

# PUTTING INFLATION TO THE TEST

W David Spergel and Ben Wandelt

PRD 89, 063537 (2014) PRD 89 083506 (2014), PRD 90 063529 (2014), arXiv:1406.0548

And **B. Hadzhiyska, R. Hlozek, J. Myers**, arXiv:1412.xxxx

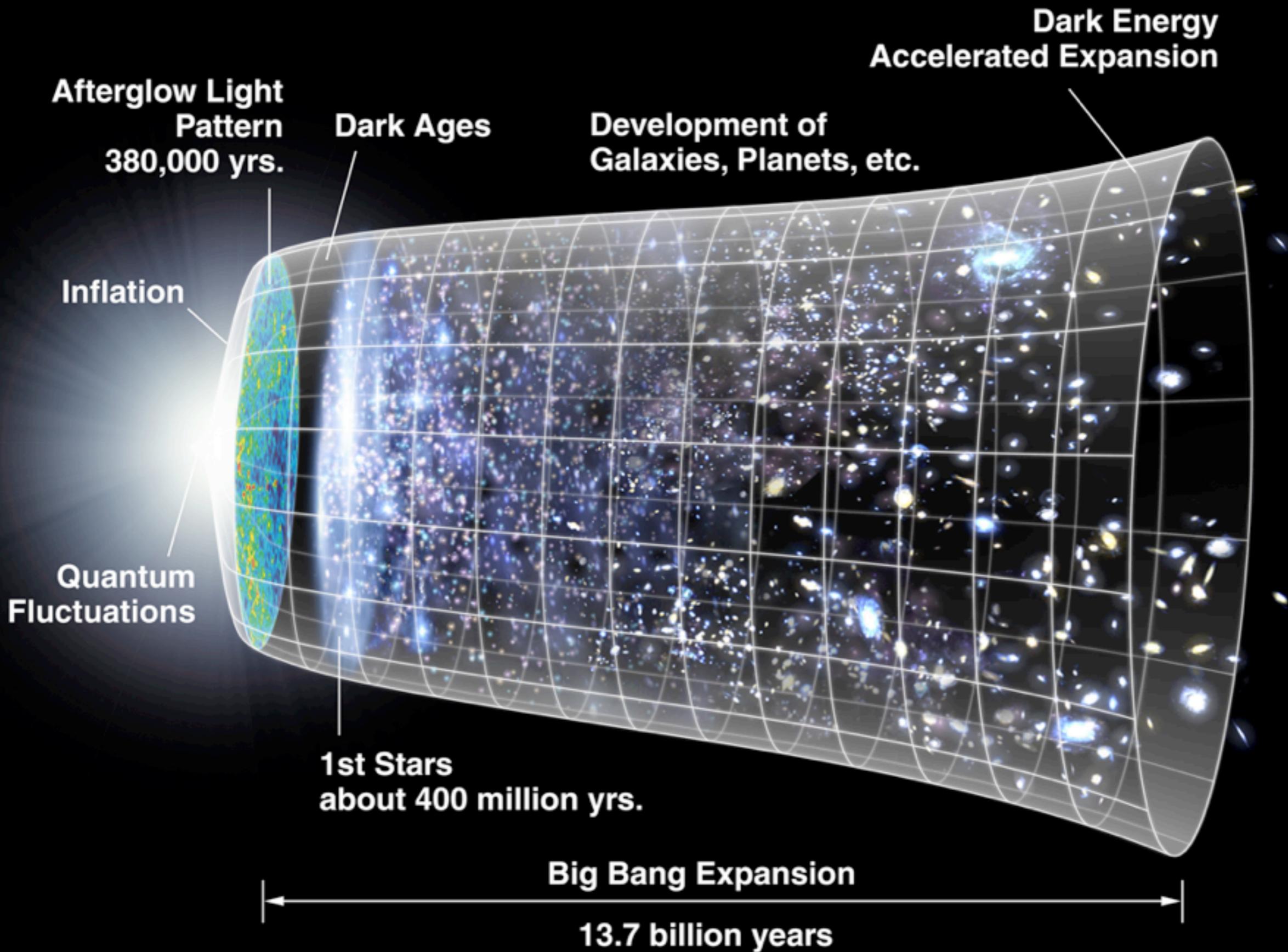
**M. Münchmeyer, B. Wandelt** arXiv:1412.xxxx

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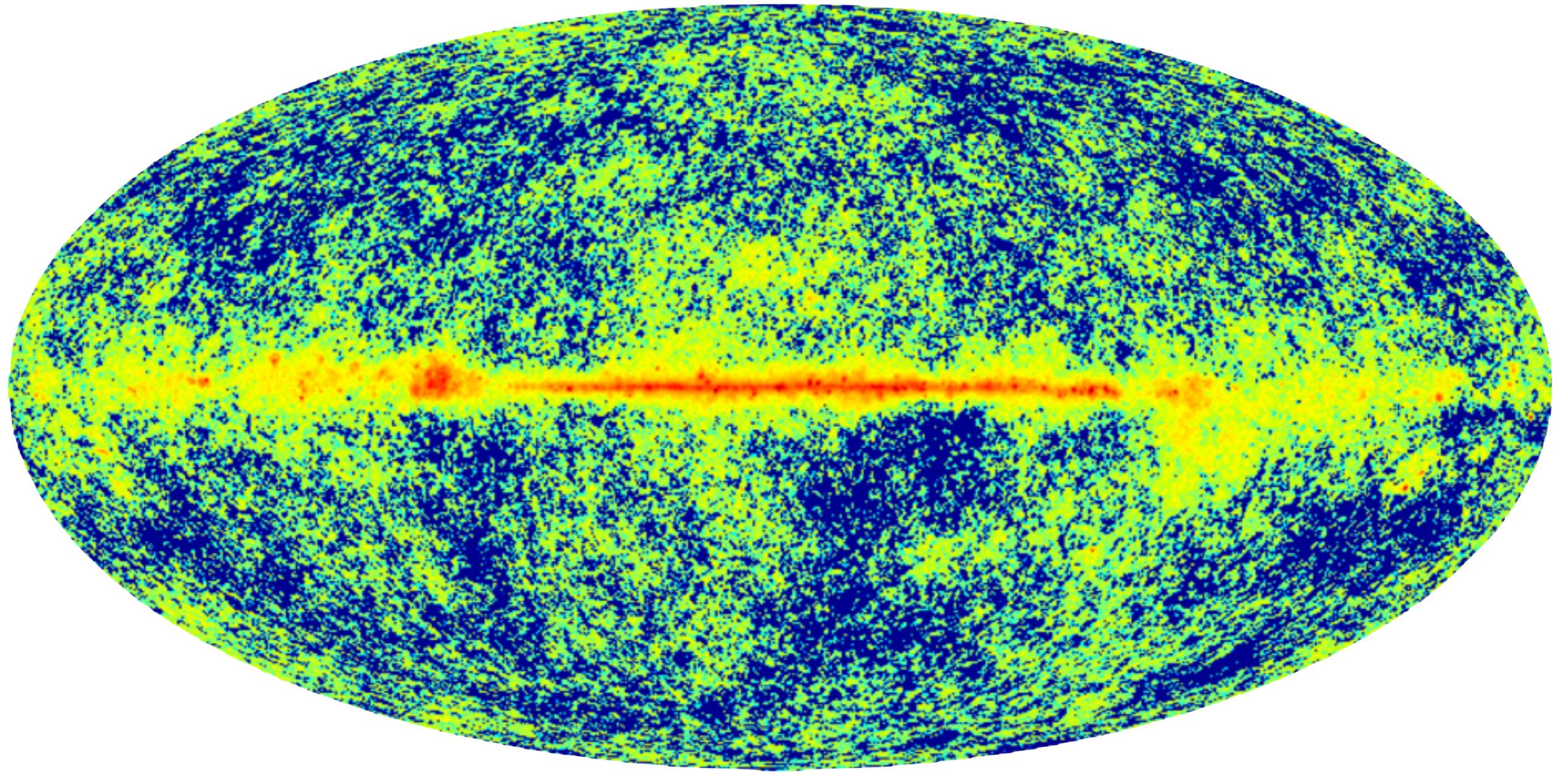
1) WHAT  
DRIVES  
INFLATION?

2) DID  
INFLATION  
HAPPEN?



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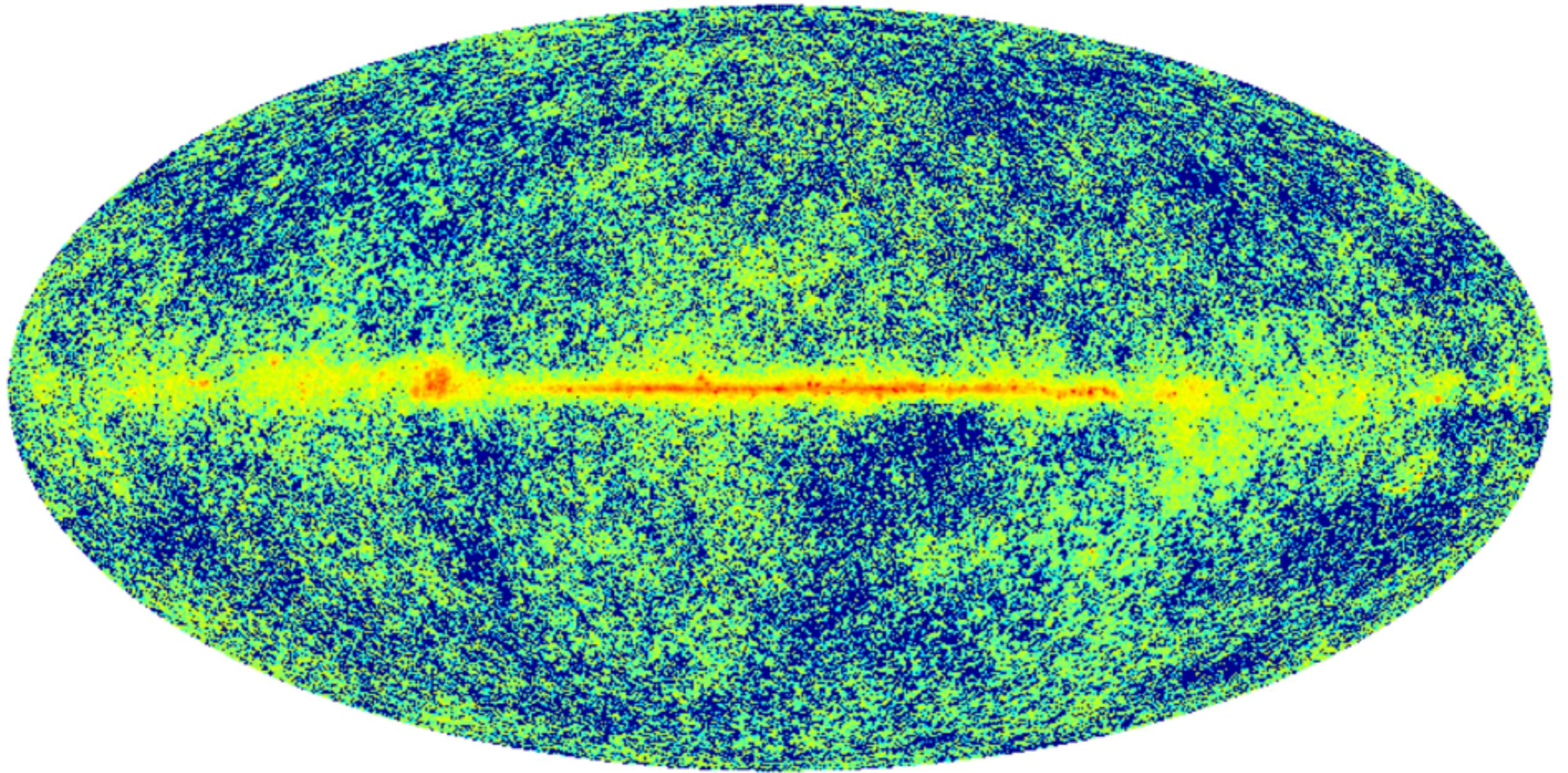
# WMAP V BAND



4.2  1.8 Log (JMK)

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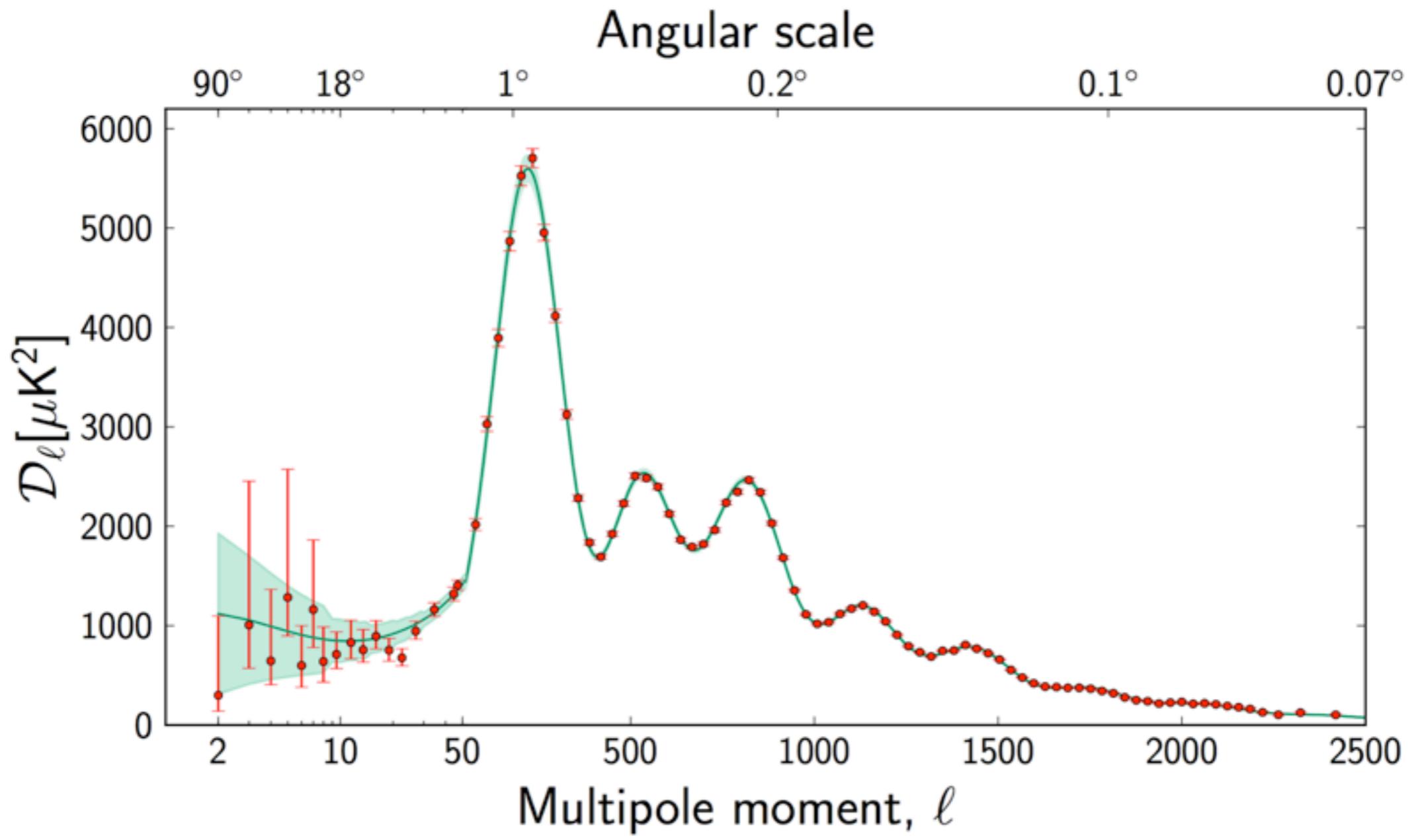
# PLANCK 70GHZ



-7.0  -1.0 Log (K\_CMB)

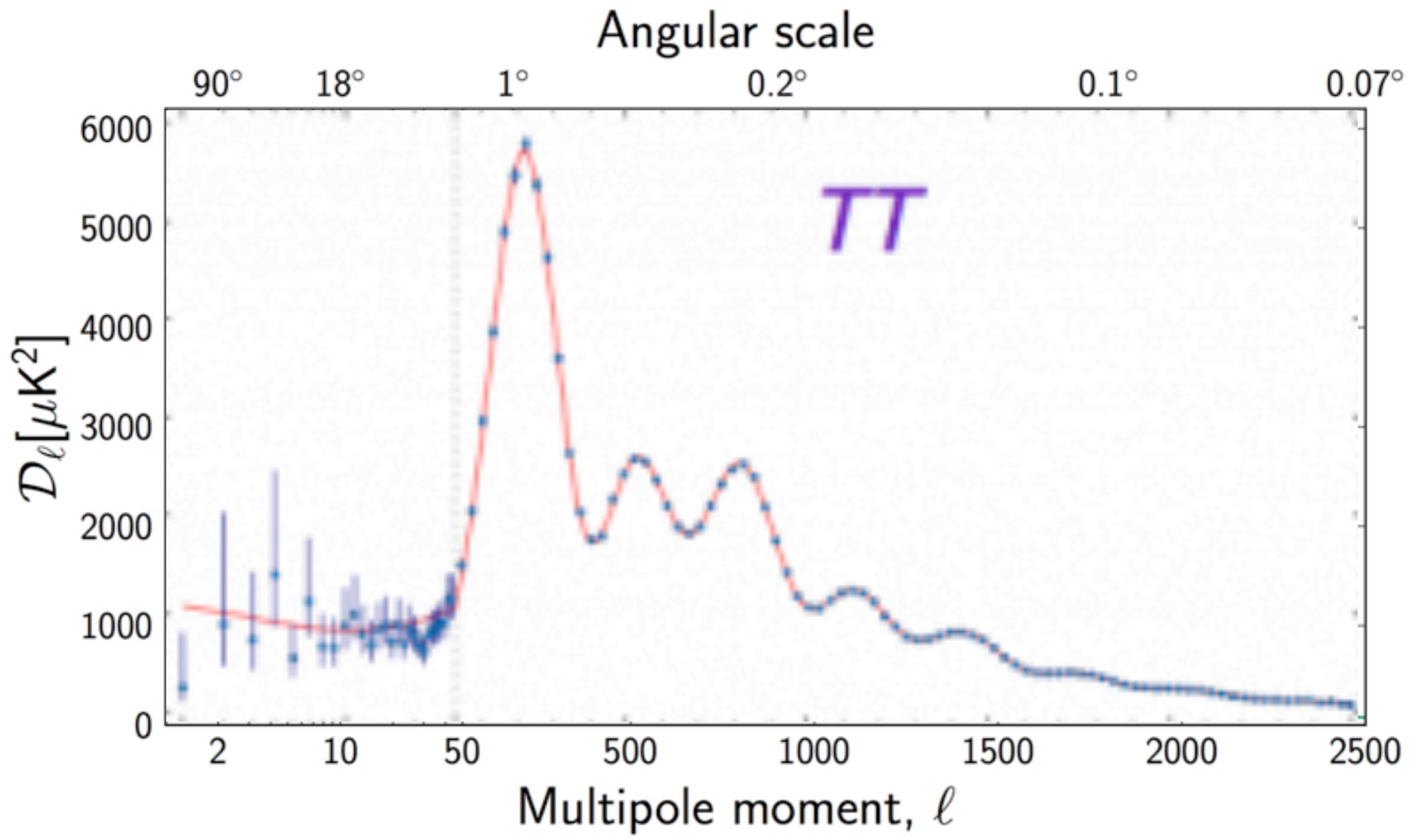


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PLANCK COLLABORATION 2013

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$\Omega_b h^2$ . . . . .	0.022068	$0.02207 \pm 0.00033$	0.022242	$0.02217 \pm 0.00033$	0.022032	$0.02205 \pm 0.00028$
$\Omega_c h^2$ . . . . .	0.12029	$0.1196 \pm 0.0031$	0.11805	$0.1186 \pm 0.0031$	0.12038	$0.1199 \pm 0.0027$
$100\theta_{MC}$ . . . . .	1.04122	$1.04132 \pm 0.00068$	1.04150	$1.04141 \pm 0.00067$	1.04119	$1.04131 \pm 0.00063$
$\tau$ . . . . .	0.0925	$0.097 \pm 0.038$	0.0949	$0.089 \pm 0.032$	0.0925	$0.089^{+0.012}_{-0.014}$
$n_s$ . . . . .	0.9624	$0.9616 \pm 0.0094$	0.9675	$0.9635 \pm 0.0094$	0.9619	$0.9603 \pm 0.0073$
$\ln(10^{10} A_s)$ . . . . .	3.098	$3.103 \pm 0.072$	3.098	$3.085 \pm 0.057$	3.0980	$3.089^{+0.024}_{-0.027}$
$\Omega_\Lambda$ . . . . .	0.6825	$0.686 \pm 0.020$	0.6964	$0.693 \pm 0.019$	0.6817	$0.685^{+0.018}_{-0.016}$
$\Omega_m$ . . . . .	0.3175	$0.314 \pm 0.020$	0.3036	$0.307 \pm 0.019$	0.3183	$0.315^{+0.016}_{-0.018}$
$\sigma_8$ . . . . .	0.8344	$0.834 \pm 0.027$	0.8285	$0.823 \pm 0.018$	0.8347	$0.829 \pm 0.012$
$z_{re}$ . . . . .	11.35	$11.4^{+4.0}_{-2.8}$	11.45	$10.8^{+3.1}_{-2.5}$	11.37	$11.1 \pm 1.1$
$H_0$ . . . . .	67.11	$67.4 \pm 1.4$	68.14	$67.9 \pm 1.5$	67.04	$67.3 \pm 1.2$
$10^9 A_s$ . . . . .	2.215	$2.23 \pm 0.16$	2.215	$2.19^{+0.12}_{-0.14}$	2.215	$2.196^{+0.051}_{-0.060}$

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PLANCK COLLABORATION 2014?

