An Effective Field Theory for Large Scale Structures

based on 1301.7182 with M. Zaldarriaga

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Outline

- Motivations
- Standard Perturbation Theory (SPT) and its problems
- Effective Field Theory for Large Scale Structures (EFToLSS)
- Resolution of the SPT problems
- Renormalization of EFToLSS
- Summary and Outlook

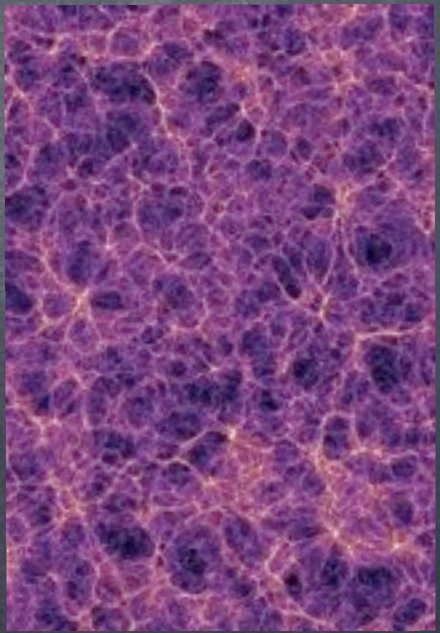
Motivations

- Define Large Scale Structures (LSS)
- LSS teach us about: Dark Matter, Dark Energy, primordial perturbations, modifications of GR, ...
- Why simulate when you can calculate?
- Analytical understanding of LSS is a milestone of our cosmological model

Large Scale Structures

- The distribution of matter in the universe is very inhomogeneous, with very dense clumps of matter (e.g. galaxies) separated by big voids

- On scales much larger than the average galaxy-galaxy distance, i.e. O(1) Mpc, the density of clumps (e.g. galaxies) is very homogeneous



