# Local primordial non-Gaussianity in large-scale structure: Halo mass function and clustering with $f_{NL}$, $g_{NL}$ and $\tau_{NL}$

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+ Simone Ferraro (Princeton) arXiv: 1106.0503

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# Outline

- Who cares about primordial non-Gaussianity?

  (i) What is Gaussian?
  (ii) Three simple models of non-Gaussianity: $f_{NL}$, $g_{NL}$, $\tau_{NL}$

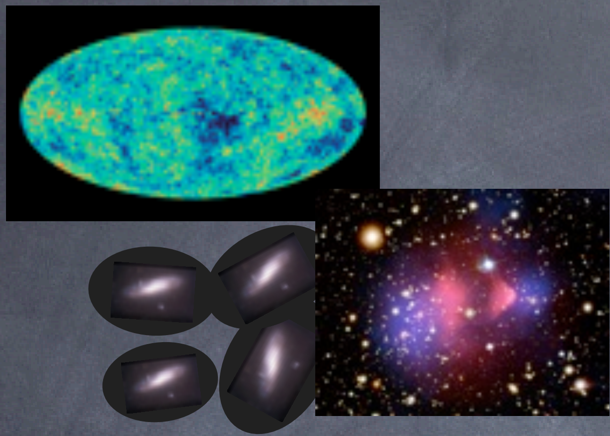
- What kinds of signatures are in large-scale structure and how do we model them?

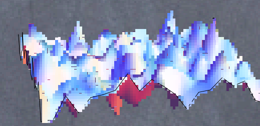
  (i) Halo mass function
  (ii) Halo clustering
  (iii) Halo stochasticity

- Conclusions

# Who cares?

* We see structure around us and we should quantify how it looks



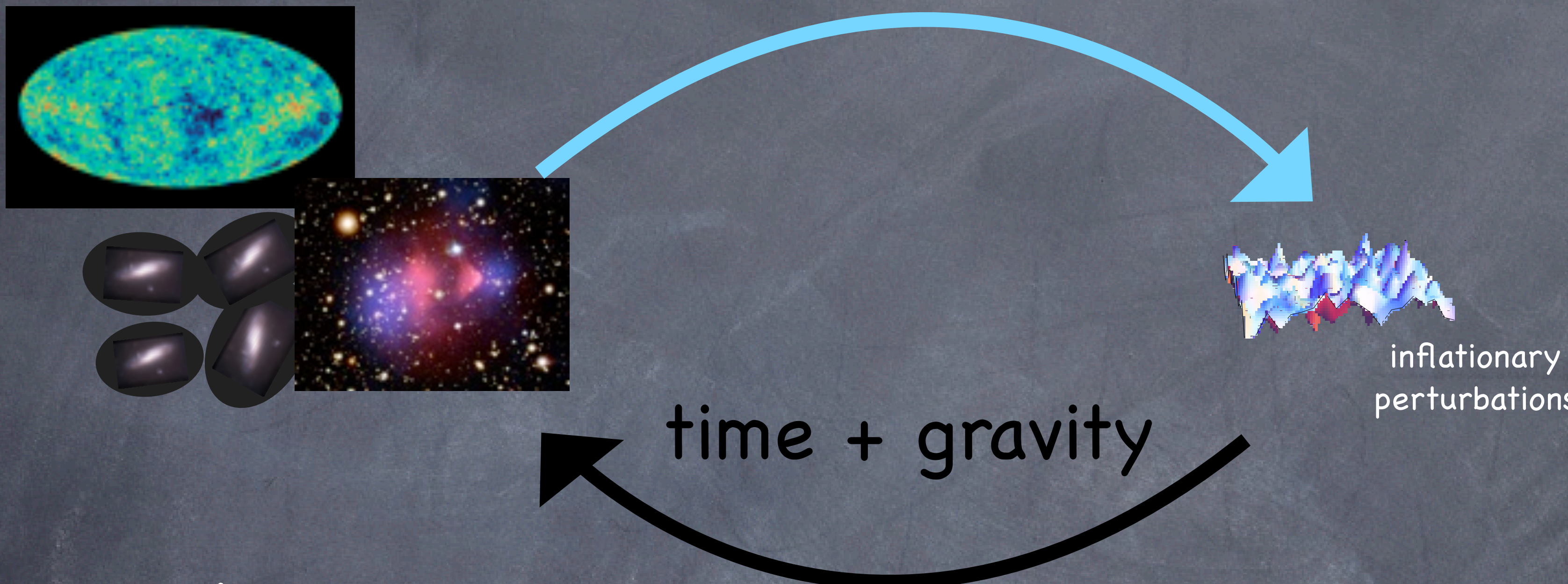


inflationary perturbations

time + gravity

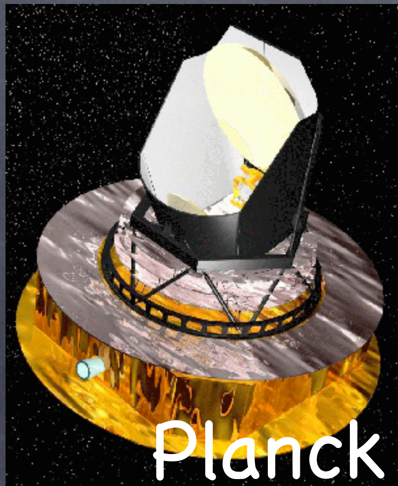
* We have a compelling framework for how structure arose (inflation) but limited handles on microphysical models

* We see structure around us and we should quantify how it looks




* We have a compelling framework for how structure arose (inflation) but limited handles on microphysical models

Different models make different predictions for <u>statistics of perturbations</u>







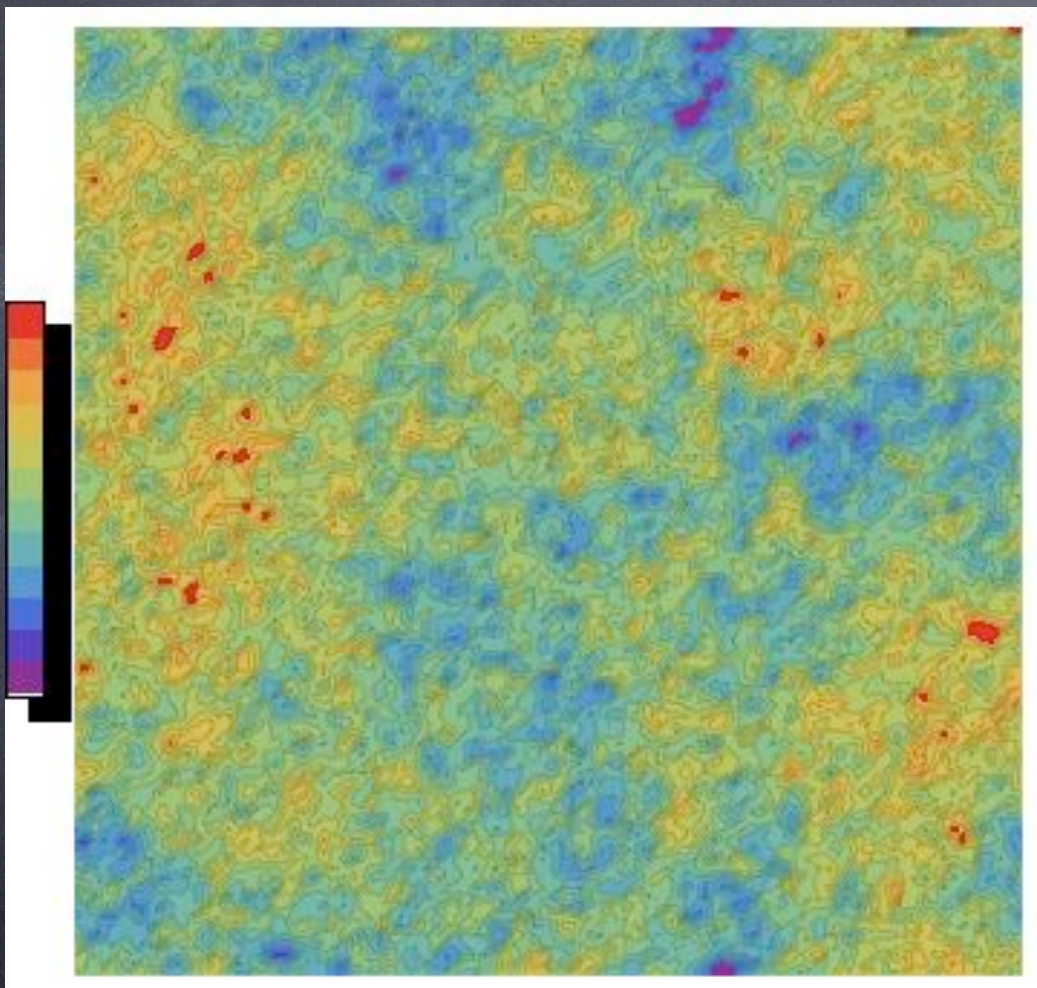


see e.g. Shandera, Dalal, Huterer 2010
Oguri and Takada 2010
Carbone, Verde, Matarrese 2009 many more . . .

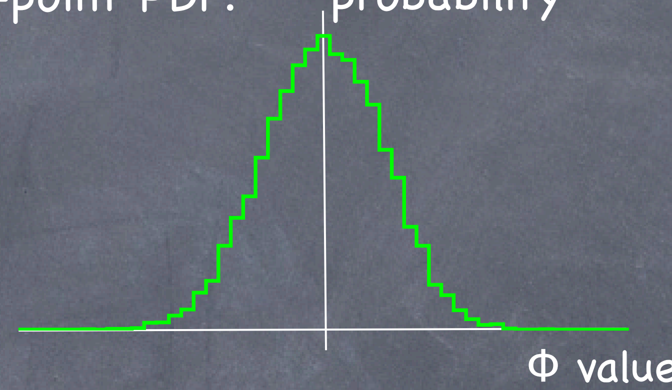
# Statistics of initial perturbations?

# Statistics of initial perturbations

a realization of a random field, Φ



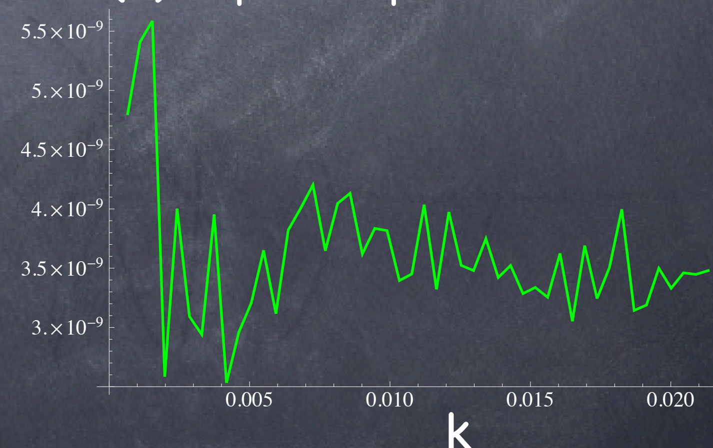
one-point PDF:




two-point function:

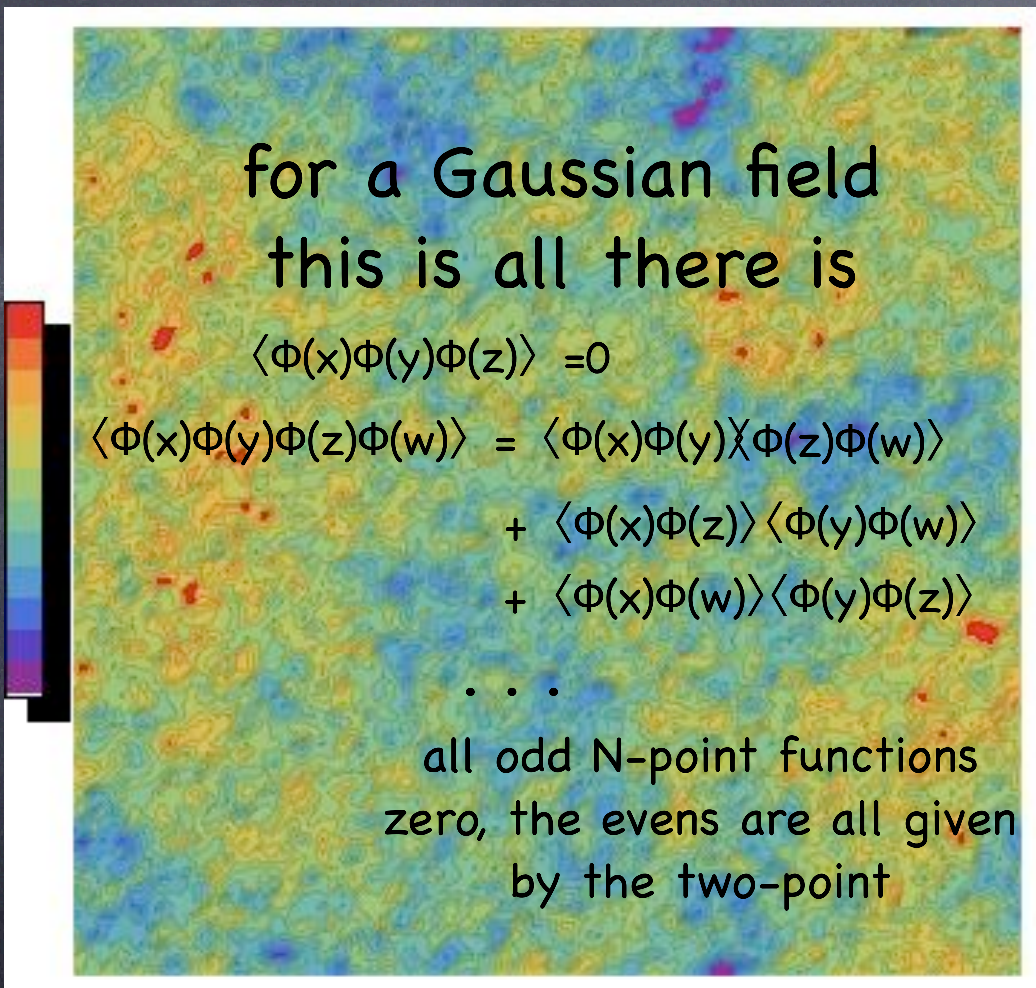
$$\langle \Phi(x)\Phi(y) \rangle \leftrightarrow P_\Phi(k)$$

$k^2 P_\Phi(k)$ power per mode

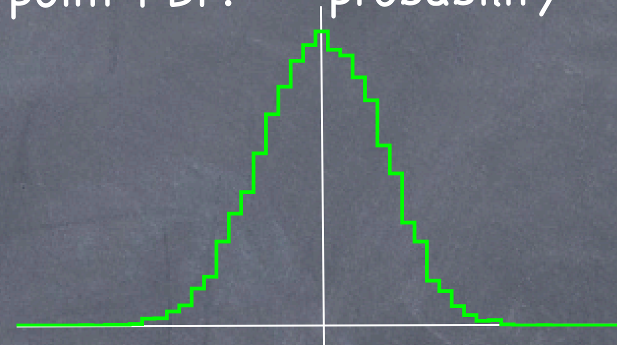


# Statistics of initial perturbations

a realization of a random field, $\Phi$




one-point PDF:

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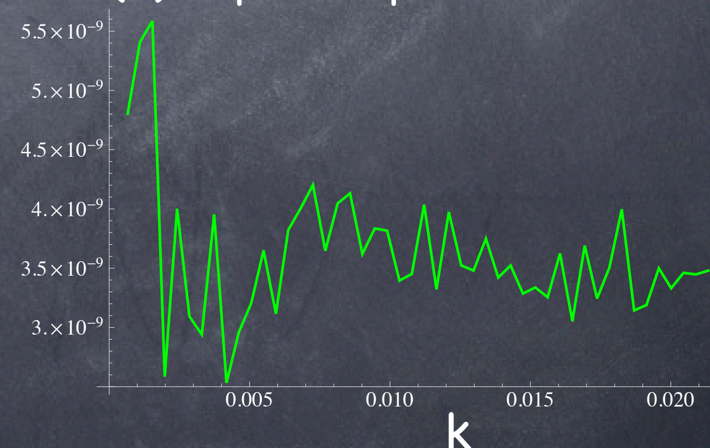
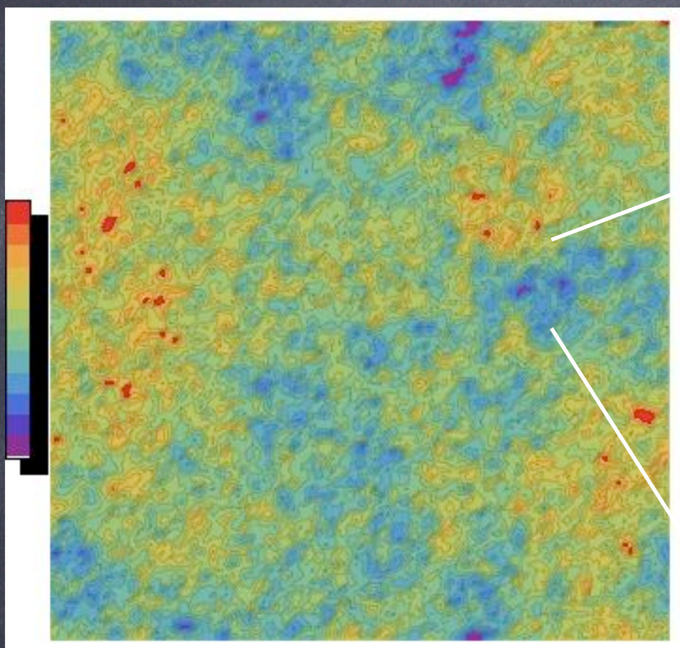
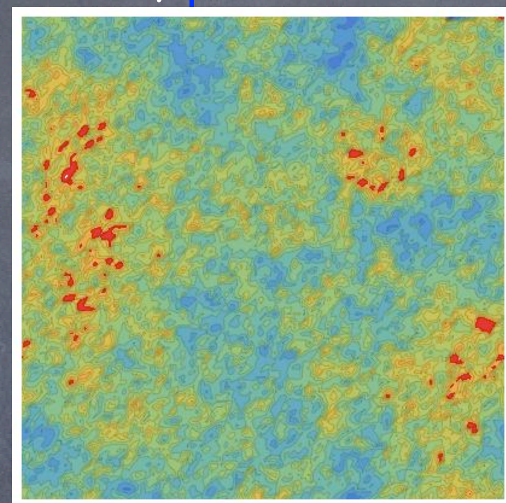


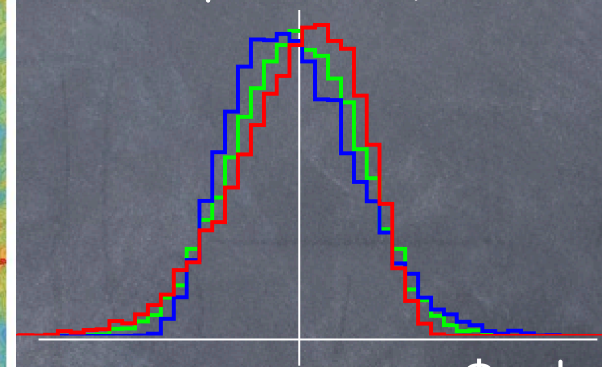
# Statistics of perturbations: a non-Gaussian example

a Gaussian random field, $\Phi$

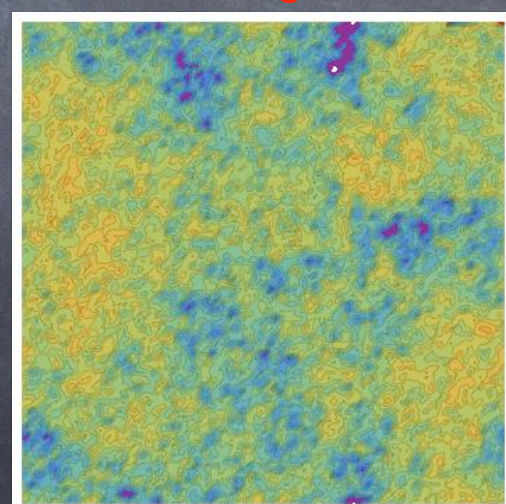


same variance, positive skewness






same variance, negative skewness



$$\Phi(x)=\Phi_G(x) + f_{NL}\,(\Phi_G{}^2(x)-\langle\Phi_G{}^2\rangle)$$

($\Phi$=primordial gravitational potential)

skewness $\langle\Phi(x)^3\rangle \sim f_{NL}\langle\Phi_G(x)^2\rangle^2$

$-10 < f_{NL} < 74$

WMAP, Komatsu et al 2010

Salopek and Bond 1990; Gangui, Lucchin, Matarrese, Mollerach 1994; Komatsu and Spergel 2001

# Statistics of perturbations

we can get more insight by splitting $\Phi$ into short and long wavelength pieces

$$\Phi_G = \Phi_{G,s} + \Phi_{G,l}$$

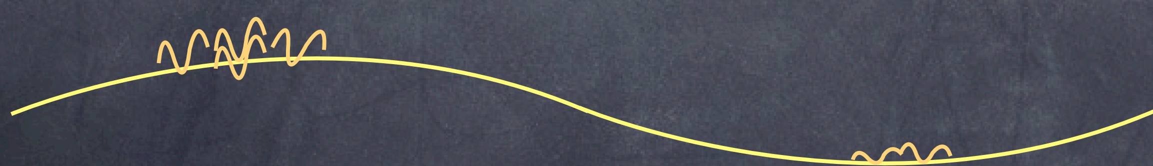
which are uncorrelated *

Locally, we see small-scale fluctuations

$$\Phi_{NG,s} = \Phi_{G,s} + f_{NL}\left(\Phi_{G,s}^2 - \langle \Phi_{G,s}^2 \rangle\right) + 2\, f_{NL}\, \Phi_{G,s}\, \Phi_{G,l}$$

with variance that varies from place to place depending on the value of $\Phi_{G,l}$

$$\langle \Phi_{NG,s}^2 \rangle = \langle \Phi_{G,s}^2 \rangle \left(1 + 4\, f_{NL}\, \Phi_{G,l}\right)$$



contrast w/ Gaussian fields where different scales are uncorrelated!

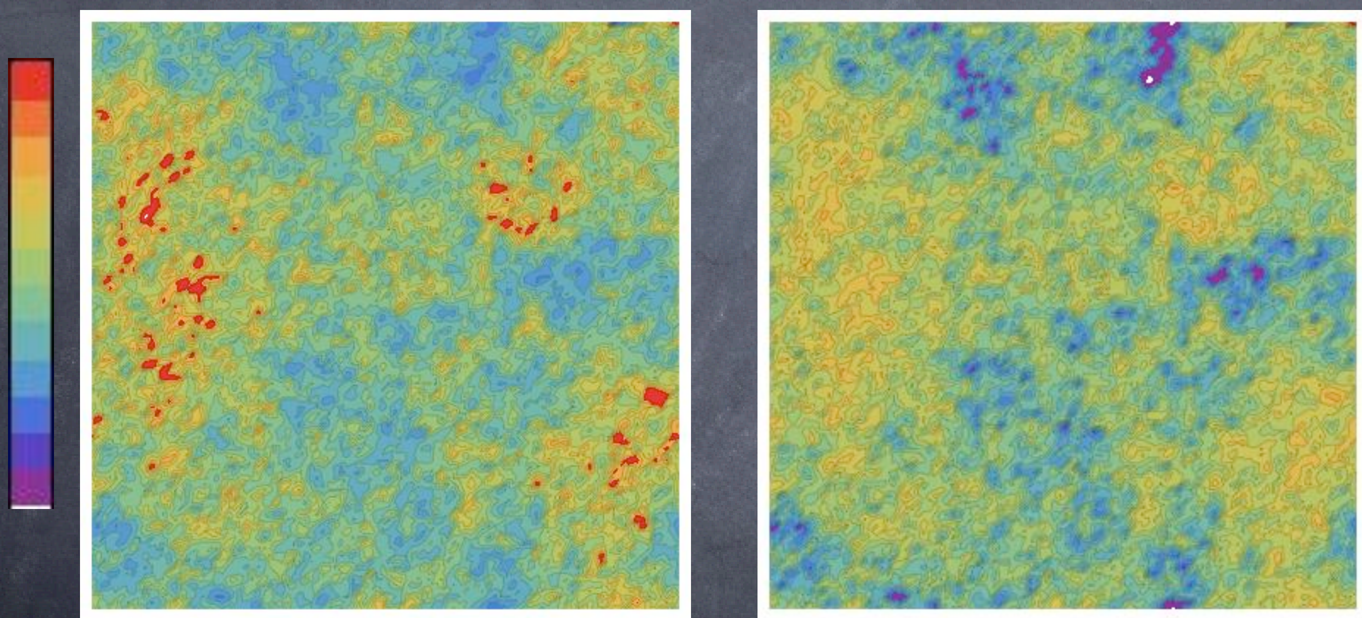
* only strictly true in fourier space, but shouldn't be a bad approximation

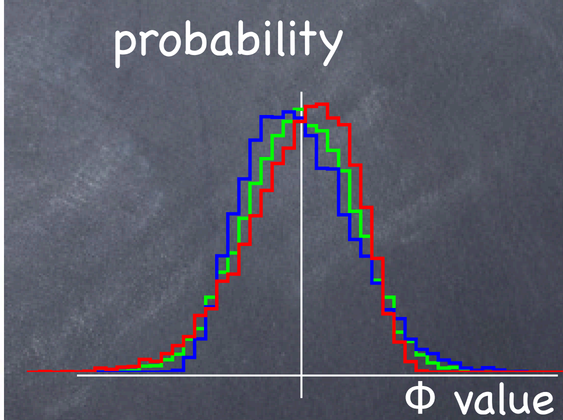
Slosar, Hirata, Seljak, Ho, Padmanabhan 2008

# Statistics of perturbations

SO, for the $f_{NL}$ model, $\Phi(x)=\Phi_G(x)+ f_{NL} (\Phi_G(x)^2-<\Phi_G{}^2>)$
we get a skewness $\langle\Phi^3\rangle \approx 6f_{NL}\langle\Phi_G{}^2\rangle^2$

and small scale power that depends on long wavelength fluctuations $\Phi_l$
via $\langle\Phi_s{}^2\rangle = \langle\Phi_{G,s}{}^2\rangle (1 + 4 f_{NL} \Phi_{G,l})$






positive skewness      same variance, negative skewness

## The $f_{NL}$ model just one example, NOT general

# ONE reason this is interesting:

single-field inflation predicts

$$\langle \Phi(\mathbf{k})\Phi(\mathbf{k}')\Phi(\mathbf{k}''\text{-->}0)\rangle \approx \underbrace{(n_s-1)}_{\approx f_{NL}}(2\pi)^3\,\delta(\mathbf{k}+\mathbf{k}')\,P_\Phi(k)\,P_\Phi(k'')$$

where $n_s = d\ln P_\Phi(k)/d\ln k + 4 \approx 1$

the so called "consistency relation"

so $f_{NL} \gtrsim$ few rules it out

Acquaviva, Bartolo, Matarrese, Riotto 2003; Maldacena 2003; Creminelli & Zaldarriaga 2004

(see also Tanaka, Urakawa 2011)

# Two more non-Gaussian models

There's an extensive literature on "$f_{NL}$" non-Gaussianity

$$\Phi(x)=\Phi_G(x)+ f_{NL} (\Phi_G(x)^2-\langle\Phi_G^2\rangle)$$

What about other models?

We'll consider two simple extensions where the non-Gaussian 4-point function is important:

$$\Phi(x)= \Phi_G(x)+ g_{NL} (\Phi_G(x)^3-3\Phi_G(x)\langle\Phi_G^2\rangle)? \quad \text{"}g_{NL}\text{"}$$

$$\Phi(x)=\varphi_G(x)+ \sigma_G(x)+ \tilde{f}_{NL}(\sigma_G(x)^2-\langle\sigma_G^2\rangle) ? \quad \text{"}\tau_{NL}\text{"}$$

(Okamoto and Hu 2002; Enqvist and Nurmi 2005)

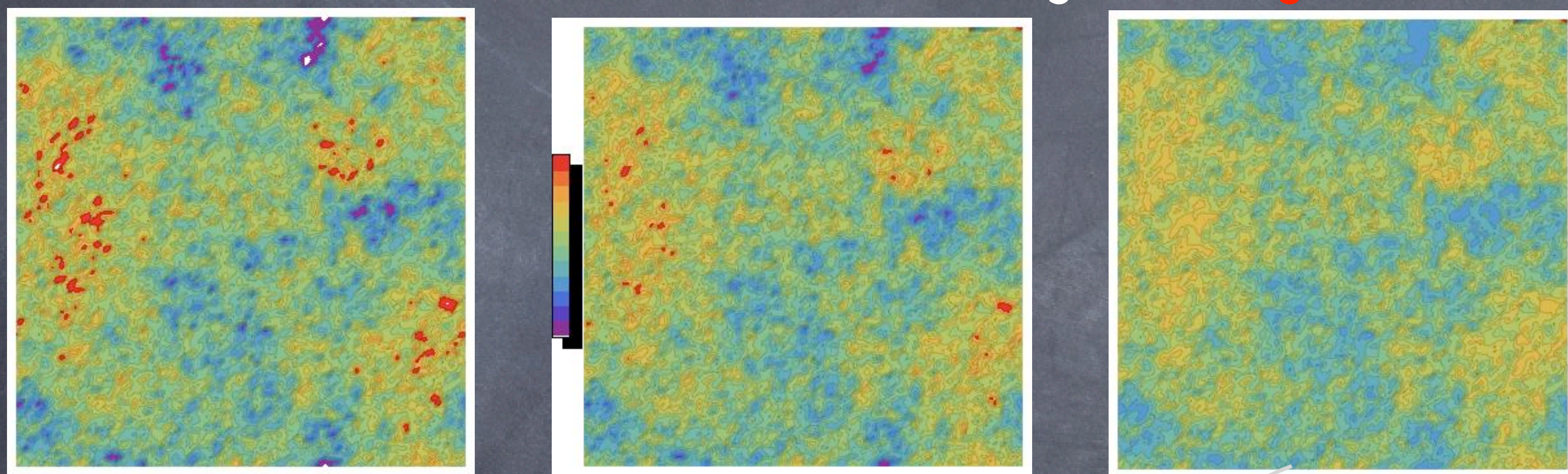
(Lyth and Wands 2002; Ichikawa, Suyama, Takahishi, Yamaguchi (2008); Tseliakhovich, Hirata, Slosar 2010)

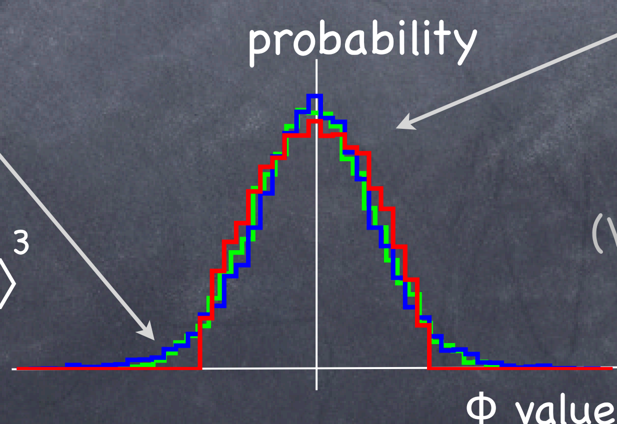
(see also Desjacques and Seljak 2010; Shandera, Dalal, Huterer 2010)

# what about

$$\Phi(x) = \Phi_G(x) + g_{NL}\,(\Phi_G(x)^3 - 3\Phi_G(x)\langle\Phi_G{}^2\rangle)? \quad \text{"}g_{NL}\text{"}$$

$g_{NL} > 0$: positive kurtosis | Gaussian | $g_{NL} < 0$: negative kurtosis






kurtosis:

$$\langle\Phi(x)^4\rangle - 3\langle\Phi(x)^2\rangle^2 \sim g_{NL}\langle\Phi_G(x)^2\rangle^3$$

current constraints:

$$-12 < g_{NL}/10^5 < 16$$

(WMAP, Fergusson et al 2010)

(Okamoto and Hu 2002; Enqvist and Nurmi 2005)

## "$g_{NL}$"

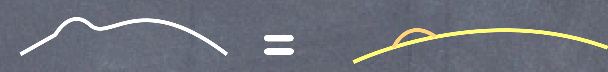
$$\Phi(\mathbf{x})=\Phi_G(\mathbf{x}) + g_{NL}\,(\Phi_G^3(\mathbf{x})-3\langle\Phi_G^2\rangle\,\Phi_G(\mathbf{x}))$$

this gives

$\langle\Phi^3\rangle = 0$ no skewness!

