Modeling the Nonlinear and Redshift-Space Behavior of

BARYON ACOUSTIC OSCILLATIONS

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In collaboration with:
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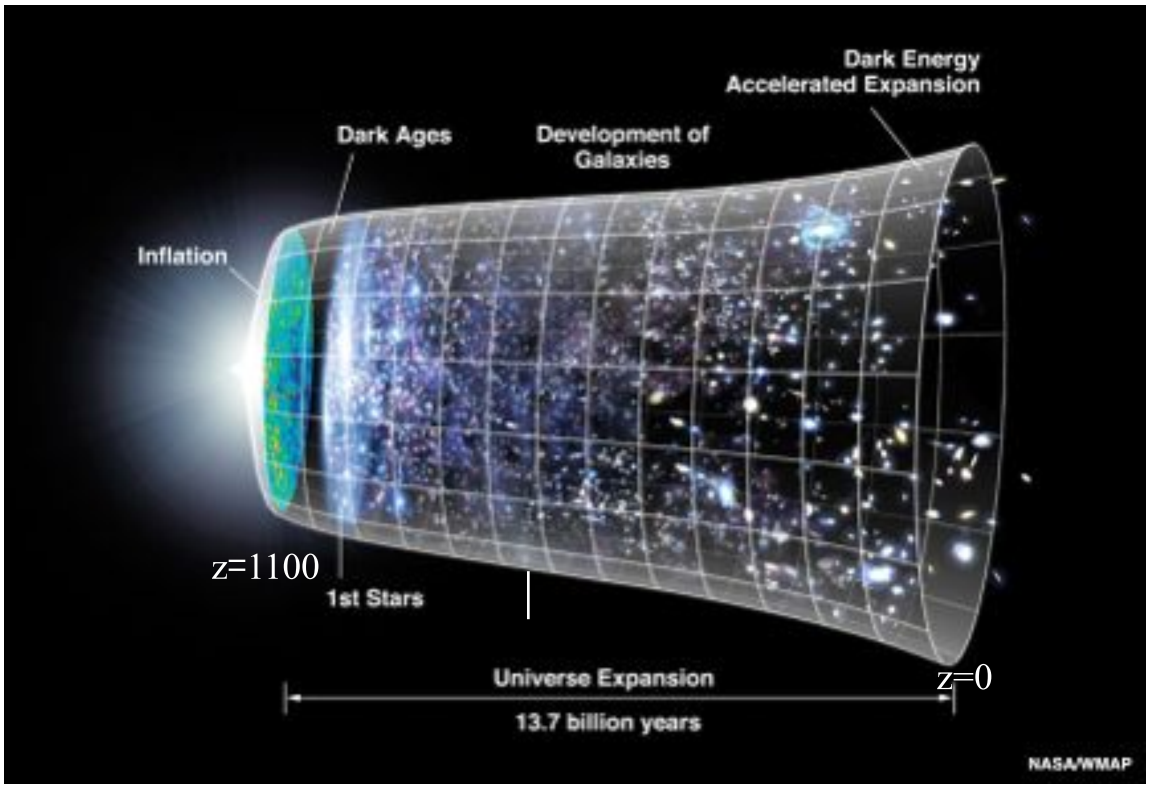
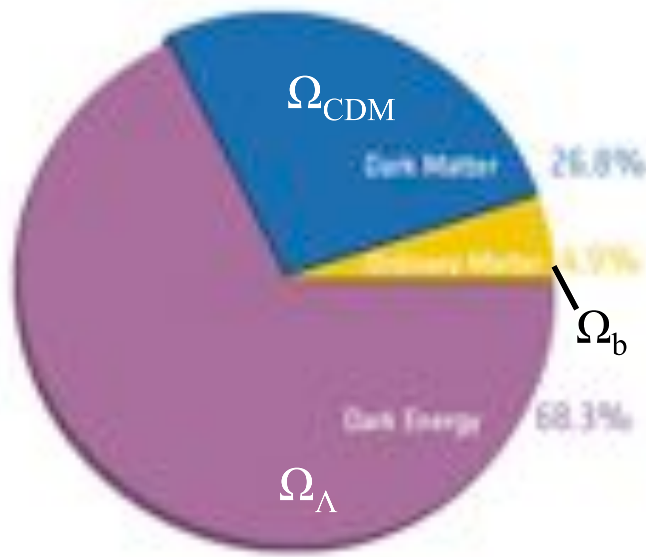
Outline

- Accelerating expansion of the Universe
  - Parameterizing dark energy
  - Cosmological distance measures
- Baryon Acoustic Oscillations (BAO) as a "standard ruler"
- Systematic Errors in BAO measurements
  - Nonlinear gravitational evolution
  - Redshift-space distortions
- New approach to perturbation theory
  - Modeling nonlinearity
  - Extension to model redshift-space distortions
- Modeling statistics of local density transformations
  - Predictions from Zel'dovich model
  - Preliminary results from tests with *N*-body simulations
- Directions for future work

Accelerated Expansion





ESA/Planck

Parameterizing Dark Energy

Simplest Model: Cosmological Constant

$$P/\rho = w = -1$$

$$H(a)^2 = H_0^2\left[\Omega_M a^{-3} + \Omega_R a^{-4} + \Omega_k a^{-2} + \Omega_\Lambda\right]$$

Constraints from *Planck*:

$$w = -1.13^{+0.24}_{-0.25} \text{ (95\%; } Planck\text{+WMAP+BAO)}$$
$$w = -1.09 \pm 0.17 \text{ (95\%; } Planck\text{+WMAP+Union2.1)}$$
$$w = -1.13^{+0.13}_{-0.14} \text{ (95\%; } Planck\text{+WMAP+SNLS)}$$
$$w = -1.24^{+0.18}_{-0.19} \text{ (95\%; } Planck\text{+WMAP+}H_0\text{)}$$

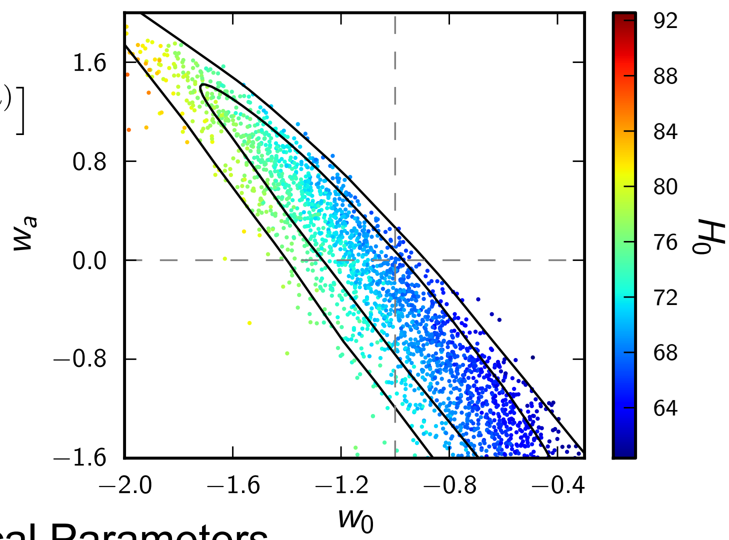
Extended Model: Time-varying Equation of State

$$w(a) = w_0 + w_a(1-a)$$
$$H(a)^2 = H_0^2\left[\Omega_M a^{-3} + \Omega_R a^{-4} + \Omega_k a^{-2} + \Omega_\Lambda a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)}\right]$$

Constraints from *Planck*:

$$w_0 = -1.04^{+0.72}_{-0.69} \quad \text{(95\%; } Planck\text{+WP+BAO),}$$
$$w_a < 1.32 \quad \text{(95\%; } Planck\text{+WP+BAO).}$$

