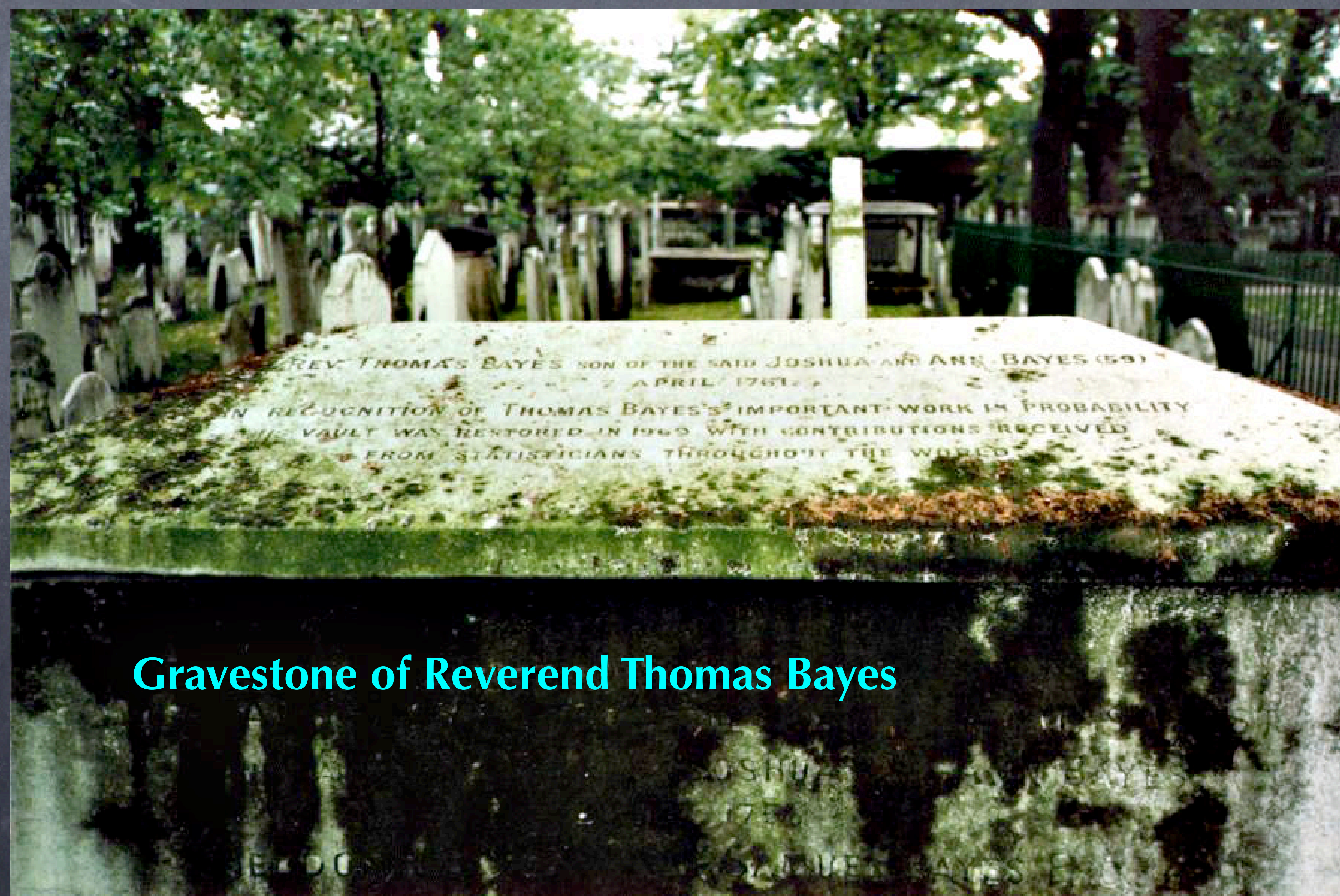
University of Hawaii
Institute for Astronomy

University of Sussex

Model Selection and Dark Energy

Andrew Liddle
February 2007

Microsoft-free presentation



Gravestone of Reverend Thomas Bayes

Parameter estimation and model selection

Parameter estimation and model selection

Bayesian parameter estimation

Choose model:

Set of parameters to be varied

Prior ranges for those parameters

Compute likelihood function

Obtain posterior parameter distribution

Interpret

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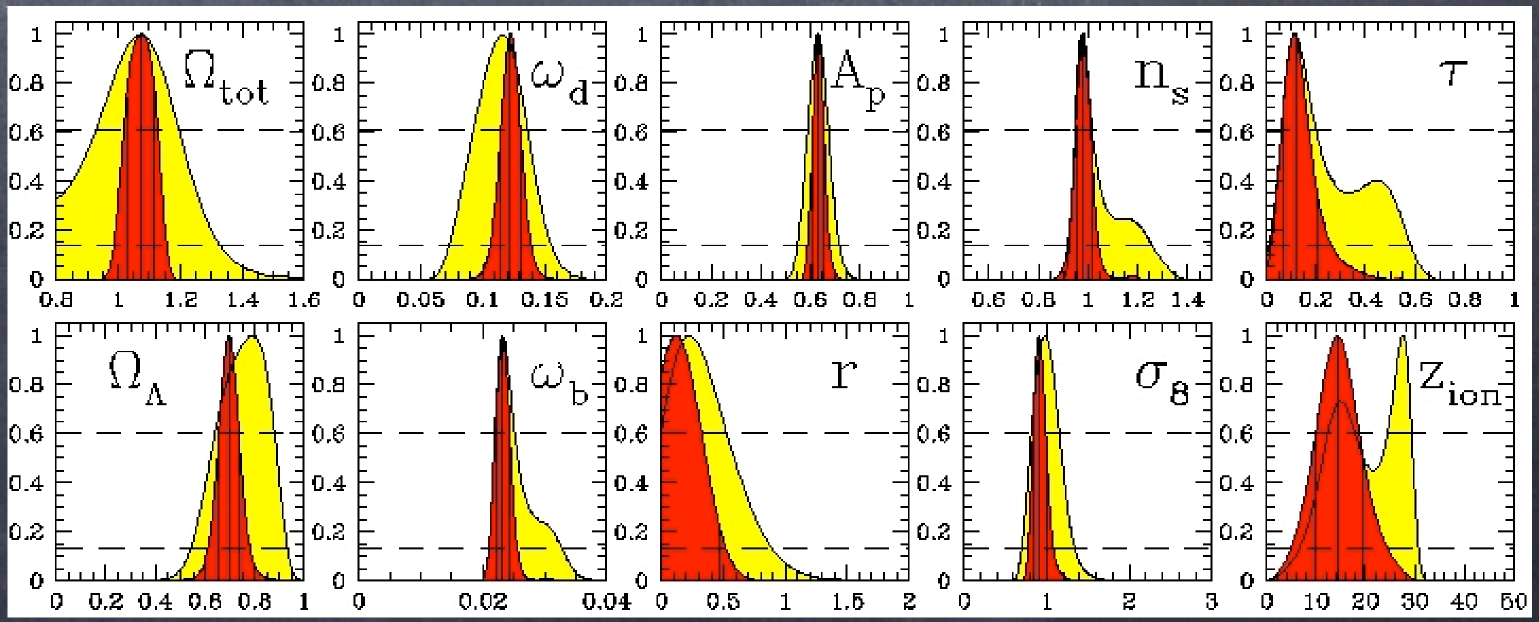

Tegmark et al.

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Assign model probability $P(M_1)$

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Compute model likelihoods, known as the Bayesian evidence
Update prior model probabilities to posterior ones
[option: multi-model inference by bayesian model averaging]
Interpret

Model Selection

Model selection is the study of **sets** of parameters. It is a higher level of inference than parameter estimation.

In many contexts we don't actually know which parameters are the relevant ones. A particular situation is deciding if a new parameter, describing some new physical effect seen in data, is actually required.

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A suitable baseline cosmological model to consider is the simplest one giving an adequate fit to current data. It is a spatially-flat adiabatic ΛCDM model with five fundamental parameters and two phenomenological ones.

$\Omega_{\rm m}$	matter density
$\Omega_{\rm b}$	baryon density
$\Omega_{\rm r}$	radiation density
h	hubble parameter
A	adiabatic density perturbation amplitude
τ	reionization optical depth
b	bias parameter (or parameters)

There are many, many ways in which this base cosmological model can be extended.