$\langle\Phi^4\rangle - 3\langle\Phi^2\rangle^2 \approx 24 g_{NL}\,\langle\Phi^2\rangle^3$ kurtosis $\propto g_{NL}$

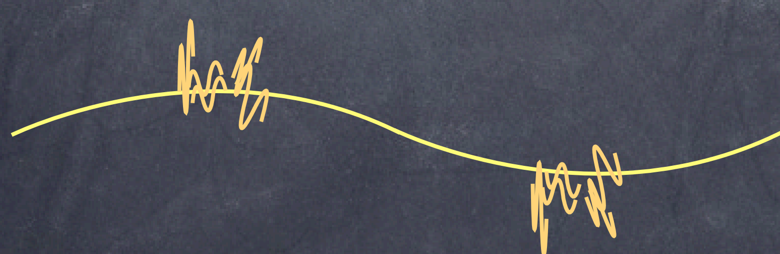
splitting $\Phi_G = \Phi_{G,s} + \Phi_{G,l}$ gives



$\langle\Phi_s^2\rangle = \langle\Phi_{G,s}^2\rangle\,(1 + 6\,g_{NL}\,\Phi_{G,l}^2)$ locally varying power

$\langle\Phi_s^3\rangle = 18\,g_{NL}\,\langle\Phi_{G,s}^2\rangle^2\,\Phi_{G,l}$ and locally varying skewness!

$\equiv f_{NL}^{eff}(x)\,\langle\sigma_{G,short}^2\rangle$



# another option

# "$\tau_{NL}$"

$$\Phi(\mathbf{x}) = \varphi_G(\mathbf{x}) + \sigma_G(\mathbf{x}) + \tilde{f}_{NL}\,(\sigma_G^2(\mathbf{x}) - \langle\sigma_G^2\rangle)$$

with $\xi^2 = P_{\varphi\varphi}(k)/P_{\sigma\sigma}(k)$ and $P_{\varphi\sigma}(k)=0$

defining

$f_{NL} = \tilde{f}_{NL}/(1+\xi^2)^2$ and $\tau_{NL} = f_{NL}^2(1+\xi^2)$ *

we get

$\langle\Phi^3\rangle \approx 6\, f_{NL}\, \langle\Phi^2\rangle^2$ BUT $\langle\Phi^4\rangle - 3\langle\Phi^2\rangle^2 \approx 48\, \tau_{NL}\, \langle\Phi^2\rangle^3$

just like $f_{NL}$ model

different!

*the conventional def of $\tau_{NL}$ is $\tau_{NL}=(6/5\, f_{NL})^2\,(1+\xi^2)$ -- but throughout this talk I drop the 6/5 for simplicity

(Lyth and Wands 2002; Ichikawa, Suyama, Takahishi, Yamaguchi (2008); Tseliakhovich, Hirata, Slosar 2010)

# another option

"$\tau_{NL}$"

$$\Phi(\mathbf{x})=\varphi_G(\mathbf{x})+\sigma_G(\mathbf{x}) + \tilde{f}_{NL}\,(\sigma_G^2(\mathbf{x})-\langle\sigma_G^2\rangle)$$

with $\xi^2 = P_{\varphi\varphi}(k)/P_{\sigma\sigma}(k)$ and $P_{\varphi\sigma}(k)=0$

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just like $f_{NL}$ model

different!

contrast w/ $\langle\Phi^4\rangle - 3\langle\Phi^2\rangle^2 \approx 48\, f_{NL}^2\, \langle\Phi^2\rangle^3$

looks like $f_{NL}$ local model but, 4-point is independent and larger than you'd expect from measuring the 3-point

(Lyth and Wands 2002; Ichikawa, Suyama, Takahishi, Yamaguchi (2008); Tseliakhovich, Hirata, Slosar 2010)

# another option

## "$\tau_{NL}$"

$$\Phi(\mathbf{x})=\varphi_G(\mathbf{x})+\sigma_G(\mathbf{x}) + \tilde{f}_{NL}\,(\sigma_G^2(\mathbf{x})-\langle\sigma_G^2\rangle)$$

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defining

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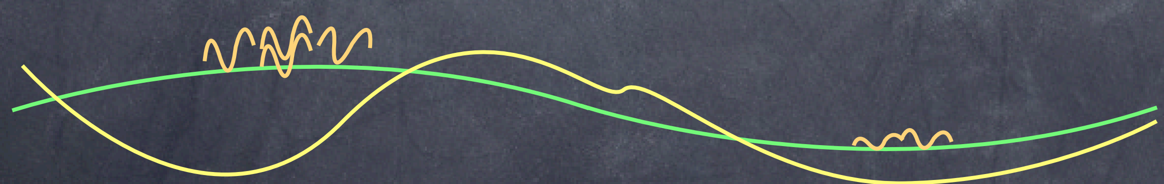
we get

$$\langle\Phi^3\rangle \approx 6\, f_{NL}\, \langle\Phi^2\rangle^2 \qquad \text{BUT} \qquad \langle\Phi^4\rangle - 3\langle\Phi^2\rangle^2 \approx 48\, \tau_{NL}\, \langle\Phi^2\rangle^3$$

AND variance varies from place to place depending on the value of $\sigma_{G,l}$ ONLY as opposed to total potential $\Phi = \varphi+\sigma$

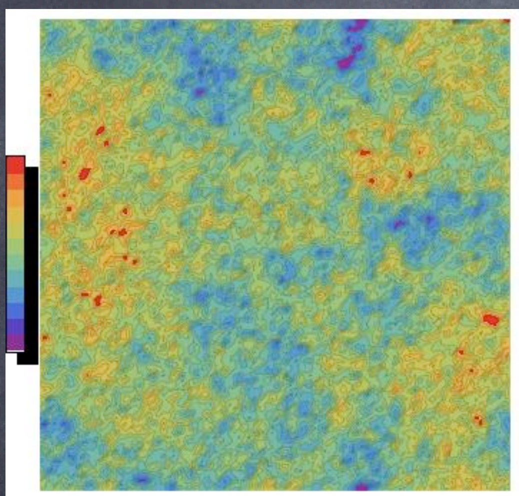
$$\langle\Phi_s^2\rangle = \langle\Phi_{G,s}^2\rangle\,(1 + 4\, f_{NL}\,(1+\xi^2)\sigma_{G,l})$$



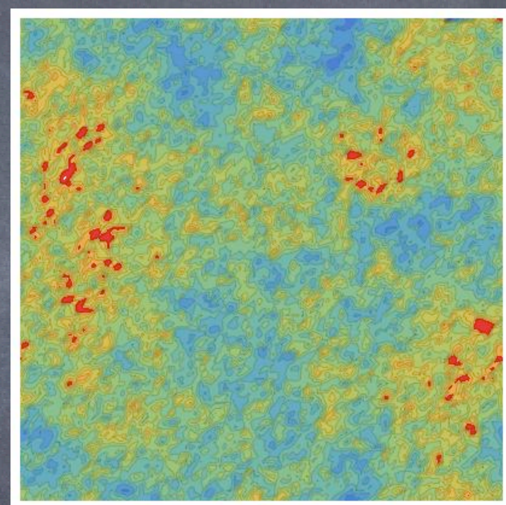
(Lyth and Wands 2002; Ichikawa, Suyama, Takahishi, Yamaguchi (2008); Tseliakhovich, Hirata, Slosar 2010)

$$\Phi(x)=\varphi_G(x)+\sigma_G(x)+\tilde{f}_{NL}(\sigma_G(x)^2-\langle\sigma_G^2\rangle) \ ? \quad \text{“}\tau_{NL}\text{”}$$

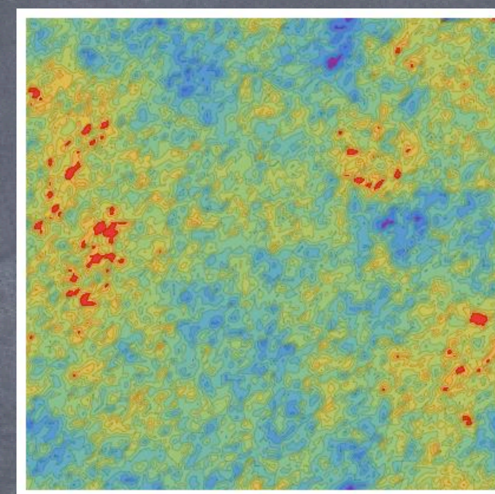
Gaussian

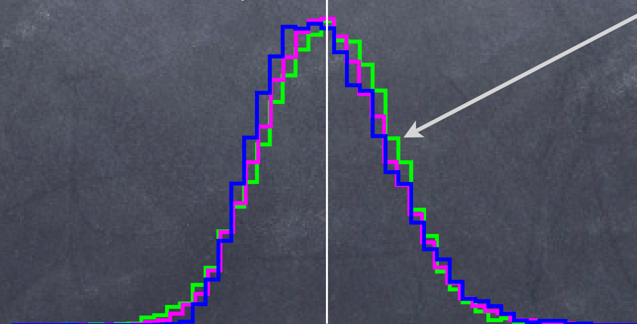


positive skewness and usual kurtosis: $\tau_{NL} = f_{NL}^2$



positive skewness and larger kurtosis: $\tau_{NL} > f_{NL}^2$






current constraints:

$-6000 < \tau_{NL} < 33{,}000$

(WMAP, Smidt et al 2010)

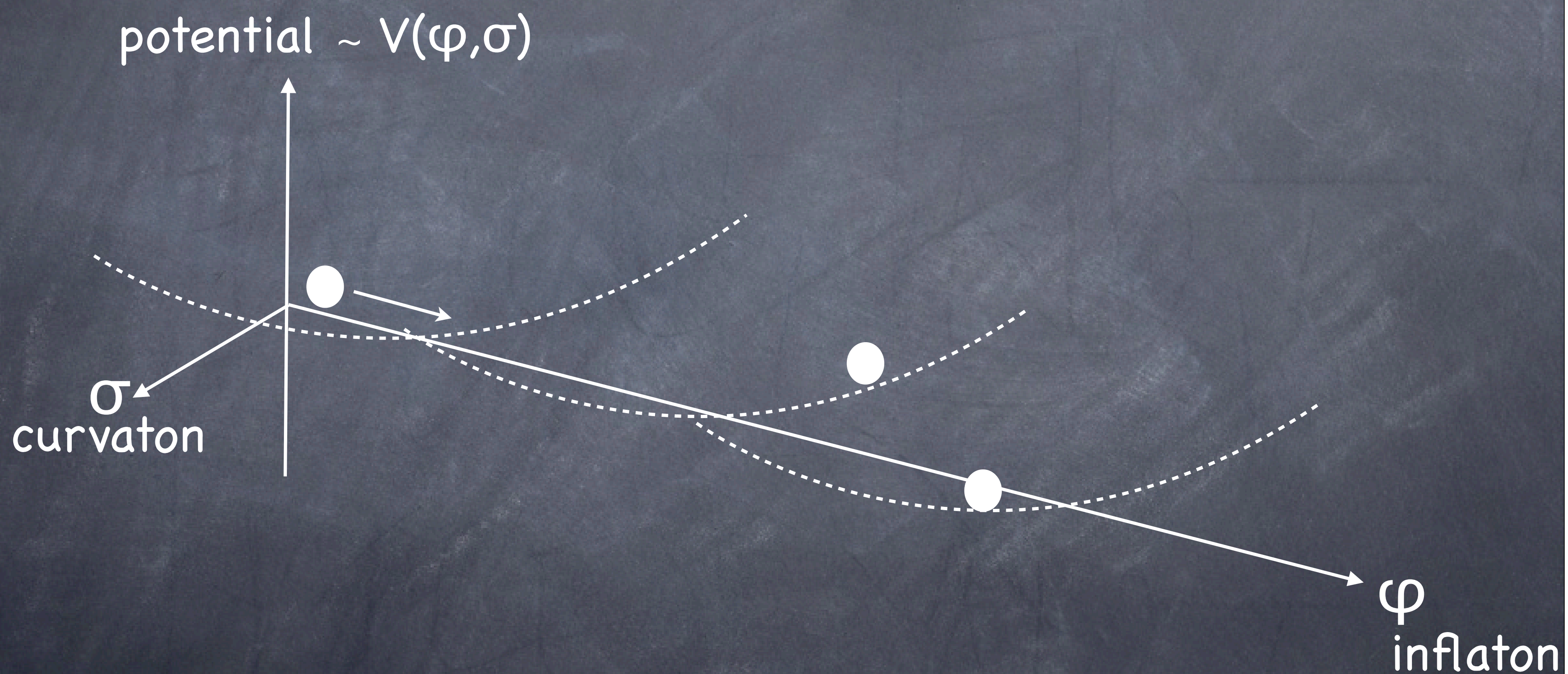
(Lyth and Wands 2002; Ichikawa, Suyama, Takahishi, Yamaguchi (2008); Tseliakhovich, Hirata, Slosar 2010)

| | "$f_{NL}$" | "$g_{NL}$" | "$\tau_{NL}$" |
|---|---|---|---|
| definition | $\Phi(x)=\Phi_G(x)+f_{NL}\ \Phi_G(x)^2$ | $\Phi(x)=\Phi_G(x)+g_{NL}\ \Phi_G(x)^3$ | $\Phi(x)=\varphi_G(x)+\sigma_G(x)$ $+f_{NL}\ (1+\xi^2)\ \sigma_G(x)^2$ <br> $\xi^2 = P_{\varphi\varphi}/P_{\sigma\sigma}$ <br> $\tau_{NL}= f_{NL}^2(1+\xi^2)$ |
| skewness | $\langle\Phi^3\rangle \approx 6f_{NL}\langle\Phi^2\rangle^2$ | $\langle\Phi^3\rangle = 0$ | $\langle\Phi^3\rangle \approx 6f_{NL}\langle\Phi^2\rangle^2$ |
| kurtosis | $\langle\Phi^4\rangle_c \approx 48\ f_{NL}^2\ \langle\Phi^2\rangle^3$ | $\langle\Phi^4\rangle_c \approx 24\ g_{NL}\ \langle\Phi^2\rangle^3$ | $\langle\Phi^4\rangle_c \approx 48\ \tau_{NL}\ \langle\Phi^2\rangle^3$ |
| short-long scale coupling | $\langle\Phi_s^2\rangle = \langle\Phi_s^2\rangle\ (1+4f_{NL}\Phi_l)$ | $\langle\Phi_s^2\rangle = \langle\Phi_s^2\rangle\ (1+6g_{NL}\Phi_l^2)$ <br> $\langle\Phi_s^3\rangle = 18\ g_{NL}\ \langle\Phi_s^2\rangle^2\ \Phi_l$ | $\langle\Phi_s^2\rangle =$ <br> $\langle\Phi_s^2\rangle\ (1+4(1+\xi^2)f_{NL}\sigma_l)$ |