- Large Scale Structures (LSS) have a small density contrast

$$\delta(x) = \frac{\rho(x)}{\bar{\rho}} - 1$$

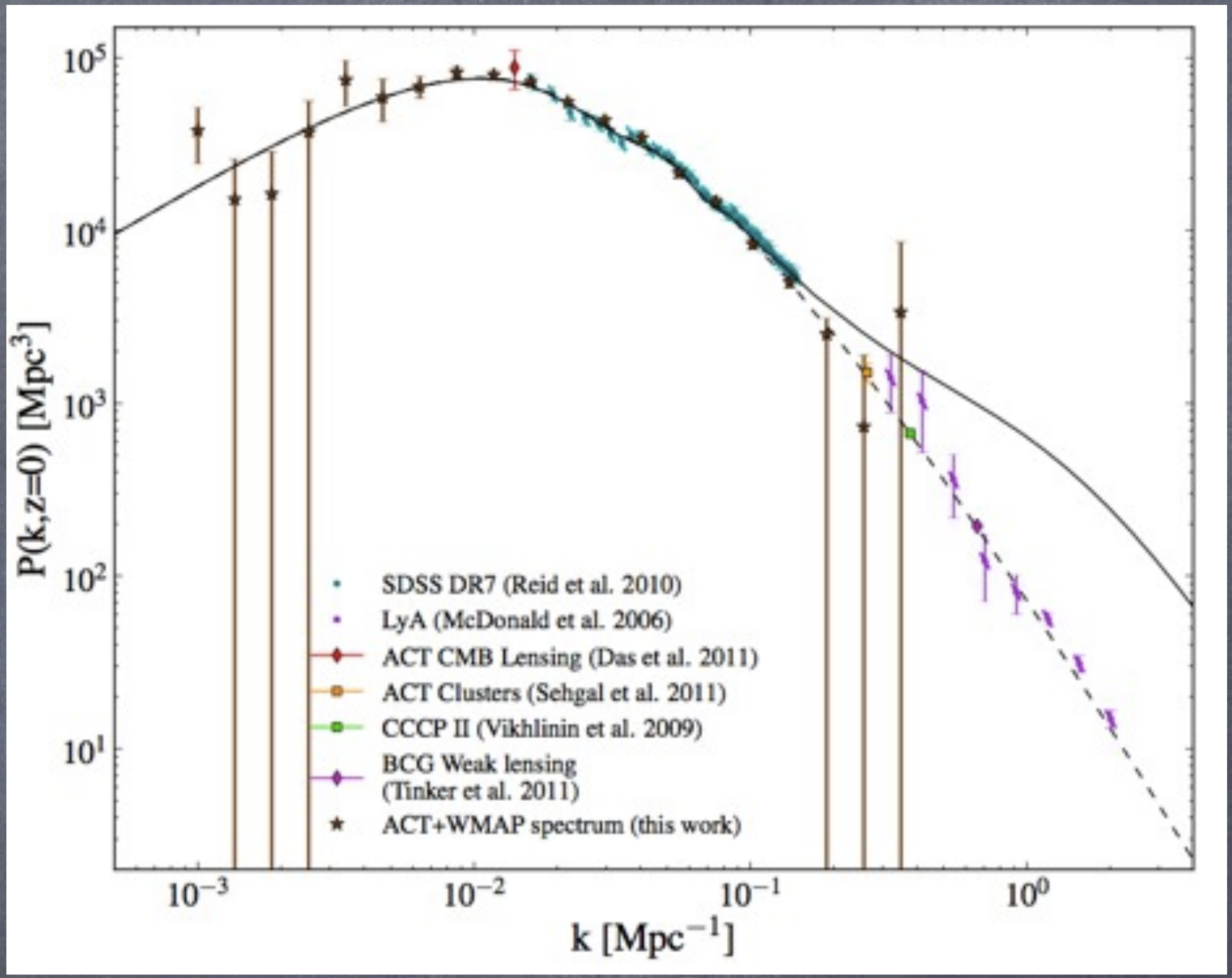
LSS and Dark Energy

- Dark Energy can be probed studying the expansion history of the universe
- The Baryon Acoustic Oscillations (BAO) provide a standard ruler of 150 Mpc
- The BAO peak has a width of O(10) Mpc which gets broaden by non-linear effects

LSS and primordial perturbations

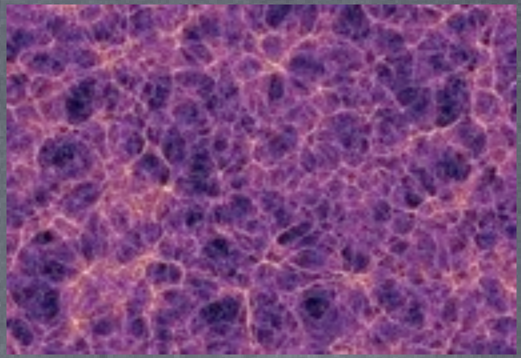
- Because of the small density contrast, LSS evolve linearly giving
  us a very clean probe of initial conditions

- LSS are compatible with 10^{-5} perturbations with a scale-invariant initial power spectrum



- Because 3D information is available through redshift, there are many more modes in LSS than in the CMB which is 2D, hence lower cosmic variance

Simulations



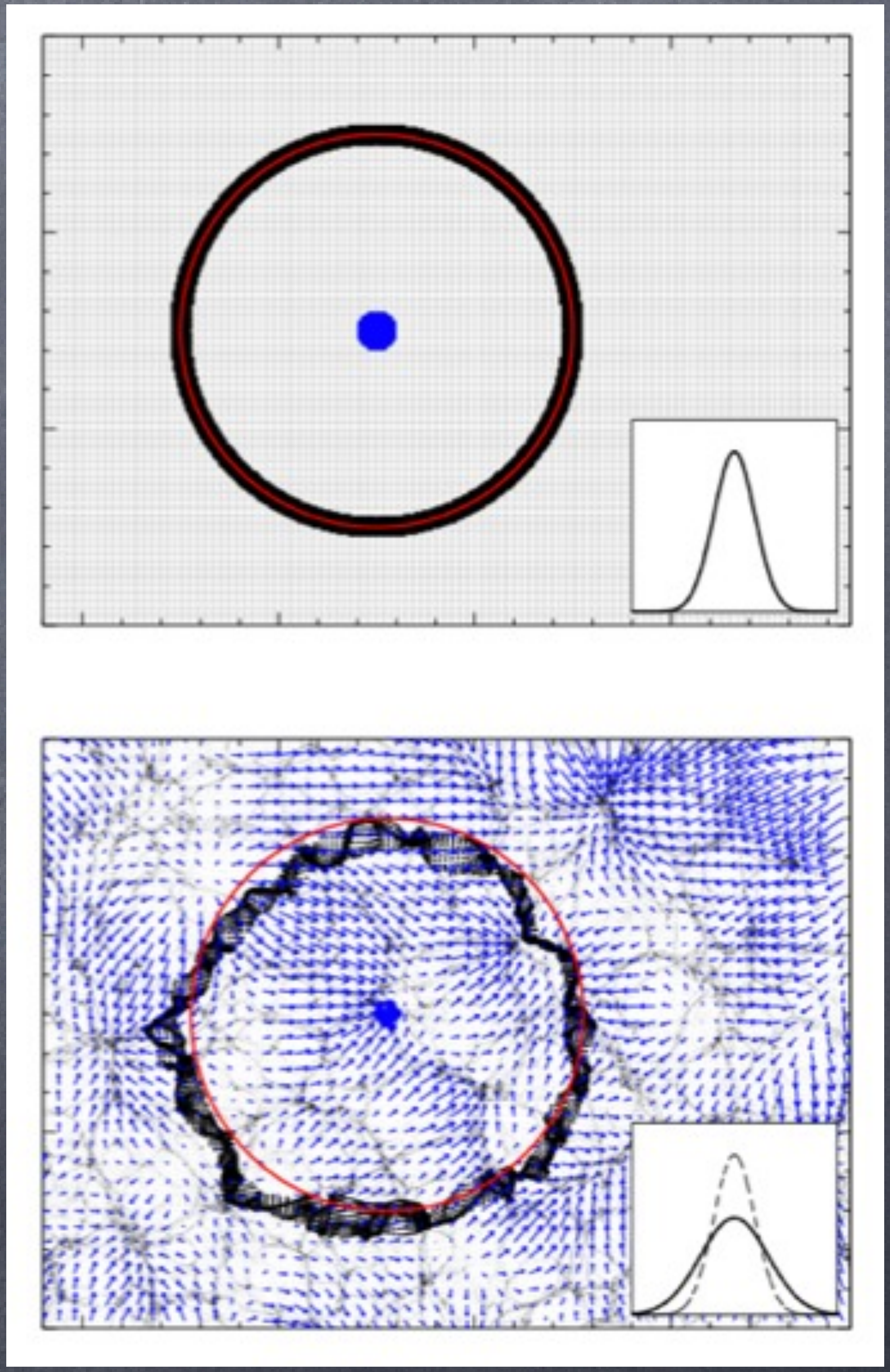
- Numerical simulations of the formation and evolution of structures have become a standard tool in interpreting new data
- Simulations are essential at short scales where the dynamics is highly non-linear
- Simulating accurately large boxes such as the observable universe requires a very large dynamical range, which is very time consuming and resource intensive
- Probing the large-dimensionality parameter space needed for cosmology makes things worse