Table 2. Candidate parameters: those which might be relevant for cosmological observations, but for which there is presently no convincing evidence requiring them. They are listed so as to take the value zero in the base cosmological model. Those above the line are parameters of the background homogeneous cosmology, and those below describe the perturbations. Of the latter set, the first six refer to adiabatic perturbations, the next three to tensor perturbations, and the remainder to isocurvature perturbations.

Ω_k	spatial curvature
$N_\nu - 3.04$	effective number of neutrino species (CMBFAST definition)
m_{ν_i}	neutrino mass for species 'i'
	[or more complex neutrino properties]
$m_{\rm dm}$	(warm) dark matter mass
$w + 1$	dark energy equation of state
dw/dz	redshift dependence of w
	[or more complex parametrization of dark energy evolution]
$c_{\rm S}^2 - 1$	effects of dark energy sound speed
$1/r_{\rm top}$	topological identification scale
	[or more complex parametrization of non-trivial topology]
$d\alpha/dz$	redshift dependence of the fine structure constant
dG/dz	redshift dependence of the gravitational constant

$n - 1$	scalar spectral index
$dn/d\ln k$	running of the scalar spectral index
$k_{\rm cut}$	large-scale cut-off in the spectrum
$A_{\rm feature}$	amplitude of spectral feature (peak, dip or step) ...
$k_{\rm feature}$	... and its scale
	[or adiabatic power spectrum amplitude parametrized in N bins]
$f_{\rm NL}$	quadratic contribution to primordial non-gaussianity
	[or more complex parametrization of non-gaussianity]
r	tensor-to-scalar ratio
$r + 8n_{\rm T}$	violation of the inflationary consistency equation
$dn_{\rm T}/d\ln k$	running of the tensor spectral index
$\mathcal{P}_S$	CDM isocurvature perturbation ...
n_S	... and its spectral index ...
$\mathcal{P}_{S\mathcal{R}}$	... and its correlation with adiabatic perturbations ...
$n_{S\mathcal{R}} - n_S$	... and the spectral index of that correlation
	[or more complicated multi-component isocurvature perturbation]
$G\mu$	cosmic string component of perturbations

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Problem 2: as we add extra parameters, the uncertainties on existing parameters increase, and eventually we learn nothing useful about anything.

We need a way of penalizing use of extra parameters - an implementation of Ockham's razor.

Model Selection Statistics

Liddle, MNRAS, astro-ph/0401198
astro-ph/0701113

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- Bayesian information criterion (Schwarz 1978)
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NB: the ratio of evidences between two models is also known as the Bayes factor.

The Bayesian evidence is the most powerful of these. It is a full implementation of Bayesian inference, and literally gives the probability of the data given the model (note, not the probability of particular parameter values). If multiplied by the prior model probability it gives the posterior model probability. However it can be hard to calculate, being a highly-peaked multi-dimensional integral.

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The Akaike Information Criterion was derived using information theory techniques. It gives an approximate minimization of the so-called Kullback-Leibler information entropy, which is a measure of the difference between two probability distributions. It is however \`dimensionally inconsistent'.

Model selection techniques are essential when considering whether or not new data requires the addition of new parameters to describe it.

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- Neglect of model dimensionality: using parameter estimation rather than model selection.

Interpretational scale

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Jeffreys′ Scale:		
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The most useful divisions are 2.5 (odds ratio of 12:1) and 5 (odds ratio of 150:1).

Application to WMAP3

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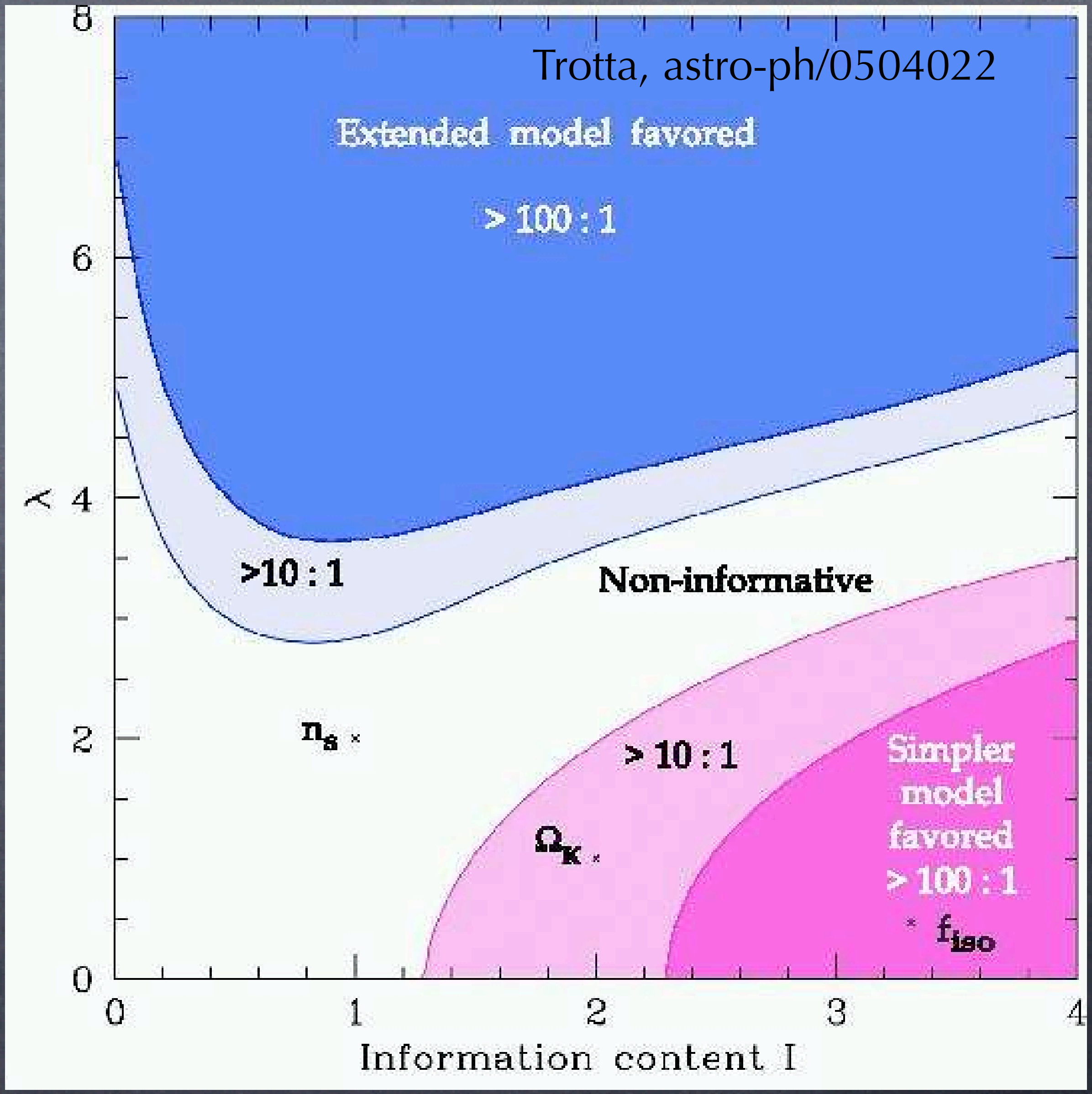

Nested Sampling: CosmoNest

Mukherjee, Parkinson and Liddle, ApJL, astro-ph/0508461
Parkinson, Mukherjee and Liddle, PRD, astro-ph/0605003
www.cosmonest.org

