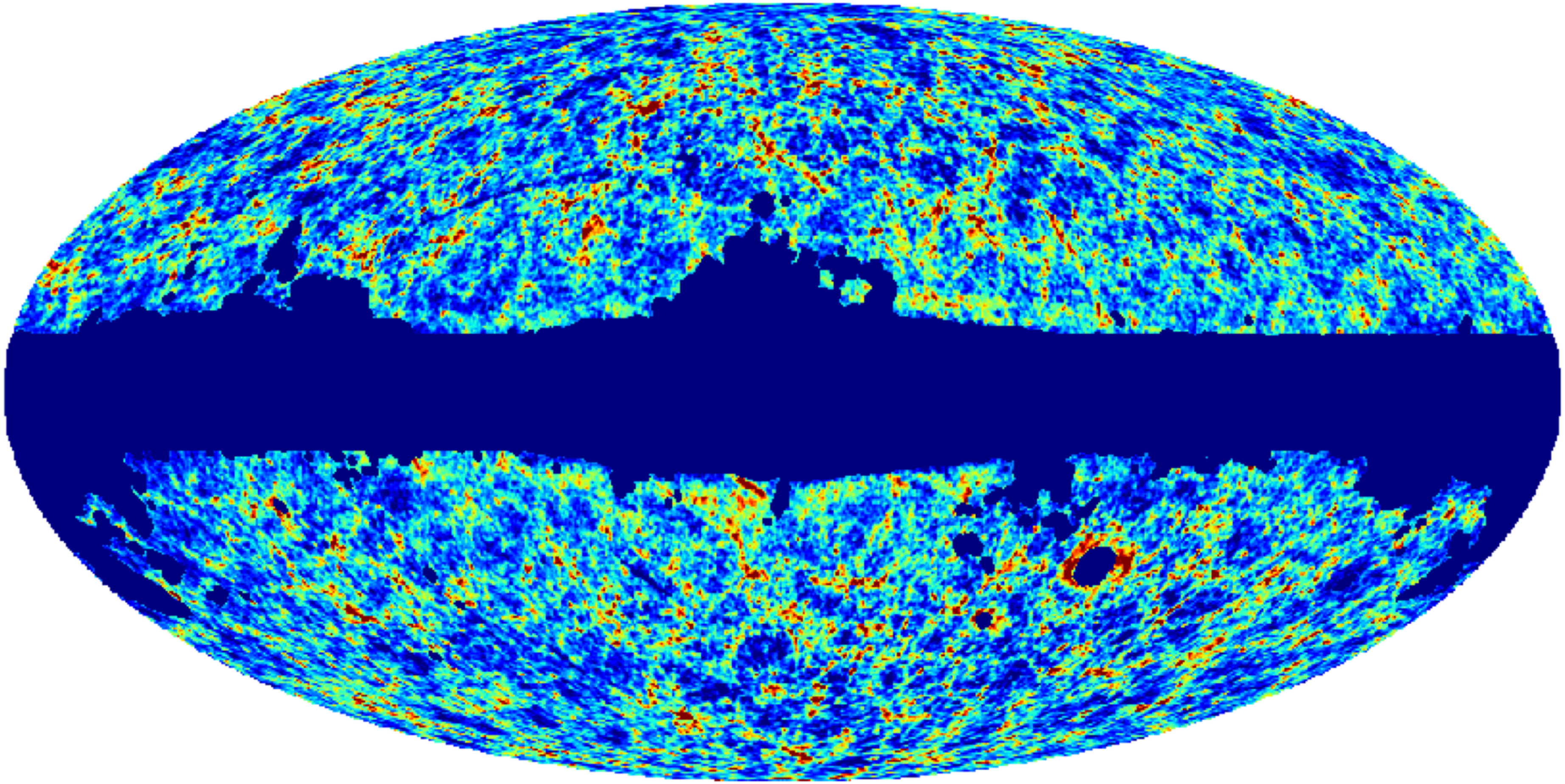
σ_z / (1+z) = 0.032 (0.015 with 2MASS)

Median z = 0.2; useful signal out to z = 0.4 (double 2MASS)

