Emergent galileons and chameleons

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Outline



Dark Energy and Modified Gravity

- Observational evidence
- A cosmological constant
- Dynamical Dark Energy



Screening mechanisms

- Quintessence
- Vainshtein screening
- Chameleon screening



- invariants
- Generalized chameleons



warp Emergent screening

- Induced metrics in higher dimensions
- Lovelock gravity
- A purely 4D Galileon



Perlmutter, Schmidt and Riess '11 - High-z Supernova Search Team & Supernova Cosmology Project



Plots from: Union2 Supernova Compilation '10, Komatsu et al. '08



Plots from: Union2 Supernova Compilation '10, UK Herschel Team '10

What's driving the expansion?

A cosmological constant?

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu} \qquad \qquad \Lambda_{\rm eff} = \Lambda + 8\pi G \langle \rho_{\rm vac} \rangle$$

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$$\Lambda = \Lambda_1 + c_e m_e^4 + c_\nu m_\nu^4 \dots$$

$$\Lambda = \Lambda_0 + c_\nu m_\nu^4 \dots$$

$$\frac{m_e^4}{\rho} \sim 10^{36}$$

Symmetry, anthropic selection, a new dynamical degree of freedom,..?



New degrees of freedom cause a large distance modification of GR

$$m_{\phi}(t_0) \sim H_0 \sim 10^{-33} eV$$

But new light degrees of freedom are very easy to excite – Fifth Forces, Equivalence Principle Violations etc..

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- Dark Energy is very weakly coupled to matter e.g. Quintessence
- Dark Energy is effectively very weakly coupled in dense environments, i.e. in regimes where we have tested GR to high precision – Vainshtein mechanism
- Dark Energy is effectively very massive in dense environments Chameleon mechanism

Very weakly coupled Dark Energy

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\rm pl}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right) + S_{\rm m}$$

The only coupling to matter is a minimal one through the metric.

cf. Peebles & Ratra '88, Wetterich, '88

Very weakly coupled Dark Energy

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The only coupling to matter is a minimal one through the metric. We can generalize this in the following ways:

- Introduce a small direct coupling (protected by shift symmetry): Brax et al. '09
- Assisted Quintessence, i.e. introduce several fields: Kim et al. '05
- Non-canonical kinetic terms k-essence: Armendariz-Picon et al. '99-'01

The Vainshtein mechanism

Weak coupling \leftrightarrow large kinetic term

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{2} (\partial \phi)^2 + c_m \phi T^{\mu}_{\mu} \right) = \int d^4x \sqrt{g} \left(-\frac{1}{2c_m^2} (\partial \bar{\phi})^2 + \bar{\phi} T^{\mu}_{\mu} \right)$$
$$\bar{\phi} = c_m^{-1} \phi$$

deRham & Tolley '10, Nicolis & Rattazzi '04, Vainshtein '72

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Large non-linear operators

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{2} (\partial \phi)^2 - c_{\rm NL} \Box \phi (\partial \phi)^2 + \phi T^{\mu}_{\mu} \right)$$
$$\Box \phi \sim T^{\mu}_{\mu} \longrightarrow \frac{1}{c_{\rm KE}^2} \sim 1 + c_{\rm NL} T^{\mu}_{\mu}$$
$$S = \int d^4x \sqrt{g} \left(-\frac{1}{2c_{\rm KE}^2} (\partial \phi)^2 + \phi T^{\mu}_{\mu} \right)$$

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Flat space galileons

A linear coupling to other matter fields via $T_{\mu\nu}$:

$$S = \int d^4x (\mathcal{L}_\pi + \pi T^\mu_\mu)$$

A Galilean shift symmetry for terms in \mathcal{L}_{π} :

$$\pi(x) \to \pi(x) + b_{\mu}x^{\mu} + c$$



Equations of motion which are at most second order in derivatives of π , cf Ostrogradski's theorem.