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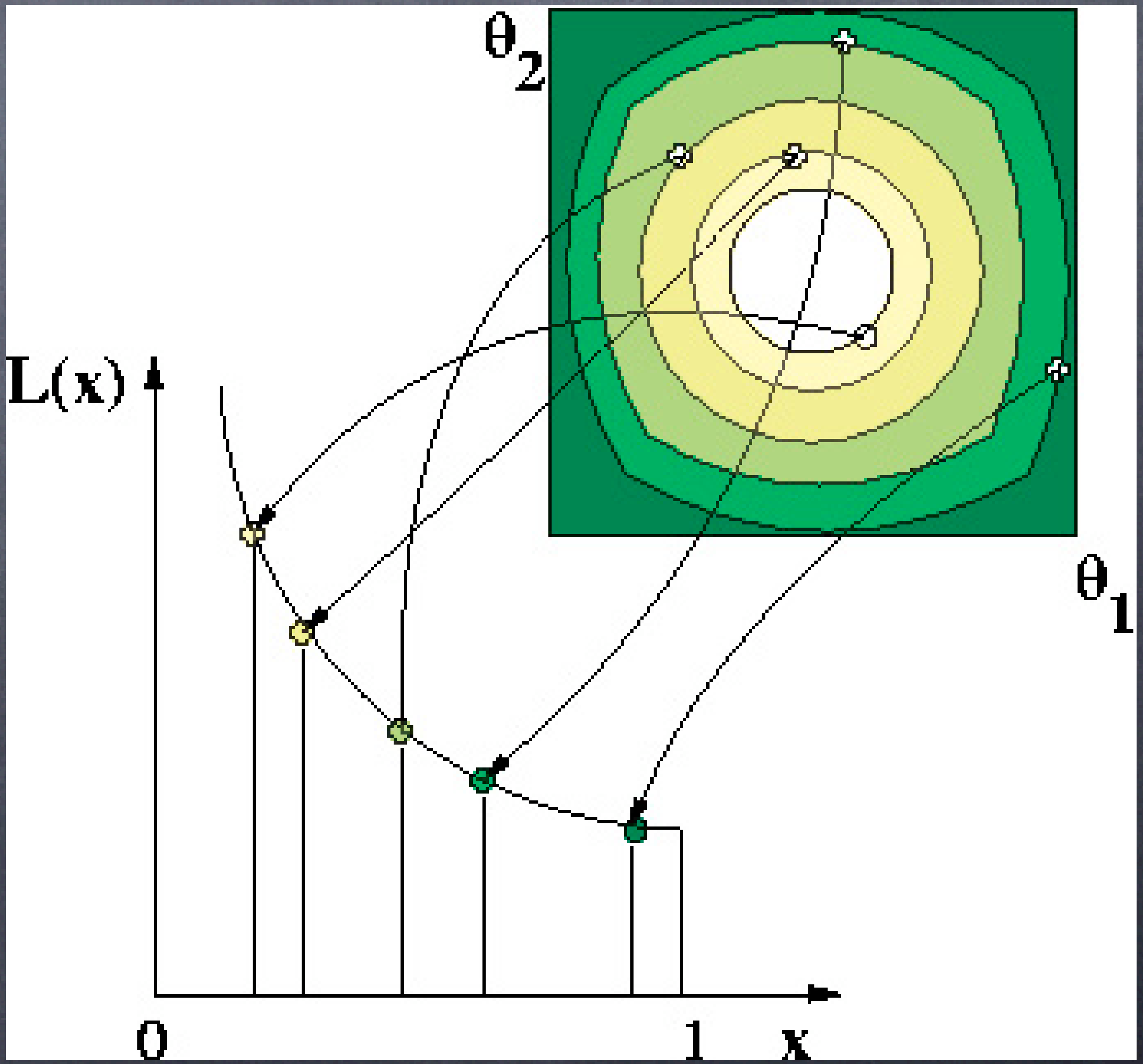

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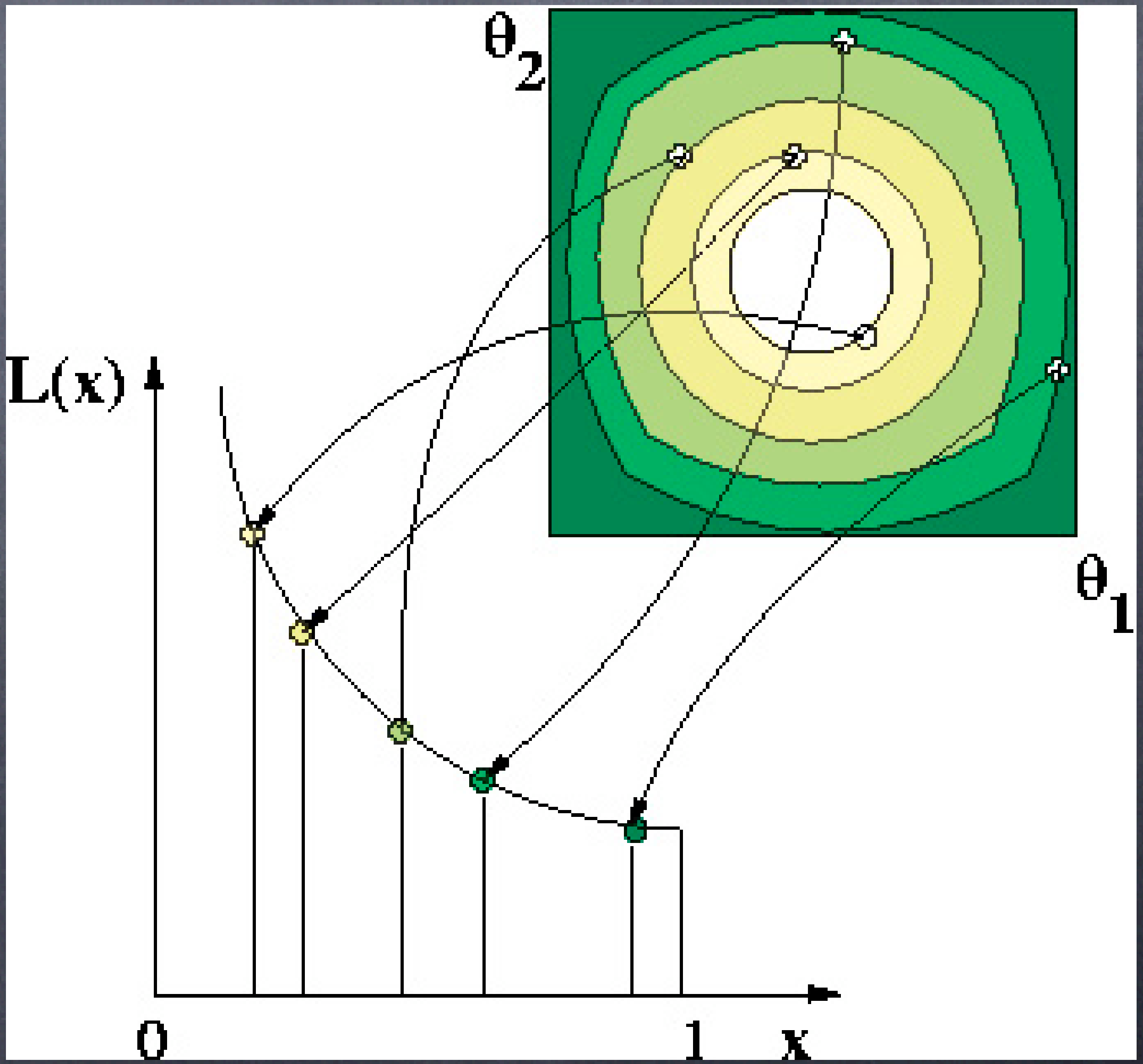