# How would this happen?
# curvaton is a way to get "local-type" non-Gaussianity

total energy dominated by <u>inflaton</u>: $H^2 = 8\pi G/3\ V(\varphi,\sigma)$

perturbations dominated by <u>curvaton</u>: $P_\Phi(k) \approx P_\sigma(k)$




Linde and Mukhanov 1997; Lyth and Wands 2002

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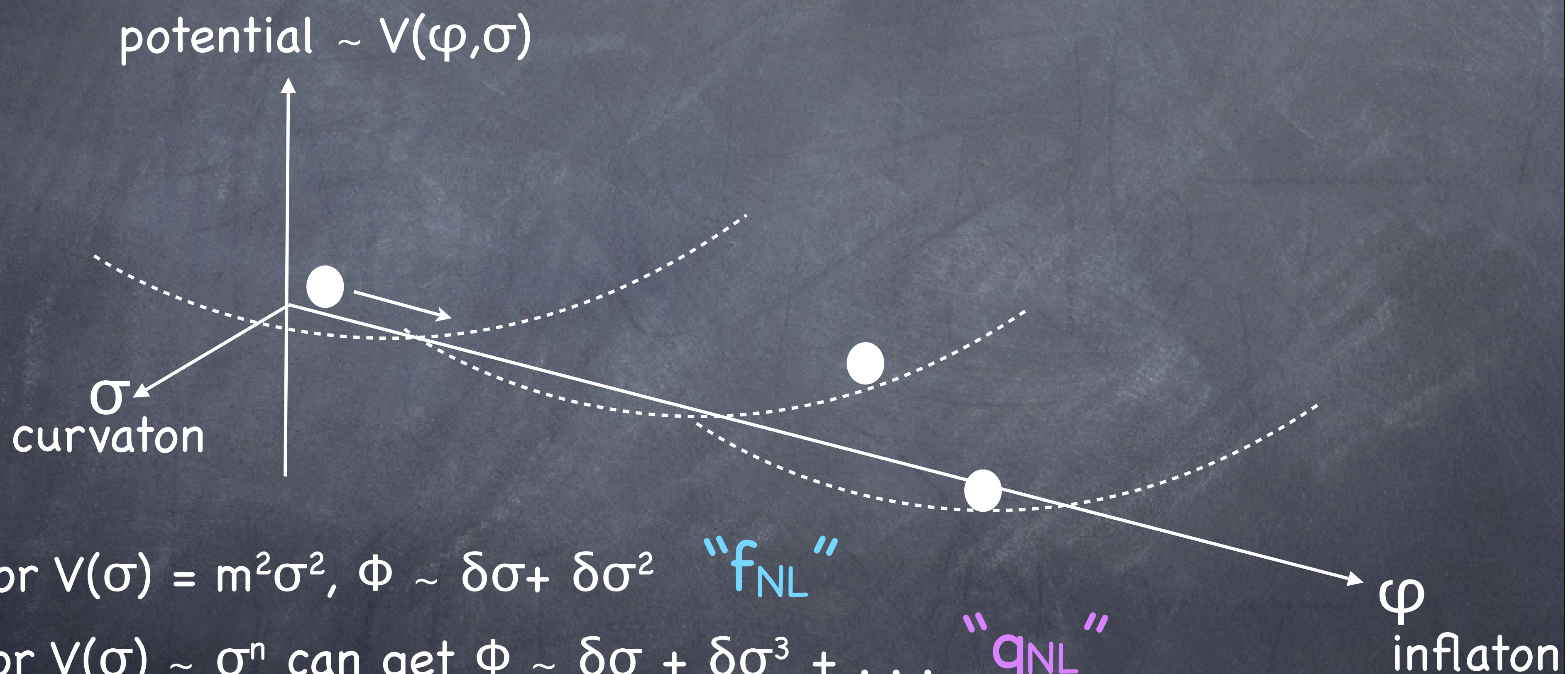



for $V(\sigma) = m^2\sigma^2$, $\Phi \sim \delta\sigma + \delta\sigma^2$ "$f_{NL}$"

for $V(\sigma) \sim \sigma^n$ can get $\Phi \sim \delta\sigma + \delta\sigma^3 + \ldots$ "$g_{NL}$"

if inflaton allowed to contribute, $\Phi \sim \delta\varphi + \delta\sigma + \delta\sigma^2 + \ldots$ "$\tau_{NL}$"

Linde and Mukhanov 1997; Lyth and Wands 2002

# Note:

single-field consistency relation

$$f_{NL} \approx \frac{\partial \ln k^3 P_\Phi}{\partial \ln k} = (n_s - 1)$$

also applies to $g_{NL}$ and $\tau_{NL}$

$$g_{NL} \approx \frac{\partial \ln k^6 B_\Phi}{\partial \ln k} = n_{NG}$$

$$\tau_{NL} \approx (n_s - 1)^2$$

e.g. Chen, Huang, Shiu 2008; Leblond & Pajer 2011
(see also Tanaka, Urakawa 2011)

also have,

$$\tau_{NL} \gtrsim f_{NL}^2$$

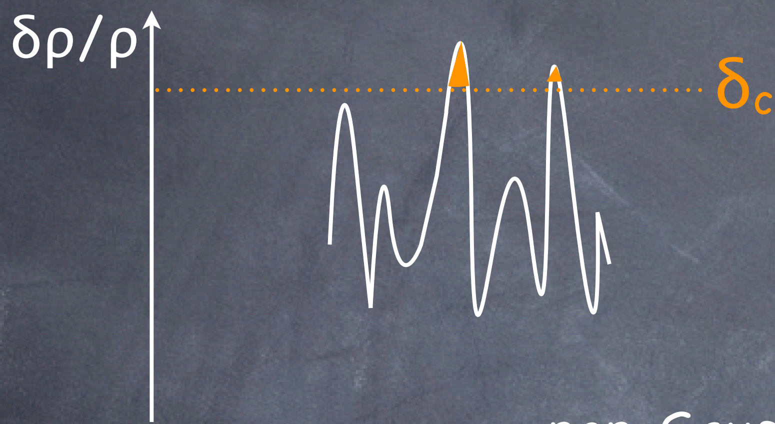
Suyama & Yamaguchi 2008; Sugiyama, Komatsu, Futamase 2011; Smith, ML, Zaldarriaga 2011

# What kinds of signatures in large-scale structure?

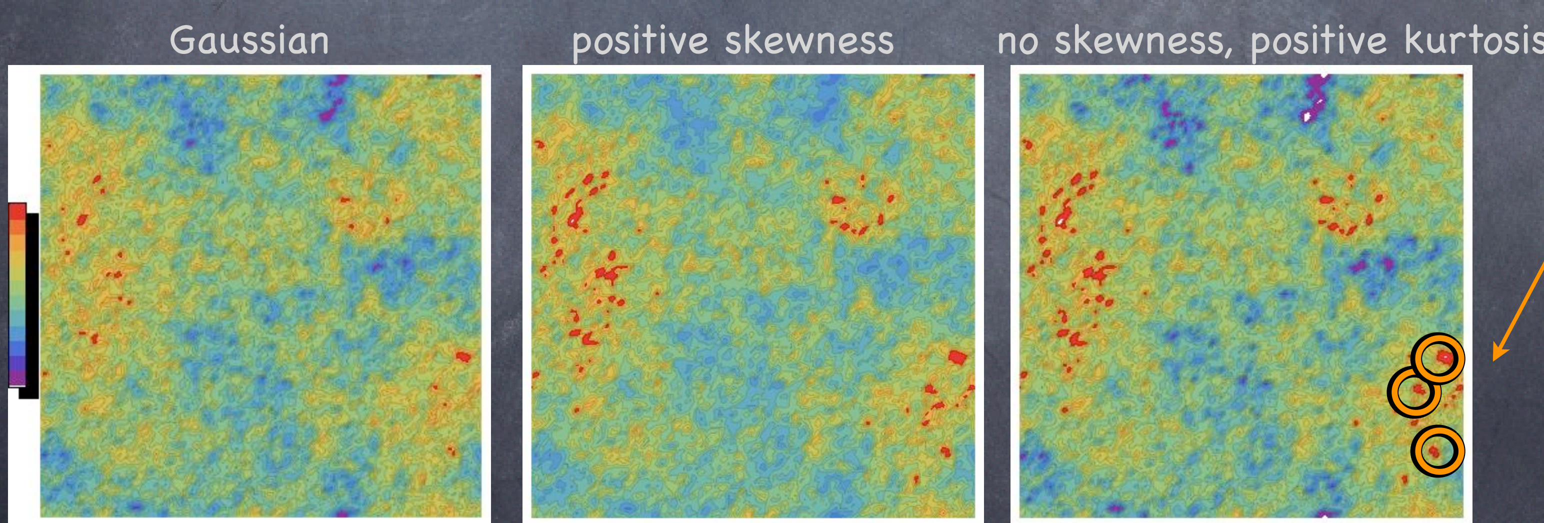
# Signatures in LSS I: more/fewer massive halos

# Signatures in LSS I: more/fewer massive halos

dark matter halos form in peaks of the density field




non-Gaussianity changes the number density of peaks




number of peaks ⇆ number of halos

Lucchin & Matarrese 1988; Chiu, Ostriker, Strauss 1998; Robinson, Gawiser, Silk 2000

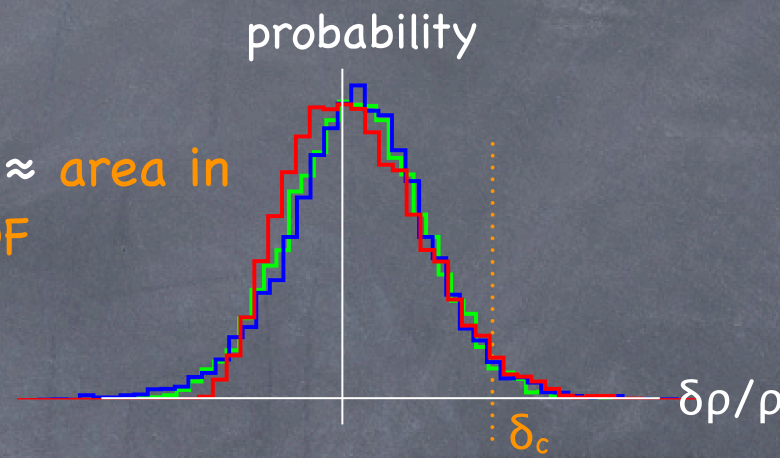
# Signatures in LSS I: more/fewer massive halos

number of peaks $\overset{\approx}{\Leftrightarrow}$ number of halos

number of peaks ≈ area in tail of PDF

PDF for $\delta(M) \leftrightarrow$ # of halos of mass M

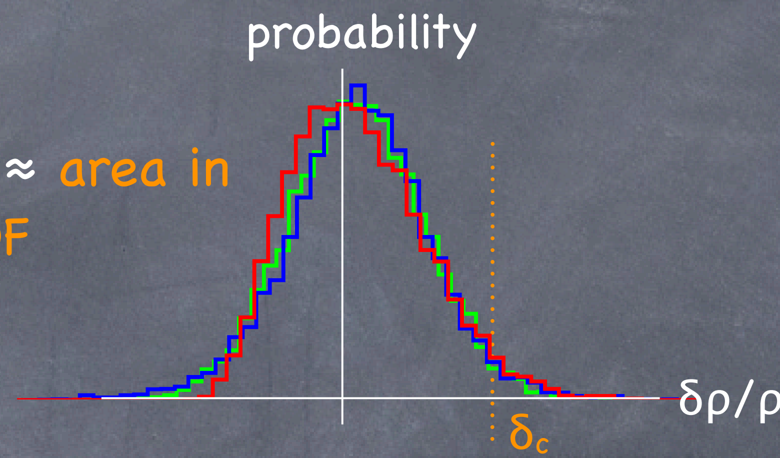
(Press & Schechter 1974)




Lucchin & Matarrese 1988; Chiu, Ostriker, Strauss 1998; Robinson, Gawiser, Silk 2000

# Signatures in LSS I: more/fewer massive halos

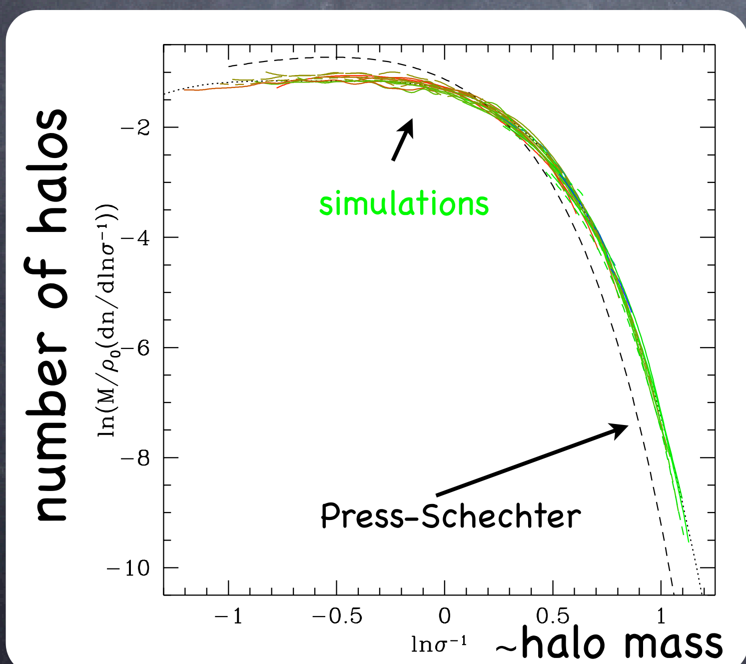
number of peaks ⇔ number of halos




number of peaks ≈ area in tail of PDF

PDF for δ(M) ↔ # of halos of mass M

(Press & Schechter 1974)



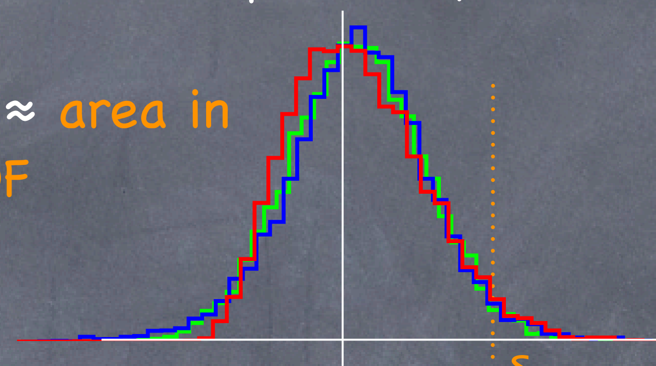

Jenkins et al 2000

Lucchin & Matarrese 1988; Chiu, Ostriker, Strauss 1998; Robinson, Gawiser, Silk 2000

# Signatures in LSS I: more/fewer massive halos

number of peaks ⇆̃ number of halos

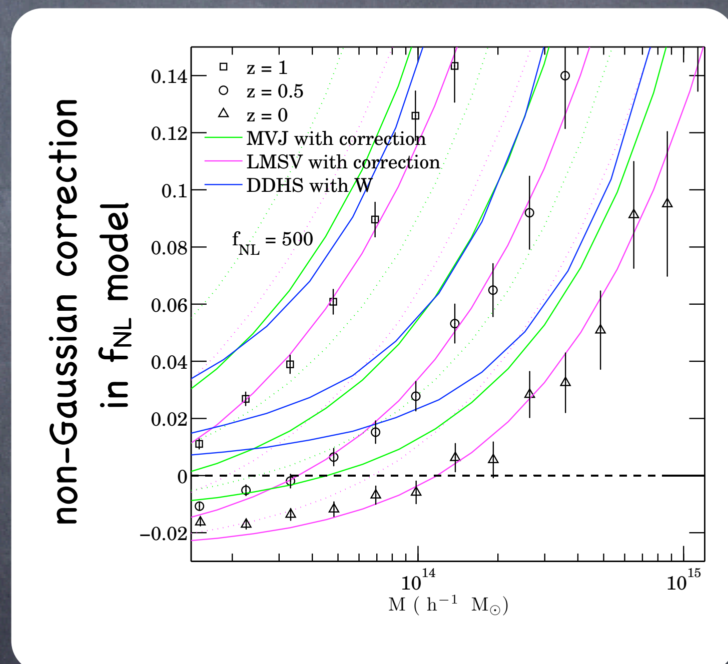
number of peaks ≈ area in tail of PDF




PDF for $\delta(M)$ ↔ # of halos of mass M

(Press & Schechter 1974)

has been applied to non-Gaussian cases by truncating an asymptotic expansion or Edgeworth series for the PDF

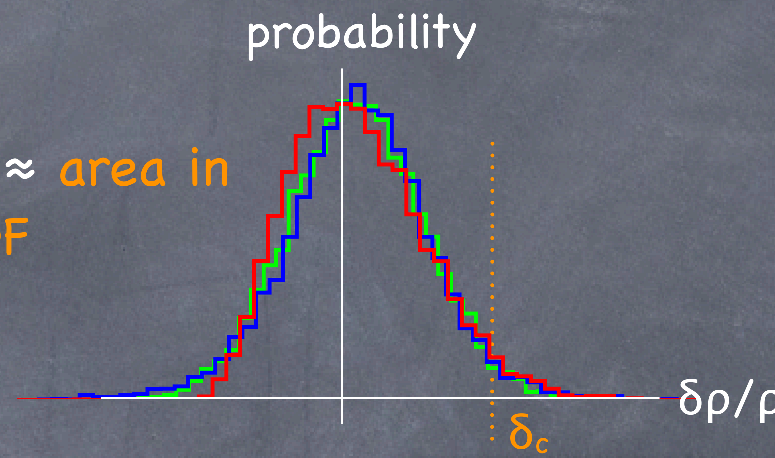



Pillepich, Porciani, Hahn 2008

Lucchin & Matarrese 1988; Chiu, Ostriker, Strauss 1998; Robinson, Gawiser, Silk 2000
Matarrese, Verde, Jimenez 2000; ML, Miller, Shandera, Verde 2007