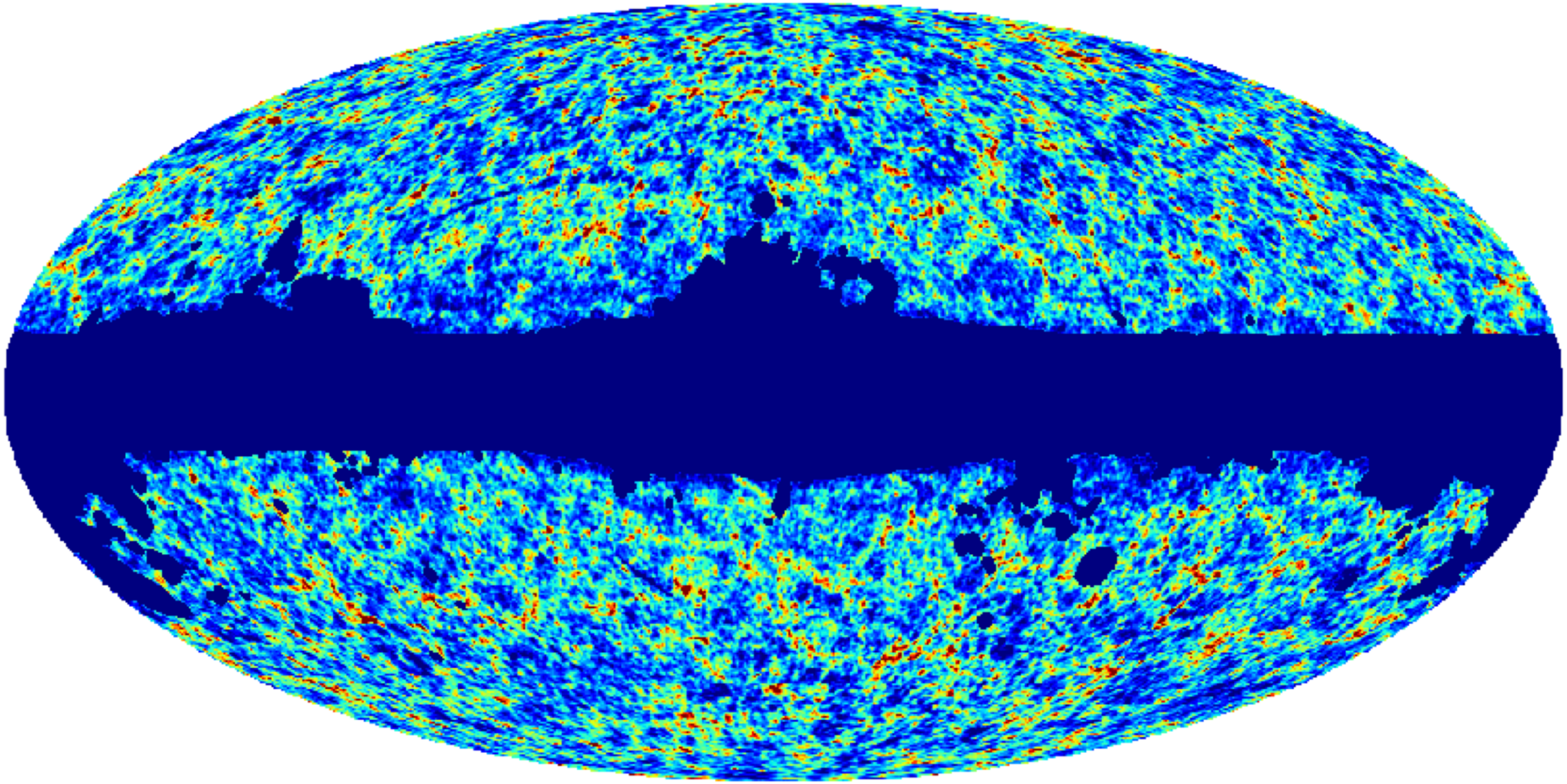




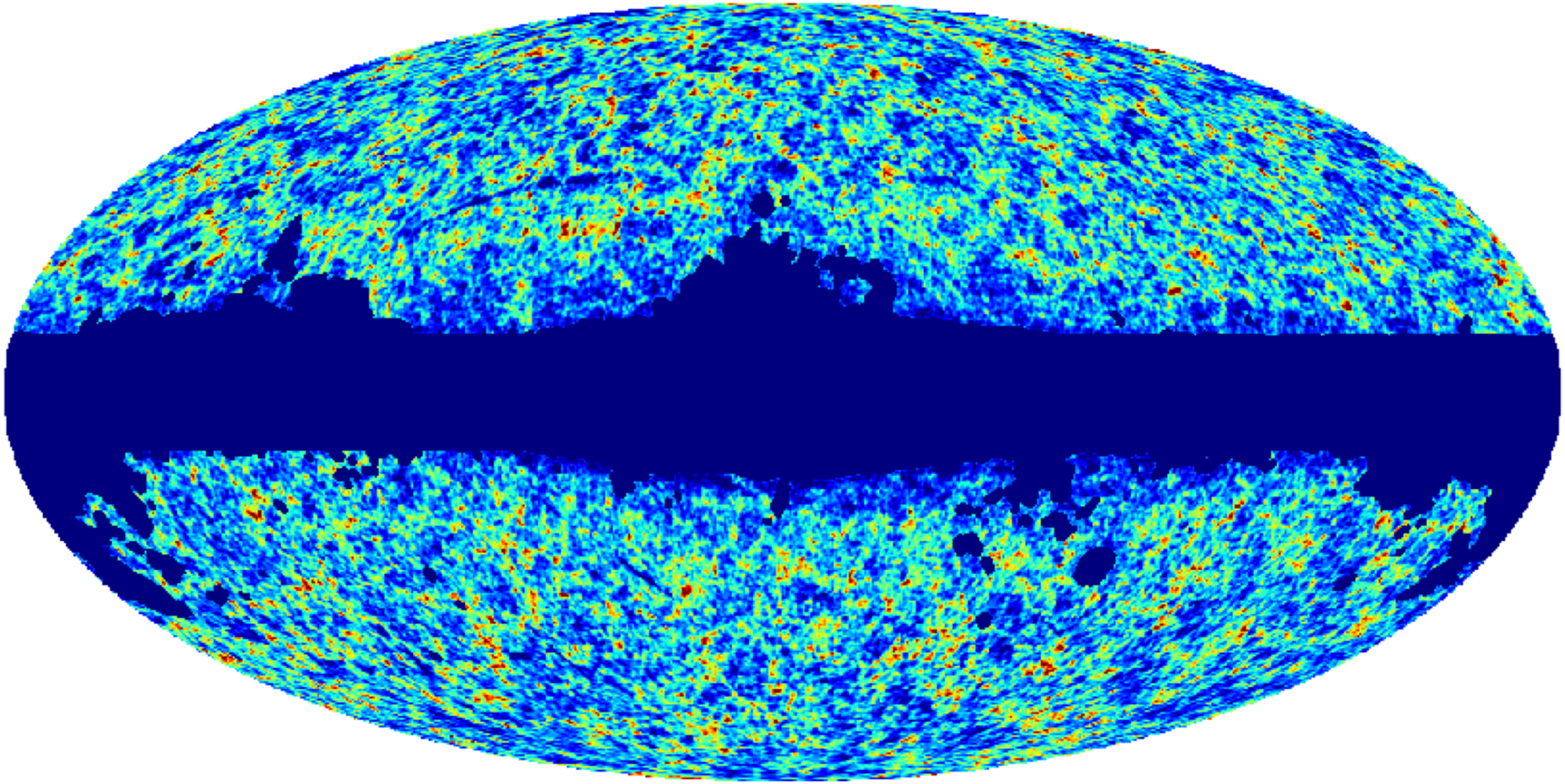
$0.1 < z < 0.15$