Planck + WMAP + BAO data

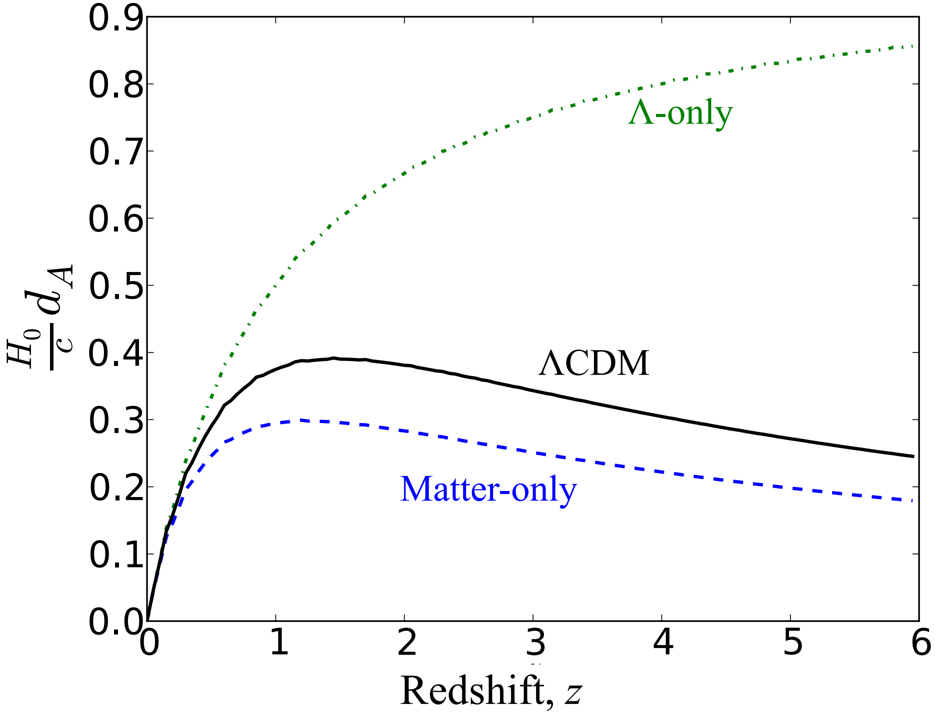


Results and figure from *Planck* 2013 results. XVI. Cosmological Parameters

Measuring Cosmological Distances

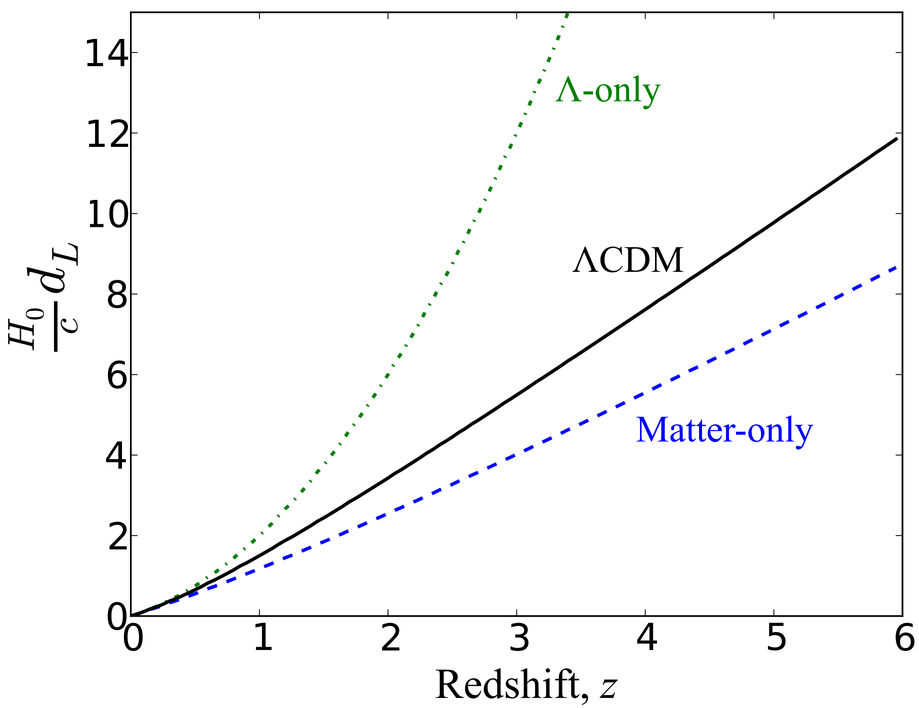
Angular Diameter Distance

$$d_A \equiv \frac{l}{\Delta\theta} = \frac{1}{1+z}\int_0^z \frac{cdz}{H(z)}$$

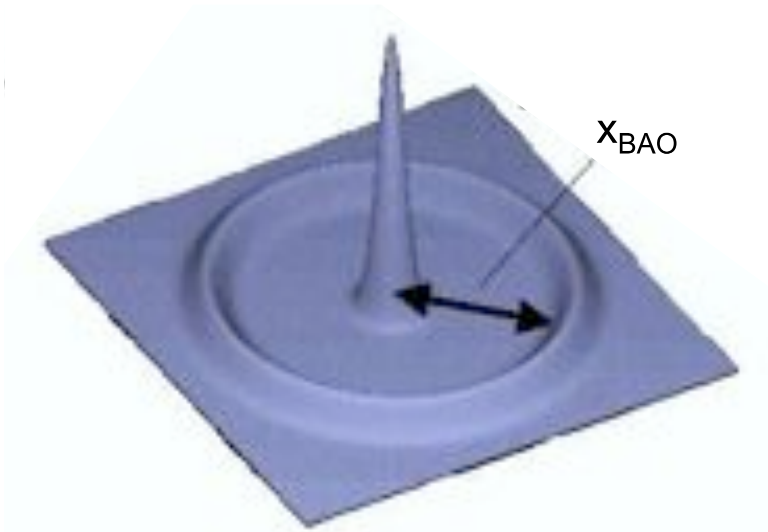


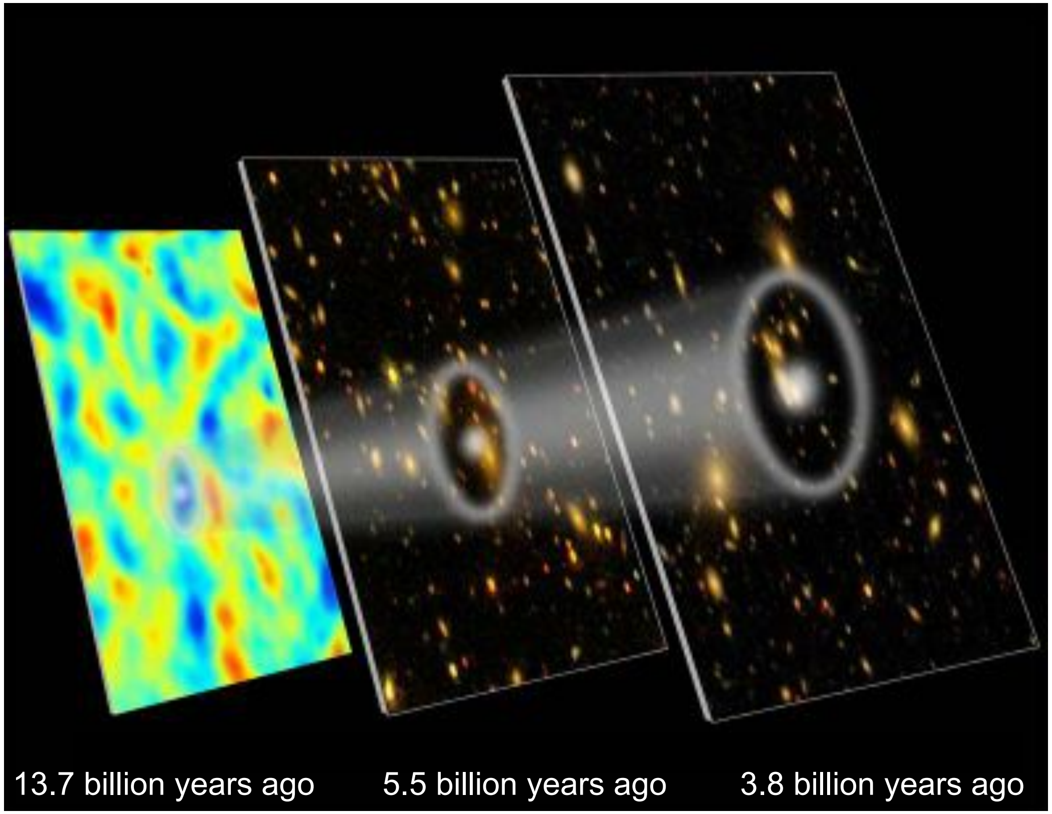
Luminosity Distance

$$d_L \equiv \sqrt{\frac{L}{4\pi F}} = (1+z)\int_0^z \frac{cdz}{H(z)}$$



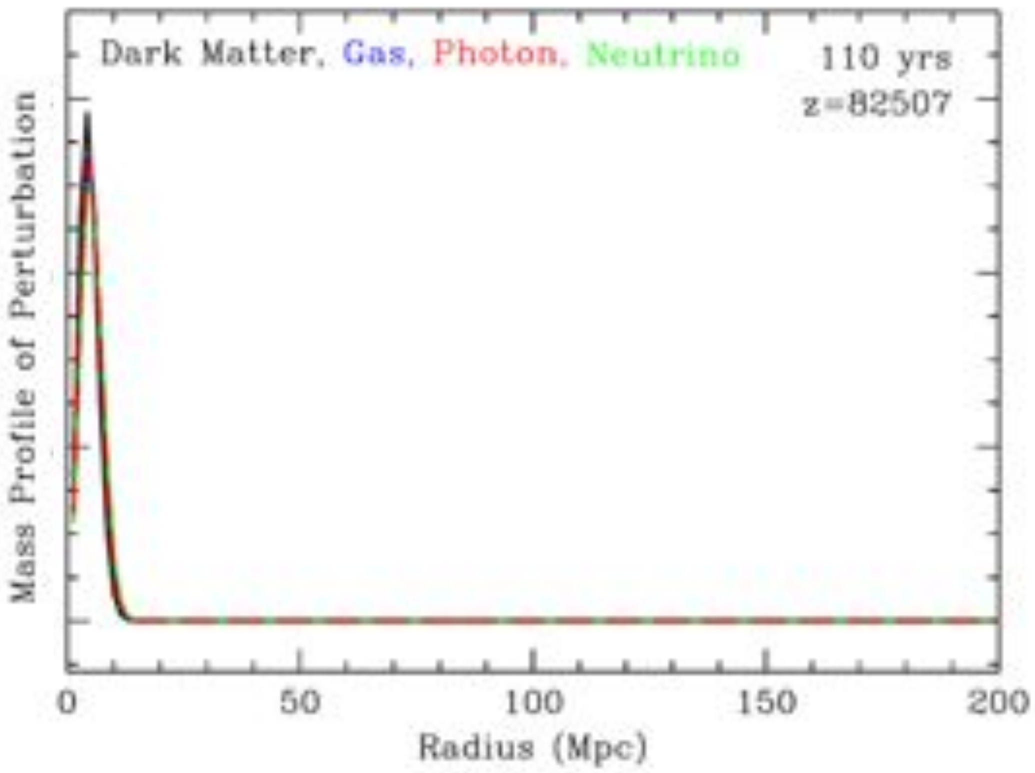
Baryon Acoustic Oscillations are a "Standard Ruler"





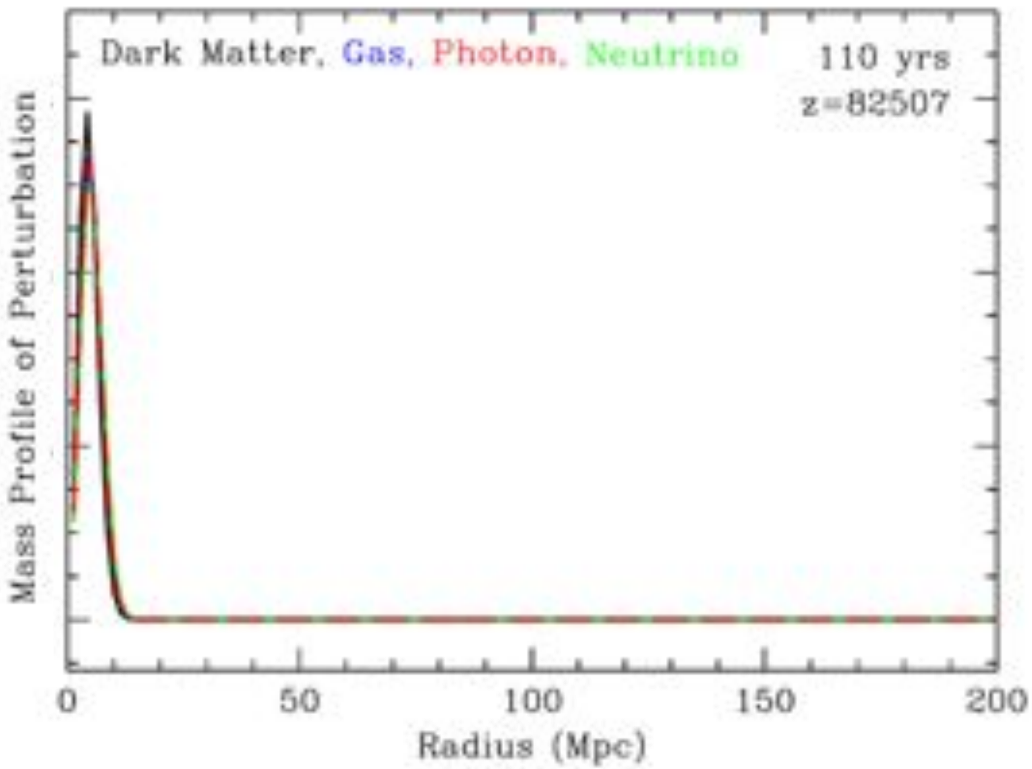
Images: SDSS-III

Baryon Acoustic Oscillations are a “Standard Ruler”



Eisenstein, http://cmb.as.arizona.edu/~eisenste/acousticpeak/

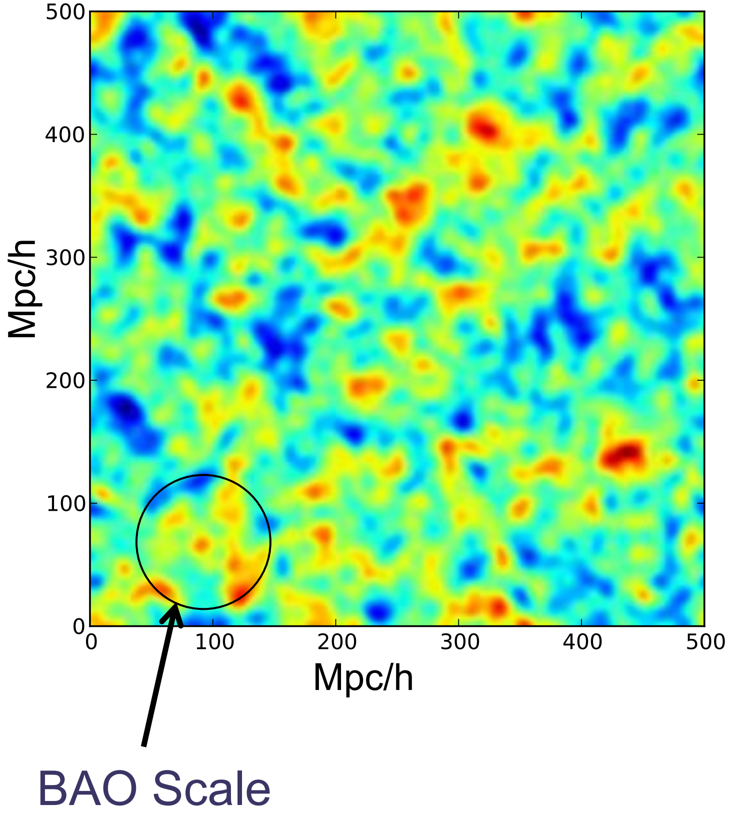
Baryon Acoustic Oscillations are a “Standard Ruler”



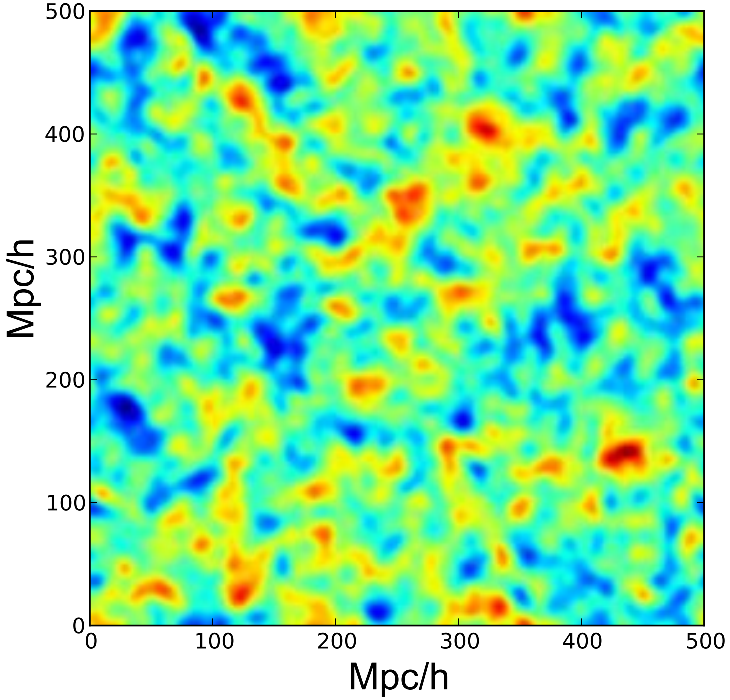
Eisenstein, http://cmb.as.arizona.edu/~eisenste/acousticpeak/

$$\delta(\mathbf{x}) \equiv \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}}$$