# Signatures in LSS I: more/fewer massive halos

number of peaks $\stackrel{\approx}{\Leftrightarrow}$ number of halos




number of peaks ≈ area in tail of PDF

PDF for δ(M) ↔ # of halos of mass M

(Press & Schechter 1974)

Motivated by some issues with asymptotic & Edgeworth mass functions

we instead tried truncating log(PDF for δ(M))

then,

$$e^{\log(\text{PDF for } \delta(M))} \leftrightarrow \text{\# of halos of mass M}$$

use Edgeworth here

see also Lam & Sheth 2009; Maggiore & Riotto 2009; D'Amico, Musso, Norena, Paranjape 2010; Chongchitnan & Silk 2010

# Signatures in LSS I: more/fewer massive halos

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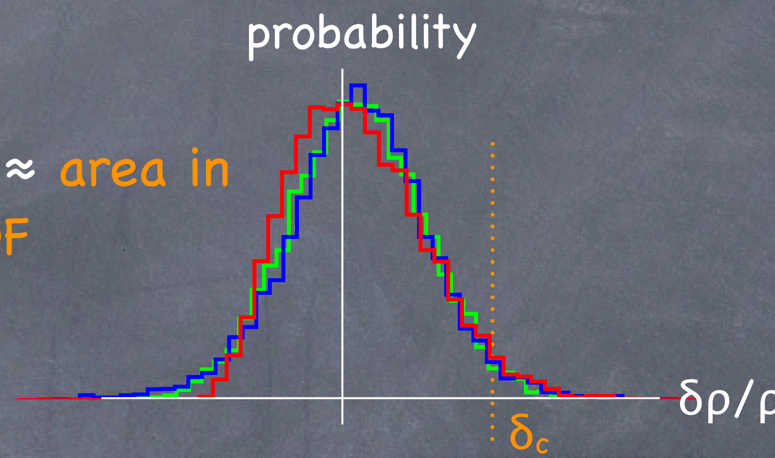


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"log-Edgeworth mass function"

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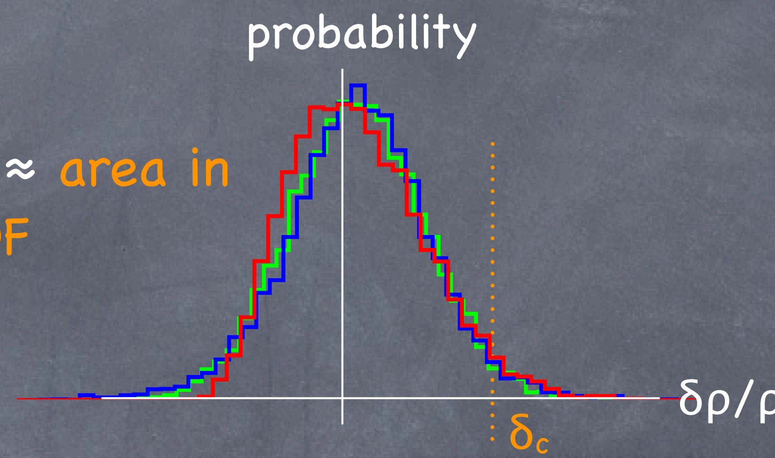


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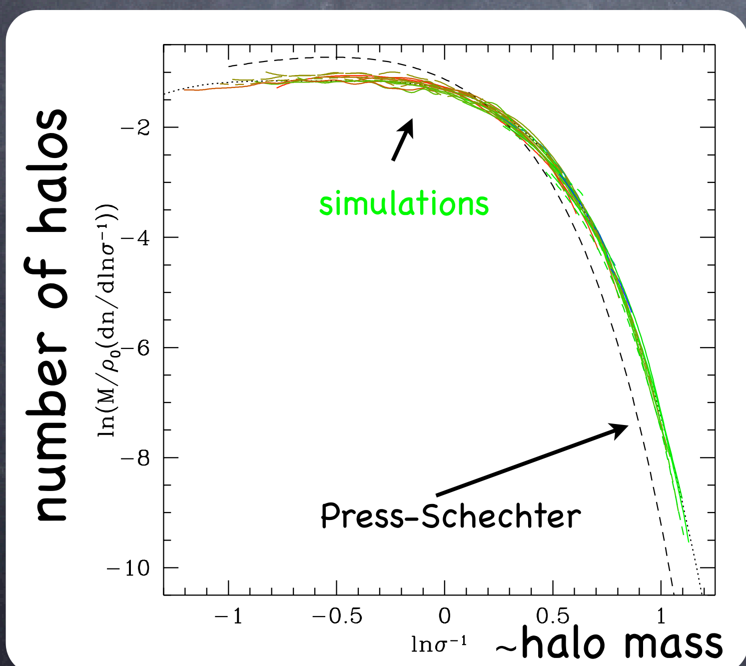
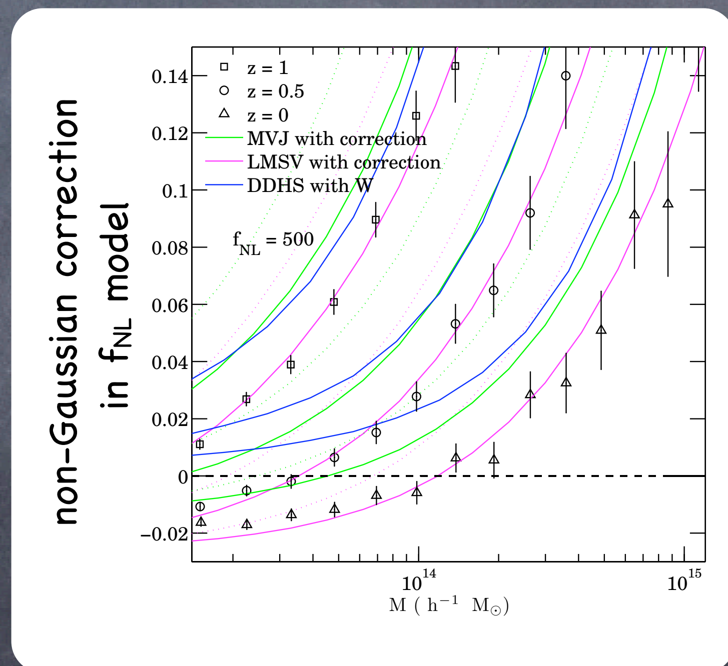



Jenkins et al 2000




Pillepich, Porciani, Hahn 2008

but anyway we need to compare with simulations!

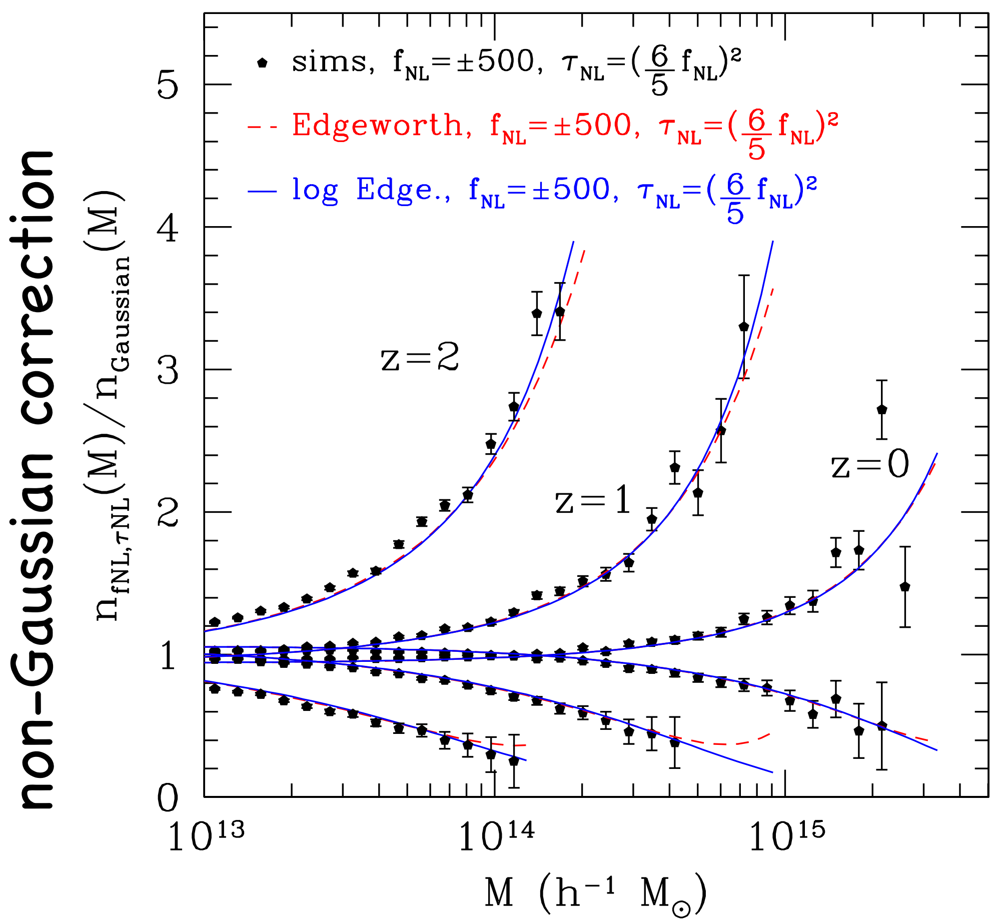
Lucchin & Matarrese 1988; Chiu, Ostriker, Strauss 1998; Robinson, Gawiser, Silk 2000 Matarrese, Verde, Jimenez 2000; ML, Miller, Shandera, Verde 2007

# Signatures in LSS I: more/fewer massive halos

## N-body simulations with $f_{NL}$, $g_{NL}$, and $\tau_{NL}$

$f_{NL}$

$\Phi(x) = \Phi_G(x) + f_{NL} (\Phi_G(x)^2 - \langle\Phi_G^2\rangle)$




ML & Smith 2010

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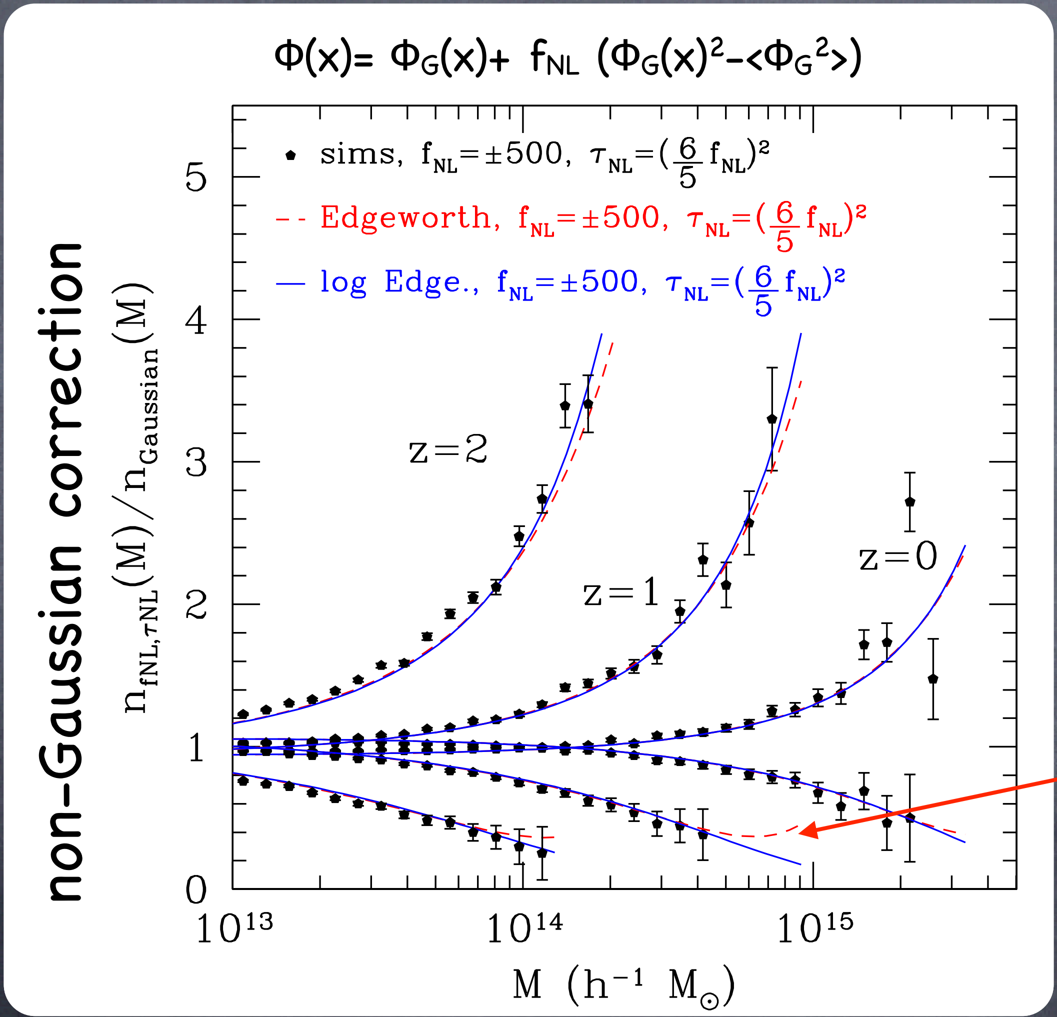



Edgeworth mass function looks good, but worry at high masses?

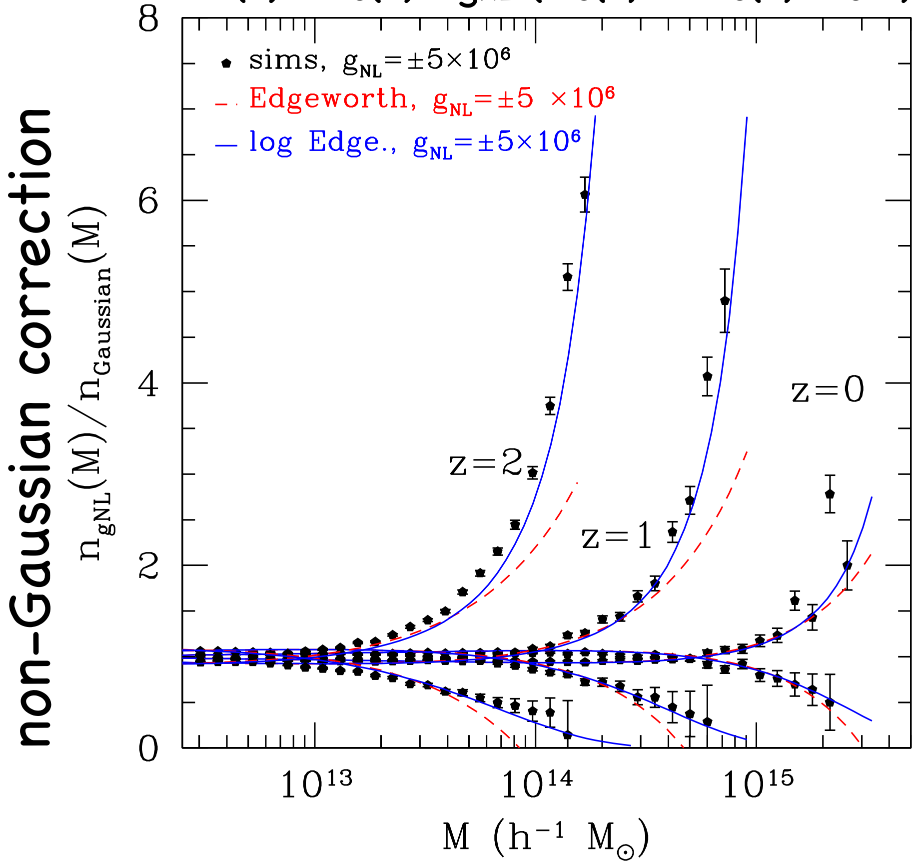
ML & Smith 2010

# Signatures in LSS I: more/fewer massive halos

## N-body simulations with $f_{NL}$, $g_{NL}$, and $\tau_{NL}$

$g_{NL}$

$\Phi(x) = \Phi_G(x) + g_{NL}\,(\Phi_G(x)^3 - 3\Phi_G(x)\langle\Phi_G{}^2\rangle)$?




kurtosis can have important effects on the mass function!