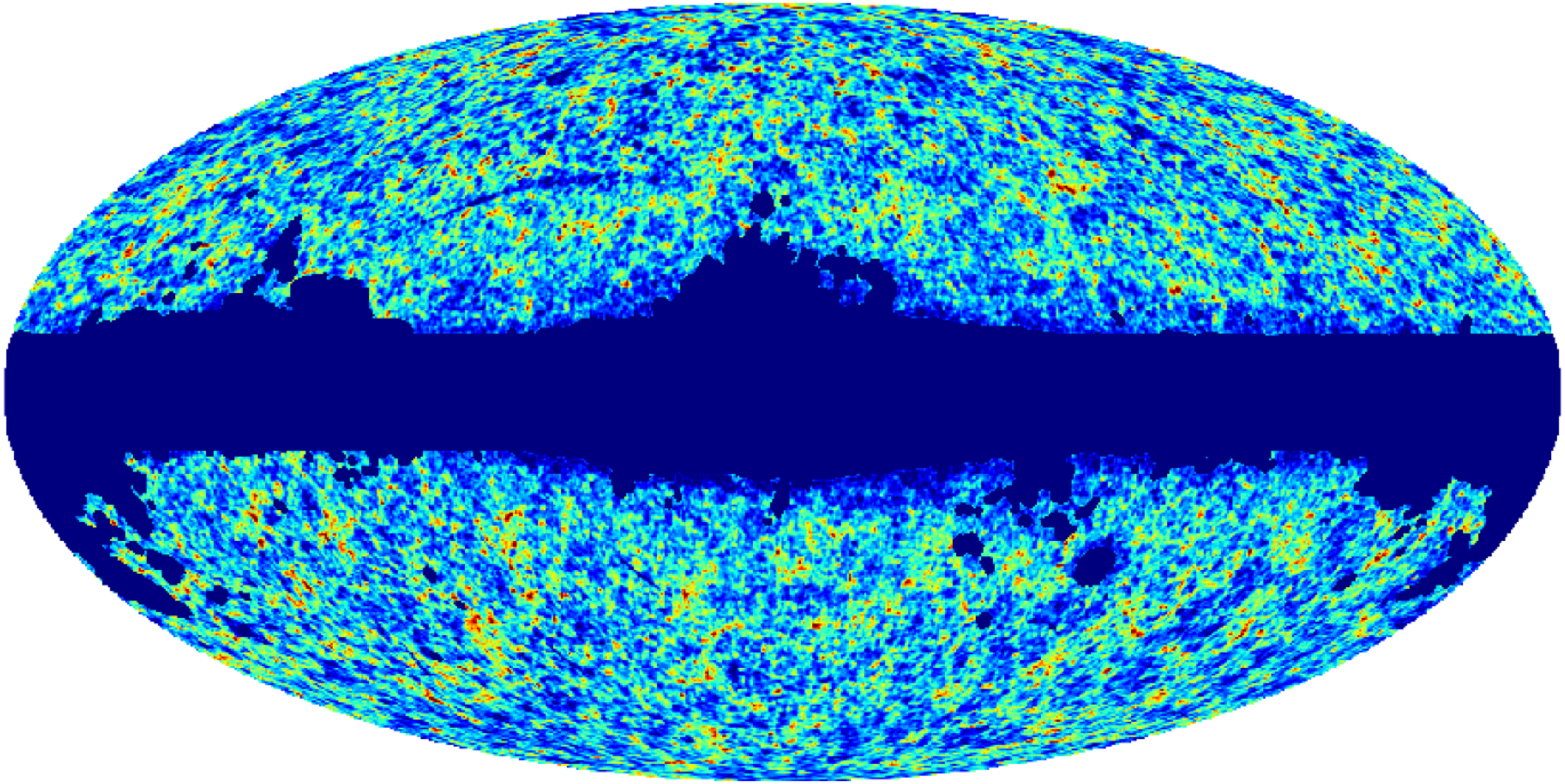
0.15 < z < 0.2



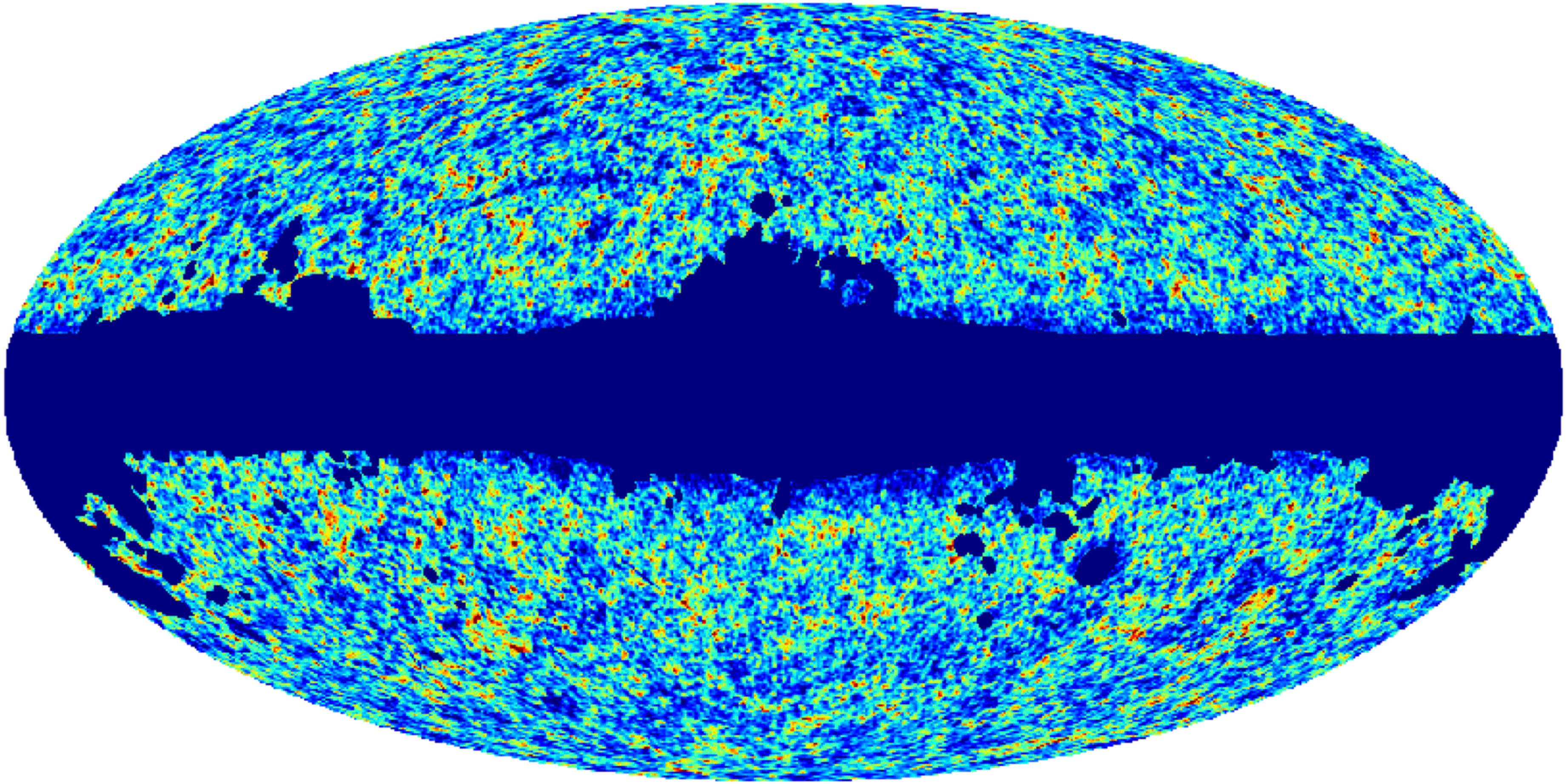
