Velocity Probe on Cosmology

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Overview

- *Primordial non-Gaussianity*

- *Dynamical mass estimate of massive clusters*

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  Phase-space distribution as an estimate of cluster (dynamical) mass.

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- *Dynamics around massive clusters is sensitive to the mass enclosed --> dynamical mass estimates.*
- *Possibility to test modified gravity.*

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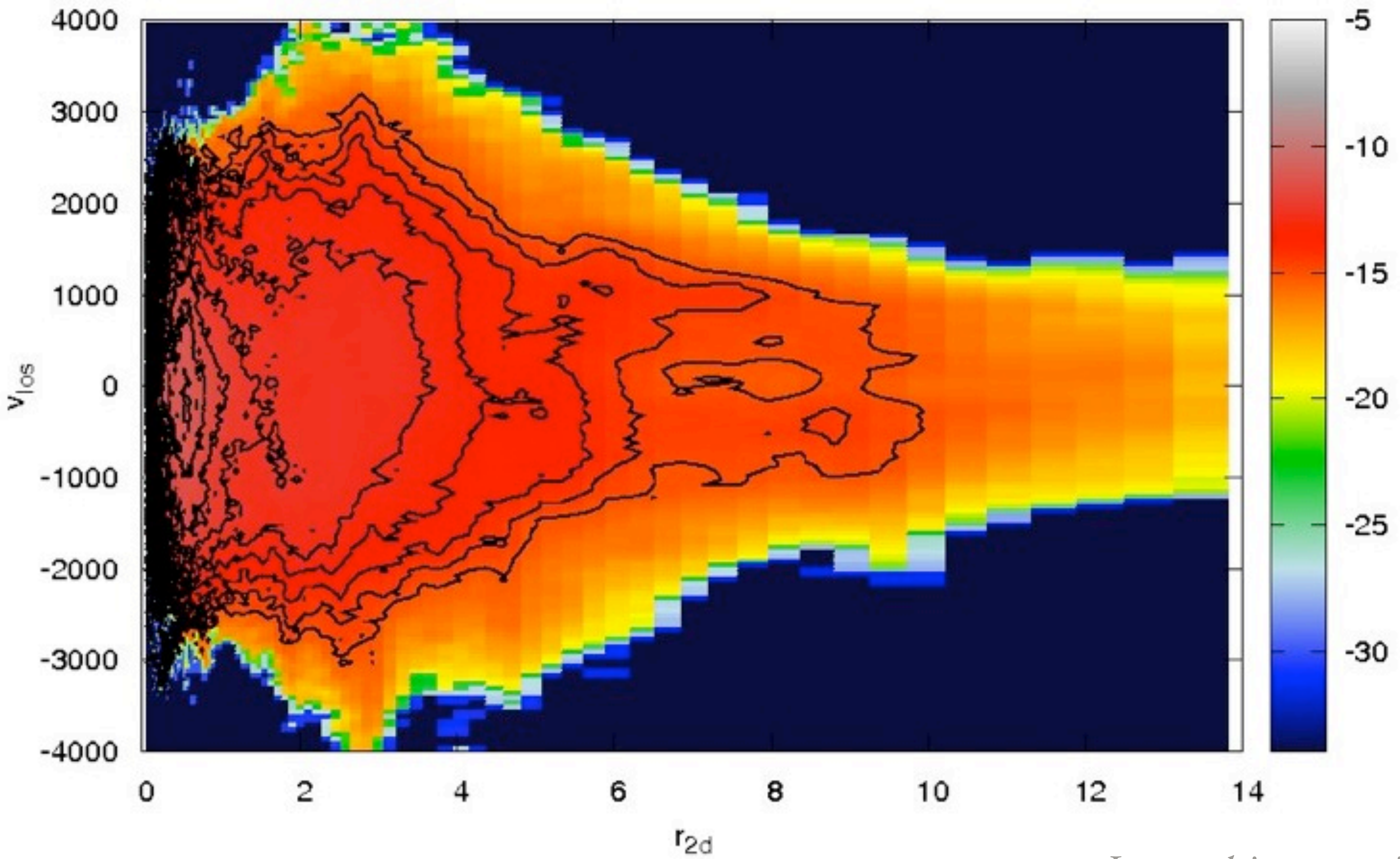
- *Use lensing technique – stacking halos.*
- *After stacking can assume spherical symmetry.*
- *Use observable quantities to form the phase space distribution: line-of-sight velocity vs projected separation from halo center.*

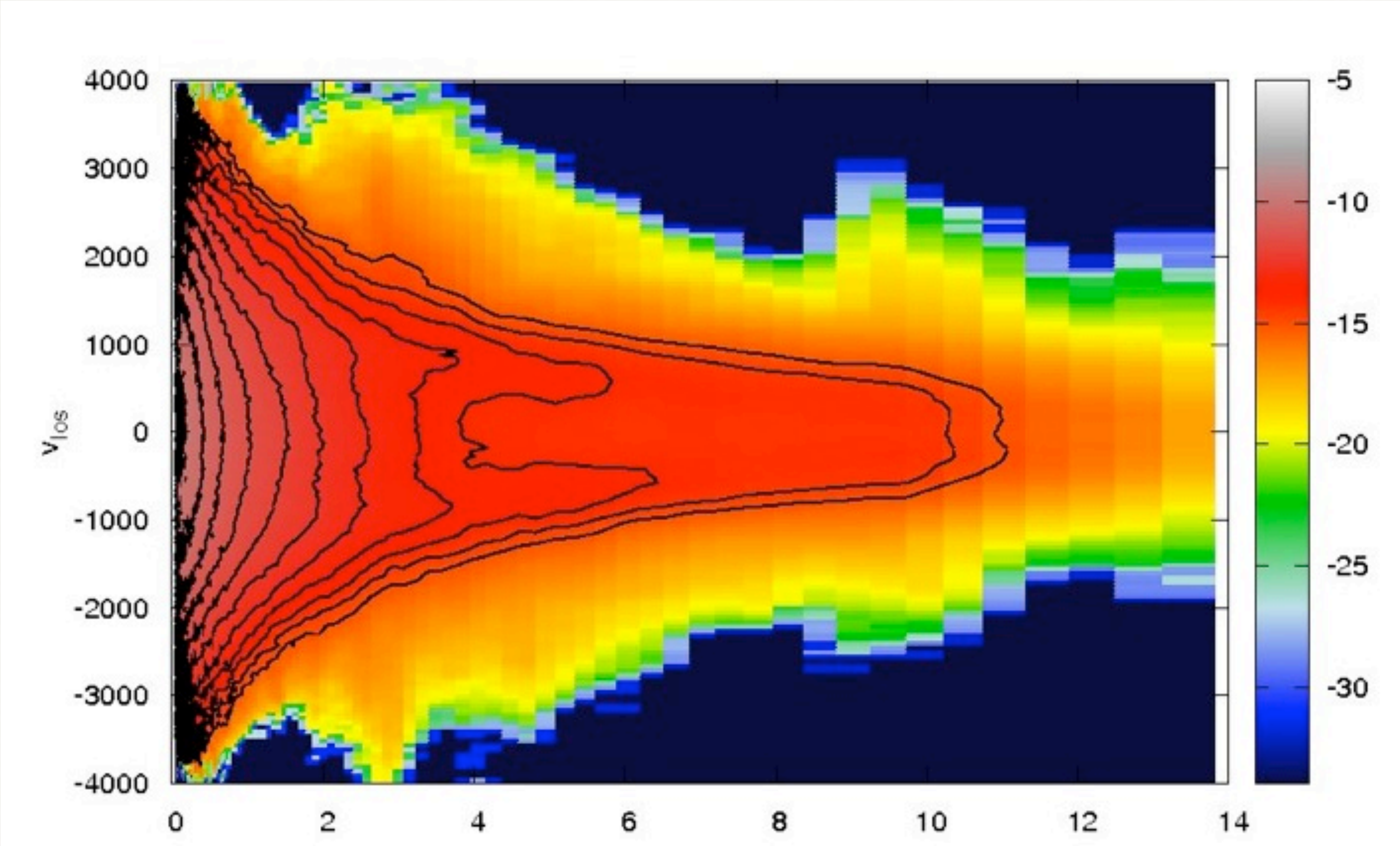
Stacking massive halos



Lam et al. in preparation

$N_{halo} = 1$

Stacking massive halos



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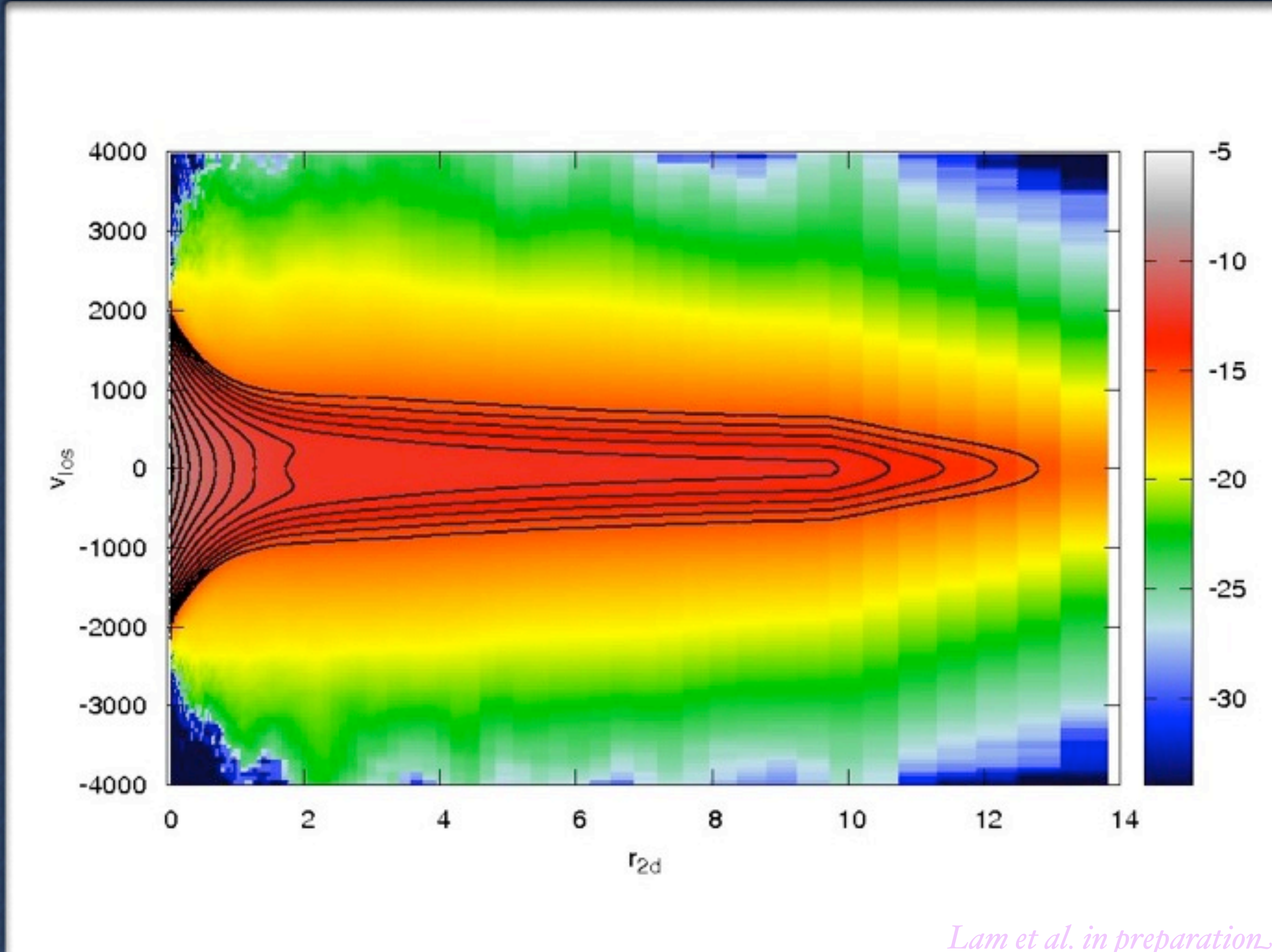
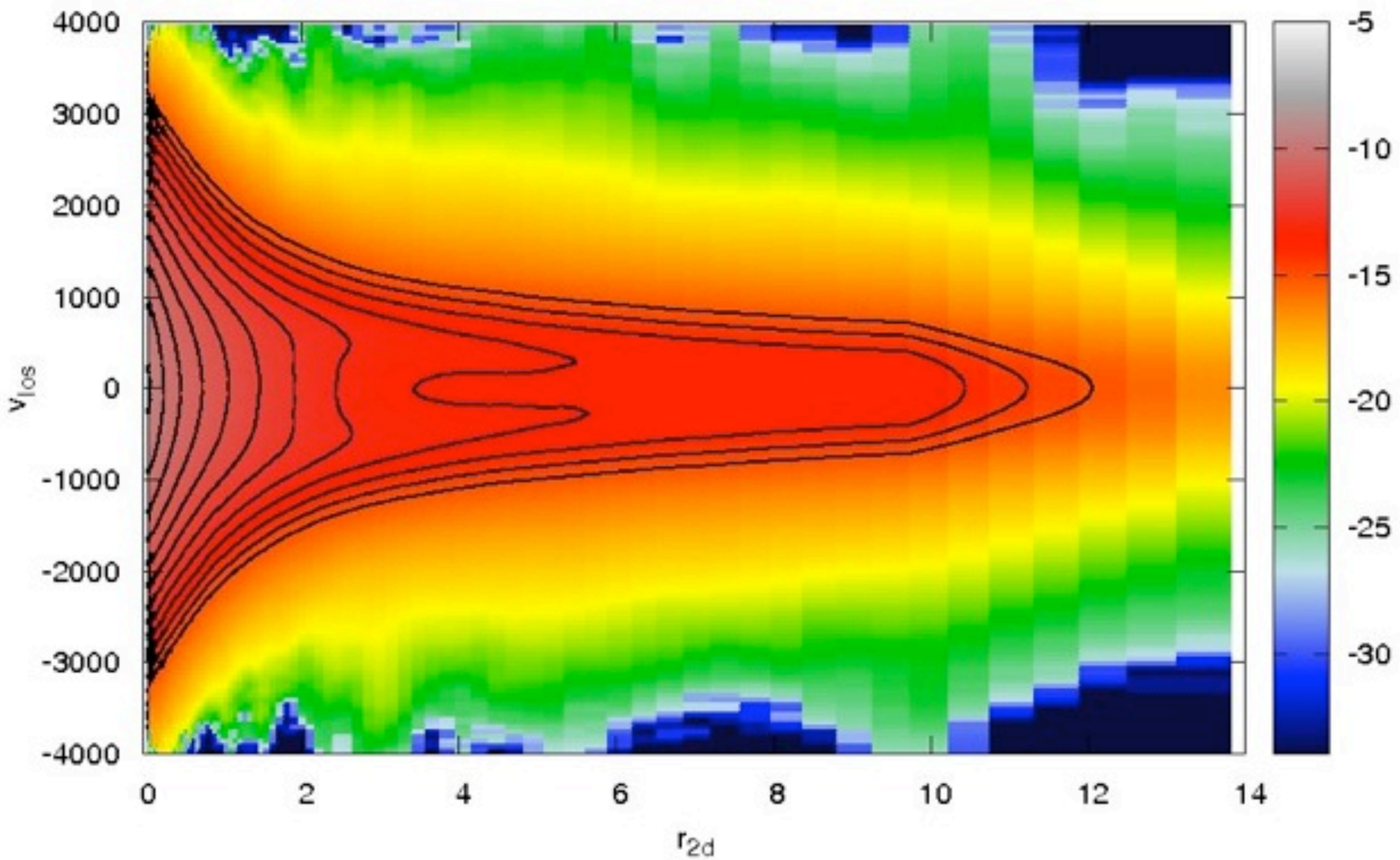
N_{halo} = 10