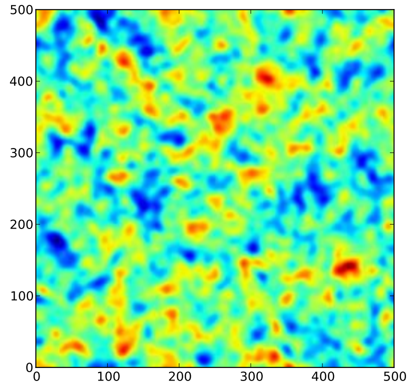
Initial over-density field with BAO



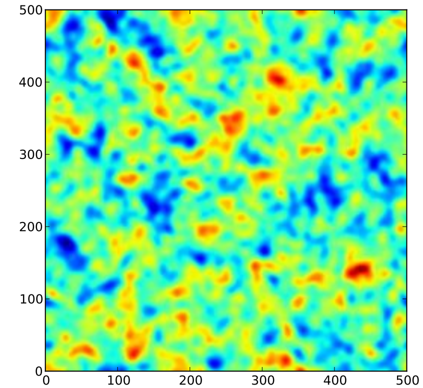
Initial over-density field without BAO



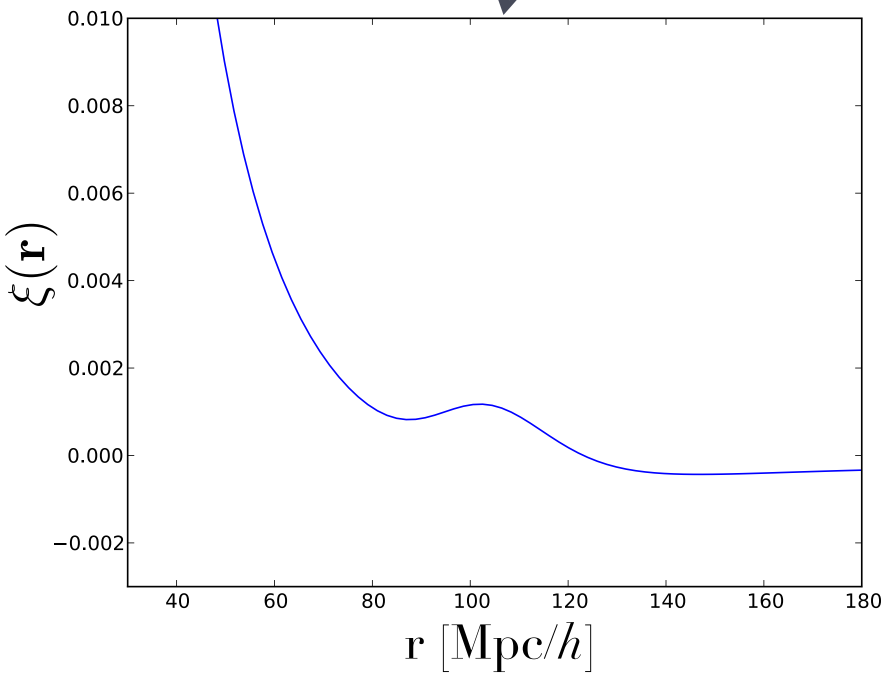
With BAO

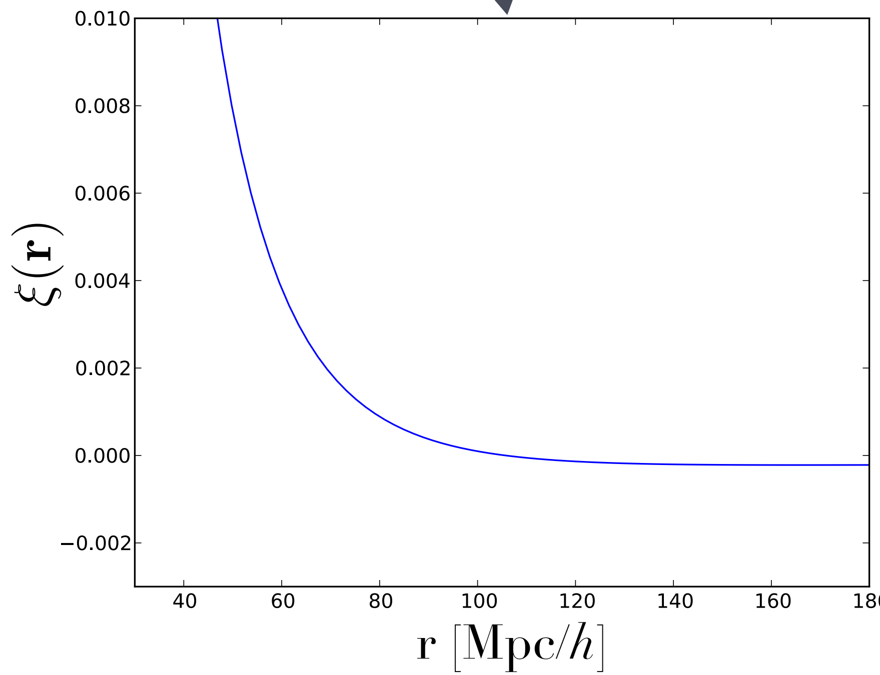


Without BAO



$$\xi(\mathbf{r}) \equiv \langle \delta(\mathbf{x}) \delta(\mathbf{x}+\mathbf{r}) \rangle$$





Baryon Acoustic Oscillations are a "Standard Ruler"

Linear Theory:

Overdensity:

$$\delta(\mathbf{x},t) = D(t)\delta_L(\mathbf{x})$$

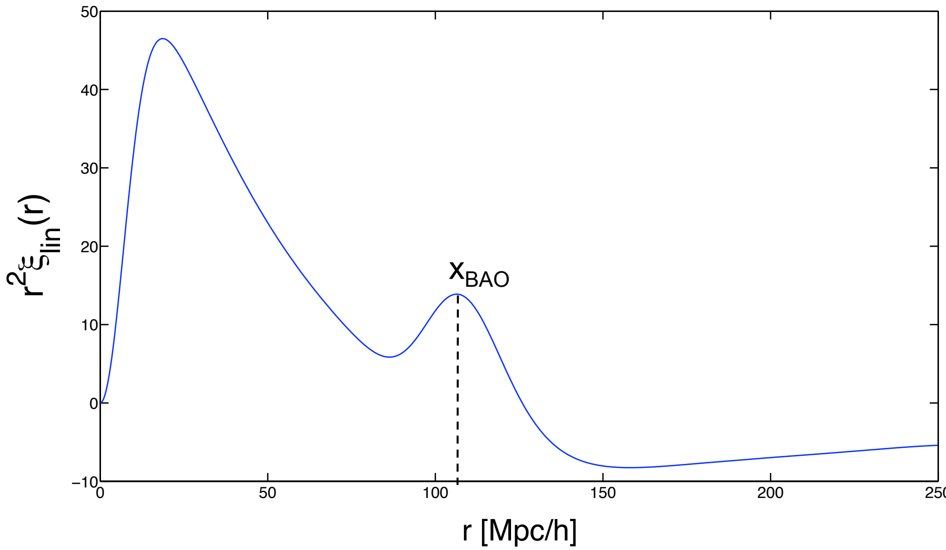
Correlation Function:

$$\xi(\mathbf{r},t) \equiv \langle \delta(\mathbf{x},t)\delta(\mathbf{x}+\mathbf{r},t)\rangle = D(t)^2\xi_L(r)$$

Power Spectrum:

$$P(k,t) = D(t)^2 P_L(k)$$

Linear Correlation Function



$$d_A = \frac{x_{BAO}}{\Delta\theta} = \frac{1}{1+z}\int_0^z \frac{c\,dz}{H(z)}$$

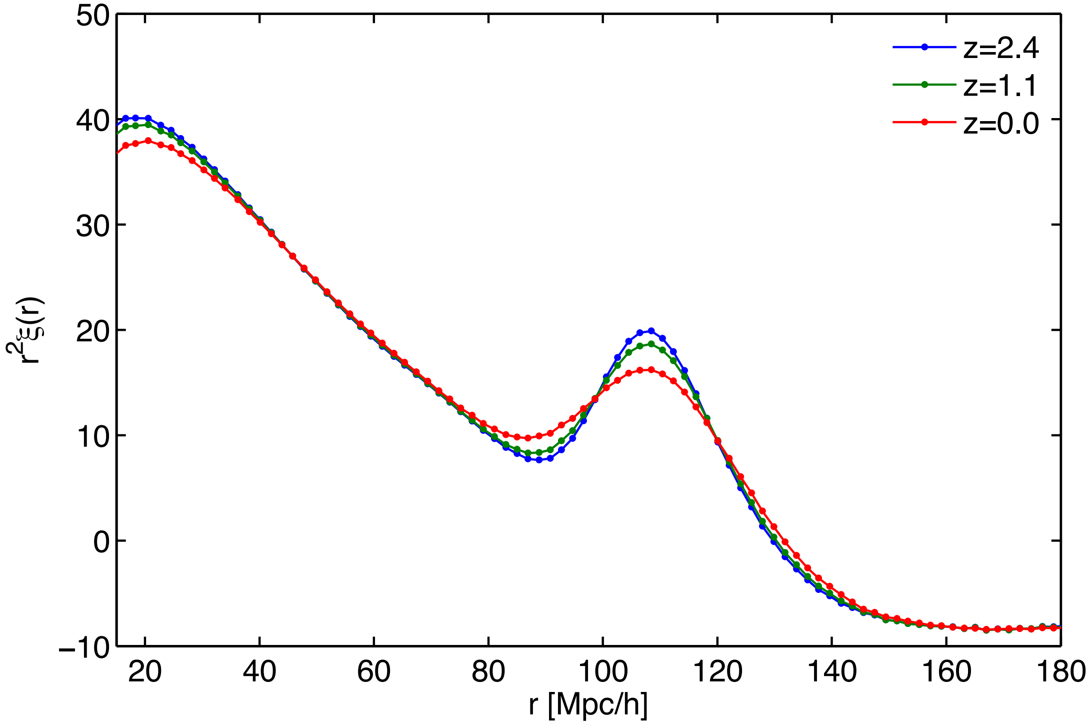
$H(z)$ → Dark Energy Parameters

BAO Systematic Uncertainties

- Nonlinear gravitational evolution
- Redshift-space distortions
- Scale-dependent galaxy bias

BAO Systematic Uncertainties

- Nonlinear gravitational evolution
- Redshift-space distortions



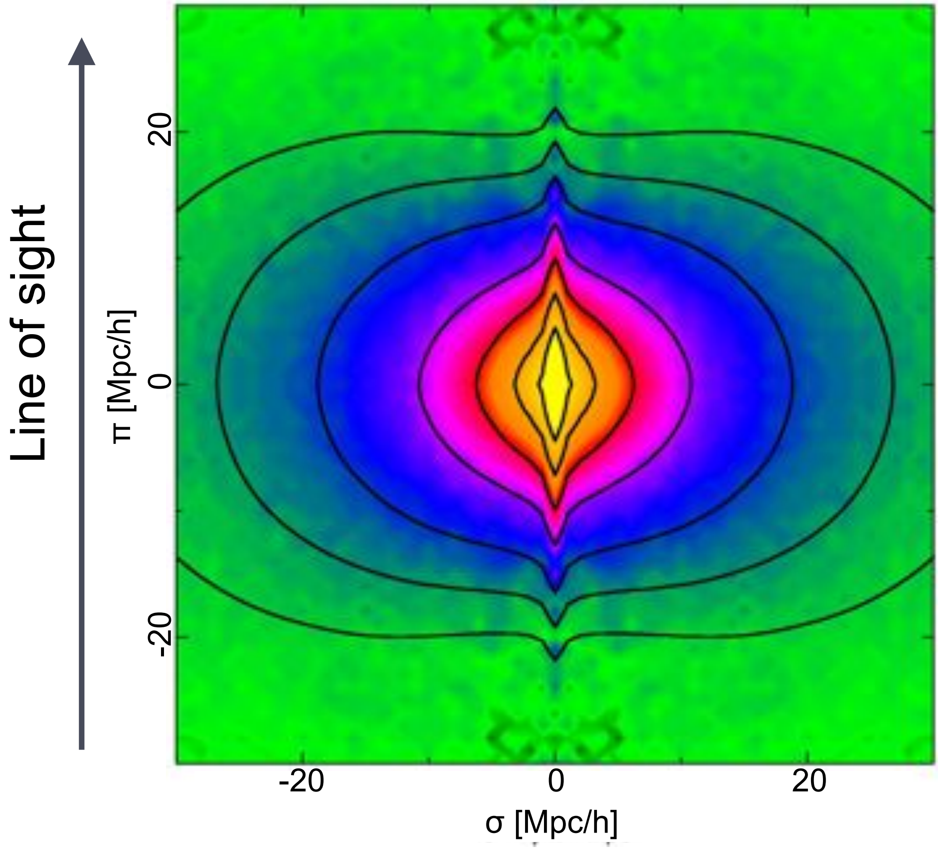
Nonlinearity leads to a smoothing and shifting of BAO peak at low redshift

BAO Systematic Uncertainties

- Nonlinear gravitational evolution
- Redshift-space distortions

  - Redshift measured from Doppler shift, used to calculate distance
    - Galaxies are not at rest in comoving frame
  - Linear in-fall on large scales
    - Flattening of correlations
  - Thermal motion on small scales
    - 'Fingers of God'

Hawkins et al. (2002), astro-ph/0212375
2dFGRS: $\beta=0.49\pm0.09$



Modeling Nonlinearity: Standard Perturbation Theory

Perturbative solution to the fluid equations in Fourier space:

$$\hat{\delta}(\mathbf{k},t) = \sum_{n=1}^{\infty} D(t)^n \hat{\delta}^{(n)}(\mathbf{k})$$

$$P_{22}(k) = 2\int d^3q\, P_{11}(q)\, P_{11}(|\vec{k}-\vec{q}|)\left[F_2^{(s)}(\vec{q},\vec{k}-\vec{q})\right]^2$$

$$F_2^{(s)}(\vec{k}_1,\vec{k}_2) = \frac{5}{7} + \frac{2}{7}\frac{(\vec{k}_1\cdot\vec{k}_2)^2}{k_1^2 k_2^2} + \frac{(\vec{k}_1\cdot\vec{k}_2)}{2}\left(\frac{1}{k_1^2}+\frac{1}{k_2^2}\right)$$