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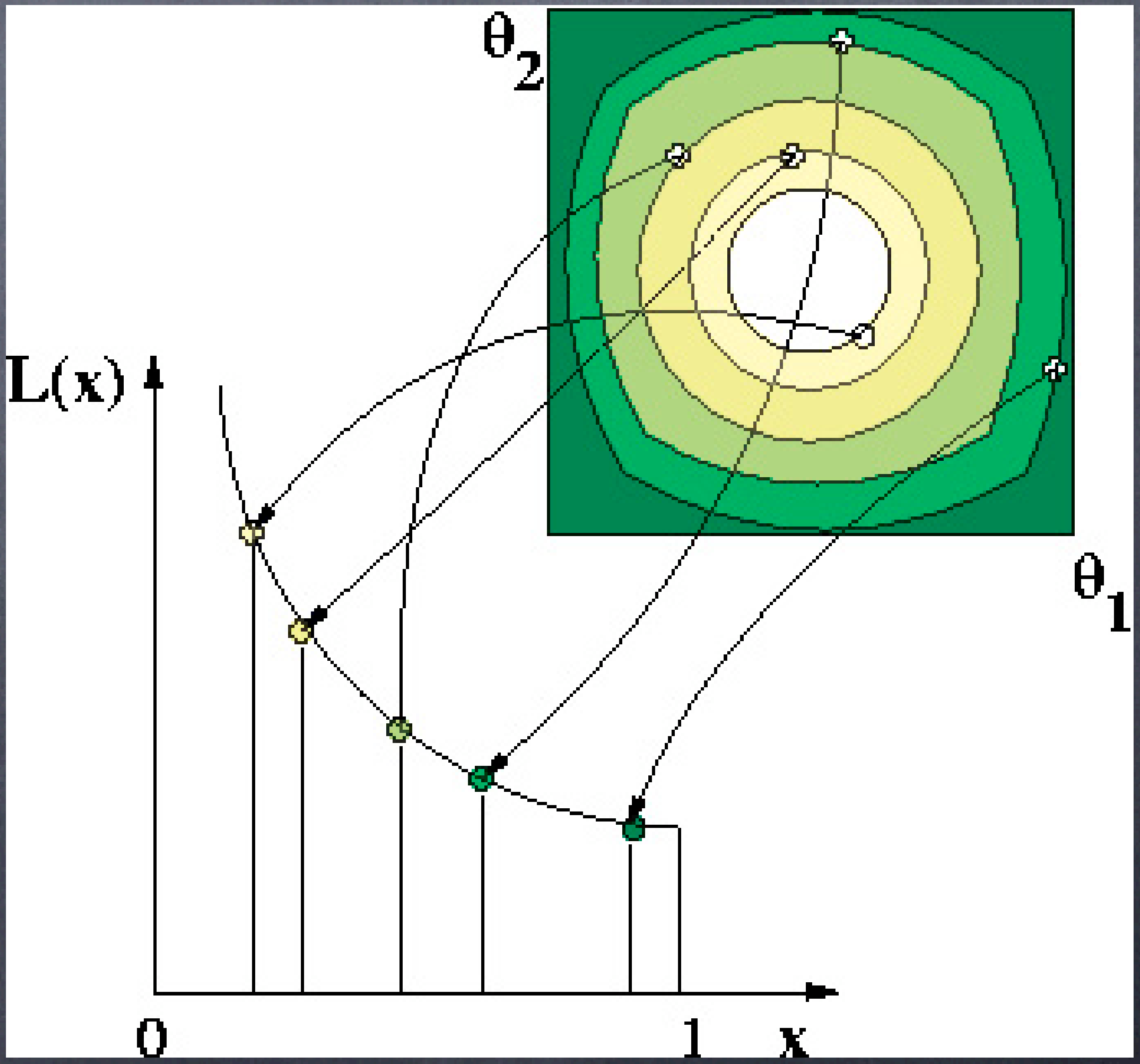
This can then be evaluated using Monte Carlo samples to trace the variation of likelihood with prior mass, peeling away thin nested isosurfaces of equal likelihood.



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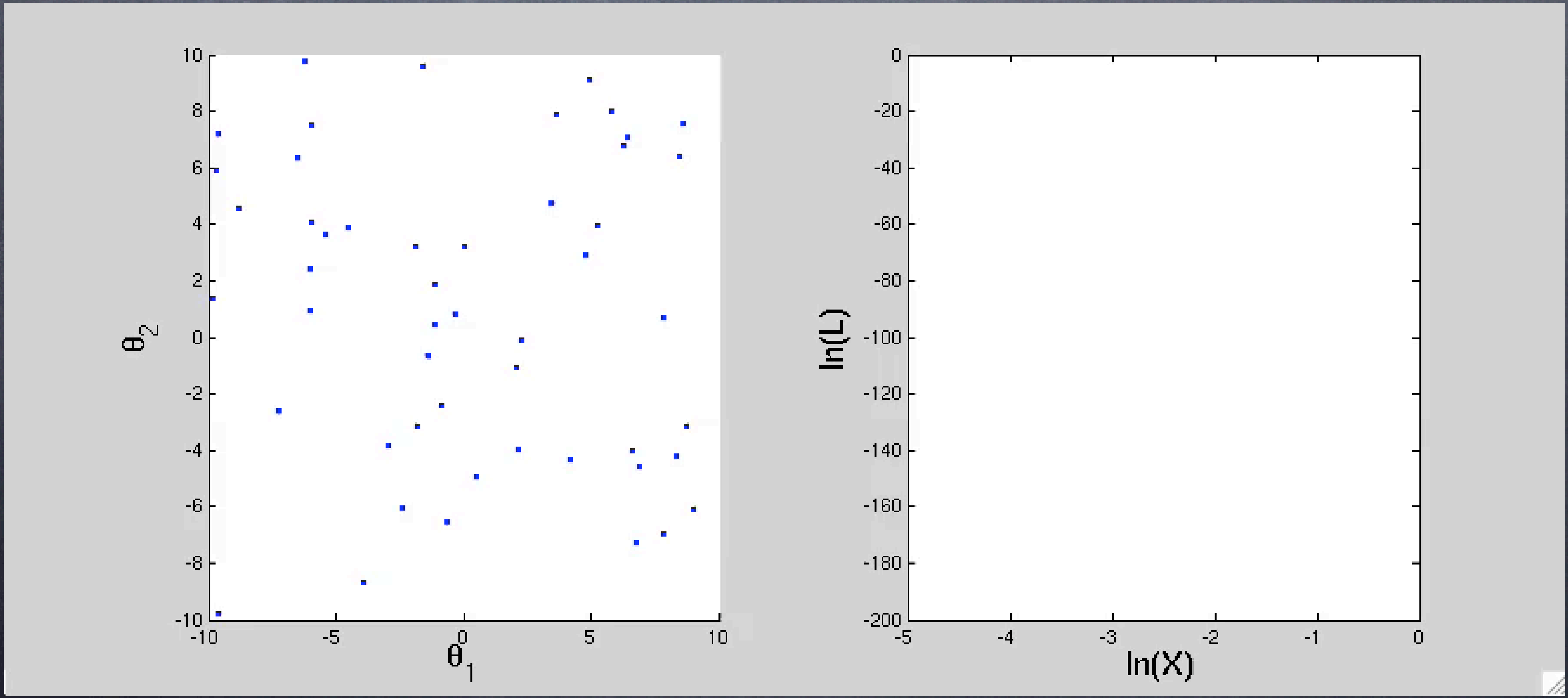

A model selection example: spectral index from WMAP3

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WMAP3 has been interpretted as giving ruling out the Harrison-Zel′dovich $n_S = 1$ spectrum and hence favouring inflation, e.g. $n_S = 0.951^{+0.015}_{-0.019}$. But this ignores model dimensionality. Using CosmoNest we find

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4. ... unless you use a logarithmic prior for r, which puts you back close to the r=0 case.

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  simulate data for a fiducial model (eg LambdaCDM);
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  simulate data for a fiducial model (eg LambdaCDM);
  estimate expected parameter uncertainties about that model;
  interpret that if the true model is outside the contours, LambdaCDM is excluded.
- Bayes factor approach:
  simulate data at each point in parameter plane;
  compute Bayes factor (ie evidence ratio) of full model versus eg LambdaCDM at each point.

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- Fisher matrix approach often assumes a gaussian likelihood.

