Quasistatic Equlibrium Models of Galaxy Formation and the Consequences of Stochasticity

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12 May 2015

Berkeley Cosmology Seminar

Or How I Learned to Stop Worrying
and
Love the Central Limit Theorem

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A Malaise in Galaxy Evolution

- Incomplete knowledge limits us to oversimplified histories
- Toy models used as a substitute for physical understanding
- Empirical constraints on average histories
- Dependent on assumption of no. dens. evolution, or
- Dependent on assumed halo merger trees and MF evol
- All such work assumes SFR-M correlation is deterministic

The result is a picture that does not make full use of the data.

The result is a picture reasonably devoid of astrophysics.

... or least a picture with too much baked into the pie from the start.

My Starting Point

- Since Tinsley & Larson (1978), through Efstathiou (2000), Dekel et al (2009) we understand galaxies grow and form stars in steady-state between inflows, outflows, feedback

- Such quasi-static equlibria imply evolving mass growth rates, with an expectation of steady-state (i.e. $\mathrm{E}[\Delta dM/dt] = 0$)

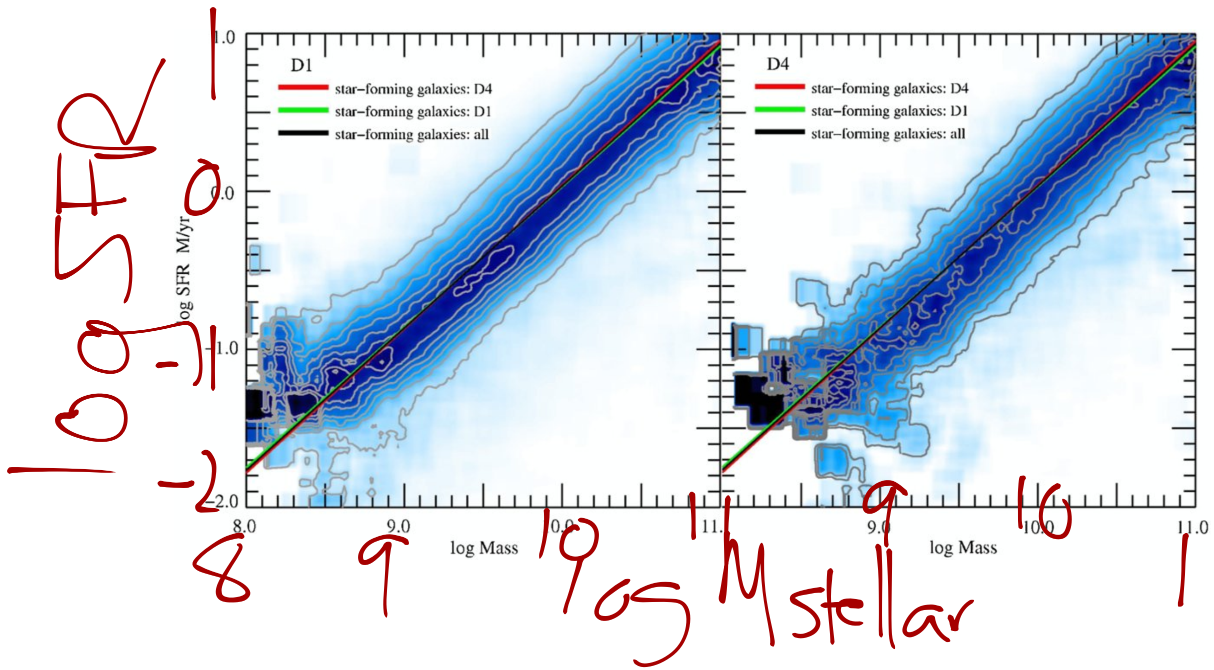
- Starting with this expectation, one can write down dM/dt as a nonnegative non-Markovian stochastic process, use the central limit theorem to derive long-term expectation values $\mathrm{E}[dM/dt]$, $\mathrm{E}[M]$, and $\mathrm{Var}[dM/dt]$

My Ending Points (i.e. Math is Very Powerful)

- The "Star-Forming Main Sequence" is emergent, and a natural consequence of stellar mass growth as a stochastic process
- Derive $\mathrm{E}[(dM/dt)/M] = 2/t$, accurately matching SSFRs over $0<z<10$
- Observed intrinsic scatter in SSFR at fixed mass falls right out
- Retrodict stellar mass functions and Madau diagram $3 \lesssim z \lesssim 10$
- Infinite set of possible SFHs, including those of local group dwarf gals, MW
- Retrodict quiescent galaxy fractions along flat part of SFMS
- Strongly limits how well one can link specific progenitors with specific descendents
- Must trace full ensembles over cosmic time, but here's the math to do it
- *and another rather amusing surprise pops out as well...*

Starting with my Main Beef

This is the/an SDSS correlation of SFR vs stellar mass



(Peng et al 2010)

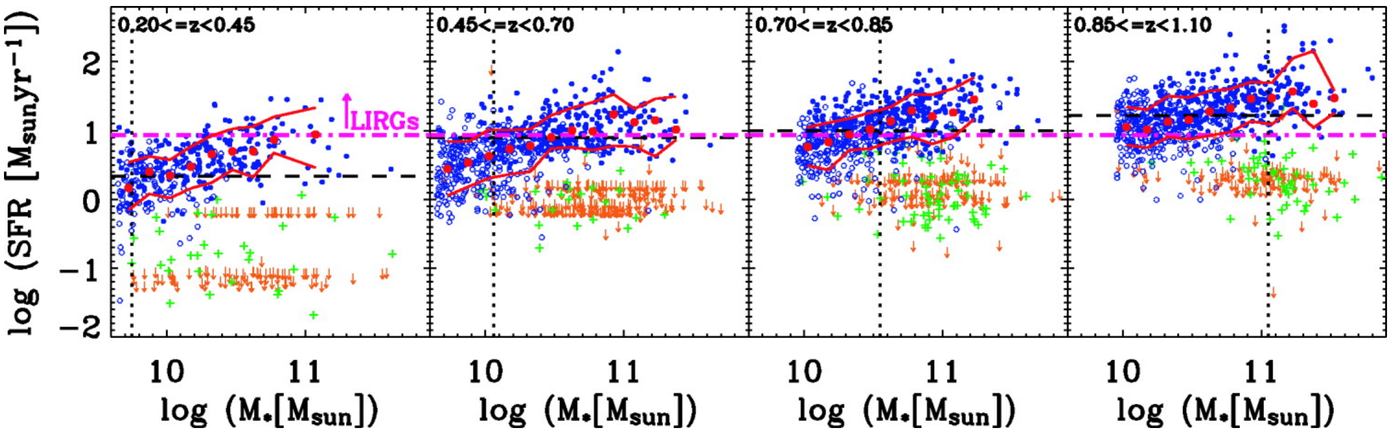
The slope in a log-log plot is not super far from unity.

The scatter in SFR at fixed stellar mass is ~0.3 dex.

The common interpretation is that more massive galaxies make more stars.

This Correlation Is Seen at High Redshift

This is the DEEP2/AEGIS correlation of SFR vs stellar mass



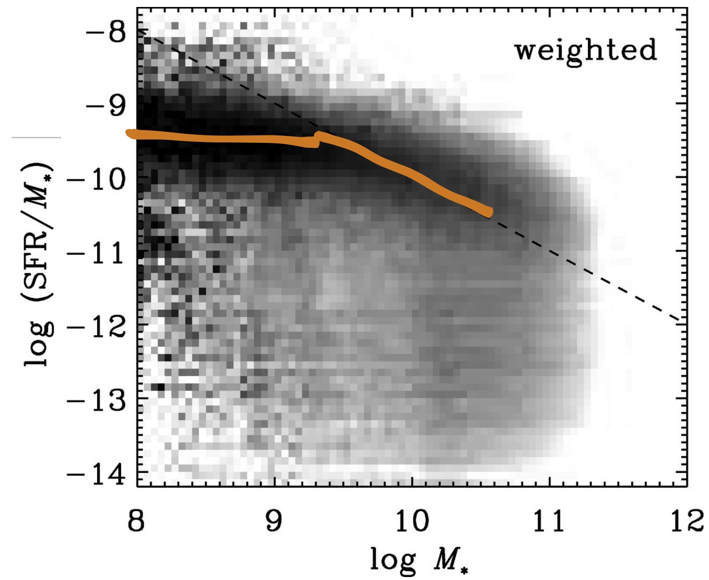
(Noeske et al 2007)

The correlation appears to be fundamental; now dubbed "Main Sequence of SF"

The scatter ~0.3 dex in SFR at fixed stellar mass *relatively* constant in M & z

How Does This Correlation Reflect Galaxy Evolution?

This is a much better SDSS view of specific SFR vs stellar mass:



(Salim et al 2007)

Intrinsic scatter ~0.4 dex in SSFR, *relatively* constant in M

SSFR vs M: relatively flat below $\log M < 9.5$, anticorrelated at higher masses

There must be mass-dependent astrophysics!