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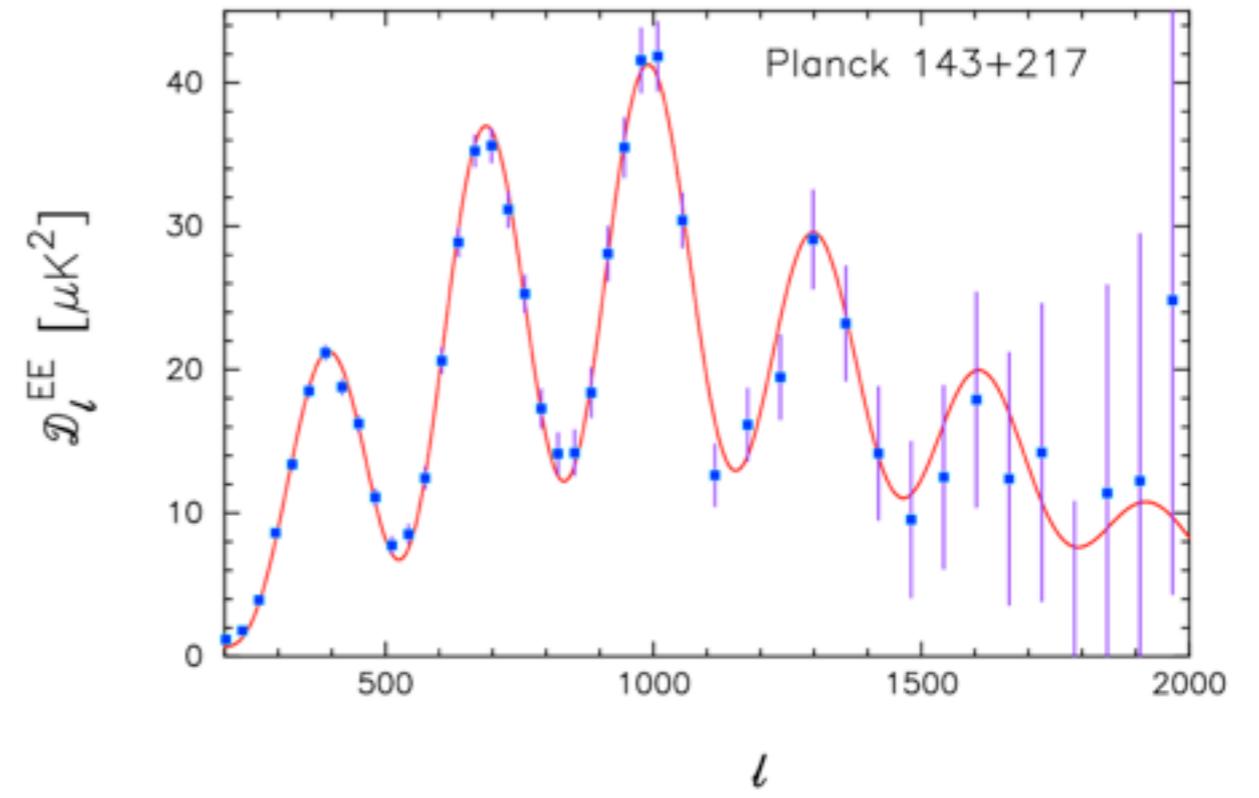
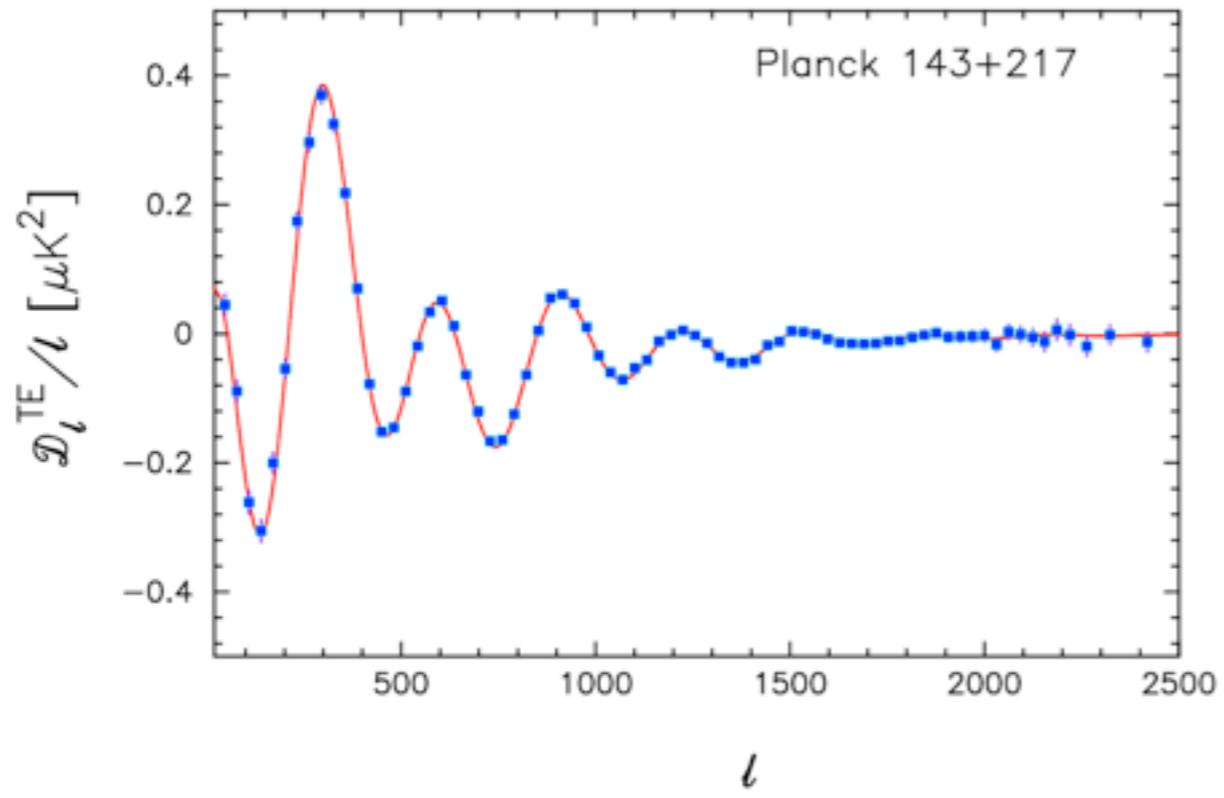
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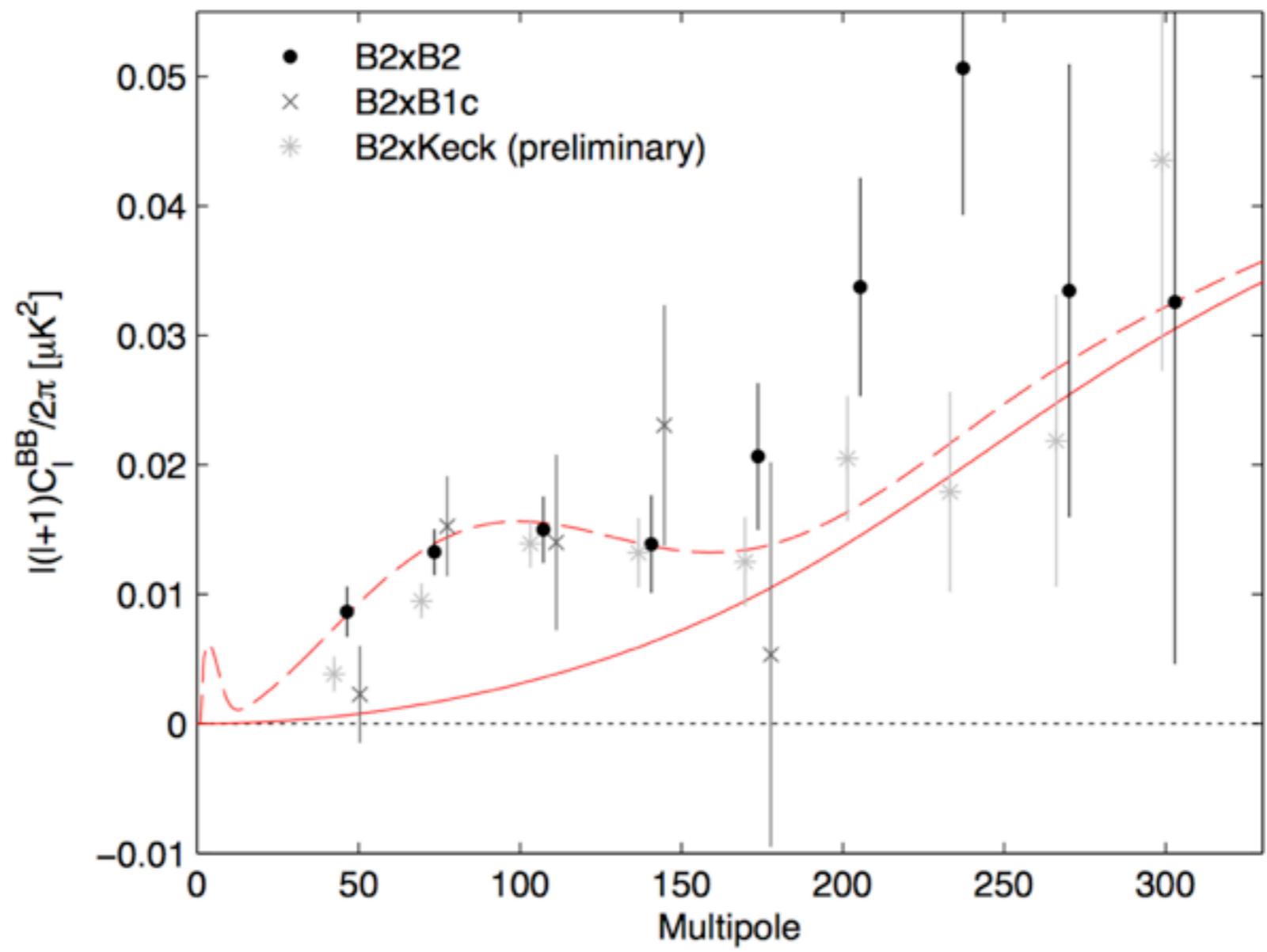
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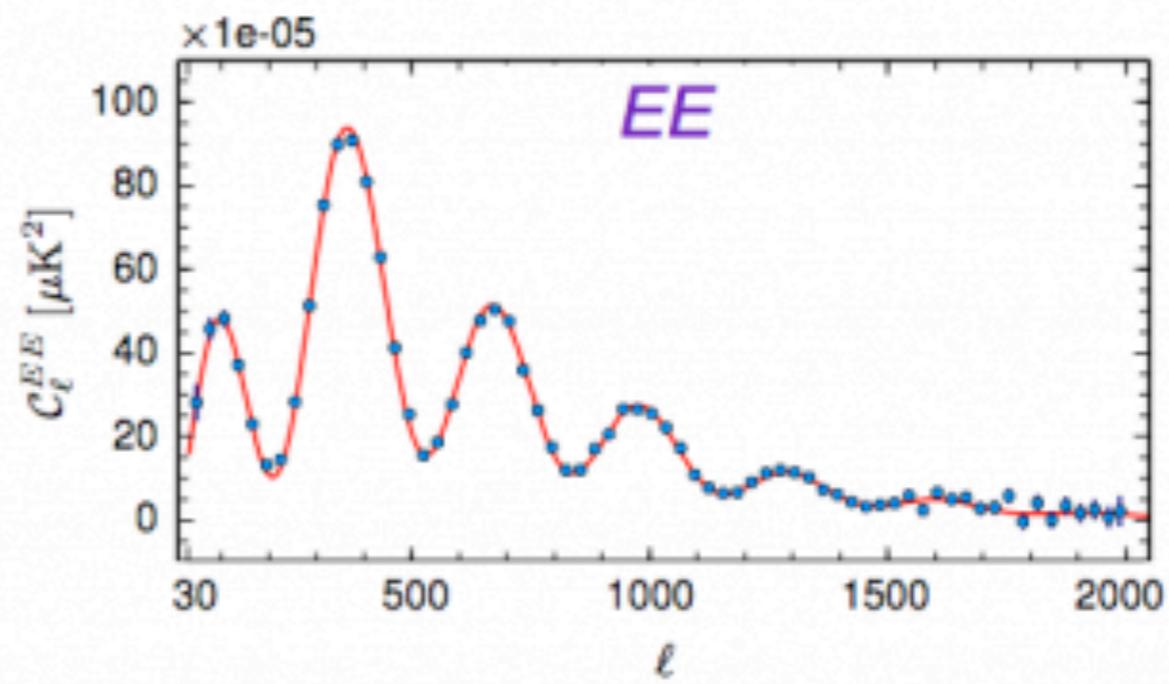
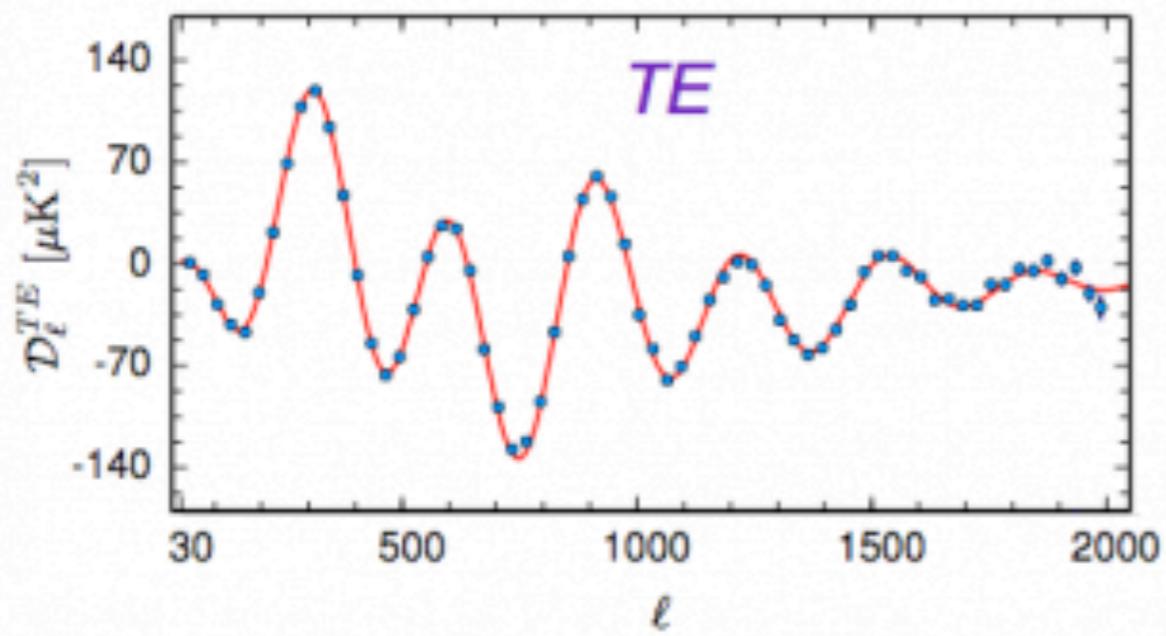
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BICEP COLLABORATION 2014

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PLANCK COLLABORATION 2014

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# NGS

## NON-GAUSSIANITIES

- Or absence thereof (KSW)

$$f_{\text{NL}}^{\text{local}} = 2.7 \pm 5.8$$

$$f_{\text{NL}}^{\text{equil}} = -42 \pm 75$$

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- And many others. In particular:

Phase Wavenumber	$\phi = 0$ $f_{\text{NL}} \pm \Delta f_{\text{NL}} (\sigma)$	$\phi = \pi/4$ $f_{\text{NL}} \pm \Delta f_{\text{NL}} (\sigma)$
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- **Fluctuations we observe are:**

- small:  $\delta T/T = 10^{-5}$
- statistically isotropic:  $P(\vec{k}) = P(k)$
- Gaussian:  $f_{NL} = 0$
- nearly scale invariant:  $P(k) = P$
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# WHAT DRIVES INFLATION?

- Motivation for features
- Results from Planck
- Discussion

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- Inflation consistency condition
- Results from Planck/BICEP/  
LIGO/PTA/BBN
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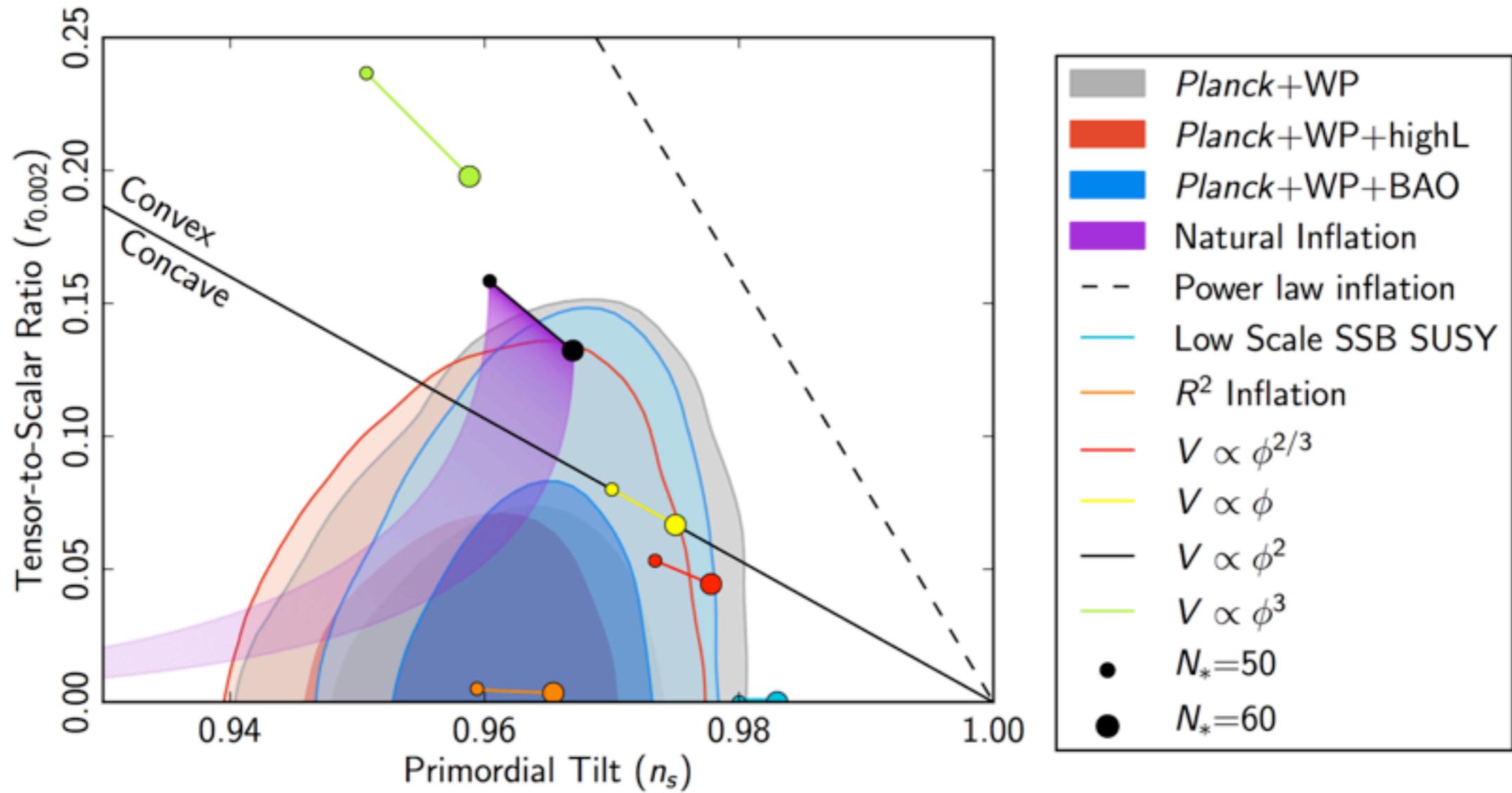
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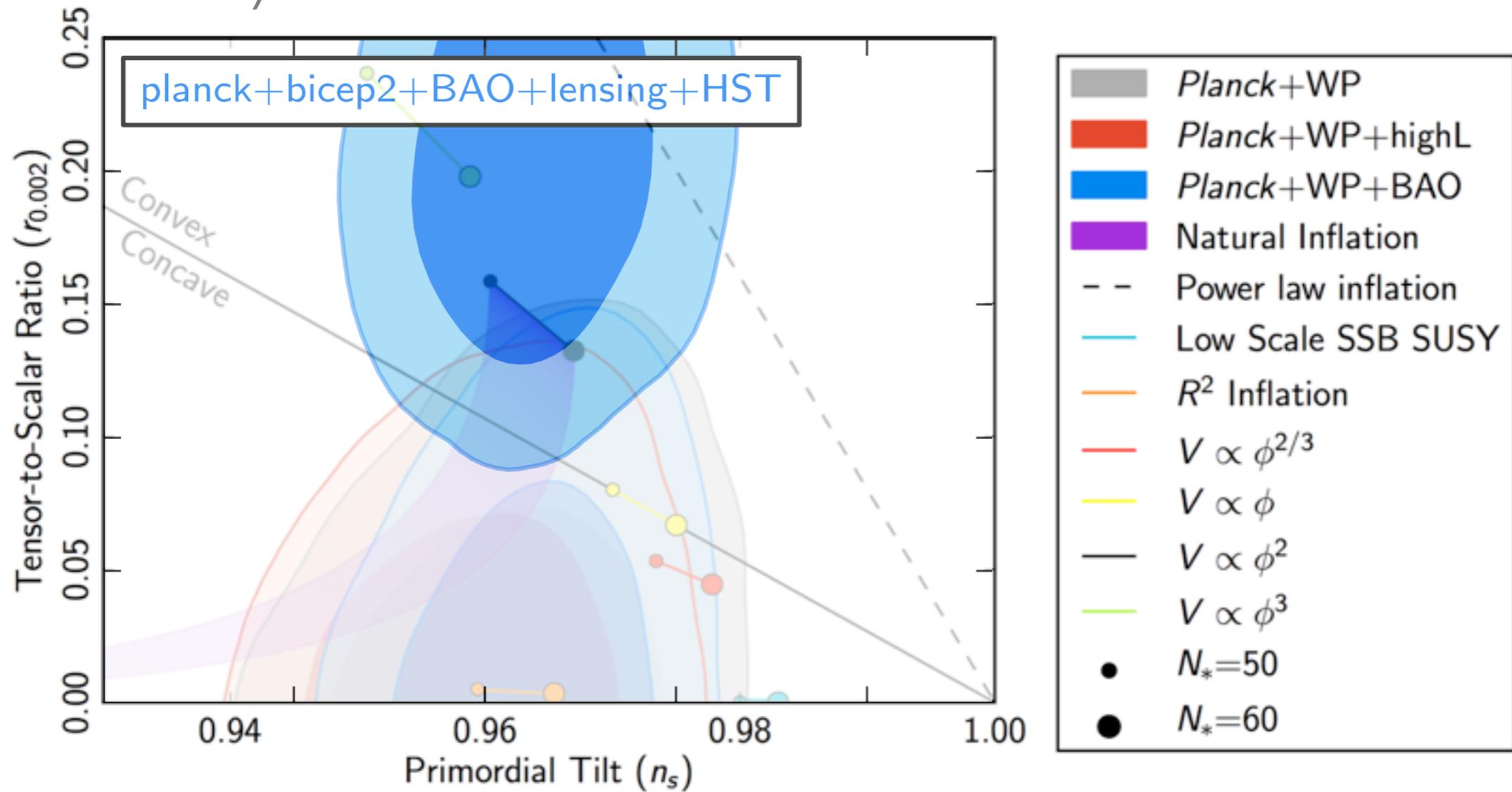
# The Universe Pre-Bicep



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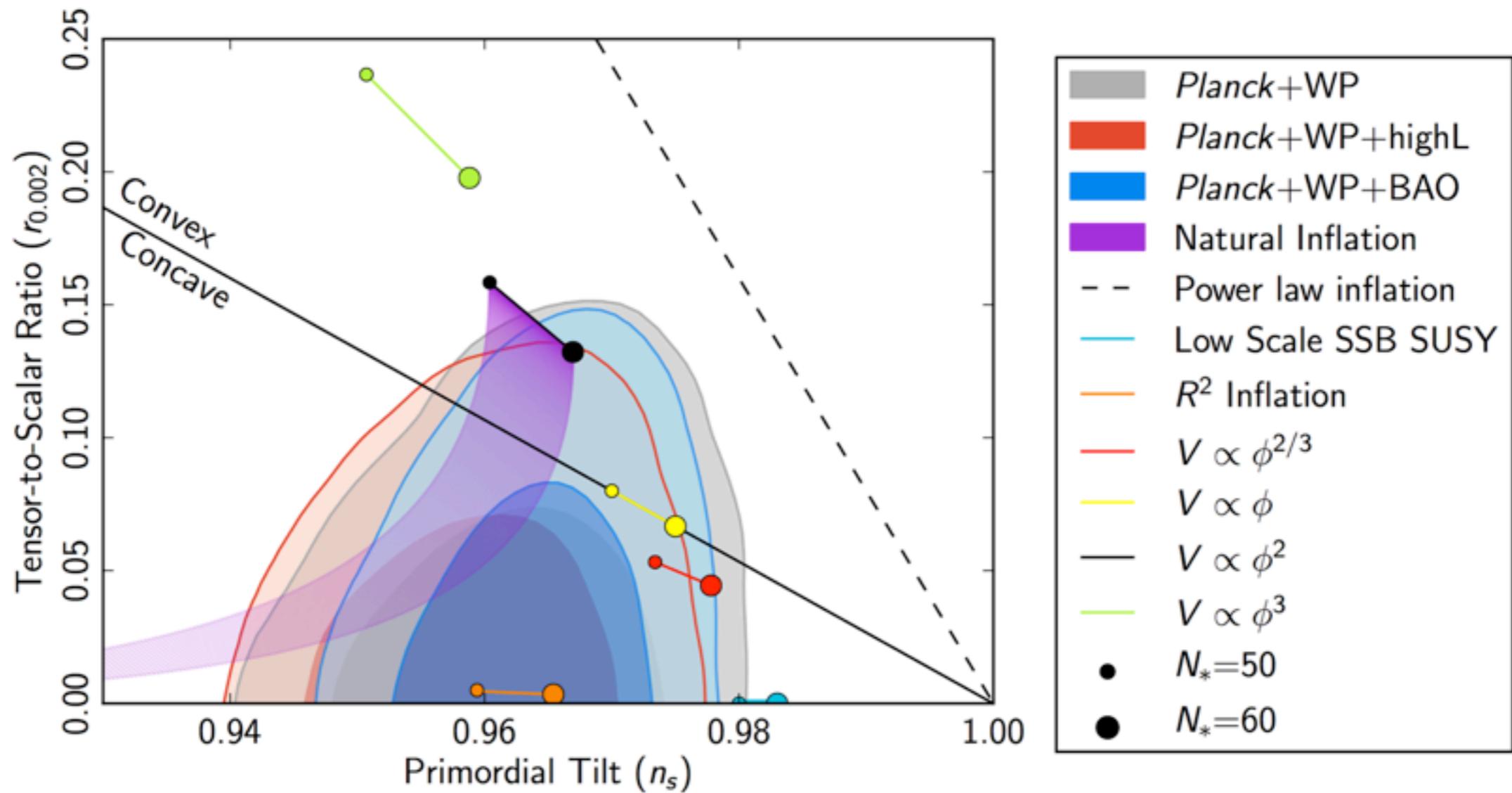
# The Universe Post-Bicep (pre-Planck-dust)



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# The Universe Post-Planck-dust?



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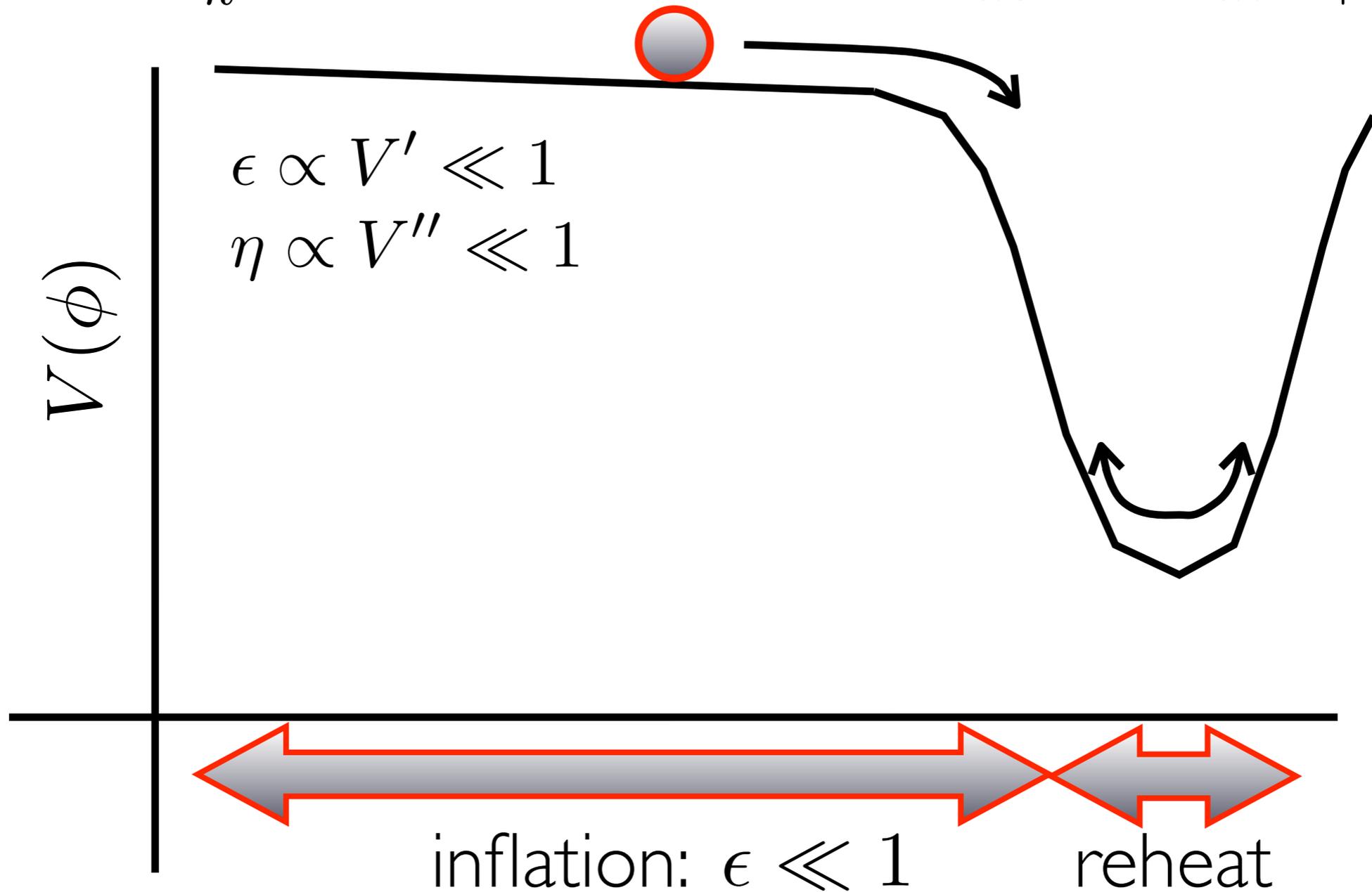
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# Inflation paradigm