Dark energy forecasting

Mukherjee, Parkinson, Corasaniti, Liddle, Kunz, MNRAS, astro-ph/0512484

$$w = w_0 + (1-a)w_a$$

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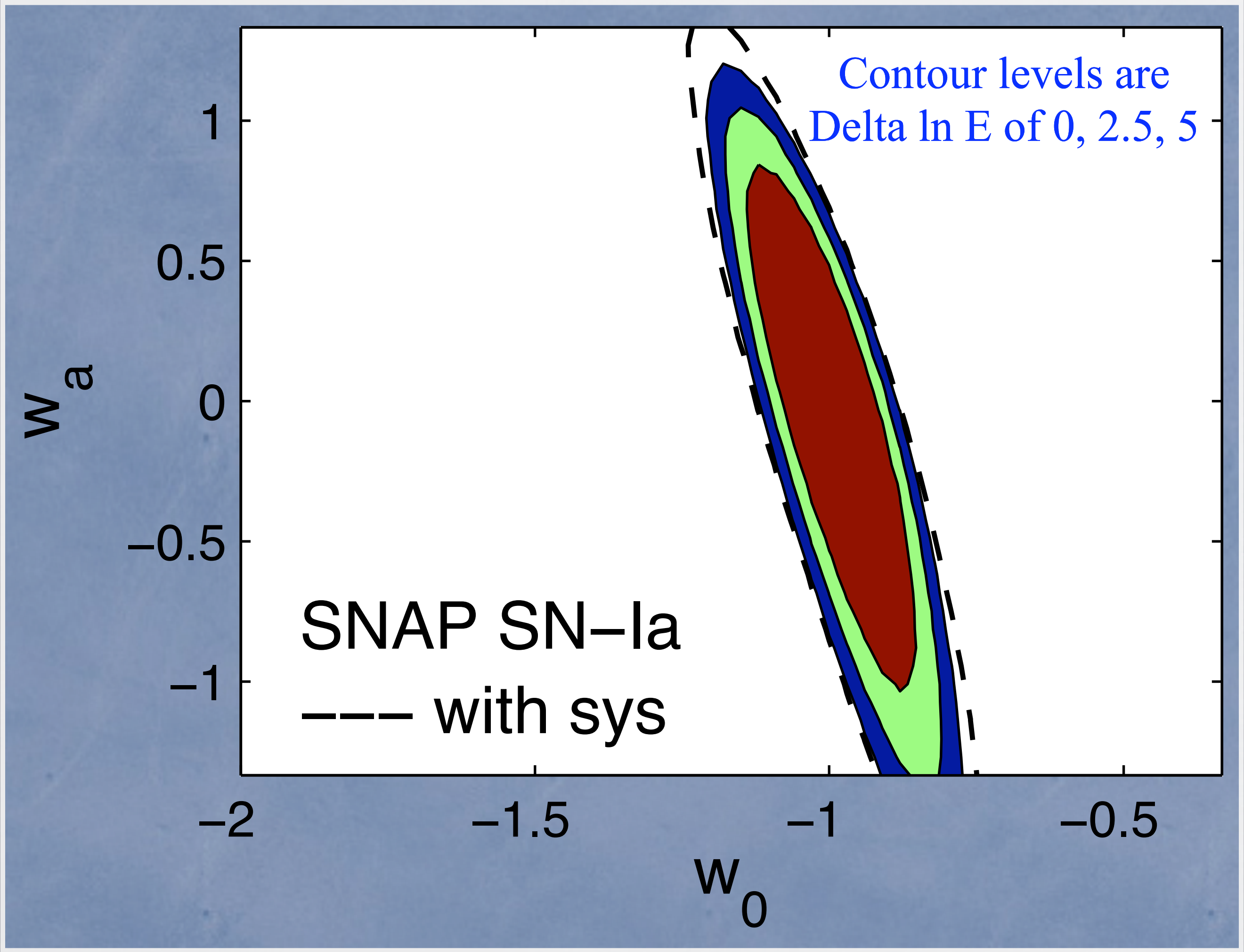

Projected Bayes factor plot against LambdaCDM, SNAP supernovae only

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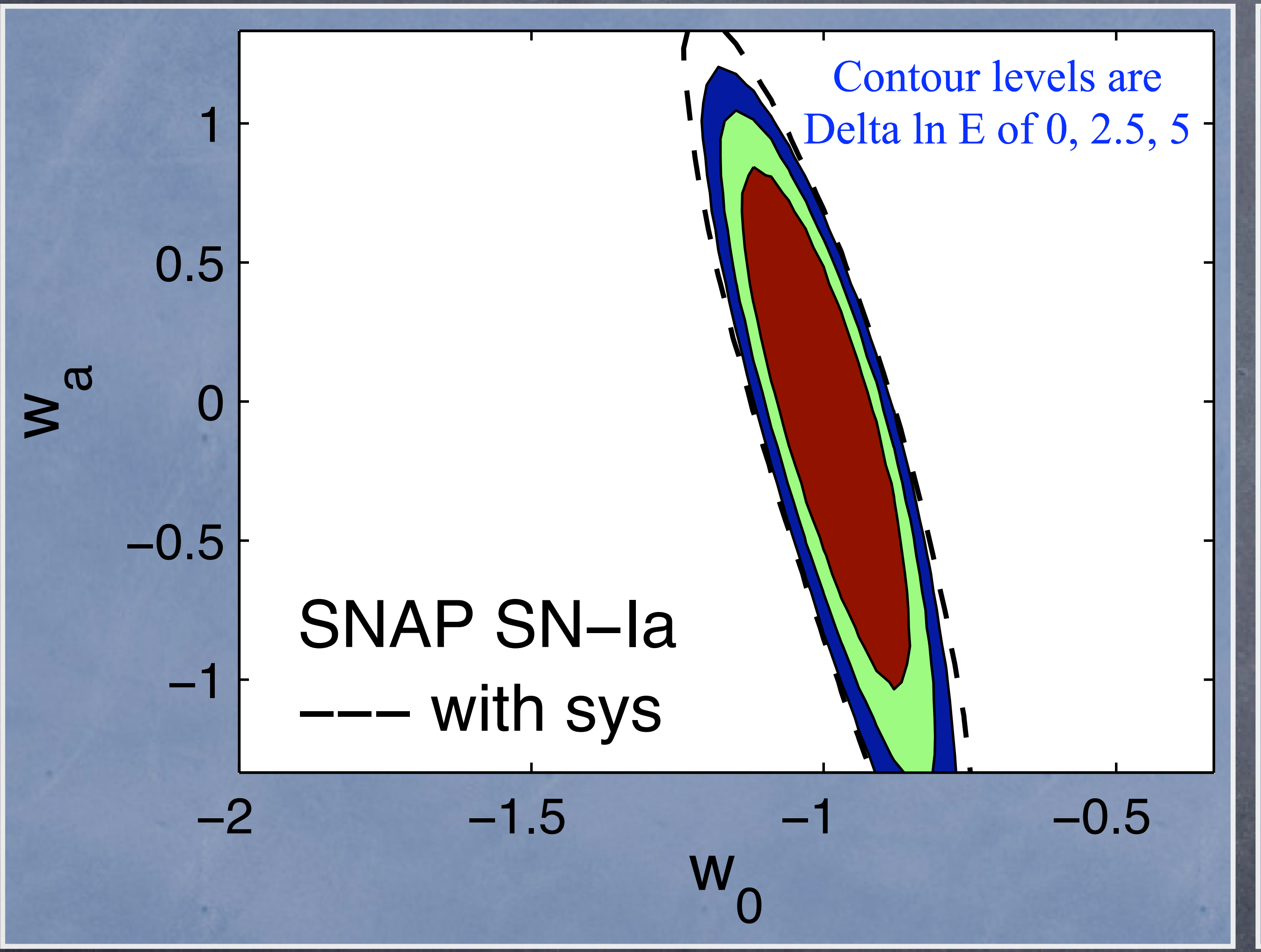