0.2 < z < 0.25



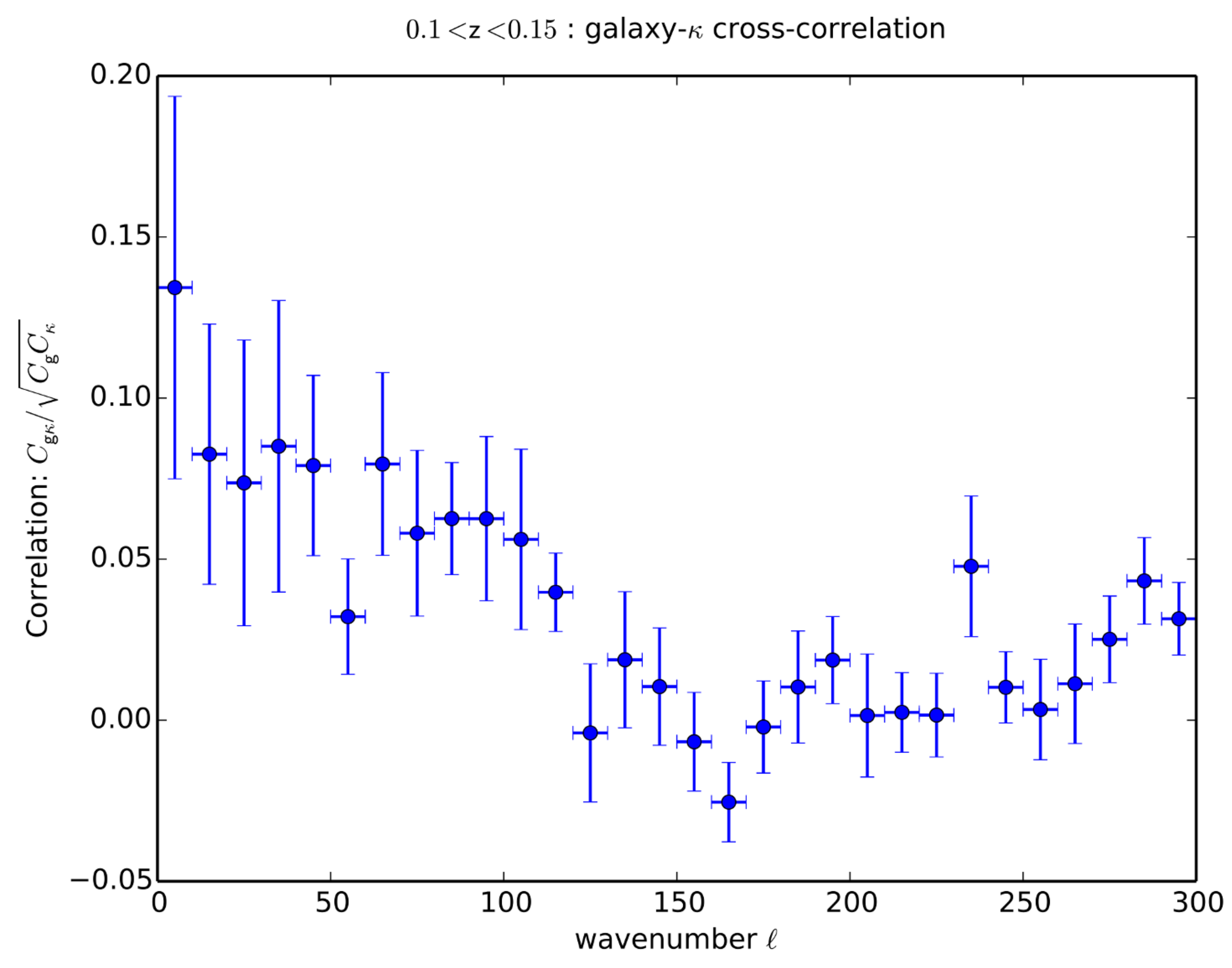
0.25 < z < 0.3



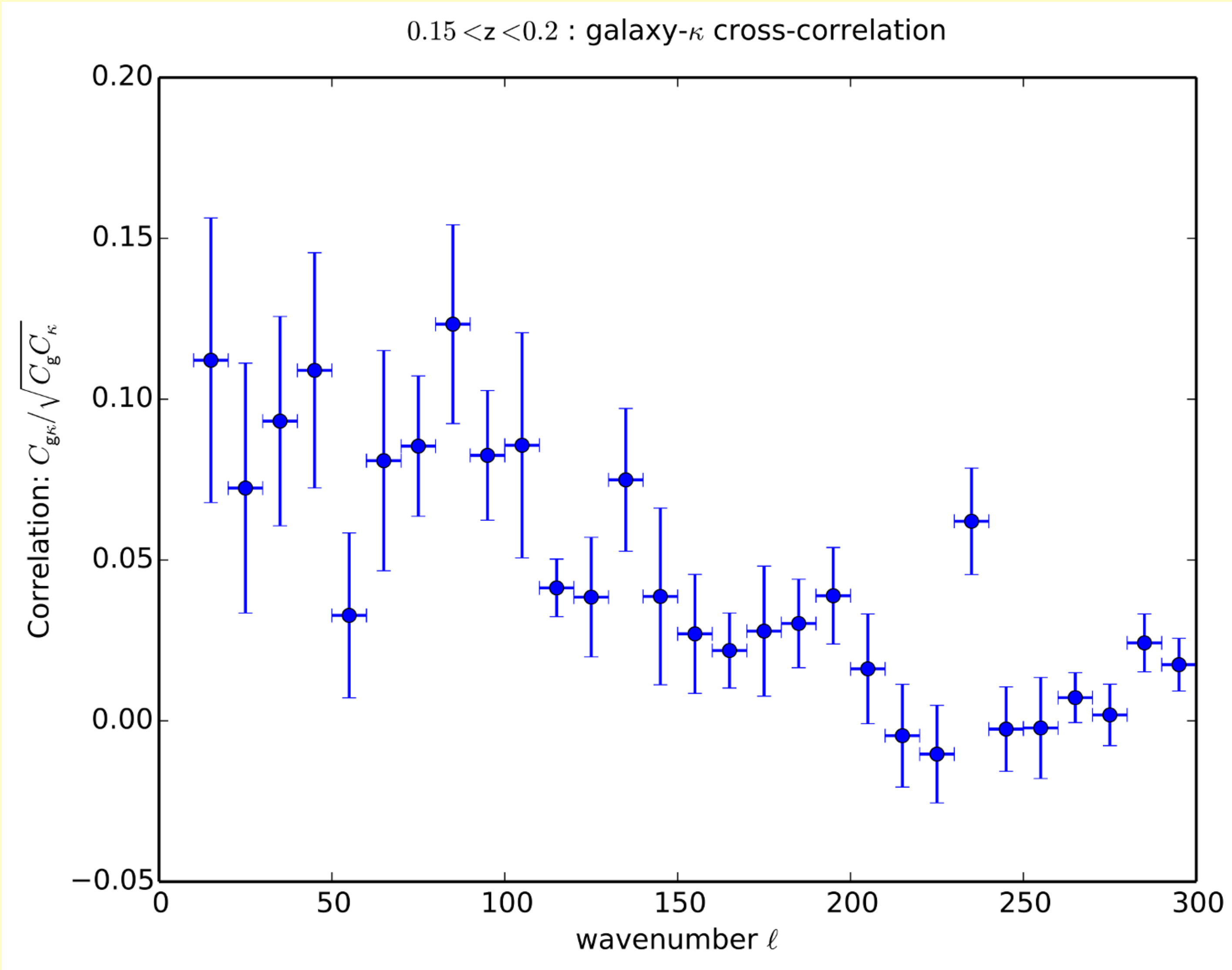