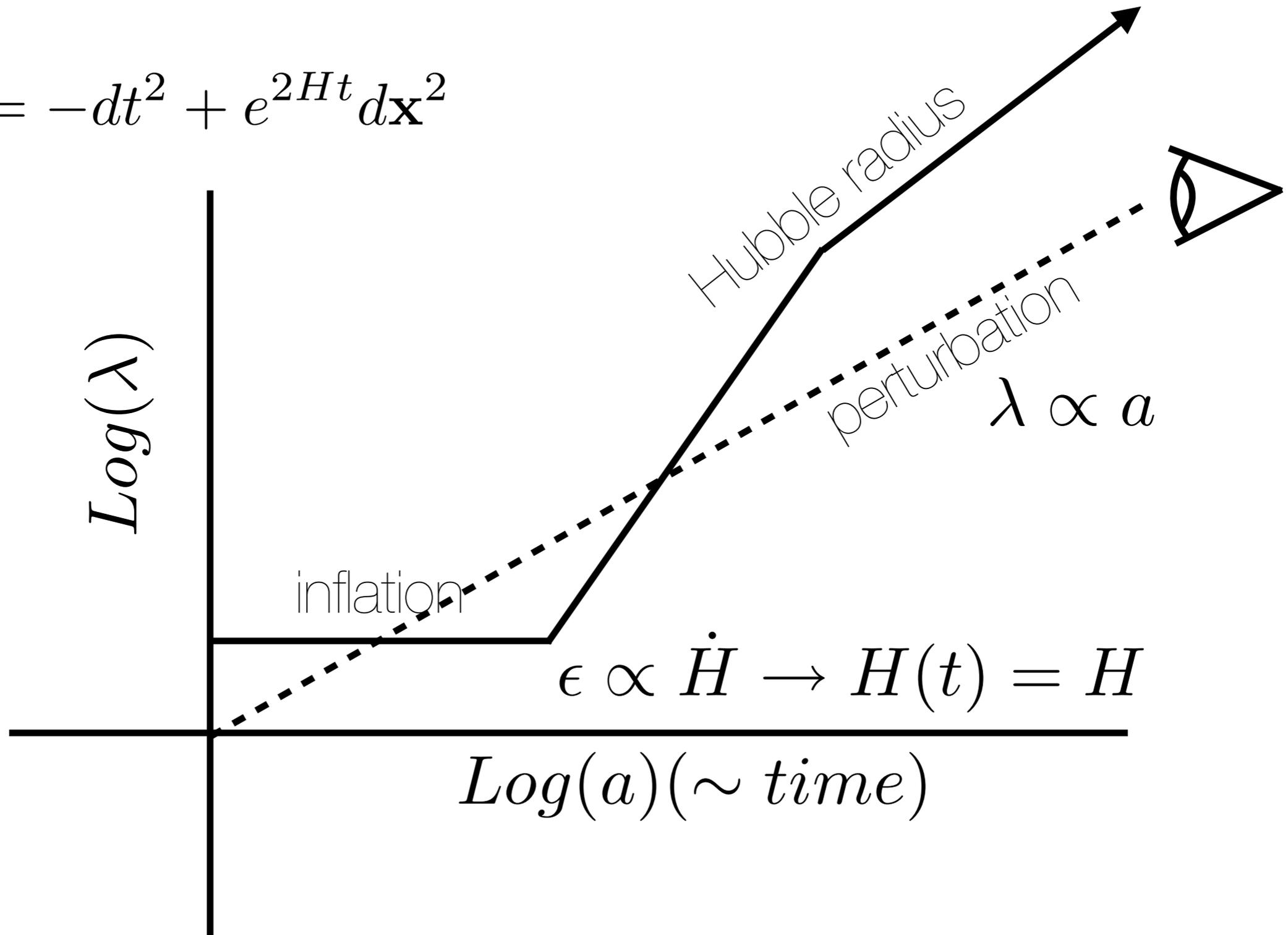
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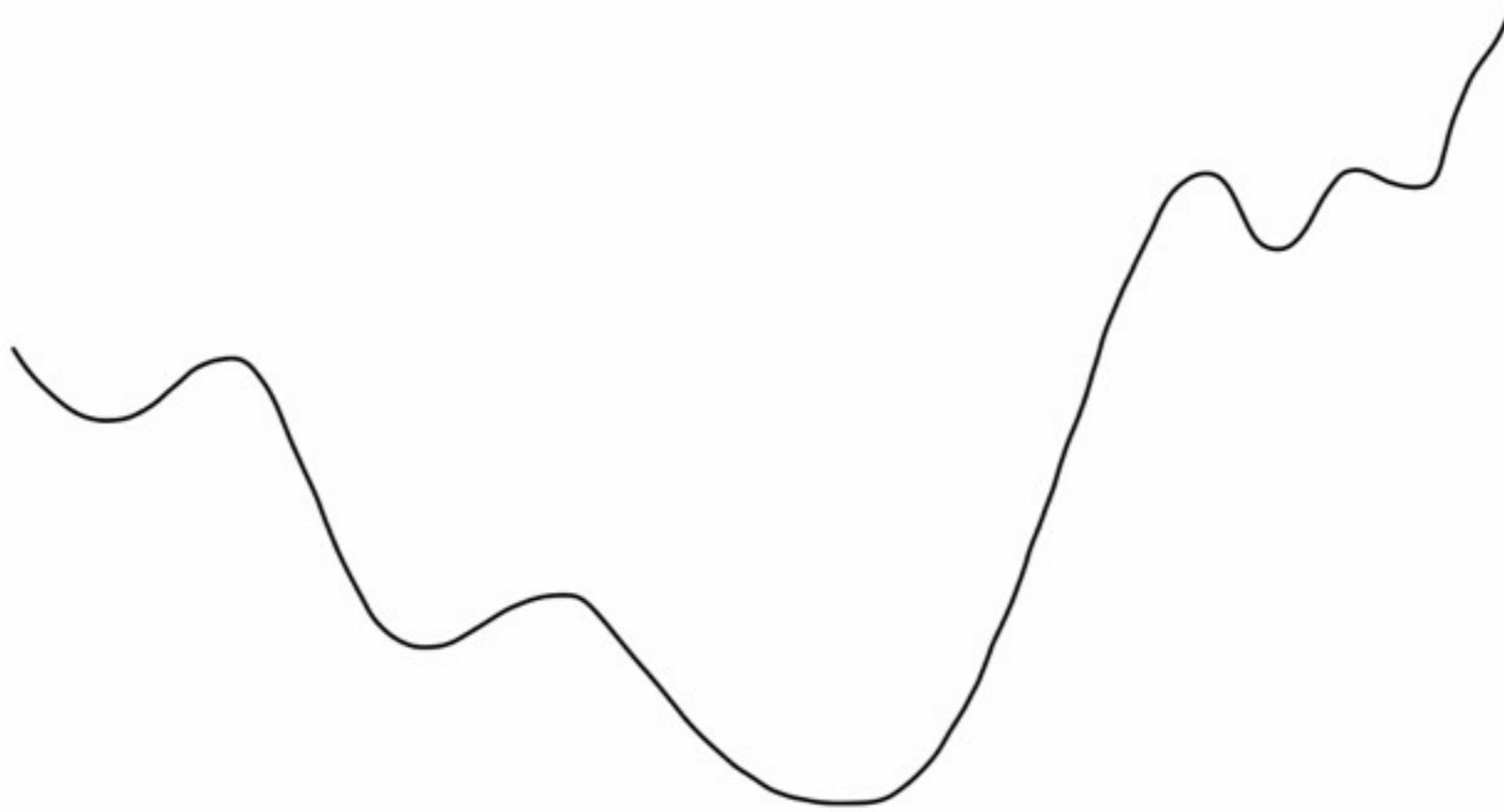


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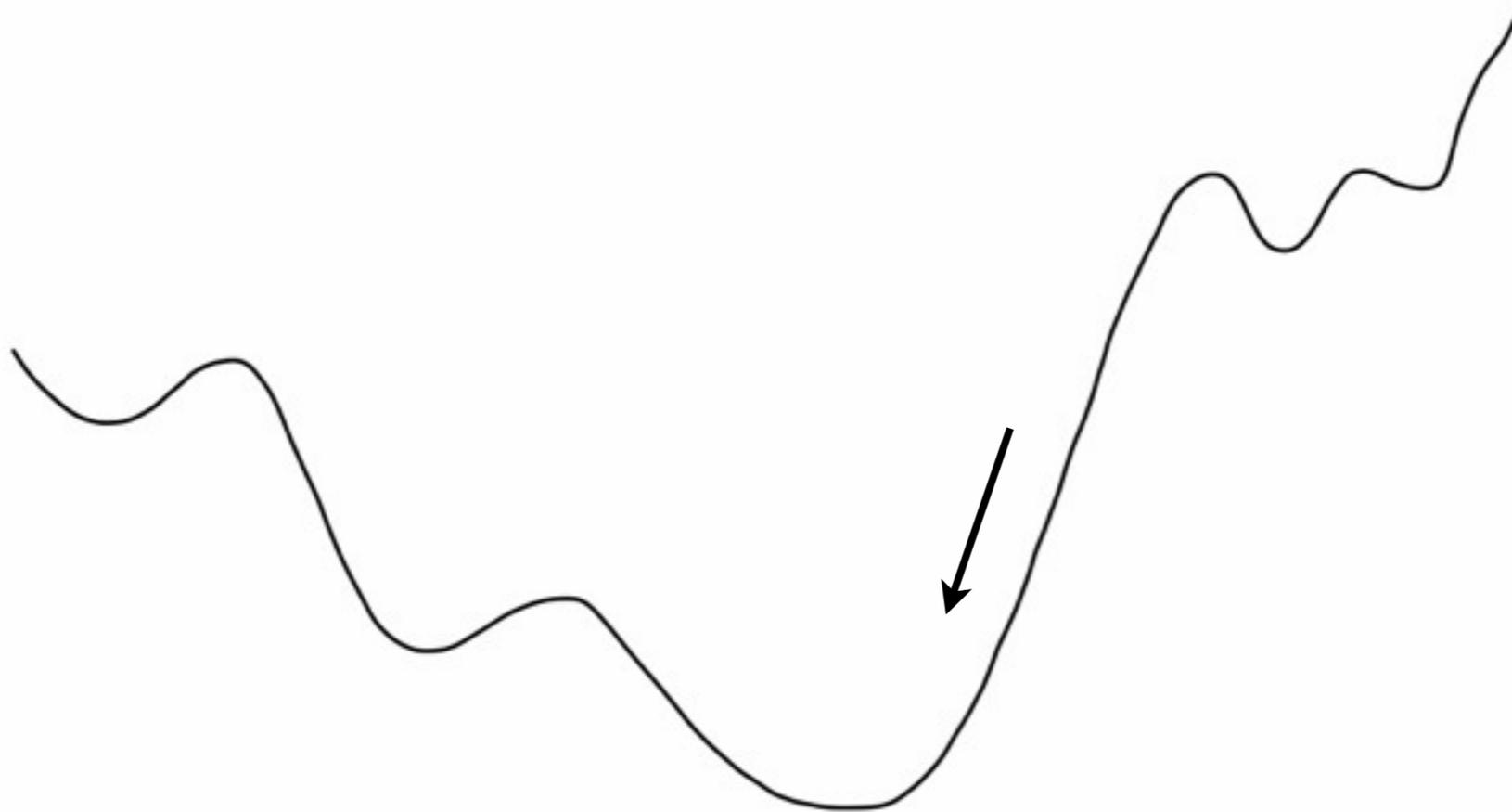
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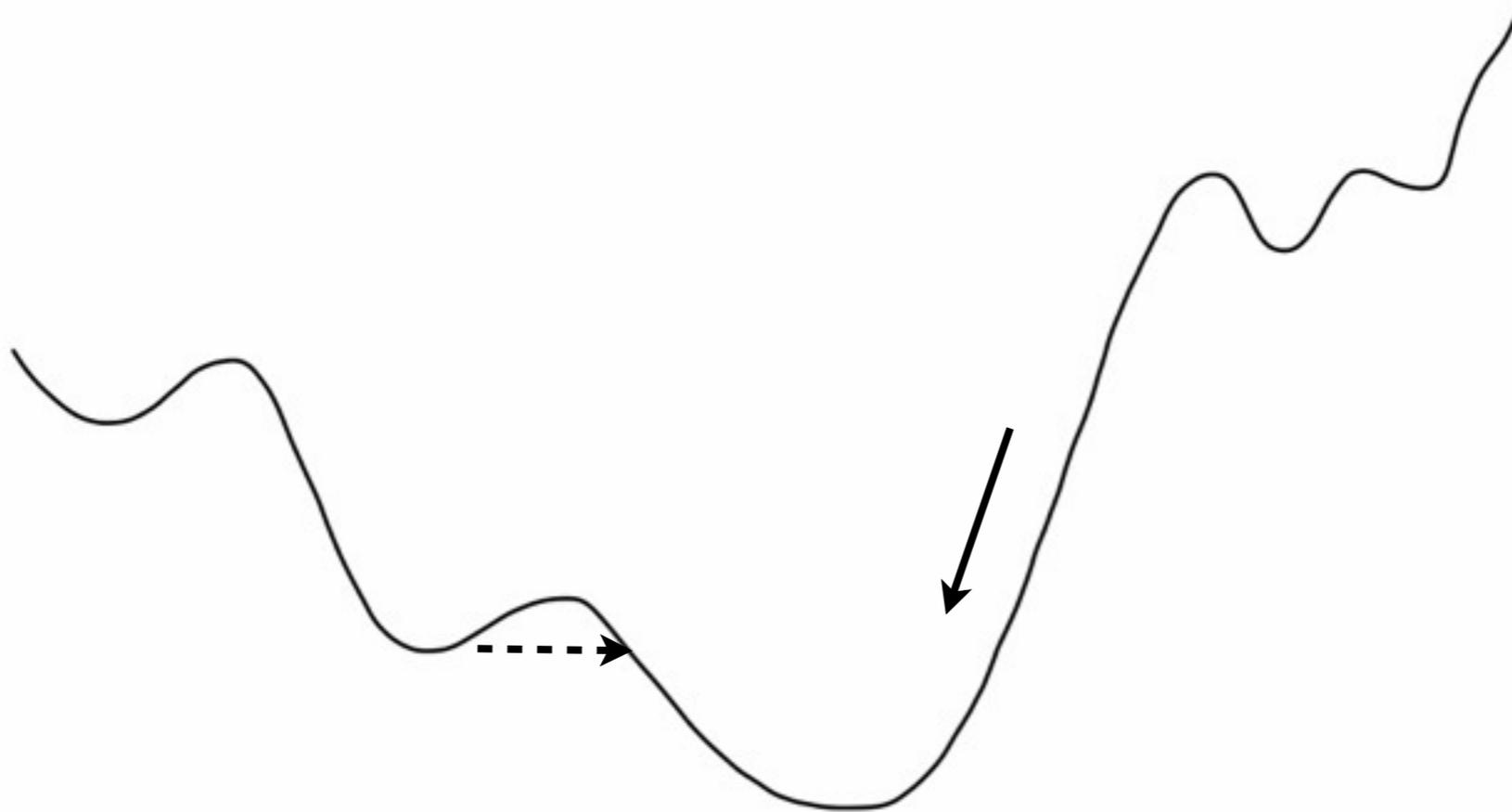
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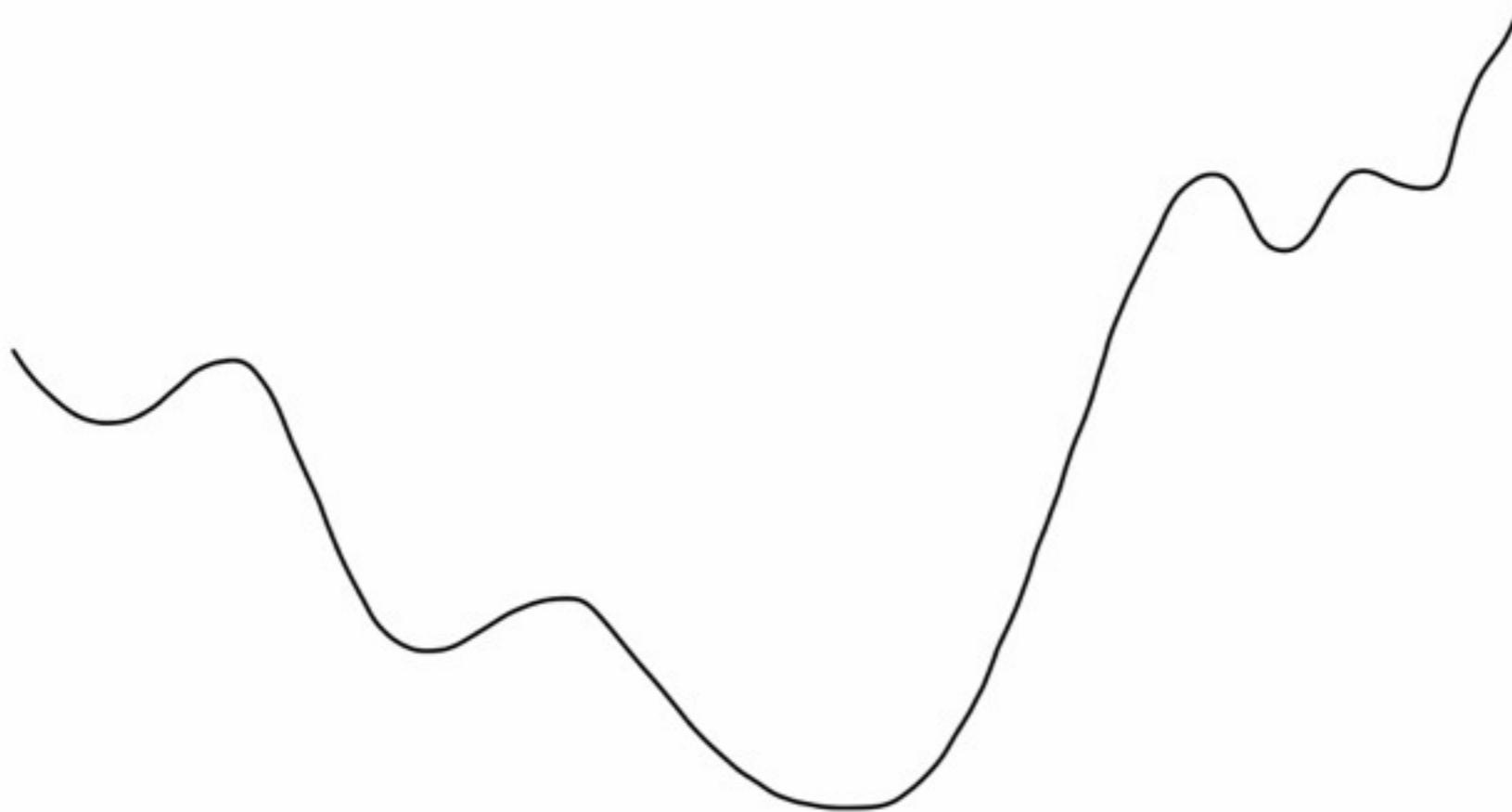
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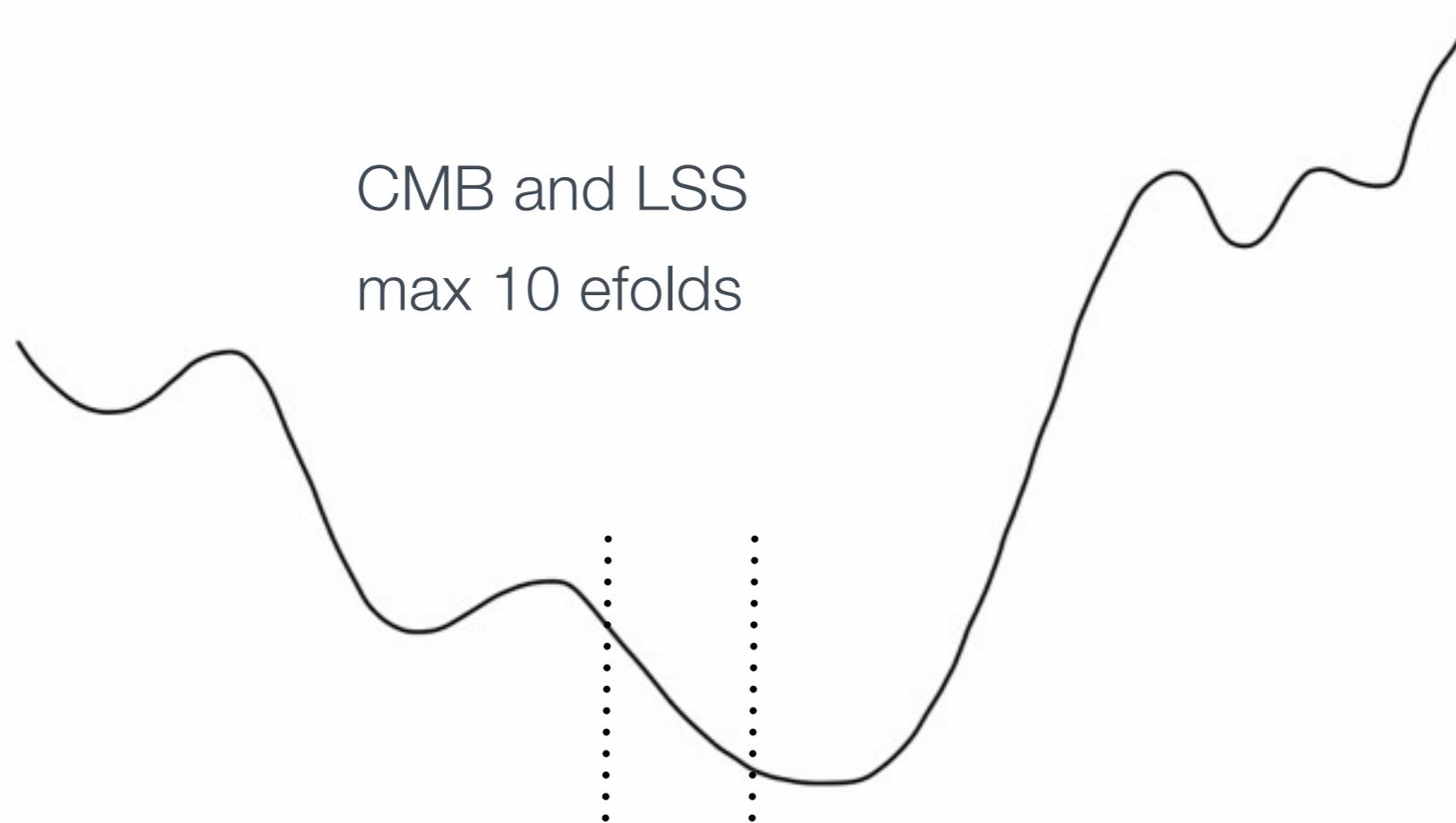
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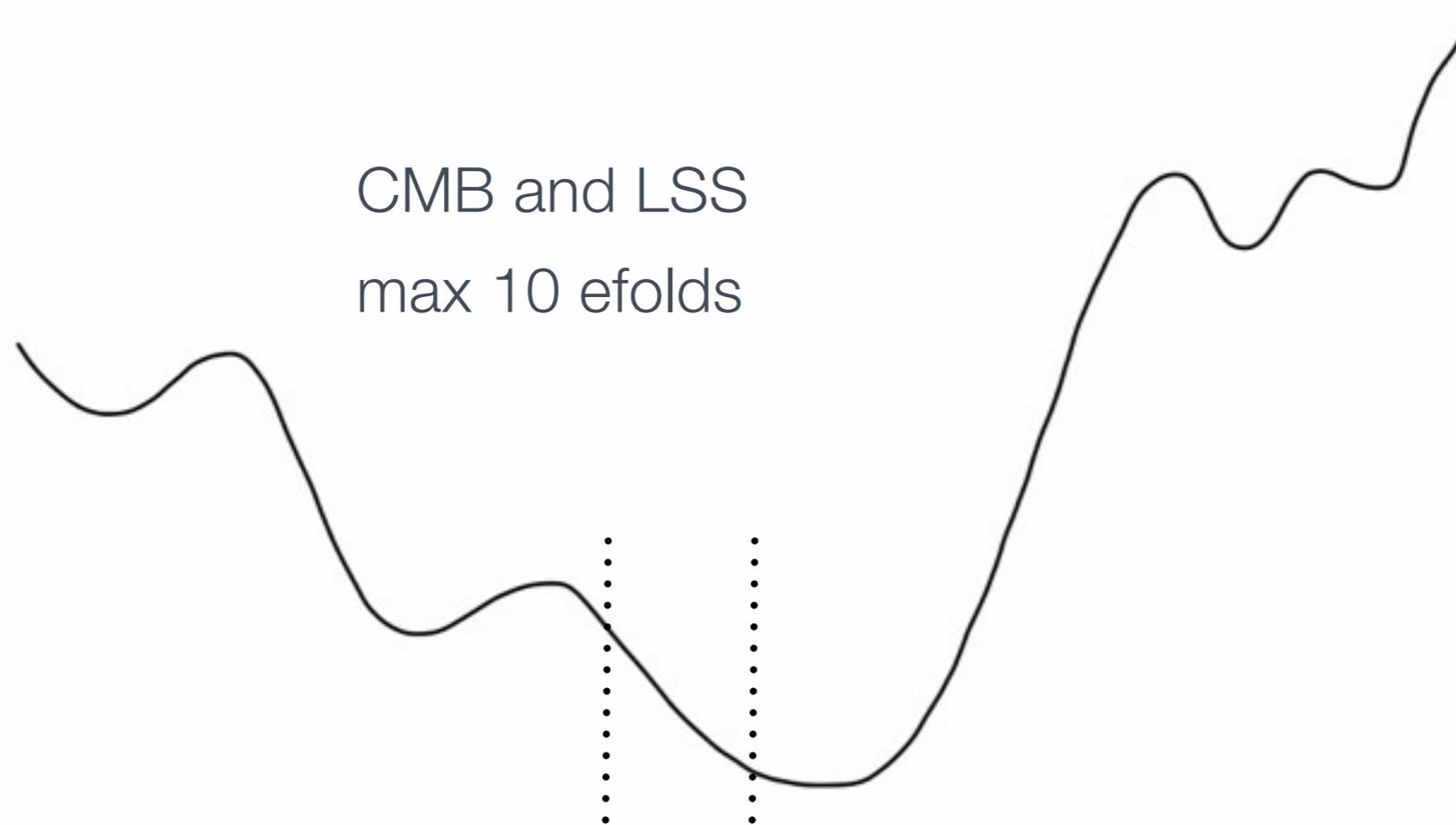
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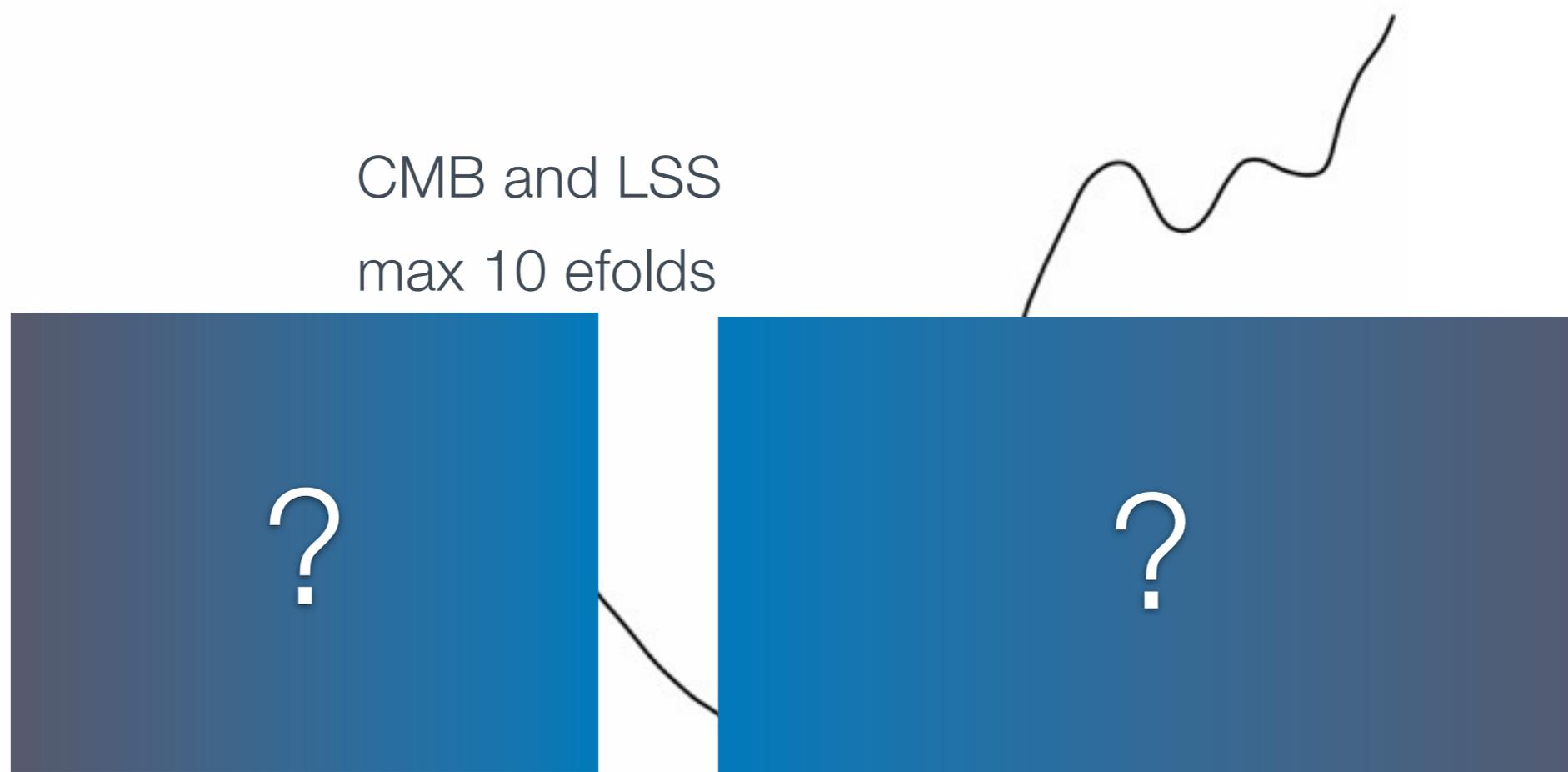