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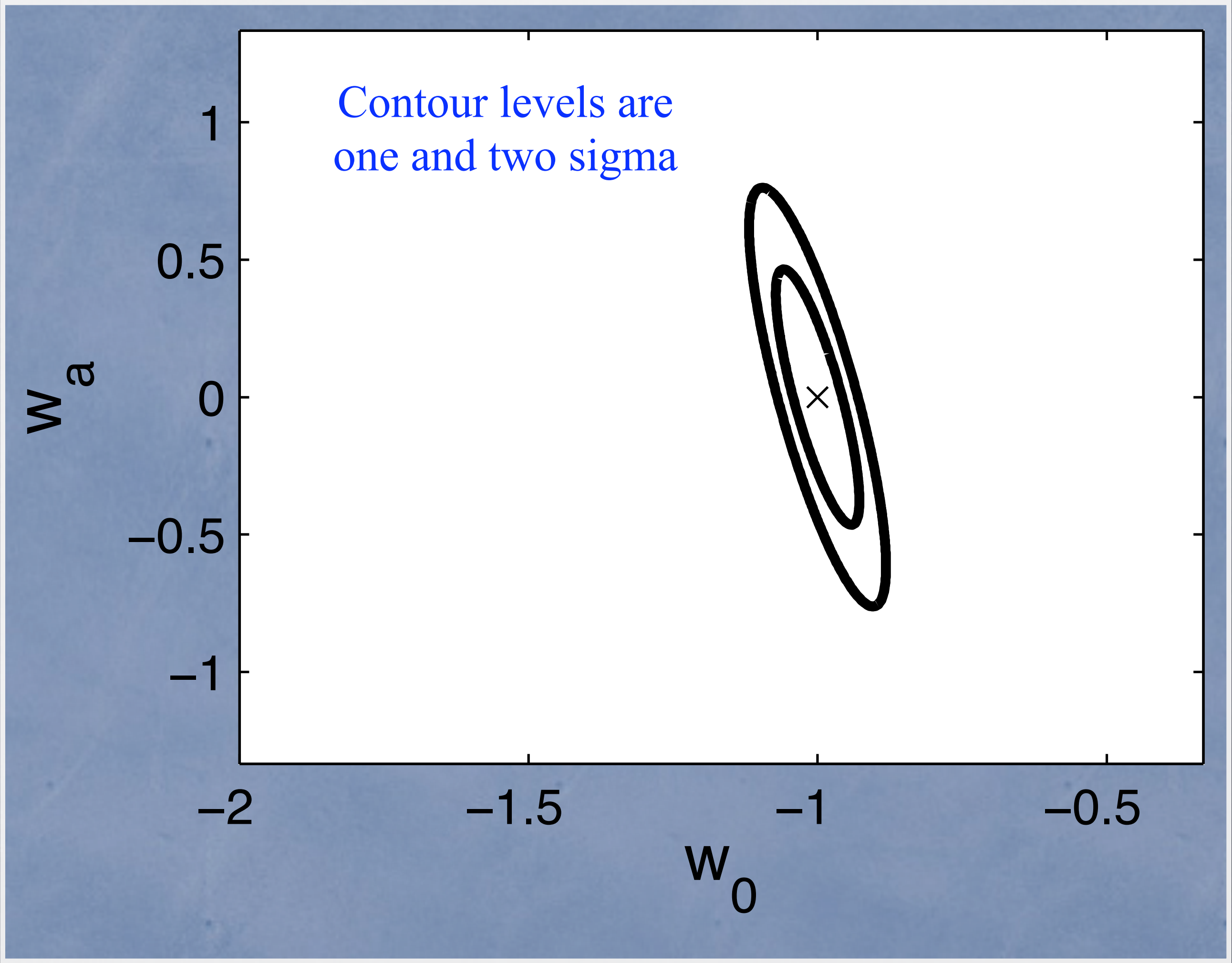
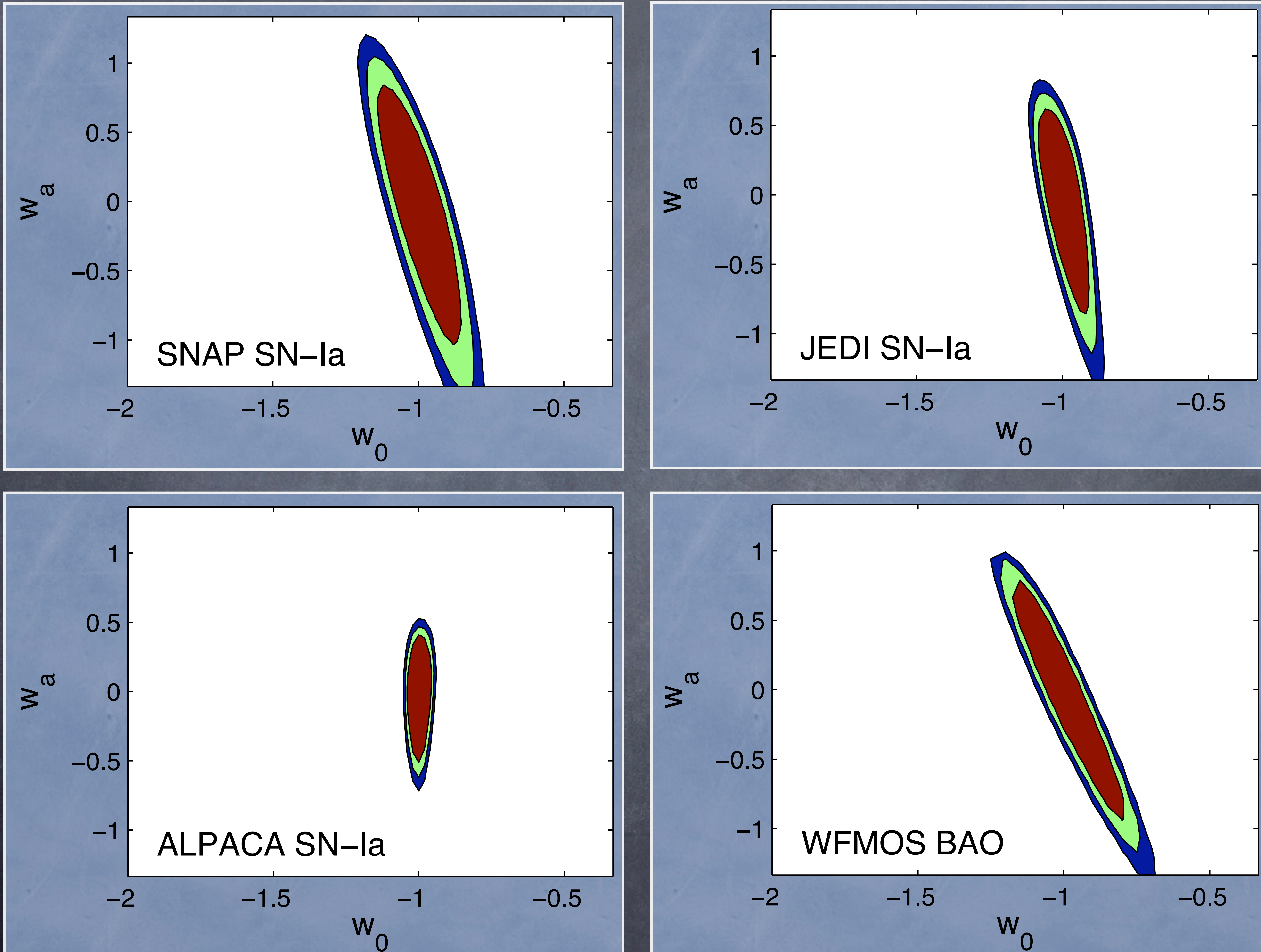


Projected Fisher matrix uncertainties about LambdaCDM



SNAP SN−Ia
JEDI SN−Ia
ALPACA SN−Ia
WFMOS BAO
w0
wa
−2
−1.5
−1
−0.5
1
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0

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Liddle, Mukherjee, Parkinson, and Wang, astro-ph/0610126

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CMB shift+BAO(SDSS)+SN

data used	Model			
WMAP+SDSS+	$\Delta \ln E$	H	$\chi^2_{\rm min}$	parameter constraints
	Model I: Λ			
Riess04	0.0	5.7	30.5	$\Omega_{\rm m} = 0.26 \pm 0.03,\ H_0 = 65.5 \pm 1.0$
Astier05	0.0	6.5	94.5	$\Omega_{\rm m} = 0.25 \pm 0.03,\ H_0 = 70.3 \pm 1.0$
	Model II: constant w, flat prior $-1 \leq w \leq -0.33$			
Riess04	-0.1 ± 0.1	6.4	28.6	$\Omega_{\rm m} = 0.27 \pm 0.04,\ H_0 = 64.0 \pm 1.4,\ w < -0.81, -0.70^a$
Astier05	-1.3 ± 0.1	8.0	93.3	$\Omega_{\rm m} = 0.24 \pm 0.03,\ H_0 = 69.8 \pm 1.0,\ w < -0.90, -0.83^a$
	Model III: constant w, flat prior $-2 \leq w \leq -0.33$			
Riess04	-1.0 ± 0.1	7.3	28.6	$\Omega_{\rm m} = 0.27 \pm 0.04,\ H_0 = 64.0 \pm 1.5,\ w = -0.87 \pm 0.1$
Astier05	-1.8 ± 0.1	8.2	93.3	$\Omega_{\rm m} = 0.25 \pm 0.03,\ H_0 = 70.0 \pm 1.0,\ w = -0.96 \pm 0.08$
	Model IV: w_0–w_a, flat prior $-2 \leq w_0 \leq -0.33,\ -1.33 \leq w_a \leq 1.33$			
Riess04	-1.1 ± 0.1	7.2	28.5	$\Omega_{\rm m} = 0.27 \pm 0.04,\ H_0 = 64.1 \pm 1.5,\ w_0 = -0.83 \pm 0.20,\ w_a = --^b$
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	Model V: w_0–w_a, $-1 \leq w(a) \leq 1$ for $0 \leq z \leq 2$			
Riess04	-2.4 ± 0.1	9.1	28.5	$\Omega_{\rm m} = 0.28 \pm 0.04,\ H_0 = 63.6 \pm 1.3,\ w_0 < -0.78, -0.60^a,\ w_a = -0.07 \pm 0.34$
Astier05	-4.1 ± 0.1	11.1	93.3	$\Omega_{\rm m} = 0.24 \pm 0.03,\ H_0 = 69.5 \pm 1.0,\ w_0 < -0.90, -0.80^a,\ w_a = 0.12 \pm 0.22$

LambdaCDM

Constant W

W_0-W_a

(Almost) current dark energy data

Liddle, Mukherjee, Parkinson, and Wang, astro-ph/0610126

CMB shift+BAO(SDSS)+SN

	data used	Model			
	WMAP+SDSS+	$\Delta \ln E$	H	$\chi^2_{\rm min}$	parameter constraints
					Model I: Λ
LambdaCDM	Riess04	0.0	5.7	30.5	$\Omega_{\rm m} = 0.26 \pm 0.03,\ H_0 = 65.5 \pm 1.0$
	Astier05	0.0	6.5	94.5	$\Omega_{\rm m} = 0.25 \pm 0.03,\ H_0 = 70.3 \pm 1.0$
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Constant W	Riess04	-0.1 ± 0.1	6.4	28.6	$\Omega_{\rm m} = 0.27 \pm 0.04,\ H_0 = 64.0 \pm 1.4,\ w < -0.81, -0.70^a$
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W_0-W_a	Riess04	-1.1 ± 0.1	7.2	28.5	$\Omega_{\rm m} = 0.27 \pm 0.04,\ H_0 = 64.1 \pm 1.5,\ w_0 = -0.83 \pm 0.20,\ w_a = --^b$
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LambdaCDM

Constant W

W0-Wa

Conclusion: LambdaCDM currently favoured but all models still alive

Future forecasts informed by current data

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⇒ Use current probabilities to forecast future experiment outcomes

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- If LambdaCDM is right, are upcoming experiments (eg DES, WFMOS, SNAP) good enough to favour it decisively? — YES
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  Tighter than you expect!