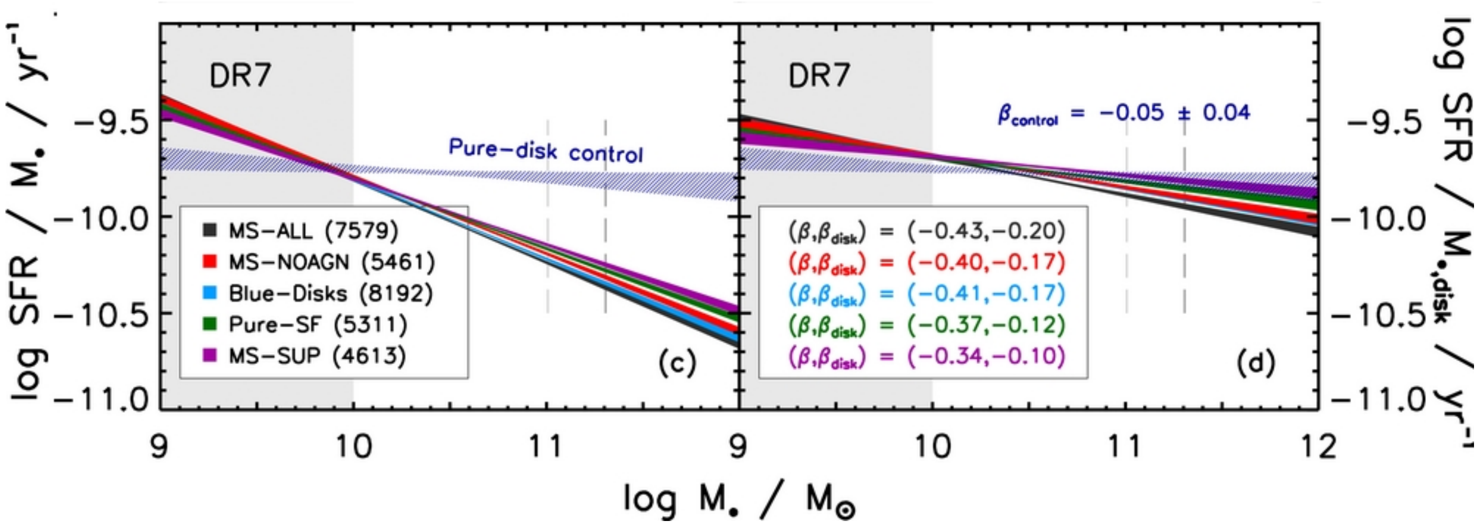
But What Is the Slope Really Saying?

Is SFR per unit stellar mass the best diagnostic of physics?

At high masses, galaxies are *bulge*-dominated

Bulges tend not to actively form a lot of stellar mass

Change SSFR to SFR per unit *disk* mass: a lot of the slope goes away



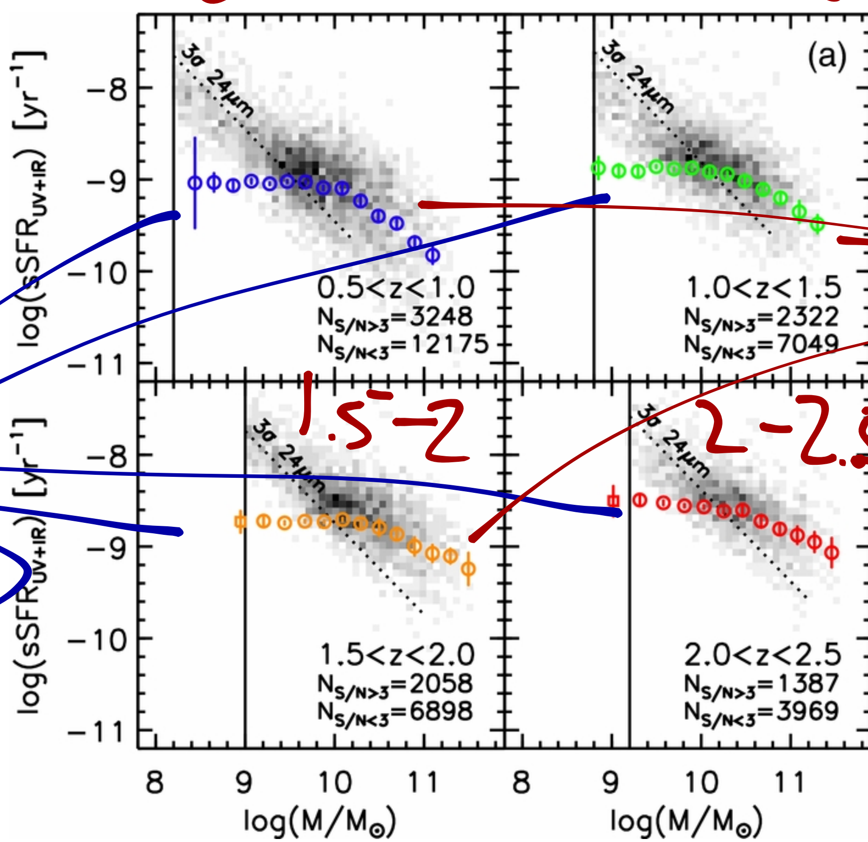
(Abramson, Kelson et al 2014)

Thus expect late-time bulge formation $\longrightarrow$ flatter SFMS at higher z,

How Did Our Suggestion Work Out?

Constant ⟨SSFR⟩ at low-mass continued to high z; break at higher mass

0.5-1 1-1.5



1.5-2 2-2.5

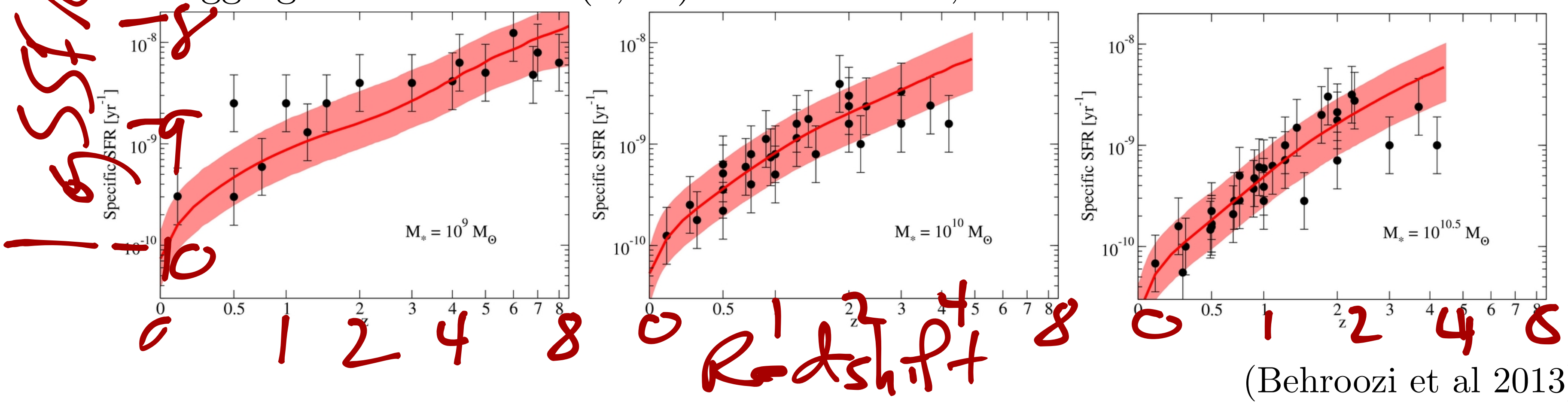
Flat ⟨SSFR⟩ vs M

note flattens at high M

(Whitaker et al 2014)

Is our children learning?

The aggregate mean SSFR(z, M) are then used, as in



(Behroozi et al 2013)

... along with halo merger trees to estimate mean histories of galaxies.

- But do average histories tell us about what individual galaxies actually do?
- And are all these $\langle \text{SSFR} \rangle = f(M, z)$ even being used properly?

Instead of starting with toy models, what if we start with far fewer assumptions?

Let us begin: Assumption 1 — Steady-state

So let us consider a process, S_t, as the amount of stellar mass formed over the tth time interval:

$$S_t = M_{t+1} - M_t$$

where M_t is the mass accumulated up to time t.

In steady-state, the expectation for S_{t+1} is

$$E[S_{t+1}] = S_t$$

We call S a "stationary process."

Let us begin: Assumption 1 — Steady-state

Given a sequence of stellar mass growth, $S_0, S_1, S_2, \ldots, S_{t+1}$, let us define X_{t+1}*,

$$X_{t+1} = S_{t+1} - S_t$$

In other words,

$$S_t = (S_t - S_{t-1}) + (S_{t-1} - S_{t-2}) + (S_{t-2} - S_{t-3}) + \cdots + S_0$$

$$S_t = \sum_{i=1}^{t} X_i$$

And remember that

$$M_{t+1} = \sum_{i=1}^{t} S_i$$

$$M_{t+1} = \sum_{i=1}^{t} \sum_{j=1}^{i} X_j$$

* Warning: astrophysics buried here.