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$$\mathcal{L}_{1} = \pi$$

$$\mathcal{L}_{2} = -\frac{1}{2}\partial\pi \cdot \partial\pi$$

$$\mathcal{L}_{3} = -\frac{1}{2}(\Box\pi)\partial\pi \cdot \partial\pi$$

$$\Pi_{\nu}^{\mu} \equiv \partial^{\mu}\partial_{\nu}\pi$$

$$\mathcal{L}_{4} = -\frac{1}{4}[(\Box\pi)^{2}\partial\pi \cdot \partial\pi - 2(\Box\pi)\partial\pi \cdot \Pi \cdot \partial\pi$$

$$-(\Pi \cdot \Pi)(\partial\pi \cdot \partial\pi) + 2\partial\pi \cdot \Pi \cdot \Pi \cdot \partial\pi]$$

$$\mathcal{L}_{5} = -\frac{1}{5}[(\Box\pi)^{3}\partial\pi \cdot \partial\pi - 3(\Box\pi)^{2}\partial\pi \cdot \Pi \cdot \partial\pi - 3\Box\pi(\Pi \cdot \Pi)(\partial\pi \cdot \partial\pi)$$

$$+ 6(\Box\pi)\partial\pi \cdot \Pi \cdot \Pi \cdot \partial\pi + 2(\Pi \cdot \Pi \cdot \Pi)(\partial\pi \cdot \partial\pi)$$

$$+ 3(\Pi \cdot \Pi)\partial\pi \cdot \Pi \cdot \partial\pi - 6\partial\pi \cdot \Pi \cdot \Pi \cdot \Pi \cdot \partial\pi]$$
(Nicolis et al. 2008)

cf. Armendariz-Picon et al. '99-'01, Nicolis et al. '08, Babichev et al. '09, Deffayet et al. '10, Kobayashi et al. '10, Gubitosi & Linder '11, ...

The chameleon mechanism

$$\mathcal{S} = \int d^4x \sqrt{g} \left(\frac{M^2}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right) + \mathcal{S}_m \left(A^2(\phi) g_{\mu\nu}, \Psi_i \right).$$

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$$V_{\text{eff}}(\phi) = V(\phi) + \hat{\rho}A(\phi)$$

 $m_{\min} \equiv \sqrt{V_{eff,\phi\phi}(\phi_{\min})}$

Khoury & Weltman '03

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Khoury & Weltman '03

The most general relation between matter and gravitational geometries?

$$S = \int d^4x \sqrt{-g} \frac{M^2}{2} R + \mathcal{S}_m \left(\tilde{g}_{\mu\nu}, \Psi_i \right) + \mathcal{S}_\phi$$

o.

Start by looking at Finsler geometry: Most general geometry in which the squared line element is homogenous of second degree in coordinate increments:

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Requiring $\mathcal{G}_{\mu\nu}$ to be:

$$\mathcal{G}_{\mu
u} = \mathcal{G}_{\mu
u} \left(g_{\mu
u}, x^{lpha}, dx^{eta}, \phi, \partial\phi \right)$$

- a function of coordinate invariants only,
- independent of dx^{β} ,
- introducing one additional degree of freedom ϕ ,
- requiring it to be a function of at most first derivatives in ϕ .

Disformal gravity

$$S = \int d^4x \sqrt{-g} \frac{M^2}{2} R + \mathcal{S}_m \left(\tilde{g}_{\mu\nu}, \Psi_i \right) + \mathcal{S}_\phi$$

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$$\mathcal{G}_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = A(\phi, X)g_{\mu\nu} + B(\phi, X)\partial_{\mu}\phi\partial_{\nu}\phi$$

In other words, the quasimetric $\mathcal{G}_{\alpha\beta}$ reduces to the Riemannian metric $\tilde{g}_{\alpha\beta}$. Bekenstein '92

$$S = \int d^4x \sqrt{-g} \frac{M^2}{2} R + \mathcal{S}_m \left(\tilde{g}_{\mu\nu}, \Psi_i \right) + \mathcal{S}_\phi$$

$$\tilde{g}_{\mu\nu} = A(\phi, X)g_{\mu\nu} + B(\phi, X)\partial_{\mu}\phi\partial_{\nu}\phi$$

- Conformal limit $B(\phi, X) = 0$: A generalized chameleon model.
- For $B(\phi, X) \neq 0$ we have a "disformal" relation. This will turn out to be intimately related to Galileon models.

• Note that $\tilde{g} = g \left(A^d + A^{d-1} B X \right)$. So in the minimal disformal model $\tilde{g}_{\mu\nu} = g_{\mu\nu} + f \partial_{\mu} \phi \partial_{\nu} \phi$, we recover a DBI-type action $S = \int d^4 x \sqrt{\tilde{g}} = \int d^4 x \sqrt{g} \sqrt{1 + f X}$.