Model selection forecasts for Planck

Pahud, Liddle, Mukherjee, and Parkinson, PRD, astro-ph/0605004

We can also do model selection forecasts for the Planck satellite. For the spectral index it looks like this:

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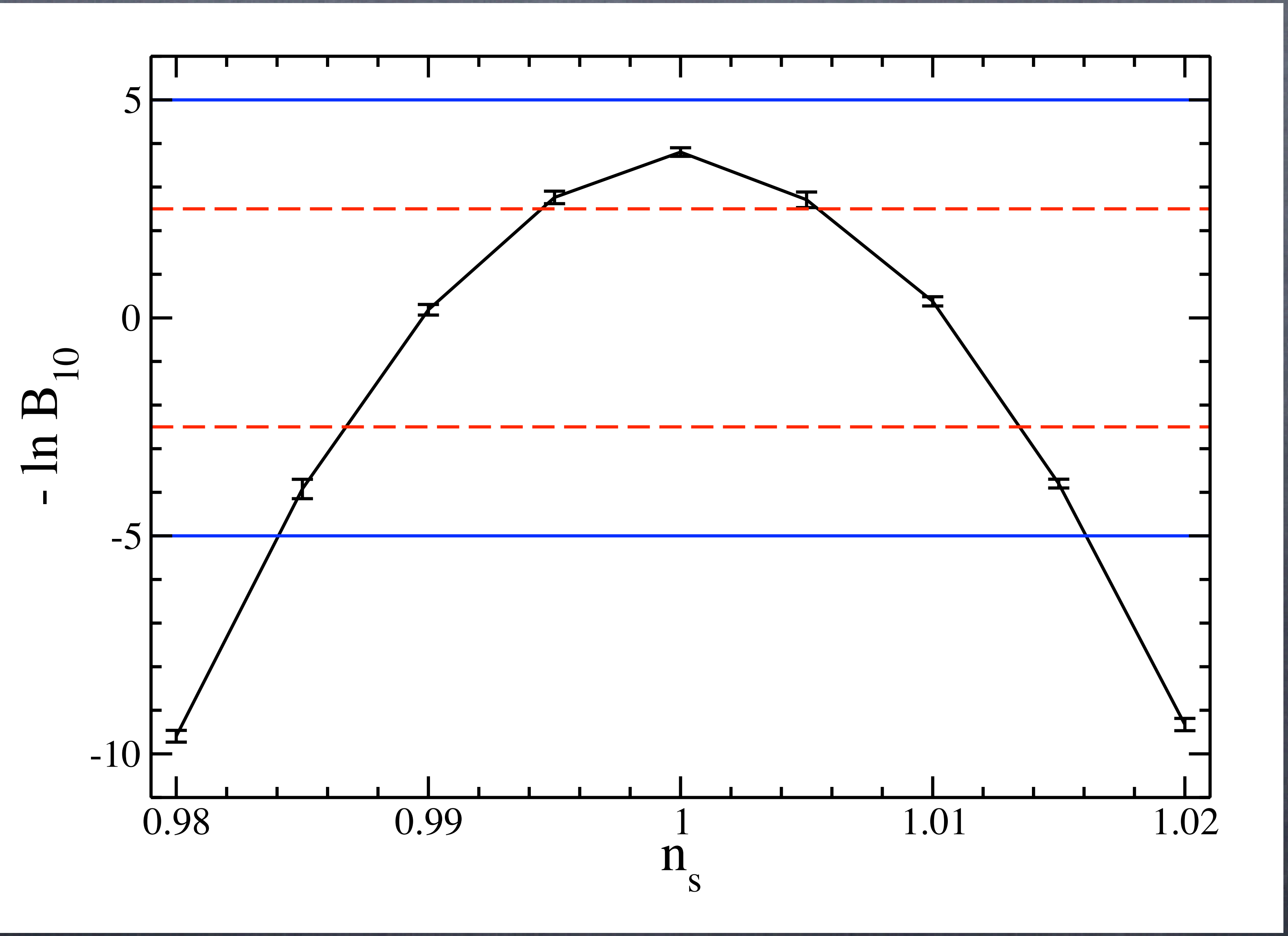

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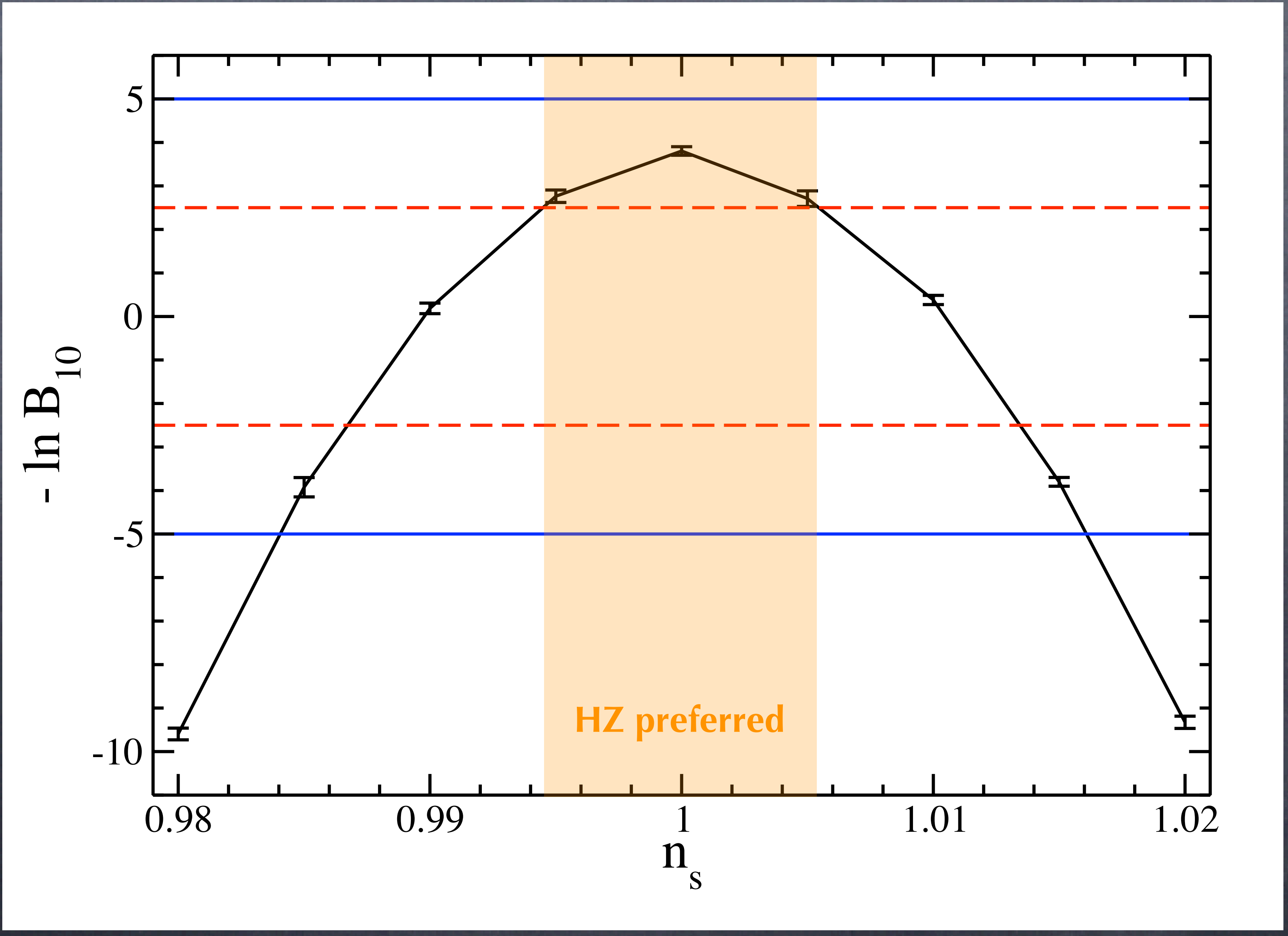