Let us begin: Assumption 1 — Steady-state

So the SFMS is apparently just a correlation between $\sum_i^t X_i$ and $\sum_i^t \sum_j^i X_j$.

Can we work out how those two sums should be correlated?

S is stationary, so $E[X] = 0$. But there is a variance σ_t^{2*}:

$$\mathrm{Var}[S_t - S_{t-1}] = \sigma_t^2$$

Believe it or not, we now have almost everything we need to compute a lot of the evolution of the cosmic ensembles of galaxies!

* Astrophysics buried here!

The Martingale Central Limit Theorem

If the stochastic differences, X, are i.r.v. centered on zero, then S is called a "martingale," and X are "martingale differences."

Why do you care about this?

Sums of sequences of such numbers obey central limit theorems.

If you have central limit theorems, you can compute probabilities!

(Strap yourself in for the ride now.)

The Martingale Central Limit Theorem

We need to compute the variance in S_t:

$$\mathrm{Var}[S_t] = E[S_t^2] - (E[S_t])^2$$

Given that S is stationary, centered on $S_0 = 0$, $E[S_t] = 0$, and thus

$$\mathrm{Var}[S_t] = \sum_{i=1}^{t} X_i^2 = \sum_{i=1}^{t} \sigma_i^2$$

where σ_i is the expected variance in the stochastic changes to S at time i.

Let's take an ensemble of N object histories $S_{n,t}$, where $n \in \{1, 2, 3, \ldots, N\}$.

Each object, n, has a history, with different variances at every timestep, etc.

We therefore define an RMS stochastic fluctuation for n's history up to $S_{n,t}$:

$$\overline{\sigma}_{n,t} = \left(\frac{1}{t} \sum_{i=1}^{t} \sigma_{n,i}^2 \right)^{1/2}$$

← important

The Martingale Central Limit Theorem

Note these RMS stochastic fluctuations for each n history up to time t,

$$\overline{\sigma}_{n,t} = \left(\frac{1}{t}\sum_{i=1}^{t}\sigma_{n,i}^2\right)^{1/2}$$

have all the physics.

The central limit theorem states that the distribution of $S_{n,t}$, normalized by these RMS fluctuations,

$$\frac{S_{n,t}}{t^{1/2}\overline{\sigma}_{n,t}} = \frac{1}{t^{1/2}\overline{\sigma}_{n,t}}\sum_{i=1}^{t}X_{n,i}$$

is a Gaussian centered in zero with a standard deviation of unity:

$$\frac{S_{n,t}}{t^{1/2}\overline{\sigma}_{n,t}} \xrightarrow{d} N(0,1)$$

Imposing Nonnegativity

Stellar mass growth is almost always nonnegative.

Imposing $S \geq 0$ turns S into a submartingale, and S, on average tends to go up.

Every submartingale can be expressed as the sum of:

(1) a martingale (yay!), and

(2) a long-term drift term

The resulting limit for $S > 0$ is the nonnegative half of the Gaussian:

$$P\left[\frac{S_{n,t}}{t^{1/2}\overline{\sigma}_{n,t}} < x\right] = \left(\frac{2}{\pi}\right)^{1/2} \int_0^x e^{-x^2/2} dx$$

WE NOW HAVE A PROBABILITY DISTRIBUTION.

Let us now skip doing the integrals and just write down the 1st and 2nd moments.

Markovian Expectation Values

So far we have derived a probability distribution for S_t assuming the timesteps are independent of each other, and galaxies at time t don't care what they've done previously.

You get 1st and 2nd moments of dP/dx, plus the integral of the 1st moment:

$$E\left[\frac{S_t}{\overline{\sigma}}\right] = \left(\frac{2}{\pi}\right)^{1/2} t^{1/2}$$

← SFR

$$\text{Var}\left[\frac{S_t}{\overline{\sigma}}\right] = \frac{1}{2} E\left[\frac{S_t}{\overline{\sigma}}\right]^2$$

← Scatter

$$E\left[\frac{M_t}{\overline{\sigma}}\right] = \left(\frac{2}{3}\right)\left(\frac{2}{\pi}\right)^{1/2} t^{3/2}$$

← Mass

A Markovian Star-Forming Main Sequence

If galaxies grow in a sort of steady-state, with stochastic changes to their growth rates, and every stochastic change to a galaxy's growth rate is independent of the other stochastic changes in its history, one gets this SFMS:

$$E\left[\frac{S_t}{M_t}\right] = \left(\frac{3}{2t}\right)$$

← sSFR

$$\mathrm{Sig}\left[\frac{S_t}{M_t}\right] = \frac{1}{\sqrt{2}} E\left[\frac{S_t}{M_t}\right]$$

← Scatter

$$\mathrm{Sig}\left[\ln \frac{S_t}{M_t}\right] \approx \frac{1}{\sqrt{2}}$$

$$\mathrm{Sig}\left[\log \frac{S_t}{M_t}\right] \approx 0.3 \text{ dex}$$

Remember how all the physics went into $\overline{\sigma}$? It canceled out when calculating S/M!

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$$\mathrm{Sig}\left[\log\frac{S_t}{M_t}\right] \approx 0.3 \text{ dex}$$

But the bad news is that galaxies aren't Markovian.

Covariant Stochasticity: Timesteps are Correlated

In reality, a galaxy's history has long- and short-term correlations between stochastic changes to its growth:

$$S_t = \sum_{i=1}^{t} \sum_{j=0}^{m} c_{i,i-j} X_{i-j}$$

There is an unknown, seemingly unconstrained set of covariances between stochastic changes in S.

Pick up football, cry, walk home?

Covariant Stochasticity: Timesteps are Correlated

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There is an unknown, seemingly unconstrained set of covariances between stochastic changes in S.

NO!

Sums of m-dependent random variables also obey limit theorems!

Covariant Stochasticity: Timesteps are Correlated

And for us, it gets cooler . . .

If the covariances have a "moving average" form, such as what one would largely find in a cosmological setting with a matter power spectrum correlating stochastic changes to the states of quasi-static equilibria over a broad/infinite range of timescales, then...

Covariant Stochasticity: Convergence in Distribution

ELSEVIER

Stochastic Processes and their Applications 90 (2000) 157–174

stochastic processes and their applications

www.elsevier.com/locate/spa

Convergence of weighted sums of random variables with long-range dependence ☆

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Boston University, Boston, MA 02215, USA

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Abstract

Suppose that f is a deterministic function, $\{\xi_n\}_{n\in\mathbb{Z}}$ is a sequence of random variables with long-range dependence and B_H is a fractional Brownian motion (fBm) with index $H \in (\frac{1}{2}, 1)$. In this work, we provide sufficient conditions for the convergence

$$\frac{1}{m^H} \sum_{n=-\infty}^{\infty} f\left(\frac{n}{m}\right) \xi_n \to \int_{\mathbb{R}} f(u)\,\mathrm{d}B_H(u)$$

in distribution, as $m \to \infty$. We also consider two examples. In contrast to the case when the ξ_n's are i.i.d. with finite variance, the limit is not fBm if f is the kernel of the Weierstrass–Mandelbrot process. If however, f is the kernel function from the "moving average" representation of a fBm with index H', then the limit is a fBm with index $H + H' - \frac{1}{2}$. © 2000 Published by Elsevier Science B.V.

Covariant Stochasticity: fractional Brownian motion

Vol. 10, No. 4, October 1968

FRACTIONAL BROWNIAN MOTIONS, FRACTIONAL NOISES AND APPLICATIONS*

BENOIT B. MANDELBROT† AND JOHN W. VAN NESS‡

1. Introduction. By "fractional Brownian motions" (fBm's), we propose to designate a family of Gaussian random functions defined as follows:[1] $B(t)$ being ordinary Brownian motion, and H a parameter satisfying $0 < H < 1$, fBm of exponent H is a moving average of $dB(t)$, in which past increments of $B(t)$ are weighted by the kernel $(t - s)^{H-1/2}$. We believe fBm's do provide useful models for a host of natural time series and wish therefore to present their curious properties to scientists, engineers and statisticians.