$10^{14} \leq M_{halo} \leq 1.5 \times 10^{14}$



Lam et al. in preparation

$4 \times 10^{14} \leq M_{halo} \leq 5 \times 10^{15}$

Modeling of phase space distribution around massive halos

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- *1-halo terms: virial motions (non-linear)*
- *2-halo terms: spherical collapse model*

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- *Fitting formula for v_sigma (Evrard et al. 2008):*

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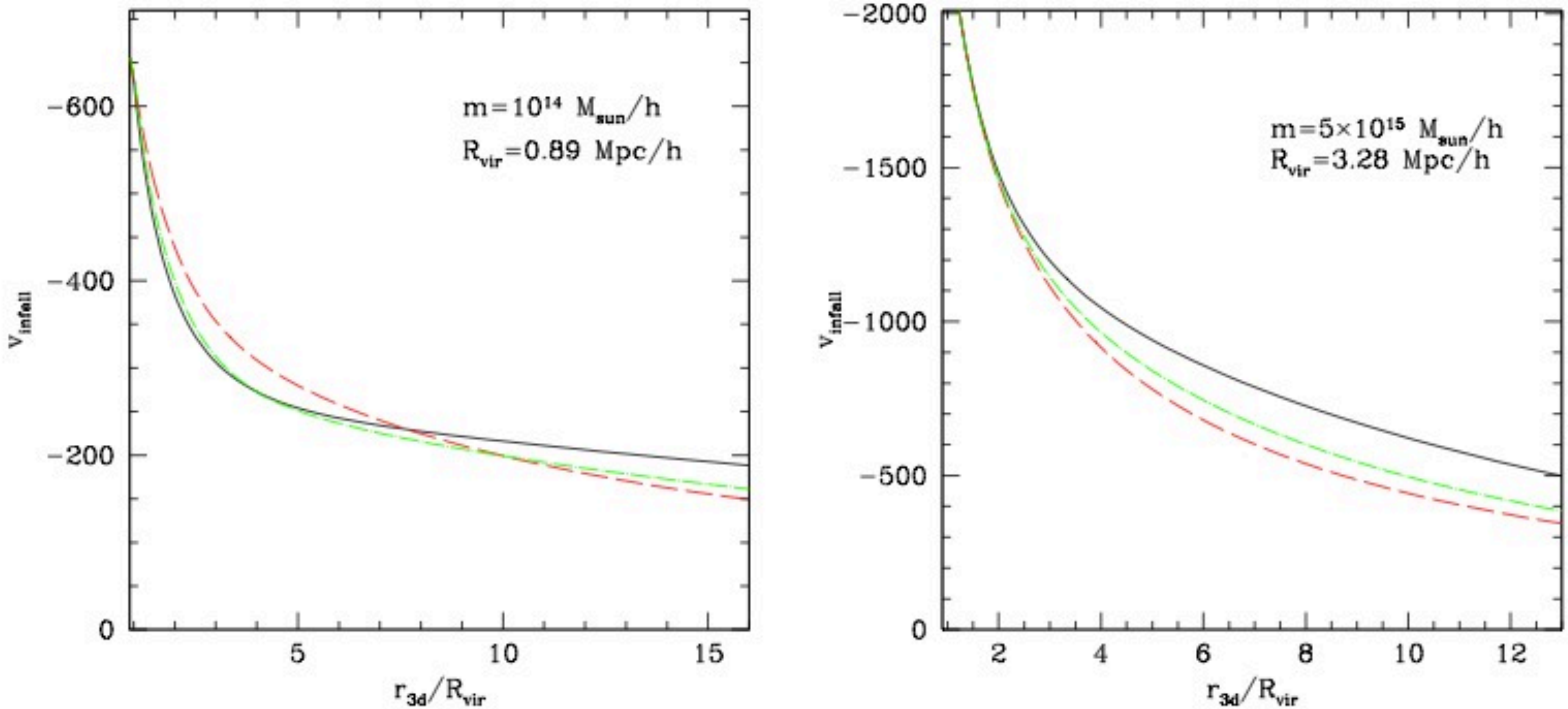
$$\sigma_{\rm DM}(M, z) = \sigma_{\rm DM,15}\left[\frac{(1+z)^{3/2}M_{200b}}{10^{15}{\rm M_{sun}}}\right]^{\alpha}$$