Analytical description

- Since LSS evolve almost linearly, we have powerful analytical tools to describe the physics, e.g. perturbation theory
- Very general results can be derived where the dependence on cosmological parameters is explicit
- We can combine analytical result with simulations on short scales, which are much less resource intensive
- Obtain a real understanding of what's going on

Mildly non-linear regime

- Below some non-linear scale k_{NL} the density perturbations are strongly coupled and not amenable to analytical computations
- $k<k_{NL}$ are mildly non-linear, that's where we can make some progress
- These scales are crucial for the (reconstruction of) the BAO peak
- The number of independent modes grow with the cube of the shortest scale. So pushing closer to k_{NL} is essential to make progress on primordial perturbations



Standard Perturbation Theory

- A Boltzmann equation for collisionless Dark Matter particles: the Vlasov equation
- On large scales (before shell crossing) one can truncate the hierarchy and get the fluid equations (Bernardeau et al '01)
- Problem 1: there is no clear expansion parameter
- Problem 2: missing deviations from a perfect pressureless fluid
- Problem 3: predictions are UV-divergent and hence unphysical

Vlasov Equation

- Since there is 6 times more DM than baryons, we focus on a system of collisionless DM particles interacting only gravitationally
- The corresponding Boltzmann equation

$$\frac{\partial f}{\partial t} + \frac{p^i}{ma^2}\frac{\partial f}{\partial x^i} - m\partial_i \phi \frac{\partial f}{\partial p^i} = 0$$

known as the Vlasov Equation, describes the evolution of the phase-space density

$$f(\vec{p}, \vec{x}) = \sum_i \delta^3(\vec{x} - \vec{x}_i)\delta^3(\vec{p} - ma\vec{v})$$

- The Poisson's equation determines ϕ

Fluid equations

- Let us define density and velocity

$$\rho \equiv m a^{-3} \int d^3 p f(x,p) \quad \rho v^i \equiv \int d^3 p \, p^i f(x,p)$$

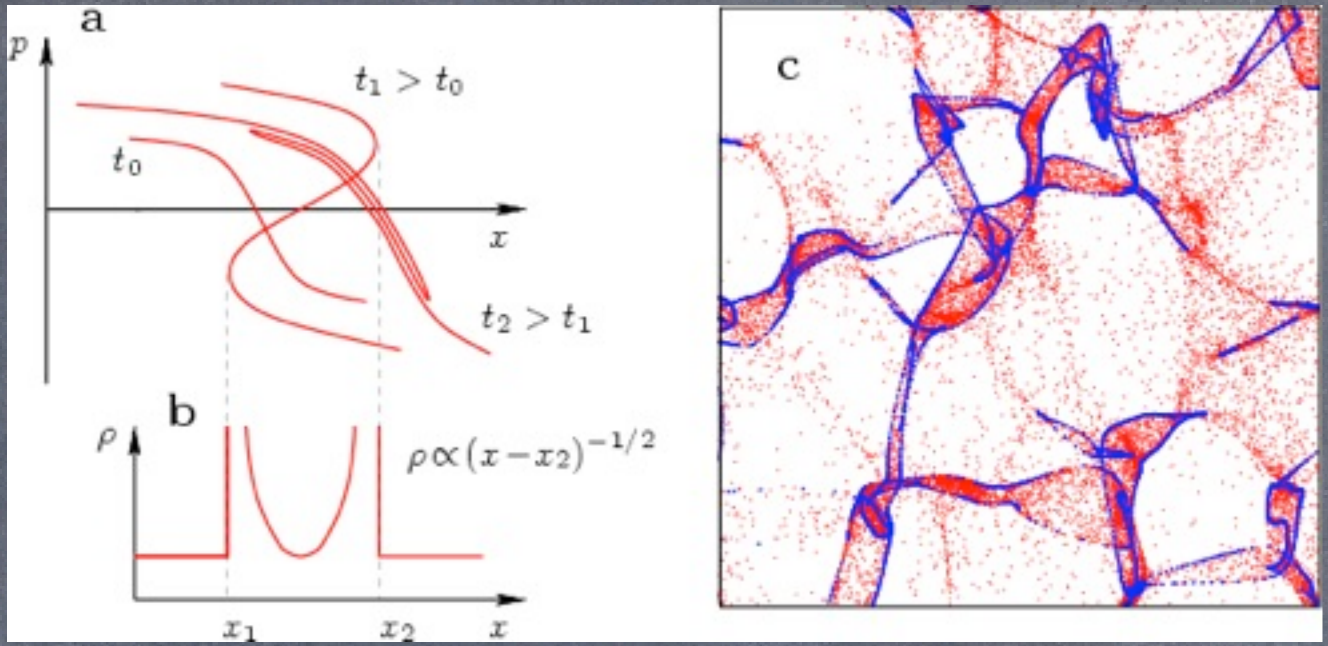
- Taking the first two moment of the Vlasov eqution leads the continuity and the Euler equations