The basic feature of fBm's is that the "span of interdependence" between their increments can be said to be infinite. By way of contrast, the study of random functions has been overwhelmingly devoted to sequences of independent random variables, to Markov processes, and to other random functions having the property that sufficiently distant samples of these functions are independent, or nearly so. Empirical studies of random chance phenomena often suggest, on the contrary, a strong interdependence between distant samples. One class of examples arose in economics. It is known that economic time series "typically" exhibit cycles of all orders of magnitude, the slowest cycles having periods of duration comparable to the total sample size. The sample spectra of such series show no sharp "pure period" but a spectral density with a sharp peak near frequencies close to the inverse of the sample size [1], [4]. Another class of examples arose in the study of fluctuations in solids. Many such fluctuations are called "1:f noises," because their sample spectral density takes the form λ^{1-2H}, with λ the frequency, $\frac{1}{2} < H < 1$ and H frequently close to 1. Since, however, values of H far from 1 are also frequently observed, the term "1:f noise" is inaccurate. It is also unwieldy. With some trepidation due to the availability of

M+vN (1968)

Covariant Stochasticity: fractional Brownian motion

2. The definition of fractional Brownian motion. As usual, t designates time, $-\infty < t < \infty$, and ω designates the set of all the values of a random function. (This ω belongs to a sample space Ω.) The ordinary Brownian motion, $B(t, \omega)$, of Bachelier, Wiener and Lévy is a real random function with independent Gaussian increments such that $B(t_2, \omega) - B(t_1, \omega)$ has mean zero and variance $|t_2 - t_1|$, and such that $B(t_2, \omega) - B(t_1, \omega)$ is independent of $B(t_4, \omega) - B(t_3, \omega)$ if the intervals (t_1, t_2) and (t_3, t_4) do not overlap. The fact that the standard deviation of the increment $B(t + T, \omega) - B(t, \omega)$, with $T > 0$, is equal to $T^{1/2}$ is often referred to as the "$T^{1/2}$ law."

Definition 2.1. Let H be such that $0 < H < 1$, and let b_0 be an arbitrary real number. We call the following random function $B_H(t, \omega)$, *reduced fractional Brownian motion* with parameter H and starting value b_0 at time 0. For $t > 0$, $B_H(t, \omega)$ is defined by

$$B_H(0, \omega) = b_0,$$

$$B_H(t, \omega) - B_H(0, \omega)$$

$$= \frac{1}{\Gamma(H + \frac{1}{2})} \left\{ \int_{-\infty}^{0} [(t - s)^{H-1/2} - (-s)^{H-1/2}] \, dB(s, \omega) + \int_0^t (t - s)^{H-1/2} \, dB(s, \omega) \right\} \qquad (2.1)$$

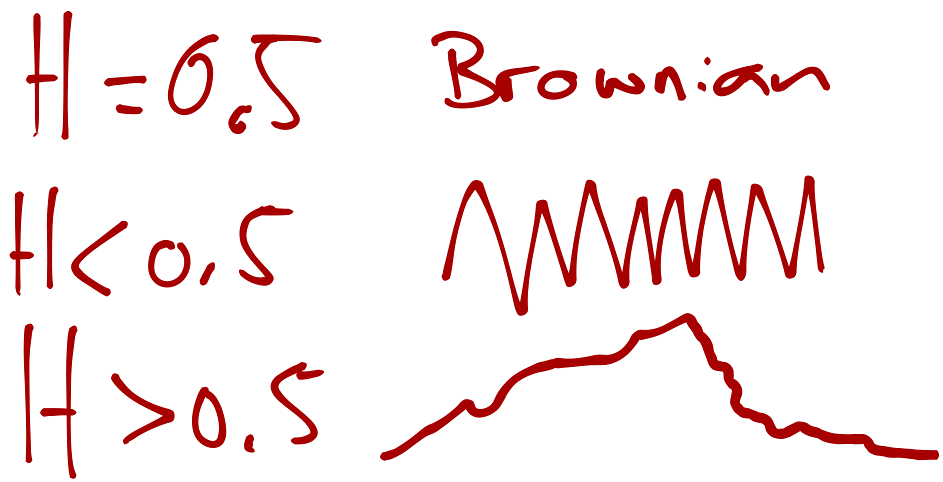
M+vN (1968)

fractional Brownian motion: the long and the short of it

We already derived what is effectively the Brownian case.

The fBm models are generalizations governed by the Hurst parameter: $0 \leq H \leq 1$.

Technically the bounds are not inclusive, but...



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For such a case, it would be like galaxies never forgot what happened to them. (Hmmmm.)

Nonnegative fBm: Expectation Values

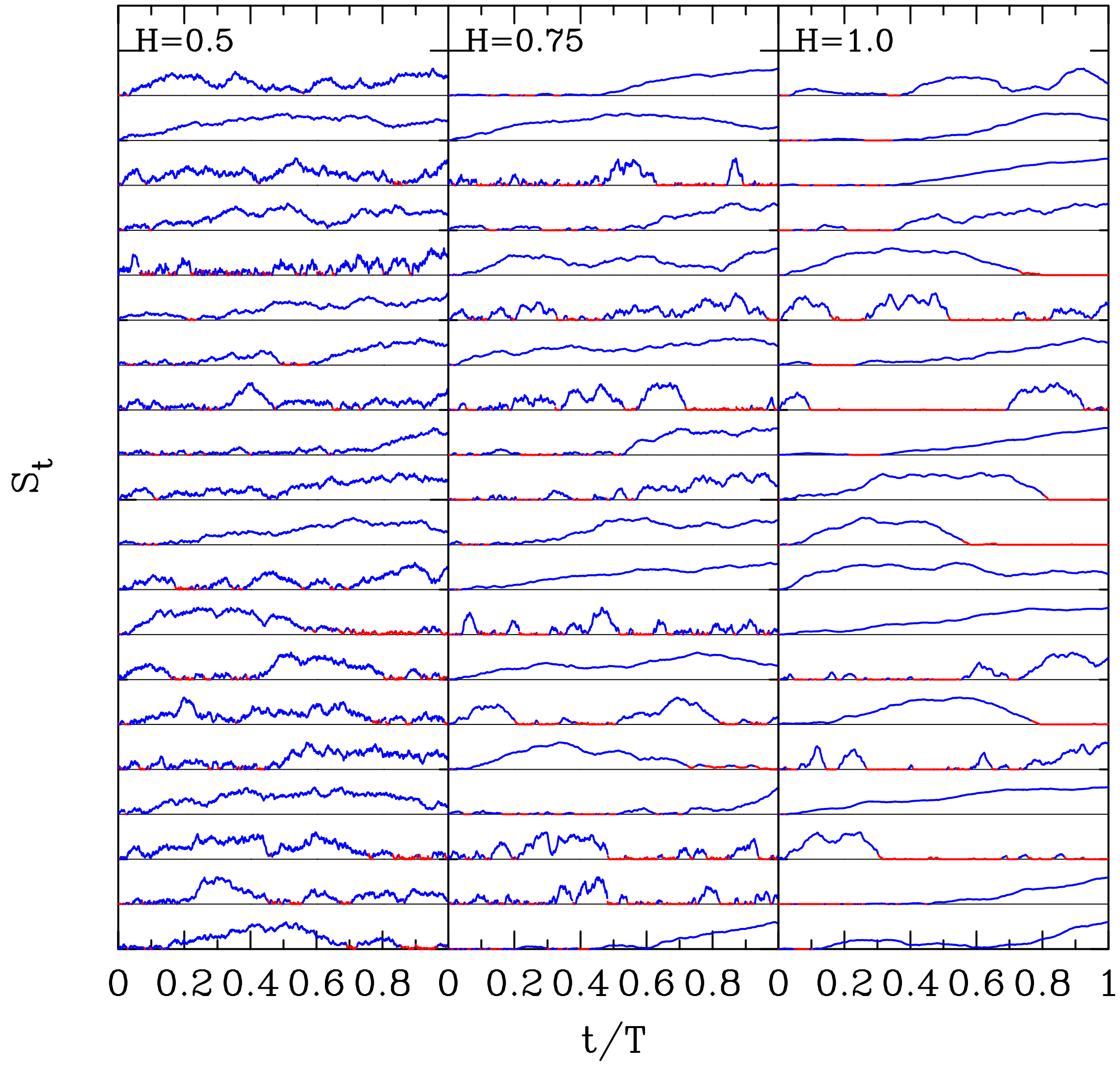
Serving both the interests of the audience and the speaker, let us just jump to:

$$E[S_t] = \overline{\sigma}\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{t^H}{2H}\right)$$ ← SFR