$$\sigma_{\rm DM,15} = 880{\rm km/s} \text{ and } \alpha = 0.355$$

M_{200b} = Mass of halo defined as 200 times background density

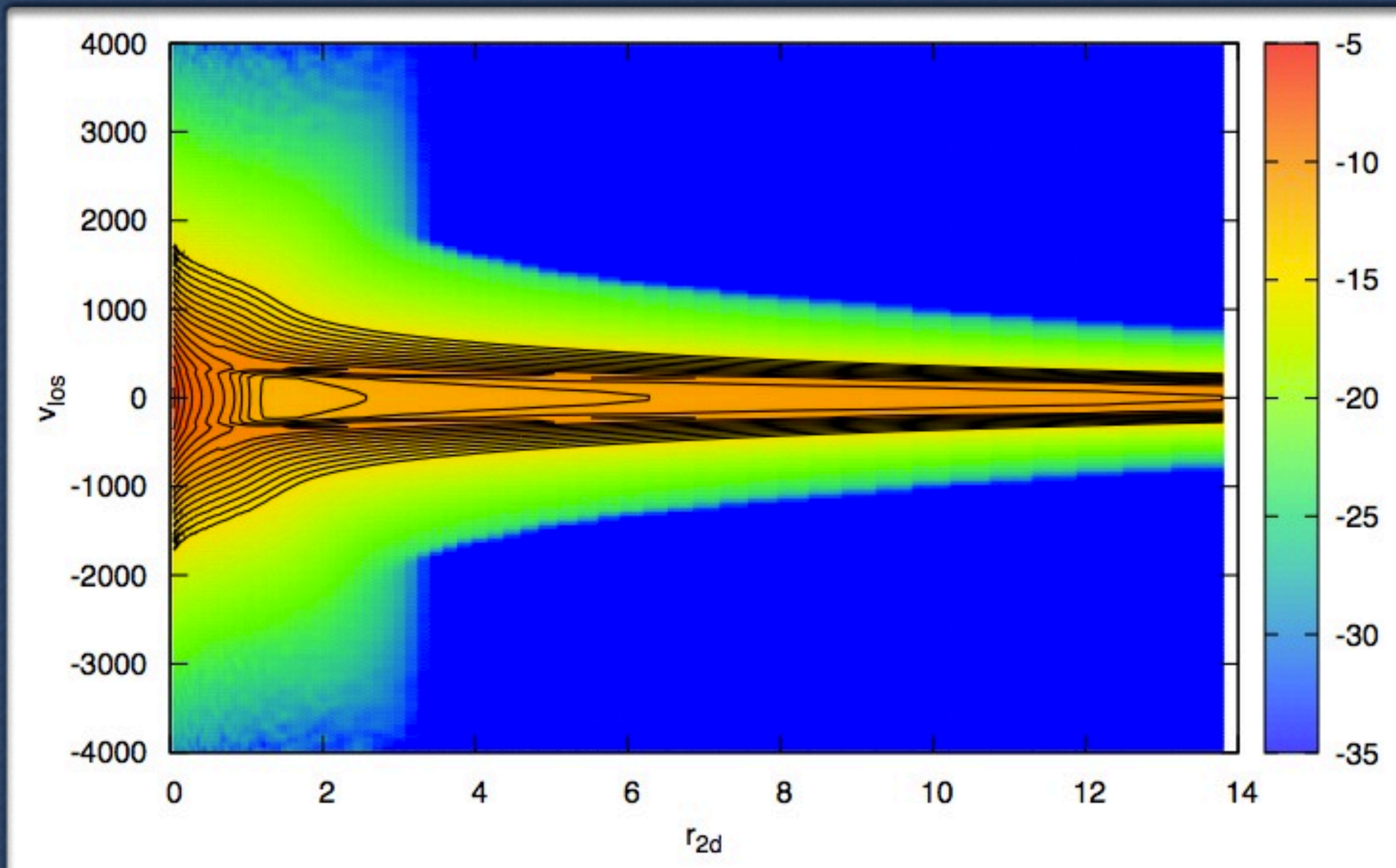
2-halo term: Radial infalling



Lam et al. in preparation

Infalling velocity predicted by spherical collapse model

BUT it does NOT work



2-halo term revisit

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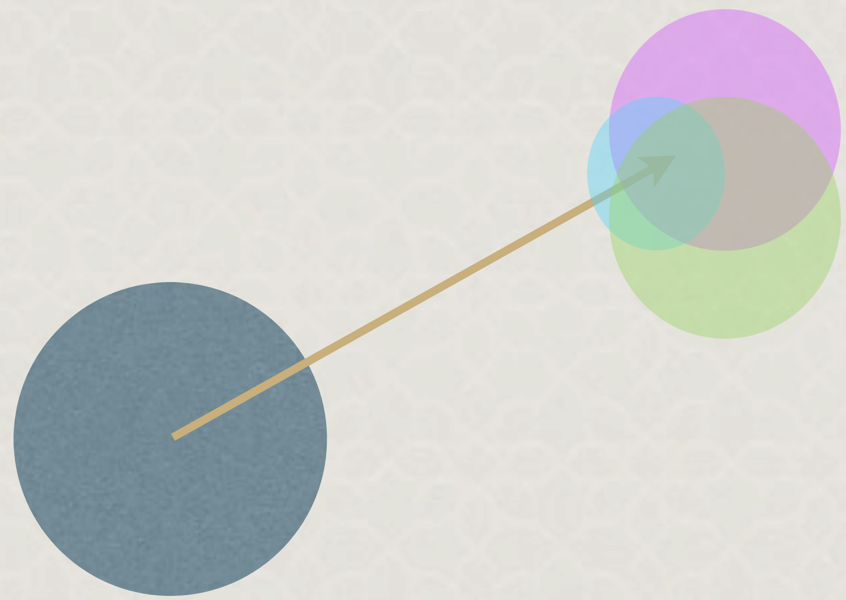
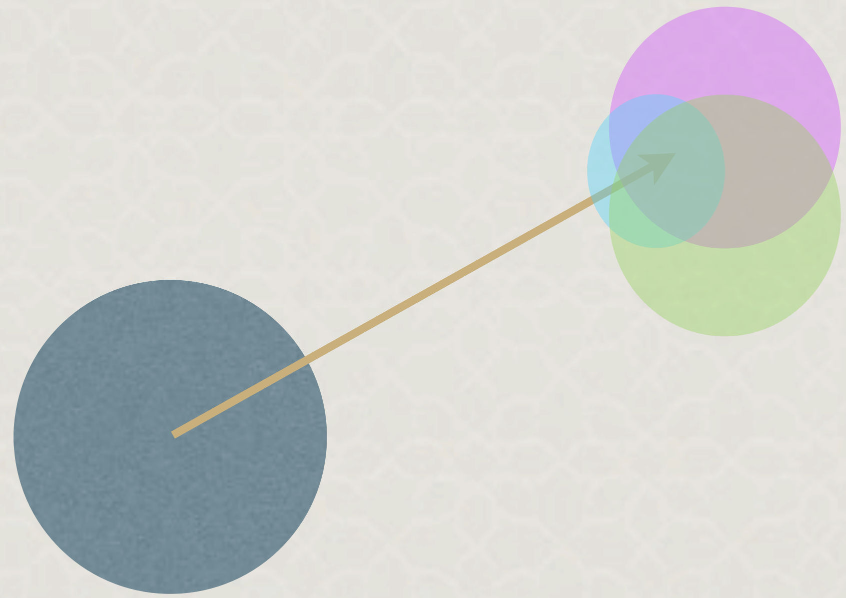
- *Virial motion inside secondary halo.*

$\sigma^2_{\mathrm{eff}}(r_{3d}, M_{\mathrm{halo}})$



Want to calculate an effective dispersion as a function of r_{3d} for a mass M_{halo} halo.

$\sigma^2_{\rm eff}(r_{3d}, M_{\rm halo})$



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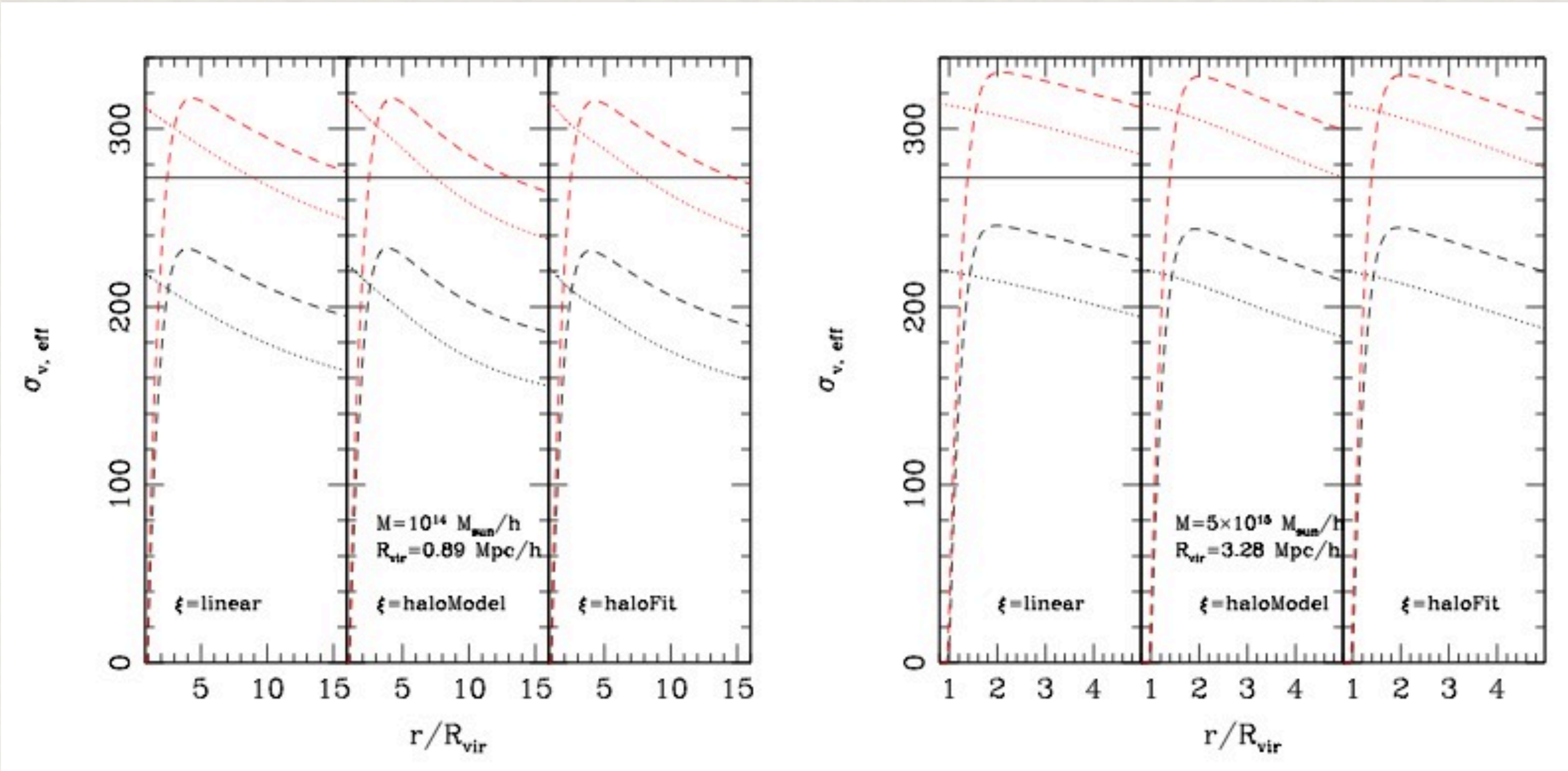
$$\sigma^2_{v,\rm eff}(r_{3d}, M) = \frac{1}{\bar{w}} \int_{m_{\rm min}}^{m_{\rm max}} {\rm d}m \int_0^{r_{\rm vir}(m)} {\rm d}^3y \; w(m, \vec{y}; \vec{r}_{3d}, M) \sigma^2_v(m, \vec{y}),$$

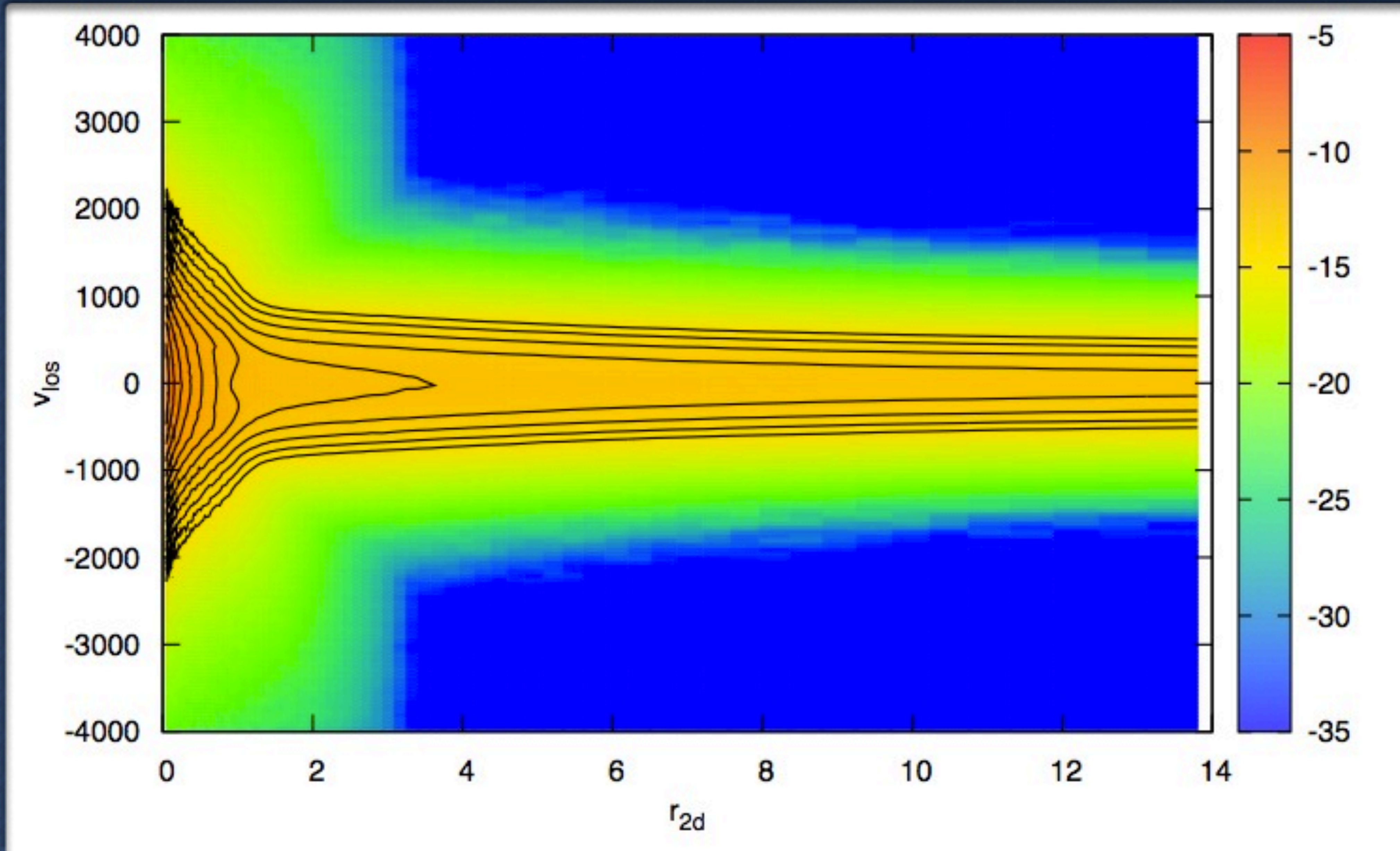
mass of secondary halo

relative position of r_{3d} to the center of secondary halo

$$w(m, \vec{y}; \vec{r}_{3d}, M) = n(m) \rho_{\rm NFW}(|\vec{y}|)[1 + \xi_{hh}(|\vec{x}|, M, m)],$$

$$\sigma^2_{\rm eff}(r_{3d}, M_{\rm halo})$$





Look better, but still not matching numerical measurements

Halo-halo velocity

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- *AND assume this radial velocity is described by spherical collapse model.*