$$\partial_\tau \delta + \partial_i \left[(1+\delta) \, v_l^i \right] = 0$$

$$\partial_\tau v^i + \mathcal{H} v^i + \partial_i \phi + v^k \partial_k v^i = 0$$

- Can we solve it perturbatively?

Problem 1



- Because of shell-crossing the density diverges a short scales
- No clear expansion parameter for perturbation theory
- Even when applying to large scales, this makes it hard to estimate the theoretical errors in the computation

Problem 2

- The fluid equations are those of a perfect pressureless fluid

- Since the short scales cannot be model correctly, there is no way to exclude non-linear exchanges of energy and/or momentum with the large scales, leading to dissipation

- More generally, there is NO symmetry forbidding a pressure term or any higher derivative corrections, e.g. viscosity.

- Why would they not be there?

Problem 3

- Perturbation theory as (dubious) expansion in δ

$$\delta_n \sim \int GF(k,k')\delta_m(k')\delta_{n-m}(k-k')$$

- Corrections to correlators, e.g. to the power spectrum P(k), are organized in loops. E.g. linear and 1-loop:

$$\begin{aligned} P =& P_{lin} + P_{22} + P_{13} + \dots \\ \langle\delta\delta\rangle =& \langle\delta_1\delta_1\rangle + \langle\delta_2\delta_2\rangle + 2\langle\delta_1\delta_3\rangle + \dots \end{aligned}$$

and similarly for v and higher n-point functions

Problem 3

- "Loop" corrections indeed have loop integrals

$$P_{22}(k\to\infty)\simeq k^4\int\frac{dq}{q^2}P_{in}^2(q)$$

$$P_{13}(k\to\infty)\simeq k^2P_{in}(k)\int dqP_{in}(q)$$

- For generic initial conditions these are UV-divergent, and hence unphysical

$$P_{in}=Ak^n$$

	UV div	IR div
P_{13}	$n\geq -1$	$n\leq -1$
P_{22}	$n\geq 1/2$	$n\leq -1$
P_{total}	$n\geq -1$	$n\leq -3$

Effective Field Theory of Large Scale Structures

- Consistently integrate out short-scales (Baumann et al '10)
- Problems 1: smoothed density and velocity are a good expansion parameters
- Problem 2: effective corrections to a perfect pressureless fluid arise (EFT philosophy)
- Problem 3: effective corrections are exactly the needed counterterms to renormalize the theory

Smoothing

- We smooth all fields on a certain scale $\Lambda < k_{NL}$

$$\delta \rightarrow [\delta]_\Lambda = \int dx' W_\Lambda(x - x')\delta(x')$$

- Short modes can combine to create long-wavelength perturbations

- We can expand short modes in the background of long modes

$$(f\, g)_l = f_l g_l + (f_s g_s)_l + \frac{1}{\Lambda^2} \nabla f_l \nabla g_l + \dots$$

- We get long, stochastic and higher derivative terms

Short scales

- We do not know how to describe the short scales, but we can parameterize our ignorance

$$(f_s g_s)_l = \langle f_s g_s \rangle_0 + \delta_l \frac{\partial \langle f_s g_s \rangle}{\partial \delta_l} + (f_s g_s) - \langle f_s g_s \rangle + \ldots$$

- There are numerical and stochastic unknown coefficients

- These coefficients can be determined by simulations or by fitting the observations (Carrasco et al '12, Hertzberg '12)

- As always in EFT, the theory becomes predictive once we have more observables than parameters

Effective corrections

- Smoothing the Vlasov equation leads to

$$\partial_\tau \delta + \partial_i \left[(1+\delta)\, v_l^i \right] = 0\,,$$

$$\partial_\tau v_l^i + \mathcal{H} v_l^i + \partial_i \phi + v_l^k \partial_k v_l^i = - c_s^2 \partial^i \delta + c_{sv}^2 + c_{bv}^2 \frac{\partial^2 v_l^i}{\mathcal{H}} + \frac{c_{sv}^2}{\mathcal{H}} \partial^i \partial_j v_l^j - \Delta J^i \ldots$$

- A pressure, viscosity and a stochastic terms, plus (infinitely many) higher derivatives

- These are all the terms allowed by the symmetries of the problem, as in the EFT philosophy

Problems 1 & 2

- Every field is now smoothed on a scale $\Lambda < k_{NL}$

  therefore δ, $v \ll 1$ providing good expansion parameters

- The short scales are now consistently accounted for, through the effective terms

- Collisionless dark matter on large scales shows indeed deviations from a perfect pressureless fluid, that vanish as k goes to 0

- What about perturbation theory?