0.3 < z < 0.35



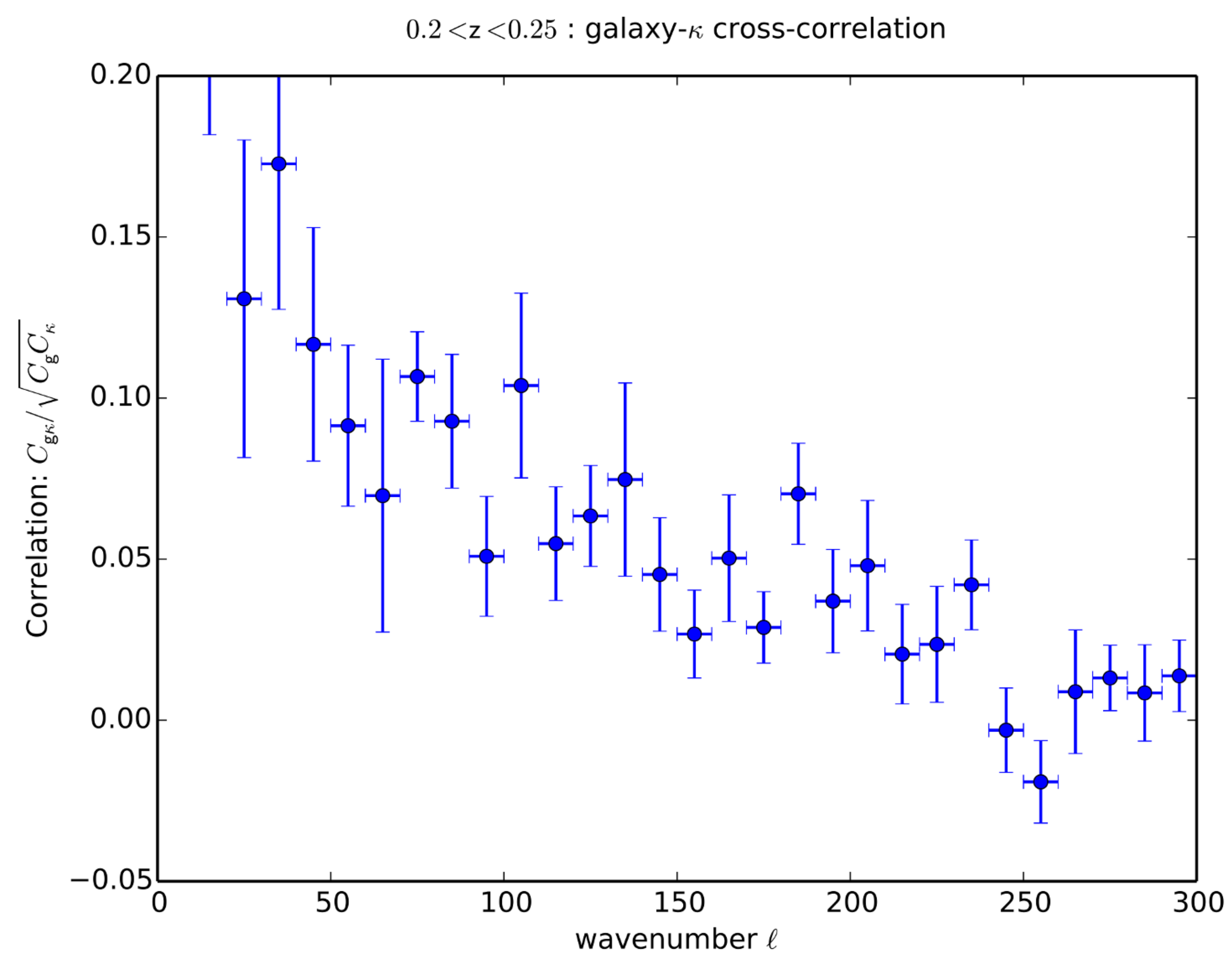
Slicing up the total convergence