ML & Smith 2010
(see also Desjacques and Seljak 2010)

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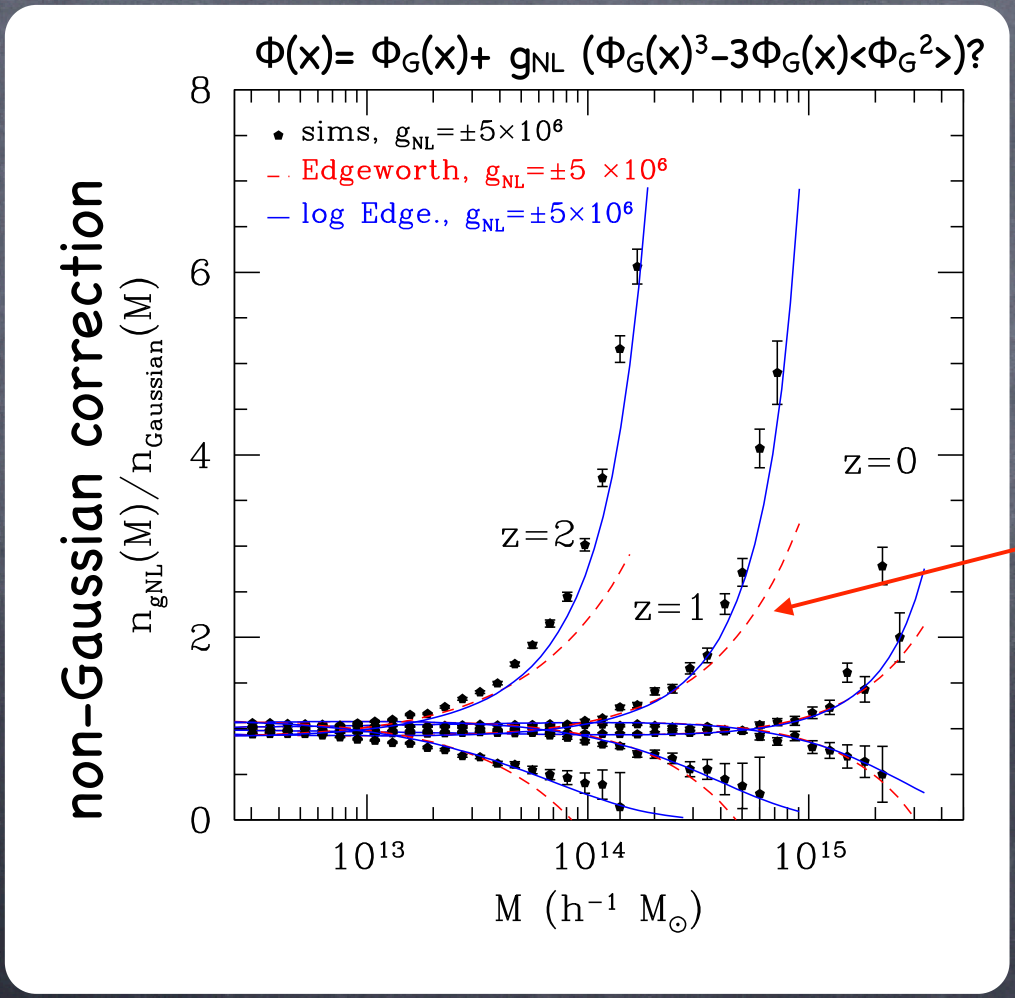


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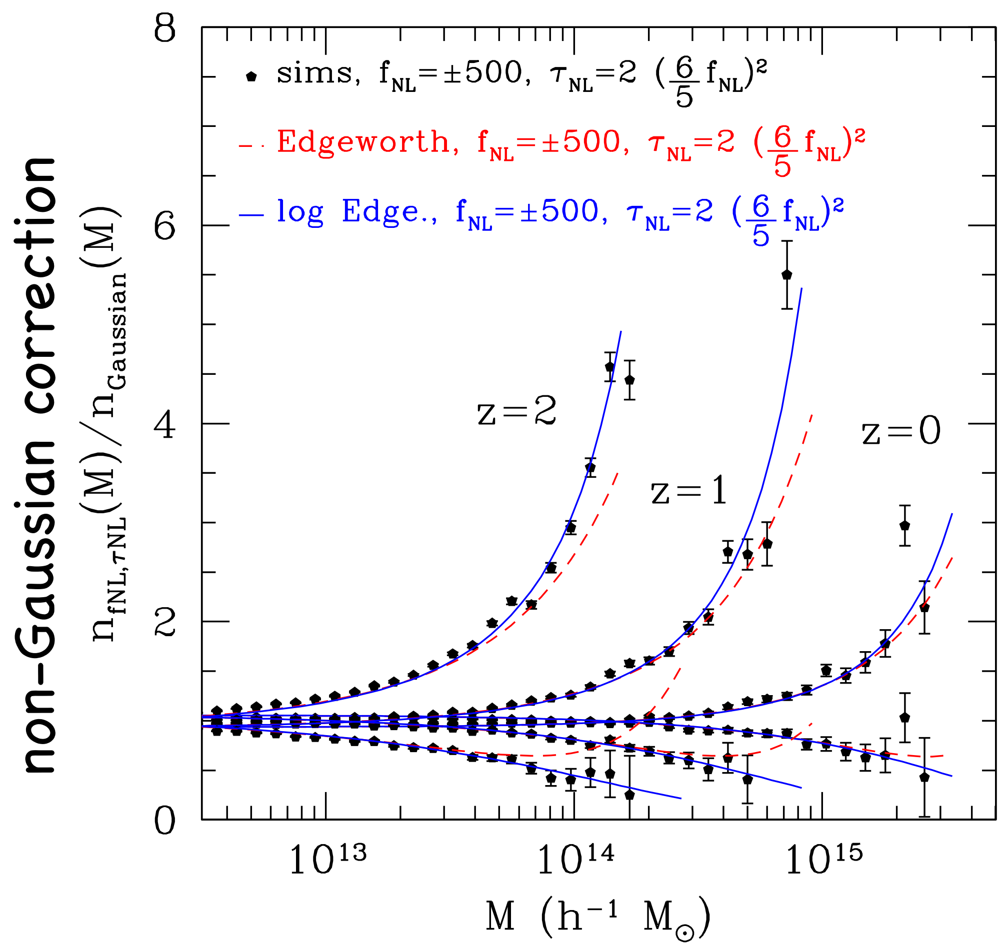
the new, log-Edgeworth expression looks a lot better!

ML & Smith 2010
(see also Desjacques and Seljak 2010)

# Signatures in LSS I: more/fewer massive halos

N-body simulations with $f_{NL}$, $g_{NL}$, and $\tau_{NL}$

## $f_{NL}$, $\tau_{NL}$ independent

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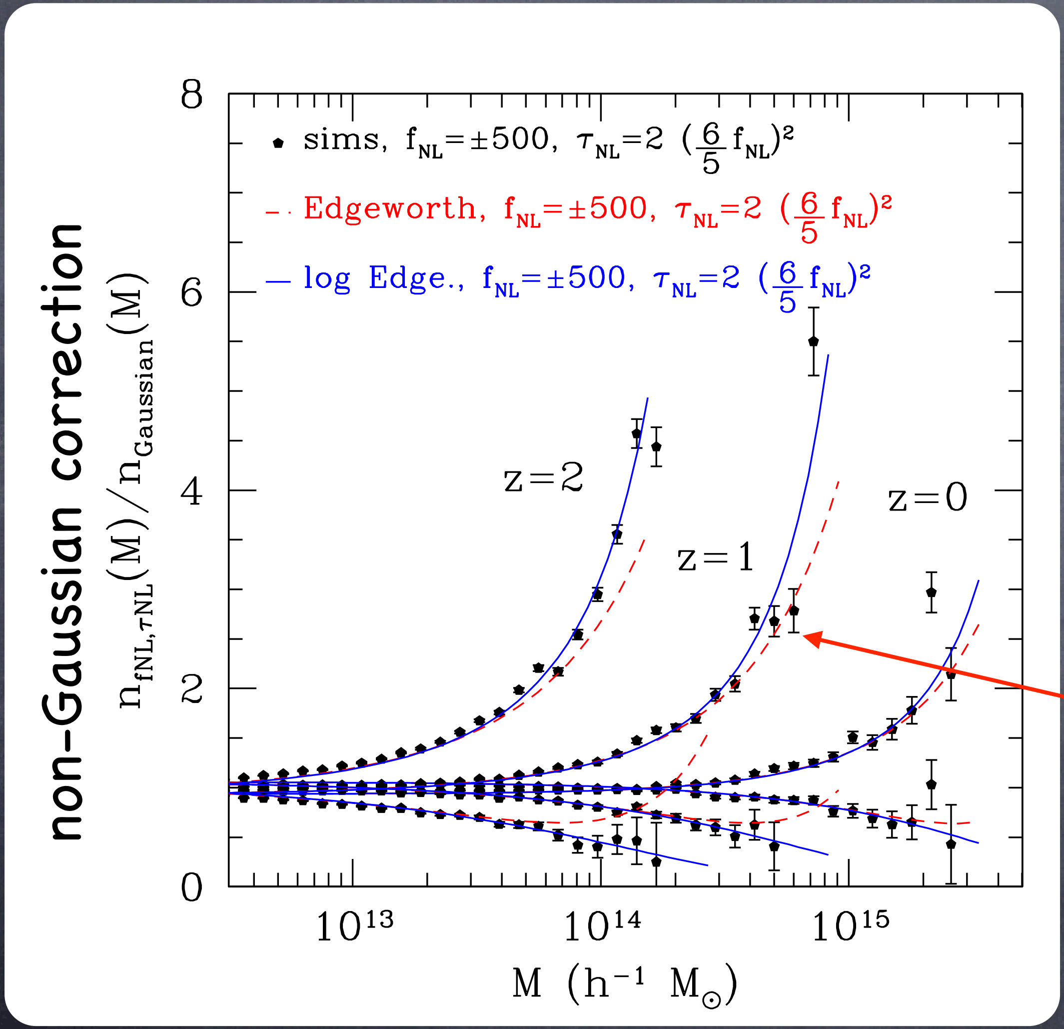


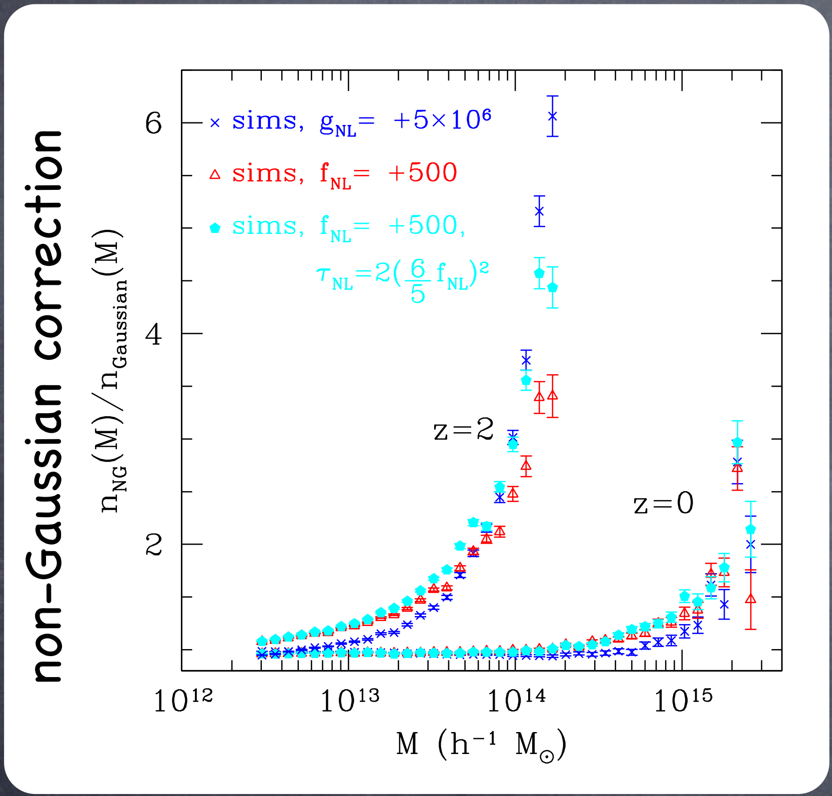

$\tau_{NL} \neq f_{NL}{}^2$ is noticeable!

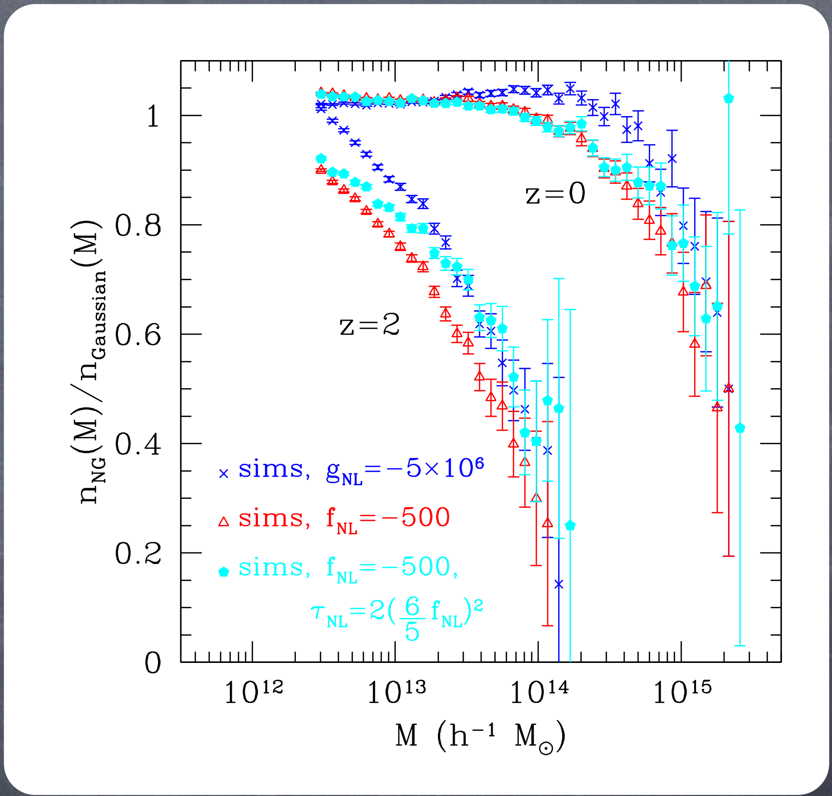
again, the new, log-Edgeworth expression looks a lot better!