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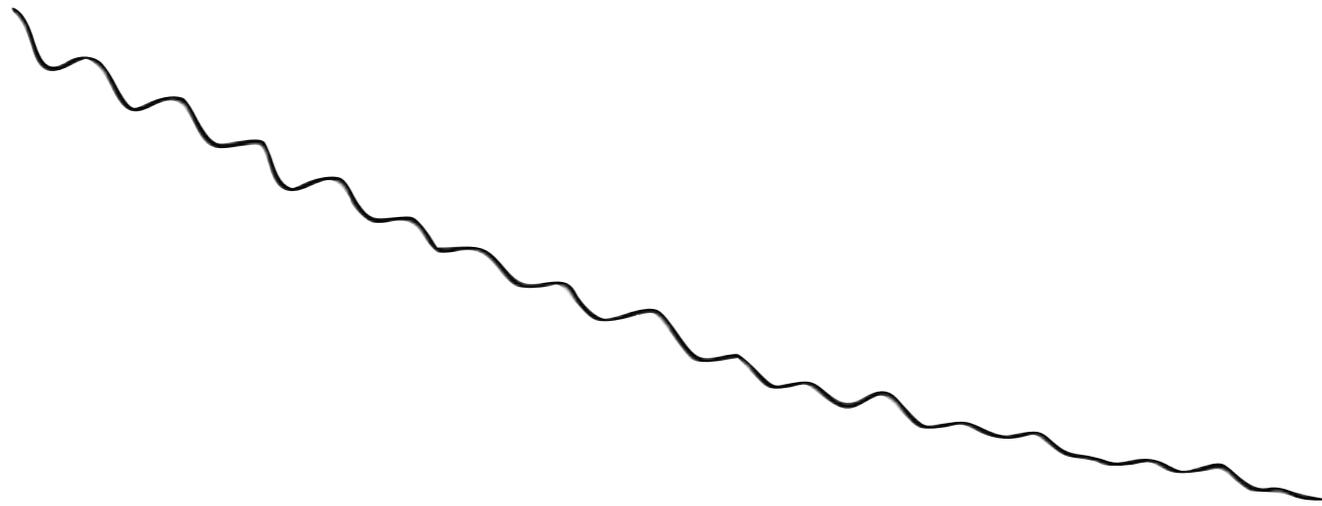
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Within these 10 E-folds, **red tilt** (less power at small scales)

Can we extract anything beyond first few derivatives?



# FEATURES

- **Roughly** (for a single clock):
  - Excitations in **time** --> **Linear oscillations (decaying)**
  - Excitations in **scale** --> **Logarithmic oscillations**
- Any **modification to initial conditions**, or **e.o.s.** during inflation (at fixed time)
- Modification of the initial condition at fixed scale, or **oscillatory potential** (linear).

# FEATURES

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- Consider following (general) **solution to the e.o.m.** :

$$\phi_k = N(k) [u_k + \beta(k)u_k^*]$$

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- Writing  $u_k = |u_k|e^{i\psi}$  and in the limit  $\beta \ll 1$

$$P_\phi(k) \simeq P_0 (1 + 2|\beta(k)| \cos(\alpha(k) + \psi))$$

# FEATURES

Non BD vacua

$$\alpha(k) \sim \Lambda/H \sim \Lambda \log k$$

Axion-monodromy

$$\alpha(k) \sim \phi_k/f \sim (\log k)/f$$

# MOTIVATION

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- More generally:
  - Phenomenological/Observational
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**Do we learn something more?** Observation--->Theory

**What do we predict?** Theory ---> Observation

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- More generally:
  - Phenomenological/Observational
  - Theoretical

**Do we learn something more?** Observation $\rightarrow$ Theory

**What do we predict?** Theory  $\rightarrow$  Observation

Generally: **Future of early Universe cosmology** is **constraining correlated variables**, i.e. Theory  $\rightarrow$  {A,B,C,...} observables

# MODELS

## Derived templates

$${}_1\Delta_{\mathcal{R}}^2(k) = A_1 \left( \frac{k}{k_*} \right)^m (1 + A_2 \cos[\omega_1 \log k/k_* + \phi_1])$$

$${}_2\Delta_{\mathcal{R}}^2(k) = B_1 \left( \frac{k}{k_*} \right)^m \left( 1 + B_2 \left( \frac{k}{\tilde{k}} \right)^n \cos[\omega_2 k + \phi_2] \right)$$

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1) e.g. Axion Monodromy, Natural inflation (similar), non-BD (NPH), unwinding inflation, (e.g. Silverstein & Westphal 2008, Flauger et al 2009, D'Amico et al 2013, Greene et al 2004, Agullo & Parker 2011, Shandera et al. 2012,2013, Battefeld et al 2013)

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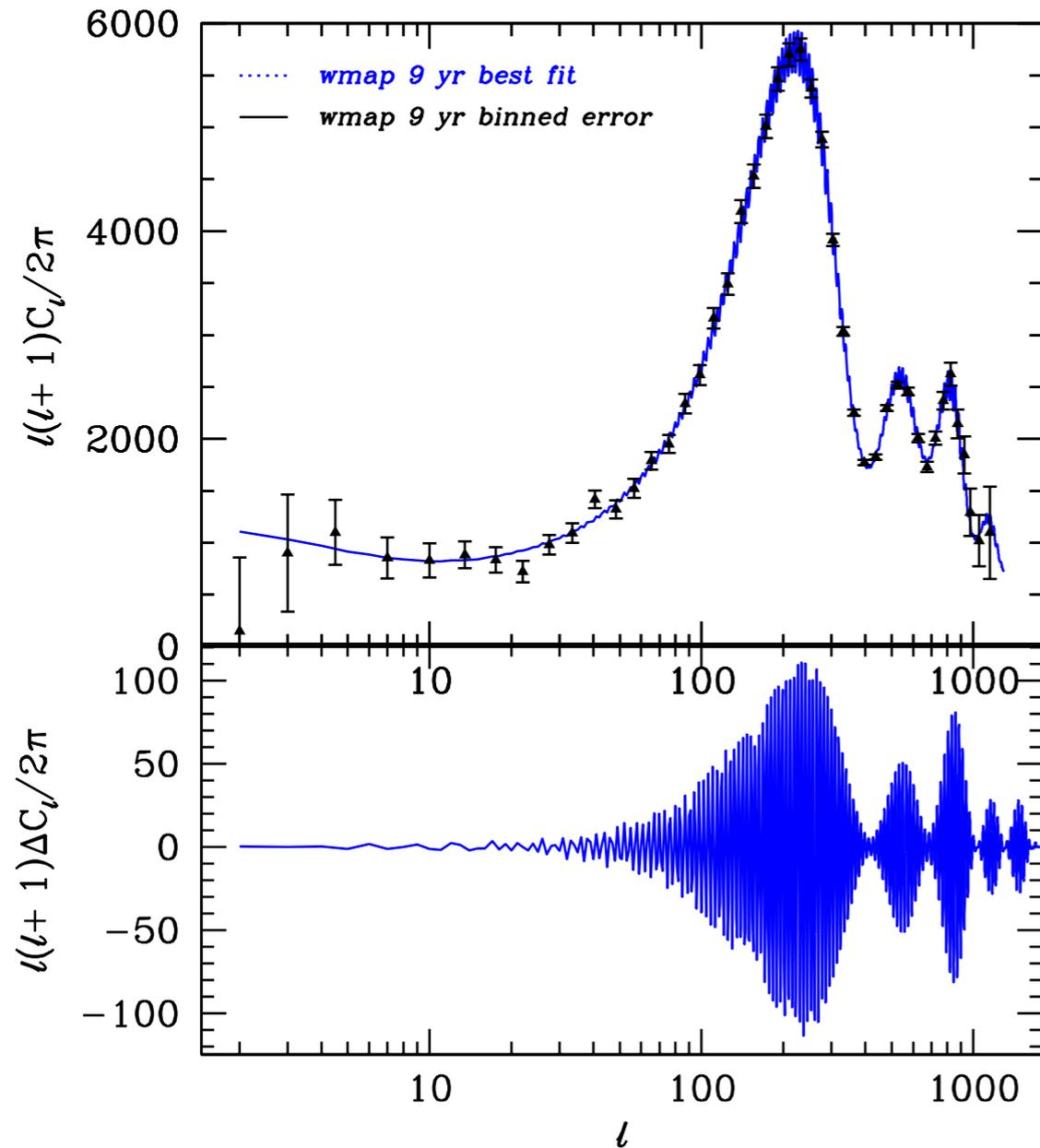
2) e.g. non-BD (BEFT) (e.g. Porrati 2004(2), Greene et al 2004) Change in e.o.s. (e.g. Joy, Sahni, Starobinsky 2007, Battefeld et al 2010, 2014 )

# DATA ANALYSIS

- 2 Issues:
  - Likelihood is **very irregular** (slow convergence)
  - Oscillations at high frequency require **high resolution** ( $k$  and  $\ell$ )
- **MonteCarloMarkovChain** (MH) generally becomes **impractical** (MULTINEST)
- Recomputing all transfer functions is **time consuming**

$$C_l = \frac{2}{\pi} \int_0^\infty \frac{dk}{k} \Delta_{\mathcal{R}}^2(k) (\Delta_l^T(k))^2$$

# PERTURBATIVE EXPANSION



$$C_l = \frac{2}{\pi} \int_0^\infty \frac{dk}{k} \Delta_{\mathcal{R}}^2(k) (\Delta_l^T(k))^2$$

Corrections are small

Perturbative expansion in  
oscillatory part

$$C_l = C_l^u + C_l^p$$

# PERTURBATIVE EXPANSION

- Expand Transfer function in oscillatory part

$$(\Delta_l^T(k))^2 = (\bar{\Delta}_l^T)^2 + 2\bar{\Delta}_l^T \sum (\Theta_i - \bar{\Theta}) \bar{\Delta}_{l,\Theta_i}^T + \mathcal{O}(\Theta_i^2)$$

- We then have for the perturbed part:

$$C_\ell^p = \bar{C}_\ell^{p(\alpha)} + \bar{C}_\ell^{p(\beta)} + \sum (\Theta_i - \bar{\Theta}_i) (\bar{C}_{\ell,\Theta_i}^{p(\alpha)} + \bar{C}_{\ell,\Theta_i}^{p(\beta)}) + \mathcal{O}((\alpha + \beta)\Theta_i^2)$$

- Power spectra and derivatives can be **precomputed**:

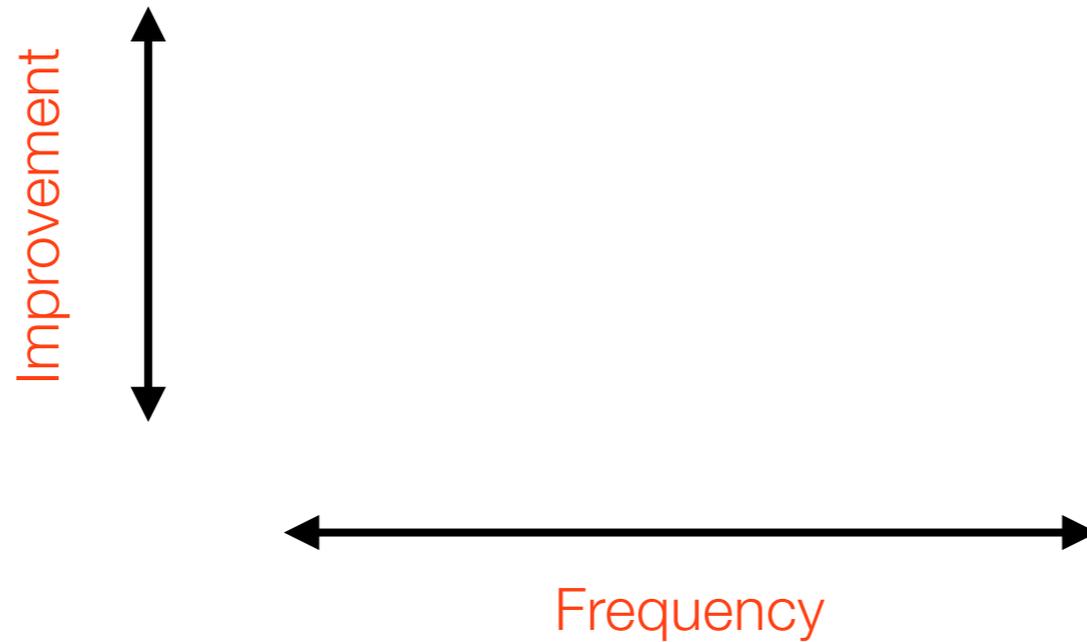
$$\bar{C}_\ell^p \quad \bar{C}_{\ell,\Theta_i}^p \quad \bar{C}_{\ell,\Theta_i\Theta_j}^p$$



Improvement in units of  $\Delta\chi^2$ . Typically,  $\chi^2$  distribution of 3 variables, requires  $\Delta\chi^2$  of  $\sim 11$  for 3 sigma significance.

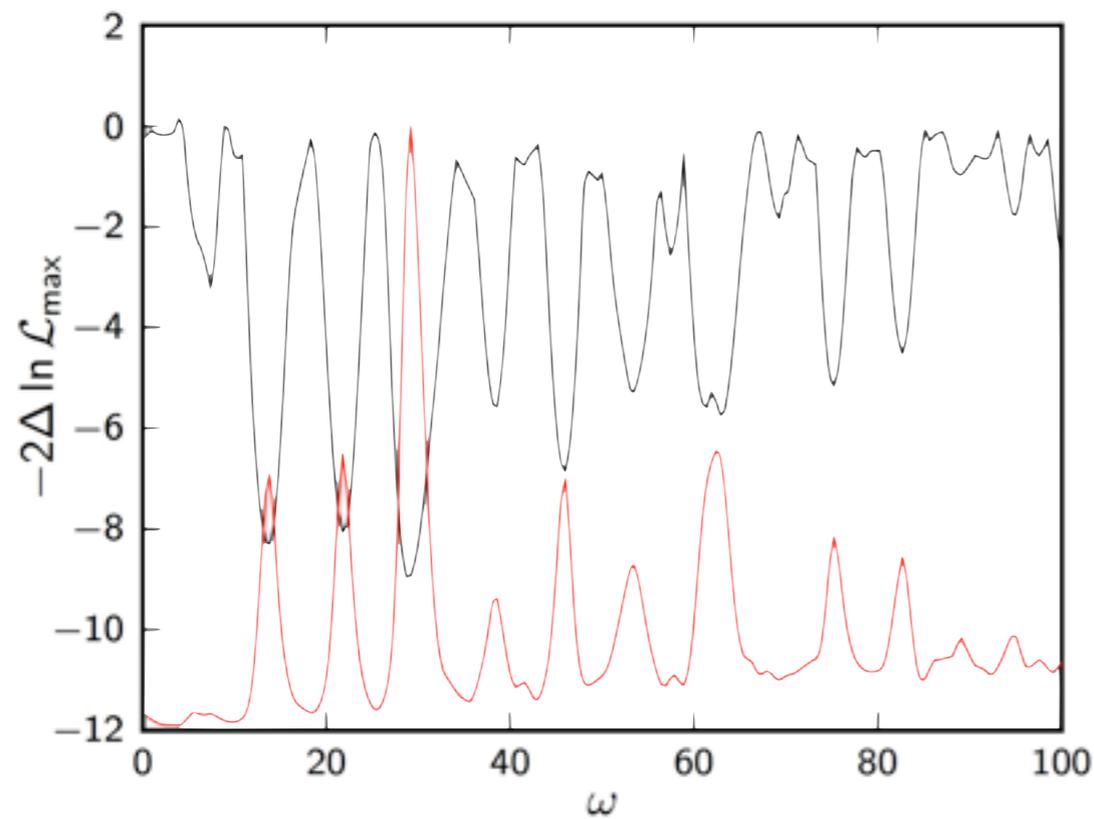
$$-2\Delta \log \mathcal{L} = \Delta\chi^2$$

## Next few slides



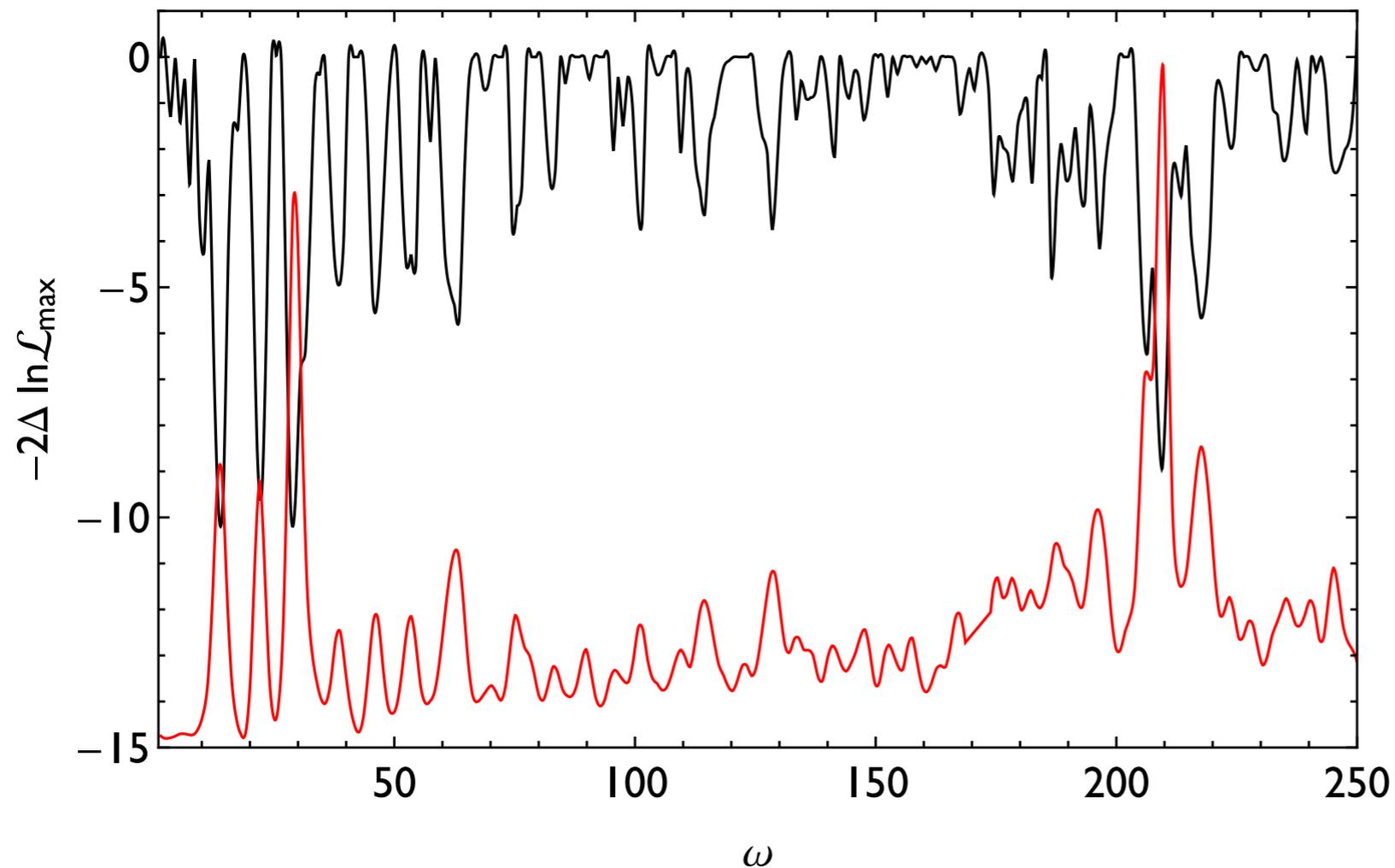
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Multinest Marginalized prob.

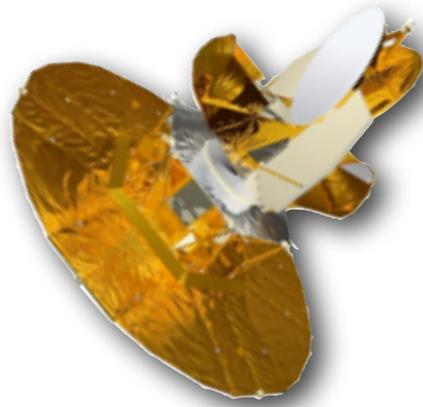
PLANCK COLLABORATION 2013



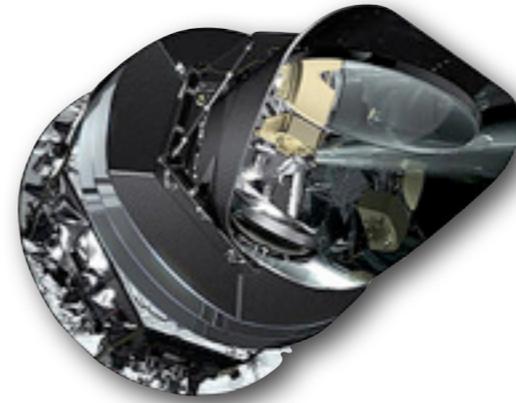
- Marginalized likelihood computed in ~16 hours, on 12 core node.