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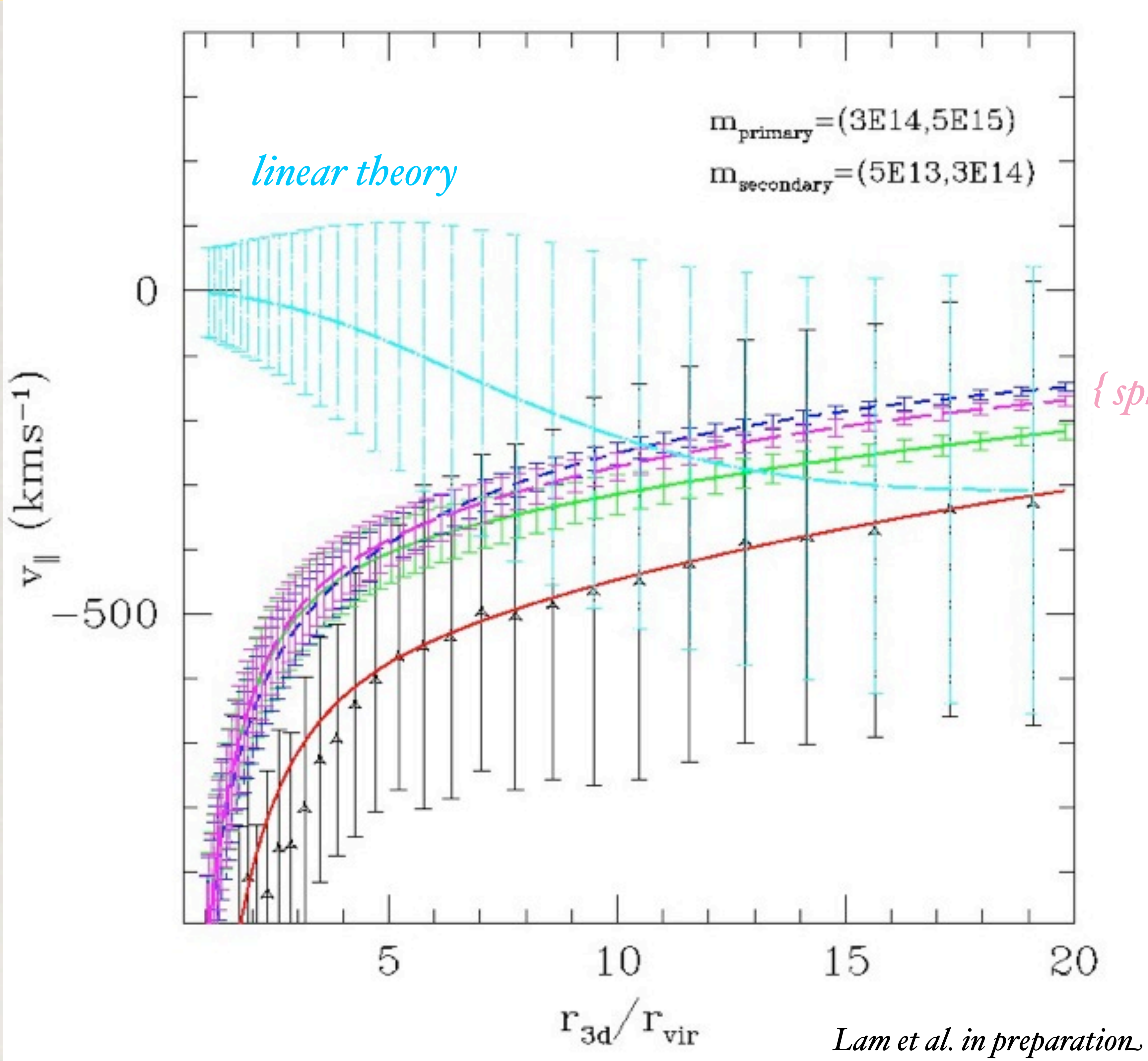
- *AND assume this radial velocity is described by spherical collapse model.*
- *Go to numerical simulations to check if it is true after stacking.*

Halo-halo velocity: Parallel to the line of separation



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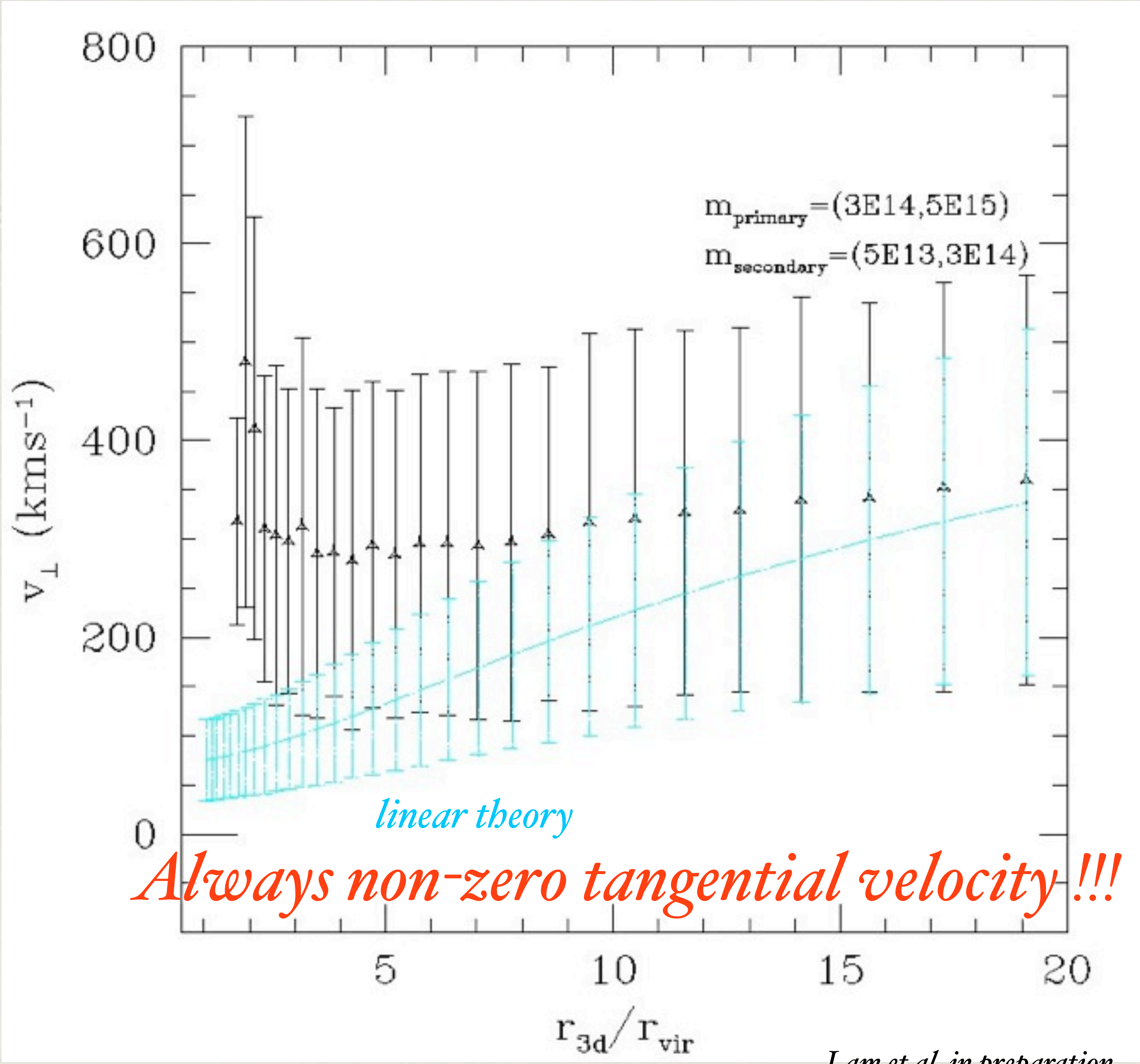
Halo-halo velocity: Perpendicular to the line of separation



Lam et al. in preparation

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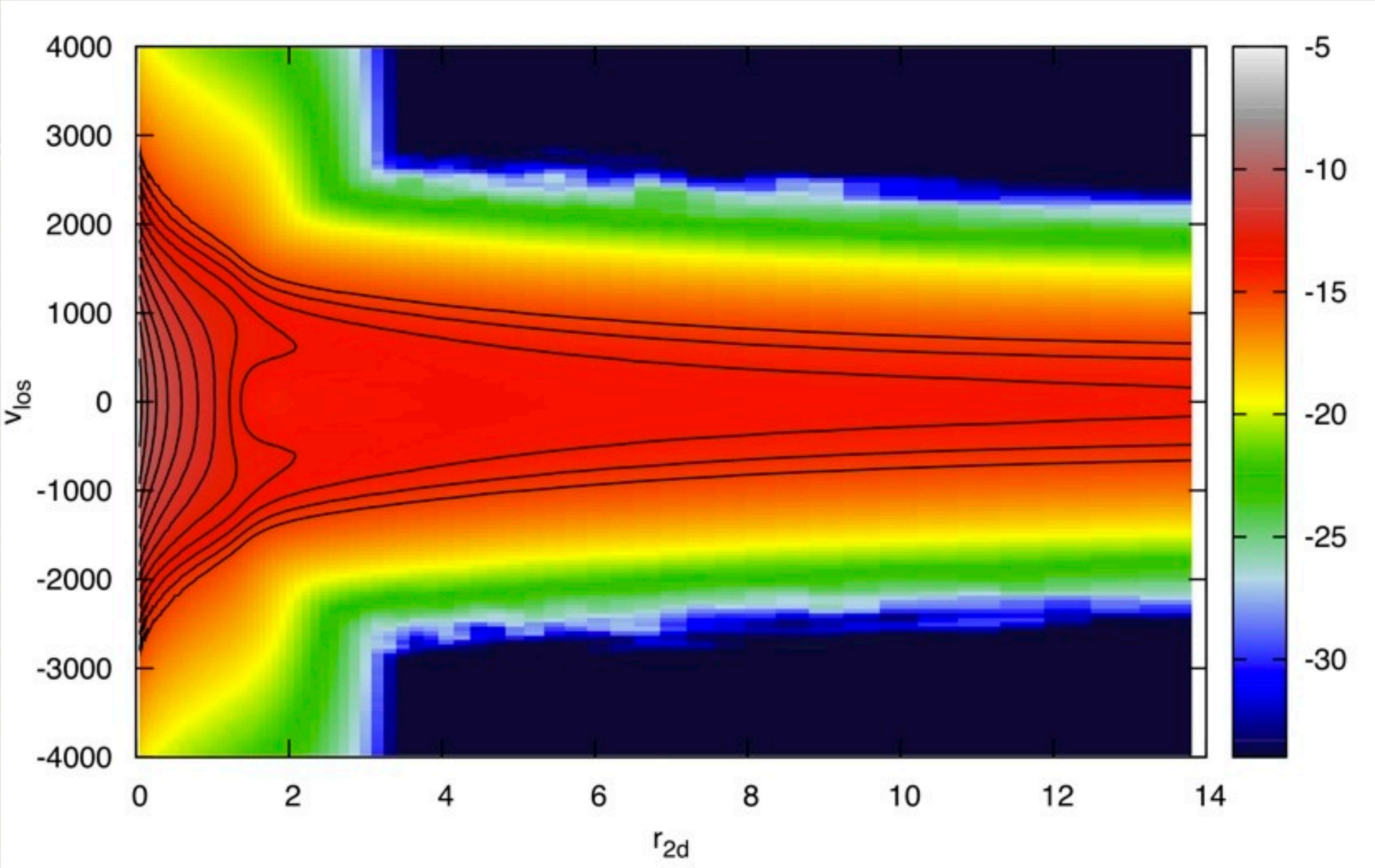
- *Tangential: constant mean and dispersion (from linear theory at large separation).*

Semi-empirical model



$3x10^{14} \leq M_{primary} \leq 5x10^{15}$

$1x10^{8} \leq M_{secondary} \leq 5x10^{15}$

Summary

- *Semi-empirical model would capture some of the features in the numerical measurements.*
- *However there is still room for improvement in the current model.*
- *The effect of the virial motion extends to radius bigger than the virial radius (see double peaks).*
- *Modified gravity: Change virial theorem (hence the virial motion) + spherical collapse (halo-halo velocity).*

Primordial non-Gaussianity

- *As a probe to inflation models*
- *Fluctuations produced by single scalar field, slow-roll inflation are described by Gaussian statistics*
- *Gaussian statistics: fully described by the two-point correlation function*