Renormalization

- For generic initial conditions, SPT predictions are UV-divergent and hence unphysical (Friemann & Scoccimarro '96)
- The effective coefficients induced by integrating out the short scales (neglected in SPT) are exactly the counterterms needed to cancel the UV-divergencies
- EFToLSS, rather than SPT, is the theoretically consistent way to do perturbation theory
- Einstein deSitter (EdS) is a simple, realistic and very instructive example

Perturbation theory

- For simplicity let us focus just on δ

$$\Box\delta \simeq -c_s^2 k^2 \delta - J + \int F(k,q)\delta(k-q)\delta(q)$$

- F is the usual interaction kernel in SPT, while J and c_s are the new effective terms

- The terms on the rhs are treated perturbatively

$$\delta_J = \int G J \quad \delta_{c_s} = \int G\, c_s^2 k^2 \delta_1$$

- New corrections to the correlators, e.g. power spectrum

$$\langle \delta_1 \delta_{c_s} \rangle \equiv P_{c_s} \quad \langle \delta_J \delta_J \rangle \equiv P_J$$

Regularization

- The smoothing has regularized the theory. For P = k^n

$$P_{22}(k \to \infty) \simeq k^4 \int \frac{dq}{q^2} P_{in}^2(q) \sim k^4 \Lambda^{2n-1}$$

$$P_{13}(k \to \infty) \simeq k^2 P_{in}(k) \int dq P_{in}(q) \sim k^2 P_{in}(k) \Lambda^{n-1}$$

- But now we have extra (conter)terms

$$P_J = \langle JJ \rangle(\Lambda) \sim k^4 f(\Lambda)$$

$$P_{c_s^2} = c_s^2(\Lambda) k^2 P_{in}(k)$$

- Precisely the right k-dependence to cancel the UV-divergences

Cancellation of UV-divergences

- Although we show it just at one loop the cancellation of divergences takes place at all loops
- This is ensured by the EFT construction: if all terms compatible with the symmetries are included, there is always a term with the same structure as the UV-divergences
- The cancellation ensures that the result is independent of the cutoff Λ, and hence physically meaningful (unlike for SPT)

Einstein de Sitter

- During 3300<z<1 our universe was matter dominated
- To first approximation most structures formed in a universe with Ω_m=1, i.e. an Einstein deSitter (EdS) universe
- The (non-relativistic) SPT fluid equations have a scaling symmetry in EdS

$$\tilde{\delta}(x,\tau) = \delta(\lambda_x x, \lambda_\tau \tau)$$

because there is no velocity in the problem

- This simple but realistic example teach us a lot about the structure of perturbation theory

Self-similarity

- For the rescaled solution to belong to the same cosmology as the original one, one needs

$$\Delta^2(k,\tau) \equiv \frac{k^3 P(k,\tau)}{2\pi^2} \qquad \Delta^2(k,\tau) = \Delta^2(k/\lambda_x, \lambda_\tau \tau)$$

- This happens only for a self-similar (no-scale) initial power spectrum $P = A\, a^2\, k^n$

- Only one scale in the problem, e.g. the non-linear scale

$$k_{NL}^{3+n} \equiv \frac{2\pi^2}{Aa^2} \propto \tau^{-4} \qquad \Delta^2_{lin} = \left(\frac{k}{k_{NL}}\right)^{n+3}$$

- Everything must be function of k/k_{NL}

Power spectrum

- Because of self-similarity, knowing the k-dependence of every term fixed the form of all correlators.

- E.g. the power spectrum is

$$\Delta^2 = \left(\frac{k}{k_{NL}}\right)^{3+n} + \beta(n)\left(\frac{k}{k_{NL}}\right)^{5+n} + \gamma(n)\left(\frac{k}{k_{NL}}\right)^{7}$$
$$+ \left(\frac{k}{k_{NL}}\right)^{2(3+n)}\left[\alpha(n) + \tilde{\alpha}(n)\ln\left(\frac{k}{k_{NL}}\right)\right] + \ldots$$