MEERBURG, SPERGEL, WANDELT 2014

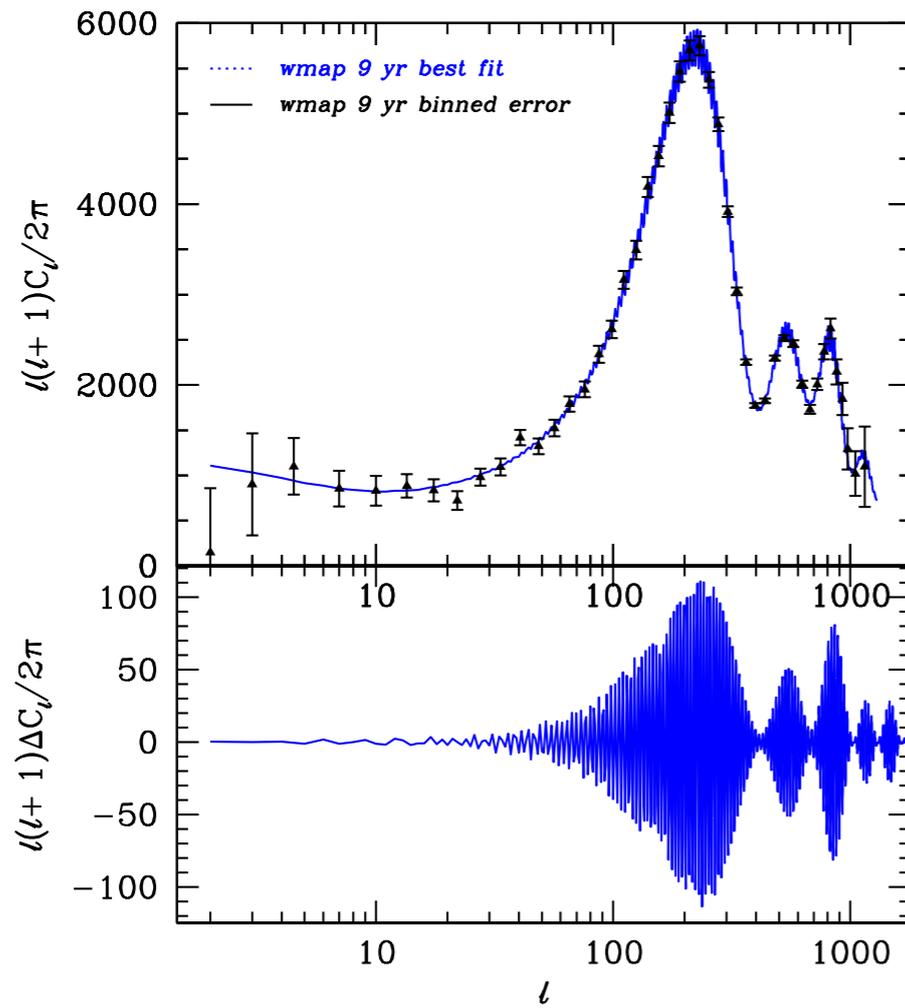
BERKELEY 2014



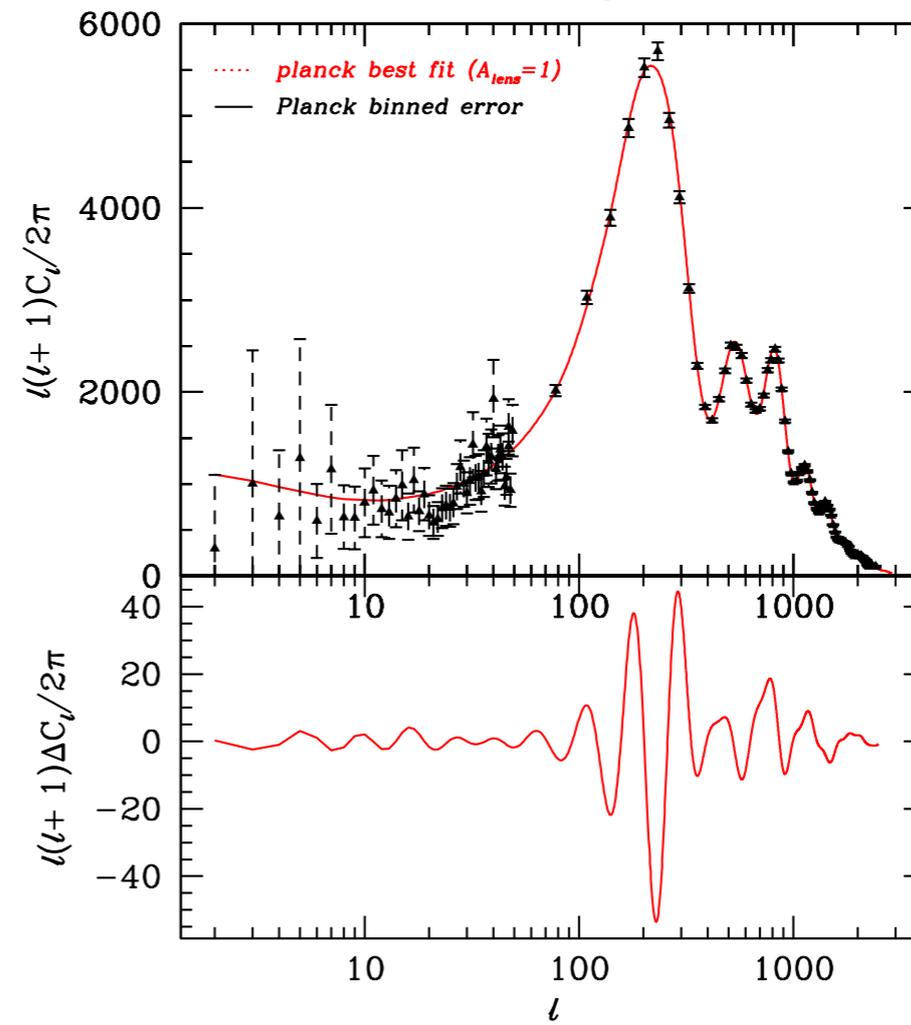
VS



WMAP 9



PLANCK 1

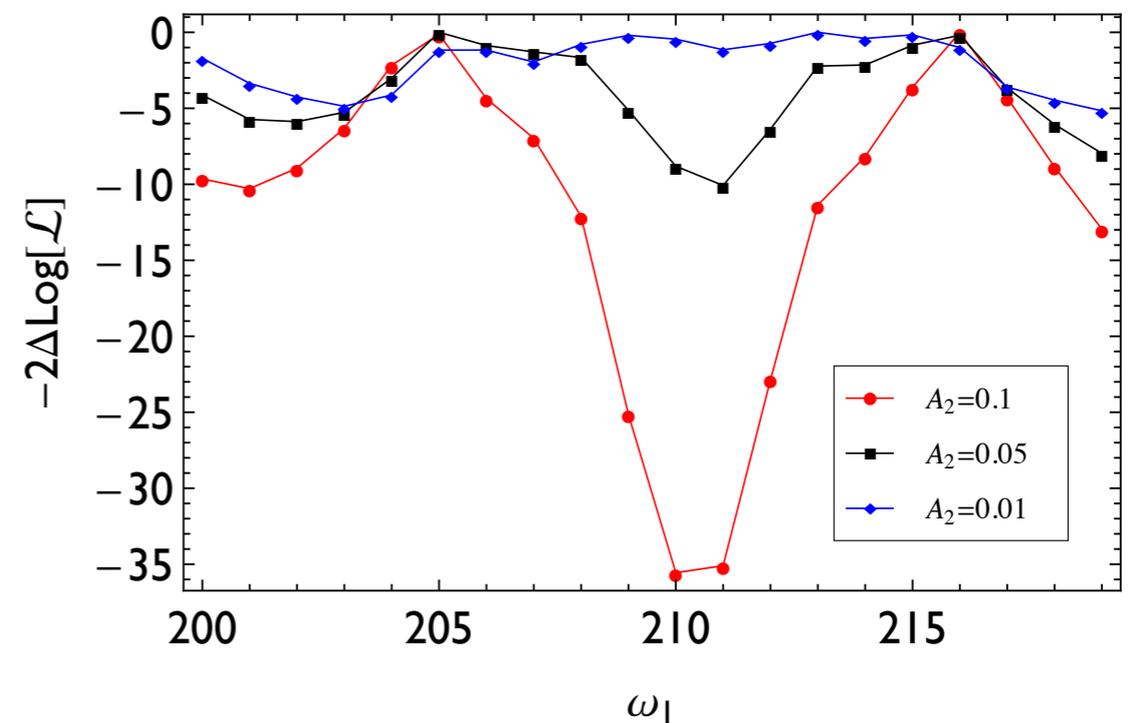
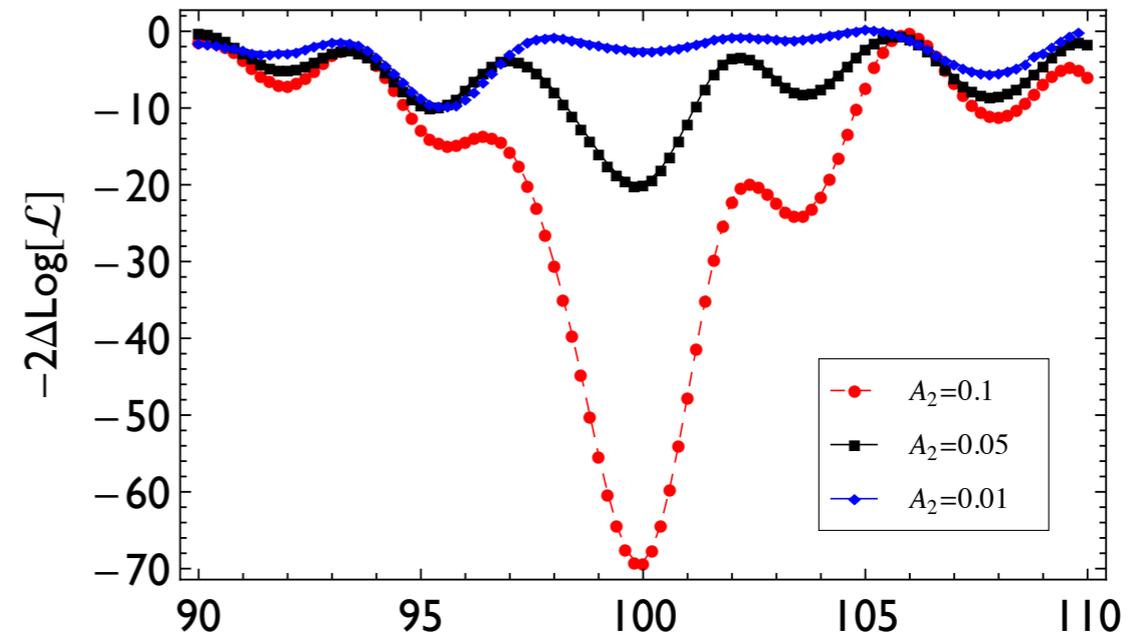
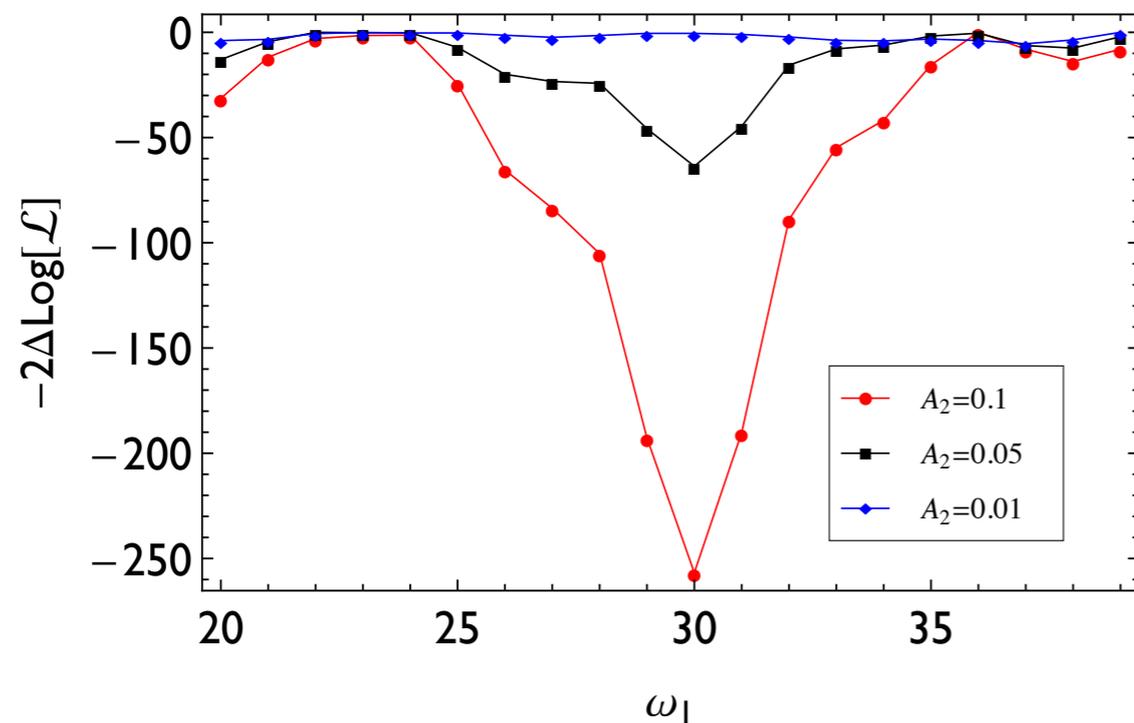


# ARE THESE REAL?

- Information criteria
- Simulations
- Circumstantial

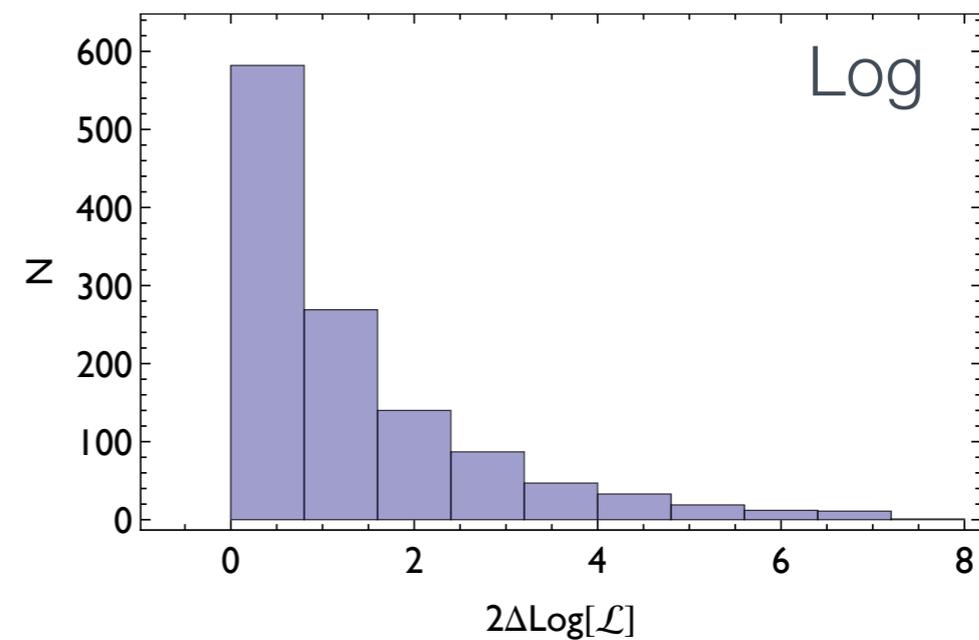
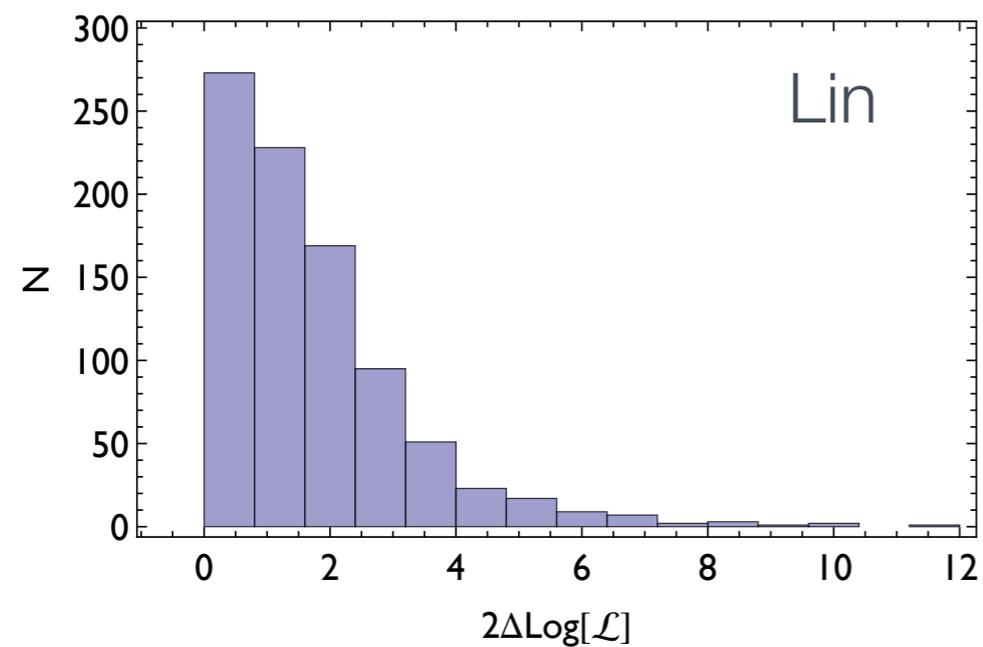
# SIMULATIONS

- Planck-like noise
- 3 different mock frequencies:  
30, 100, 210
- 3 different amplitudes:  
0.1%, 1%, 10%



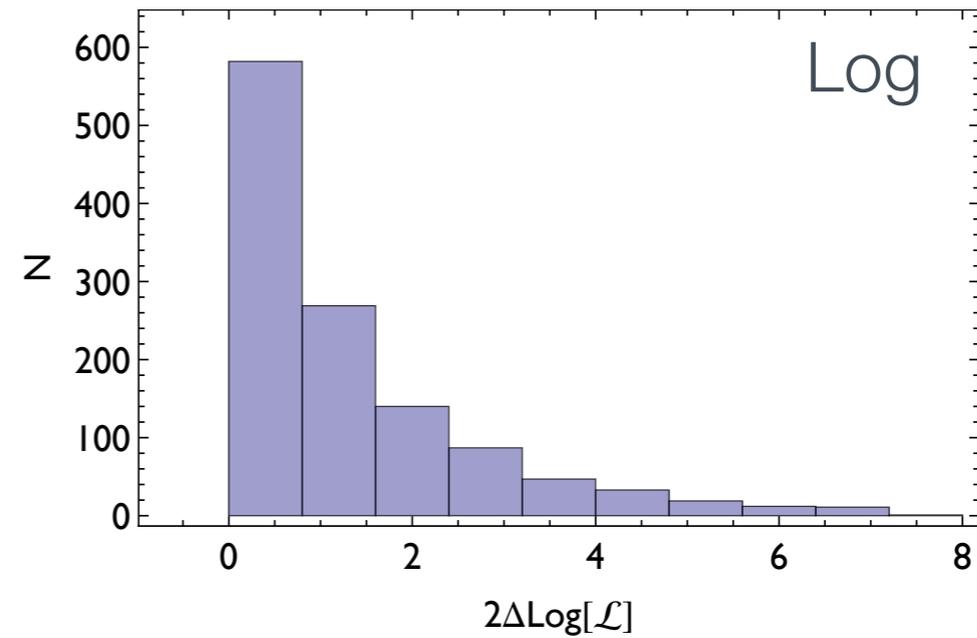
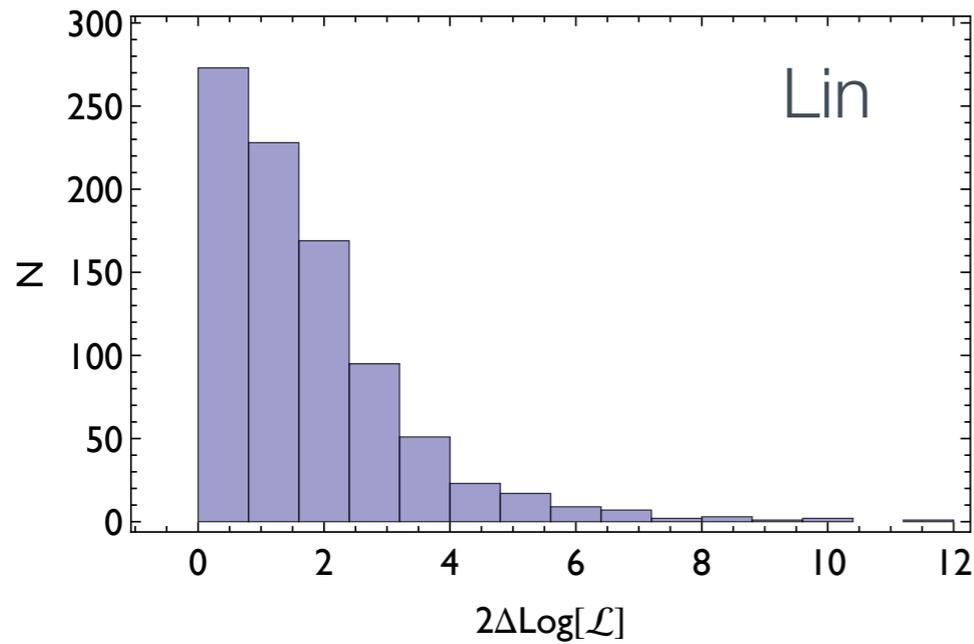
# SIMULATIONS

**Full MCMC** (planck noise) with mock data, no signal

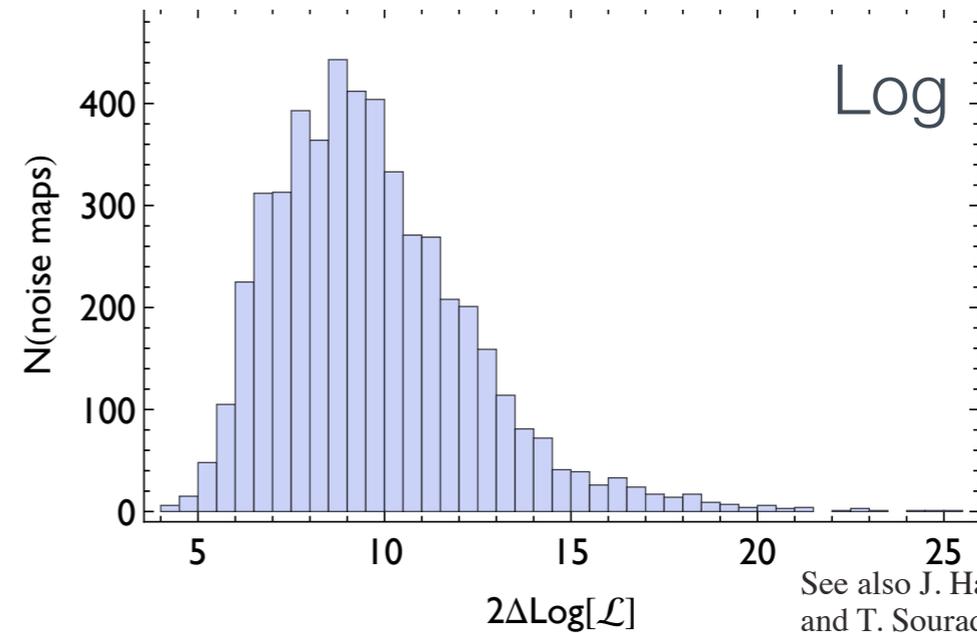
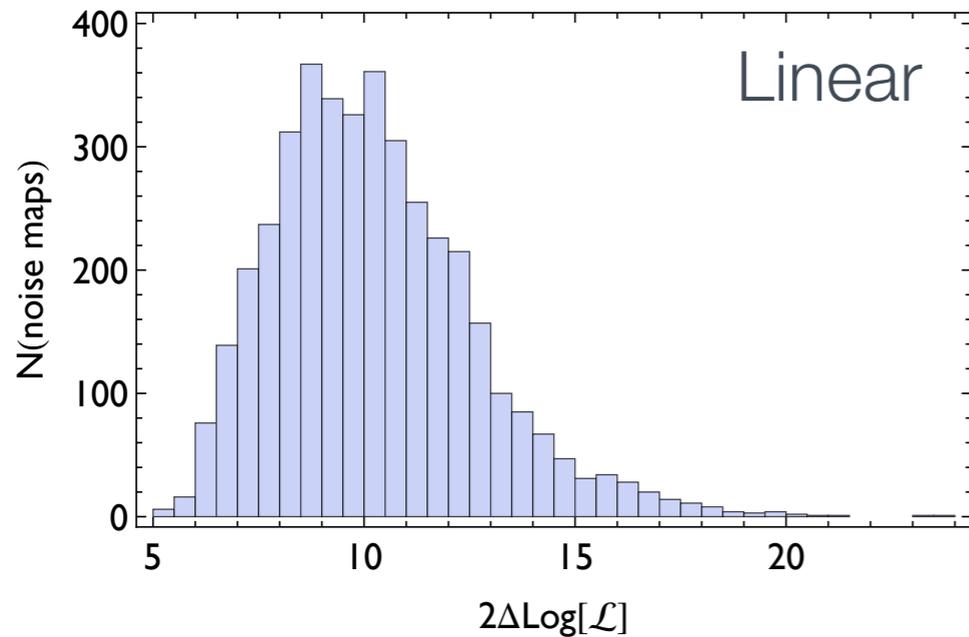


# SIMULATIONS

**Full MCMC** (planck noise) with mock data, no signal



**5000 Universes; WMAP 9 noise** and cosmic variance, no signal

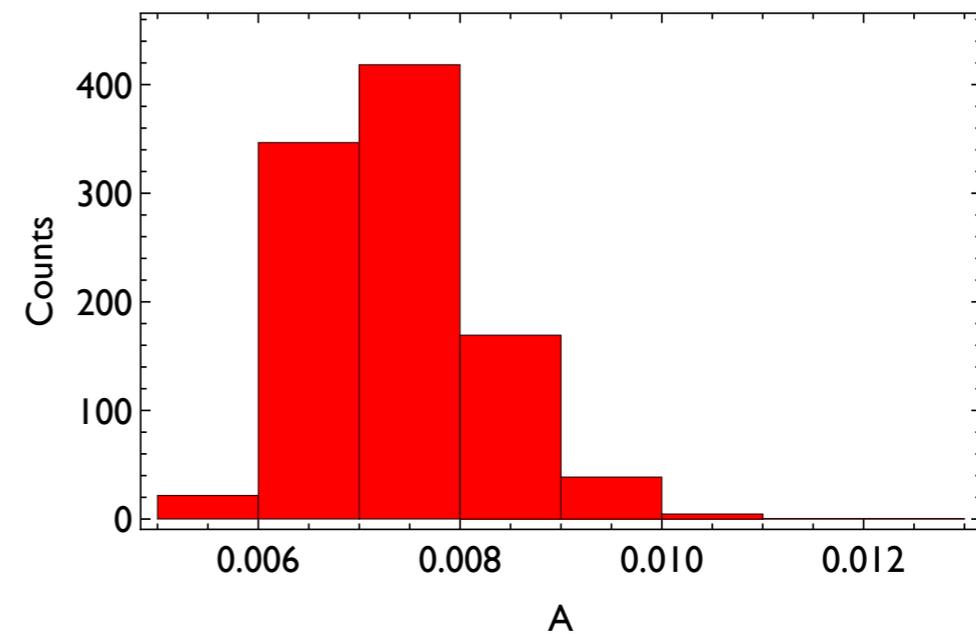
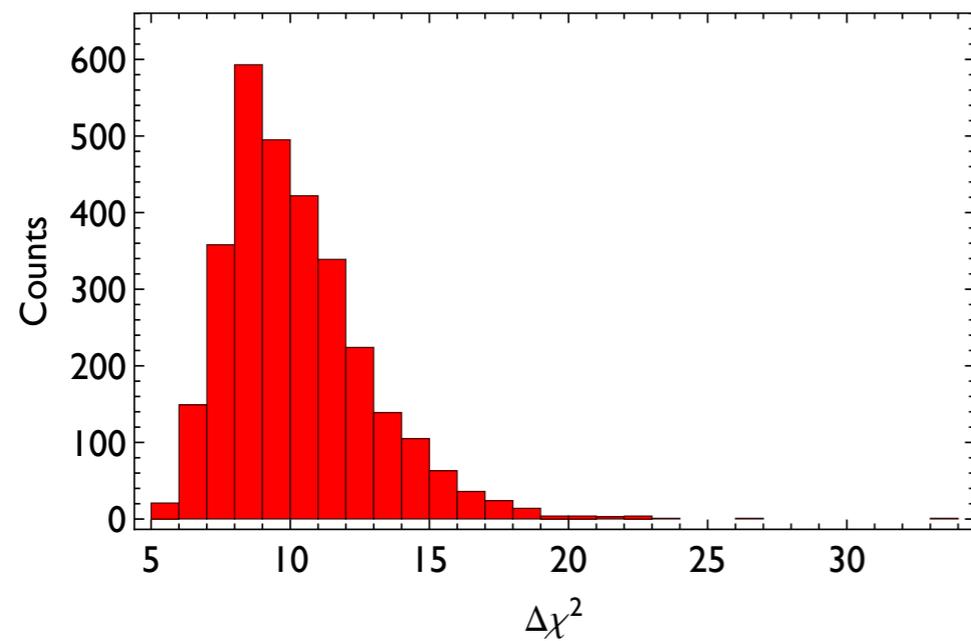