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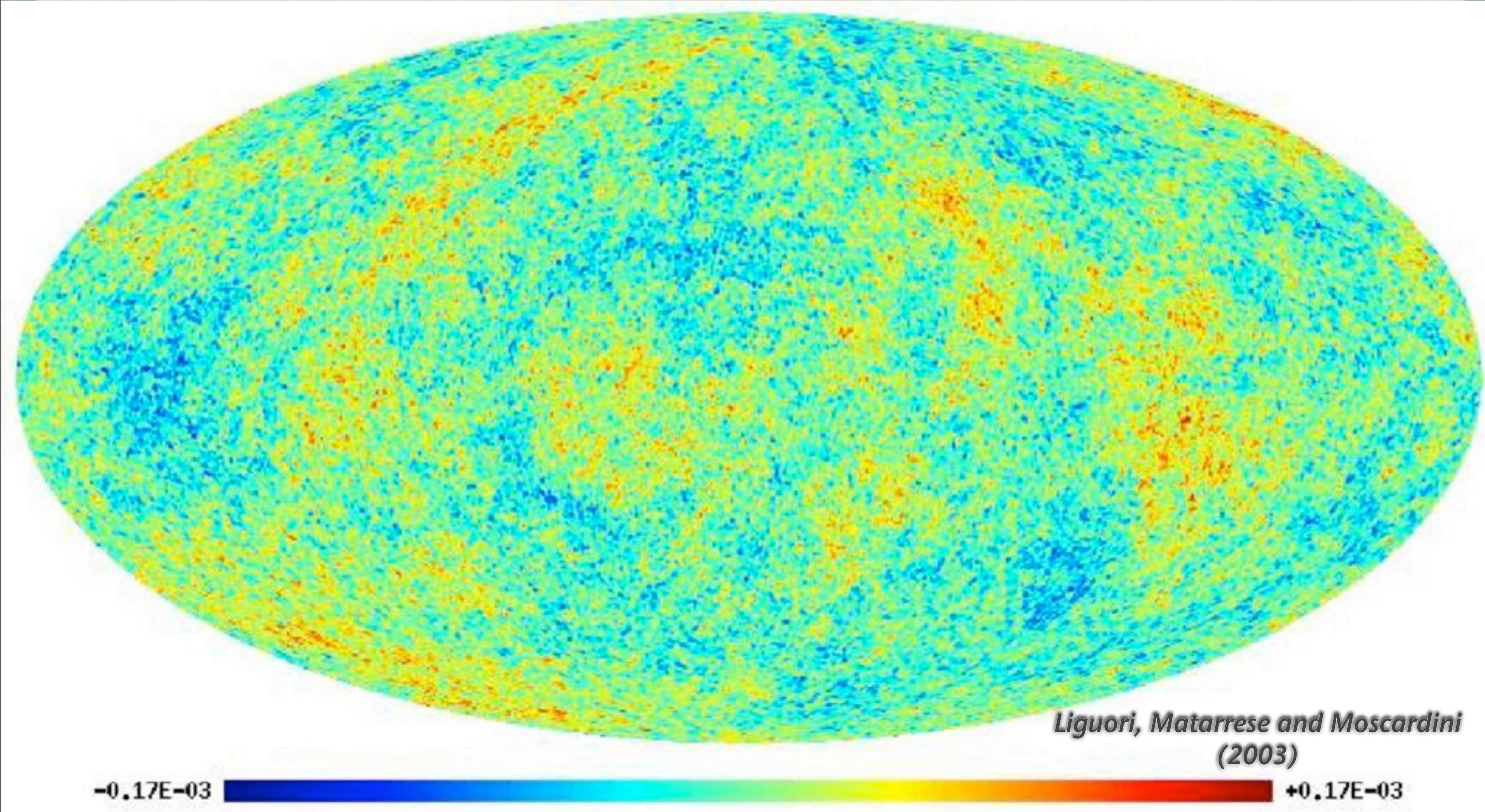
Fig. 1. Representation of the connected part of the moments.
Bernardeau et al. (2002)

CMB Probe



$f_{nl} = 0$

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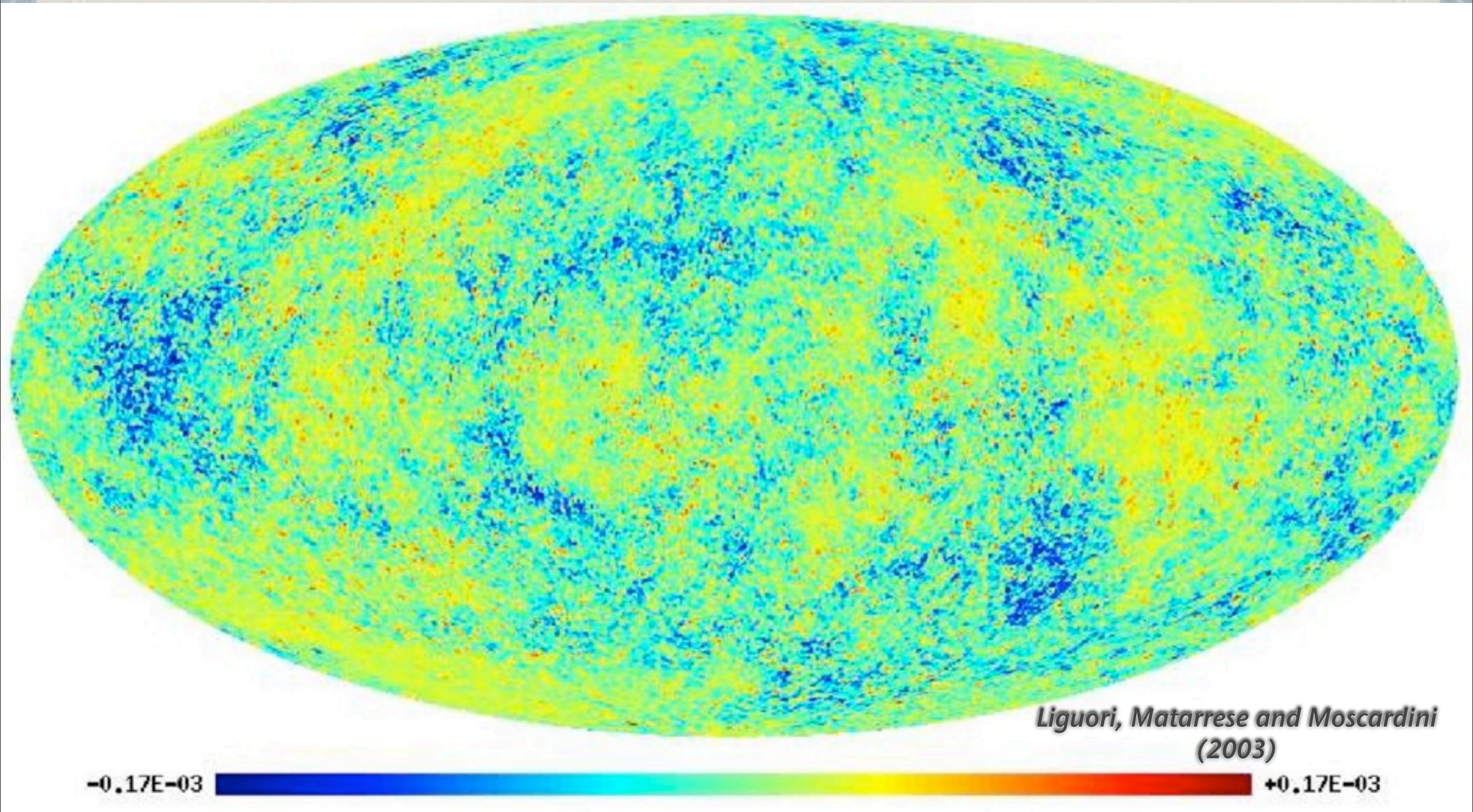

$f_{nl} = 3000$

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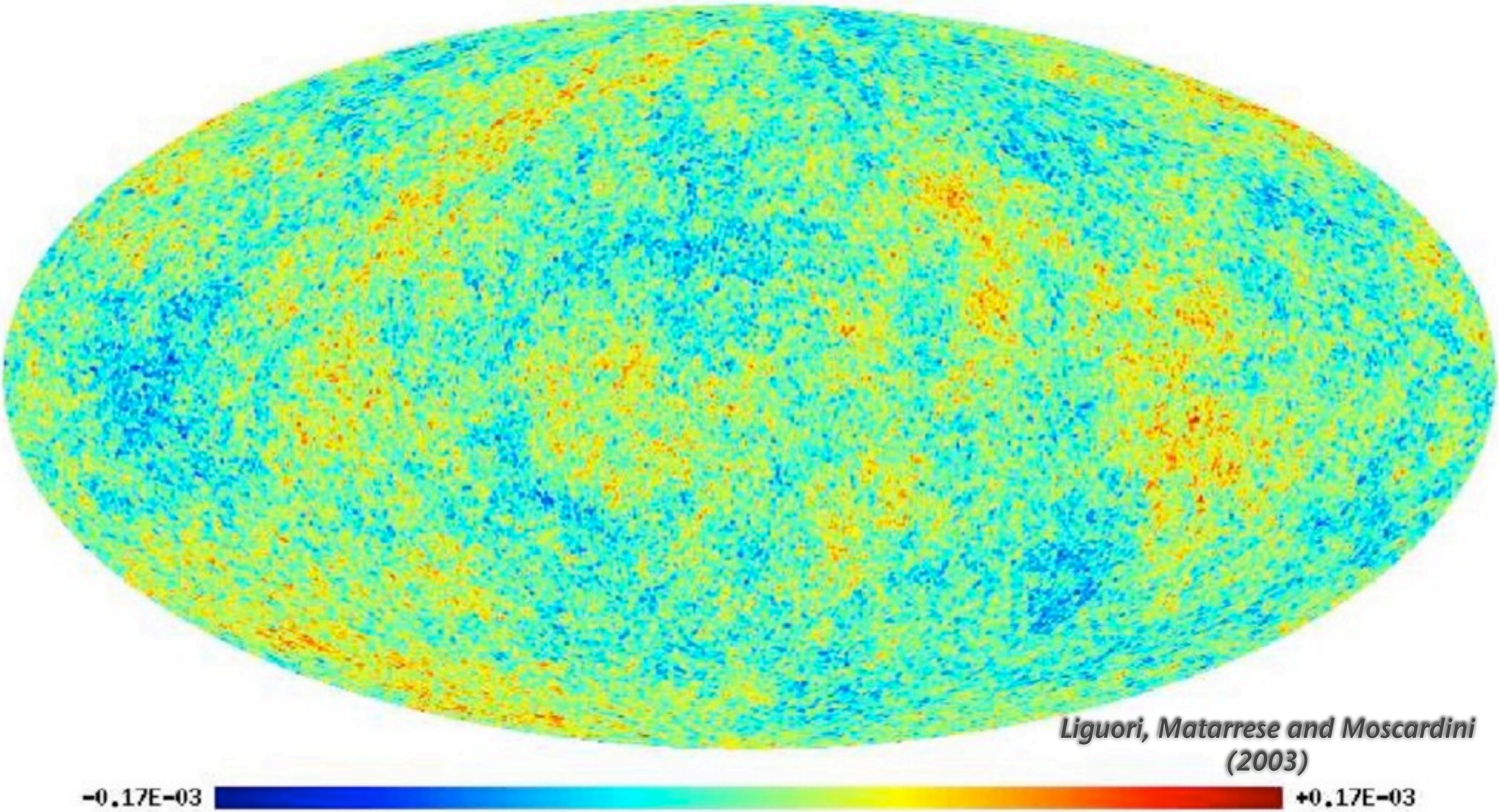

Liguori, Matarrese and Moscardini (2003)