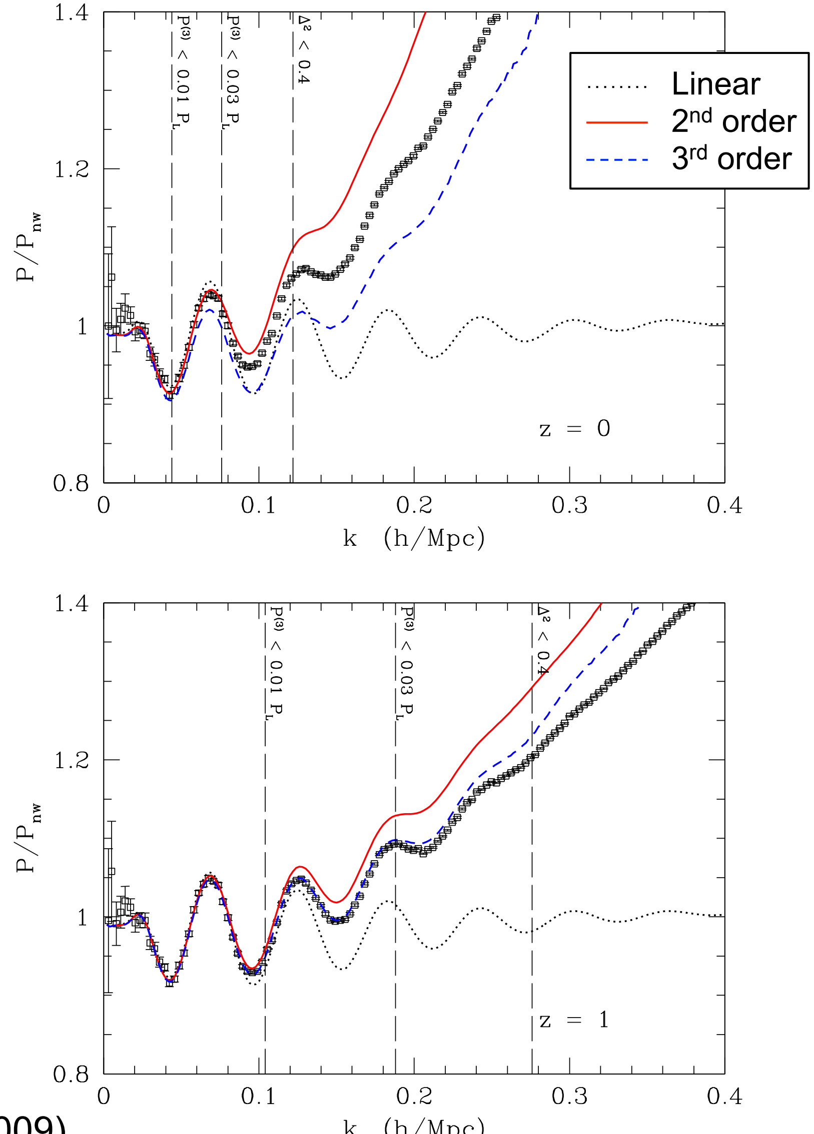


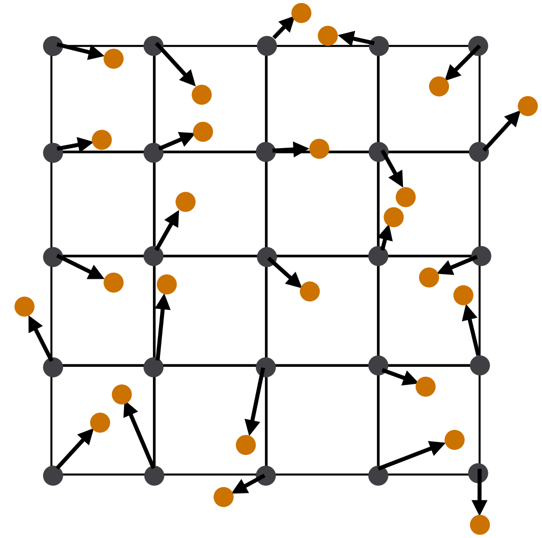
Figure: Carlson, White, Padmanabhan, arXiv:0905.0497 (2009)

New Approach: Motivation

- Structure of the Fourier space kernels suggests that in configuration space, the result may be simpler
- Terms beyond 2^{nd} order may be simplified in configuration space compared to Fourier space
- Configuration space can be more easily extended to redshift space

New Approach: Method

$$\boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},\tau) = \boldsymbol{x}(\tau)$$



LPT:

$$\boldsymbol{x}(\tau) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},\tau).$$

$$\boldsymbol{\Psi}(\boldsymbol{q},\tau) = \boldsymbol{\Psi}^{(1)}(\boldsymbol{q},\tau) + \boldsymbol{\Psi}^{(2)}(\boldsymbol{q},\tau) + \cdots .$$

1st order Lagrangian perturbation theory (Zel'dovich approximation):

1LPT: $\mathbf{x}(\mathbf{q},t) = \mathbf{q} - D(t)\vec{\nabla}\phi(\mathbf{q})$

Poisson: $\nabla^2\Phi(\mathbf{q}) = 4\pi G\bar{\rho}\delta_L(\mathbf{q})$

$4\pi G\bar{\rho}\phi(\mathbf{q}) \equiv \Phi(\mathbf{q})$

$$\frac{\rho(\mathbf{x},t)}{\bar{\rho}} = \left|\frac{\partial x_i}{\partial q_j}\right|^{-1} = \frac{1}{J(\mathbf{q},t)} = 1 + \delta(\mathbf{q}(\mathbf{x}))$$

New Approach: Method

Jacobian in terms of invariants of deformation tensor:

$$J(\mathbf{q},t) = 1 - D I_1(\mathbf{q}) + D^2 I_2(\mathbf{q}) - D^3 I_3(\mathbf{q})$$

$$d_{ij}(\mathbf{q}) = \frac{\partial^2 \phi(\mathbf{q})}{\partial q_i \partial q_j}$$

$$\delta_q(\mathbf{q},t) = D I_1(\mathbf{q}) + D^2 \left(I_1(\mathbf{q})^2 - I_2(\mathbf{q})\right) + D^3 \left(I_1(\mathbf{q})^3 - 2 I_1(\mathbf{q}) I_2(\mathbf{q}) + I_3(\mathbf{q})\right) + \ldots$$

Coordinate transformation from ***q*** to ***x***:

$$\mathbf{q}(\mathbf{x}) = \mathbf{x} + D\, \nabla_q \phi(\mathbf{q}(\mathbf{x}))$$

$$\delta_x(\mathbf{x},t) \equiv \delta_q(\mathbf{q},t) = \delta_q\left(\mathbf{x} + D \nabla_q \phi(\mathbf{q}(\mathbf{x}))\right)$$

Expansion of the density in terms of linear quantities:

$$\delta(\mathbf{x},t) = \left(\delta(\mathbf{q},t) + D \sum_i \frac{\partial \phi(\mathbf{q})}{\partial q_i} \frac{\partial \delta(\mathbf{q},t)}{\partial q_i} + D^2 \sum_{i,j} \frac{\partial^2 \phi(\mathbf{q})}{\partial q_i \partial q_j} \frac{\partial \phi(\mathbf{q})}{\partial q_j} \frac{\partial \delta(\mathbf{q},t)}{\partial q_i} + \frac{1}{2} D^2 \sum_{i,j} \frac{\partial^2 \delta(\mathbf{q},t)}{\partial q_i \partial q_j} \frac{\partial \phi(\mathbf{q})}{\partial q_i} \frac{\partial \phi(\mathbf{q})}{\partial q_j} \right)\Bigg|_{\mathbf{q}=\mathbf{x}} .$$

McCullagh & Szalay. ApJ, 752, 21 (2012)

New Approach: Method

Perturbatively calculate the nonlinear correlation function:

$$\xi(\mathbf{r},t) \equiv \langle \delta(\mathbf{x},t)\delta(\mathbf{x}+\mathbf{r},t)\rangle = \xi^{(1)}(\mathbf{r})D^2 + \xi^{(2)}(\mathbf{r})D^4 + ...$$

We use Wick's theorem for expectation values of Gaussian quantities:

$$\langle a(\mathbf{x}_1)b(\mathbf{x}_2)c(\mathbf{x}_3)d(\mathbf{x}_4)\rangle = \langle a(\mathbf{x}_1)b(\mathbf{x}_2)\rangle\langle c(\mathbf{x}_3)d(\mathbf{x}_4)\rangle + \langle a(\mathbf{x}_1)c(\mathbf{x}_3)\rangle\langle b(\mathbf{x}_2)d(\mathbf{x}_4)\rangle + \langle a(\mathbf{x}_1)d(\mathbf{x}_4)\rangle\langle b(\mathbf{x}_2)c(\mathbf{x}_3)\rangle$$

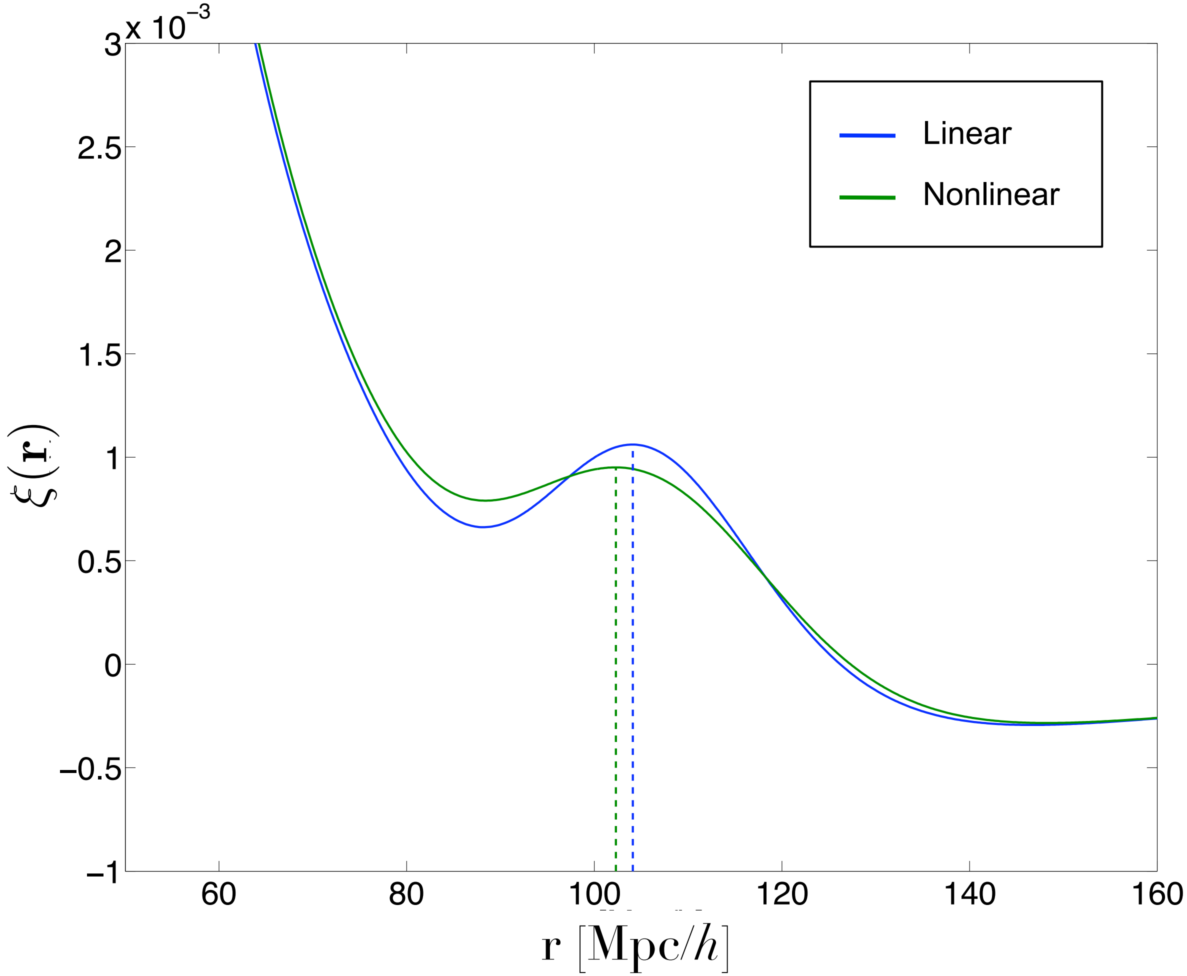
First nonlinear contribution to the correlation function in terms of initial quantities:

$$\xi^{(2)}(\mathbf{r}) = -\frac{1}{3}\xi_0^{-2}(0)\xi_0^2(r) + \frac{19}{15}\xi_0^0(r)^2 + \frac{34}{21}\xi_2^0(r)^2 + \frac{4}{35}\xi_4^0(r)^2 - \frac{16}{5}\xi_1^{-1}(r)\xi_1^1(r) - \frac{4}{5}\xi_3^{-1}(r)\xi_3^1(r) + \frac{1}{3}\xi_0^{-2}(r)\xi_0^2(r) + \frac{2}{3}\xi_2^{-2}(r)\xi_2^2(r)$$