ML & Smith 2010

# Signatures in LSS I: more/fewer massive halos

comparison of $f_{NL}$, $g_{NL}$, and $\tau_{NL}$




ML & Smith 2010

# Signatures in LSS I: more/fewer massive halos

The log-Edgeworth is a good fit for $f_{NL}$, $g_{NL}$, and $\tau_{NL}$ , even at high masses and redshifts!

but cosmology with clusters is <u>hard</u>

# Signatures in LSS I: more/fewer massive halos

poss. advantage is insensitivity to "shape" of NG

$\langle\delta_M{}^2\rangle$ ,$\langle\delta_M{}^3\rangle$, $\langle\delta_M{}^4\rangle_c$ ⟶ $n_{NG}(M)$

smoothed variance,
skewness, kurtosis

Don't need to know $B(k_1,k_2,k_3)$, $T(k_1,k_2,k_3,k_4)$; "local", "equilateral" info integrated out

(see also Wagner, Verde, Boubekeur 2010)

more to explore: halo finders, mass-observable relation

(these issues apply to using clusters for dark energy also)

# Signatures in LSS II: scale-dependent halo bias

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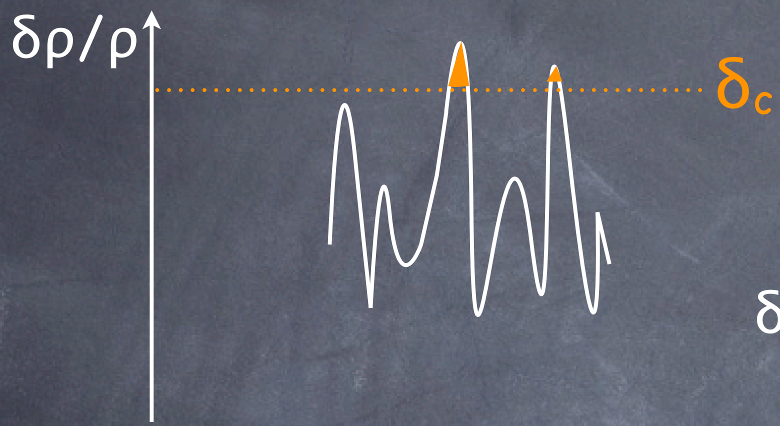
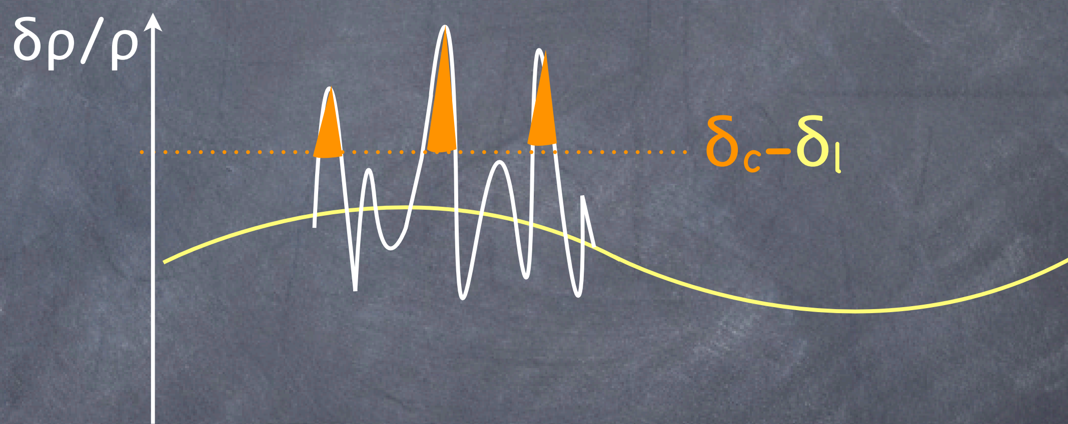
a dark matter halo forms when $\delta\rho/\rho$ is larger than the collapse threshold




which is easier to reach on top of a long wavelength density perturbation




so the number of halos fluctuates depending on $\delta_l$

$$\delta n/n = \frac{\partial n}{\partial \delta}\,\delta_l \;.\;.\;.$$

# Signatures in LSS II: scale-dependent halo bias

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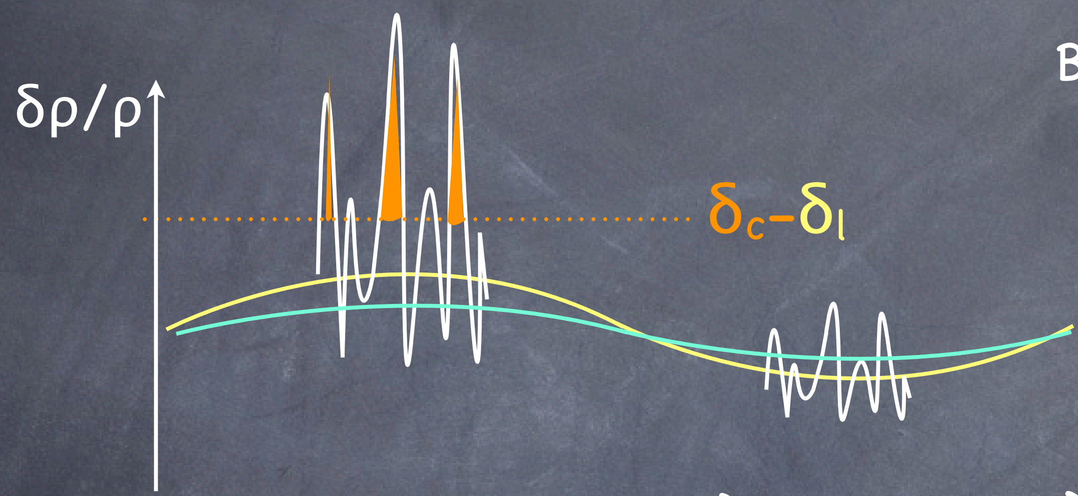


BUT with $f_{NL}$, the small-scale power fluctuates also depending on $\Phi_l$

$$\delta n = \frac{\partial n}{\partial \delta}\,\delta_l + 4 f_{NL} \frac{\partial n}{\partial P_s}\,\Phi_l \ldots$$

Poisson's

$$\nabla^2 \Phi_l \sim 4\pi G\, \delta_l$$

$$\delta n \sim \left( \frac{\partial n}{\partial \delta} + \frac{4 f_{NL}}{k^2} \frac{\partial n}{\partial P_s} \right) \delta_l$$

Dalal, Doré, Huterer, Shirokov 2007

Matarrese & Verde 2008; Slosar, Hirata, Seljak, Ho, Padmanabhan 2008; Afshordi & Tolley 2008; McDonald 2008

# Signatures in LSS II: scale-dependent halo bias

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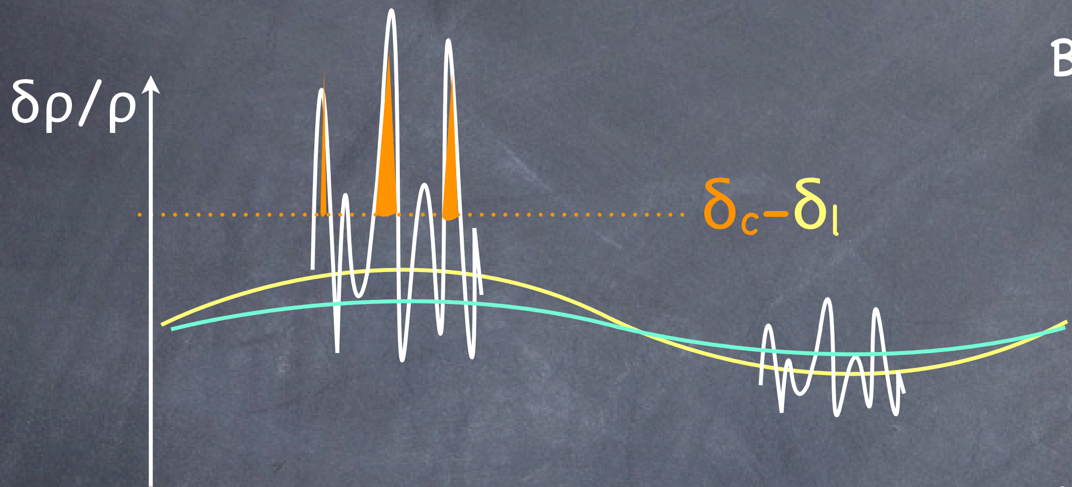


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this $1/k^2$ scaling is hard to generate with local (post-inflationary) processes ⟶ powerful test!

Dalal, Doré, Huterer, Shirokov 2007

Matarrese & Verde 2008; Slosar, Hirata, Seljak, Ho, Padmanabhan 2008; Afshordi & Tolley 2008; McDonald 2008

# Signatures in LSS II: scale-dependent halo bias

so on large scales $P_{n\delta} \sim \left(b + \frac{2 f_{NL} \delta_c}{k^2}(b-1)\right) P_{\delta\delta}$

where, halo bias $\mathbf{b} = 1 + \frac{1}{n}\frac{\partial n}{\partial \delta}$ and $P_s \frac{\partial n}{\partial P_s} = \frac{\delta_c}{2}\frac{\partial n}{\partial \delta}$

(need simulations to accurately predict these derivatives)

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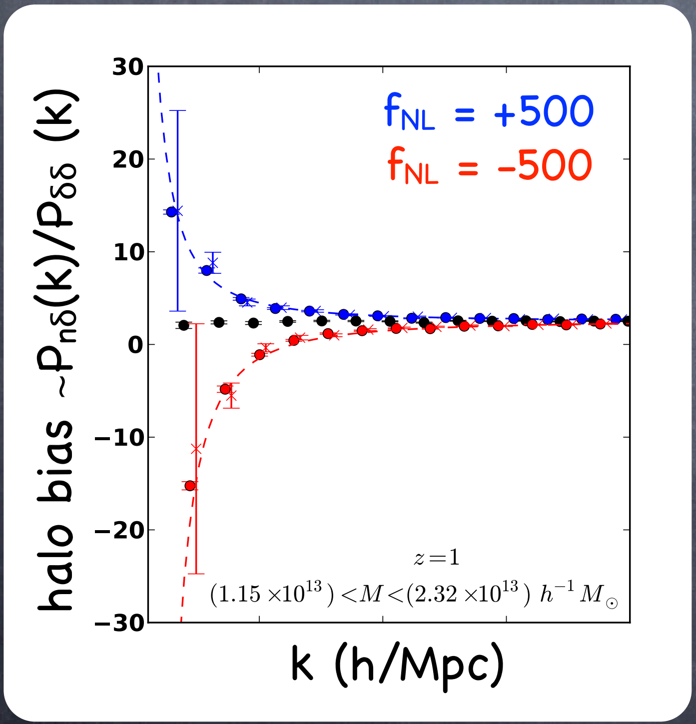
for example,

(our sims)

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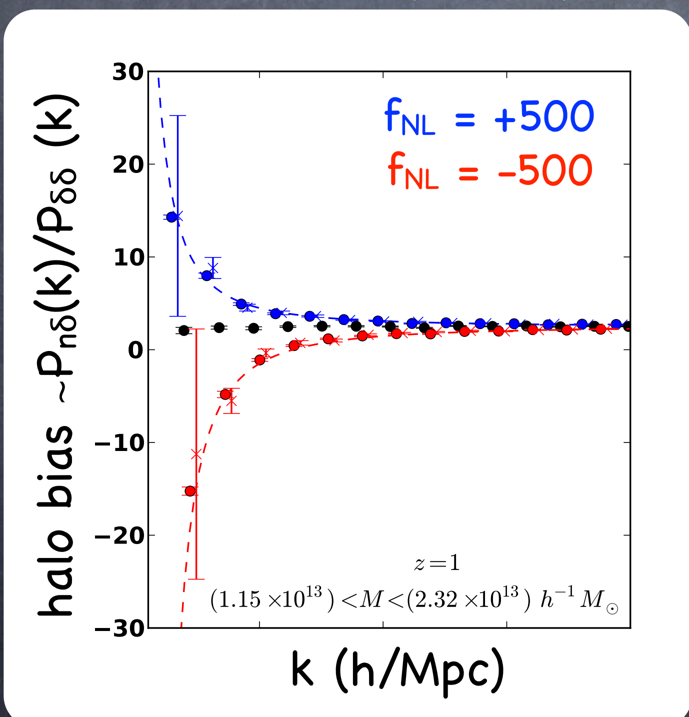
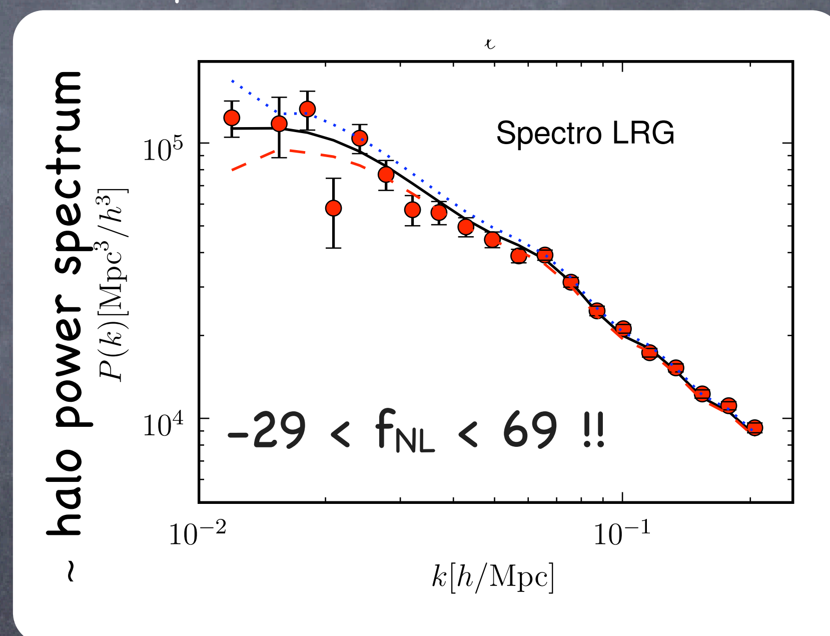



example data




Slosar, Hirata, Seljak, Ho, Padmanabhan 2008

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# Signatures in LSS II: scale-dependent halo bias

for the $g_{NL}$ model, the local skewness fluctuates depending on $\Phi_l$ : $\langle \Phi_s^3 \rangle = 18\, g_{NL} \,\langle \Phi_s^2 \rangle^2\, \Phi_l$

so halo numbers fluctuate as

$$\delta n/n = b\, \delta_l + 3\, g_{NL} \frac{\partial \ln n}{\partial f_{NL}} \Phi_l \ldots$$

(recall, for $f_{NL}$: skewness ~ $6 f_{NL} \,\langle \Phi_s^2 \rangle^2$)

Smith, Ferraro, ML 2011

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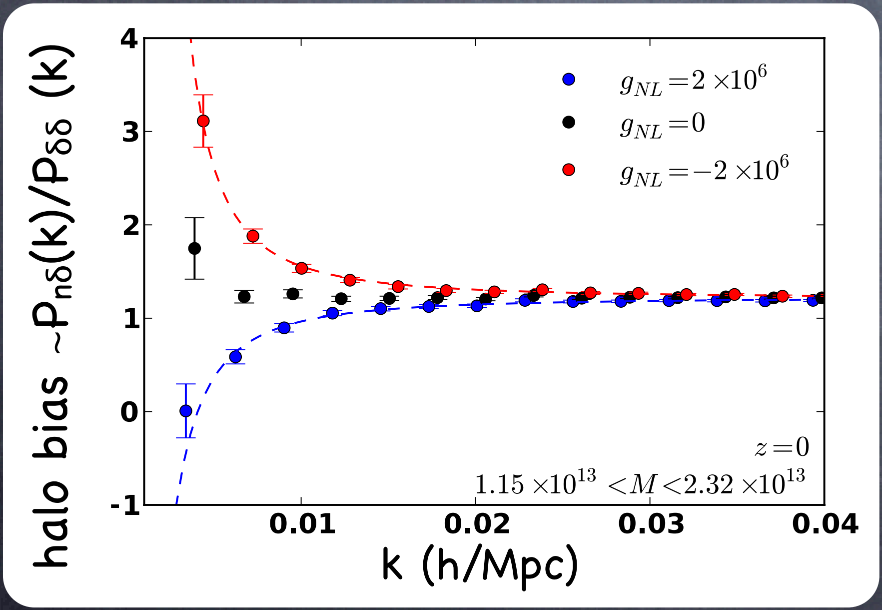


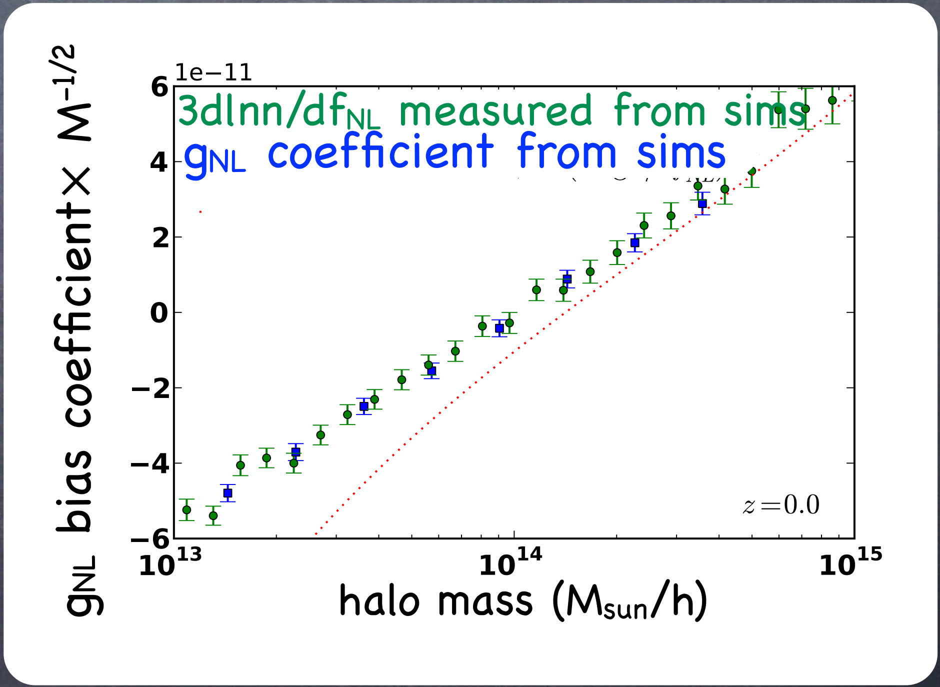


Smith, Ferraro, ML 2011

(see also Desjacques and Seljak 2010; Desjacques, Jeong, Schmidt 2011)