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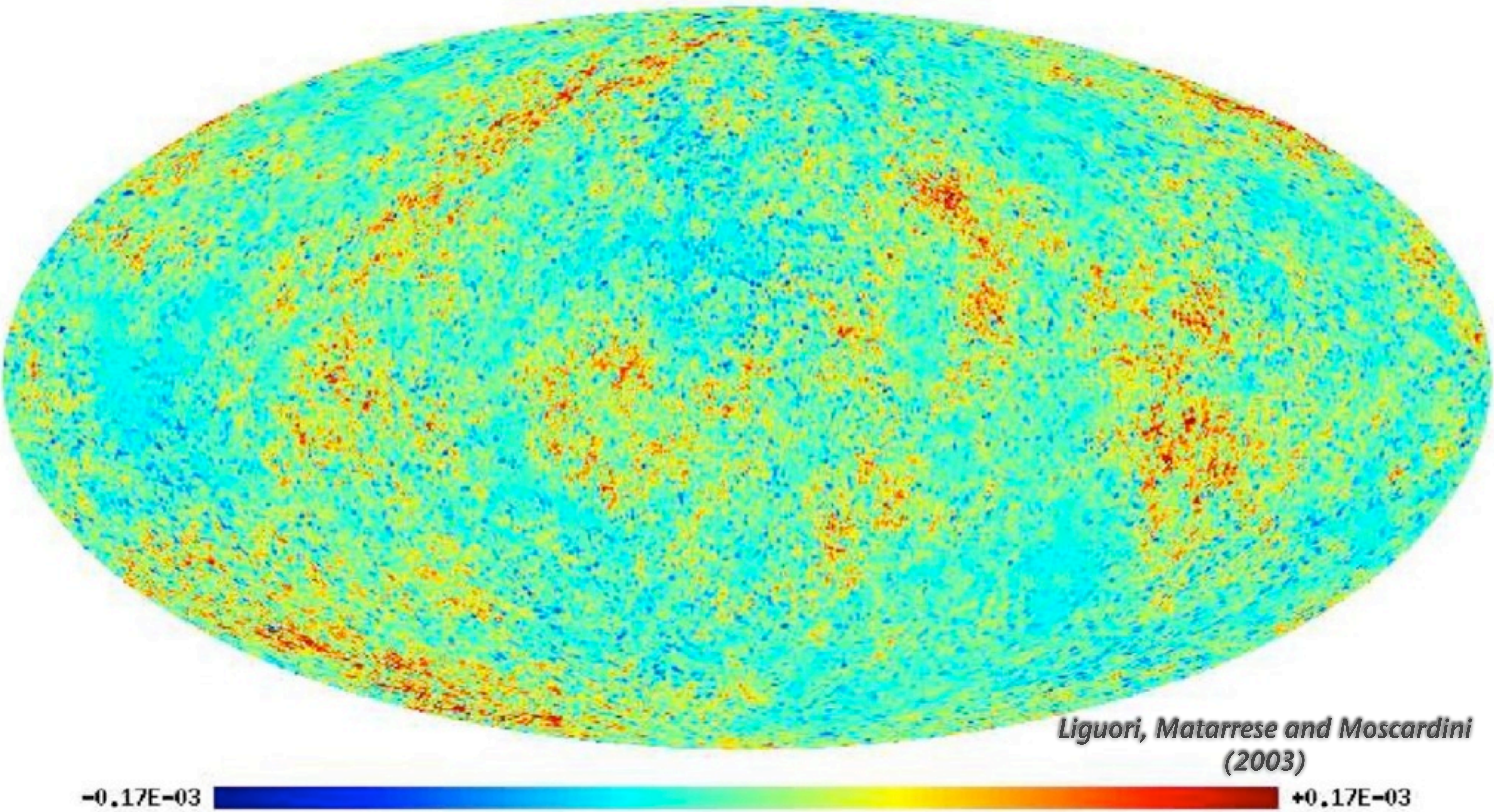
CMB Probe



$f_{nl} = -3000$

Large-scale structures probes

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- *Extremum sensitive to primordial non-Gaussianity (see CMB maps above)*
- *Abundances of rare objects to constrain primordial non-Gaussianity.*
- *Scale-dependent halo bias – distinctive signal at large scale (small k).*

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- *Peculiar velocity field - density field in linear theory.*
- *Using velocity field to probe primordial non-Gaussianity: pairwise velocity distribution, redshift space distortion.*

Pairwise Velocity Distribution

- *Measure probability of the relative velocity of tracers separated by some separation r.*

$$p(v_\parallel, v_\perp; r, f_{nl}) = p_g(v_\parallel, v_\perp; r)?$$

Pairwise velocity distribution

- *Look at linear theory prediction first*

Pairwise velocity distribution

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  - *Gaussian initial conditions: Multivariate Gaussian*

$$p_0(\mathbf{v}; r) = \frac{1}{(2\pi)^{3/2}\sqrt{|A|}} \exp\left(-\frac{1}{2}\mathbf{v}^T A^{-1}\mathbf{v}\right) \qquad \mathbf{v} = (v_\parallel, v_{\perp_a}, v_{\perp_b})$$

Linear Pairwise Velocity Distribution

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- *No correlation between different v's: only non-vanishing correlations are auto-correlations*

$$\langle v_{||}^2 \rangle, \ \langle v_{\perp_a}^2 \rangle, \text{ and } \langle v_{\perp_b}^2 \rangle$$

Linear Pairwise Velocity Distribution

- *No correlation between different v's: only non-vanishing correlations are auto-correlations*

$$\langle v_{\|}^2 \rangle,\ \langle v_{\perp_a}^2 \rangle, \text{ and } \langle v_{\perp_b}^2 \rangle$$

- *Can separate into a product of three univariate Gaussian*

$$p_0(\mathbf{v}; r) = p_0(v_{\|}; r) p_0(v_{\perp_a}; r) p_0(v_{\perp_b}; r)$$

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Pairwise velocity distribution with primordial non-Gaussianity

- *Higher order correlations as well as cross correlations are non-vanishing.*
- *Multivariate Gaussian does not work.*
- *Main corrections are in the order of bispectrum.*

Characteristic Function

- *Fourier transform of the probability density function*

Moment generating function: $\mathcal{M}(\vec{\lambda}; r) \quad = \sum_{p=0}^{\infty} \frac{\langle (\vec{\lambda}\cdot\vec{v})^p \rangle}{p!} \quad = \langle \exp(\vec{\lambda}\cdot\vec{v}) \rangle$

Cumulant generating function: $\mathcal{C}(\vec{\lambda}; r) \quad = \sum_{p=2}^{\infty} \frac{\langle (\vec{\lambda}\cdot\vec{v})^p \rangle_c}{p!} \quad = \langle \exp(\vec{\lambda}\cdot\vec{v}) \rangle_c$

And they are related by: $\mathcal{M}(\vec{\lambda}; r) \quad = \exp\left[\mathcal{C}(\vec{\lambda}; r)\right] \quad = \exp\langle \exp(\vec{\lambda}\cdot\vec{v}) \rangle_c$

Pairwise velocity distribution with primordial non-Gaussianity

Characteristic function:

$$\exp\left[\sum_j \frac{\langle v_j^2\rangle}{2}(i\lambda_j)^2 + \frac{\langle v_\parallel^3\rangle}{3!}(i\lambda_\parallel)^3 + \frac{\langle v_\parallel v_{\perp_a}^2\rangle}{2}(i\lambda_\parallel)(i\lambda_{\perp_a})^2 + \frac{\langle v_\parallel v_{\perp_b}^2\rangle}{2}(i\lambda_\parallel)(i\lambda_{\perp_b})^2 + \dots\right]$$

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Fourier's Transform to get the probability density function

Pairwise velocity distribution with primordial non-Gaussianity

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Fourier's Transform to get the probability density function

But in general it is difficult to do the transform.

Approach: Taylor expand around Gaussian density function

$$\mathcal{F}_0(i\boldsymbol{\lambda}; r) = \exp\left[\sum_j \frac{\langle v_j^2 \rangle}{2}(i\lambda_j)^2\right]$$

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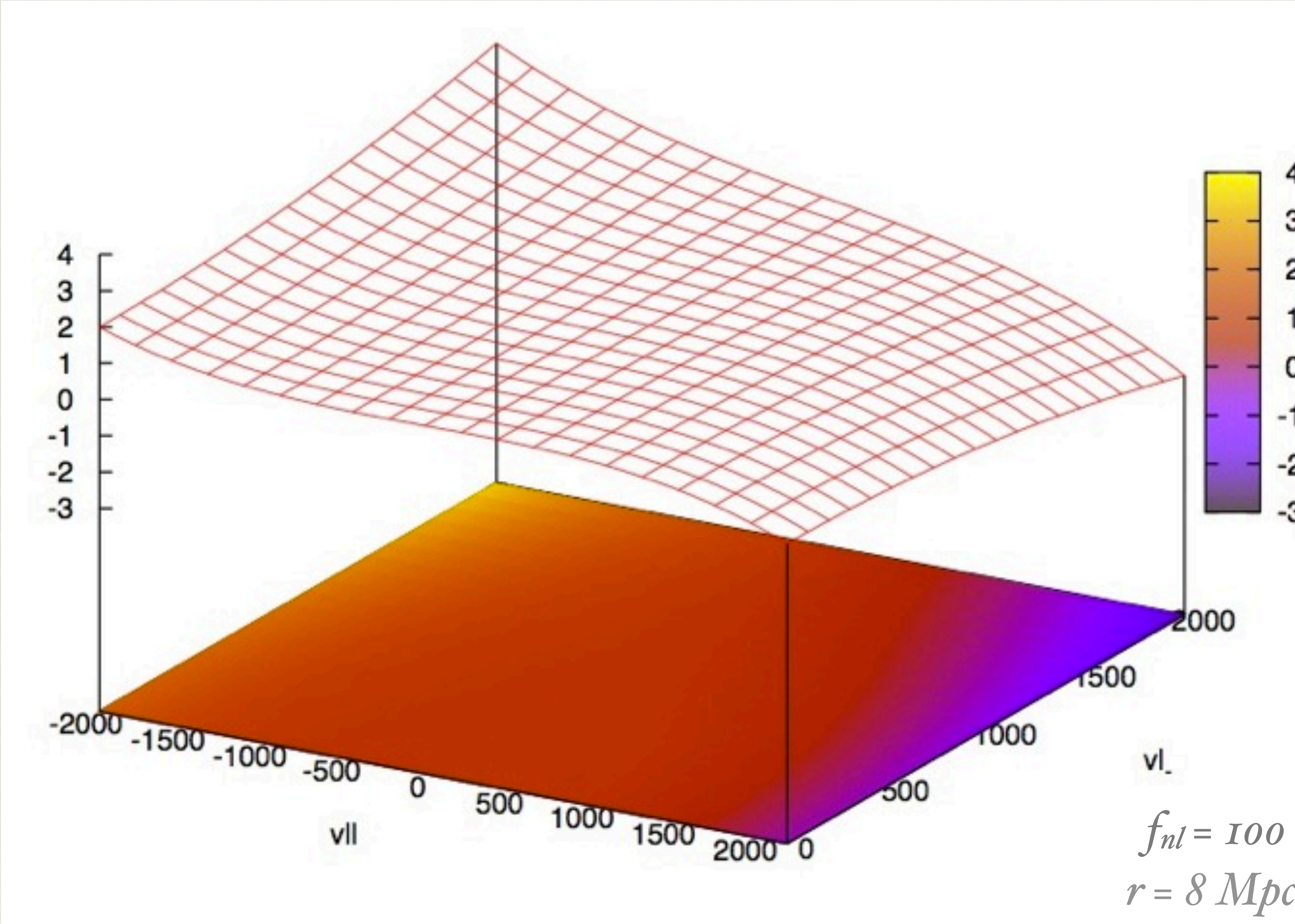
The linear relative velocity density function:

$$p(v_\parallel, v_{\perp_a}, v_{\perp_b}; f_{nl}, r) = p_0(v_\parallel, v_{\perp_a}, v_{\perp_b}; r)[1 + \alpha_{300} h_{300} + \alpha_{120}(h_{120} + h_{102})],$$

$$\alpha_{300} = \frac{1}{6}\frac{\langle v_\parallel^3 \rangle}{\langle v_\parallel^2 \rangle^{3/2}}, \qquad \alpha_{120} = \frac{1}{2}\frac{\langle v_\parallel \mathfrak{v}_\perp^2 \rangle}{\langle v_\parallel^2 \rangle^{1/2} \langle \mathfrak{v}_\perp^2 \rangle},$$