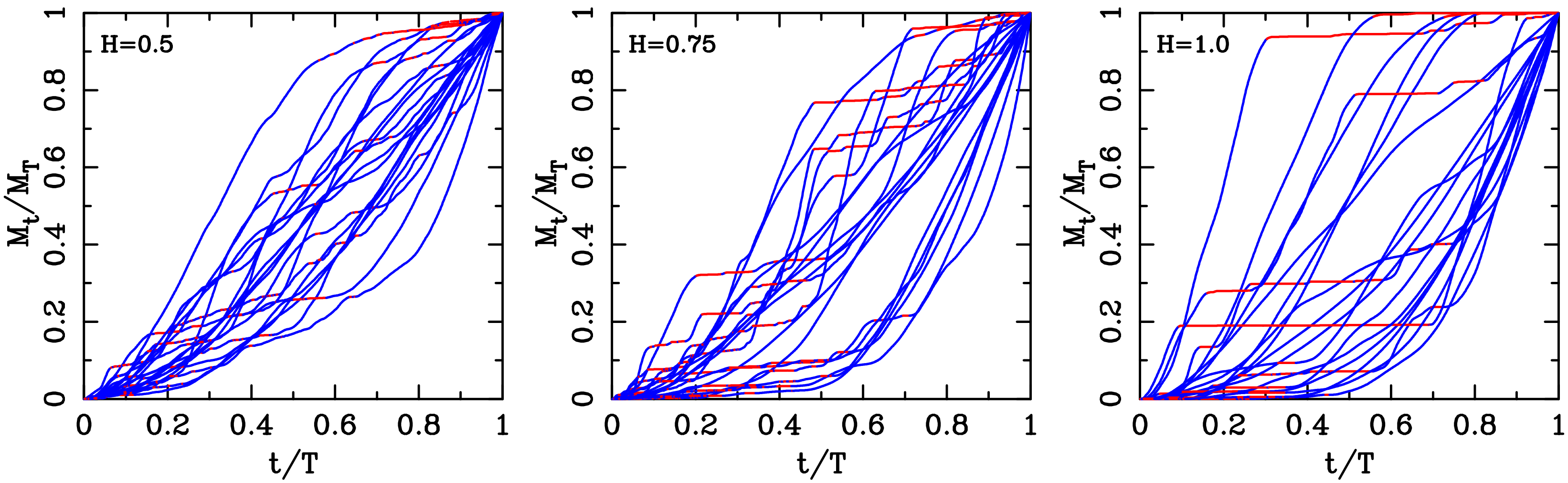
$$\mathrm{Sig}[S_t] = H^{1/2}E[S_t]$$ ← Scatter

$$E[M_t] = \overline{\sigma}\left(\frac{2}{\pi}\right)^{1/2}\left[\frac{t^{H+1}}{2(1+H)H}\right]$$ ← Mass

Nonnegative fBm: Example Scale-Free Histories



Nonnegative fBm: Example Scale-Free Growth Histories



Nonnegative fBm: The Star-Forming Main Sequence

The expectation values for S_t and M_t, again, are both proportional to $\overline{\sigma}$.

Thus one obtains a generalized SFMS of:

$$E[S_t/M_t] = \frac{(H+1)}{t}$$

← SSFR

$$\mathrm{Sig}[S_t/M_t] = H^{1/2} E[S_t/M_t]$$

← scatter

The amazing thing about this result is that the predicted scatter is independent of any long-term drift changes in expectations (such as when galaxy environments evolve to sufficiently modify long-term expectations of gas supply, etc).

Nonnegative fBm: The Star-Forming Main Sequence

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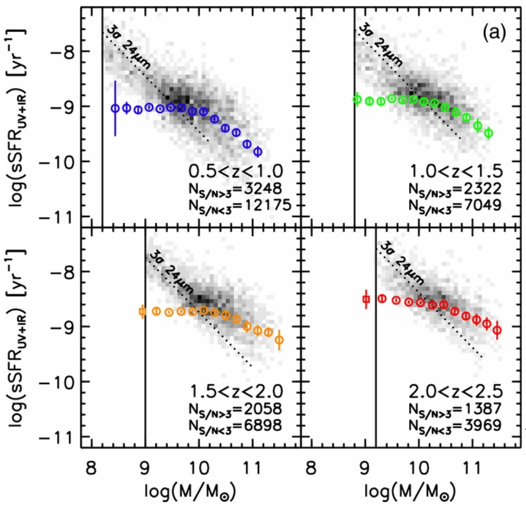
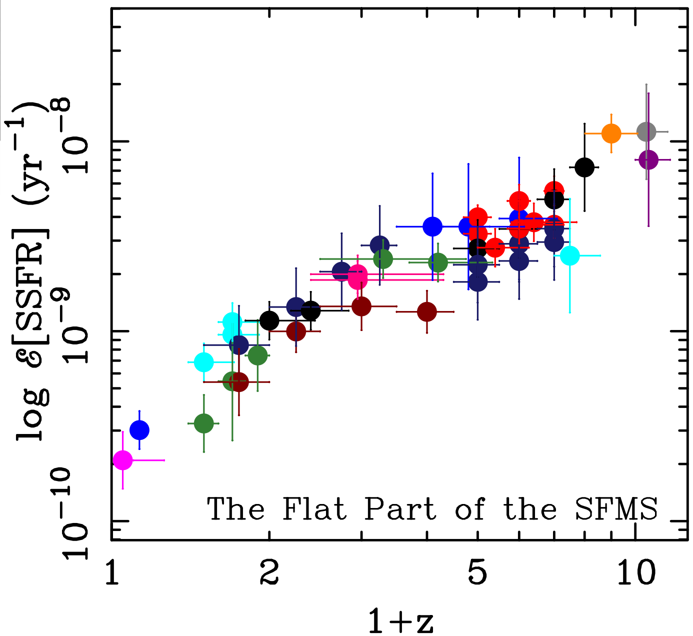
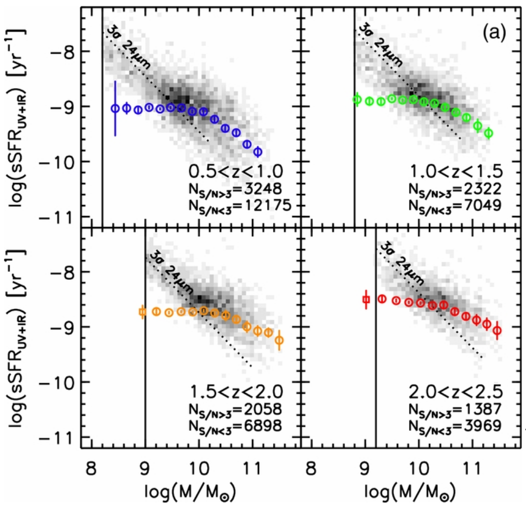
In other words, systematic changes in long-term expectations will not affect the relative scatter.

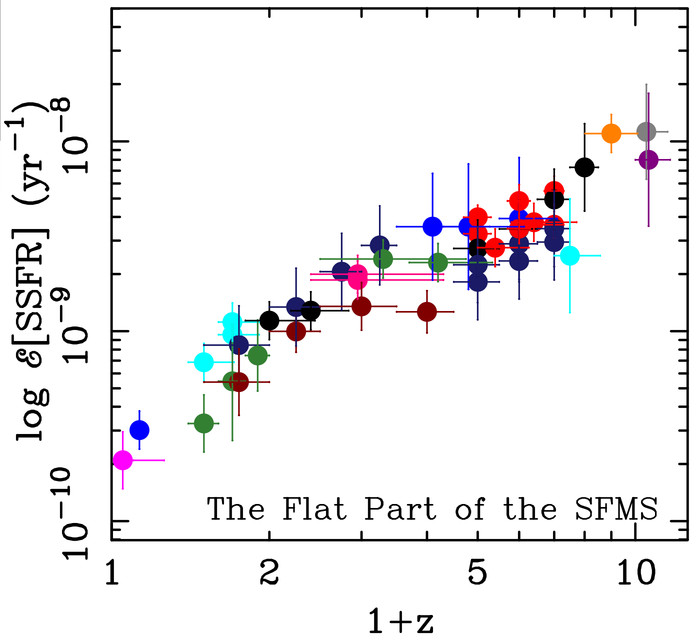
Back to the Star-Forming Main Sequence





Back to the Star-Forming Main Sequence

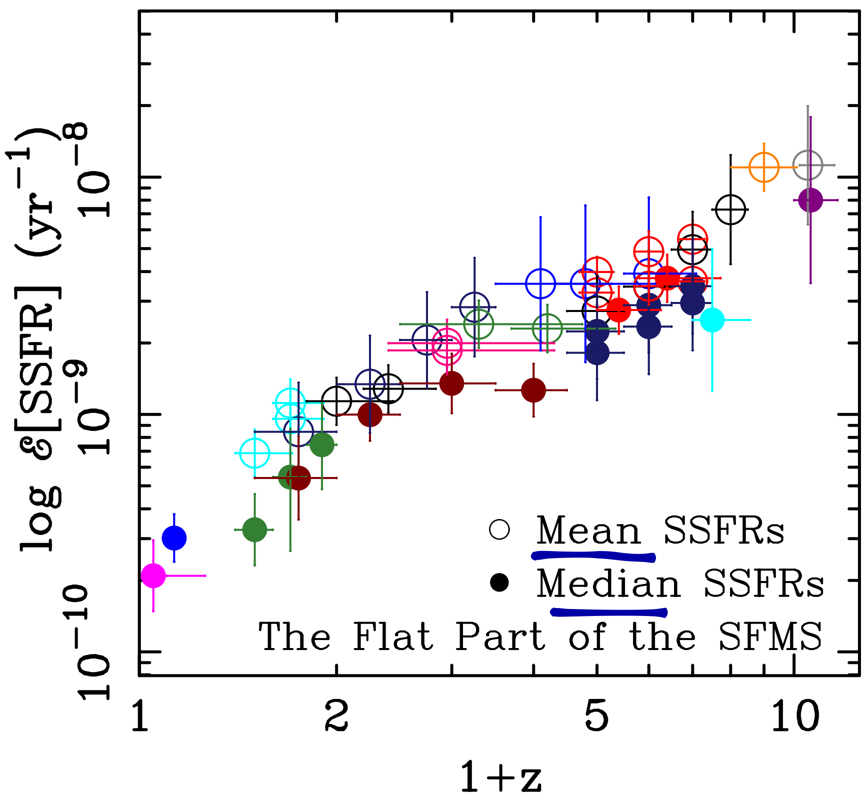




These data look like a fracking mess. How would when even begin to test whether the predictions are correct?