# Signatures in LSS II: scale-dependent halo bias

bias coefficient for $g_{NL}$ in terms of mass

$$b_{g_{NL}}(k) = b + \frac{3 g_{NL}}{k^2} \frac{\partial \ln n(M)}{\partial f_{NL}}$$

contrast w/$f_{NL}$ where coefficient in terms of bias

$$b_{f_{NL}}(k) = b + \frac{2\,\delta_c\, f_{NL}\,(b-1)}{k^2}$$

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we have a fit for $g_{NL}$ in terms of bias:

$$b_{g_{NL}}(k) \sim b + g_{NL} \frac{\text{non-linear function}(b)}{k^2}$$

form will depend on selection of population in M, z

# Signatures in LSS II: scale-dependent halo bias
## Summary:

local non-Gaussianity

$$\Phi(x)=\Phi_G(x)+ f_{NL} (\Phi_G(x)^2-<\Phi_G{}^2>) + g_{NL}(\Phi_G(x)^3-\Phi_G<\Phi_G{}^2>)$$

$\longrightarrow$ scale dependent halo bias

$$b_{f_{NL},g_{NL}} (k) \sim b + \frac{f_{NL},g_{NL} \times \text{constant}}{k^2}$$

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impossible to generate
with single field inflation!

e.g. Creminell, D'Amico, Musso, Noreña 2011

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precise values of $f_{NL}$, $g_{NL}$ will require care -- but seeing $1/k^2$ is the most exciting part

# Signatures in LSS III: stochastic halo bias

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$f_{NL}$ model: $\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{NL}\,(\Phi_G^2(\mathbf{x}) - \langle\Phi_G^2\rangle)$

* $\langle\Phi_s^2\rangle = \langle\Phi_{G,s}^2\rangle\,(1 + 4\,f_{NL}\,\Phi_{G,l})$ and $\delta = \nabla^2\Phi/4\pi G\rho$

$\delta n/n = b\,\delta_l + 2f_{NL}(b-1)/\delta_c\,\Phi_l\ldots$

$\tau_{NL}$ model, $\Phi(\mathbf{x}) = \varphi_G(\mathbf{x}) + \sigma_G(\mathbf{x}) + f_{NL}\,(1+\xi^2)^2(\sigma_G^2(\mathbf{x}) - \langle\sigma_G^2\rangle)$

* $\langle\Phi_s^2\rangle = \langle\Phi_s^2\rangle\,(1 + 4\,f_{NL}\,(1+\xi^2)\,\sigma_{G,l})$ BUT $\delta = \nabla^2(\varphi+\sigma)/4\pi G\rho$

* $\delta n/n = b\,\delta_l + 2f_{NL}\,(1+\xi^2)\,(b-1)\sigma_l\ldots$

$\xi^2 = P_{\varphi\varphi}(k)/P_{\sigma\sigma}(k)$

Tseliakhovich, Hirata, Slosar 2010

# Signatures in LSS III: stochastic halo bias

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* $\delta n/n = b\,\delta_l + 2f_{NL}\,(1+\xi^2)\,(b-1)\sigma_l \ldots$

$\xi^2 = P_{\varphi\varphi}(k)/P_{\sigma\sigma}(k)$

σ fluctuates independently of φ and therefore δ → halos stochastic w.r.t dark matter

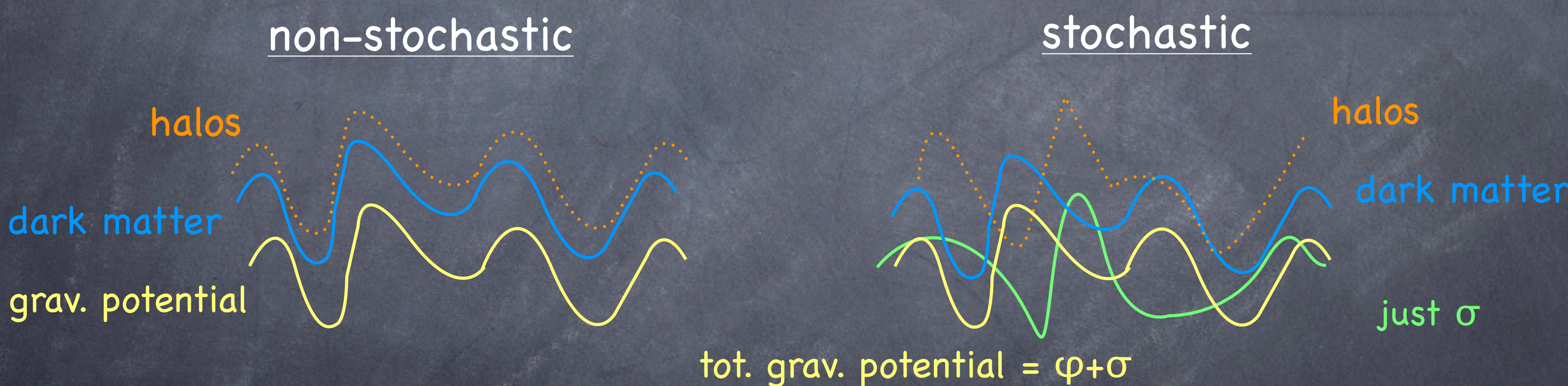
Tseliakhovich, Hirata, Slosar 2010

# Signatures in LSS III: stochastic halo bias

halos are now stochastic w.r.t. dark matter $\delta$

$$\delta n/n = b\ \delta_l + 2f_{NL}(1+\xi^2)(b-1)/\delta_c\ \sigma_l\ldots$$

because $\sigma$ fluctuates independently of $\delta$




Tseliakhovich, Hirata, Slosar 2010
Smith & ML 2011

# Signatures in LSS III: stochastic halo bias

## what does it look like?

$P_{n\delta}(k) \sim (b + 2f_{NL}(b-1)/\delta_c k^2)\, P_{\delta\delta}$

$P_{nn}(k) \sim (b + 2f_{NL}(b-1)/\delta_c k^2)^2 P_{\delta\delta} + (2f_{NL}(b-1)/\delta_c k^2)^2\, \xi^2 P_{\sigma\sigma}$

stochasticity $\propto (\tau_{NL} - f_{NL}^2)$

$\xi^2 = P_{\varphi\varphi}(k)/P_{\sigma\sigma}(k)$

$\tau_{NL} = (1 + \xi^2)\, f_{NL}^2$

Smith & ML 2011

# Signatures in LSS III: stochastic halo bias

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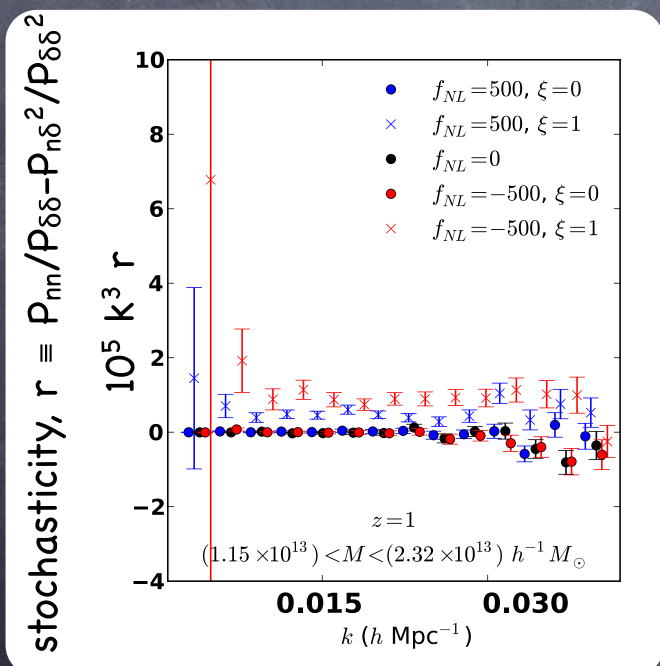
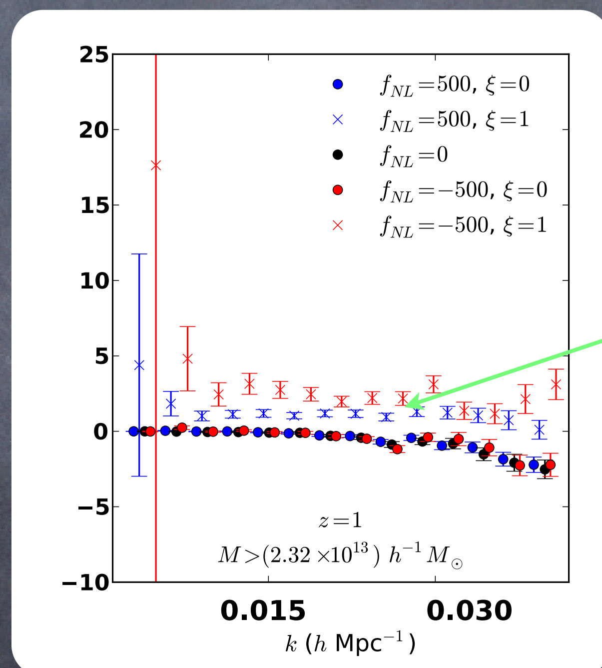
$\xi^2 = P_{\varphi\varphi}(k)/P_{\sigma\sigma}(k)$

$\tau_{NL} = (1+\xi^2)\, f_{NL}^2$




models with $\xi \neq 0$ indeed stochastic

$(\tau_{NL} \neq f_{NL}^2)$

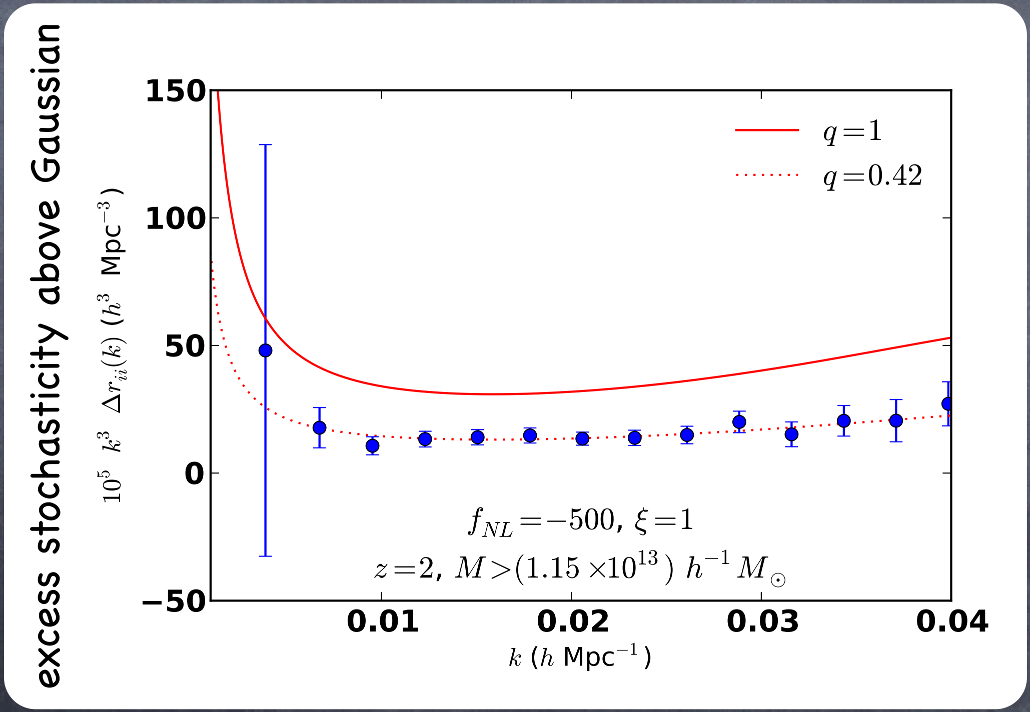
N.B. the bias factor in $P_{n\delta}$ is unchanged from $f_{NL}$-only model

Smith & ML 2011

# Signatures in LSS III: stochastic halo bias

## does stochasticity agree with predictions?

$P_{nn}(k) \sim (b + 2f_{NL}(b-1)/\delta_c k^2)^2 P_{\delta\delta} + (2f_{NL}(b-1)/\delta_c k^2)^2 \xi^2 P_{\sigma\sigma}$




um, shape looks good but not amplitude

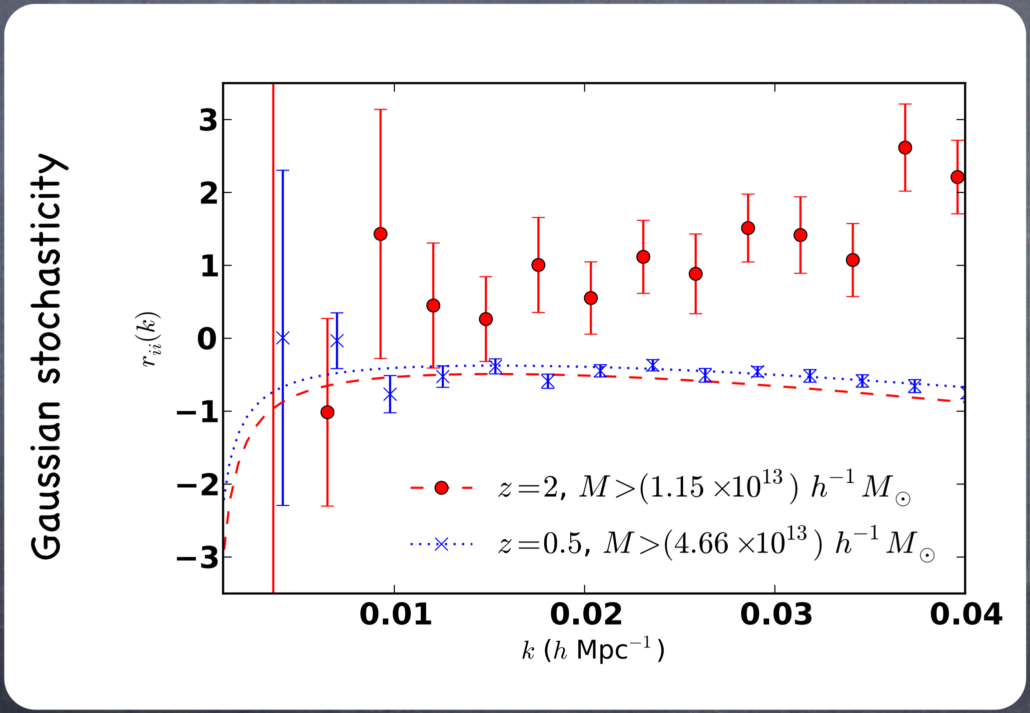
tends to look better at low masses, low $f_{NL}$

Smith & ML 2011

# Signatures in LSS III: stochastic halo bias

## does stochasticity agree with predictions?

not great even in the Gaussian case . . .



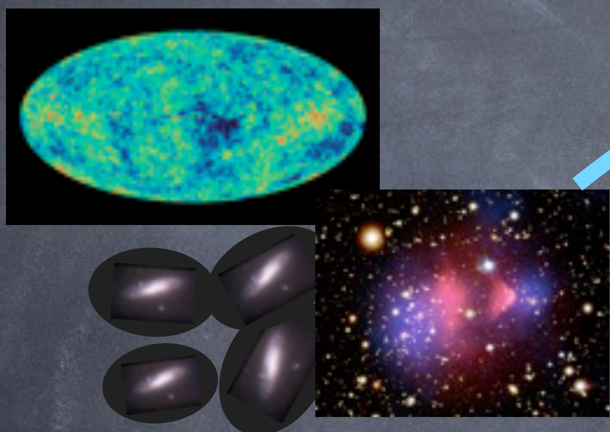

(curves are the halo model predictions)

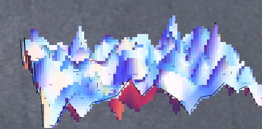
Smith & ML 2011

see also Hamaus, Seljak, Desjacques, Smith, Baldauf 2010

# Summary

galaxy surveys + clever theory + N-body simulations





inflationary perturbations

time + gravity

# Summary

- Non-Gaussian initial conditions can significantly change the abundance of dark matter halos
- We've found an analytic description for the halo mass function that compares well to N-body for $f_{NL}$, $g_{NL}$ and $\tau_{NL}$ -- perhaps it works for more general forms of NG?
- Large-scale halo bias and stochasticity can be dramatically altered by non-Gaussianity.
- Analytic descriptions of bias agree well with sims (but still need to determine Gaussian parameters from sims)
- If two-fields generate perturbations (and only one is non-Gaussian) halo bias becomes stochastic, but the analytic description typically overpredicts the amplitude