$$h_{ijk} \equiv H_i(\nu_\parallel) H_j(\nu_{\perp_a}) H_k(\nu_{\perp_b})$$

$$p(v_{\parallel}, v_{\perp}; r, f_{nl})/p_g(v_{\parallel}, v_{\perp}; r)$$



No change

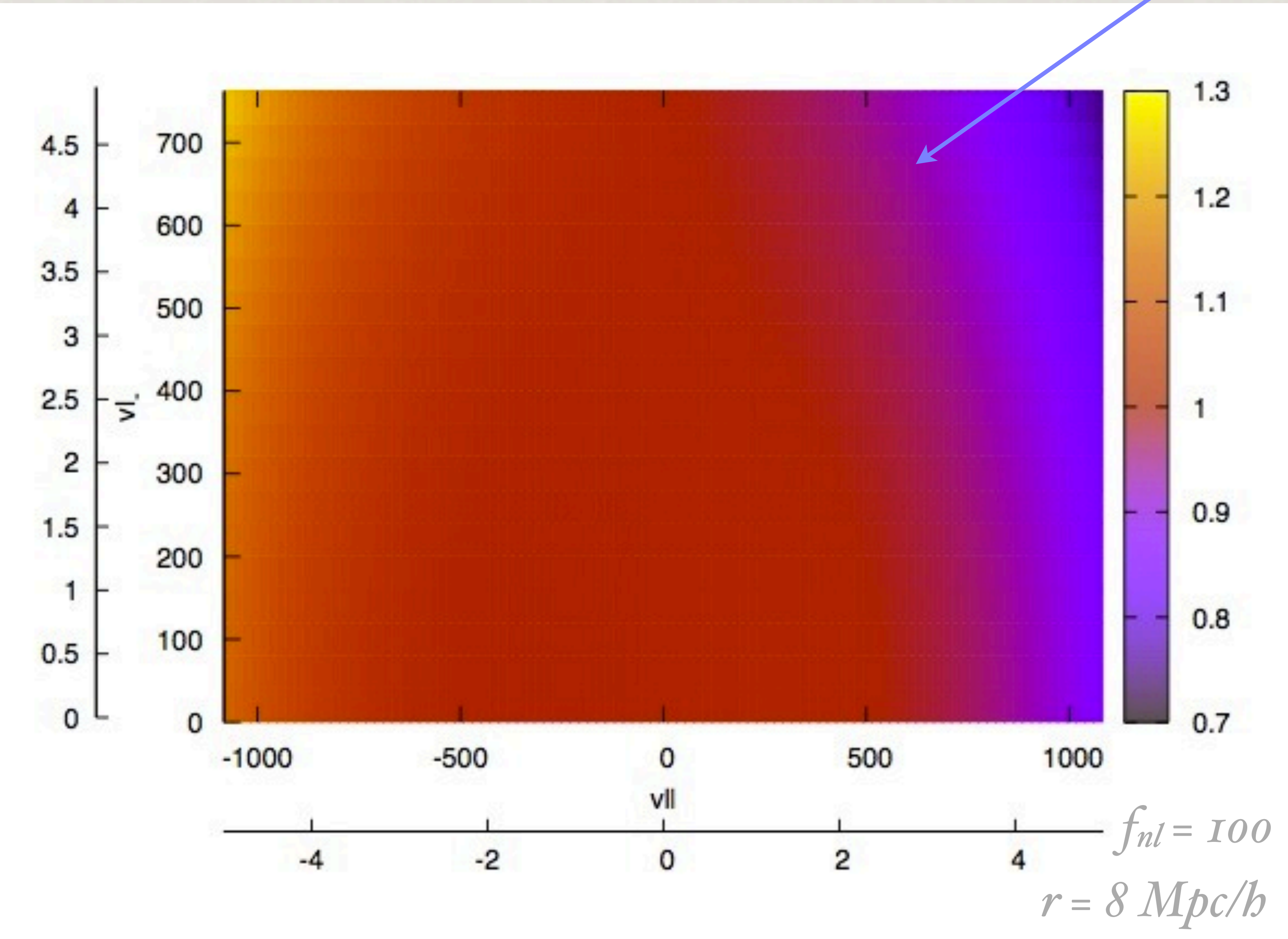
f_{nl} = 100

r = 8 Mpc/h

Lam, Nishimichi & Yoshida 2011

$$p(v_{\parallel}, v_{\perp}; r, f_{nl})/p_g(v_{\parallel}, v_{\perp}; r)$$

Change due to cross correlation



f_{nl} = 100

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Lam, Nishimichi & Yoshida 2011

Linear theory predictions:

Marginalized over $v_\perp$, ratio < 1 when $v_{||} \gg 0$ for $f_{nl} > 0$

ratio > 1 when $v_{||} \ll 0$ for $f_{nl} > 0$

Marginalized over $v_{||}$, ratio $= 1$ for all $v_\perp$ and f_{nl}

Evolution of velocity distribution

Two particles separated by r at z with relative velocity v have different r_0 and v_0 at earlier redshift z_0.

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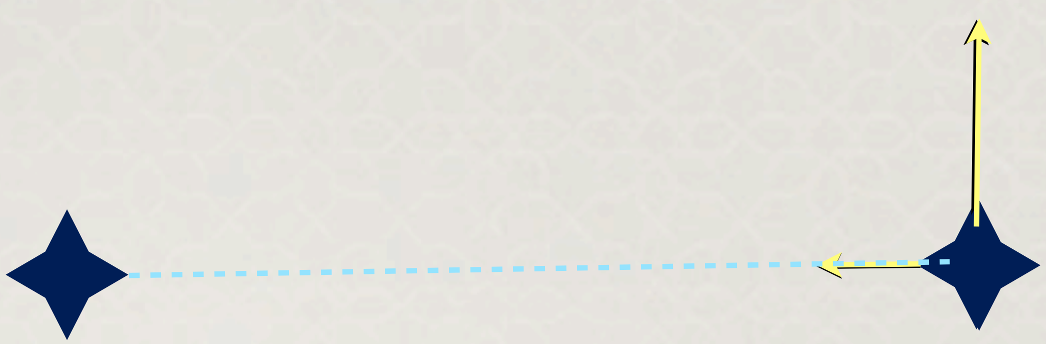
Zeldovich Approximation: Velocity unchanged.



Evolution of velocity distribution

Two particles separated by r at z with relative velocity v have different r_o and v_o at earlier redshift z_0.

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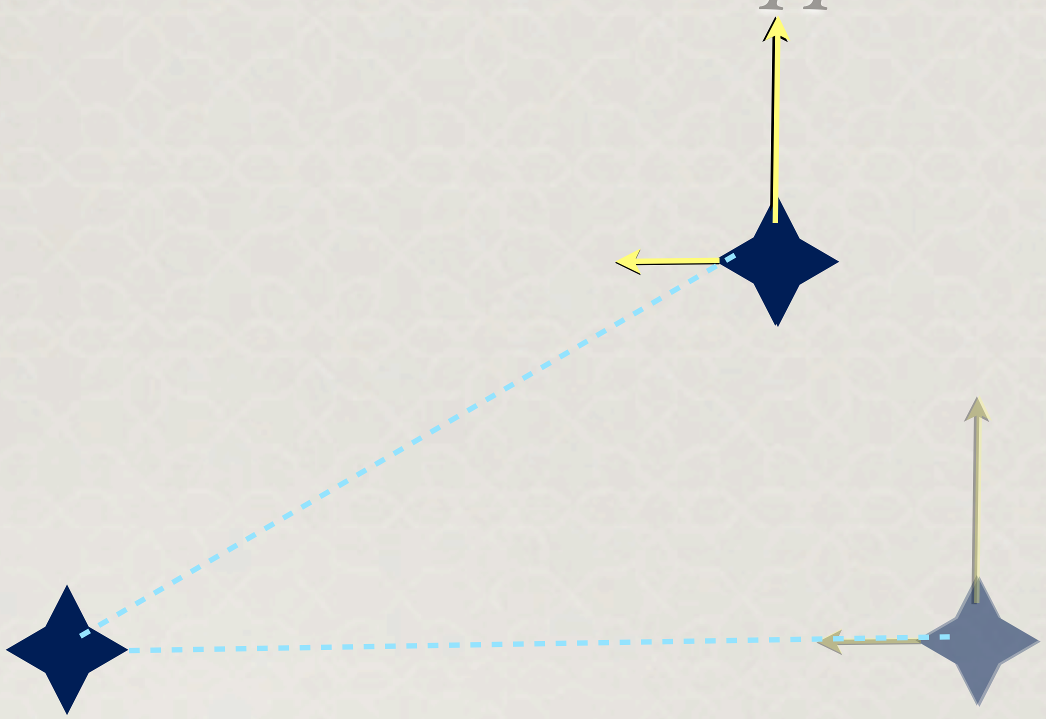
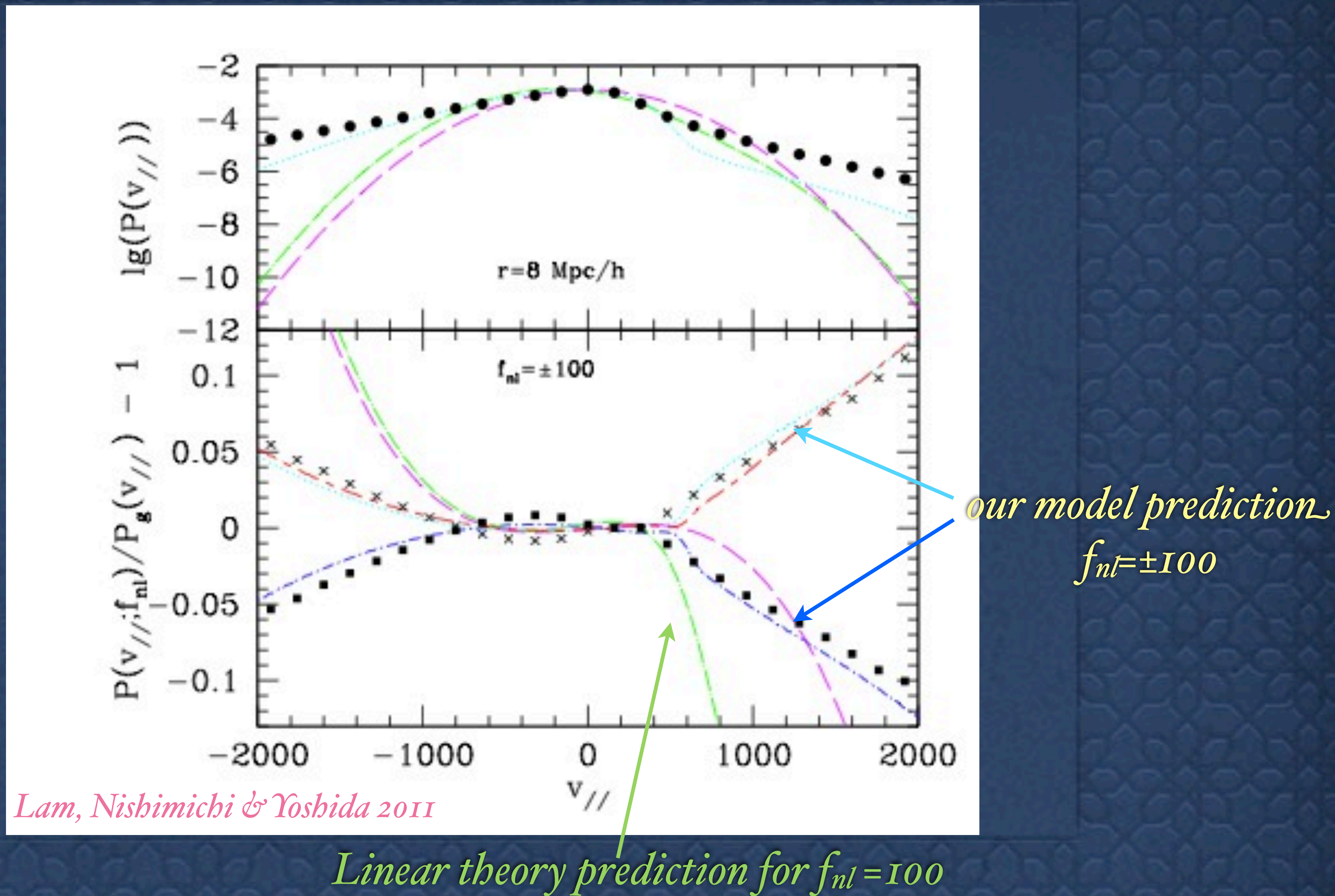


$$r^2 = \left(r_i + \frac{D_0}{\dot{D}_i} v^i_{\parallel}\right)^2 + \left(\frac{D_0}{\dot{D}_i}\right)^2 \left({v^i_{\perp_a}}^2 + {v^i_{\perp_b}}^2\right)$$

$$v_{\parallel} = \frac{\dot{D}_0}{r}\left(\frac{r_i v^i_{\parallel}}{\dot{D}_i} + \frac{D_0}{\dot{D}_i^2} v^{i\,2}\right)$$

$$|v_{\perp}|^2 = v^2_{\perp_a} + v^2_{\perp_b} = \left(\frac{\dot{D}_0}{\dot{D}_i} v^i\right)^2 - v^2_{\parallel}$$