Rethinking those Measurements

Turns out that different people measure different things, artificially inflating the apparent disagreement among datasets.



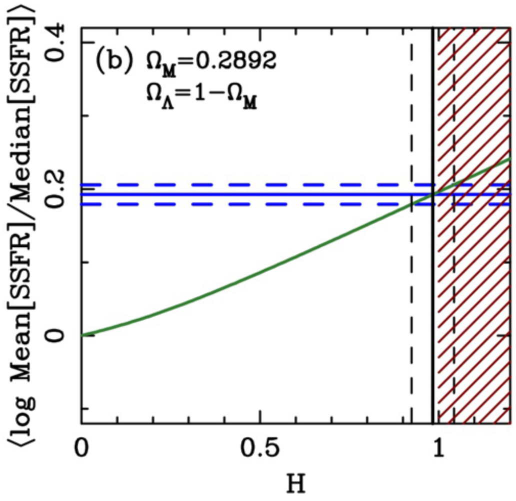
The difference between a mean and a median

Recall that

$$E[S_t/M_t] = \frac{(H+1)}{t}$$
$$\text{Sig}[S_t/M_t] = H^{1/2} E[S_t/M_t]$$

This scatter translates directly to an offset between the mean and median SSFR.

Let's fit A/t to the mean SSFRs and B/t to the medians and compute $\log A/B$:



$H = 0.98 \pm 0.06$

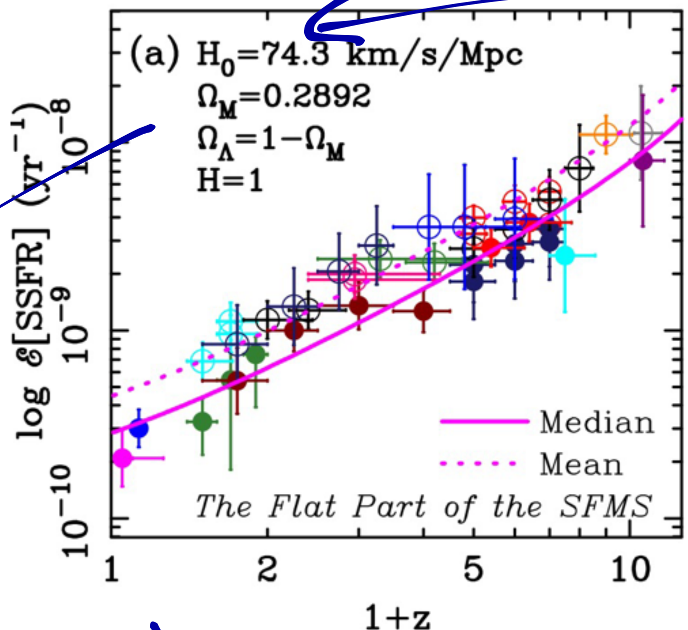
The green line is what one predicts for different values of H.

So galaxies are a bit like elephants

Here the violet solid line is the predicted locus for Median[SSFR] vs redshift.

The violet dashed line is the predicted locus for the Mean[SSFR] vs redshift.

H_0 from Freedman et al (2012)



Ω, Λ from Hinshaw et al (2013)

You might ask how unique this is...

And I would answer, in part, thusly

We derived that the Median$[S/M]$ on the flat-part of the SFMS is identically $2/t$.

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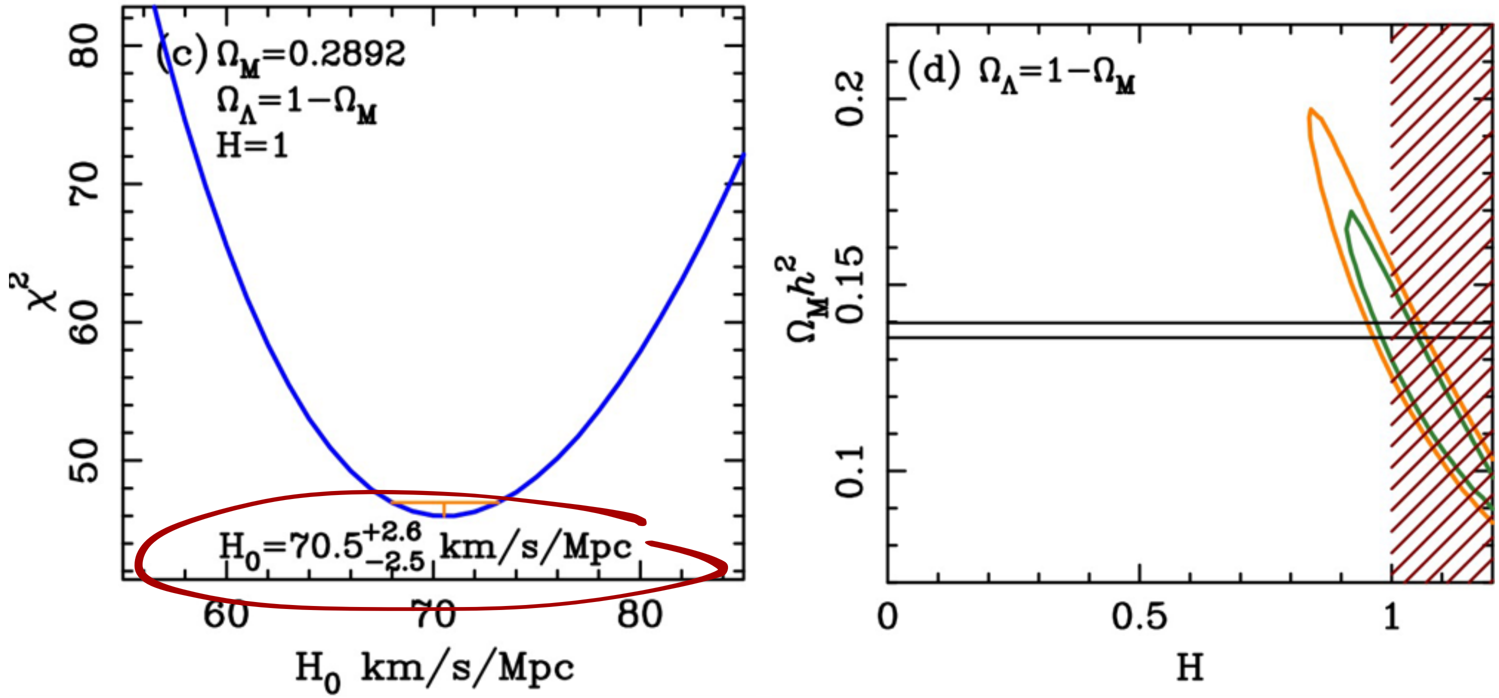
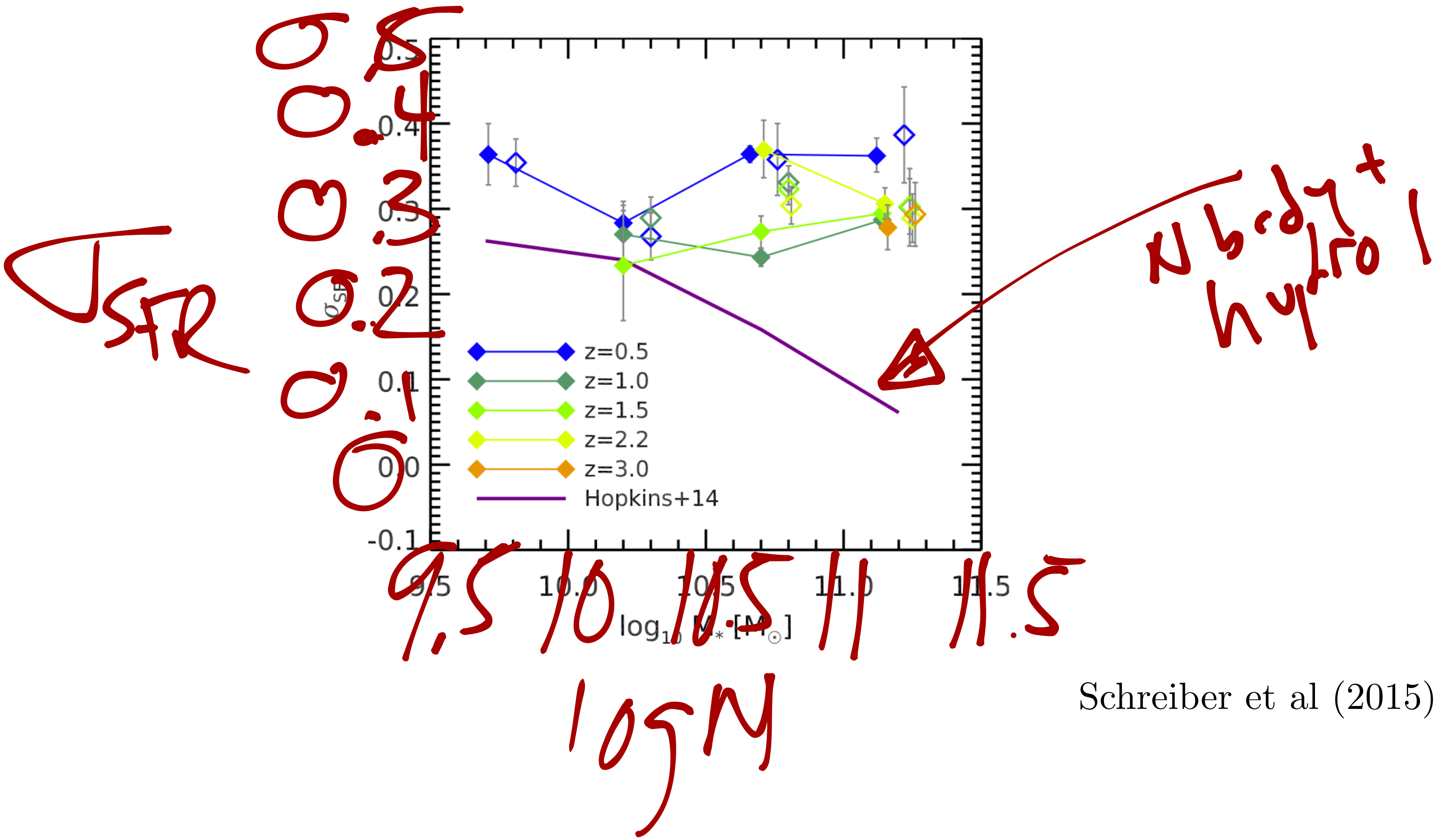