Apparent violation of self-similarity

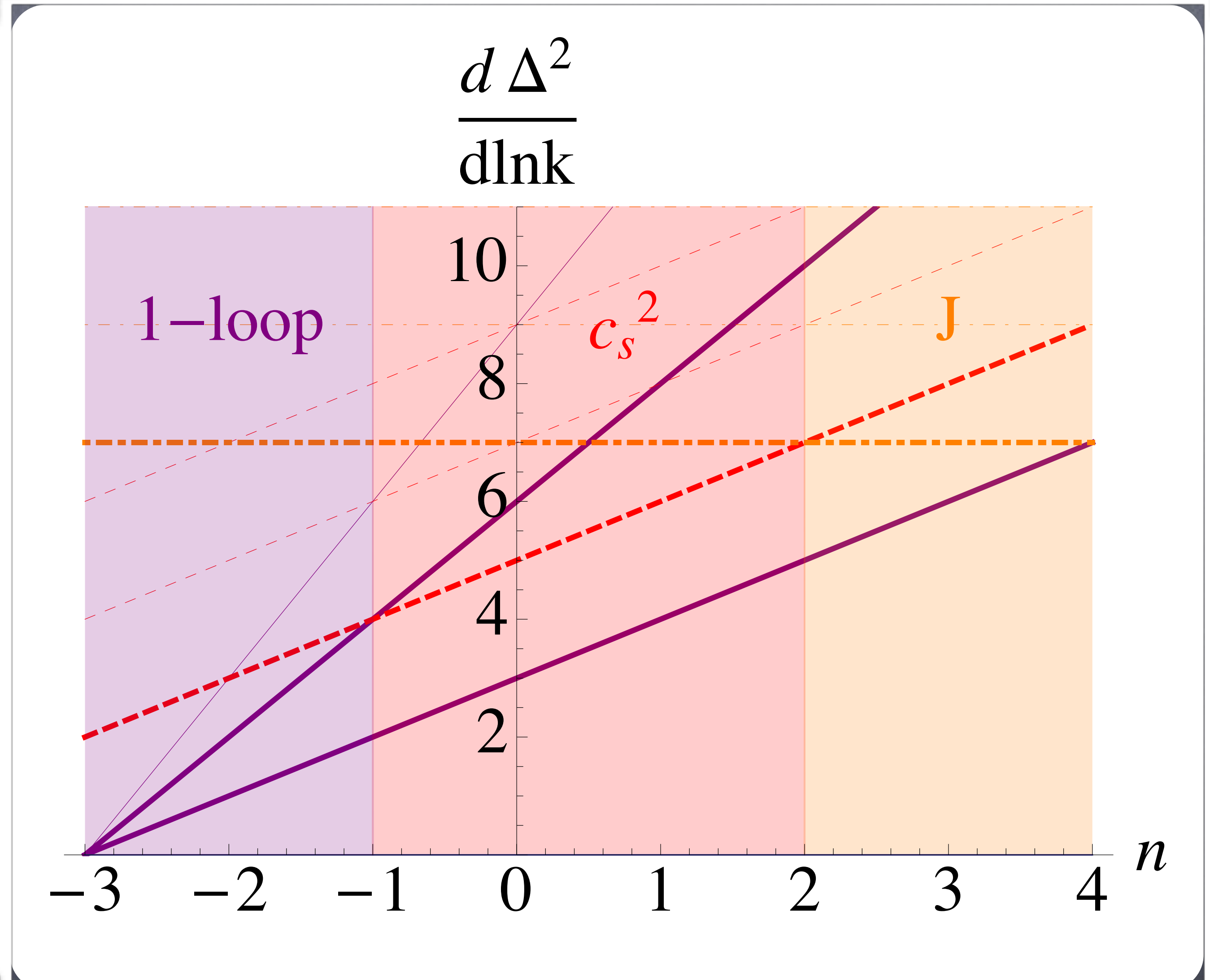
- When UV-divergences are present, in cutoff regularization terms appear of the form

$$+\left(\frac{\Lambda}{k_{NL}}\right)^{\#\#}\left(\frac{k}{k_{NL}}\right)^{\#}$$

- These violate self-similarity (Frieman & Scoccimarro '96)
- But also the counterterms violate self-similarity in such a way that the final result, after the cancellation, is self-similar
- Dimensional regularization instead preserves self-similarity in all steps of the computation

Relative importance of corrections

- Relative importance of terms as k->0 depends on n
- For our universe n = 1.5 hence c_s is more important than 2-loops J is less important than 3-loops