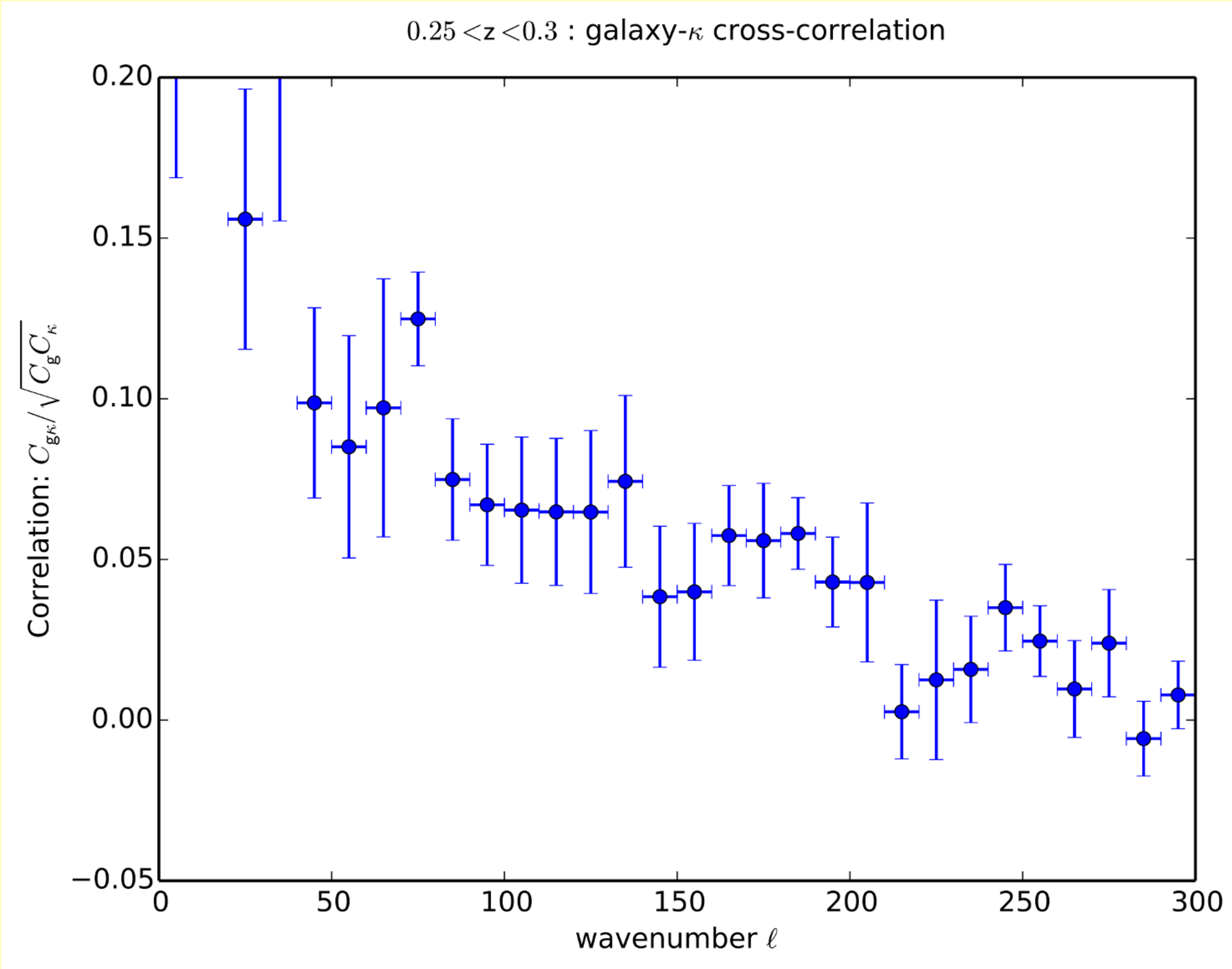
Harmonic-space correlation coefficient: independent of bias