IOW: $2/t$ goes right through the medians, and $2/t \times 1.57$ right through the means.
To a few pct.

What about the expectation value for the scatter?



Schreiber et al (2015)

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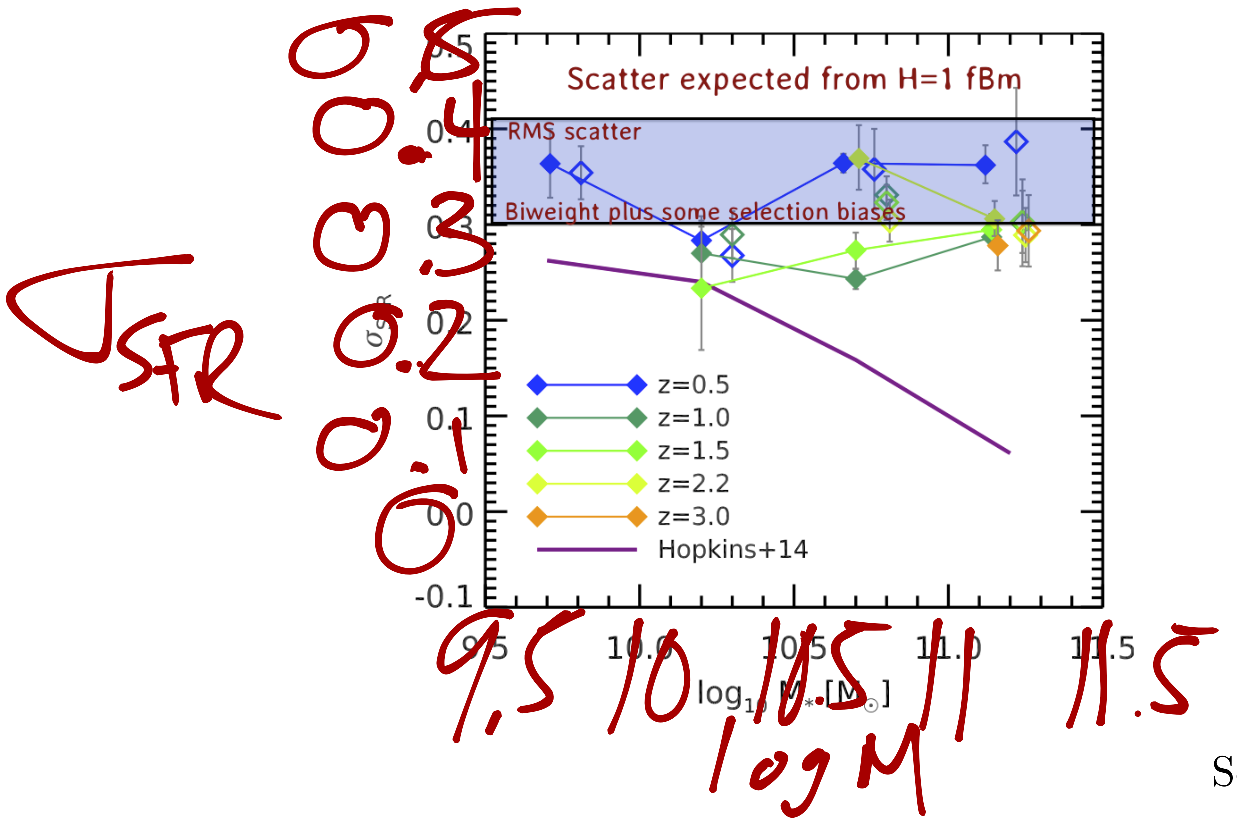

Schreiber et al (2015)

In SDSS Salim et al (2007) quote 0.4 dex intrinsic.

At high redshift Gonzalez et al (2014) quote ~0.5 dex.

Very difficult to measure right; selection biases matter a lot.

Do not measure for SF gals only!

Let Us Breath And Quickly Take Stock

- The SFMS is emergent.
- The SFMS does not imply that more massive galaxies form stars at greater rates!
- Rather: in order for a galaxy of mass M to have formed by z, it had to have formed stars more vigorously than lower mass galaxies over a given time
- Correlation does not imply causation, except in this case, star-formation causes stellar mass.
- The set of SFHs implied by fBm is quite diverse (and infinite).
- Implied histories show activity on a range of timescales, such that, e.g., "quiescent" SFHs aren't actually dead. Some of them get better!

At Early Times SSFR is Not Correlated with Mass

At low redshift and moderately high galaxy masses, SSFR is anticorrelated with M.

That means we need to input some physics to alter long-term expectations under the hood.

The lack of dependence of SSFR on M at early times implies we have a fully formed model of galaxy ensembles at those epochs.

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Except that we derived: $E[M_t] = \overline{\sigma} t^2/(2\sqrt{2\pi})$.

Up until now, we have treated $\overline{\sigma}$ as a nuisance, as something we can ignore.

But $\overline{\sigma}$ normalizes the SFRs and stellar masses, and is thus critical for computing stellar mass functions over time!

Can we calculate $\overline{\sigma}$ a priori?

A Characteristic Stochastic Fluctuation Amplitude

Let us start with

$$E\left[\frac{dM}{dt}\right] = \frac{\overline{\sigma}}{\sqrt{2\pi}}t$$

Let us then take the first derivative, and investigate ensembles for which the RMS fluctuation is roughly independent of time:

$$\frac{d}{dt}E\left[\frac{dM}{dt}\right] = \frac{\overline{\sigma}}{\sqrt{2\pi}}$$

$$E\left[\frac{d^2M}{dt^2}\right] = \frac{\overline{\sigma}}{\sqrt{2\pi}}$$

Let us simplify dM/dt as the rate of accretion of baryons, converted to stars with some fraction ϵ, where v_b is the infall velocity and ρ_b is the ambient density:

$$\frac{dM}{dt} = \epsilon \rho_b v_b$$

A Characteristic Stochastic Fluctuation Amplitude

We'll use a simple top-hat approximation, and other assumptions about the density of the ambient medium being relatively constant over a short enough timescale at the start of the stochastic process S, so that:

$$\frac{d^2M}{dt^2} = \epsilon \rho_b \frac{dv_b}{dt}$$

$$= \epsilon \rho_b \frac{GM_h}{R_h^2}$$

gravity

which eventually will look like

$$\frac{d^2M}{dt^2} = \epsilon f_b \left(\frac{4\pi 178}{3}\right)^{2/3} G M_h^{1/3} \rho^{5/3}$$

usual halo overdensity factor

Using characteristic halo mass at the onset of star-formation, and the matter density at that epoch, one then has a characteristic d^2M/dt^2, and thus a characteristic $\overline{\sigma}^*$

A Characteristic Stochastic Fluctuation Amplitude

Popular halo mass functions for $z \sim 10$ have characteristic $M_h \sim 6 \times 10^9 M_\odot$ (e.g. Warren et al 2006, Tinker et al 2008).