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See also J. Hamann, A. Shafieloo, and T. Souradeep (2010), R. Easther and R. Flauger (2013)

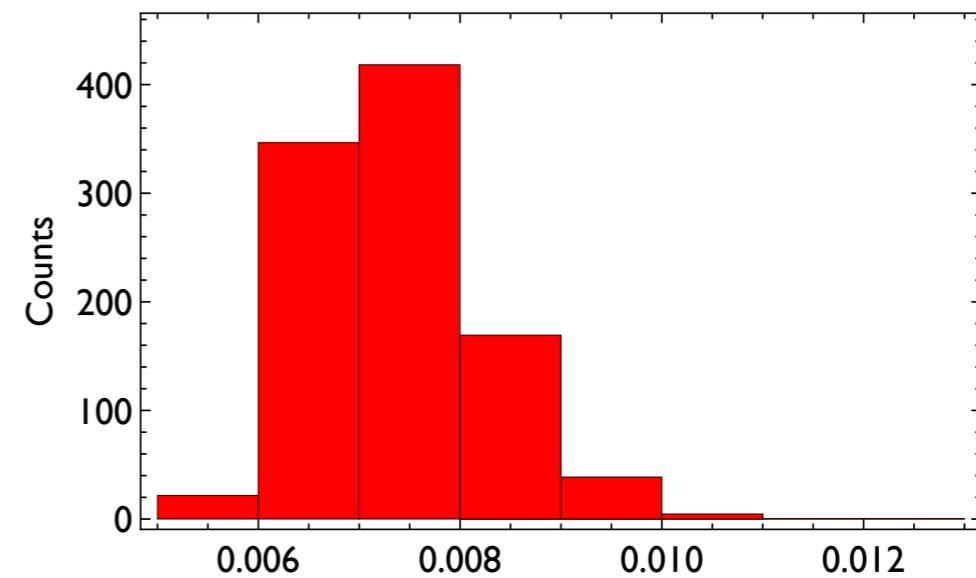
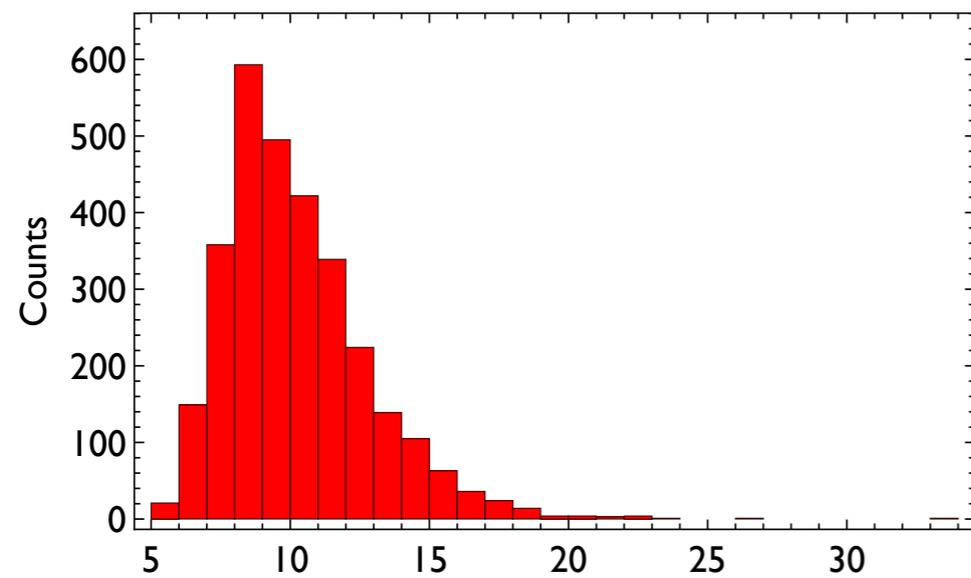
# SIMULATIONS

**5000** Universes; **Real Planck noise** and cosmic variance, **no signal**

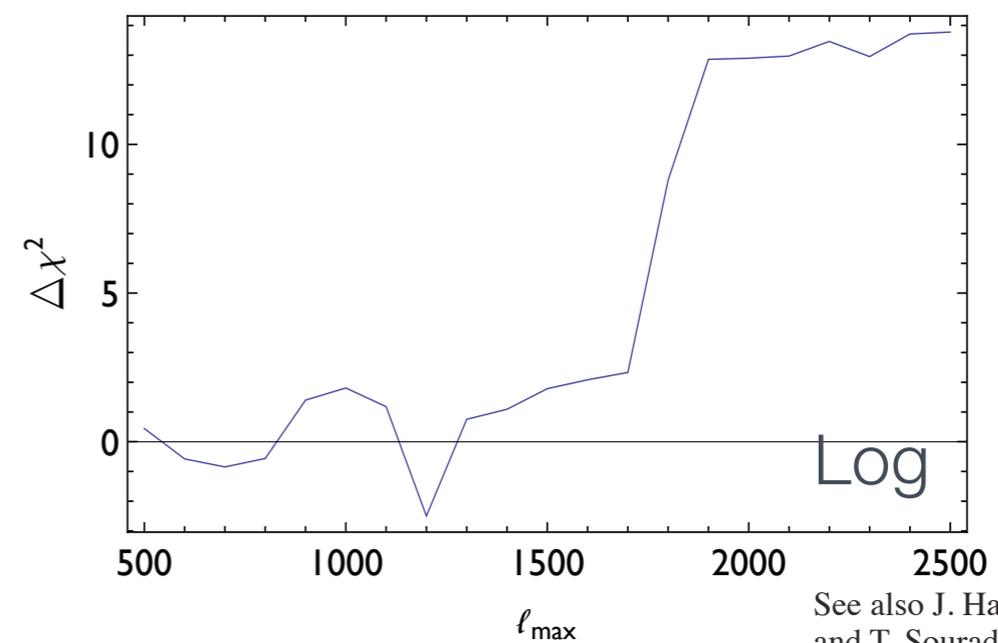
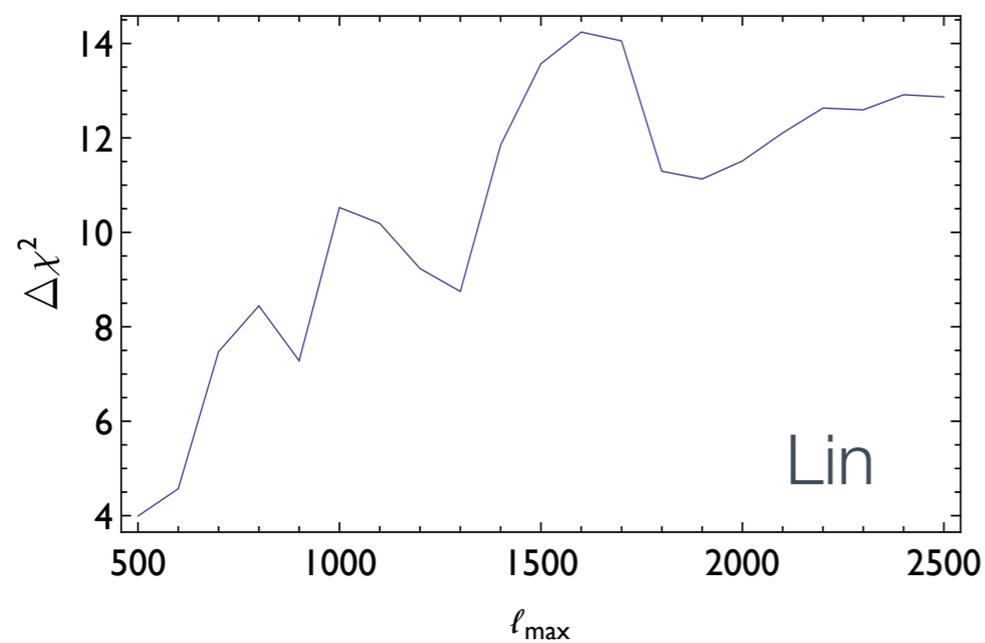


# SIMULATIONS

5000 Universes; Real Planck noise and cosmic variance, no signal



Improvement as a function of maximum multipole:



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See also J. Hamann, A. Shafieloo, and T. Souradeep (2010), R. Easther and R. Flauger (2013)

# ARE THESE REAL?

- Information criteria --> A mess
- Simulations --> Most likely not
- Circumstantial --> Most likely not

# BICEP

- What about BICEP? 2 Things:
  - $r = 0.2$  would put extra tension between TT data and TT theory
  - Also,  $r = 0.2$  suggests Super Planckian displacement of field / violates Lyth bound
- Is there a model that would solve these 2 things in one go? Maybe: Axion monodromy. Shift symmetry
  - Solves first
  - and could induce a large feature, to solve second?

$$\Delta_{\mathcal{R}}^2(k) = A_s \left( \frac{k}{k_*} \right)^{n_s - 1} (1 + \delta n_s \cos[\omega \log k/k_* + \phi])$$

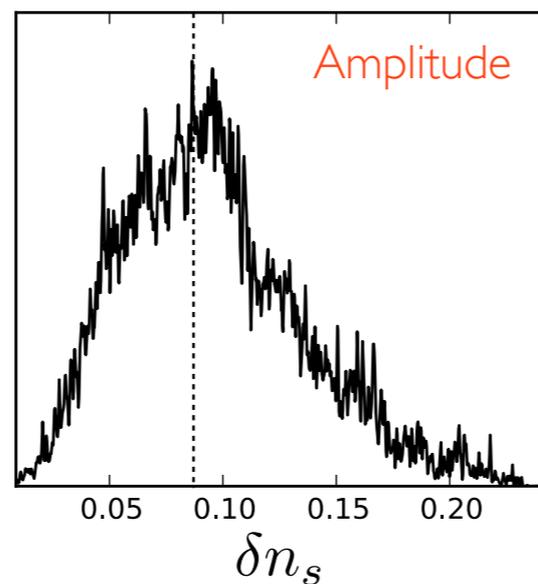
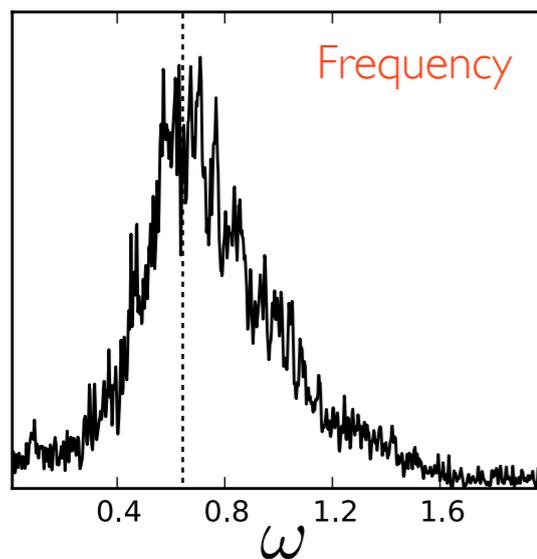
$$\Delta_{\mathcal{D}}^2(k) = A_s \frac{r_*}{4} \left( \frac{k}{k_*} \right)^{-r_*/8} \left( 1 - \delta n_s \frac{r_*}{8\omega} \sin[\omega \log k/k_* + \phi] \right)$$

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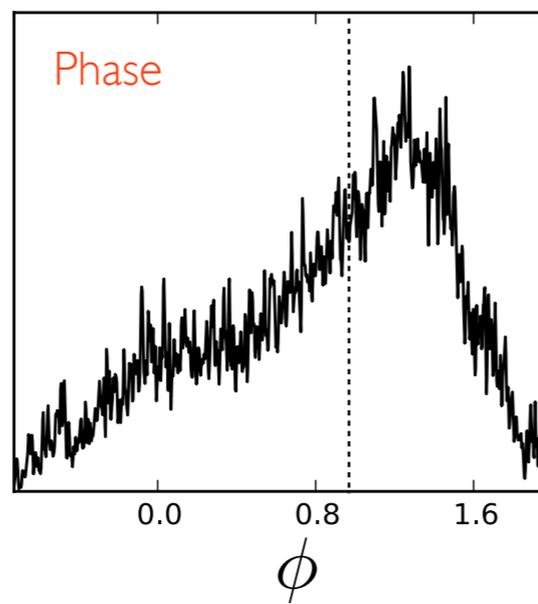
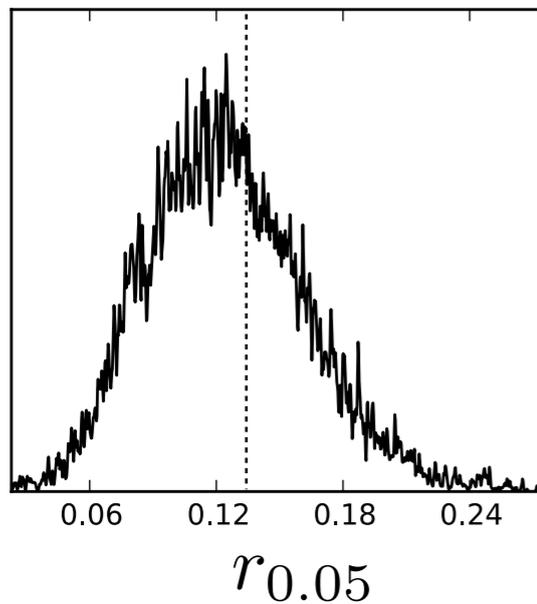
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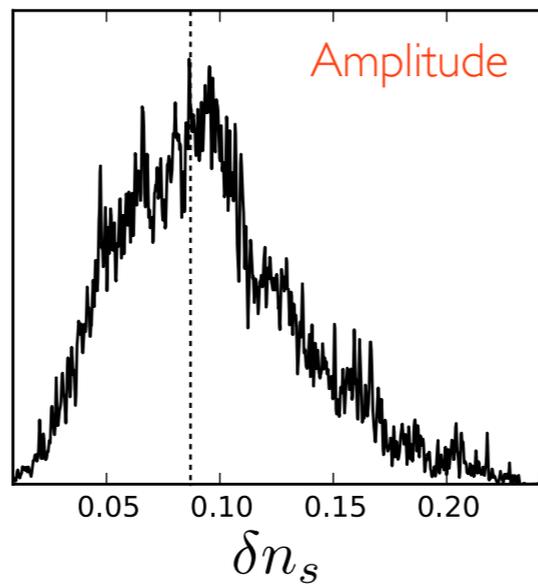
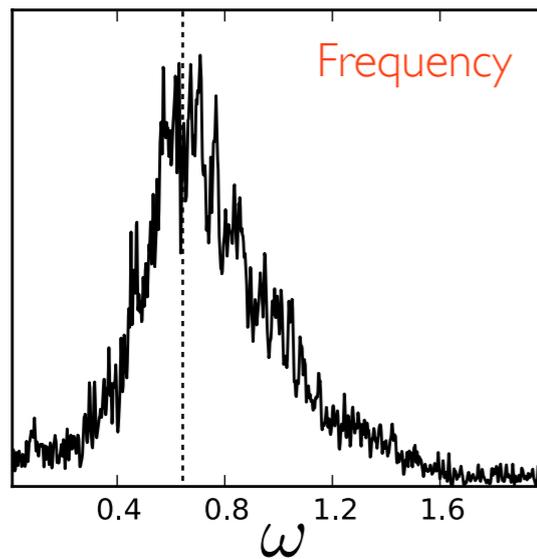
Planck+BICEP+SPT  
+ACT



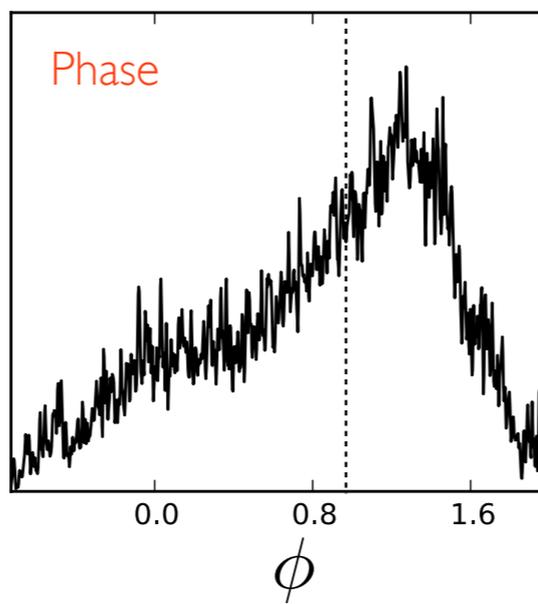
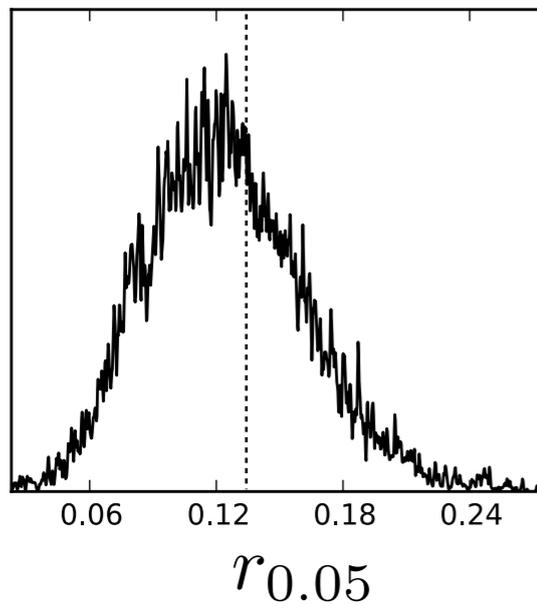
$$f/M_p \sim \mathcal{O}(.01)$$

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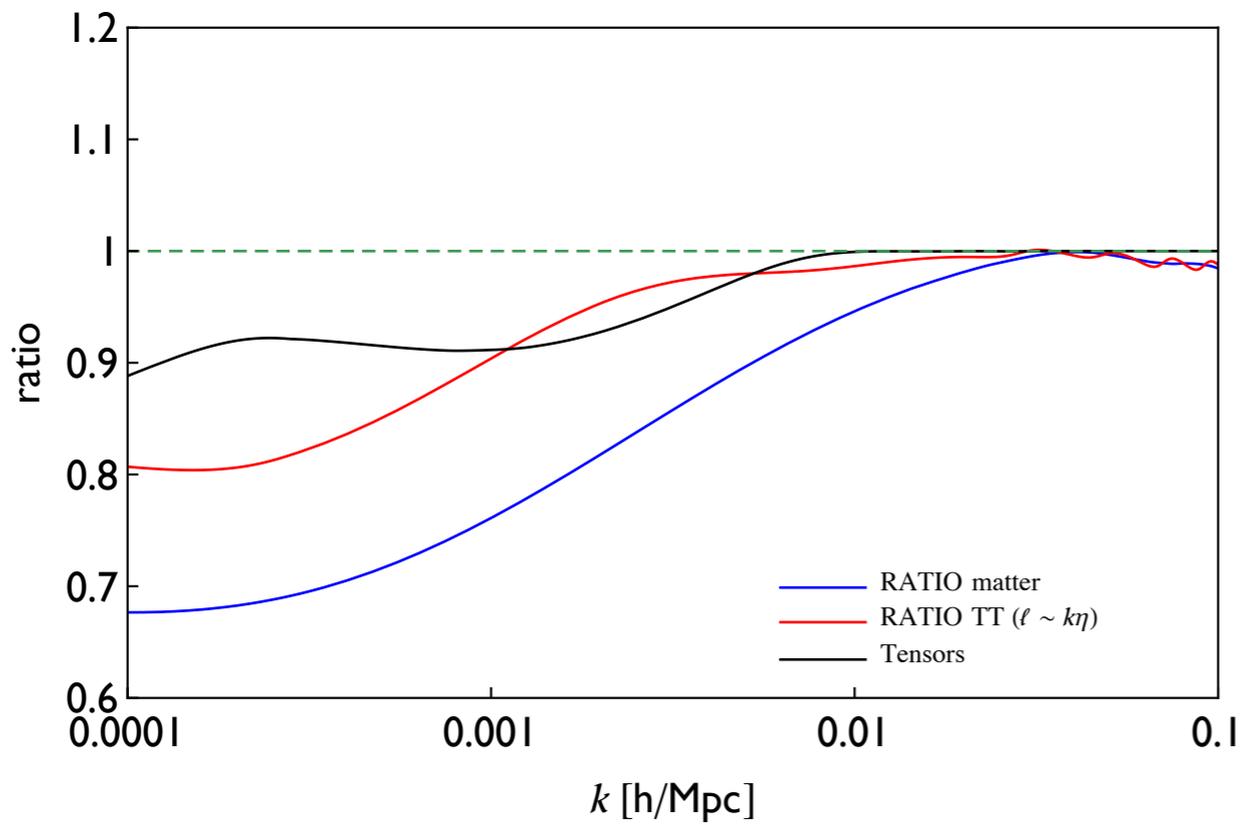
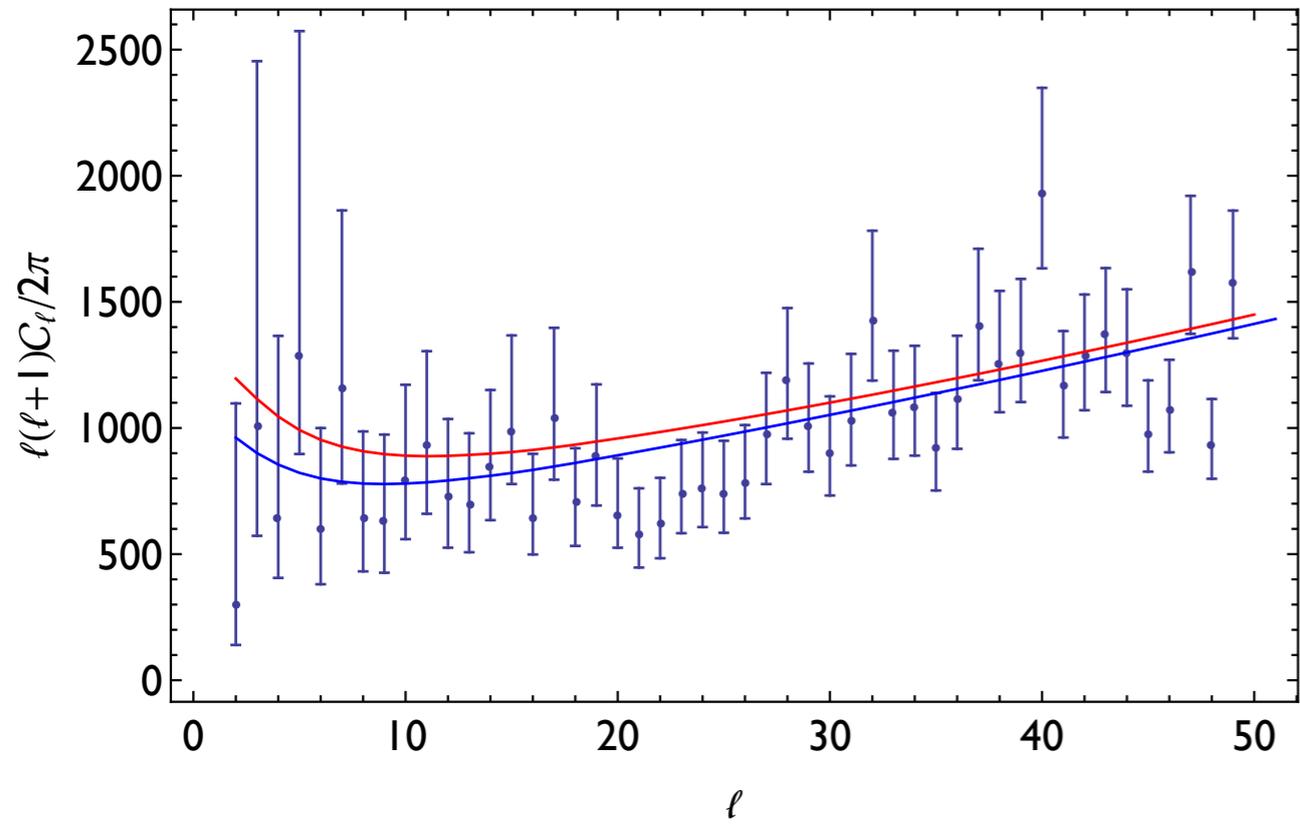