- This shows which terms can be consistently included

Dimensional regularization

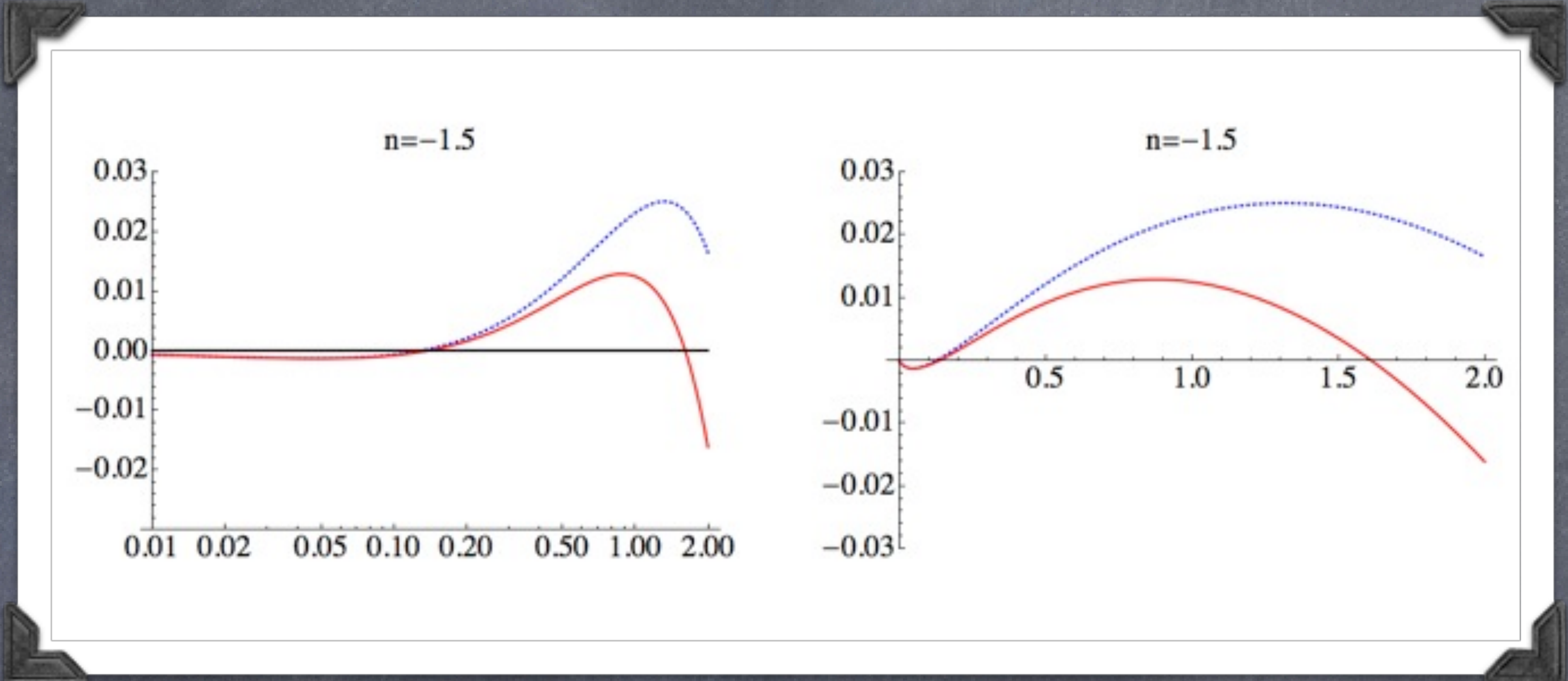
- β and γ are n-dependent fitting parameters, that can be determined comparing with observations or simulations (e.g. Carrasco et al '12, Hertzberg '12)
- α and α-tilde are n-dependent numbers predicted by perturbation theory. They are most easily computed in dimensional regularization (dim reg)
- Dim reg preserved the scaling symmetry (unlike the cutoff regularization) of EdS, hence no violation of self-similarity appears anywhere in the computation