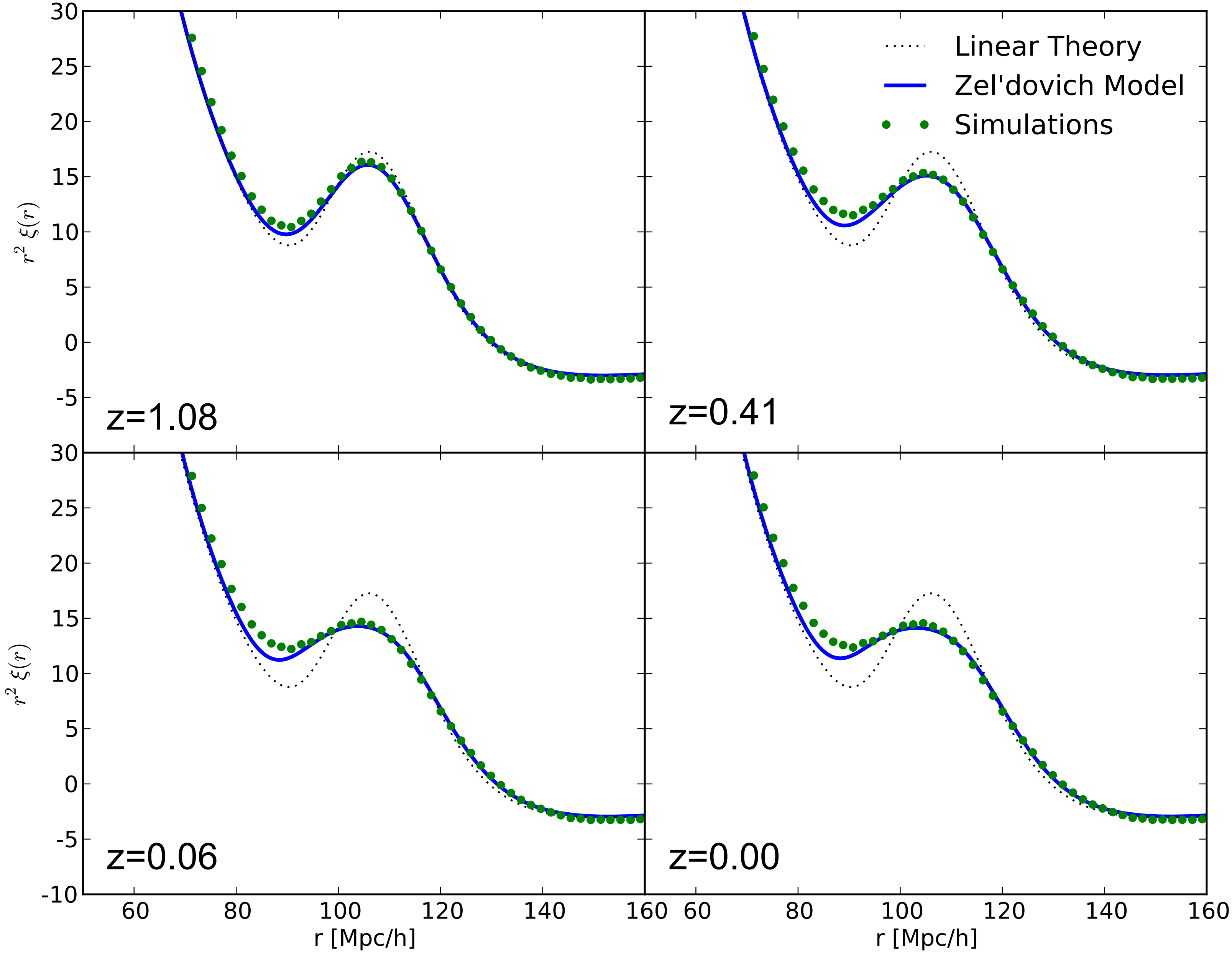
Where: $\xi_n^m(r) = \frac{1}{2\pi^2}\int P_L(k)j_n(kr)k^{m+2}dk$

McCullagh & Szalay. ApJ, 752, 21 (2012)

New Approach: Results



In the Zel'dovich model, at z=0, the peak is damped by about 10% and shifted to lower radius by ~1%



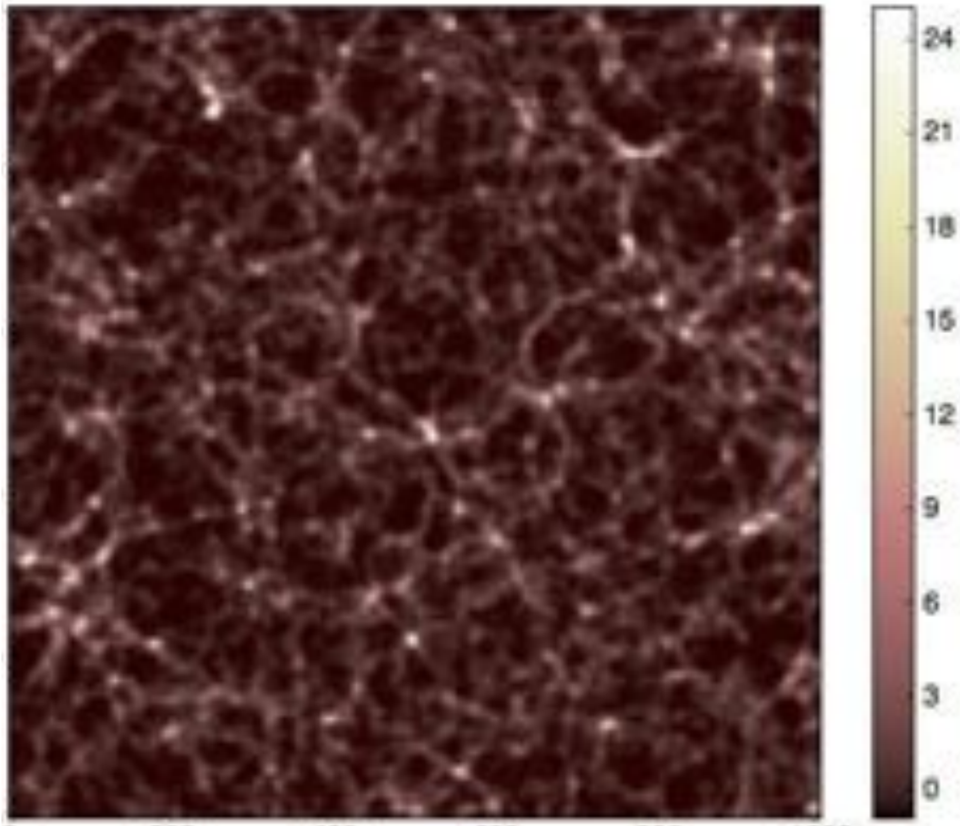
77 Indra simulations

T. Budavári, S. Cole, D. Crankshaw, L. Dobos, B. Falck, A. Jenkins, G. Lemson, M. Neyrinck, A. Szalay, and J. Wang

Real Space vs Redshift Space: Density

Line of sight

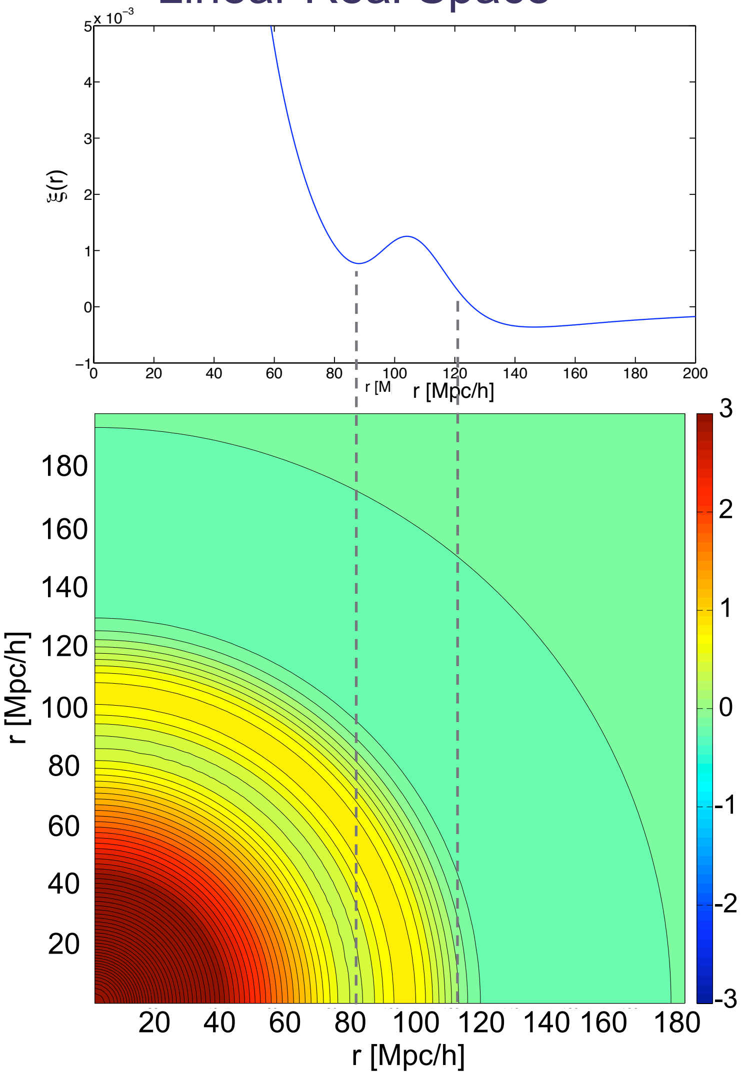
500 Mpc/*h*



500 Mpc/*h*

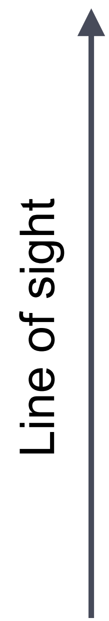
Real Space vs Redshift Space: Correlation Function

Linear Real Space

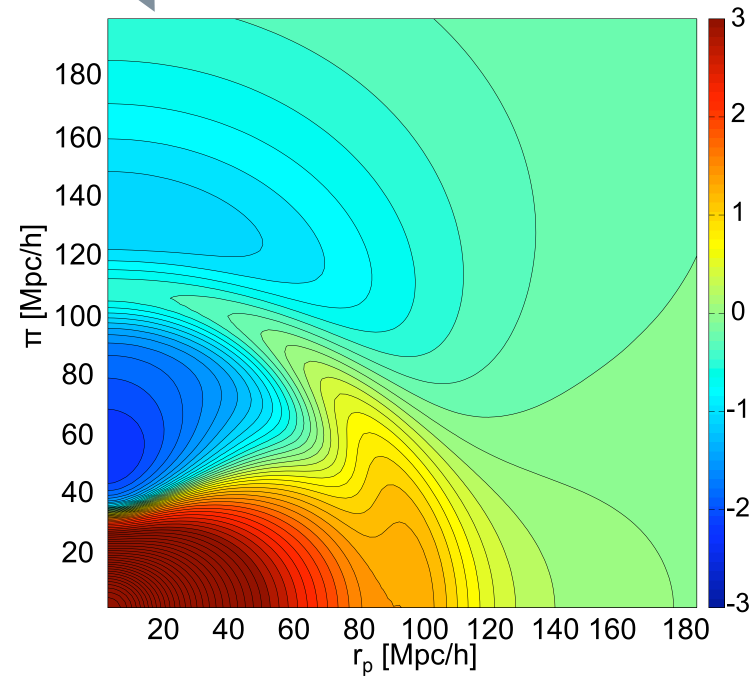


$$f \equiv \frac{d \ln D}{d \ln a}$$

$$\xi^{(1)}(\mathbf{s}) = \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right)\xi_0^0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mathcal{P}_2(\mu)\xi_2^0(s) + \frac{8f^2}{35}\mathcal{P}_4(\mu)\xi_4^0(s) \ ,$$



Linear Redshift Space

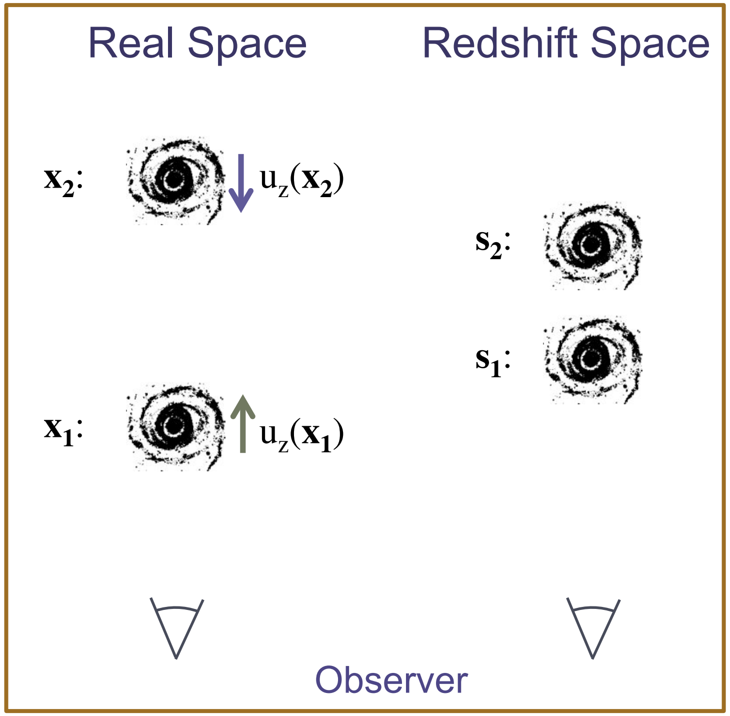


Redshift-space Correlation Function

Transformation to redshift-space coordinates:

$$\mathbf{s} = \mathbf{x} - u_z(\mathbf{x})\hat{\mathbf{z}}$$

$$u_z(\mathbf{x}(\mathbf{q})) = fD\nabla_q \phi(\mathbf{q}) \cdot \hat{n}$$



Redshift-space overdensity in terms of the Jacobian:

$$\frac{\rho(\mathbf{s},t)}{\bar{\rho}} = \left|\frac{\partial s_i}{\partial q_j}\right|^{-1} = \frac{1}{J(\mathbf{q},t)} = 1 + \delta(\mathbf{q}(\mathbf{s}))$$