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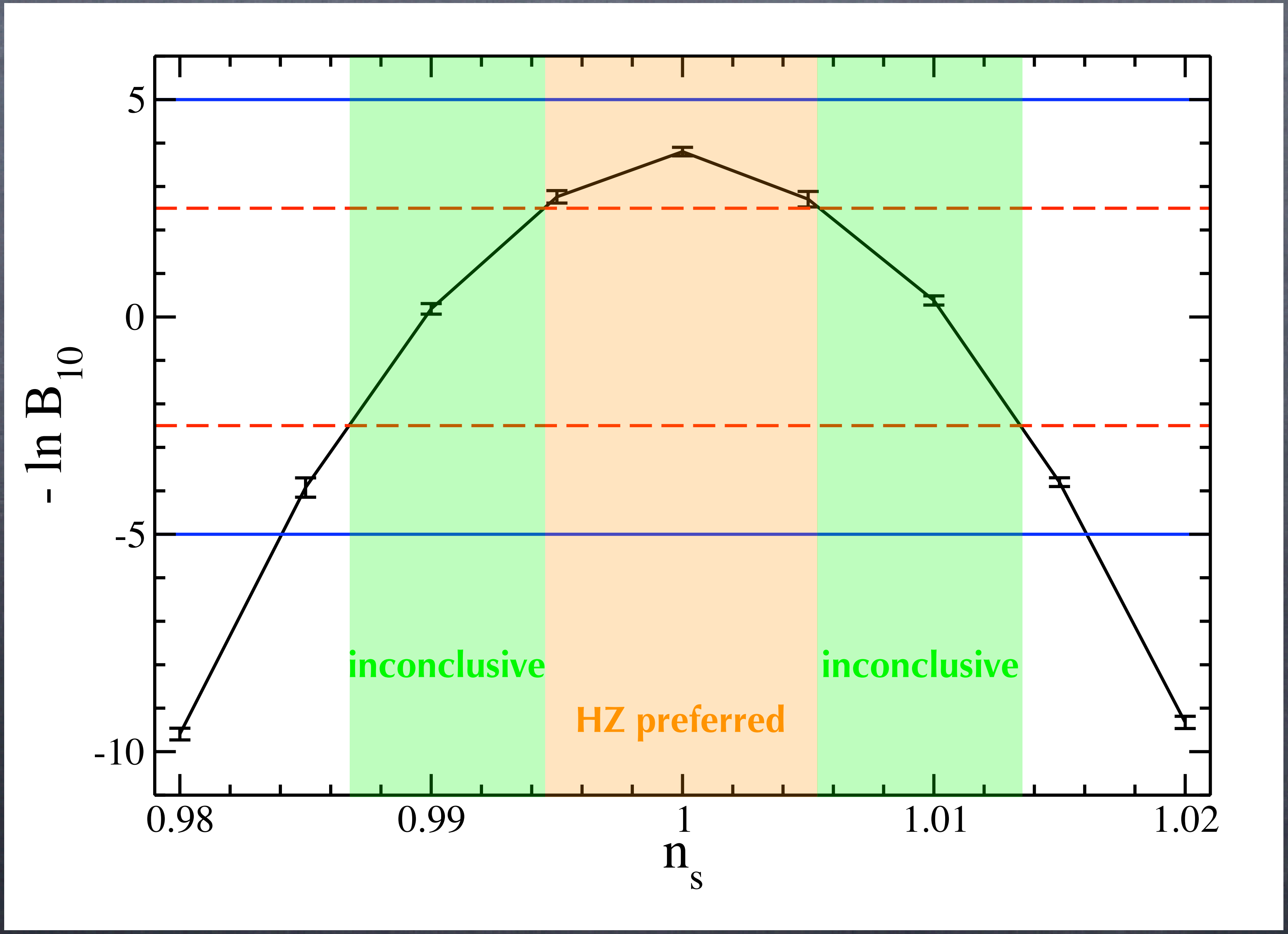

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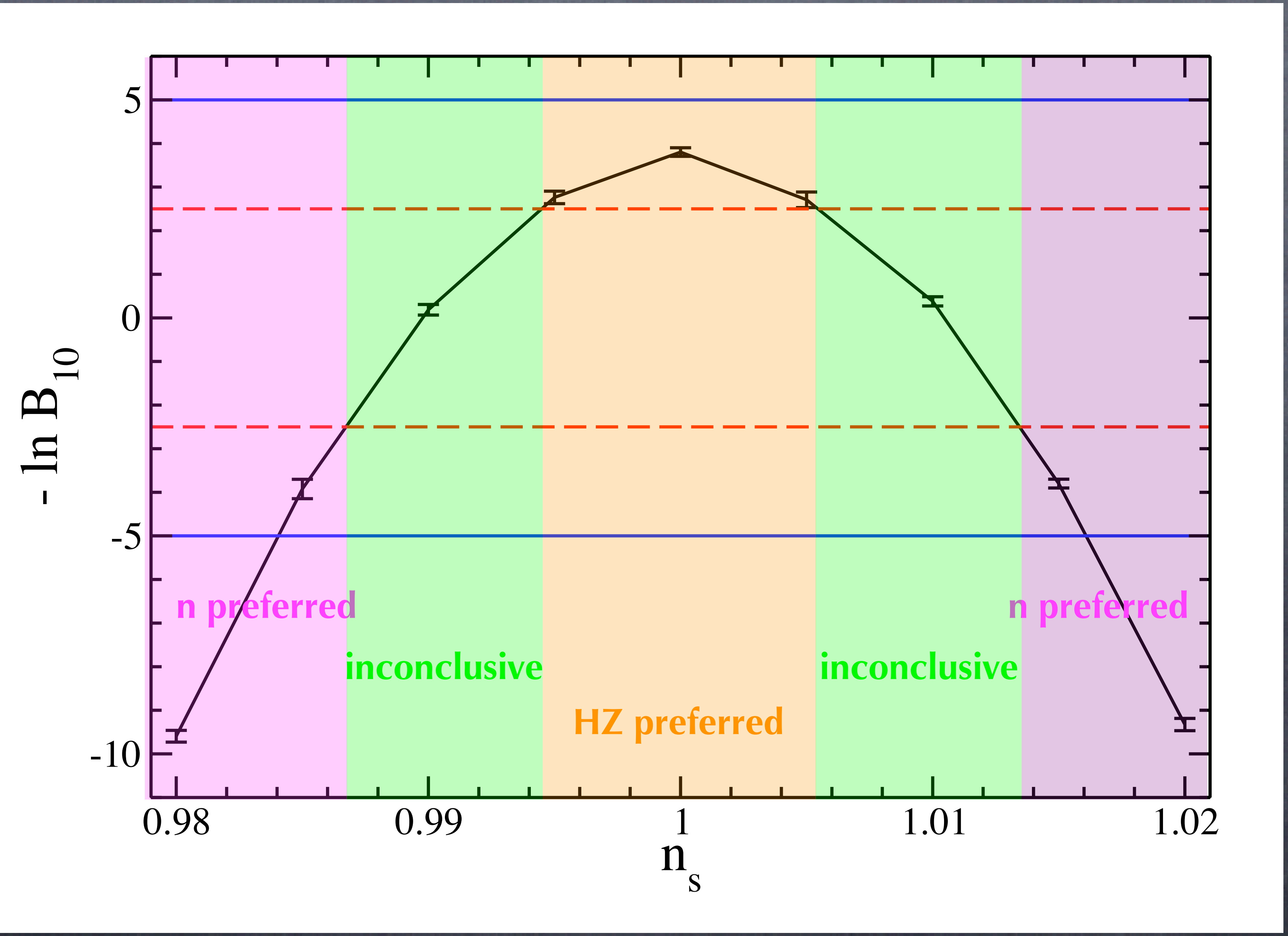

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- Model selection forecasting is a powerful new tool for experimental design and comparison, and is readily applied to dark energy experiments.