x = ratio -1 for f_{nl}=100
■ = ratio -1 for f_{nl}=-100



Parallel to the line of separation

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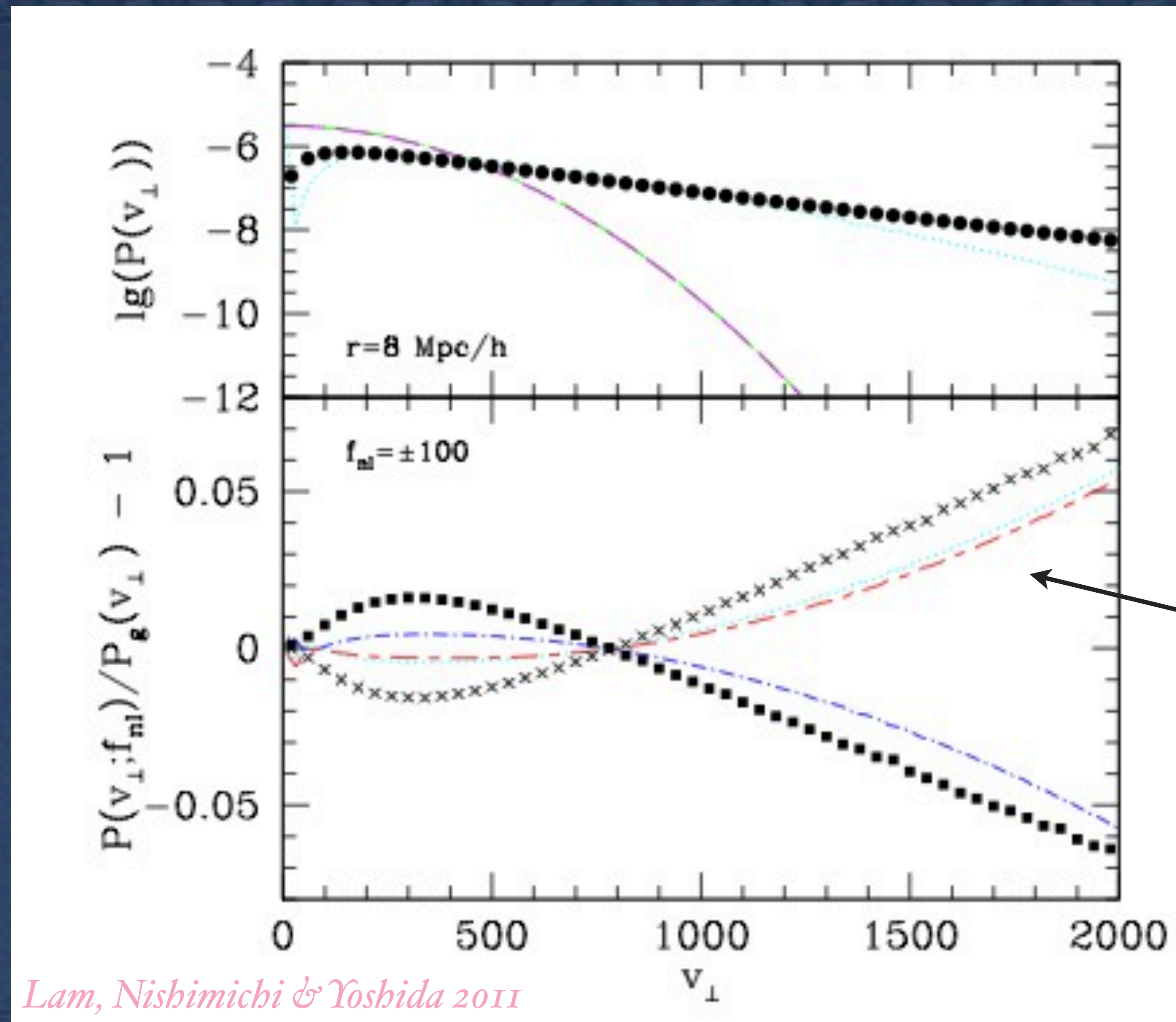
■ = ratio -1 for f_{nl} =-100



Lam, Nishimichi & Yoshida 2011

Linear theory predicts no change in PDF ~when marginalized over v_parallel

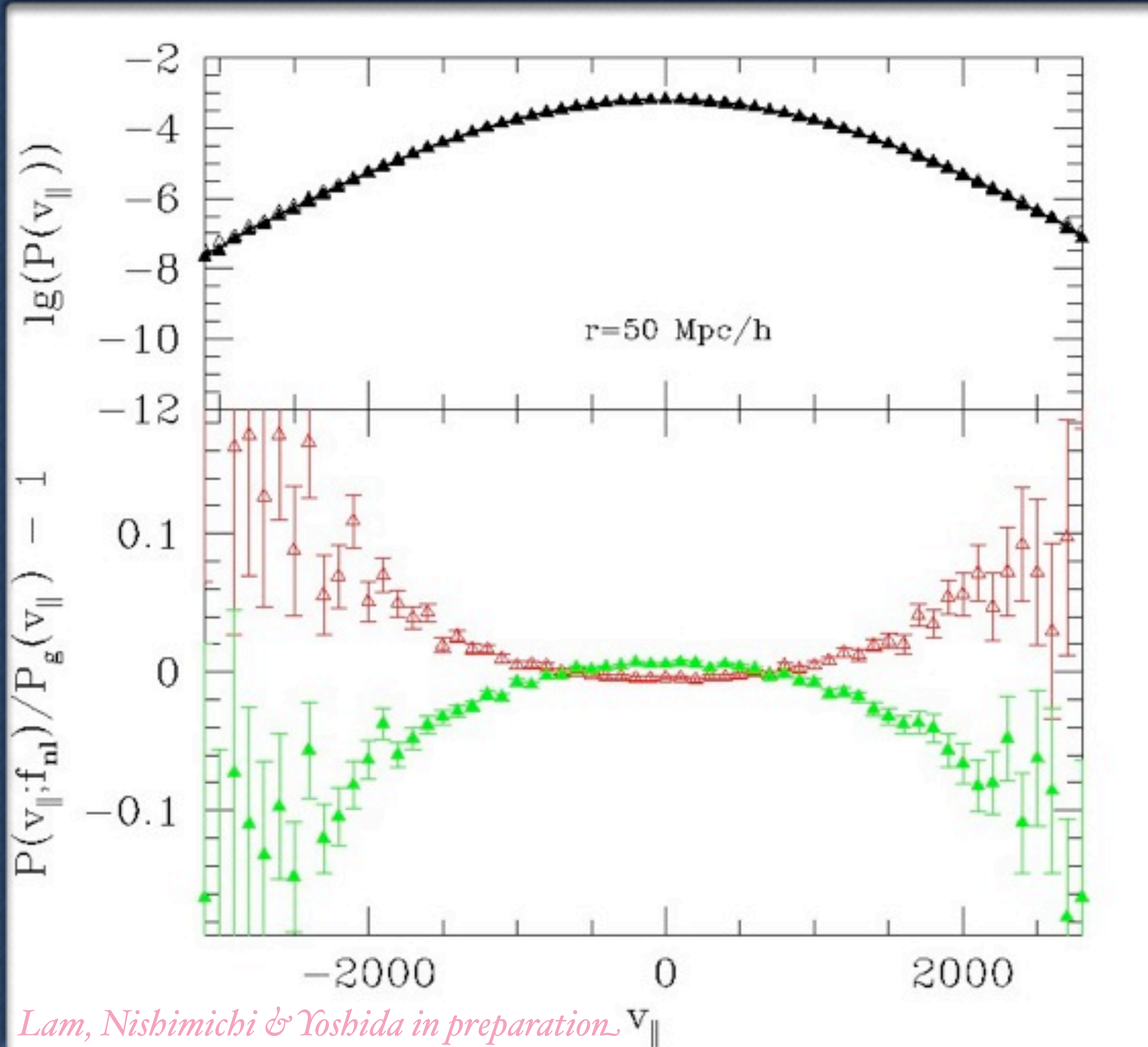
Perpendicular to the line of separation

Short summary

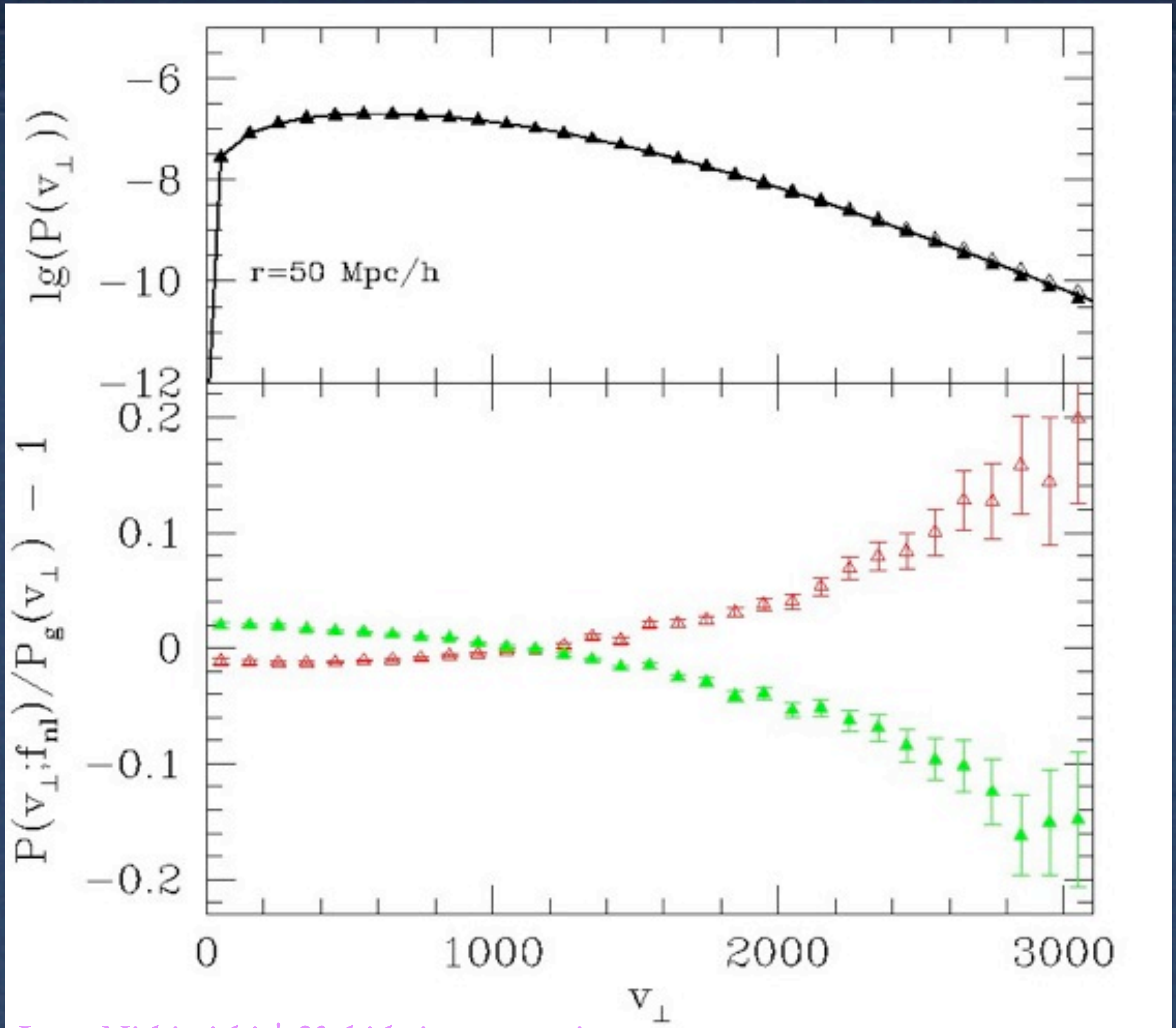
- *Primordial non-Gaussianity leaves signatures on the velocity field: ~we use the pairwise velocity distribution to demonstrate this effect.*
- *Primordial non-Gaussianity induces non-vanishing three-point correlation functions: auto correlation of v_parallel AND cross correlation of v_parallel and v_perp.*
- *Linear theory does not work – wrong predictions for both v_parallel and v_perp.*
- *Evolution model implementing Zeldovich Approximation works reasonably well.*

What's next?

- *Redshift space distortion to probe primordial non-Gaussianity.* *(see Lam, Desjacques & Sheth 2010 or Schmidt 2010 for linear theory prediction)*
- *Peculiar velocity of biased tracers – massive clusters*



Halo-Halo: Parallel to the line of separation



Lam, Nishimichi & Yoshida in preparation

Halo-Halo: Perpendicular to the line of separation