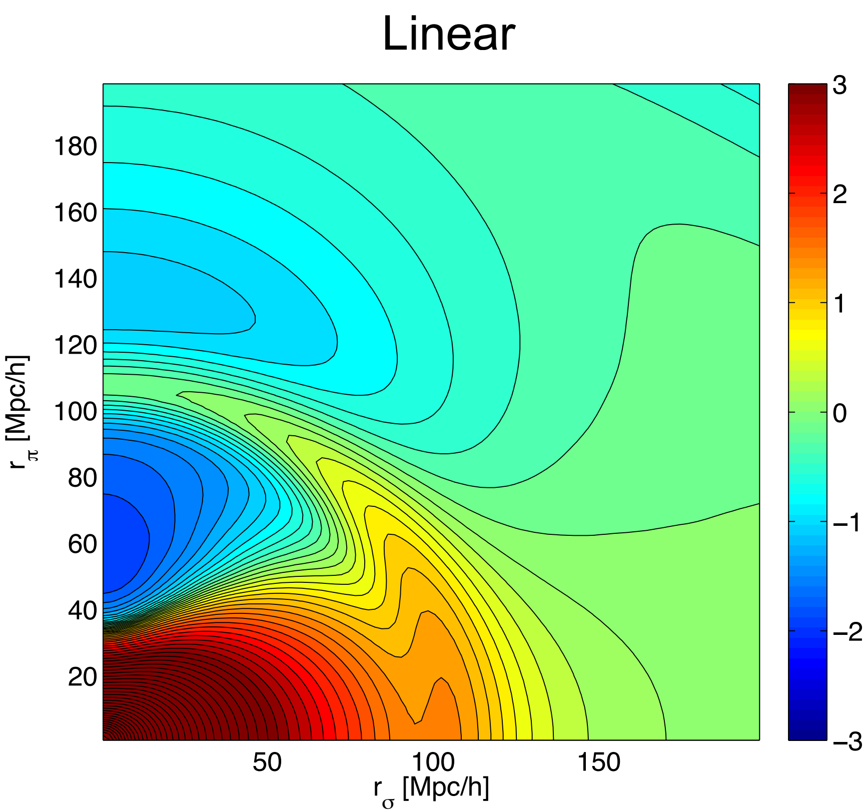
$$J(\mathbf{q},t) = 1 - D(I_1(\mathbf{q}) + f d_{nn}(\mathbf{q})) + D^2\big((1+f)I_2(\mathbf{q}) - fM_{nn}(\mathbf{q})\big) - D^3(1+f)I_3(\mathbf{q})$$

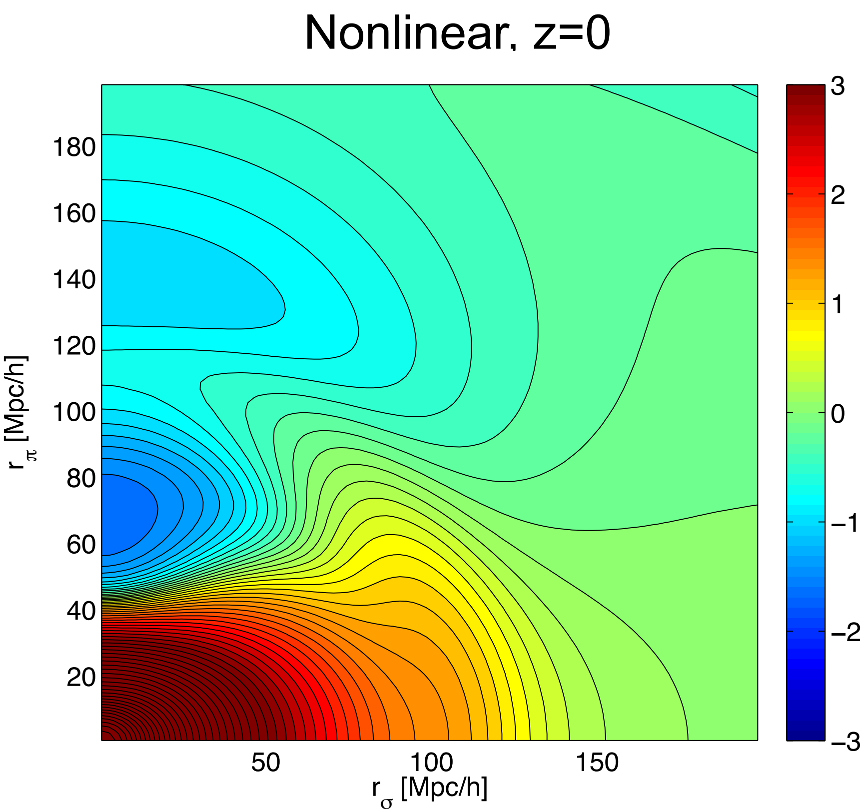
Nonlinear correlation function:

$$\xi^{(2)}(\mathbf{s}) = \left(\frac{19}{15} + \frac{76f}{45} + \frac{64f^2}{45} + \frac{16f^3}{25} + \frac{3f^4}{25}\right)\xi_0^0(s)^2 + \Bigg(\left(\frac{34}{21} + \frac{136f}{63} + \frac{638f^2}{315} + \frac{244f^3}{245} + \frac{48f^4}{245}\right. + \left(\frac{748f}{441} + \frac{7144f^2}{3087} + \frac{452f^3}{343} + \frac{96f^4}{343}\right)\mathcal{P}_2(\mu) + \left(\frac{3672f^2}{1715} + \frac{3312f^3}{1715} + \frac{864f^4}{1715}\right)\mathcal{P}_4(\mu)\Bigg)\xi_2^0(s)^2 + \cdots$$

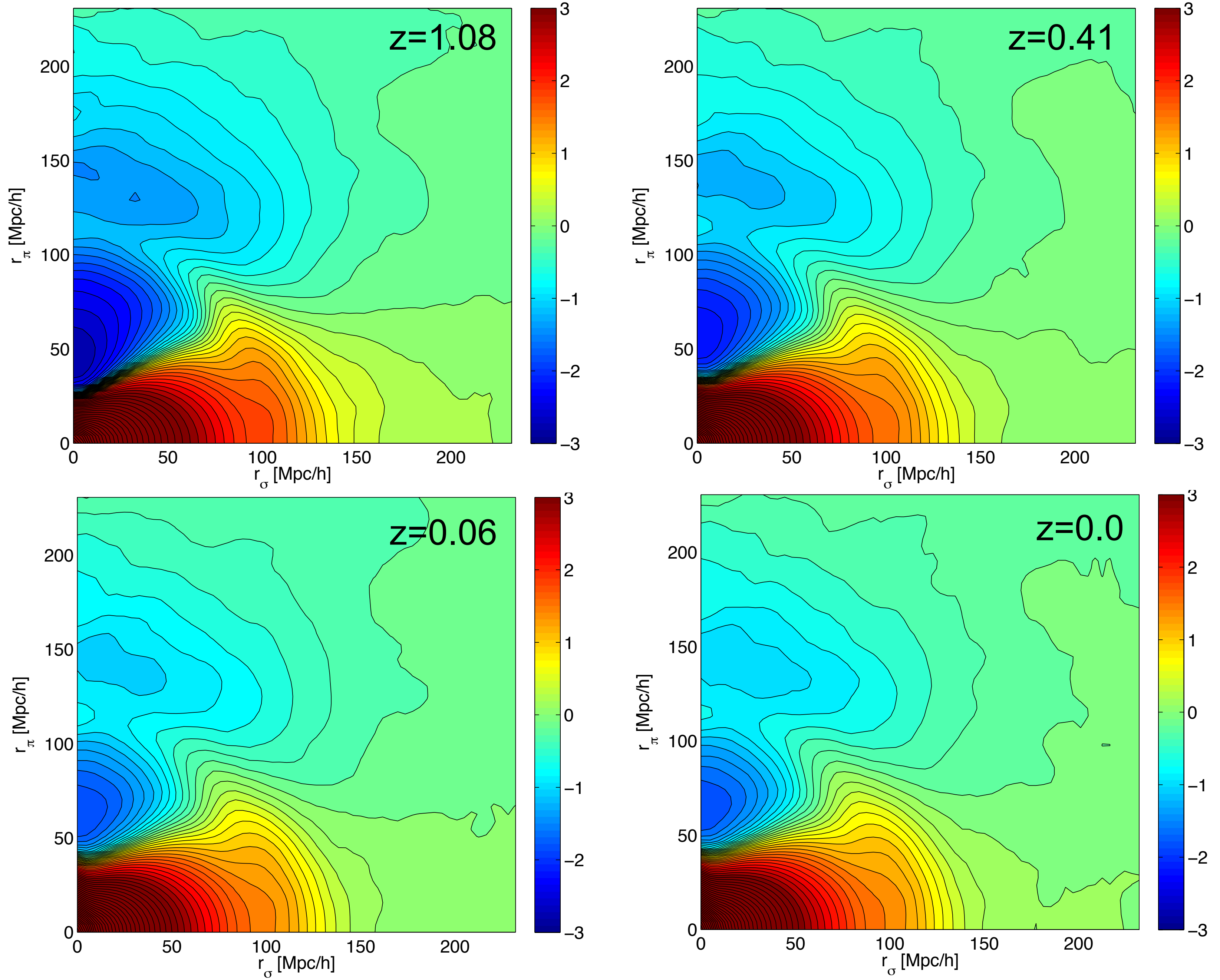
Redshift-space Correlation Function

Zel'dovich model prediction:



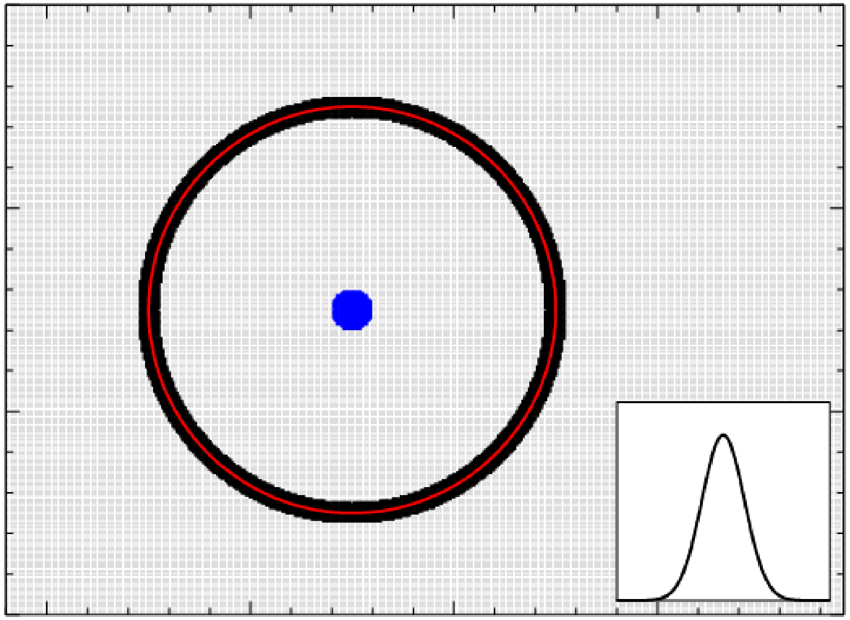


Indra *N*-body Simulations, 77 realizations



Understanding the BAO Peak Shift

Initial particle distribution



Evolved particle distribution

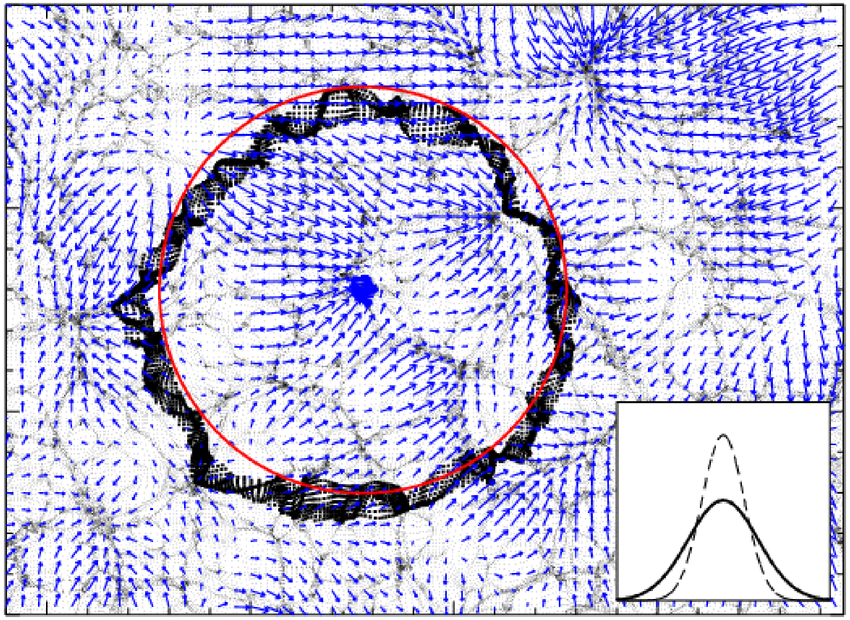
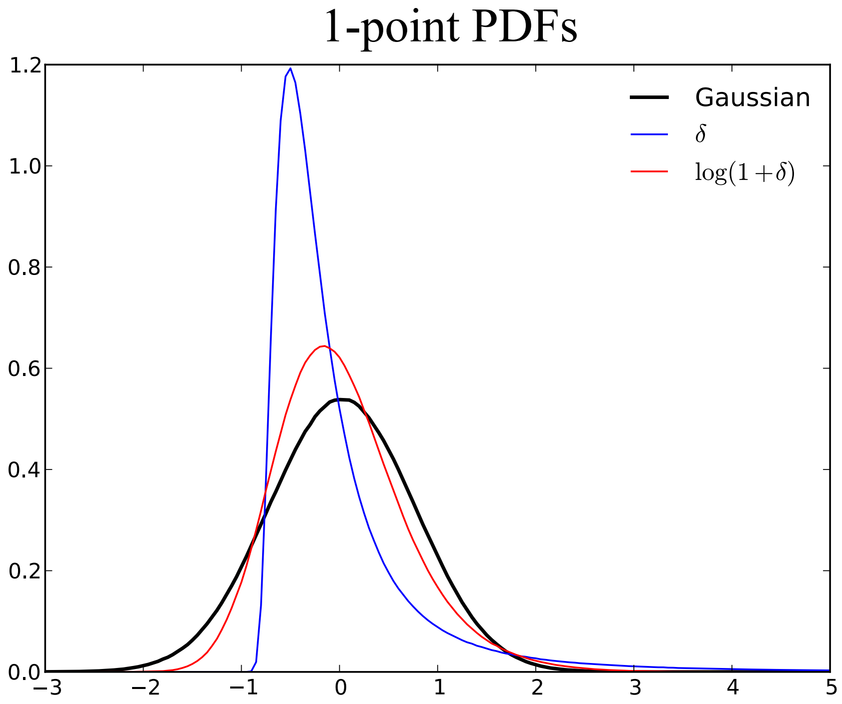