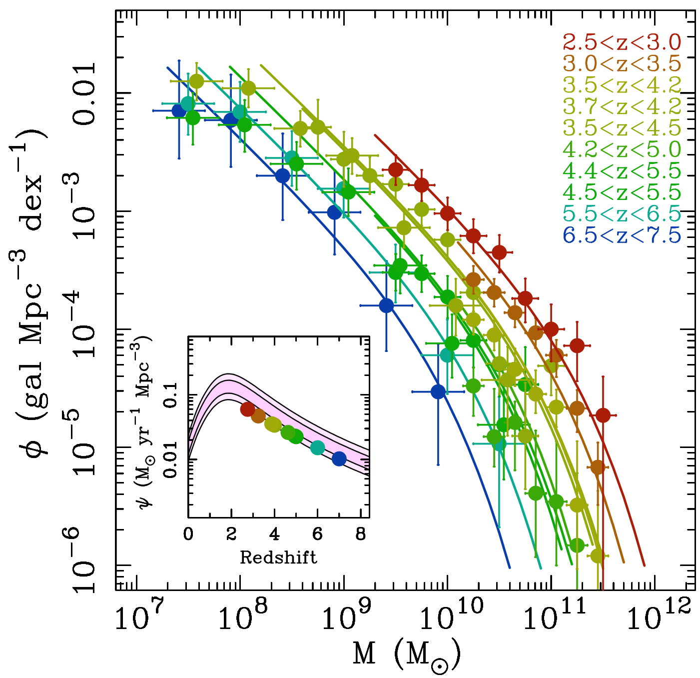
Let us adopt a rate of conversion of baryons to stars of 2%, and baryon fraction $f_b = 0.2$.

This number is what goes in front of, e.g., $E[M_t] = \overline{\sigma} t^2/(2\sqrt{2\pi})$:

$$\overline{\sigma}^* \approx \left(\frac{\epsilon}{0.02}\right)\left(\frac{f_b}{0.2}\right)\left(\frac{1+z}{1+10}\right)^5\left(\frac{M_h}{6 \times 10^9 M_\odot}\right)^{1/3} \times \left(1.4 \times 10^{-7} M_\odot/\mathrm{yr}^2\right)$$

High-z Stellar Mass Functions and Madau Diagram

One can also work out that the expected spectrum of $\overline{\sigma}$'s should have a low-$\overline{\sigma}$ slope of $\alpha \sim -2$.



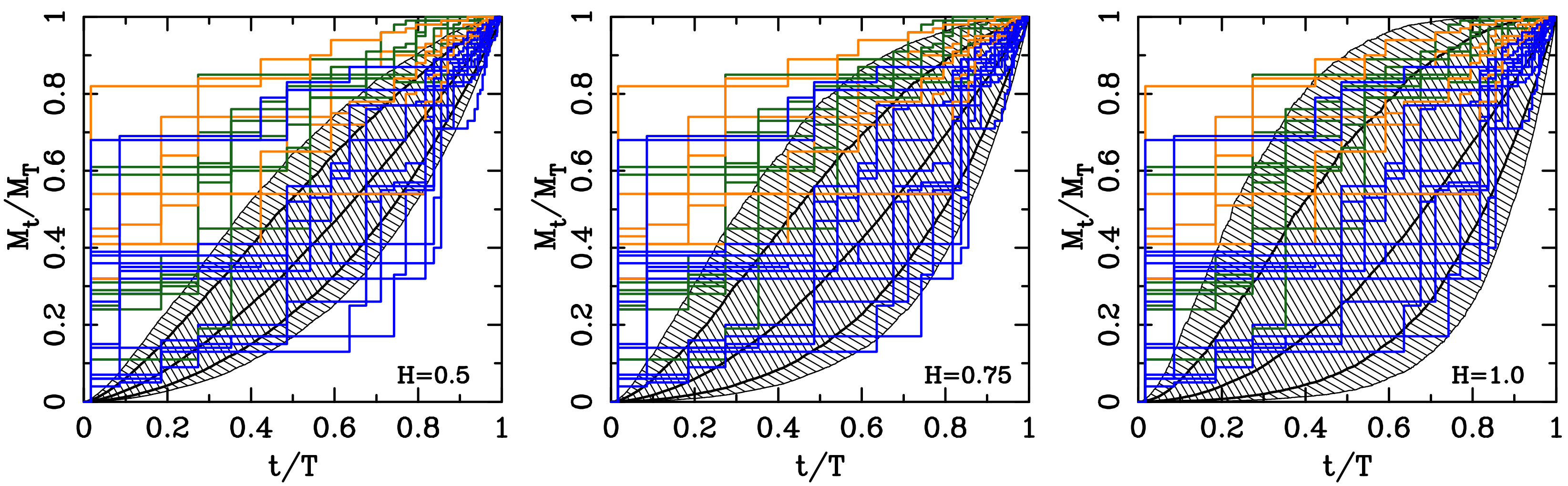
(SFRD from Madau & Dickinson 2014; mass functions from various.)

Given systematics in SFRDs, and high-z MFs, we're doing pretty well.

In the Local Universe, Low Mass Galaxies

Our derivations should also explicitly match those low-z galaxies that have stellar masses where the SFMS is still flat today.

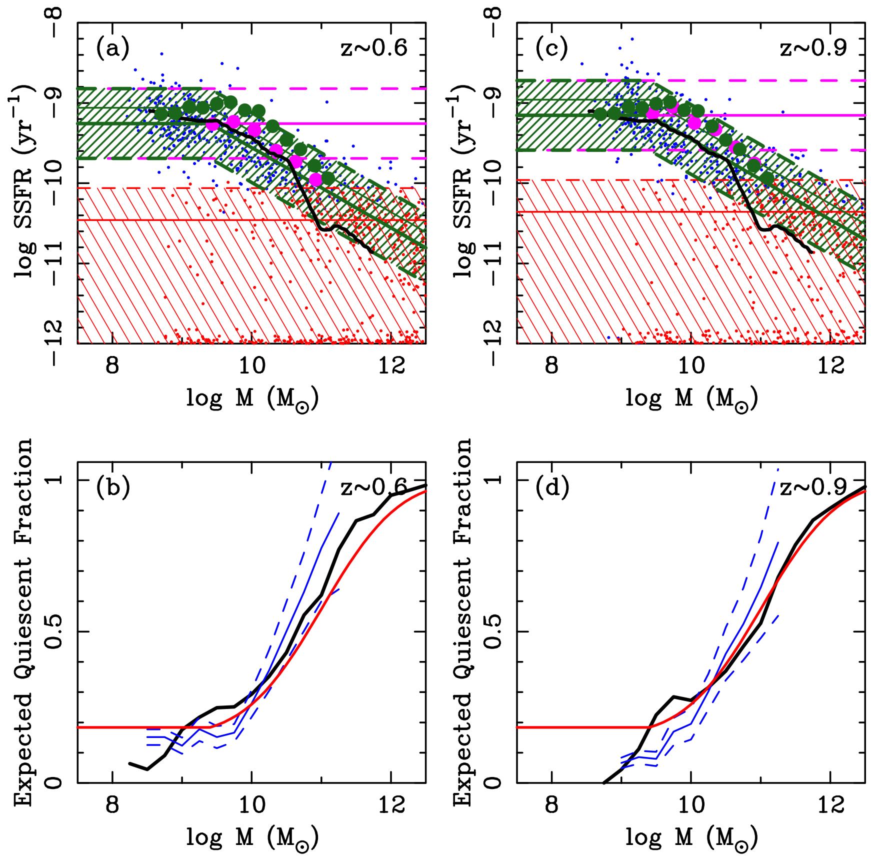
Here are some growth histories derived from local dIrr, dTrans, dE galaxies:



(Weisz et al 2014)

Implications for the Scatter in SSFR at Fixed Mass

The intrinsic scatter in SSFR means "quiescent" often $\neq$ *dead*



(Data from Tomczak et al 2014)

Over long baselines in z, galaxies below the median will move above, and vice versa.

My Ending Points (i.e. Math is Very Powerful)

- The "Star-Forming Main Sequence" is emergent, and a natural consequence of stellar mass growth as a stochastic process
- Derive $\mathrm{E}[(dM/dt)/M] = 2/t$, accurately matching SSFRs over $0<z<10$
- Surprise! We used the published SSFRs to derive, e.g., H_0 to ~3%!
- Observed intrinsic scatter in SSFR at fixed mass falls right out
- Retrodict stellar mass functions and Madau diagram $3 \lesssim z \lesssim 10$
- Infinite set of possible SFHs, including those of local group dwarf gals, MW
- Retrodict quiescent galaxy fractions along flat part of SFMS
- Strongly limits how well one can link specific progenitors with specific descendents
- Must trace full ensembles over cosmic time, but we now have math to help us!
- This framework is not yet complete — must incorporate a little more physics to get long-term evolution of massive galaxies (merging? gas depletion? AGN?)