A generalized chameleon

Higher derivative chameleon:

$$\mathcal{S} = \int d^4x \sqrt{g} \left(\frac{M^2}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right) + \mathcal{S}_m \left(A^2(\phi, X) g_{\mu\nu}, \Psi_i \right).$$

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Effective potential $V_{\rm eff}$ gets renormalized. Schematically:

 $\Box \phi (1 - \hat{\rho}) = V_{,\phi} + \dots$

Conformal factor $A^2(\phi, X)$ still needs to retain ϕ -dependence in order to create an effective minimum for V_{eff} . But the amplitude of the potential and hence the effective mass of the chameleon field is affected by higher derivative terms.





Lovelock gravity: The most general metric theory of gravity yielding conserved second order equations of motion in an arbitrary number of spacetime dimensions D.





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In the non-relativistic limit $(\partial \pi)^2 \ll 1$, we recover the galileon terms (here shown for flat space $g_{\mu\nu} = \eta_{\mu\nu}$):



deRham & Tolley '10



Return to generic disformal relation: $\tilde{g}_{\mu\nu} = q_{\mu\nu} + B(\pi, X)\partial_{\mu}\pi\partial_{\nu}\pi$

A minimal 4D galileon

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For a purely 4D theory we only have two Lovelock invariants:

$$S_{\lambda} = -\lambda \int d^{4}x \sqrt{-\tilde{g}}$$
$$S_{R} = \frac{M_{4}^{2}}{2} \int d^{4}x \sqrt{-\tilde{g}}\tilde{R}$$

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and the corresponding Galileon terms:

$$S_2 = S_{\lambda}^{NR} = -\frac{\lambda}{2} \int d^4 x \, (\partial \pi)^2$$
$$S_4 = S_R^{NR} = \frac{M_4^2}{4} \int d^4 x \, (\partial \pi)^2 \, \left((\Box \pi)^2 - (\partial_\mu \partial_\nu \pi)^2 \right)$$

 $(\partial \pi)^2 \Box \pi$ and stability against small wavelength perturbations?

The Vainshtein effect

Study spherically symmetric, static equations of motion, near an object of mass M:

$$d_2\left(\frac{\pi'}{r}\right) + 2d_4\left(\frac{\pi'}{r}\right)^3 = \frac{M}{4\pi r^3}$$

 d_i are linear combinations of Lagrangian coefficients c_i .

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A minimal 4D disformal theory

Up to here have always assumed that we have a "galileon"-type theory with:

$$S = \int d^4x \sqrt{-q} \frac{M^2}{2} R + \sum_i c_i \mathcal{S}_i(g_{\mu\nu})$$

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But:

$$S_{4} = \frac{M_{4}^{2}}{2} \int d^{4}x \sqrt{-\tilde{g}}\tilde{R}$$

= $\frac{M_{4}^{2}}{2} \int d^{4}x \sqrt{-q} \left[R \left(1 - \frac{1}{2} (\partial \pi)^{2} - \frac{1}{8} (\partial \pi)^{4} \right) - 2 \partial^{\mu} \pi \partial^{\nu} \pi R_{\mu\nu} \left(1 - \frac{1}{2} (\partial \pi)^{2} \right) - \frac{1}{2} \left([\Pi]^{2} - [\Pi^{2}] \right) (\partial \pi)^{2} - 2 \left([\Pi] [\phi] - [\phi^{2}] \right) \right]$

So the non-linear scale R_2 is also a measure of the relative contribution from S_4 to the Einstein-Hilbert R.

$$R_{\rm NL}^6 = \frac{M^2 d_4}{8\pi^2 d_2^3}$$

The full 4D disformal theory

(Induced) matter metric for "galileon" was: $\tilde{g}_{\mu\nu} = q_{\mu\nu} + \partial_{\mu}\pi \partial_{\nu}\pi$

What happens when we make use of the full disformal relation and have: $\tilde{g}_{\mu\nu} = q_{\mu\nu} + B(\pi, X)\partial_{\mu}\pi\partial_{\nu}\pi$?

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"The most general scalar field theories which have an action that depends on derivatives of order two or less, and have equations of motion that stay second order and lower on flat space-time." cf. Deffayet et al. '11

Schematically:
$$\mathcal{L}_n^{\text{gen}} = f(\pi, X) \cdot \mathcal{L}_n^{\text{Gal}}$$

Most general extension, under the condition that field equations stay second order, of k-essence, Galileons, k-Mouflage, kinetically braided scalars...

cf. Armendariz-Picon et al. '99-'01, Nicolis et al. '08, Babichev et al. '09, Deffayet et al. '10, Kobayashi et al. '10





Thank you!

JN, in progress, JN & Mazumdar, in progress