Figure: Padmanabhan, arXiv:1202.0090 (2012)

Local Density Transformations: Log Density



$A = \log(1+\delta(x))$

$$\xi_A^{(1)}(r) = \xi_0^0(r)$$

$$\xi_A^{(2)}(r) = -\frac{2}{3}\xi_0^0(0)\xi_0^0(r) + \frac{13}{30}\xi_0^0(r)^2 + \frac{20}{21}\xi_2^0(r)^2 + \frac{4}{35}\xi_4^0(r)^2 - \frac{6}{5}\xi_1^{-1}(r)\xi_1^1(r) - \frac{4}{5}\xi_3^{-1}(r)\xi_3^1(r) + \frac{1}{3}\xi_0^{-2}(r)\xi_0^2(r) - \frac{1}{3}\xi_0^{-2}(0)\xi_0^2(r) + \frac{2}{3}\xi_2^{-2}(r)\xi_2^2(r)$$

McCullagh, Neyrinck, Szapudi, & Szalay. ApJL, 763, L14 (2013)

Local Density Transformations: Displaced Initial Density

Initial density:

$$\delta_0(\mathbf{q})$$

Displaced initial density:

$$\delta_L(\mathbf{x},t) = \delta_0(\mathbf{x} - \Psi(\mathbf{q}(\mathbf{x}),t))$$

Correlation function:

$$\xi^{(1)}_{\delta_L}(r) = \xi^0_0(r)$$

$$\xi^{(2)}_{\delta_L}(r) = -\frac{2}{3}\xi^0_0(0)\xi^0_0(r) + \frac{1}{3}\xi^0_0(r)^2 + \frac{2}{3}\xi^0_2(r)^2 + \frac{1}{3}\xi^{-2}_0(r)\xi^2_0(r) - \frac{1}{3}\xi^{-2}_0(0)\xi^2_0(r) + \frac{2}{3}\xi^{-2}_2(r)\xi^2_2(r)$$

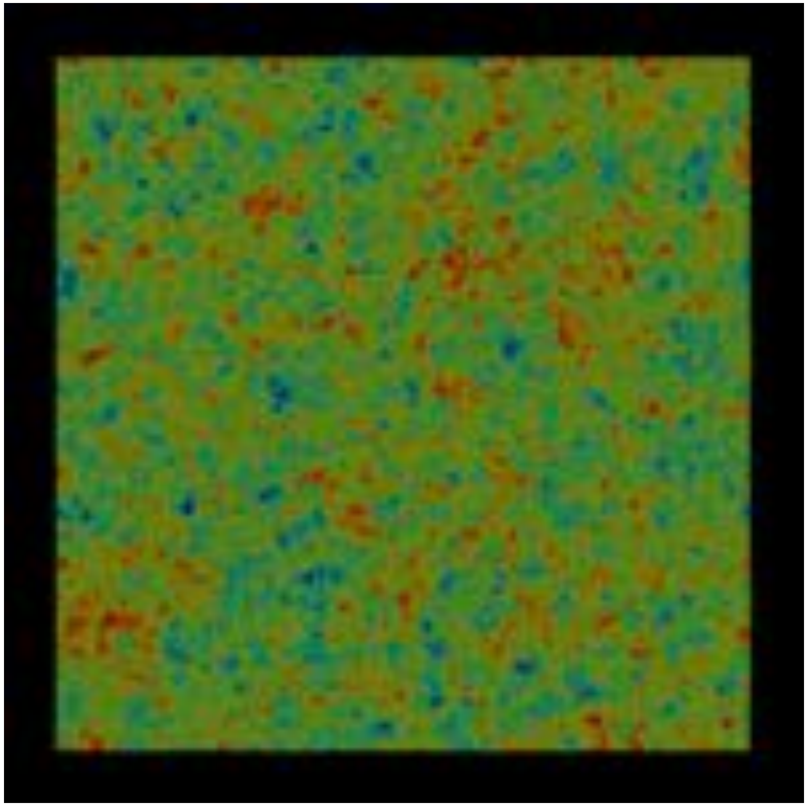
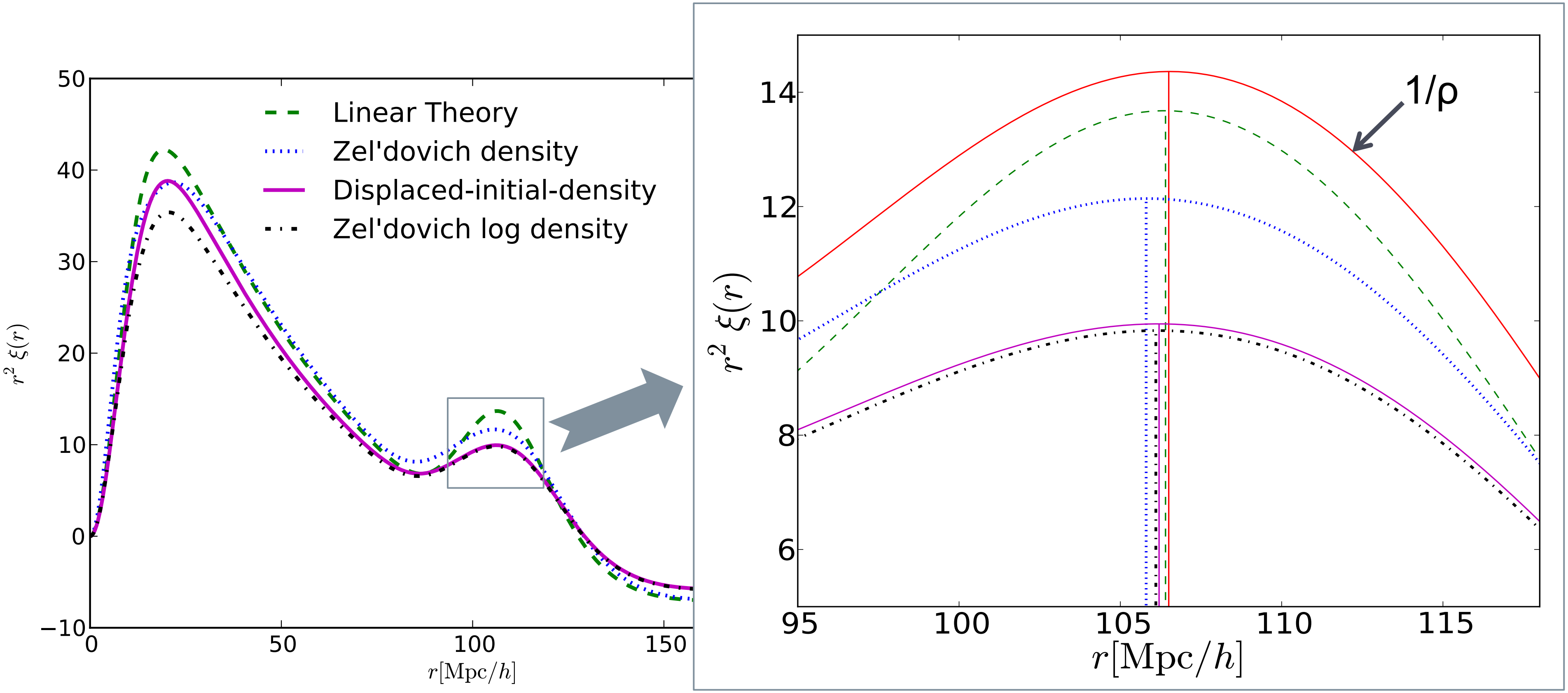


Figure: Mark Neyrinck

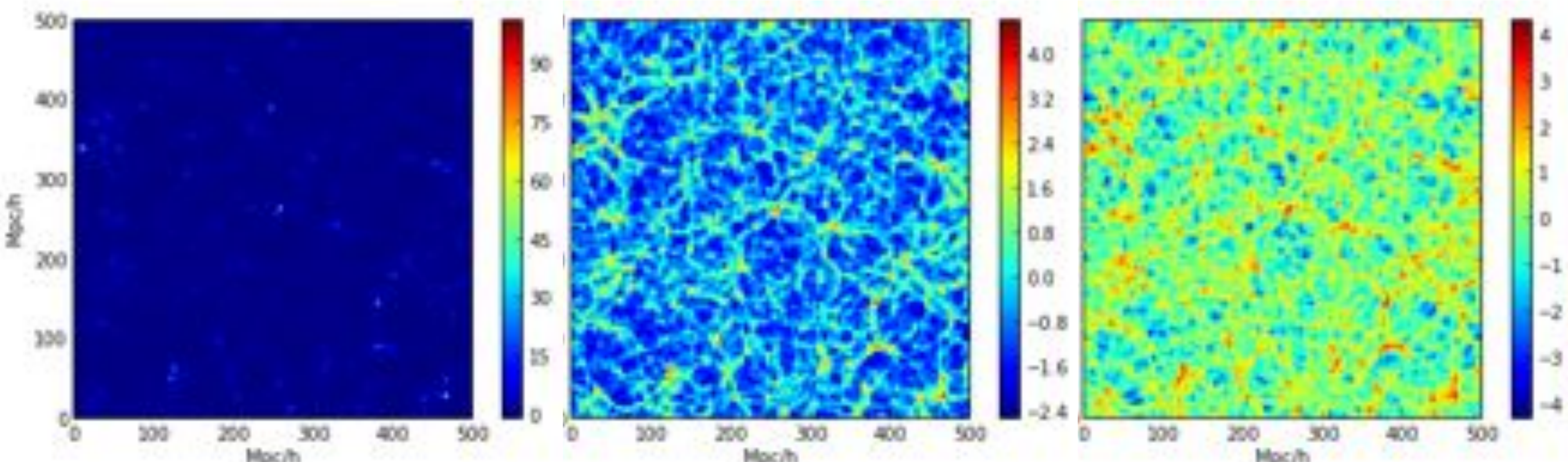
McCullagh, Neyrinck, Szapudi, & Szalay. ApJL, 763, L14 (2013)



- -	Linear Theory:	r_{peak}=106.4 Mpc/h	
·····	Zel'dovich density:	r_{peak}=105.8 Mpc/h	-0.6 Mpc/h
——	Displaced-initial-density:	r_{peak}=106.2 Mpc/h	-0.2 Mpc/h
-·-·	Zel'dovich log-density:	r_{peak}=106.1 Mpc/h	-0.3 Mpc/h

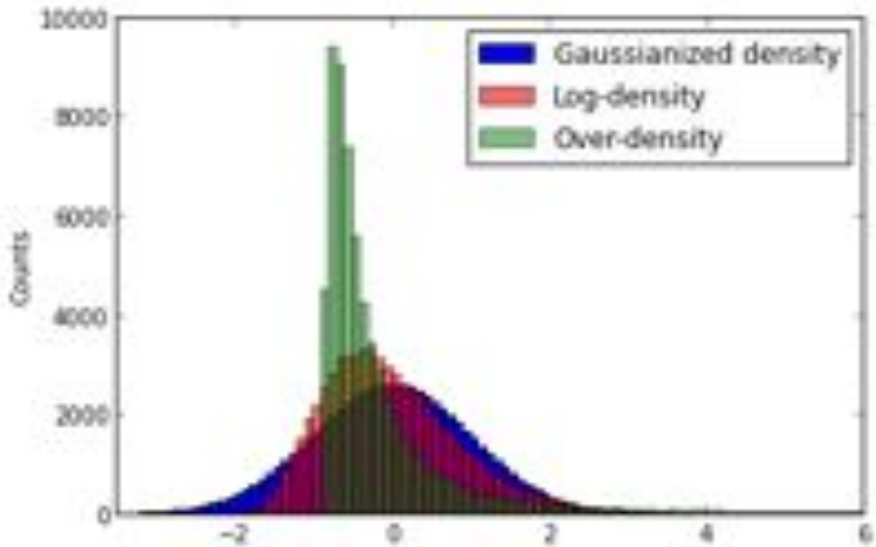
McCullagh, Neyrinck, Szapudi, & Szalay. ApJL, 763, L14 (2013)