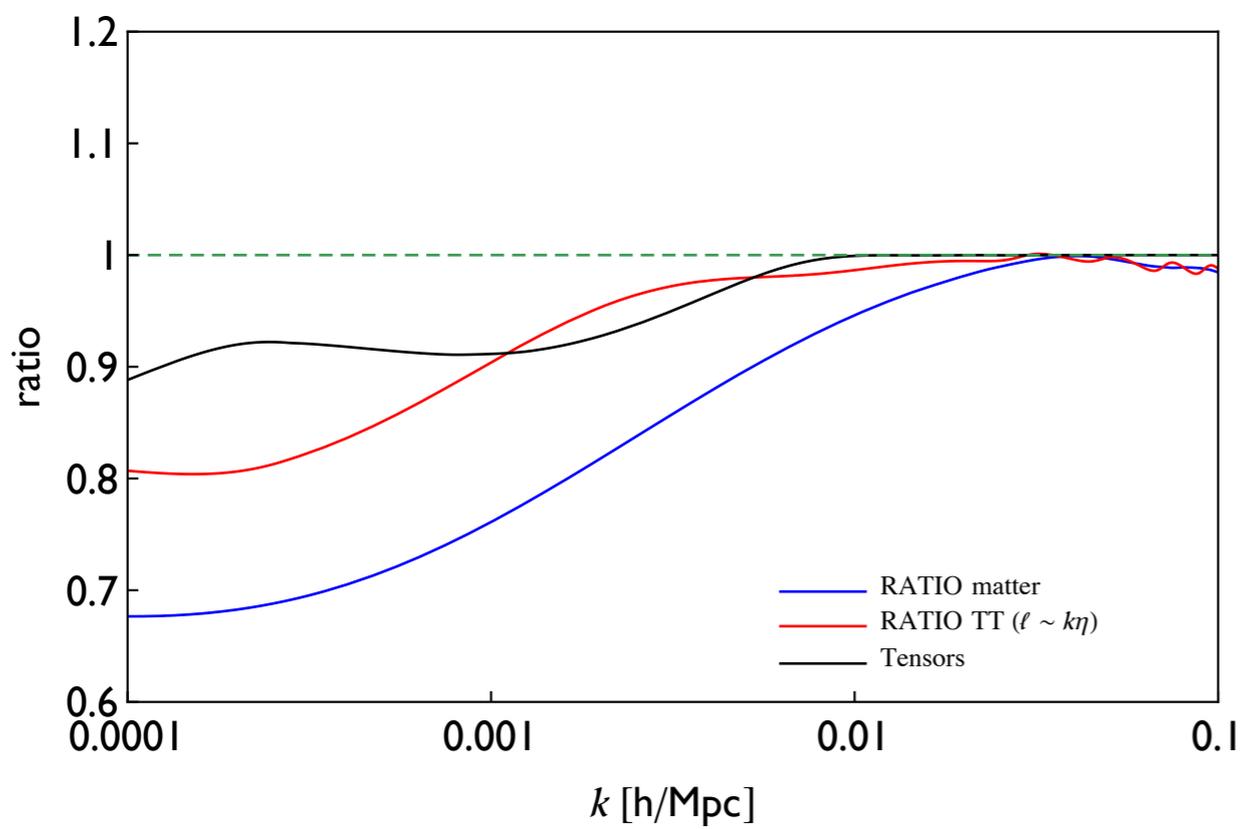
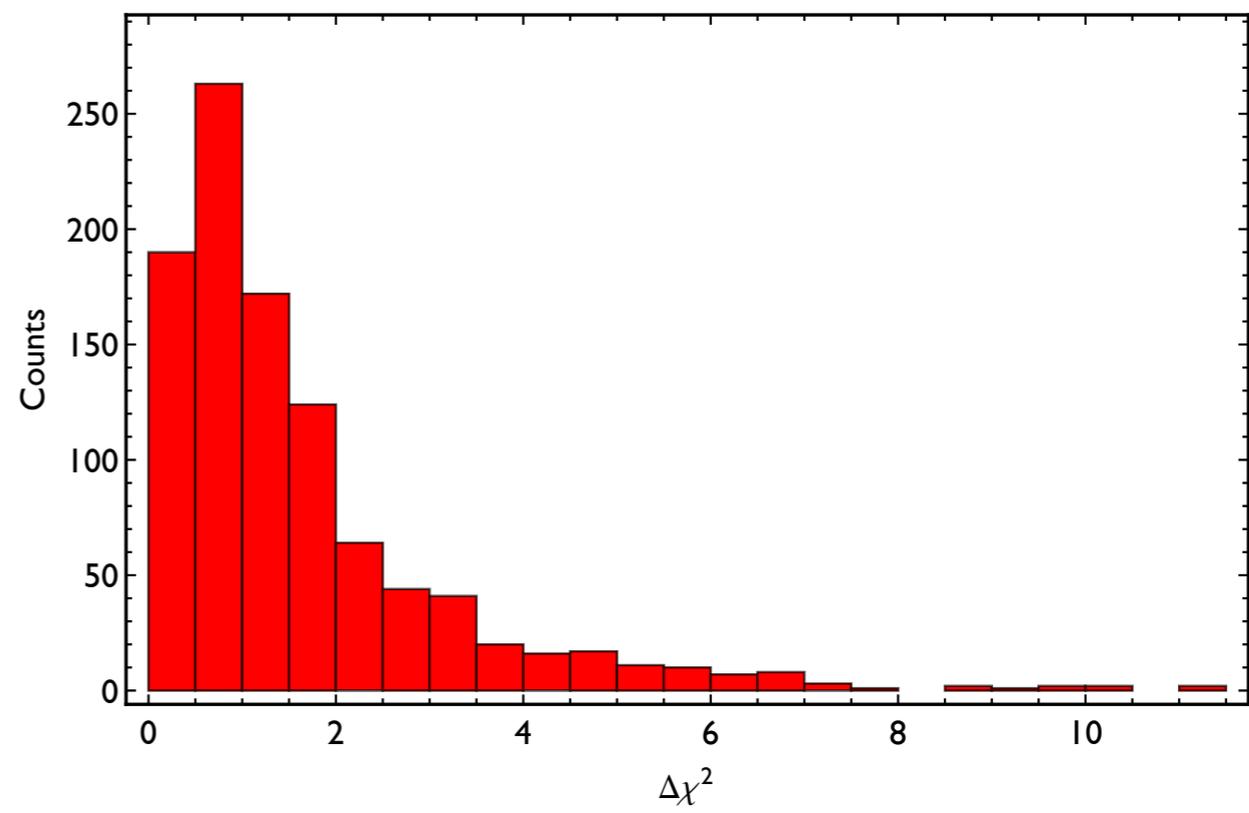
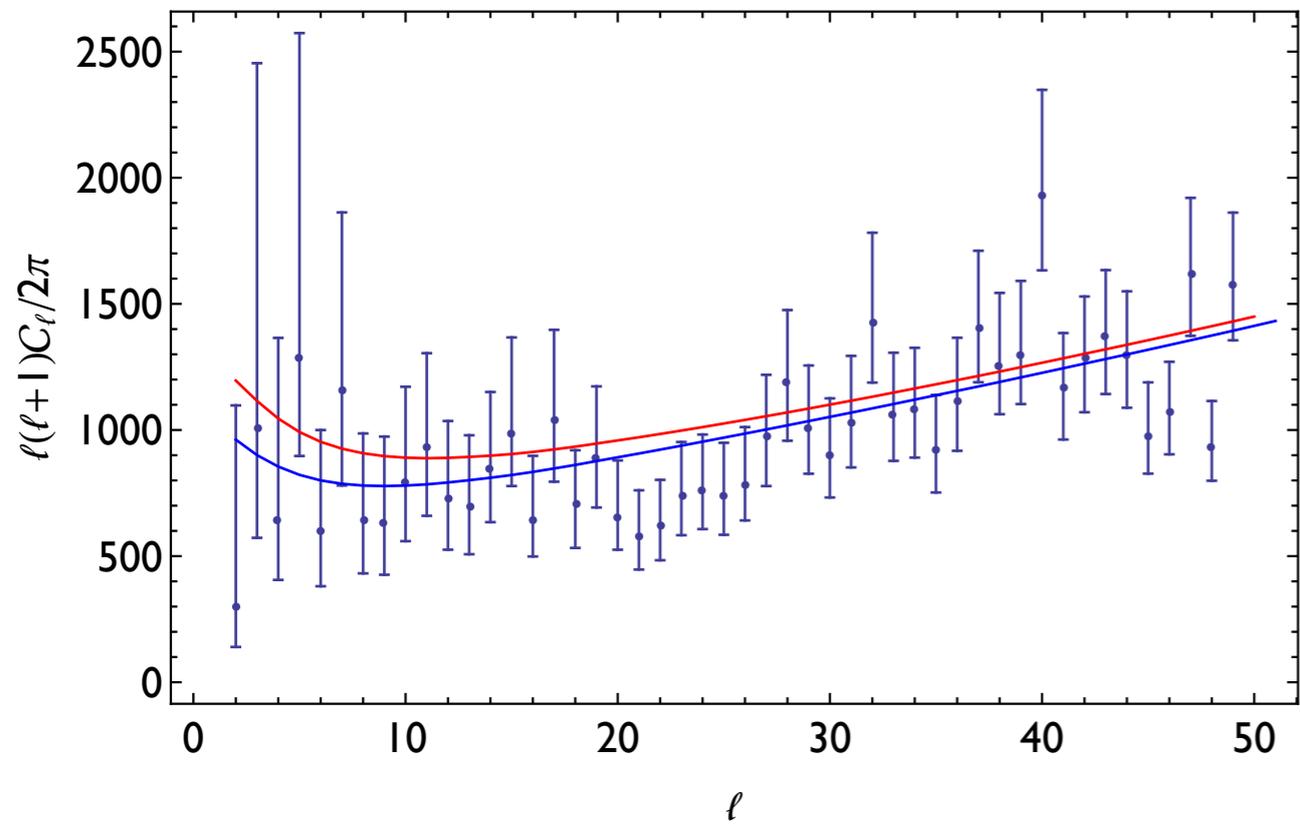
Planck+BICEP+SPT  
+ACT



component	$\Delta\chi^2$
Lowlike	$\sim -0.3$
Lensing	$\sim 0.6$
BICEP2	$\sim -3.4$
Commander	$\sim -8$
CAMspec	$\sim 1.1$
ACTSPT	$\sim -1.2$
total	$\sim -11.2$

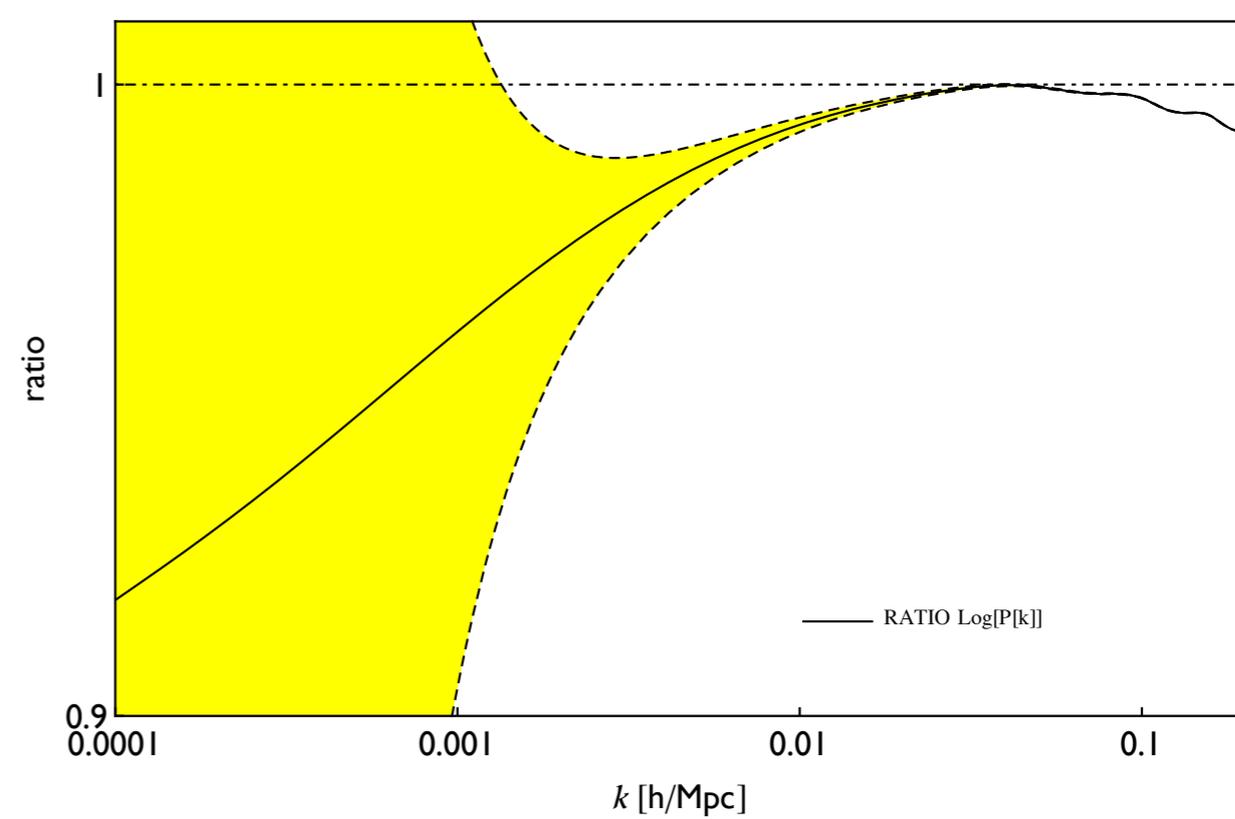
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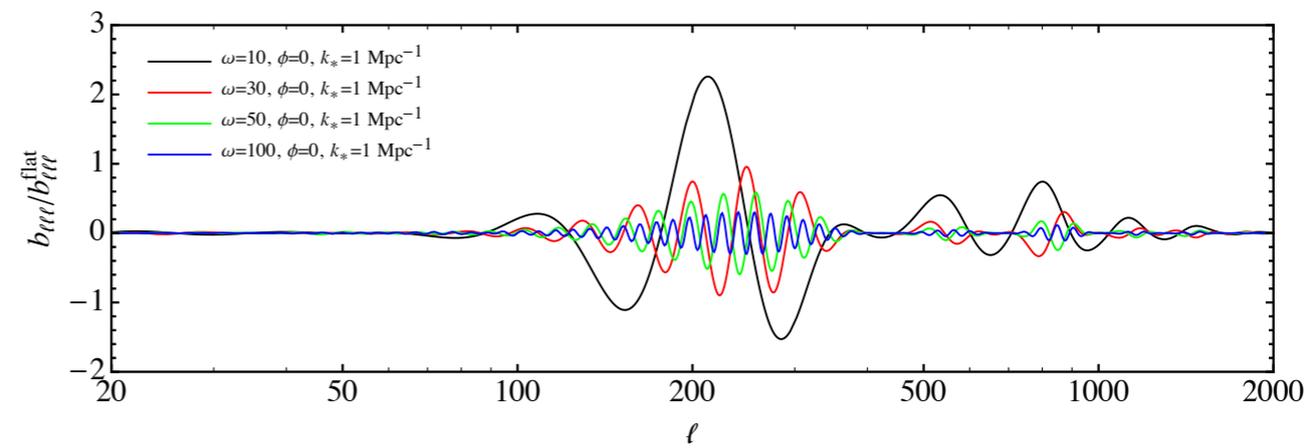
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WFIRST

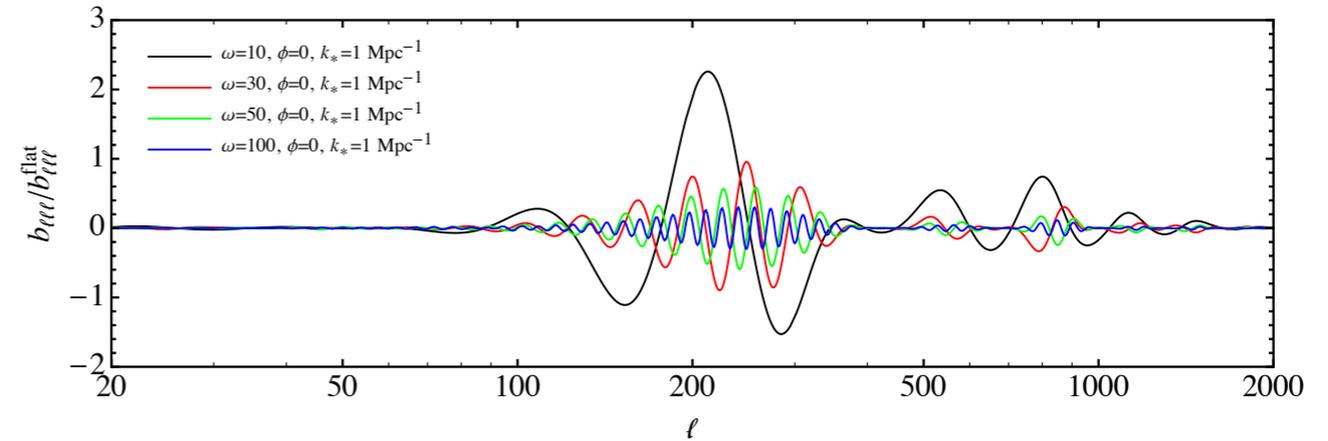


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# BISPECTRUM



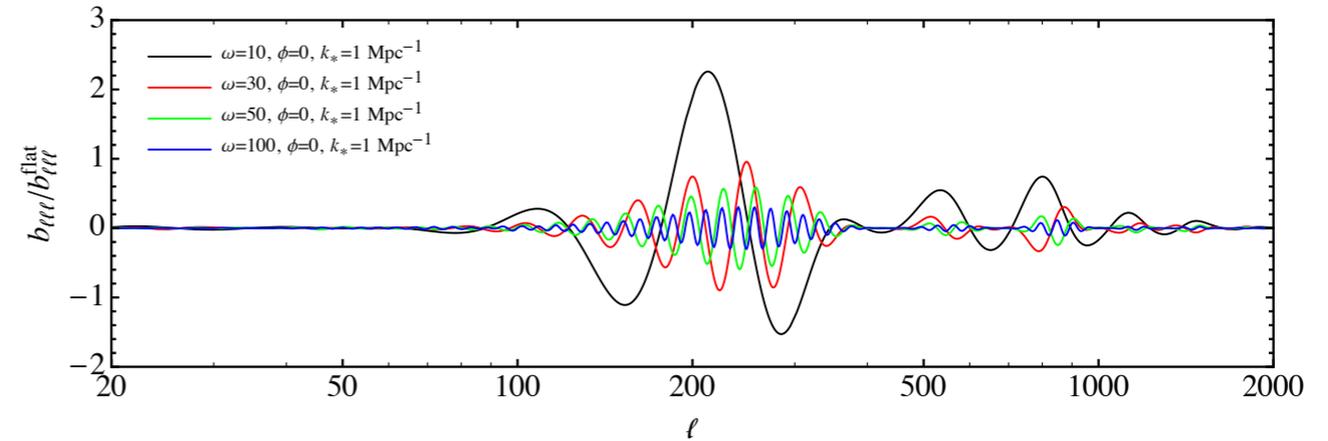
# BISPECTRUM



- Bispectrum. Difficult

$$b_{l_1 l_2 l_3} = \left(\frac{2}{\pi}\right)^3 \int x^2 dx \prod_i \int dk_i k_i^2 B(k_1, k_2, k_3)^{1/3} \Delta_{l_i}(k_i) j_{l_i}(k_i x)$$

# BISPECTRUM

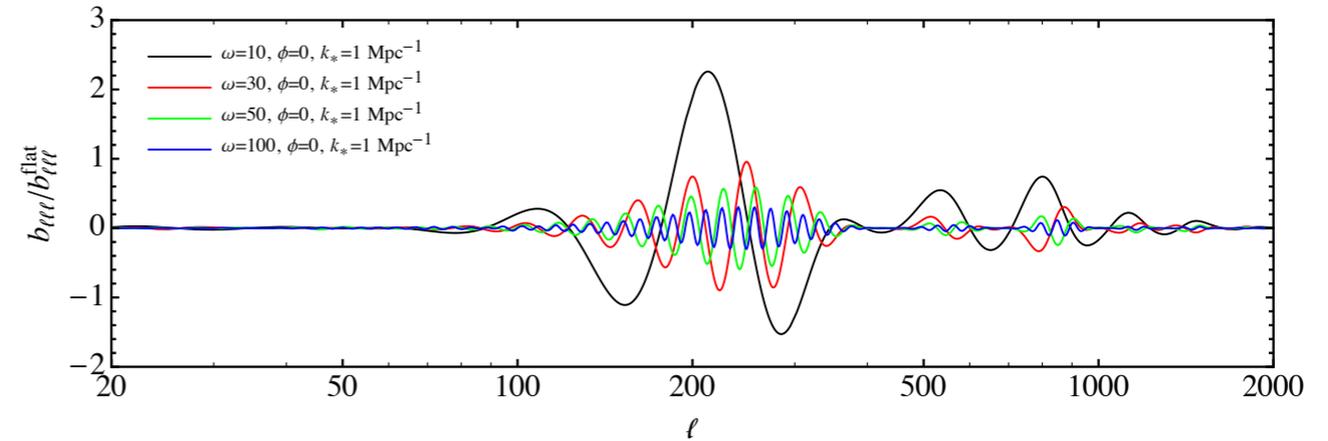


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- **Too low signal** to noise in a single mode: sum over all modes:  $\sum_{\text{all } \ell} b_{l_1 l_2 l_3} \star b_{l_1 l_2 l_3}^{\text{temp}}$

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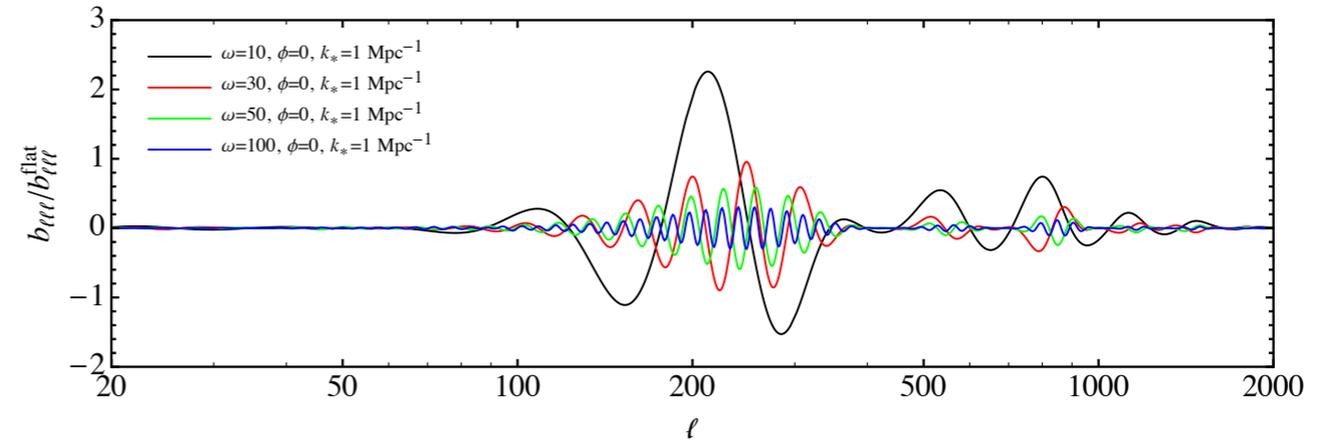


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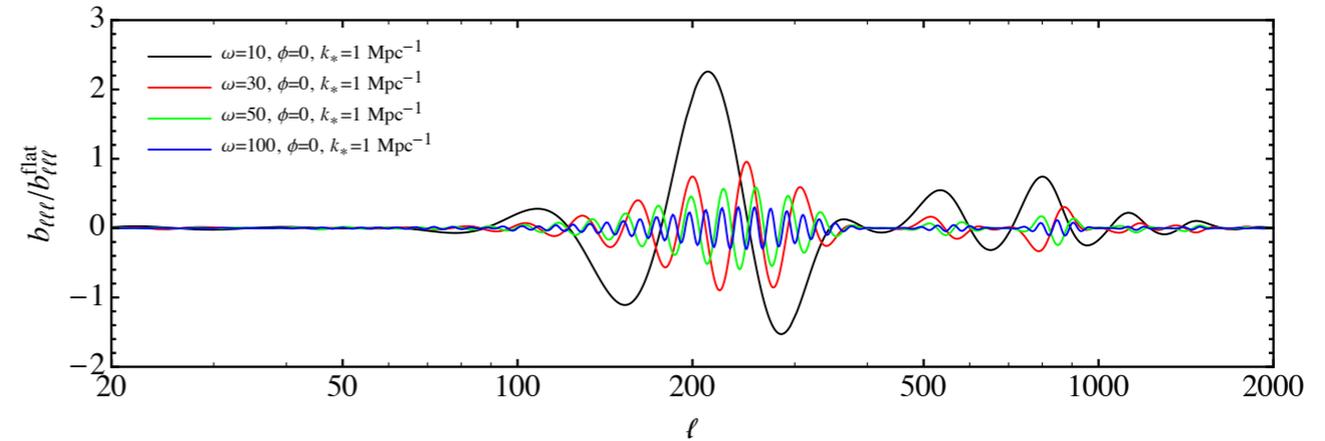


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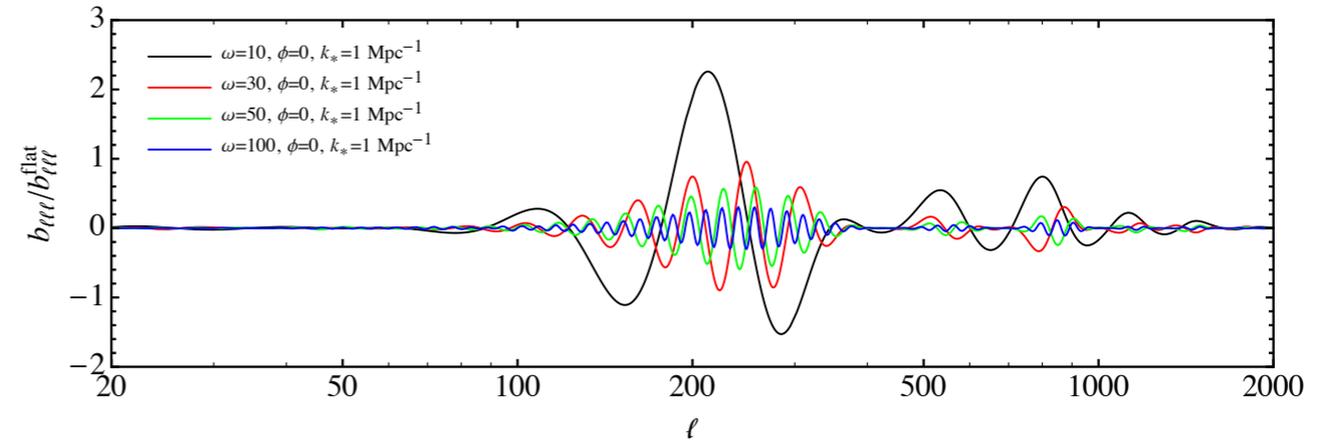


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- also, computationally, **prefer** something that is **factorizable**, because building an estimator would be super slow (i.e. KSW estimator)

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- many templates are needed for **different frequencies/phases**
- also, computationally, **prefer** something that is **factorizable**, because building an estimator would be super slow (i.e. KSW estimator)
- Using **Fourier expansion** seems to be the most efficient