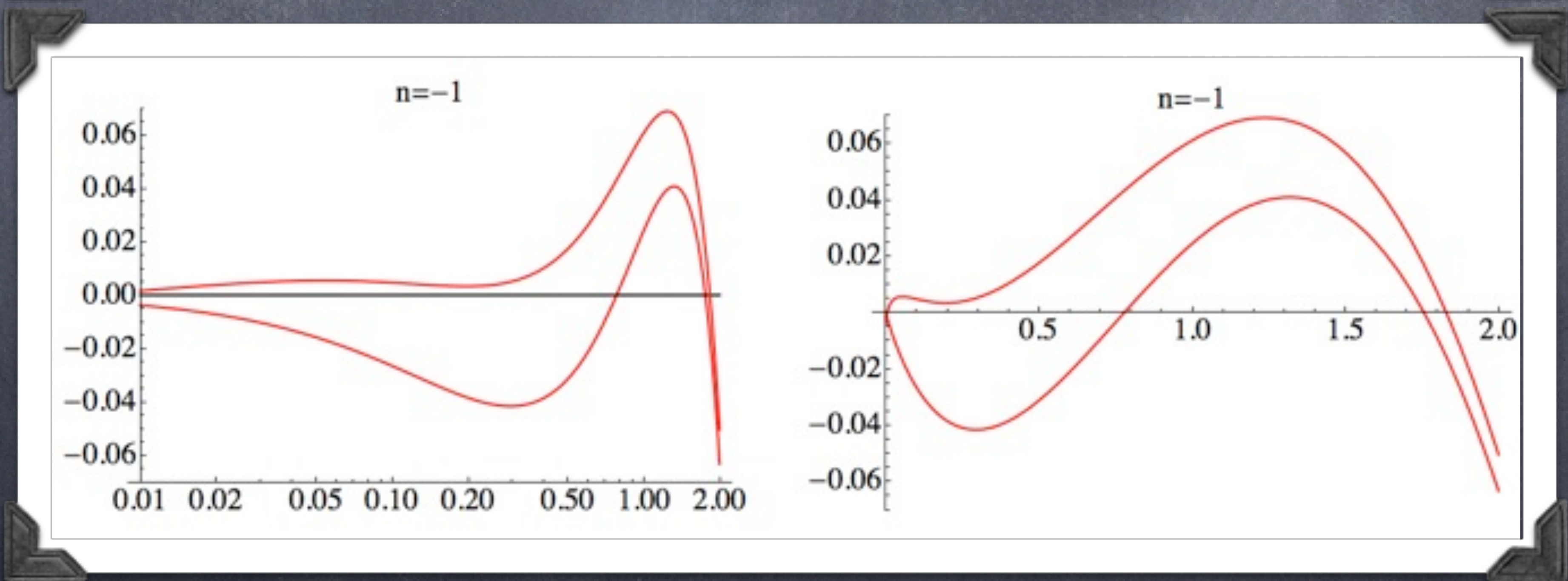
Dim reg computation

$$P_{22}(k,\tau) = \left(\frac{\Gamma[4-\frac{d}{2}-n]\Gamma^2[(-4+d+n)/2]}{2\Gamma^2(2-n/2)\Gamma[-4+d+n]} + \frac{3\Gamma[3-\frac{d}{2}-n]\Gamma[(-4+d+n)/2]\Gamma[(-2+d+n)/2]}{\Gamma[1-n/2]\Gamma[2-n/2]\Gamma[-3+d+n]} \right.$$
$$+ \frac{29\Gamma[2-\frac{d}{2}-n]\Gamma^2[(-2+d+n)/2]}{4\Gamma^2[1-n/2]\Gamma[-2+d+n]} - \frac{11\Gamma[2-\frac{d}{2}-n]\Gamma[(-4+d+n)/2]\Gamma[(d+n)/2]}{4\Gamma[2-n/2]\Gamma[-n/2]\Gamma[-2+d+n]}$$
$$- \frac{15\Gamma[1-\frac{d}{2}-n]\Gamma[(-4+d+n)/2]\Gamma[(2+d+n)/2]}{2\Gamma[-1-n/2]\Gamma[2-n/2]\Gamma[-1+d+n]} + \frac{15\Gamma[1-\frac{d}{2}-n]\Gamma[(-2+d+n)/2]}{2\Gamma[1-n/2]\Gamma[-n/2]}$$
$$\times \frac{\Gamma[(d+n)/2]}{\Gamma(-1+d+n)} - \frac{25\Gamma[-d/2-n]\Gamma[(-2+d+n)/2]\Gamma[(2+d+n)/2]}{\Gamma[-1-n/2]\Gamma[1-n/2]\Gamma[d+n]} + \frac{25\Gamma[-d/2-n]}{4\Gamma[-2-n/2]}$$
$$\left. \times \frac{\Gamma[(-4+d+n)/2]\Gamma[(4+d+n)/2]}{\Gamma[2-n/2]\Gamma[d+n]} + \frac{75\Gamma[-\frac{d}{2}-n]\Gamma^2[(d+n)/2]}{4\Gamma^2[-n/2]\Gamma[d+n]} \right) \frac{A^2 a^{4(d-2)}}{49} \frac{1}{8\pi^{3-d/2}}$$
$$\times k^{2n+d}, \qquad (A.7)$$

$$P_{13}(k,\tau) = \Gamma[-1+d/2] \left(-\frac{\Gamma[(4-d-n)/2]\Gamma[(-2+d+n)/2]}{84\Gamma(1-n/2)\Gamma[-2+d+n/2]} - \frac{19\Gamma[-(d+n)/2]\Gamma[(2+d+n)/2]}{84\Gamma(-1-n/2)\Gamma(d+n/2)} \right.$$
$$+ \frac{\Gamma[-(2+d+n)/2]\Gamma[(4+d+n)/2]}{12\Gamma[-2-n/2]\Gamma[1+d+n/2]} + \frac{5\Gamma[(2-d-n)/2]\Gamma[(d+n)/2]}{28\Gamma[-1+d+n/2]\Gamma[-n/2]}$$
$$\left. - \frac{\Gamma[(-4+d+n)/2]\Gamma[(6-d-n)/2]}{42\Gamma[2-n/2]\Gamma[-3+d+n/2]} \right) \frac{1}{8\pi^{3-d/2}} A^2 a^{4(d-2)}\ k^{2n+d}. \qquad (A.8)$$

Comparison with simulations





Comparison with simulations

- Depending on n there are 0, 1 or 2 fitting parameters
- EFToLSS provide a better fit to simulations than than SPT (not surprisingly)
- Once fitting parameters e.g. c_s^2 or J are fitted, their value is fixed for all other predictions, e.g. velocity correlators and higher n-point functions
- Also there is much more information in each individual realization

Conclusions

- SPT is unsatisfactory for at least three reasons

  1. there is no clear expansion parameter
  2. deviation from perfect pressureless fluid are missing
  3. predictions are UV-divergent and hence unphysical

- The EFT approach is to consistently integrate out the short scales. This addresses all the above problems

  1. smoothed fields are small everywhere
  2. pressure, dissipation and stochastic noise arise as fitting parameters
  3. couterterms cancel UV-divergences a make the theory predictive

Conclusions

- EdS is a simple but phenomenologically relevant example
- We found a very simple results for the 1-loop power spectrum using self-similarity
- This example teaches us the relative importance of loop and effective corrections, which depends on the power spectrum
- For our universe the effective pressure is more important than 2-loop corrections

Outlook

- Generalize to velocity correlators and higher n-point function. Is the relative importance of operators the same?

- The effective coefficients coefficients have been estimated (Carrasco et al '12, Hertzberg '12) fitting the power spectrum. But there is more information in each individual realization

- Lagrangian perturbation theory (LPT) improves SPT accounting for bulk flow (Tassev & Zaldarriaga '12). Develop an EFT of LPT