Log-transform vs Gaussianization transform



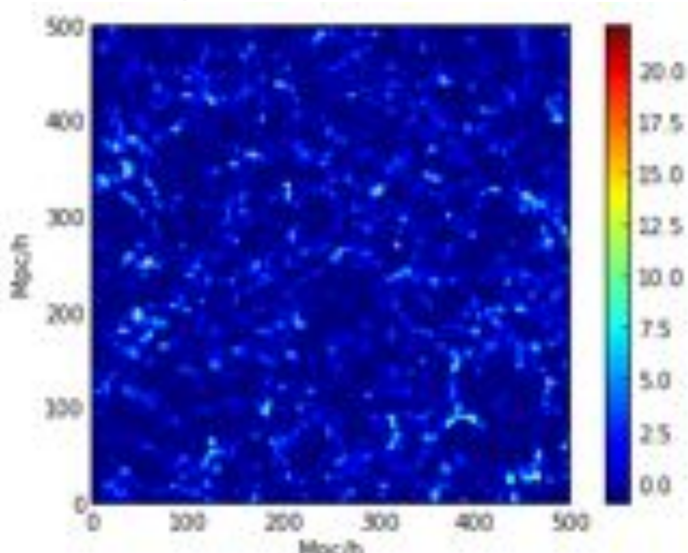
DM density DM log-density DM gaussianized-density

1-point PDFs:

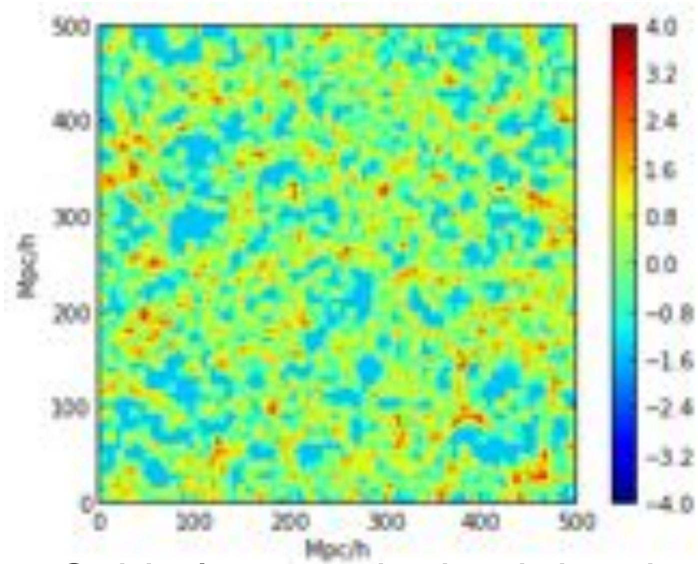


Indra simulations:
1024^3 DM particles,
1 Gpc/h box

Testing Prediction with Simulations

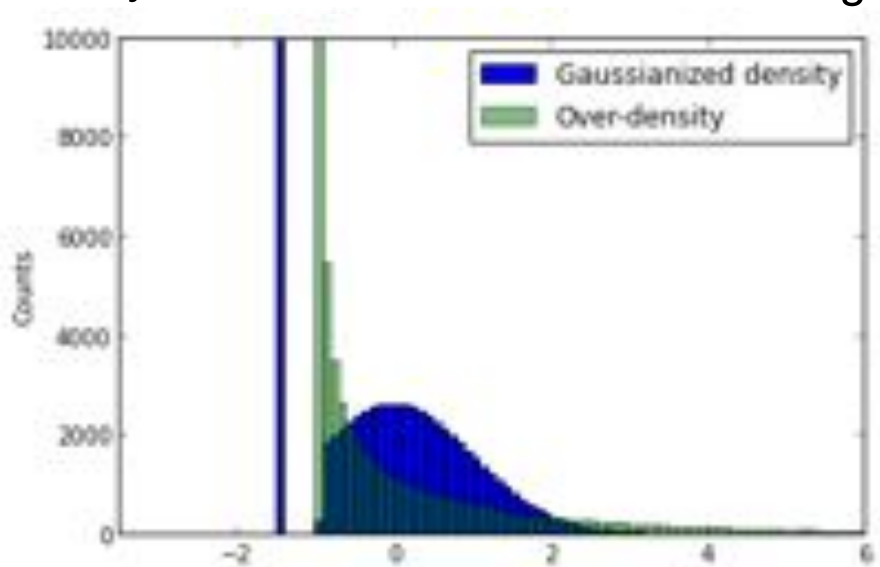


Subhalo density



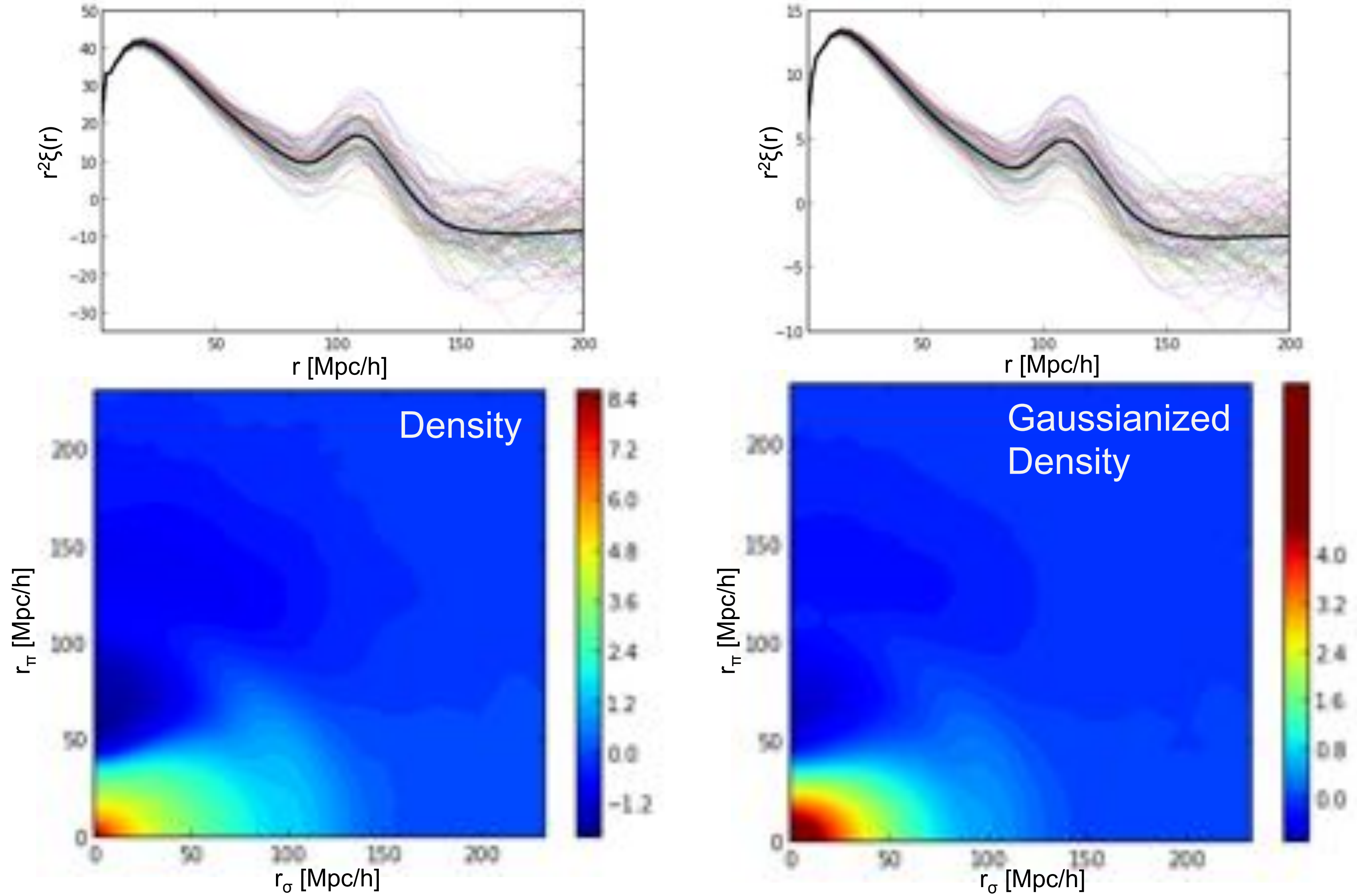
Subhalo gaussianized density

1-point PDFs:



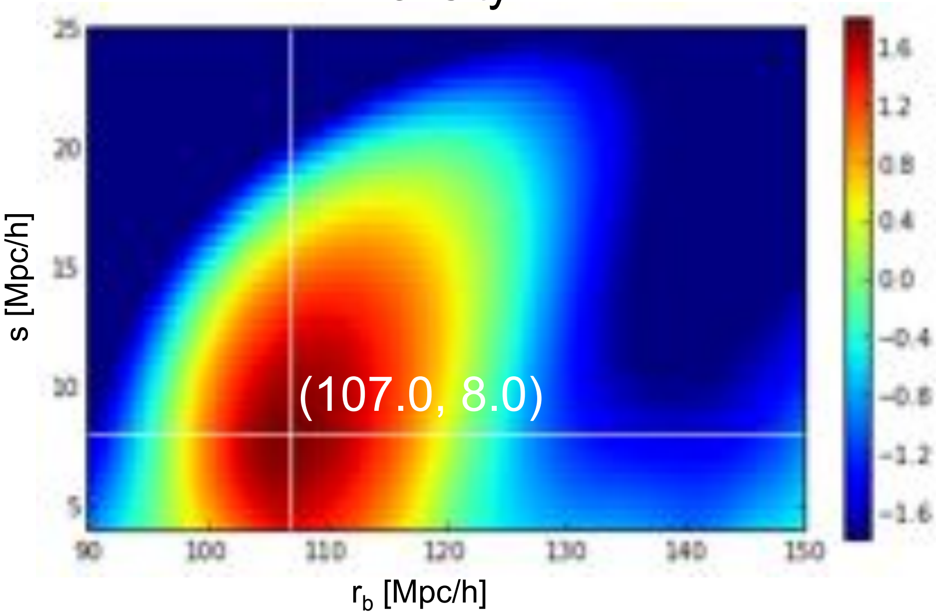
BAO Peak in Indra Subhalo Density Field

98 realizations

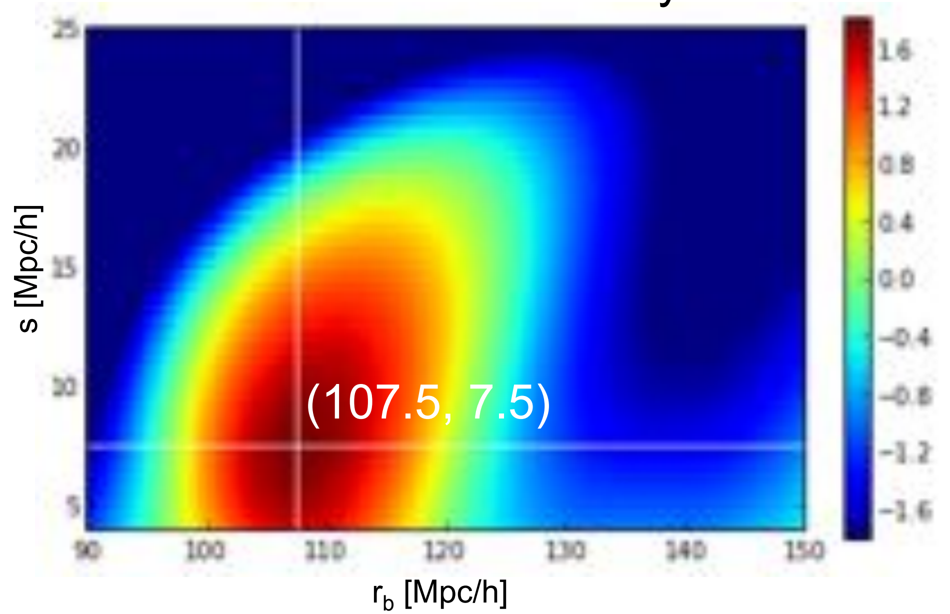


BAO Peak in Indra Subhalo Density Field

Density



Gaussianized Density



std of peak=8.7 Mpc/h

std of peak=7.6 Mpc/h

Peak (un)shifted by ~0.5 Mpc/h
S/N increased by ~1%

Linear r_b=107.7 Mpc/h

Directions for Future Work

Transformed-density statistics

- Developing pair-counting estimator for the log-density correlation function
- Measuring the log-density correlation function from SDSS data

Configuration-space perturbation theory

- Extending model to higher orders in LPT
- Modeling higher-order statistics in configuration space
- Including galaxy bias in model

2LPT:

$$\vec{x} = \vec{q} - D_1 \nabla_q \phi^{(1)} + D_2 \nabla_q \phi^{(2)} \qquad D_2(\tau) \approx -\frac{3}{7} D_1^2(\tau)$$

$$\begin{aligned}\xi_{(22)}(\vec{r}) &= \langle \delta^{(2)}(\vec{x}) \delta^{(2)}(\vec{x}+\vec{r}) \rangle \\ &= \frac{1219}{735} \xi_0^0(r)^2 + \frac{1}{3} \xi_0^{-2}(r) \xi_0^2(r) - \frac{124}{35} \xi_1^{-1}(r) \xi_1^1(r) + \frac{1342}{1029} \xi_2^0(r)^2 + \\ &\quad \frac{2}{3} \xi_2^{-2}(r) \xi_2^2(r) - \frac{16}{35} \xi_3^{-1}(r) \xi_3^1(r) + \frac{64}{1715} \xi_4^0(r)^2\end{aligned}$$

Conclusions

The BAO signal is a powerful tool for constraining Dark Energy equation-of-state parameters

As survey precision improves, theoretical models of BAO systematics must improve to accurately constrain Dark Energy parameters

Zel'dovich Model:

- Provides simpler higher-order corrections than Fourier-space equivalent
- Models nonlinear behavior of the correlation function
- Models redshift-space distortions in the nonlinear correlation function

Application: Local density transformations

- Log (or Gaussianized) density field gives less-biased peak position and slightly higher signal-to-noise of peak detection

Thank you!