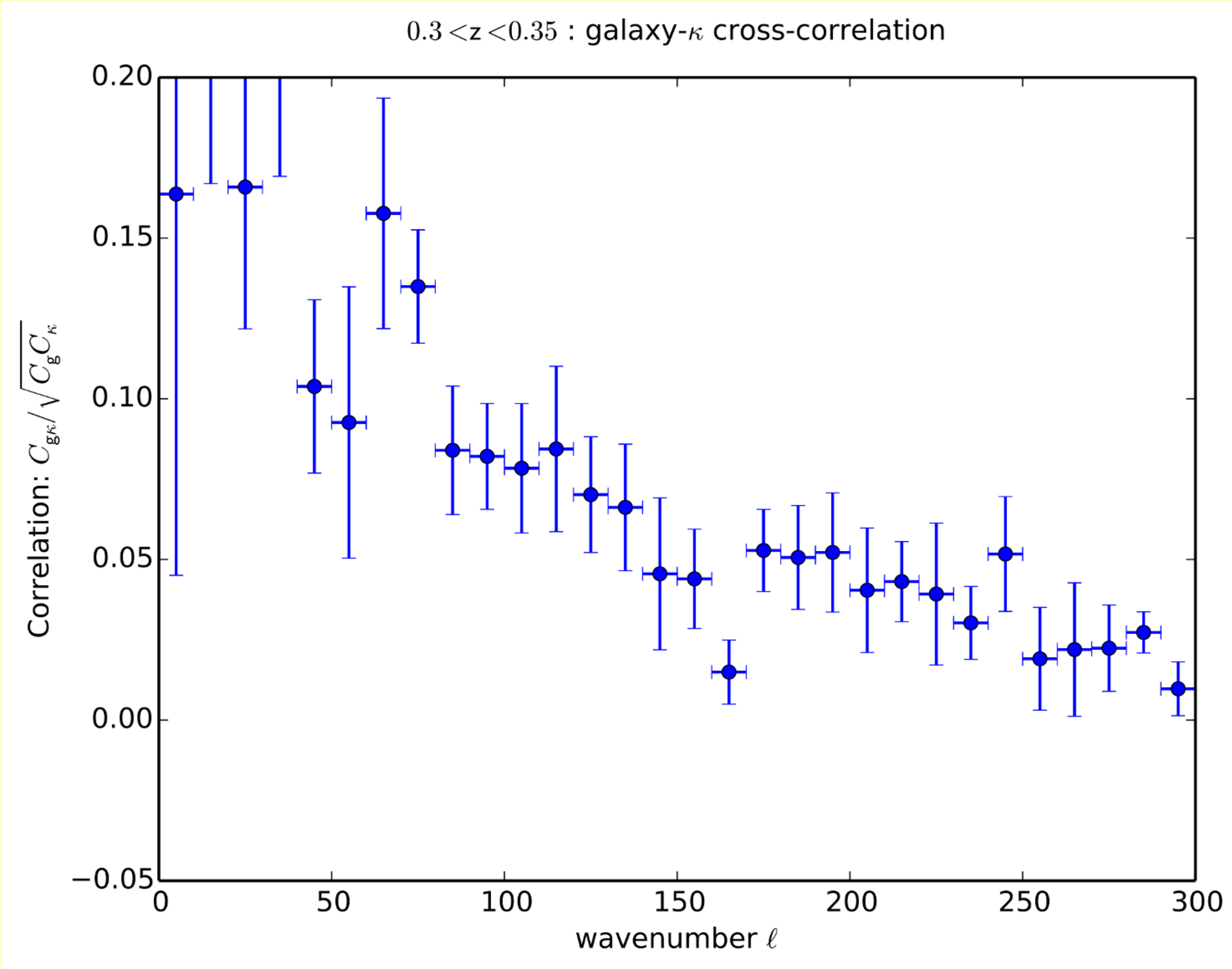
$0.15 < z < 0.2$: galaxy-κ cross-correlation
Correlation: $C_{g\kappa}/\sqrt{C_g C_\kappa}$
wavenumber ℓ
0.20
0.15
0.10
0.05
0.00
−0.05
0
50
100
150
200
250
300