# BISPECTRUM

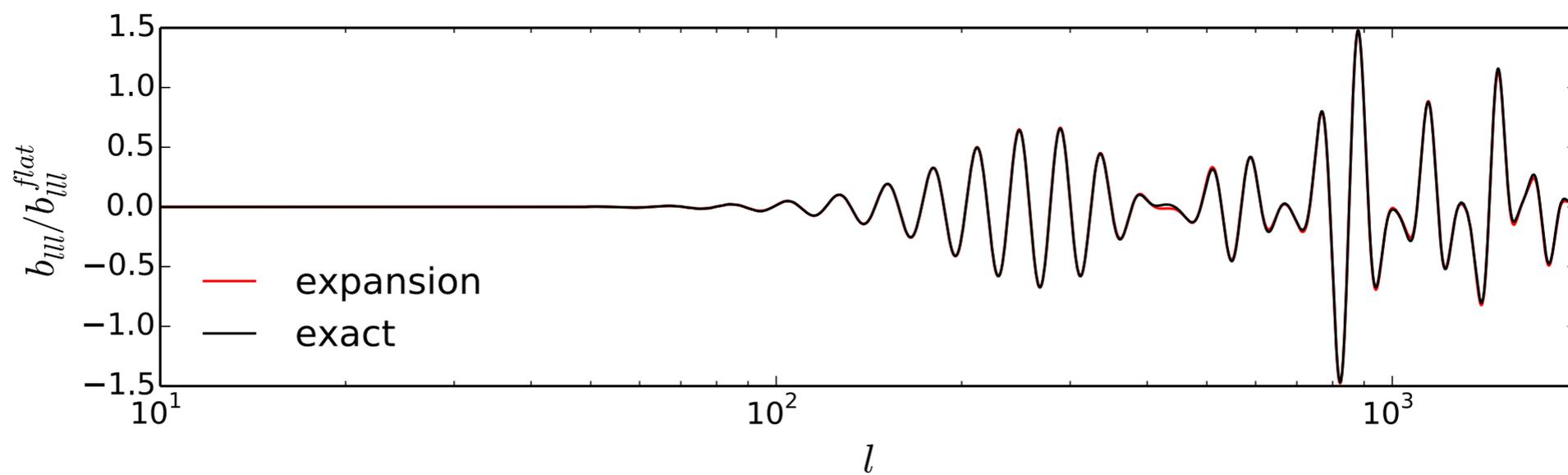
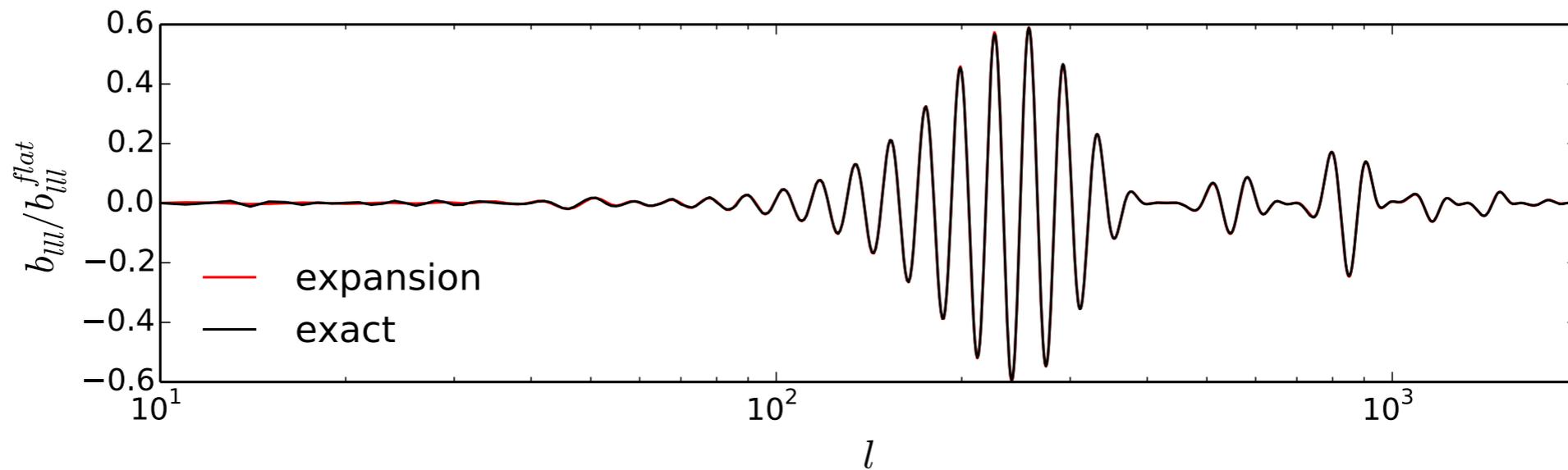
- Bispectra will have **preserved scaling argument** (log vs lin), but
  - Bispectra measure interactions, so the exact scale dependence will generally be different. Could lead to mixed effects.
  - Simplest is bispectrum of the resonant form:

$$B_{\Phi}(k_1, k_2, k_3) = \frac{6\Delta_{\Phi}^2 f_{\text{NL}}}{(k_1 k_2 k_3)^2} \sin\left(\omega \log \frac{k_t}{k_*} + \phi\right).$$

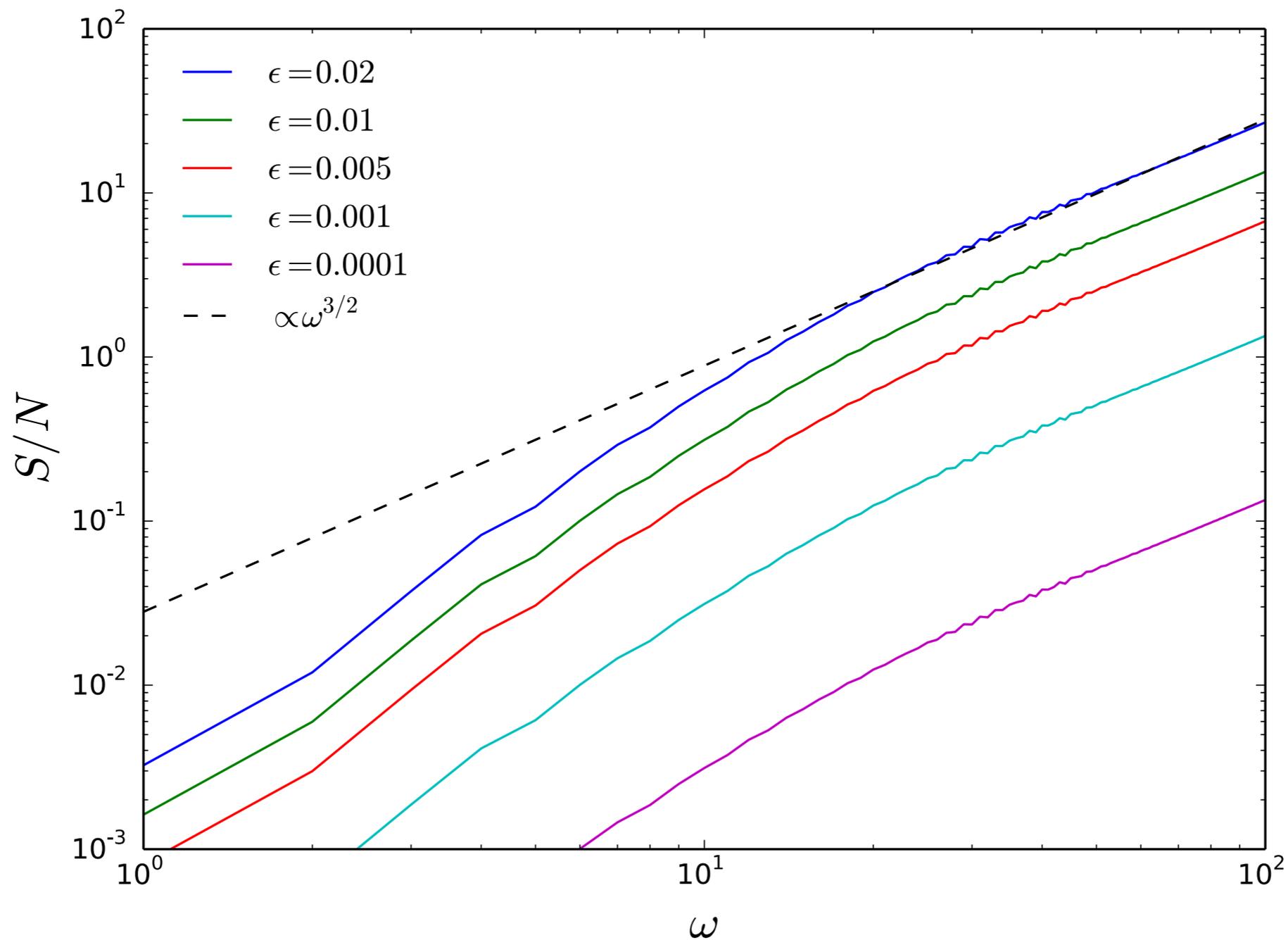
- Not factorized, several ways, most efficient way:

$$S(k_t) = \sum_{n=0}^N \left( a_n \cos \frac{2\pi n k_t}{\Delta k_t} + b_n \sin \frac{2\pi n k_t}{\Delta k_t} \right)$$

# BISPECTRUM



# BISPECTRUM



# WHAT DRIVES INFLATION?

- Motivation for features
- Results from Planck
- Discussion

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- Inflation consistency condition
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# INFLATION CONSISTENCY CONDITION

- What?
  - Relation between power in tensors and scale dependence in tensors:  $\mathbf{n}_t = -r/8$
- Why?

# INFLATION CONSISTENCY CONDITION

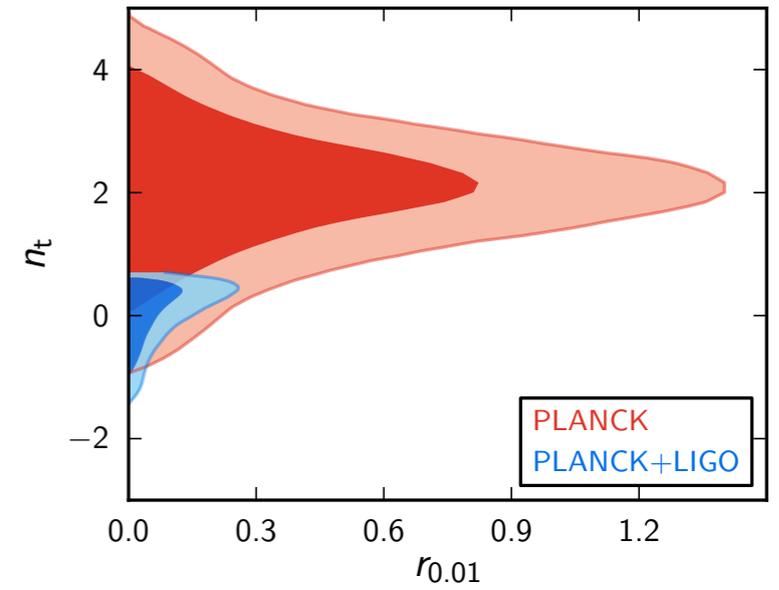
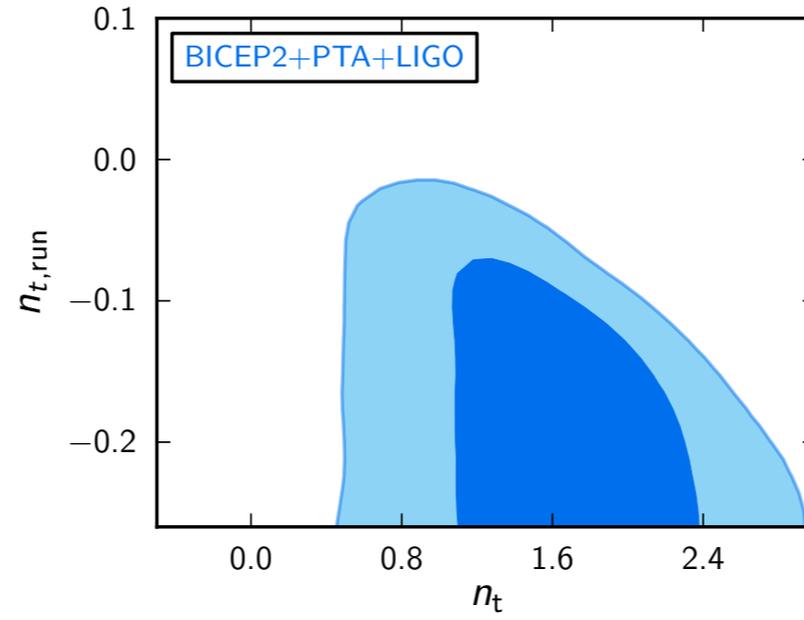
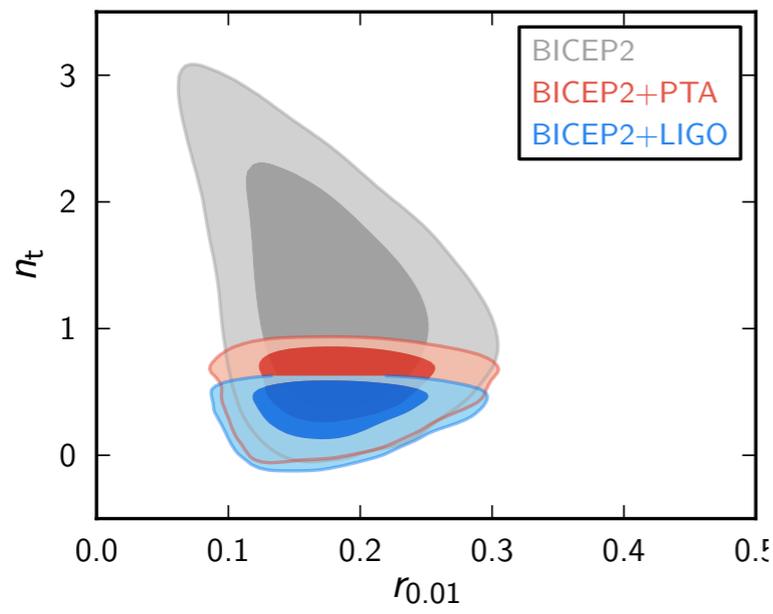
- **What?**
  - Relation between power in tensors and scale dependence in tensors:  $\mathbf{n}_t = -r/8$
- **Why?**
  - Unique **prediction from 'inflation'**. Even for more fields. At least tilt is always blue. Would violate null energy condition.
  - **Counter examples**: Ekpyrotic, or inverse decay of Gauge quanta.

# INFLATION CONSISTENCY CONDITION

- For (accurate) tilt measurement (at level of this relation), **would need other measures** (then CMB).
- Direct detection? Probably. Hard, see e.g. Dodelson (2014), Boyle et al (2014). BBO? Elisa?
- **What do we know now?**
- CMB only, CMB+LIGO+PTA, CMB++ (BBN?), -BICEP?

# RESULTS (1)

Testing **inflation consistency condition** w current data:  $n_t = -r/8$

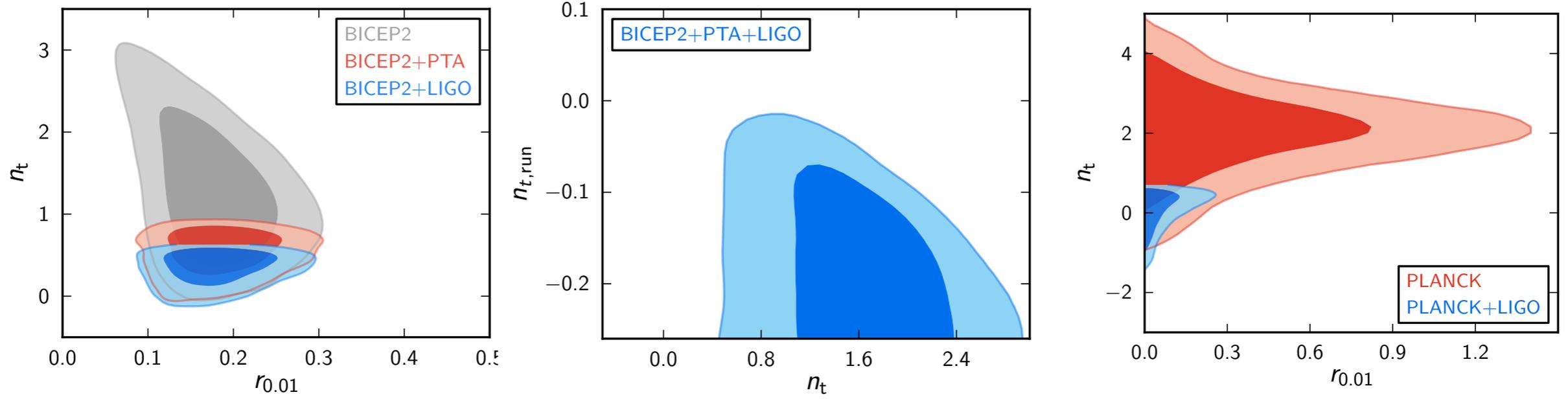


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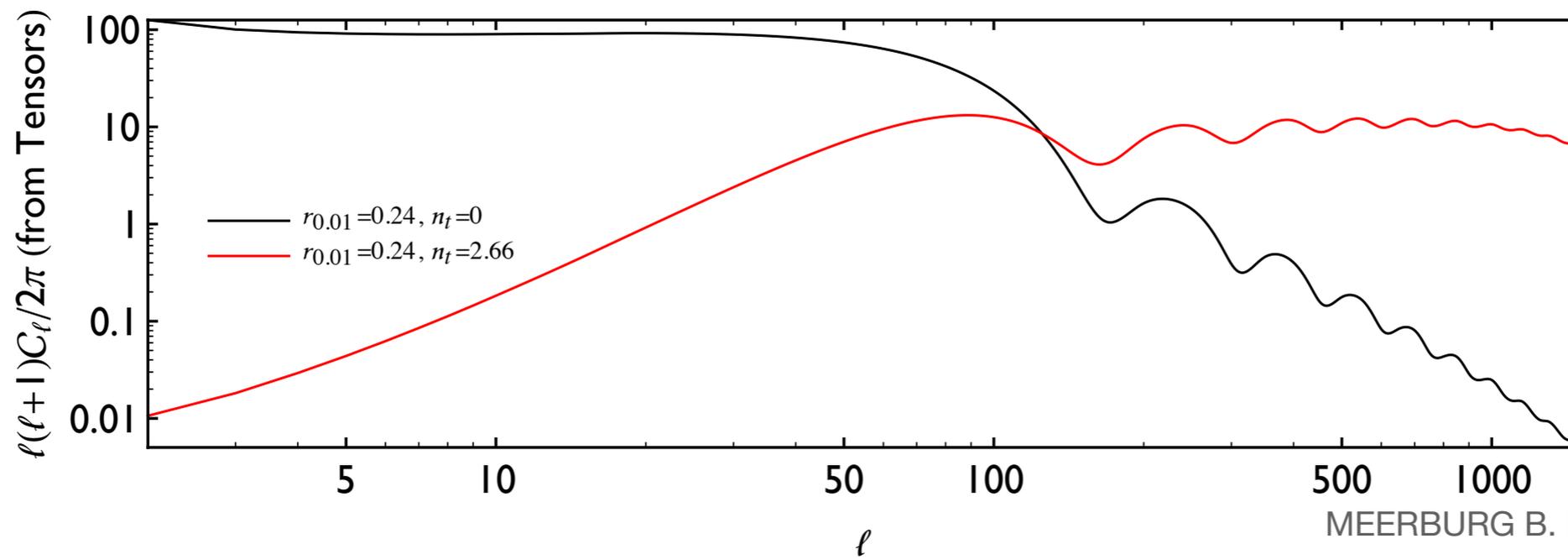
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# RESULTS (1)

Testing **inflation consistency condition** w current data:  $n_t = -r/8$



Why?



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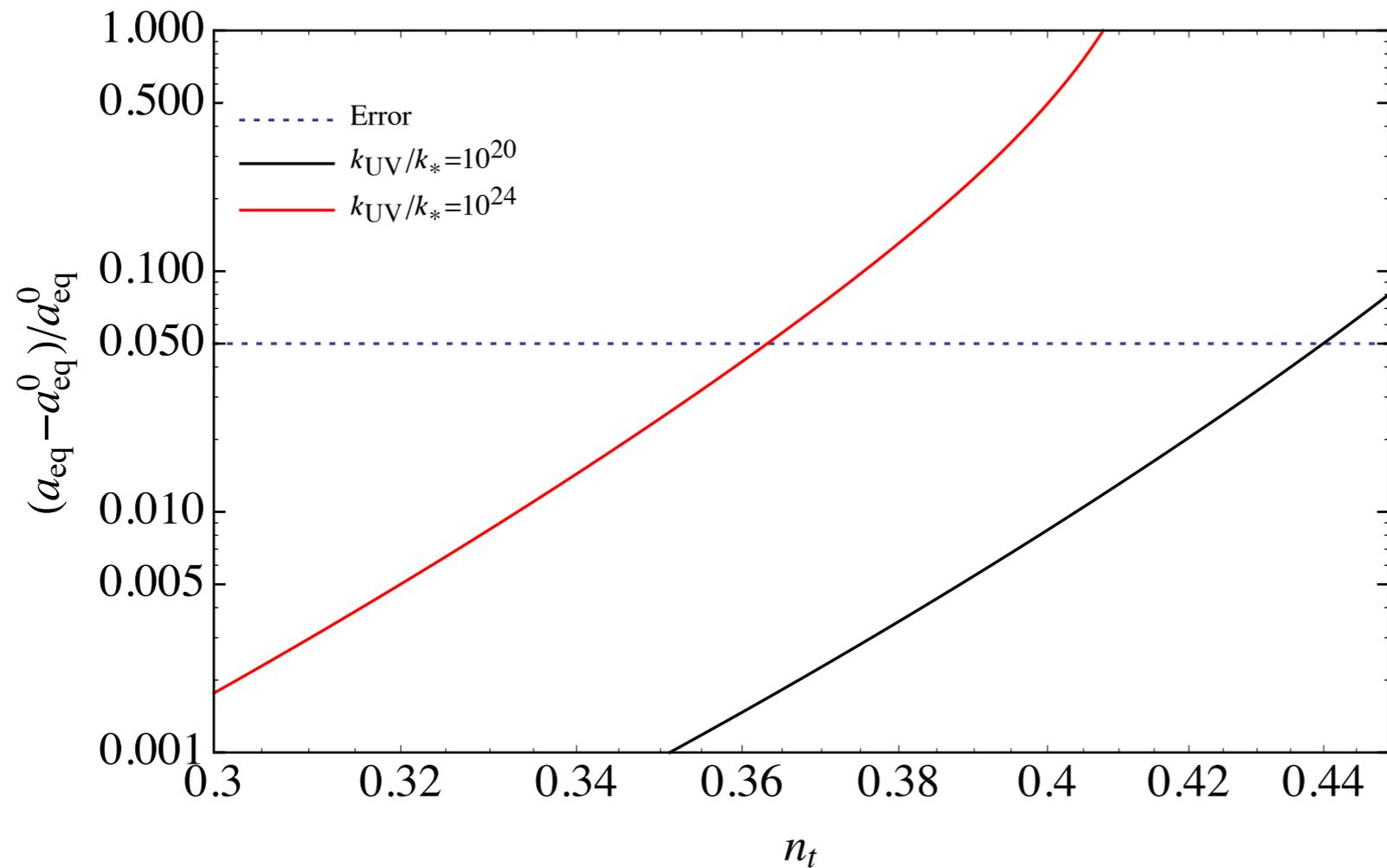
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# RESULTS (2)

What about background expansion? Too much GW's  
would change expansion history, BBN?

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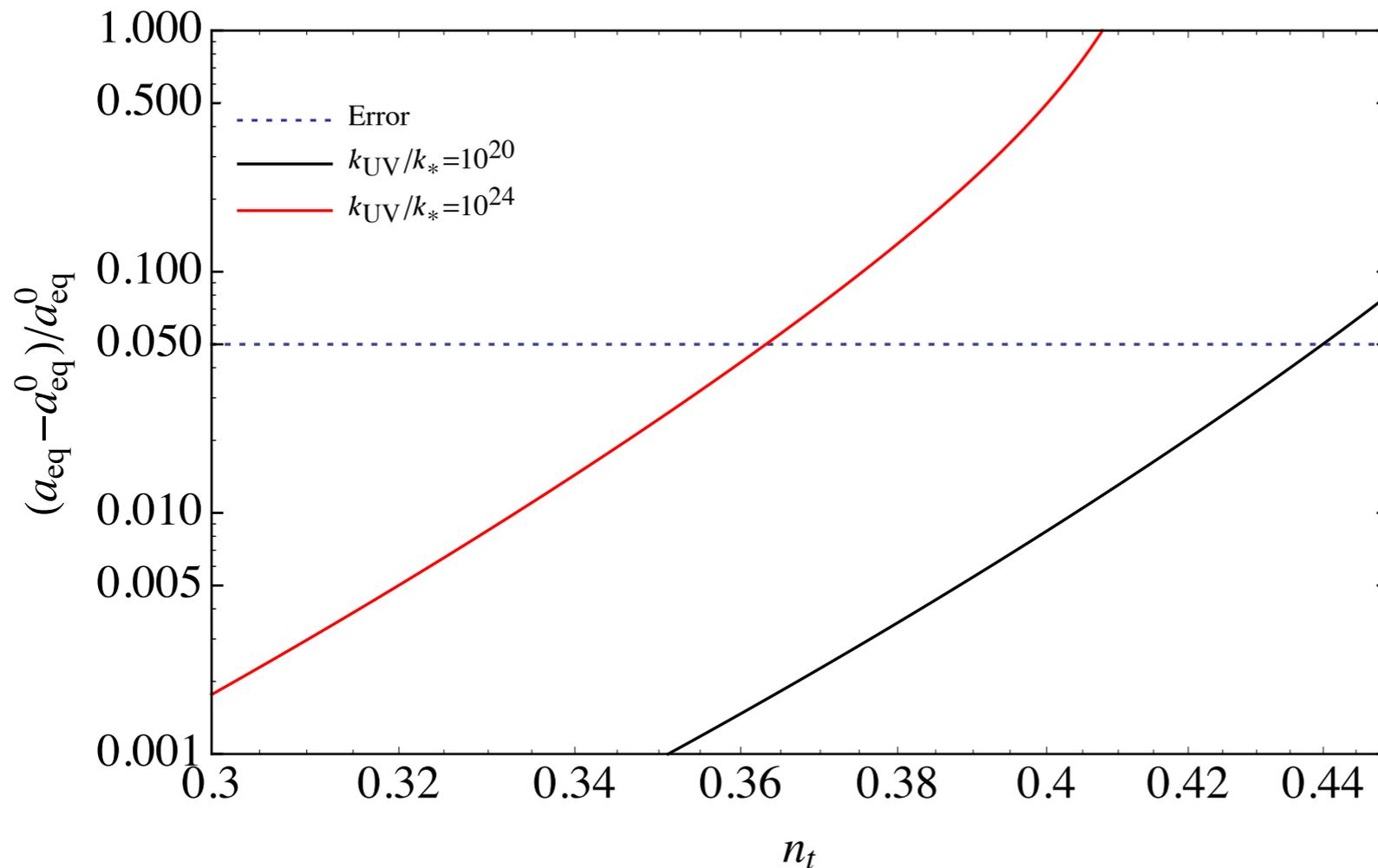


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# RESULTS (2)

What about background expansion? Too much GW's would change expansion history, BBN?



So, yes, matters. But **depends strongly on UV cutoff**. Also, above constraint should really come from Neff (which is more complicated, see e.g. Bashinsky & Seljak 2003). **Error bar is bigger**, about 10 percent w Planck 2014.

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# WHAT DRIVES INFLATION?

- Features? Somewhat natural, measurement would be profound.  
**Perturbative approach; fast and accurate**
- Log spaced oscillations:
  - WMAP 9 signature has mostly disappeared.
  - New low freq. signatures. Mild correlation with lensing amplitude.
- Linear spaced oscillations.
  - WMAP 9 and Planck are consistent
- **Are these real?** Most likely not (at 95% C.L.)
- **Bicep would favor running/long wavelength oscillation** at almost 3 sigma

# DID INFLATION HAPPEN?

- **Test inflation consistency condition**
  - LIGO and expansion history give very similar constraints
  - Near future, LIGO will be better. Long term, BBO.
- There exists some model dependence, i.e. cutoff scale etc.
- LIGO will also be dominated by mergers, so constraint is conservative.
- For (too) negative tilt, IR divergence. Is this a bigger problem (what about super horizon modes?) GW energy density no longer well defined? (only back reaction on the metric).