$0.2 < z < 0.25$: galaxy-κ cross-correlation
Correlation: $C_{g\kappa}/\sqrt{C_g C_\kappa}$
wavenumber ℓ
0.20
0.15
0.10
0.05
0.00
−0.05
0
50
100
150
200
250
300

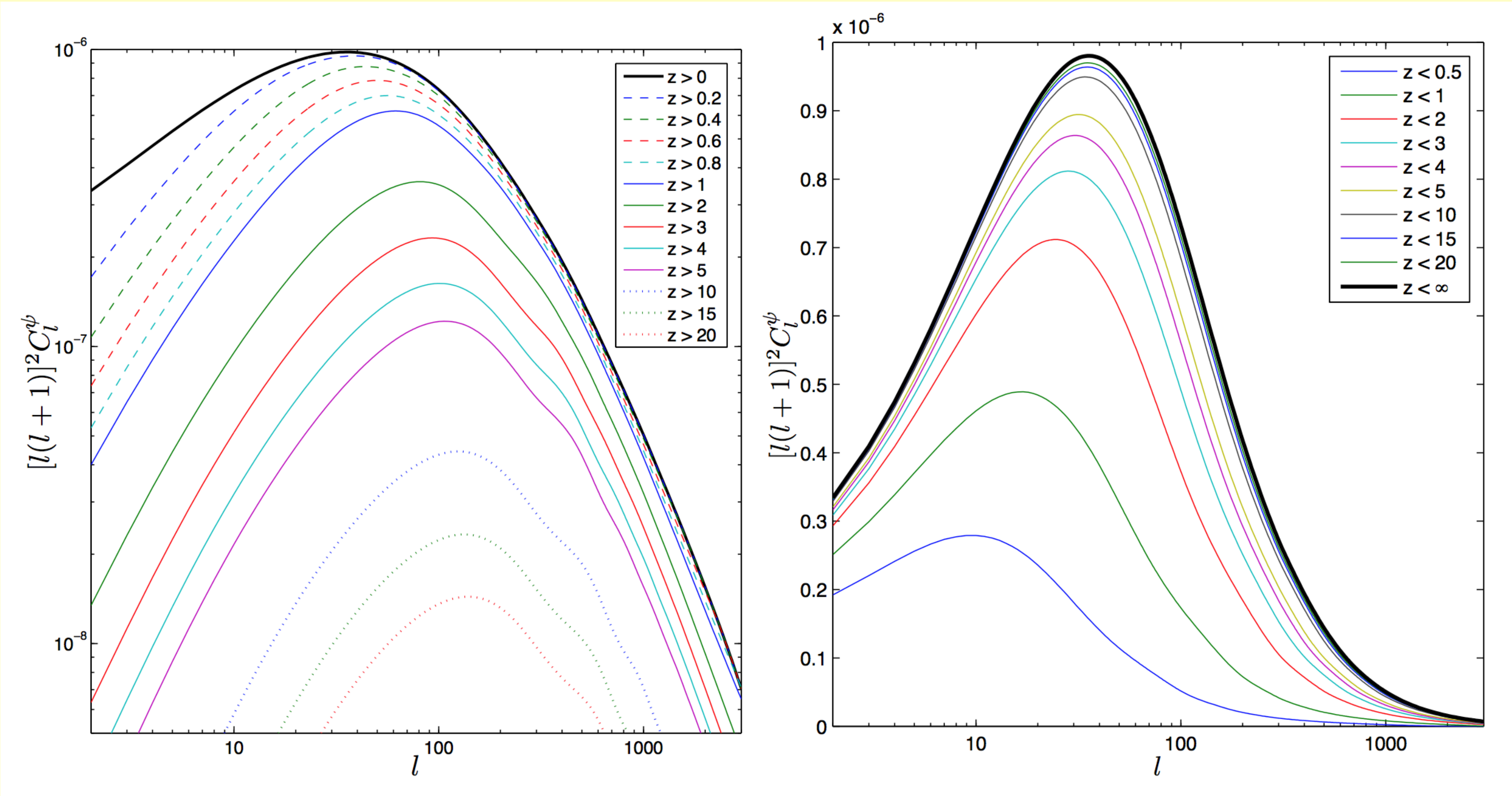


$0.25 < z < 0.3$: galaxy-κ cross-correlation
Correlation: $C_{g\kappa}/\sqrt{C_g C_\kappa}$
wavenumber ℓ
0.20
0.15
0.10
0.05
0.00
−0.05
0
50
100
150
200
250
300



$0.3 < z < 0.35$: galaxy-κ cross-correlation
Correlation: $C_{g\kappa}/\sqrt{C_g C_\kappa}$
wavenumber ℓ
0.20
0.15
0.10
0.05
0.00
−0.05
0
50
100
150
200
250
300

Theory (Hu; Lewis & Challinor)



Low z: $C(<z) / C = 0.1\, z\, (l / 100)^{-0.8}$

Suggests correlation 0.07 $(l / 100)^{-0.4}$ in all dz = 0.05 slices

Conclusions & outlook

- Statistical biases in distance from combined effect of gravitational lensing can be understood
  - Need to distinguish sky and source averaging
  - Needed to prove that constant-z area unchanged
  - No post-Newtonian effects (beyond light bending itself)

- Photographic astronomy lives
  - Only source for all-sky galaxy catalogues
  - Pairing with WISE cleans nicely: 20M galaxies to $z<0.4$
  - Good photo-z's allow CMB lens tomography
    - Probe growth of fluctuations at $z < 0.4$